PALACKY UNIVERSITY OLOMOUC

FACULTY OF ARTS

THE RECONSTRUCTION OF A. N. PRIOR'S ONTOLOGY

Dissertation Thesis

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Olomouc 2016

Abstract: Although the first analytic philosophers were primarily focused on a logical analysis of language, some of their initial works also contained ontological discussions. One of the most distinct ontological positions of the twentieth century was defended by Arthur Norman Prior. The unusual nature of the position could be demonstrated with the fact that he was ascribed such divergent positions as nominalism and platonism. This might have been caused by his atypical combination of ontological views. He was, on the one hand, nominalist in his mature works. On the other hand, he advocated intensional logic and presentism. The aim of this dissertation is to reconstruct the ideas which influenced him when he formulated his ontological positions. Not only are Prior's ideas introduced but also the ideas of his precursors and the contemporaries which influenced him. In contrast, the ideas of logicians and philosophers, whom Prior opposed, are also presented. The dissertation consists of four parts, which deal with Prior's concepts of possible worlds, theories of quantifications, propositions and individuals. It is merely a historical work and therefore the polemics which arose after Prior's death are not discussed.

Key words: A. N. Prior, Ontology, Logic, Possible Worlds, Quantification, Propositions, Individuals, Nominalism, Intensional Logic, Presentism

Abstrakt: Přestože se první analytičtí filosofové zaměřovali především na logickou analýzu jazyka, už jedny z jejich prvních knih a článků obsahují také ontologické diskuse. Jednu z nejzajímavějších ontologických pozic ve 20. století, pak zastával novozélandský logik a filosof Arthur Norman Prior. Jeho pozice je natolik neobvyklá, že bývá jedněmi označován za platonistu, zatímco jiní jej označují za nominalistu. To je patrně způsobeno tím, že v jeho filosofii se objevuje neobvyklá kombinace přístupů k ontologii. Ve svých pozdních dílech byl Prior nominalista, tento přístup pak kombinoval s presentismem a obhajobou intensionálních logických systémů. Tato disertace si především klade za cíl zrekonstruovat původ z jakých myšlenkových tradic, ze kterých Prior při vytváření své ontologie vycházel. Jsou zde tak představeny nejen Priorovy myšlenky, ale také myšlenky jeho předchůdců a současníků, kteří jej inspirovali. Na druhou stranu jsou uvedeny také názory filosofů a logiků, kterým Prior ve svém díle oponoval. Práce je rozdělena do čtyř větších celků, které se věnují Priorově pojetí možných světů, teorii kvantifikace, propozicím a jeho přístupu k individuím. Jedná se o práci historickou, a proto zde až na výjimky nejsou uvedeny polemiky, které Priorova ontologie vyvolala.

Klíčová slova: A. N. Prior, ontologie, logika, možné světy, kvantifikace, propozice, individua, nominalismus, intensionální logika, presentismus

I hereby declare that I wrote this dissertation on my own and that I quoted all used sources.

In Olomouc ..... ……..

Zuzana Rybaříková

Acknowledgements

Although, I am the sole author of this dissertation, it could not have arisen without the contribution and help of many others. I would therefore like to thank them from the bottom of my heart. I cannot mention everybody who helped me during my work but let me at least thank those who helped me the most:

- My parents and relatives who supported me and forgave me for not always being a perfect daughter, sister or granddaughter and for not necessarily meeting their expectations.
- My students, especially those from the course "Logic and ontology", since they raised questions which would have occurred to me and thus helped me reach a deeper understanding of a number of problems.
- Jan Rybařík who helped uncover a number of faults in my dissertation.
- Colleagues from my department who provided advice, amongst other things, especially Filip Tvrdý, Tomáš Nejeschleba and Hana Krylová.
- Vladimír Marko for the valuable advice he provided at the beginning of my research.
- The researchers from the Virtual Lab for Prior Studies who allowed me to work with Prior's unpublished texts and who organised the Arthur Prior Centenary Conference, which was deeply stimulating for my work. I would also like to thank Peter Øhrstørm, David Jakobsen and Jørgen Albretsen for their kind acceptance of me in Aalborg.
- My supervisor Jan Štěpán, who carefully read the entire text and revealed various troublesome parts of it. Thank you for caring about me and stimulating me throughout the whole four years when I was doing my Ph.D.

This work was supported by the student project "Historical Sollutions of Contemporary Philosophical Problems" No. IGA_FF_2015_004 of Palacký University.

Contents

1, Introduction

> In the language of William James, extensionalism and nominalism both count as "tough-minded" philosophies, and tend to be propagated by the same philosophical gang, while intensionalism and Platonism both count as "tender-minded", and again have a tendency, though in this case not such strong tendency, to be held by the same people. From the sociological point of view, I am probably a rather deviant type in advocating a combination of intensionalism and nominalism. But I think this particular deviation can be supported by reasons.[1]

Although, the first philosophers who belong to the analytic tradition were focused primarily on the logical analysis of language, certain aspects of ontology are also discussed in their initial works. When dealing with language they had to face the problems with the entities which occur in them but which are abstract. On the one hand, a philosopher could try to avoid them and use language as a tool for this purpose.[2] One the other hand, their postulation could have certain advantages which could compensate for the overpopulated universe.[3] One of the outstanding positions in this discussion was suggested in the mid twentieth century by Arthur Prior. As follows from the introductory quotation, he was one of the logicians who tried to avoid extensively the ontological commitments of his theory.

Prior's ideas are still lively in philosophy and logic. Arthur Prior's logic and philosophy has been intensively discussed of late since Prior's centenary was celebrated. Additionally, some of his ideas are still influential in philosophy,

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[1] Arthur N. Prior, "Intentionality and Intensionality," in A. Prior, *Papers in Logic and Ethics*, ed. P. T. Geach and A. J. P. Kenny (London: Duckworth, 1976), 190.
[2] E.g. B. Russell, "On Denoting," *Mind* 14 (1905): 479–493.
[3] Gottlob Frege, "Über Sinn und Bedeutung," *Zeitschrift für Philosophie und philosophische Kritik* 100/1 (1892): 25–50, accessed July 9, 2015 <http://www.deutschestextarchiv.de/book/view/frege_sinn_1892?p=11>.

logic or even computer science.[4] Arthur Prior combined nominalism with intensionality in his mature works. In addition, he was also an exponent of presentism. These three positions are difficult to advocate together as will be demonstrated in this brief introduction. Prior, however, tried to interface them. This dissertation also focuses on the question as to whether he was actually successful in his attempt.

Arthur Prior was a philosopher and logician from New Zealand, where he studied at Otago University, inter alia under John Findlay. After World War II he was appointed as a lecturer at Canterbury University College in Christchurch. His first book *Logic and the Basis of Ethics* was published in Clarendon Press in 1949 and achieved great success in Oxford. This was one of the reasons why he was invited by Gilbert Ryle to deliver the John Locke Lectures in Oxford. Copeland describes his work here as follows:

> On Mondays during Hilary and Trinity terms Prior lectured on modal logic, his great passion, and on tense logic, his great invention.[5]

Prior invented modern tense logic shortly before his departure to Oxford. Over the following years he intensively developed modal and tense logic. When his lectures in Oxford ended, he returned to New Zealand. His connections with English philosophers were so close, however, that he came back to England when he had an opportunity to apply for a vacant chair at Manchester University. After seven years in Manchester, Anthony Kenny recommended him at Balliol College in Oxford, where he spent his last years. Prior died prematurely in 1969 in the age of 54.[6] Several of his theories were not settled at that time. One of them is ontology which caused difficulties in its reconstruction.

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[4] Peter Øhrstørm and Per Hasle. *Temporal Logic: From Ancient Ideas to Artificial Intelligence* (Dodrecht: Kluwer Academic Publisher, 1995), 344.
[5] B. Jack Copeland, "Arthur Prior," in *The Stanford Encyclopedia of Philosophy*. Fall 2008 edition. Edited by Edward N. Zalta, accessed March 21, 2016, http://plato.stanford.edu/archives/fall2008/entries/prior/.
[6] Ibid.

Prior's ontology is distinct. This can be seen in the fact that some authors ascribe to him certain platonist views[7] while others maintain that he is a nominalist[8]. The reasons why they claim that he was a platonist can be found in his systems of intensional logic, which, as will be mentioned later, appear to require entities from a platonist universe. Prior is quite consistent, however, in his denial of abstract entities and also argued that he was a nominalist. Thus, the latter authors seem to be right. Nonetheless, this query provides an opportunity to discuss Prior's ontology thoroughly.

As follows from the previous paragraph, there are several works[9] which deal with Arthur Prior's ontology. They are usually focused, however, on merely part of Prior's theory. As far as the current author knows, Prior's ontology, i.e. his concepts of possible worlds, time instants, several theories of quantification, his understanding of propositions and theories which are linked with names and individuals have not been introduced in one work. In addition, the impact which Prior's ontology had on logicians and philosophers from the Lvov-Warsaw school has not been discussed properly, even though, in certain aspects this influence is crucial.

I would like to assert in my dissertation that Prior's concept was considerably impacted by the adoption of certain views of his precursors and contemporaries. I will focus primarily on logicians from the Lvov-Warsaw

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[7] E.g. Peter Loptson, "Prior, Plantinga, Heacceity, and the Possible," in *Logic and Reality: Essays on the Legacy of Arthur Prior*, ed. B. J. Copeland (Oxford: Clarendon Press), 420.
[8] E.g. Roger Teichmann, "Statements of Property-identity and Event-identity," in Copeland, J (ed.): *Logic and Reality: Essays on the Legacy of Arthur Prior*. 462.
[9] E.g Kit Fine, Modality and Tense (Oxford: Oxford University Press, 2005); Philip Hugly and Charles Sayward, Intensionality and Truth (Dodrecht: Kluwer Academic Publisher, 1996); David Jakobsen, Peter Øhrstørm and Henrik Schärfe, **"**A. N. Prior's Ideas on Tensed Ontology," in *Conceptual Structures for Discovering Knowledge*, ed. Simon Andrews et al (Berlin: Springer, 2011), 118 – 130; David Jakobsen, "A. N. Prior's Notion of the Present," in *Multidisciplinary Aspects of Time and Time Perception,* ed. Argiro Vatakis et al (Berlin: Springer, 2011), 36 –45; David Jakobsen, "Arthur Norman Priors bidrag til metafysikken," (PhD diss., Aalborg University, 2012); Karel Lambert, "Russellian Names: Notes on a Theory of Arthur Prior", in *Logic and Reality: Essays on the Legacy of Arthur Prior,* ed. Jack B. Copeland (Oxford: Clarendon Press, 1996), 411–417; Loptson, "Prior, Plantinga, Heacceity, and the Possible," 419–435; Peter Øhrstørm, "Two Essays on Temporal Realism: Introduction," in *Logic and Reality: Essays on the Legacy of Arthur Prior*, ed. B. Jack Copeland (Oxford: Clarendon Press, 1996), 43–44; Teichmann, "Statements of Property-identity and Event-identity," 461–476; James van Cleve, "Objectivity without objects: a Priorian program," *Synthese* (forthcoming).

school and Frank P. Ramsey. My dissertation, which is merely historical, will therefore also briefly introduce the most important features of the theories of logicians who influenced Prior's ontology. In addition, philosophers whose theories Prior opposed will also be presented.

There are also several logicians who greatly affected the development of Prior's system of logic as Prior elsewhere admitted.[10] These include C. S. Peirce or Saul Kripke. In contrast to the previously mentioned philosophers, their ideas, as far as I am aware, were not a significant component of Prior's ontology. This is the reason why their theories will not be presented in my dissertation.

My dissertation is divided into four sections. It firstly introduces Prior's concept of possible worlds and time instants, its origins and the development of Prior's thoughts. I will consequently discuss Prior's theories of quantification which were essential for his ontology. Apart from an objectual quantification, Prior handled propositional quantification and viewed various operators as quantifiers. Some of his ideas were allowed by the adoption of Leśniewski's ideas as will be demonstrated. Thirdly, Prior's concept of propositions will be presented and the ideas of the philosophers who influenced it. Lastly, Prior's concept of names, which he created in discussion with Russell and Leśniewski, will be examined along with Prior's concept of individuals. In order to illuminate certain problematic features of Prior's ontology, let me introduce the three positions which were defended by him.

1. 1 Nominalism

Nominalism as a metaphysical or epistemological position has a long history. When dealing with this position, however, one must first discuss what it means when one is labelled as a nominalist. The answers to this question have differed greatly throughout the history of philosophy as well as within various philosophical traditions and amongst philosophers themselves. As Rodriguez-

[10] E.g. Arthur N. Prior, *Past, Present and Future* (Oxford: Clarendon Press, 1967), 20–31 and 117–137.

Pereyra points out, this term contains several meanings, which are more or less compatible.[11]

Nominalism was originally formed as an opposite to realism and focused on the problem of universals. The dispute between both positions flared violently in the Middle Ages, even though, there are actually some hints of it in classical philosophy. As Simons points out, it was the period where nominalism received its name and the first nominalists claimed that there are only particulars and that the universals are only names, "nomina" in Latin.[12]

However, at the final stage of medieval logic, nominalists began to also reject abstract entities such as for instance *complexe significablia*.[13] The term nominalist began to be ambiguous because, as Hanke demonstrates, various philosophers whose acceptance of abstract entities and universals differed entirely called themselves nominalists.[14] This discrepancy has continued to the present day. Modern logic, mathematics, sciences and philosophy even complicate the situation, since the number of entities, whose real existence is doubtful, increases greatly in these fields of study.

I have chosen Simons's definition for identifying Prior's position, because he created it connection with the theories of logicians from the Lvov-Warsaw school,[15] who were, as will be discussed further in my dissertation, significant for Prior's ontology. Simons distinguishes between four meanings of nominalism in the history of philosophy. The first relates only to universals and is similar to the medieval theory entitled conceptualism. It claims that there are

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[11] Gonzalo Rodriguez-Pereyra, "Nominalism in Metaphysics," in *The Stanford Encyclopedia of Philosophy*. Summer 2014 edition, ed. Edward N. Zalta, accessed January 17, 2015, http://plato.stanford.edu/archives/sum2014/entries/nominalism-metaphysics/.
[12] Peter Simons, "Nominalism in Poland," in *Leśniewski's Systems: Protothetic*, ed. J. T. J. Srzednicki and Z. Stachniak (Dordrecht: Kluwer, 1996), 1.
[13] The definition of the term "nominalism" was nevertheless ambiguous even prior to the rejection of other abstract entities. Apart from radical nominalists, such as Anselm's opponent Roscelin, there were also moderate nominalists who maintained that universals are only mental concepts. This view was also called conceptualism and was sometimes presented as an independent school of thought. (E.g. Pavel Floss, *Cesty evropského myšlení. 1, Architekti křesťanského středověkého vědění* (Praha: Vyšehrad, 2004), 155–159.)
[14] See Miroslav Hanke, "Problém univerzálií v pozdně středověkém nominalismu: Případ Martina Magistriho (1432-1472)," (paper presented at the conference Pluralita tradic ve středověké a pozdně novověké filosofii, Nové Hrady, December 8–9, 2014.)
15 Simons, "Nominalism in Poland," 1–22.

no universals which can be independent of the mind but the universals can be identified with the concepts in our minds. The second copes with universals more radically, since it follows Roscelin in the opinion that universals are merely *flatus vocis*, i.e. words.[16] However, only the existence of universals continues to be rejected.

In contrast to previous concepts, the existence of any abstract entities is denied completely in the third meaning of nominalism. Finally, in the last meaning it is claimed that there are no classes. The two former opinions have their origins in the Middle Ages, although Simons finds the latter two in modern logic, namely in the work of Quine and Goodman.

In order to describe properly the various different usage of nominalism, Simons enables a combination of these meanings. He specifically points out that certain philosophers have considered universals or classes to be abstract entities while some do not and other regard universals as classes, while their opponents do not accept it.[17] Simons also points out that if the author is a representative of nominalism, defined as a denial of abstract entities, there is a need differentiate precisely what an "abstract entity" means for him or her, as there is no consensus among authors as to which entity is abstract and which is not.

Prior appears to be an exponent of the third type of nominalism,[18] as it was suggested by Simons. It can be argued that his nominalism is primarily focused on a denial of abstract entities. Further chapters will demonstrate, however, whether he was truly consistent in this view and which entities he considered abstract and how he coped with their non-existence.

1. 2 Intensional Logic

Although hints of a division between intensional and extensional logic can be found earlier, it is primarily linked with modern logic. Fitting claims that Frege

[16] Floss, *Architekti křesťanského středověkého vědění*, 120–121.
[17] Simons, "Nominalism in Poland," 1–3.
[18] Prior, "Intentionality and Intensionality," 188–189.

differentiated between the sense of a term and its reference. Using Frege's example, the statement "The morning star is the evening star" is not trivial information, even though both words have the same reference i.e. the planet Venus. The term "meaning" has been used in analytic tradition for both aspects of Frege's analysis, sense and reference. In order to avoid confusions there are other titles for these two phenomena. From the logical point of view, Carnap names are important since the reference was entitled by him "extension" and the sense "intension".[19] Consequently, the classical logic which does not differentiate between the different senses was entitled extensional logic and various non-classical logics which require this differentiation were entitled intensional logics.

Fitting points out that the classical first order was designed for the purpose of mathematics, which does not need to differentiate between extensions and intensions and hence is extensional.[20] However, certain philosophers[21] maintained that the formalization of ordinary language requires a more detailed analysis including intensions. One of them was Prior, as he clearly maintained in his paper *Intentionality and Intensionality*. He nonetheless differentiated between intensional functions and intensional objects or intensions. While he admitted the former he denied the existence of the latter.[22]

Prior's denial is in accordance with his nominalism. This would mean that Prior intended to handle intensional context and intensional logic without the ontological commitments which this surrounding seems to require. Namely, he had to deny the real existence of a possible world and possibilia, even though he postulated systems of modal logic. In addition, he had to avoid the postulation of the real existence of time instants, and some way of existence of individuals which appear in the past or in the future. In the case of epistemic logic and truth-theory, he also had to handle the existence of propositions. If Prior had a

[19] Melvin Fitting, "Intensional Logic," in *The Stanford Encyclopedia of Philosophy*. Summer 2015 edition, ed. Edward N. Zalta, accessed March 20, 2016, http://plato.stanford.edu/archives/sum2015/entries/logic-intensional/.
[20] Fitting, "Intensional Logic."
[21] E.g. Prior, "Intentionality and Intensionality," 187–189.
[22] Ibid., 187.

preference for extensional logic, he would not have an obligation for the majority of these questions.

1. 3 Presentism

Although intensional logic does not fit properly with a nominalistic world-view, presentism could also cause certain queries. Jakobsen argues that presentism is a view which claims that the past and future is not real and consequently, only present objects exists.[23] Prior advocated this view when he discussed the Barcan formula. As will be presented further in the section focused on individuals, the eternalists' solution appeared to be paradoxical to Prior.[24]

As Jakobsen points out there are several different types of presentisms and some of them are not compatible with Prior's view. A distinct feature of Prior's presentism was that he allowed quantification over objects which are not present.[25] This was enabled by Prior's concept of quantification which differed from Quine's as will be introduced further.

Prior's presentism was linked with his temporal realism. Øhrstørm argues that by holding this position Prior maintained that the distinction between the past, present and future is real, i.e. that time is not completely present in all the moments.[26] This position is in contrast with a tapestry view of time, where time is viewed from the position of God's eye and counts on the distinction between earlier and later. This division of two ways of understanding of time was introduced in modern philosophy by John Ellis McTaggart in his paper *The Unreality of Time*. McTaggart differentiated between the A-series which corresponds with temporal realism and the B-series which corresponds with a

[23] Jakobsen, "A.N. Prior's Notion of the Present," 37 – 38.
[24] Arthur N. Prior, *Time and Modality* (Oxford: Clarendon Press, 1957), 29–32.
[25] Jakobsen, "A.N. Prior's Notion of the Present," 38 and 41.
[26] Øhrstørm, "Two Essays on Temporal Realism," 43.

tapestry view of time, although he maintained that neither of them actually exists.[27]

Markosian points out that there are difficulties which affected presentism from the ontological point of view. Firstly, there are a large number of statements which deal with non-present objects as "Socrates lived in Ancient Greece" or "Thomas Aquinas was a member of the Dominican Order", etc. The majority of the individuals, who appear in these statements, are non-existent presently, thus they cannot be referred to based on presentism, although the statements are meaningful.

Secondly, there are statements which deal with the relations of non-existent objects such as "Abelard was the lover of Heloise" or "Wittgenstein died earlier than Russell". There is a similar problem as in the first case. The statements are considered meaningful but the references to the names, which they included, are opaque. Thirdly, there is the problem of truth-makers. The problem is similar to the previous two. If there are no passed or not-yet-existent individuals, there are no truth makers of statements which deal with them. Lastly, presentism is linked with temporal realism as was mentioned previously, which is not compatible with the theory of relativity. [28] This is also dubious from a scientific point of view although Prior tried to cope with this problem. He dealt with time from the human perspective and for that perspective he maintained that temporal realism is prior than the tapestry view of time.[29]

In summary, Arthur Prior was one of the prominent figures of modern logic. As a philosophic logician he was also interested in the ontological commitments of his systems of logic and discussed them in several of his works. Prior claimed in his mature works that he was a nominalist. He also argued for presentism and

[27] John Ellis McTaggart, "The Unreality of Time," *Mind* 17 (1908): 457–474.
[28] Ned Markosian, "Time," in *The Stanford Encyclopedia of Philosophy*. Spring 2014 edition, ed. Edward N. Zalta, accessed March 21, 2016, http://plato.stanford.edu/archives/spr2014/entries/time/.
[29] Arthur N. Prior, "Some Free Thinking about Time," in *Logic and Reality: Essays on the Legacy of Arthur Prior,* ed. Jack B. Copeland (Oxford: Clarendon Press, 1996), 49–51.

developed systems of intensional logic. As was presented previously, the combination of these assumptions appears to be problematic. Therefore, the aim of my dissertation is to maintain how and why Prior could formulate his view. In addition, I would like to introduce theories, which inspired Prior in the formulation of his concept. There are also several critical studies to Prior's approach, but in order to keep my work to its limits, my dissertation is merely a historical work focused on the evolution of certain aspects of Prior's ontological views and origins.

2, Possible Worlds and Time Instants

> It is an oddity of current thinking about modality that it has been heavily influenced, one might even say dominated, by two extreme and highly implausible views. The first of these, associated with the name of Quine, is that modal notions are lacking in sense. There is no intelligible distinction to be drawn between what is necessarily and what is contingently the case or between an object's essential and accidental features. The second of these two views, associated with the name of David Lewis, is that the possible and the actual are on an ontological par. Other possible worlds and their inhabitants are just as real as the actual world and its inhabitants; and there is no difference between them in regard either to the degree or to the kind of reality that they possess.[30]

This quotation from Fine's book appropriately illustrates Fine's but also Prior's concepts of such an abstract entities as possible worlds or time instants. Prior was not one of the philosophers who followed Quine's criticism of modal logic nor did he postulate such an overpopulated universe which is characteristic for David Lewis's modal metaphysics. As was mentioned in the introduction, he combined intensional logic with nominalism. Several ontological queries arose, however, Prior demonstrated that it can be combined in his concept of possible worlds and time instants, as will be shown in this part of my dissertation. Furthermore, as will be shown in the last chapter of this part, the transcription of his ontological positions into his system of logic led to a hybridisation of his logical system which inspired the current development of hybrid logic.[31]

In this section I will introduce Prior's concept of possible worlds and time instances as one concept, even though, differences between them can be found.

[30] Kit Fine, *Modality and Tense* (Oxford: Oxford University Press, 2005), 1.
[31] Patrick Blackburne, "Arthur Prior and Hybrid Logic," *Synthese* 150 (2003): 329–372.

However, Prior in several places in his works approximates these two concepts and also his final form of concepts of possible worlds and time instants are quite similar. In addition, the same variables are used for both of them in his systems of logic.[32]

Prior's concept of possible worlds is characterised by two main features. Firstly, it is the claim that possible worlds consist of propositions. A possible world, according to Prior, is a world proposition which contains a conjunct of true propositions about this world. Secondly, Prior as a nominalist denied the real existence of possible worlds and time instants. The first feature of his theory was undoubtedly inspired by Wittgenstein's *Tractatus Logico-Philosophicus*[33] while Prior had probably never mentioned the source of the inspiration for the second feature. Nevertheless, I would like to claim in my dissertation that he might have been influenced in this step by the logicians from the Lvov-Warsaw school.

Since Wittgenstein and Polish logicians seem to be significant for Prior's views, their theories will be introduced in the first two chapters of this part. Carew Meredith's modal logic will consequently be discussed, since he affected Prior and in his concept interfaced Wittgenstein's and Łukasiewicz's ideas. The fourth chapter contains the development of Prior's view. His mature concept of possible worlds is discussed in the following chapter. The last chapter focuses on rediscovering Prior's hybridisation. Prior's ideas, which have been inspiring for current hybrid logicians, are shown here with the reasons why Prior is considered the founding father of hybrid logic.

2. 1 Wittgenstein

Wittgenstein introduced several topics into analytic philosophy, with the concept of possible worlds being one of them. Suszko argues that the concept of possible worlds played an important role in the ontology which was presented

[32] See e.g. Prior, *Past Present and Future*, 188–189.
[33] Ibid., 99.

by Wittgenstein in *Tractatus*. He claims that Wittgenstein introduced two different ontological conceptions here. They are entitled s-ontology and o-ontology by Suzsko. The letter "s" means that the former is the ontology of situations and consequently the letter "o" shows that the latter is the ontology of objects. Possible worlds serve as the link between them.[34]

2. 1. 1 S-ontology

Although the description "possible worlds" cannot be found in *Tractatus*, Suszko claims that this concept is implicitly contained in Wittgenstein's ontology. The refinement of Wittgenstein's ontology also leads directly into the definition of possible worlds. In *Tractatus,* Wittgenstein used the description "possible states of affairs", which according to Suszko build possible worlds. Hence one possible world consists of many possible states of affairs in the same way that the real world consists of real states of affairs, which correspond to true propositions in logic.[35] As Wittgenstein maintained:

> 4. 26. The specification of all true elementary propositions describes the world completely. The world is completely described by the specification of all elementary propositions plus the specification, which of them are true and which false.[36]

Therefore, in Wittgenstein's concept, every true proposition corresponds to some states of affairs and all true propositions build the description of the entire world. Chrudzimski points out, in contrast, that Wittgenstein did not understand the propositions in a Fregean sense but that they determine the

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[34] Roman Suzsko, "Ontology in Tractatus of L. Wittgenstein," *Notre Dame Journal of Formal Logic* 9 (1968): 8.
[35] Ibid., 19–20.
[36] 4. 26. Die Angabe aller wahren Elementarsätze beschreibt die Welt vollständig. Die Welt ist vollständig beschrieben durch die Angaben aller Elementarsätze plus der Angabe, welche von ihnen wahr und welche falsch sind.
Ludwig Wittgenstein, *Tractatus Logico-Philosophicus,* McGuinness English translations. Side-by-side-by-side edition, version 0.41, accessed September 21, 2014, http://people.umass.edu/klement/tlp/.

truth-value of the world which they represent. If the proposition is particularly true then the state of affairs, which it describes, is the case in the actual world.[37]

Wittgenstein's quotation is even more interesting from the semantic point of view of the possible worlds. Although Wittgenstein is not the first who dealt with possible worlds, Copeland considers him the first logician who introduced in his *Tractatus* hints of the possible semantic worlds. It was not only Prior who was influenced by Wittgenstein's concept of possible worlds. Copeland asserts that Wittgenstein's well known ideas affected the works of most of pre-Kripke's modal logicians, particularly Carnap's modal logic. [38]

2. 1. 2 O-ontology

In spite of the importance of this concept it is not the sole description of possible worlds to be found in *Tractatus*. Wittgenstein's usage of possible worlds in o-ontology is still lacking, which is also essential for the reconstruction of Wittgenstein's concept of possible worlds, according to Suzsko's interpretation.[39] This subject is even talked over even in some chapters before Wittgenstein introduces his renowned concept, outlined in the previous paragraphs.

> 2.0211 If the world had no substance, then whether a proposition had sense would depend on whether another proposition was true.
>
> 2.0212 It would then be impossible to form a picture of the world (true or false). [40]

[37] A. Chrudzimski, "Contentless Syntax, Ineffable Semantics and Transcendental Ontology. Reflections on Wittgenstein's Tractatus," *KRITERION* 17 (2003): 2–3.
[38] Jack B. Copeland, "The Genesis of Possible World Semantics," *Journal of Philosophical Logic* 31 (2002): 100–101.
[39] Suzsko, "Ontology in Tractatus," 8–9.
[40] 2.0211 Hätte die Welt keine Substanz, so würde, ob ein Satz Sinn hat, davon abhängen ob ein anderer Satz wahr ist.
2.0212 Es wäre dann unmöglich, ein Bild der Welt (wahr oder falsch) zu entwerfen.
(Wittgenstein, *Tractatus.*)

The important term here, with the exception of "possible worlds", is the term "substance". Proops points out that possible states of affairs were necessary for Wittgenstein's conception of substance. He maintains that Wittgenstein was at this point deeply affected by Kant, who in his *Critique of Pure Reason* assumes substance as the principle which is impervious to change, and this preserves the individual's identity. Proops argues that the same concept of substance is contained in Wittgenstein's ontology. It is not used, however, as a tool that enables us to identify different forms of an individual through their development but rather serves as the identifier of an individual through possible worlds.[41]

2. 2 Logicians from the Lvov-Warsaw School

As was mentioned previously, the second feature of Prior's possible worlds' ontology is more opaque. Notwithstanding, this chapter will present certain arguments in favour of the claim that Prior might have been influenced by the logicians from the Lvov-Warsaw school. More specifically, the discussion is focused on the logicians from the Warsaw part of the Lvov-Warsaw school. As Woleński points out, these logicians inclined to extensional logic and nominalism.[42] Specifically, Łukasiewicz's had several doubts concerning nominalism and was not nominalist[43], but Leśniewski and some of his students were nominalists.[44]

One reason why there could be some impact on Prior is the fact that they influenced many aspects of Prior's ontology. As will be demonstrated in the

[41] I. Proops, "Wittgenstein on the Substance of the World," *European Journal of Philosophy* 12 (2004): 106–109.
[42] Jan Woleński, "Mathematical Logic in Poland 1900–1939 People, Circles, Institutions, Ideas," in *Essays in the History of Logic and Logical Philosophy* (Cracov: Jagiellonian University Press, 1999), 75–77.
[43] Jan Leopold Łukasiewicz, "In Defence of Logistic," in J. L. Łukasiewicz, *Selected Works*, ed. L. Borkowski (Amsterdam: North Holland Publishing Company – Warsaw: Polish Scientific Publisher, 1970), 241.
[44] E.g. Czesław Lejewski, *Letter from 11th July 1955 to A. N. Prior*. Unpublished manuscript stored in the Bodleian Library. Box 2. In *Virtual Lab for Prior Studies*, accessed April 13, 2016, http://research.prior.aau.dk/user.php?show=prior_letters&edit_correspondence_id=905&from=Lejewski&to=Prior&both=.

further parts of my dissertation there is the apparent influence of Leśniewski and his students on Prior's theory of quantification and his concept of names. Specifically, Prior was convinced that the adoption of some features of Leśniewski's theory enabled him to be both an intensional logician and a nominalist.[45] In addition, Prior was for a certain period of his life deeply influenced by Łukasiewicz. Apart from Łukasiewicz's approach to the history of logic, which Prior appreciated[46], they both sided with indeterminism which affected their systems of logic.[47]

In contrast, as Woleński emphasizes, Łukasiewicz was an extensional logician. There is no need to postulate the existence of possible worlds in his logic since if a future contingence is dealt with and there is nothing to refer to, the truth-value of the formula is ½. Possible worlds are consequently replaced by truth-values in Łukasiewicz's system of modal logic.[48] Similarly, Leśniewski did not incline to intensional logic, arguing that reality could be described by just one system of logic, which is two-valued and extensional.[49] Hence, they could not influence directly Prior's concept of possible worlds. Nonetheless, as could be shown using the example of Łukasiewicz's student Carew Meredith, Prior's concept of possible worlds could still be to some extent an original development of their ideas. The denial of the abstractness of possible worlds is entirely in accordance with Łukasiewicz and Leśniewski.

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[45] Arthur N. Prior, *Papers on Time and Tense*, ed. Per Hasle et. al. (Oxford: Oxford University Press, 2003), 220–221.
[46] Arthur N. Prior, "Łukasiewicz's Symbolic Logic," *Australian Journal of Philosophy* 30 (1952): 37.
[47] Jan Leopold Łukasiewicz, "Farewell Lecture by Professor Jan Łukasiewicz, delivered at the Warsaw University Lecture Hall on March 7, 1918," in J. L. Łukasiewicz. *Selected Works*, 85; Arthur Prior, "Reaction to Determinism," Virtual Laboratory for Prior Studies, ed. P. Øhrstrøm et al., accessed October 28, 2014, http://research.prior.aau.dk/cms/uploads/pdf/proofread/20110208202553.pdf.
[48] Jan Woleński, *Logic and Philosophy in Lvov-Warsaw School* (Dordrecht: Kluwer, 1989), 133–134.
[49] Ibid. 145.

2. 3 Meredith's System of Logic

Carew Meredith was Łukasiewicz's student in Dublin, with whom Prior corresponded from 1952 and with whom he elaborated his system of modal logic. They began their cooperation in 1956 when Prior visited Great Britain as an invited speaker for the John Locke Lectures in Oxford. He also intended to visit Łukasiewicz in Dublin but the old and ill Professor Łukasiewicz had died several weeks before Prior's arrival.

Despite Łukasiewicz's death, Prior came to Dublin and began close cooperation with Meredith which resulted inter alia in the publication of a paper where Meredith introduces his formalization of Wittgenstein's possible worlds ideas in Łukasiewicz's system of modal logic. Meredith's paper was known to some of his colleagues before the publication of this paper[50] through copies which circulated between them, since Meredith developed this system in 1953.[51] It was Prior's effort, however, which persuaded him about the official publication of it because, as Copeland claims, he was not prepared to publish any of his papers, especially after Łukasiewicz's death.[52]

The Meredith system is based on Lewis' system of modal logic but is extended by the law of extensionality, which was originally introduced by Łukasiewicz $\delta p \rightarrow (\Box(p \leftrightarrow q) \rightarrow \delta q)$ and by the variables *n* and *ṅ*. The law of extensionality was founded as a two-valued system of logic, but Meredith's addition of variable *n* makes this system many-valued.[53] This is also an important feature of Meredith's system. Despite this system being based on Łukasiewicz's system of modal logic, the Wittgensteinian "possible states of affairs" occurs here. This does not mean, however, that Meredith postulates the existence of such entities here or even that this system is intensional. Meredith's systems follow precisely Łukasiewicz's rules for modal logic and if variables as *n* which represents

[50] E.g. Prior quoted it in his *Possible Worlds* (Arthur N. Prior, "Possible Worlds." *Philosophical Quarterly* 46 (1962): 37).
[51] C. A. Meredith and Arthur N. Prior, "Modal Logic with Functorial Variables and a Contingent Constant," *Notre Dame Journal of Formal Logic* 2 (1965): 99.
[52] Jack B. Copeland, "Meredith, Prior and the History of Possible World Semantics," *Synthese* 150 (2006): 376.
[53] Meredith and Prior. "Modal Logic," 99–100.

possible states of affairs occur here. They stand for truth-values not for specific entities. Meredith's system of logic is therefore still many-valued and extensional, even though possible states of affairs are part of it.

More precisely, this system is four-valued. Apart from the values 1 and 0 for true and false he introduces the values *n* and *ṅ*. "1" in that system means necessarily true, e.g., true in the actual as well as in the alternative world. In contrast, "0" is interpreted as necessarily false, thus false in the actual and the alternative world. Meredith's newly introduced values deal with contingency. To be exact, *n* means contingently true (true in the actual world but false in an alternative world) and *ṅ* stands for contingently false (false in the actual world but true in an alternative world).[54]

Prior admired the preciseness of Meredith's system and adopted it in his works.[55] In spite of this fact, when the manuscript of this system was finally published in the paper which Prior and Meredith wrote together, Prior's part contains the crucial criticism of Meredith's system. The critique of Meredith's mistakes gave Prior an opportunity to provide a system of logic, which finished as a formulation of world-propositions as will be presented in the next chapter.

2. 4 The Historical Development of Prior's Ideas

Copeland points out that modern modal logic, which began to develop soon after the formation of modern logic, lacked an appropriate semantics for a long period. Due to this deficiency it inclined more to inaccuracies, which was exploited by their critics to its refutation. There was initially the idea of quantification over the possibilia, which is included in the Barcan formula, and which was also mentioned by Peirce.[56]

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[54] Prior, *Past, Present and Future*, 78.
[55] For instance: Prior, "Possible Worlds," 37; or Arthur N. Prior. and C. A. Meredith, "Interpretations of Different Modal Logics in the "Property Calculus"," in *Logic and Reality: Essays on the Legacy of Arthur Prior,* ed. Jack B. Copeland (Oxford: Clarendon Press, 1996), 133–134.
[56] Copeland, "The Genesis of Possible World Semantics," 99–100.

Prior was one of the logicians who became involved in the improvement of the formulation of modern formal logic and whose ideas influenced its further development. However, his own ideas advanced a great deal over the course of his life. This chapter will therefore be focused on the changes in Prior's ideas which precluded the formulation of the final form of his concept of possible worlds.

2. 4. 1 The Craft of Formal Logic

The first work in which Prior discusses modality is according to Copeland *The Craft of Formal Logic*,[57] which was designed as Prior's textbook because he lacked an English textbook in his logic lessons. Makoska-Cubrinovska, who investigates this unpublished manuscript, asserts that his concept of possible worlds was influenced a great deal by Wittgenstein's *Tractatus* and also by Wittgenstein's followers inters alia R. Carnap.[58]

Prior might have primarily been influenced by Wittgenstein's idea that there is a correspondence between propositions and states of affairs and that the description of the actual world consists of all true propositions. However, there are propositions, false at the moment, which can play a role in the construction of various possible worlds.

Other similarities can be found between Prior's and Wittgenstein's ontology in the papers where Prior discusses the existence of individuals. Firstly, that the identification through different possible worlds (and also through different time instances in Prior's case) is possible due to existence of substances which serve as an identifier of individuals.[59] Prior only touched on these ideas, however, in his paper and it is not certain whether he actually introduced it into his ontology.

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[57] Copeland, "Meredith, Prior," 378.
[58] Aneta Markoska-Cubrinovska, "Possible Worlds in "The Craft of Formal Logic"," *Synthese* (forthcoming).
[59] Comp. Wittgenstein. *Tractatus*, 2. 0211–2.0212; Prior, *Papers on Time and Tense*, 78; and Proops, "Wittgenstein on Substances of World," 106–111.

Secondly, it is the idea that whatever occurs as an object in the possible worlds has to be based on something existent in the actual world.[60] Nevertheless, also in this case there is no evidence that Prior was inspired by Wittgenstein when he formulated that position. It is more likely, as Betti claims in her paper, that the logicians who were in some way influenced by traditional logic share the same ideas, however, there is no direct influence between them in such a case.[61]

2. 4. 2 Formal Logic

The publication of *The Craft of Formal Logic* was actually rejected by a publishing house due to its length. Prior did not shorten it but instead wrote another textbook entitled *Formal Logic* which was based more on the logical systems of Polish logicians than *The Craft of Formal Logic*.[62] While explaining modality, Prior also deals here with logical tradition, primarily with Aristotle's ideas and with the traditional division of *modality de re* and *modality de dicto*. However, Prior merely uses Łukasiewicz's system of modal logic in order to formalise traditional ideas and in the whole book the logical formalism is provided in Polish notation.[63]

Prior remarkably inclined to the systems of Polish logicians especially to Łukasiewicz's in the years before the publication of *Formal Logic*. He discussed intensively Łukasiewicz's many-valued systems and also had several critical remarks concerning it, being truly amazed by them.[64] This resulted not only in the adoption of the Polish notation but Prior also began to be a defender of Łukasiewicz's three-valued logic.

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[60] Comp. Wittgenstein, *Tractatus*, 2.022–2. 023 and Prior, *Papers on Time and Tense*, 77.
[61] In fact, Betti compares in her paper Bolzano and Leśniewski, even though, this assertion is meant more universally. Adrianna Betti, "*De Veritate*: Another Chapter the Bolzano-Leśniewski Connection," in *The Lvov – Warsaw School and Contemporary Philosophy*, ed. Katarzyna Kijania-Placek and Jan Woleński (Dordrecht: Kluwer Academic Publisher, 1998), 115.
[62] Copeland, "Arthur Prior."
[63] E.g. Arthur N. Prior, *Formal Logic* (Oxford: Clarendon Press, 1955), 191–193.
[64] Comp. Arthur N. Prior, "In What Sense is Modal Logic Many-Valued?" *Analysis* 12 (1952): 138–143; Prior, "Łukasiewicz's Symbolic Logic," 33–46; and Arthur N. Prior, "On Propositions Neither Necessary nor Impossible," *Journal of Symbolic Logic* 18 (1953): 105–108.

In his *Three-Valued Logic and Future Contingents*, he wrote that Łukasiewicz's logic "is admirably adapted to the expression of this way [Aristotle's one] of regarding statements about contingent future events".[65] Although Prior expressed some doubts about three-valued logic, explicitly that in meta-logic Aristotle's approach has to be more two-valued than many-valued, he claims in the end of his paper that three-valued logic provided a new accuracy for an understanding of modal statements and its ontology.

As Prior adopted Łukasiewicz's three-valued logic it can be assumed that he also accepted the rejection of the possible world. Prior nevertheless still found Łukasiewicz's three-valued logic too deterministic as he demonstrated in his texts.[66] Being an ardent defender of indeterminism,[67] he could not accept even a hint of determinism and therefore offered another solution.

2. 4. 3 The Beginning of Prior's Tense Logic

Prior, who was also a prominent historian of logic, introduced his two-valued indeterministic system of modal logic when he discussed the Master argument of Diodorus Cronus in the paper,[68] which was published the same year as *Formal Logic*. However, it was in fact temporal logic which was established this paper because two of the operators which were used here are also linked with time, i.e. the operators *F*, which stands for "It will be the case that …" and the operator *G* which stands for "It will always be the case that…" . Over a short time Prior presented further tense operators: *P* i.e. "It was the case that…", *H* i.e. "It was necessarily the case that…". Prior began to definitively abandon many-

[65] Arthur N. Prior, "Three-Valued Logic and Future Contingents," *Philosophical Quarterly* 3 (1953): 323.
[66] Arthur N. Prior, "Diodoran Modalities," *The Philosophical Quarterly* 20 (1955): 212.
[67] Øhrstørm and Hasle. *Temporal Logic*, 167–171.
[68] Øhrstørm and Hasle pointed out that Prior claimed that it was Findlay's article *Time: Treatment of Some Puzzles* which inspired him to postulate this form of temporal logic. In addition, also B. Mates enquiry on Diodoran modality is important for a formulation of the temporal logic in Prior's article. The temporal logic which is introduced here is, nonetheless, more of a draft than a final system of logic and it endured several substantial changes in Prior's later works. (Øhrstørm and Hasle. *Temporal Logic*, 170–171.)

valued logical systems, even though, he created certain many-valued logical systems after that.

This was the beginning of Prior's temporal logic and also of the explicitly expressed connection between time and modality which is typical for Prior's systems of logic. In addition, the title of the book where these systems are described with more preciseness is *Time and Modality*. This connection also has one by-product specifically the closeness which is between possible worlds and time instances in Prior's ontology. This closeness is also known by Prior who asserted it clearly in the appendix of his book *Past, Present and Future*: "To be the case at such-and-such an instant is simply to be the case in such-and-such a world."[69]

The first definition of temporal and modal logic in *Diodoran Modalities* contained several mistakes which were for a long period an object of discussion between Prior and other logicians.[70] This discussion as Prior reported in his papers and books[71] was very lively and inspiring for all the sides which joined in. Since, however, Łukasiewicz's extensional approach was rejected here the question of possible worlds could come back.

Prior discussed them again in his book *Time and Modality.* When Prior dealt with possible states of affairs here, he rejected that they could have some kind of existence as also possibilia have. He claims:

> Possible states of affairs are no more arranged in such an order about the actual state of affairs, than they are capable of being given proper names. The difficulty is, in short, that the non-existence of an object at some time or in some state of affairs can only be referred to *from* the time at which, or state of affairs in which, the object *does* exist; and whereas we can say exactly how the time now present would be identified *from* some other time, we cannot say how the state of affairs which happens to be the actual one would be identified *from* some

[69] Prior, *Past, Present and Future*, 189.
[70] Vladimír Marko, "Some Pioneering Formal Reconstructions of Diodorus' *Master Argument*," *Logica and Methodologica* V (1999): 67–72.
[71] See Prior, *Time and Modality*, 8–28; Arthur N. Prior, "Diodorus and Modal Logic: A Correction," *The Philosophical Quarterly* 32 (1958): 226–230; Prior, *Past, Present and Future*, 22–23.

> other possible state of affairs, and I do not know that we can be said to 'identify' possible states of affairs anyway.[72]

Discussing non-existent individuals, Prior admitted that possible states of affairs could be a useful tool. He was, however, rather sceptical of the need to postulate their existence as well as to postulate the existence of possible individuals in them. Although he used possible states of affairs for the semantics of his modal propositions[73], he did not explain their ontological status. Hence, there is a rather negative than positive definition of possible states of affairs in Prior's *Time and Modality*. He claimed that they did not exist, but he did not clarify what they were like. Notwithstanding, since Prior used here the term "possible states of affairs" which clearly refers to Wittgenstein's *Tractatus* it can be assumed that Prior returned here to Wittgenstein's ontology. However, the attempt of reduction of abstract entities which Prior demonstrates in this book might have been influenced by logicians from the Lvov-Warsaw school.[74]

2. 4. 4 Possible Worlds

Prior paid his debt to ontology of possible worlds in the previously mentioned paper *Possible Worlds*, where the concept of possible worlds is explained more precisely. Nevertheless, it could seem dubious whether this attempt, which is based on Geach's rather sci-fi then scientific ideas, can represent a serious definition of possible worlds, since Prior presented them:

> Suppose we define a 'possible' state of affairs or world as one which can be reached from the world we are actually in. What is meant by reaching or travelling to one world from another need not here be amplified; we might reach one world from another merely in thought, or we might reach it more concretely in some dimension-jumping vehicle dreamed up by science fiction (the case originally put by Geach), or we might reach it simply by the passage of

[72] Prior, *Time and Modality,* 52.
[73] Ibid., 143.
[74] Comp. Woleński, *Lvov-Warshaw School*, 133–134; Prior, *Time and Modality,* 73–78.

time (one important sense of ' possible state of affairs ' is ' possible out-come of the present state of affairs ').[75]

As Copeland points out, however, this formalization was presented for the first time in Prior and Meredith's paper *Interpretations of Different Modal Logics in the "Property Calculus"* from 1956. The paper was not published during Prior's and Meredith's life but circulated among their colleagues in several copies. It contains only a formal logical system and the function of the operator *U* is not explained here. In spite of this fact, this short paper, which was finally published in 1996 as part of Copeland's book *Logic and Reality*, demonstrates that Meredith and Prior developed possible worlds' (or more precisely possible states of affairs') semantics even in the year 1956 and Prior's interpretation of possible worlds in the paper *Possible Worlds* is not dependent on Geach's sci-fi ideas.[76]

In spite of the playful note of the entire paper, it is an important step in the formulation of Prior's concept of possible worlds. Prior adopts here Meredith's notation, where possible worlds are formalized as variables such as *a*, *b* or *c*. Each variable is described by Meredith in the Wittgensteinian sense as "everything that is the case."[77] The operator *U* is introduced here which can be described as a "jumping" operator. *Uab* means an individual jump from possible world *a* into possible world *b*.

Furthermore, from the ontological point of view one of Prior's remarks is important. He claimed that possible states of affairs (or possible worlds) could only occur in the future but not in the past. It is well known that Prior as an indeterminist advocated a linear past but an open future. Moreover, in this paper Prior uses a branching time structure for the explanation of the open

[75] Prior, "Possible Worlds," 36.
[76] Copeland, "Meredith, Prior," 378–380; Prior and Meredith, "Interpretations of Different Modal Logics", 133–134.
[77] Meredith and Prior, "Modal Logic," 99.

future which was introduced to him shortly before by Saul Kripke in his letter to Prior.[78]

There is one distressing feature in Prior's dealing with possible states of affairs and possible worlds, however, in his paper *Possible Worlds*. It seems that the distinction between possible states of affairs and possible worlds which is important for Suszko's interpretation of Wittgenstein's ontology,[79] plays no role in Prior's paper since Prior skipped fluently between the usage of "possible states of affairs" and "possible worlds". He began with a description of possible states of affairs but continued explaining how possible worlds could be formalized without drawing any line between these entities.[80]

In summary, Prior assumed a Wittgenstein ontology here. It seems to be enriched, however, by some of Łukasiewicz's ideas, Meredith's system of logic, Wittgenstein's concept of possible states of affairs, Kripke's branching structure and his own operator. Amongst them especially Meredith's system of logic has had a substantial impact on Prior's concept of possible worlds. The next chapter will consequently focus on Meredith's system of logic in which Meredith formalizes his ideas.

2. 5 Prior's Criticism of Meredith – the Formulation of World-Propositions

Meredith's system of logic was "elegant and ingenious" according to Prior from the formal point of view[81]. There is another aspect of it, however, the philosophical one, which makes this system more problematic as Prior further demonstrated it contains paradoxes. The main difficulty lies in the definition of possible states of affairs and its representation via variables. The description of the *n* variable, which represents a possible state of affairs as "the totality of

[78] Thomas Ploug and Peter Øhrstørm, "Branching time, indeterminism and tense logic: Unveiling the Prior-Kripke letters," *Synthese*. 188 (2012): 367–379.
[79] Suzsko, "Ontology in Tractatus," 19–20.
[80] Prior, "Possible Worlds," 36–43.
[81] Meredith and Prior, "Modal Logic," 100.

what is the case" is vague and allows for inter alia introducing the formula *id(p) = □(p → q)* into Meredith's system, which is described by Prior as "that proposition can be identical with a logical complication of itself".[82]

Prior demonstrates that this violation can raise a paradox via an analysis of a Medieval self-reference paradox, originally created by Strode:

Stage I.	The argument in Stage I is sound (= *p)*
	Therefore, I am the Pope (or anything at all) (= *q)*
Stage II.	(1) If the argument in Stage I is not sound, then possibly (*p* and not *q*). $((\neg\Box(p \rightarrow q)) \leftrightarrow (\Diamond(p \wedge \neg q)))$
	(2) If possibly (*p* and not *q*), then possibly *p,* i.e. possibly Stage I is sound. $(\Diamond(p \wedge \neg q) \rightarrow \Diamond p)$
Therefore	(3) If Stage I is not sound, it possibly *is* sound (syllogistically from (1) and (2)).[83]
Therefore	(4) Stage I is possibly sound (by $((\neg p \rightarrow \Diamond p) \rightarrow \Diamond p)$, a modal law obtainable syllogistically from $((\neg\Diamond p \rightarrow \Diamond p) \rightarrow \Diamond p)$, and $\neg\Diamond p \rightarrow \neg p$.
	But (5) Stage I *could* be sound only if it *is* sound ($\Box\Diamond(p \rightarrow q) \rightarrow \Box(p \rightarrow q)$[84], from $\Diamond\Box p \rightarrow \Box p$, S5).
Therefore	(6) Stage I is sound, i.e. *p* [(4), (5), modus ponens]
Therefore	(7) I am the Pope (from (6), by Stage I).[85]

Additionally, Prior mentioned another reason why the formula *id(p) = □(p → q)*, should be rejected. This was demonstrated by Prior's former student Cresswell. Cresswell proved, as Prior claims in his part of the paper, that a system of

[82] Ibid., 100.
[83] There Prior uses the formula $id(p) = \Box(p \rightarrow q)$ for the first time. The antecedent of formula $(\neg\Box(p \rightarrow q) \rightarrow \Diamond p))$, is substituted by *p*.
[84] In addition, Prior replaces there *p* with the formula $\Box(p \rightarrow q)$.
[85] Meredith and Prior. "Modal Logic," 100.

arithmetic could be created, where no individual variables are postulated. The lowest types of variables of this system are propositional variables. Then if the formula *id(p) = □(p → q)* is introduced, it causes an infinite regress.

In conclusion, Prior claimed, that *n* in Meredith's system of logic had to be replaced in order to avoid the paradoxes which it integrated. Prior therefore postulated a new functor *W*, which bound propositions. The meaning of *Wp* is "p comprehends all truths"[86] and also introduces propositional quantifiers.

How the replacement in the Meredith system works is shown in this table:[87]

Meredith	Prior	Meaning in ordinary language
n	Wp → p	"the world is the case"
p → □(n → p)	Wp → (q → □ (p → q))	"the world is everything what is the case"
¬□n	Wp → ¬□p	"the world is not necessary"

Moreover, the Meredith description of *n:*

> For the conjunction of all truths would have to contain as conjuncts (a) itself, (b) its own double negation, (c) every fact as to what it implies; to name only a few of the impossibilities.[88]

was substituted by Prior's explanation of world-propositions as:

> "world" proposition is a *maximum* proposition; if we conjoin with it the least thing that it does not imply we shall have a contradiction, since among the things it does imply will be the negation of the added item.[89]

It is worth emphasizing that the system of logic which is an output of Prior's transformations is both intensional and also two-valued. It therefore violated Łukasiewicz's ideas which are preserved in the Meredith system of logic.

[86] Ibid., 101.
[87] Prior. *Past, Present and Future*, 78.
[88] Meredith and Prior, "Modal Logic,"101.
[89] Ibid, 104.

Notwithstanding, it correlates with Prior's own preferences of intensional and two-valued logic.

The world-propositions which were introduced in Prior's and Meredith's paper *Modal Logic with Functorial Variables and a Contingent Constant* were further developed in Prior's book *Past*, *Present and Future*. Prior provided here more formalism for his concept. To be precise, the operator *W* is defined here as:[90]

$$Wp \overset{def}{\equiv} (p \wedge \forall q\,(q \rightarrow \Box(p \rightarrow q))$$

The operator *Wp* is introduced once again in the system of logic which is based on Lewis' S5 system of logic and which consists of the axioms:

1, $Wp \rightarrow p$

2, $Wp \rightarrow (q \rightarrow \Box(p \rightarrow q))$

From which he deduces the theorems:

3, $Wp \rightarrow (\Box p \leftrightarrow Wp)$

4, $\Box Wp \rightarrow p$

5, $\Box(p \rightarrow Wp)$

In the same fashion as Aristotle, Prior in his *On Interpretation* also tried to specify a contradiction to the world-proposition. Prior claimed, in accordance with Aristotle's solution, that the negation of the world-proposition cannot be another world-proposition, and therefore the negation of the world-proposition cannot be a world-proposition. In addition, if *p* is a world-proposition, $\neg p$ is also a world-proposition, but *p* and $\neg p$ are not a pair of contradictions.[91]

Furthermore, Prior introduced another operator in his book *Past, Present and Future*, the operator *Q* in his system of logic. *Qp* means "*p* is totality of the truth at some time", which had, according to Prior, also the meaning "*p* is a possible world".[92] In the system described above this operator is introduced as:

[90] Prior. *Past, Present and Future*, 79.
[91] Ibid., 82.
[92] Prior. *Past, Present and Future*, 80.

$$Wp \rightarrow (p \wedge Qp)$$

Fine described *Q* operator, especially *Qp*, as a world-proposition later in his *Postscript*. *Qp* is value in only the world, or more precisely *Qp* means that *p* is contingently true.[93] This operator should not be mingled, however, with the system of logic Q for a non-permanent existence, which Prior also discussed in this book. The system and the operator both signed with *Q* have nothing in common (except for the fact that they were introduced by Prior).

2. 6 The Final Form of World-Propositions

From the formal point of view Prior's step is even more interesting as Prior combines this system of modal logic with his operators of temporal logic. As was mentioned previously, his concepts of possible worlds and time instants are extremely close, namely incorporated in one system of logic. Furthermore, since Prior was a defender of indeterminism, it is not surprising that the temporal operators were introduced when he discussed Laplace's determinism and the branching time structure. He explicitly stressed here that in order to preserve the important features of the system, not every combination of time operators would be allowed. Namely, in the branching structure of time $\Box p$ could be replaced by *GHp* but not by *HGp*.[94]

As was discussed previously, McTaggart pointed out in his paper *The Unreality of Time* that there are two ways of conceptualizing time. The concept of time as past, present and future, which was called the A-series by McTaggart and the concept of time where only earlier and later is distinguished. This concept is called the B-series by McTaggart.[95] When Prior formulated his systems of temporal logic, systems which formalize time in accordance with A-series can be found, which using the operators *F*, *P*, *G*, and *H*, which were mentioned in the previous chapter, he called these systems A-logical. In addition, Prior also

[93] Kit Fine, "Postscript," in Arthur Prior, *World, Times and Selves*, ed. Kit Fine (London: University of Massachusetts Press/Duckworth, 1977), 119.
[94] Prior. *Past, Present and Future*, 84–85.
[95] McTaggart, "The Unreality of Time," 458–459.

formulated systems in accordance with the B-series, which were called B-logical. The U-calculus introduced in *Possible Worlds* is one of them.

Although Prior discussed both types of systems of temporal logic, he preferred the A-logical ones. Firstly, Prior was a realist in the case of time. He believed that time is real and to that conviction the A-logical systems of logics is better suited.[96] Presentism which he defended is also more easily combined with the A-series, as was presented in the introduction. Secondly, from the ontological point of view, the B-logical systems are highly controversial, especially for Prior, who did not want to maintain any type of existence of entities as possible worlds or time instants. The variables which are used in this system of logic indicate a certain kind of reality of time instants. At least if some time instant is claimed to be earlier than the other, it has to have a certain beginning and certain end and it has to be bounded in some way.[97]

When doing this Prior had to face one essential problem. He had to prove that A-logic has a priority before B-logic because A-logic has less expressive power than B-logic. Furthermore, this calculus even demonstrated its usefulness in Prior's "world-jumping" paper *Possible Worlds* and as Copeland shows was introduced by Prior and Meredith several years earlier.[98] Prior admitted that U-calculus seems to be from the philosophical point of view simpler than systems of A-logic, but points out that U-calculus needs to use two sorts of variables where tense logic dealt only with one.[99] He consequently had to prove that these intuitions are justifiable.

He did it by suggesting a hybridised system of logic. In this system two different sorts of variables are used: *p*, *q*, *r*, etc. for propositions and *a*, *b*, *c* etc. for world-propositions. There are also two different sorts of quantifiers *G, H* and $\forall, \exists$. The introduction of systems of logic which deal with all these variables and quantifiers is viewed as Prior's contribution to the development of hybrid

[96] Øhrstrøm: "Two Essays on Temporal Realism," 43.
[97] Prior. *Past, Present and Future*, 189–190.
[98] Copeland, "Meredith, Prior," 376.
[99] Prior. *Papers on Time and Tense*, 120.

logic.[100] Notwithstanding, Prior's hybridisation of logic is dealt with in the next chapter.

Lastly, it is worth showing how Prior's concepts of possible worlds and time instants is constructed from the ontological point of view. As was mentioned previously, Prior did not really differentiate in his ontological concept between possible worlds and time instants. The important features of both of them are stressed by Prior as:

> a, things-quality metaphysics is fine, but that
> b, instants [and possible worlds] are not things, and
> c, quantifying over instant-variable [and world-variable] does not commit us to the view that instants [and possible worlds] are things.[101]

This quotation also indicates that no possibilia are involved in Prior's ontology. Hence, if possible worlds or time instants consist of individuals, these individuals refer to objects that now exist in the actual world. Prior also spoke about world-propositions and instant-propositions as opposed to possible worlds and time instants, with only the former occurring in his ontology. As he maintained:

> A world-state proposition in the tense-logical sense is simply an *index of an instant*; indeed, I would like to say that it *is* an instant, in the only sense in which 'instants' are not highly fictitious entities.[102]

The proposition is as Fine claims the "one which is *maximally possible*; it is possible and implies any proposition or its negation."[103] The reason why Wittgenstein's description of this proposition "the world is everything that is the case", which was used by Meredith, is not suitable was shown in the previous chapters. Finally, as Prior pointed out in the quotation above, neither possible-worlds, nor time instants are individuals in any sense. This violates Quine's concept of quantification, namely that "to be is to be the value of a

[100] Blackburn, "Arthur Prior and Hybrid Logic," 350.
[101] Prior, *Papers on Time and Tense*, 209.
[102] Prior, *Past, Present and Future*, 188–189.
[103] Prior, *Worlds, Times and Selves*, 138.

variable".[104] Prior did not agree, however, with this strict concept of quantification as will be demonstrated further. Hence, he did not consider himself to be obliged here to some sort of ontological commitment.[105]

There is still the remarkable impact of the theories of great logicians, Wittgenstein and Łukasiewicz, which influenced Prior's previous concept of possible worlds, in his postulation of world-propositions and instant-propositions. Prior in his book and later articles actually developed a complicated system of logic. His reduction of B-logical U-calculus into the A-logical system of logic K_t resulted in a hybridisation of this logical system.[106]

Although, the hybridisation of Prior's logic is remarkable primarily from the formal point of view it has also an impact on ontology, because one criticized feature of the B-logical systems occurs again in the hybridised systems. They namely contain the variables, which represent possible worlds or time instants.

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[104] W. V. O. Quine, "On what, there is," accessed September 21, 2014, <http://en.wikisource.org/wiki/On_What_There_Is>.

[105] Prior, *Papers on Time and Tense*, 220–221.

[106] Since Prior's approach was truly unusual the question arose as to who might have inspired Prior and affected Prior's concept of possible worlds. Uckelman points out that Buridan's solution of one of sophismas might have influenced Prior a great deal. (Sara L. Uckelman, "Prior on an *insolubilium* of Jean Buridan," *Synthese* 188 (2012): 487–498.) Buridan, a prominent nominalist from the fourteenth century, solves several paradoxical propositions in his book *Sophisms*. The one which might have inspired Prior, is a self-reflexive paradox, one of the paradoxes which was called *Insoluble* in Medieval logic. Insoluble is a proposition which seems to be true but a detailed exclamation clarifies that it is self-contradictory. The insoluble which Prior focused on has the form "Every proposition is affirmative, therefore no proposition is negative." This sole hypothetical proposition, known also as consequences, is contradictory since it is a self-reference proposition, which claims that there is no negative proposition but its second part contains negative proposition. Buridan differentiates between two kind of possible propositions, one of them is possible and the second possible-true. The representation of the former type is the second part of the consequence; "no proposition is negative." The state of the world, which it describes, is possible, even though, it can never be true or even valid. The represent of the latter is every contingent proposition which can be true, i.e. the first part of the consequence "Every proposition is affirmative." Prior dealt with this Buridan's insoluble in his paper *The Possibly True and the Possible*. (Arthur N. Prior, "The Possibly-True and the Possible," *Mind* 78 (1969): 481–492.) Uckelman stresses that Prior here in order to reconstruct Buridan's solution postulates two types of language. The object language L, which is represented by Latin and the meta-language M which is represented by standard English. Uckelman provides here insight into Prior ideas, when she is able to find hints of hybridisation of logic in this Prior's analysis of Buridan's insoluble. Prior did not refer to Buridan, however, when he discussed hybridisation of his logic. He instead mentioned the influence which Peirce's concept of quantification and Zeman's interpretation of it had on his systems of logic. (Prior, *Papers on Times and Tense*, 235–236). As far as I know, however, this possible source of Prior's inspiration was not examined. In addition, from the point of view of Polish logic the impact which Leśniewski's Protothetic had on Prior's hybridisation is also remarkable. Since, my work focuses on another field of Prior's work I cannot answer these questions properly here, even though, it is not without significance.

Hence the question arises once again whether Prior is prepared to accept some way of existence of these entities. It seems that he does not since even in his last and posthumously published book he insisted on the unreality of these kinds of entities. In contrast, hybridised systems imply some way of their existence. Prior might have coped with this problem if he lived longer, but unfortunately, he did not. Hence, it should be claimed that he is not really consistent in his ontology of possible worlds and time instants. This is primarily caused by his hybrid logic, which actually has many advantages. It is a field of Prior's study which is developed extensively at present as is also shown in the next chapter.

2. 7 Hybrid Logic as a Further Development of Prior's Ideas

It was argued in the previous chapters that Prior's final system of modal and tense logic was hybridised. However, it was not precisely explained what this means and why this hybridisation might be interesting for logicians at present, especially for those who work in the field of hybrid logic. Hybrid logic will therefore be briefly described in this chapter and the features of Prior's system of logic which are important for it are emphasized.

Indrzejczak claims that modern hybrid logic evolved in the 1990s in between logicians from the Sophia school, who were primarily interested in Combinatory Propositional Dynamic Logic, when they enriched this logic with addition of nominals. They were initially not aware of Prior's work on the same problem. Soon after when hybrid logic expanded, however, it was pointed out that this logical system was created for the first time and used by Arthur Prior in the last years of his life.[107]

Prior's main contribution to hybrid logic is that he was the first who used formulas (world-propositions and instants-propositions) as terms in his logical system. He did not create nominals in the hybrid logic sense of the word, however. They were introduced by Prior's former pupil Bull in his paper *An*

[107] Andrzej Indrzejczak, *Natural Deduction, Hybrid Systems and Modal Logics* (Dordrecht: Springer, 2010), 366–367.

Approach to Tense Logic published soon after Prior's death.[108] In contrast, the variables that bound nominal and only nominal also occur in Prior's papers.

As can also be seen in Prior's case, hybrid logic is in some way an extension of modal logic. Hence it may also be entitled modal hybrid logic as it is in Indrzajczak's book. It is a system of logic, however, or more precisely they are systems of logic[109], which in some way extended the possibilities of modal logic. The advantages of hybrid logic in comparison with ordinary modal logic were summarised by Indrzejczak as:[110]

- more expressive language
- better behavior in completeness theory
- more natural and simpler proof theory
- good behavior in decidability, complexity, interpolation and other important features

The differences between hybrid logic and ordinary modal logic can be described with the example provided by Blackburn in his paper. Blackburn compares the approaches of hybrid and modal logic to the A-series and B-series concept of time. As the B-series describe time from the outside and the A-series is the description of the one who is embedded it time. Modal logic is consequently more likely to be the system which holds the modal relations from the outside and hybrid logic is capable of grasping the relations of individuals, time instants and possible worlds inside the systems and also allows for a reference to them.[111]

Bräuner asserts that the main features which differentiate the hybrid logics of ordinary modal logics are the usage of nominals and the addition of two sorts of quantifiers, one which binds nominals in the actual world and the second which binds them over worlds (or over time instants in Prior's case). Nominals are

[108] Prior was even one of the reviewer of this paper but unfortunately died before the publication of it. Bull, however, expressed clearly in this paper his gratitude to Prior, who substantially influenced this paper. (See Richard Bull, "An Approach to Tense Logic," *Theoria* 36 (1970): 282–300.).
[109] Since several logical systems which are hybridised were postulated, one of them is Prior's.
[110] Indrzejczak, *Natural Deduction*, 365.
[111] Blackburn, "Prior and Hybrid Logic," 332–333.

terms which stand for the entire formula but apart from this behave as terms.[112] Moreover, as Blackburn stresses, they have to "be true exactly in one point of model". They can, however, be combined with other variables in formulas of hybrid logic. This can be demonstrated with one of the formulas of hybrid logic $@_i\varphi$, which asserts φ that is true in the point *i* as Blackburn claims "at i,φ". The nominal in this formula is variable *i* and the operator which binds it @.[113] Bräuner argues that @ in Prior's formalisation is replaced by operators *P* and *F*, however as can be seen from Prior's system of logic introduced in the previous chapters these operators could also bind different variables than nominal.[114]

Blackburn points out that Prior's hybridisation of his system of logic might have been too successful. It demonstrated its usefulness not only in the transcription of U-calculus to T-calculus, but as Prior discovered soon after, a similar approach could be used in egocentric logic, e.g. in the logic which describes the world from the position of the speaker (or inventor, or first person). Apart from the interest of this result, Prior is forced to admit that it contradicted his goal to prove that A-logic is primer than B-logic. Since Prior died soon after the publication of this paper and even before the publication of the papers where this problem is further discussed, he never satisfactorily solved this query.[115]

Notwithstanding, Prior is viewed as not only the founding father of modern temporal logic but also as the founding father of hybrid logic. Furthermore, as was previously emphasised, the origins of the hybridisations of his system of logic are connected with his ontological views.

Taking everything into account, the aim of this chapter was to introduce Prior's concept of possible worlds and time instants, which are linked in his ontology. This was carried out firstly by introducing those logicians and philosophers who

[112] Torben Bräuner, "Modal Logic, Truth and the Master Modality," *Journal of Philosophical Logic* 31 (2002): 360.
[113] Blackburn, "Prior and Hybrid Logic," 344–345.
[114] Bräuner, "Master Modality," 383
[115] Blackburn, "Prior and Hybrid Logic," 350–364.

affected Prior. The development of Prior's ideas concerning possible worlds was consequently described. Prior's final form of the concept of possible worlds was considerably influenced by his ontological positions.

Firstly, only actual objects exist, possible worlds and time instants are not objects, and hence there are no such entities as possible worlds or time instances. Secondly, it is a priority of A-logic beneath B-logic, which was to some extend demonstrated by Prior via a hybridisation of his system of logic. Prior's hybridisation of his systems of logic is currently highly influential in the field of study of hybrid logic, even though, as was also pointed out, it did not solve satisfactorily Prior's problem.

It cannot be claimed convincingly in this chapter that Prior is actually a nominalist in the case of possible worlds and time instants. In contrast, his presentism is clearly recognisable in his preference for A-logical systems. It was demonstrated that he tried to reduce them to world-propositions and instant-propositions. He argued that no such objects as possible worlds or time instants exist in the real world. Nonetheless, there are still qualms concerning the form of being of propositions and about the reference of entities which occurs in possible worlds or which were inhabitants of any other time instant than the actual one. This is particularly the case when they perished or when they will not begin to exist now. The reasons for these queries might be found in Prior's theory of quantification, which is also the main topic of the next section.

3, Quantification

'To be a value of a bound variable is to be' is just a piece of unsupported dogma.[116]

The theory of quantification has played a significant role in analytic philosophy. It began with Brentano and Venn's analyses of the existential import and culminated with Quine's ontology.[117] Prior disagreed with this renowned position since he was inspired by other approaches to the theory of quantification. The Leśniewskian quantification, which is often used in Prior's ontological papers, allowed him, as he assumed, to separate a quantification and an ontological commitment.[118]

The theory of quantification began to be important for ontology in the nineteenth century, when the concept of the existential import changed. It was originally problematized in one of the first books dedicated to logic, in Aristotle's *On Interpretation*. Aristotle claims that the copula "is" is not only the connector of a subject and a predicate but also implies an existential import. The proposition "Socrates is white" means not only that Socrates' body has a white colour but also that an individual called "Socrates" exists there. Thus the affirmative propositions, if they are true, indicate that their subject is existent in Aristotle's interpretation.[119]

According to the authors of the Logic Museum, the change in existential import was provided by two logicians of the nineteenth century. Firstly, Mill pointed out that the copula has merely a linking function and does not imply the existence of the subject of the proposition. Secondly, Brentano interpreted a

[116] Arthur N. Prior, *Object of Thought*, ed. P. T. Geach and A. J. P. Kenny (Oxford: Clarendon Press, 1971), 48.
[117] The Logic Museum. "Existential Import," accessed April 23, 2015, http://www.logicmuseum.com/wiki/Existential_import
[118] Prior, "Nonentities," in A. N. Prior, *Papers in Logic and Ethics*, ed. P. T. Geach and A. J. P. Kenny (London: Duckworth, 1976), 114–115.
[119] *De interp.* c. 1 p.16a12–15.

way which is not all that distant from the modern formulation of them in the predicate logic. Although Peirce's paper *On the Algebra of Logic* also contained the formalization of the square's propositions where quantifiers are used, the most renowned might have been Venn's transformation of square's propositions into Venn's diagrams.[120] From this it is only a step to Russell's formalisation in Peano-Russell's symbolism in which the square's propositions are formalized as:

$$\forall x\,(S(x) \rightarrow P(x))$$
$$\forall x\,(S(x) \rightarrow \neg P(x))$$
$$\exists x\,(S(x) \wedge P(x))$$
$$\exists x\,(S(x) \wedge \neg P(x))$$[121]

The existential import is also contained here in the particular propositions instead of the affirmative propositions which were the bearers of existential import in Aristotle's conception of the square. Furthermore, it was precisely this formalisation which allowed Quine to claim, that "to be is to be a value of the variable" and thus linked the nominal quantification with ontology.

There are several types of quantifiers which are used in Prior's theories of quantification and several variables are bound by them. There are two reasons why he did not use only the standard two quantifiers. Firstly, he was also inspired by pre-Russellian and non-Russellian traditions in which different quantifiers are used to some extent. Secondly, he maintained in one of his last papers *The Parallel Between Modal Logic and Quantification Theory*, that modal operators are similar to quantifiers. Furthermore, his temporal operators can be (and also already are) presented as quantifiers.[122]

Two important features can be identified in Prior's theory of quantification. In the first place, it tends to lack an ontological commitment, entities which do not exist presently can only occur as the values of bound variables, but nothing

[120] The Logic Museum. "Existential Import."
[121] Bertrand Russell, "The Existential Import of Propositions," *Mind* 14 (1905): 400.
[122] Prior, *World, Times and Selves*, 9–10.

more.[123] Secondly, quantifiers can bind not only variables which represent names but also those which stand for propositions.[124] As was mentioned previously, this feature is particularly important for Prior's concept of possible worlds. Both these features are, however, in direct conflict with Quine's attitude.

Furthermore, as was shown in the previous part of this dissertation, Prior asserted that quantification had to also appear within modality. Once again this position is against Quine's view.[125] Prior worked with an open future, in which the existence of individuals was not guaranteed. Another time period, the past, also includes problems because its individuals sometimes did not exist in the present any longer. Since Prior wanted to quantify here, he had to admit that something which did not exist at present could also be the value of bound variables.

3. 1 The Nominal Quantification

3. 1. 1 Quine

From the Priorean point of view the precise description of Quine's attitude is necessary since Prior referred several times to Quine as his prominent opponent. Quine formulated his well-known statement in the context of the discussion with his two imaginary opponents, McX and Wyman. McX and Wyman intended to maintain some way of existence of entities, which do not exist in reality. Their main objection against Quine's denial of existence of entities as Pegasus is that such entities must in some way exist since we have to refer to something if we claimed their non-existence. While McX's ideas are rejected immediately by Quine, Wyman's concept requires a deeper analysis in order to be denied.

[123] Prior, *Papers on Time and Tense*, 220.
[124] Hugly and Sayward, *Intensionality and Truth*, 177.
[125] For instance: Quine, *Word and Object*, 197–198.

Wyman postulated the existence of entities as is Pegasus as *unactualized possible*. Pegasus is not present in reality but still can be, if the development of the world was different.[126] Thus, if Pegasus is entitled as non-existent, it is just assumed that Pegasus lacks the attribute of actuality.[127] Quine, of course, did not want to accept Wyman's position and has two comments about it. Firstly, it inappropriately increases the number of entities in the universe. As he asserts:

> Wyman's overpopulated universe is in many ways unlovely. It offends the aesthetic sense of us who have a taste for desert landscapes, but this is not the worst of it. Wyman's slums of possibles are a breeding ground for disorderly elements.[128]

Secondly, Quine points out that there are specific types of terms which could cause a serious problem in Wyman's theory. They are the terms which include a contradiction in themselves as a square-circle, a chimera or a round-square cupola on Berkeley College. Although Wyman is aware of them and regards them as meaningless, Quine stressed that the distinction between meaningful and meaningless is indistinguishable. There is no test as to how to exclude the terms which refer to the unactualized possible from the contradictory terms which have no reference at all.[129]

In addition, Quine did not need to postulate any type of existence of Pegasus in order to claim that Pegasus did not exist meaningfully. When propositions of that type are transcribed into Russell's descriptions, all the inconsistency which arose in the previous analysis disappears. This theory is able to describe each entity in its uniqueness without any condition of postulation of some way of its existence.[130] Pegasus can therefore be described as "the winged horse that was captured by Bellerophon"[131] or if this term was too opaque or unknown it can be transformed into the verb "is-Pegasus", or "pegasizes". We can then consider the

[126] I mean here especially the theory of evolution.
[127] Quine, "On what, there is."
[128] Ibid.
[129] Ibid.
[130] Quine, *Word and Object*, 107.
[131] Quine, "On what, there is."

proposition "Pegasus does not exist" to be meaningful due to the theory of description, even though, we are not forced to admit the existence of Pegasus.[132]

It is linked with the formal form of this proposition. According to Russell, the proposition "Pegasus sleeps" would be *∃x (x is Pegasus and x sleeps)*. Additionally, as the authors in the Logic Museum emphasize, two propositions are created from one[133] and its truth value can be resolved without Pegasus' existence. This proposition is obviously not valid in Quine's interpretation, but if it was, then it implies that there is some entity which is Pegasus and this entity sleeps. It enables Quine to maintain that to be is to be the value of the bound variable (of the true proposition of course). It means precisely that if we assert that some proposition is true, then we express implicitly, that its subject is part of our theory.

Quine called this subject "posit", and it was in his view, an object of ontological commitment.[134] Hence everything which has an ontological commitment can be a value of a bound variable of a true proposition and the theory of quantification is the only criterion which distinguishes among entities, that which belongs to our theory or universe and that which does not. As Quine admitted, the bearer of existential sense was not the name itself or the variable which stood for it but the existential quantifier.[135] Notwithstanding, since asserting the truth of proposition is more plausible than the location of an object in reality, this theory allows the philosopher to enlarge his or her universe enormously, even though, Quine rises against it.[136]

It should be emphasised that Quine's and Russell's ontology differ considerably. Russell's position is based on the view that language is linked with the external world and only those entities are existent, which appear in them.[137] However, as

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[132] Ibid.
[133] The Logic Museum. "Existential Import."
[134] Quine, *Word and Object*, 22.
[135] W. V. O. Quine, "Existence and Quantification," in *Ontological Relativity and Other Essays* (New York: Columbia University Press, 1969), 91–95.
[136] Quine, "On What, There Is."
[137] W. V. O. Quine, "Russell's Ontological Development," *The Journal of Philosophy* 63 (1966): 665–667; Bertrand Russell, *Human Knowledge* (London: George Allen and Unwin, 1948), 89–92 and 98–99.

was presented previously, it is not a case in Quine's ontology. The theory of quantification is consequently the only criterion and it has no immediate link to reality.[138] As Hylon claims, in Quine's interpretation our language as well as ontology is embedded in the theory we accept.[139]

Although other objections have been raised among logicians, who are historians of philosophy or preferred modal logic,[140] this analysis is highly influential in modern logic. It is also based on the ascription of the existential import to the particular and singular propositions. It forms an unprecedented relationship between the theory of quantification and ontology, which is not acceptable for several philosophers, inter alia Prior.[141] Quine's theory consequently functions in Prior's works chiefly as the representation of the opposite side.

3. 1. 2 Leśniewski's Ontology

Prior deeply appreciated Leśniewski's concept of quantification in comparison with Quine's, even though, these three philosophers shared the same approval of desert landscapes. Prior claimed, however, that unlike Quine, there is no ontological commitment in Leśniewski's quantification which suited better his own ontology.

When Prior discussed Leśniewski's quantification, however, he dealt with theory which was mostly mediated to him by Leśniewski's students and colleagues. Among them he was primarily influenced by Czesław Lejewski who was also for a certain period Prior's colleague in Manchester, but who also kept in touch with Bolesław Sobociński and Jan Łukasiewicz. He was also familiar with Słupecki's papers.[142]

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[138] Quine, "Existence and Quantification," 93–97.
[139] Peter Hylon, "Quine on Reference and Ontology," in *The Cambridge Companion to Quine*, ed. R. F. Gibson (Cambridge: Cambridge University Press, 2006), 131–132.
[140] E. g. Joseph Melia, "On What, There Is Not," *Analysis* 55 (1995): 223–229; Czesław Lejewski: "Logic and Existence", *The British Journal for the Philosophy of Science* 18 (1954): 114–119; Fine: *Modality and Tense*, 118; Prior, *Papers on Time and Tense*, 219–221.
[141] Prior, *Papers on Time and Tense*, 220–221.
[142] Zuzana Rybaříková, "The Reception of Stanisław Leśniewski's Ontology in Arthur Prior's

Leśniewski originally described his theory of quantification in his paper *Grundzüge eines neuen System der Grundlagen der Mathematik* and in certain other papers published prior to World War II.[143] Additionally, as Sobociński stressed, only a universal quantifier occurred in Leśniewski's system of logic. He claimed that the use of an existential quantifier was adopted by Leśniewski's students in order to simplify the system.[144] Lejewski informed Prior that the lack of an existential quantifier was caused by Leśniewski's awareness that he had no correct theory of quantifiers.[145]

Consequently, Urbaniak points out that Leśniewski never described his theory of quantification sufficiently. Hence there are several unanswered questions and several concepts of it which differ dramatically from each other. Firstly, Leśniewski did not specify entities which are bound by quantifiers. Secondly, although Leśniewski constructed his quantification to be without an ontological import, he did not postulate how it could be possible[146]. Therefore, authors arose who were either aware that this quantification can be interpreted as substitutional quantification,[147] or asserted that apart from Leśniewski's view, it has an ontological commitment[148].

Urbaniak demonstrates why Leśniewski's quantification has no ontological commitment using the example of one axiom of Ontology. He points out that existence in Leśniewski's system is bared on the operator *ex*. Ontology consequently contains the axiom *[∃a].¬ex(a),* which means some *a* does not

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Logic," *Organon F* 23 (2016): 244–245.

[143] The essential works of S. Leśniewski are published and translated in English in Stanisław Leśniewski, *Collected Works*, ed. by S. J. Surma, et al. 2 vols (Dordrecht: Kluwer Academic Publishers, 1992).

[144] Bolesław Sobiciński, *Letter from 16th September 1953 to A. N. Prior*. Unpublished manuscript stored in the Bodleian Library. Box 3. In *Virtual Lab for Prior Studies*, accessed April 13, 2016, http://research.prior.aau.dk/user.php?show=prior_letters&edit_correspondence_id=1159&from=Sobocinski&to=Prior&both=.

[145] Czesław Lejewski, *Letter from 14th October 1955 to A. N. Prior*. Unpublished manuscript stored in the Bodleian Library. Box 2. In *Virtual Lab for Prior Studies*, accessed April 13, 2016, http://research.prior.aau.dk/user.php?show=prior_letters&edit_correspondence_id=906&from=Lejewski&to=Prior&both=.

[146] Rafal Urbaniak, *Leśniewski's Systems of Logic and Foundations of Mathematics*, Cham: Springer, 2014, 189.

[147] Jacek Pasniczek, "Meinong's Ontology vs. Leśniwski's Ontology: Towards Meinongian Calculus of Names," *Axiomathes* 1–2 (1996): 279–280.

[148] Peter M. Simons, "On Understanding Leśniewski," *History and Philosophy of Logic* 3:2 (1982): 183–184.

exist. For instance, if *a* stands for Pegasus, the formula is a formalization of the sentence "Some Pegasus does not exist".[149]

Form the Priorean point of view, Lejewski's theory, which he presented in his paper *Logic and Existence*, is important.[150] It was a unique attempt since Lejewski tried here to approximate Leśniewski's non-Russellian approach to philosophers who grew up in the Russellian tradition. The difference between Quine's and Leśniewski's quantification is modelled on a thought experiment:

> To have a still simpler though fictitious example let us think of the universe as limited to two objects **a** and **b**. Then the corresponding expansions would be: F**a** ∨ F**b** and F**a** ∧ F**b**. Our language, which for reasons of simplicity needs not synonyms, may leave room for noun-expressions other than the singular names "a" and "b". We may wish to have a noun-expression "c" which would designate neither of the two objects, in other words which would be empty, and also a noun-expression "d" which would designate either.[151]

In contrast with Quine, the existential quantifier has no existential import in Lejewski's interpretation of Leśniewski. This is the reason why Lejewski called this type of quantifier "particular" rather than "existential". The formula *∃x(Fx)* means either *a* or *b* or *c* or *d* have the property *F* while in Quine's interpretation there is something (*a* and *b* in this case), which have property *F*. In the both cases the formula can be true in the universe which Lejewski suggested, if we assume that at least one of the objects of the universe has this property but this is not the case of formula *∀x(Fx)*.

In Quine's analysis it can be true, if *a* and *b* have property *F*. However, due to the emptiness of the noun-expression *c*, it cannot be true in Lejewski's since it means *a* and *b* and *c* and *d* all have the property *F*. Therefore, at least the noun-expression *c* can never fulfill the conditions. In addition, there is a difference between their analyses of the formula *∃x (x does not exist)*. According to Lejewski, this formula is true, because there is the noun-expression *c*, which is

[149] Urbaniak, *Leśniewski's System*, 28 and 189–192.
[150] Lejewski, "Logic and Existence," 104–119.
[151] Ibid., 109.

empty. In contrast, Quine views this formula as false, since it says there exists an *x*, which does not exist.[152]

As Lejewski demonstrated above, the variables which are bound by a quantifier represent noun expressions in his concept which refer to a concrete object or objects (as in the noun-expression *d*). It therefore seems that there are objects which are the values of variables in Lejewski's concept of quantification. However, it is a wider sense of the word "object" than in Quine's theory. Nonetheless, it was precisely this idea which was approved by Prior and lead him to an adoption of the Leśniewskian quantification.

3. 1. 3 Prior's Nominal Quantification

Although Prior's thought underwent substantial development, the discussion on the existential import and hence quantification occurred as early as in his *Craft*, especially in the part which were later published as *The Doctrine of Propositions and Terms*. It met with the special attention of Gaech and Kenny, who published the posthumous edition. As they claim:

> …there is a widely prevalent ignorance of what alternative views are coherently tenable about the existential import of categorical propositions […] One of those things which 'everybody knows' is that if empty terms are admitted, then it is not possible to have a system preserving the square of opposition, the laws of conversion and the traditionally valid syllogism all together: as Prior remarks, they can be easily preserved if we read affirmative categoricals as having, and negatives as not having, existential import for the subject term.[153]

Prior was consequently aware of Aristotle's formulation as he also demonstrated in his text.[154]

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[152] Ibid., 107–113.
[153] P. T. Geach and A. J. P. Kenny, "Introduction," in A. N. Prior, *The Doctrine of Propositions and the Terms* (London: Duckworth, 1976), 10.
[154] Prior, *The Doctrine of Propositions and the Terms*, 122–126.

Even in the first of Prior's books which was devoted to his newly invented systems *Time and Modality*, Prior introduced some of his ideas about quantification which were relevant for his temporal ontology. Firstly, he presented von Wright's identification of Łukasiewicz's modal operators with quantifiers, which Prior also agreed with.[155] Secondly, he discussed the Barcan formula, which according to him implied excessive conditions to ontology, if it is combined with the ordinary quantification theory.[156] Thirdly, he rejected here the Russellian quantification and favoured Leśniewski's.[157]

In light of the fact that Prior approved of Leśniewski's quantification, it is not surprising that Prior's rule concerning how to distinguish between existent and non-existent entities in ontology was taken from Leśniewski's theory. He claims:

> The 'entities' which we 'countenance' in our 'ontology' do *not* depend, as Quine says they do, on what kind of variables we are prepared to bind by quantifiers. They depend on what variables we take seriously as individual variables in a first-order theory, i.e. as subjects of predicates rather than as assertibilia which may be qualified by modalities.[158]

There are consequently not quantifiers which are essential for division of entities from the ontological point of view but various variables. Some of them, as individual variables, must stand for existent entities, other, as propositional variables, do not imply the existence of the entity which they represent.[159] The same answer was provided by Leśniewski, who distinguished between several types of variables in his papers, i.e. A for names and Λ for empty names.[160]

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[155] Prior, *Time and Modality,* 6.
[156] Ibid., 27–36.
[157] Ibid., 62; A. Prior, "English and Ontology," *British Journal for the Philosophy of Science* 21 (1955): 64–65.
[158] Prior, *Papers on Time and Tense*, 220.
[159] Ibid, 115–116, 220. Also Hugly and Sayward stressed this peculiarity of Prior's theory of quantification:

> One dominant feature of Prior's entire approach is his willingness to quantify in respect to an extraordinarily wide range of terms. In this he was very much influenced by the Polish logicians, and a prime instance of this influence is found in his paper "Nonentities" of which we spoke in Chapter 5. In that paper Prior evinces a readiness to introduce letters bound by quantifiers in respect to virtually *any* class of terms. (Hugly and Sayward, *Intensionality and Truth*, 264.)

[160] Czesław Lejewski, "On Leśniewski's Ontology." In *Leśniewski's Systems: Ontology and Mereology*. Edited by J. T. J. Srzednicki and V. F. Rickey (The Hague: Nijhoff, 1984), 129–133.

Prior assumed Lejewski's reading of quantifiers (which was also Aristotle's), that $\forall$ is a universal quantifier but the quantifier $\exists$ is not an existential, but a particular quantifier. He introduced this view in his book *Past, Present and Future*. He additionally emphasised that the usage of a proper variable was essential for his theory, since only some of them implied the existence of the entities to which they referred.[161] In further discussion Prior admitted that individuals who did not exist yet or who had ceased to exist could also be the values of bound variables. This is actually the only possible way of their occurrence in the theory since Prior did not permit their existence in the real world.[162]

Having formulated these denouncing statements Prior seemed to have been Quine's keen opponent, but this is not his last solution. In the paper *Recent Advantages in Temporal Logic* and in his posthumously published books *World Time and Selves* and *Object of Thought*, Prior differentiated among individuals bound by a quantifier, which referred only to some of the existent individuals and among these who are free and can also stand for non-existent entities. He therefore confirmed Quine's condition that whatever is a value of a bound variable in the objectual quantification is supposed to be existent.

He still employed, however, different variables for each type of entity. Furthermore, the difference between bound and free variables is only essential for the universal and the particular (existential) quantifier. Additional functors, which Prior viewed as quantifiers as the modal functors $\Box$ and $\Diamond$ or the temporal *F* and *P*, could also bind variables which stand for actually non-existent individuals.[163] Prior seemed to at least appreciate Quine's objectual quantification, but only in the case of the objectual quantification. Prior's sentential quantification, which is more important for some of his systems of logic, by no means suits Quine's approach to quantification.

[161] Prior, *Past, Present and Future*, 161–163.
[162] Prior, *Papers on Time and Tense*, 220–221.
[163] Prior, *World, Times and Selves*, 52–53.

3. 2 Non-nominal Quantification

Although Prior's approach to the nominal quantification was in a certain way unique, his theory of quantification is particularly renowned due to his non-nominal quantification. It is the part of his theory of quantification in which he opposed Quine most significantly. It specifically led Prior to express that Quine's theory of quantification: "...is just a piece of unsupported dogma"[164] as was expressed in the introductory quotation.

There are two main reasons why Prior appreciated the non-nominal quantification. Firstly, it enabled him to formulate his world-propositions, as quantification over propositions. Secondly, it is an essential part of his hybrid logic as the quantification over possible worlds. Furthermore, Prior also used modal and temporal operators as quantifiers which often bind non-nominal variables in his systems of logic.

In comparison with Quine, non-nominal quantification was not something unusual for Prior. Moreover, he stressed in several places in his works[165] that there are precursors for this approach among such significant logicians as Peirce, Russell, Ramsey and Leśniewski. As he built his theory primarily on Ramsey's and Leśniewski's theory of quantification, their ideas will be introduced in the first two parts of this chapter. Prior's own theory will then be presented.

3. 2. 1. Frank Plumpton Ramsay's *The Foundations of Mathematics*

Ramsay who was originally a mathematician was influenced a great deal by Wittgenstein, whose *Tractatus* he translated into English. His work *The*

[164] Prior, *Object of Thought*, 48.
[165] Prior, *Formal Logic*, 91–103; Prior, *Time and Modality*, 127; Prior, *Object of Thought*, 33–47.

Foundations of Mathematics, which was inspirational for Prior, is composed as a Wittgensteinian critique of Russell's *Principia Mathematica*.[166]

Tucker points out that Ramsey found three defects in *Principia Mathematica* with which Ramsay tried to cope with in his exhaustive paper. From the Priorean point of view, the most important is Ramsey's second objection. While the first and third defects are linked with classes and identity, the second is Ramsey's denial of Russell's Axiom of Reducibility. This axiom contains the condition that a first-order function can be found which is extensionally equivalent for every higher order function. It was specifically employed in Russell's and Whitehead's ramified theory of types which requires a restriction of types of variables. According to Ramsay, the power of logic is reduced in this fashion. Ramsay therefore attempted to demonstrate that the Axiom of Reducibility is superfluous. He also intended to develop a non-ramified theory on which mathematics could be based.[167]

Ramsey's entire analysis was based on a slightly different understanding of propositions. He symbolized an elementary proposition as ϕa, where a stands for an individual and ϕ is the name of the quality.[168] In other words, a can be identified with a subject and ϕ with a predicate of traditional subject-predicate sentences.[169]

Ramsay introduced propositional functions which bind together each atomic proposition. Moreover, he did not consider a nominal quantification to be a nominal function but he claimed that it only bound together several propositions which maintained a similar fact. Specifically, $\exists x$ *(x is a man)* means that certain propositions of the form "this x is a man" are true (and some can be false), while by saying $\forall x$ *(x is a man)* it is said that every proposition of the form "x is a man" is true.[170] Respectively, it is maintained that the disjunction of

[166] F. P. Ramsey, *The Foundation of Mathematics and Other Logical Essays*, ed. R .B. Braithwaite, fourth edition (London: Routledge, 1965), 11–12.
[167] Denis Tucker, "Intensionality and Paradoxes in Ramsey's The Foundation of Mathematics," *Review of Symbolic Logic* 3 (2010): 1–2.
[168] Ramsey, *The Foundation of Mathematics*, 5.
[169] Ibid., 6.
[170] Ibid., 8–9.

propositions of the form "x is a man" is true in the first case and the conjunction of the same propositions is true in the second case.

Additionally, Ramsey did not limit the variables which are bound by quantifiers to individual variables, but he also bound the predicate variable. The formula $\forall\phi.F!(\phi!\hat{z})$ means that all values of $F!(\phi!\hat{z})$ are true and $\exists\phi.\ F!(\phi!\hat{z})$ means that at least one value of $F!(\phi!\hat{z})$ is true. Ramsey further added that none of these propositions are elementary. This form of quantification was introduced in order to cope with contradictions which arose in *Principia*. Russell solved these contradictions with the postulation of the Axiom of Reducibility, but as was mentioned earlier, Ramsey wanted to dismiss this rule.[171]

Finally, he also involved propositional functions.[172] At first, it is only the truth functions but when he discussed "the Liar paradox" he also introduced the propositional quantification. All these functions are derived from individual functions and as propositions which are their values appeared here in the form ϕx. Ramsey maintained, however, that only individuals referred to existent entities.[173]

Although the formula ϕx was essential for Ramsey's concept, this analysis of propositions contained an inaccuracy. Tucker points out that Ramsey's ϕx, corresponds in most cases with the present formulation $\lambda x(\phi(x))$ although this analysis did not appear in Ramsey's work.[174] Ramsey was, however, convinced that his analysis was correct and that it enabled him to avoid the Axiom of Reducibility. As was mentioned previously the criticism of this axiom, was one of the main motivations for writing his *The Foundation of Mathematics*.[175]

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[171] Ramsey, *The Foundation of Mathematics*, 26–29.
[172] Ibid., 37.
[173] Ibid., 38–40, 48–49.
[174] This objection might have been anachronistic, since the operator was introduced after publication of Ramsey's book. Alama points out, however, that the first hints of it even appear in Frege's work (Jesse Alama, "The Lambda Calculus," in *The Stanford Encyclopedia of Philosophy*. Spring 2015 edition, ed. Edward N. Zalta, accessed July 15, 2015, http://plato.stanford.edu/archives/spr2015/entries/lambda-calculus/.). In addition, Tucker is correct in his assertion that Ramsey was not precise in the definition of the meaning of his newly introduced concept. (Tucker, "Intensionality and Paradoxes," 3–5).
[175] Ramsey, *The Foundation of Mathematics*, 49.

3. 2. 2 Leśniewski – Prototheticˇ

The critique of Russell's *Principia Mathematica* is the common thread of Ramsey and Leśniewski. Leśniewski also tried to establish a new foundation for mathematics, when he developed his own system of logic. It consists of three parts: Mereology, Ontology[176] and Prototheticˇ. Mereology deals with the parts and whole and is based on two systems of logic, Ontology which handles the semantic category of names and Prototheticˇ which handles the semantic category of propositions. Słupecki pointed out that the division into semantic categories, which is distinctive for Prototheticˇ, prevented Leśniewski's system from falling into antinomies.[177]

From the point of view of propositional quantification, Prototheticˇ is most significant. It is Leśniewski's equivalent for the logic of propositions. Stachniak emphasizes that when Leśniewski formulated it he placed an emphasis that it should be: "…universally valid, consistent, decidable, complete, expressively powerful, logically economic, pure, and elegant, in other words perfect."[178] Although this plan was primarily Leśniewski's, it was Tarski who provided the formal system which was able to fulfil these conditions. Additionally, Wajsberg and Sobociński contributed to the final form of Prototheticˇ.[179]

Prototheticˇ is built on a system of semantic categories and on the rules for their introduction into the system.[180] According to Urbaniak, propositional variables and constants belong to the same semantic category in Leśniewski's system, the category of sentences. There is also the category of names which name variables and name constants belong to. Ontology primarily handles this category as was introduced in the first paragraph. Finally, functors do not belong in any of these

[176] Although the term "ontology" has a long tradition in philosophy, Leśniewski entitled his system of logic similarly. He was also aware of the fact that his system of logic is not entirely similar to this philosophical discipline. He also claimed that the term "ὄντος", means "being" in Greek and Aristotle described ontology as a general theory of object, which corresponded with his understanding of his system of logic, which dealt with the semantical category of names. (S. Leśniewski, "On the Foundation of Mathematics," in S. Leśniewski, *Collected Works I.*, trans. D. I. Barnett, ed. S. J. Surma et. al (Dordrecht: Kluwer Academic Publishers, 1992), 373–374.)
[177] Słupecki, "St. Leśniewski's Prototheticˇ," 45.
[178] Zbigniew Stachniak, "Editor's Forword," in *Leśniewski's System of Prototheticˇ*, ed. Jan T. J. Srzednicki and Zbigniew Stachniak (Dordrecht: Kluwer Academic Publisher, 1998), ix.
[179] Ibid., x–xi.
[180] Sobociński, *Letter from 16th November 1953.*

categories.[181] The semantic category of a functor is dependent on the semantic category of the variables which it is bound to.

In addition, the quantifiers belong to the same semantic category as the variables which they bind.[182] These semantic categories are primitive because they appear in the axioms of Prototheticc. A new semantic category can be introduced in accordance with definitions, which are, however, limited by certain rules, i.e.:

> In the protothetical definitions (in prototheticc, ontology, a.s.o.) the first sign of definiendum must be a defined constant. [183]

Definitions are the only way in which a new category, which does not appear in the axioms, could be introduced. [184] As Rickey points out, Prototheticc is strong enough to contain the usual laws of quantifiers and bivalency for each semantical category[185]

The rule which particularly appealed to Prior was that in Prototheticc, quantifying is allowed over every type which was generated in accordance with given rules. Prior was critical, however, of Leśniewski's "creative definitions" which are necessary for the entire theory.[186]

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[181] Urbaniak, *Leśniewski's Systems*, 60; J. Słupecki, "St. Leśniewski's Prototheticc," *Studia Logica* 1 (1953): 46–47.
[182] Słupecki, "St. Leśniewski's Prototheticc," 47.
[183] B. Sobociński, *Letter from 6th November 1953.* Unpublished manuscript stored in the Bodleian Library. Box 3. In *Virtual Lab for Prior Studies*, accessed April 13, 2016, http://research.prior.aau.dk/user.php?show=prior_letters&edit_correspondence_id=1158&from=Sobocinski&to=Prior&both=.
[184] Sobociński, *Letter from 16th September 1953 to A. N. Prior*.
[185] V. Frederick Rickey, "A Survey of Leśniewski's Logic," in *Leśniewski's System of Prototheticc*, ed. Jan T. J. Srzednicki and Zbigniew Stachniak (Dordrecht: Kluwer Academic Publisher, 1998), 25.
[186] Arthur N. Prior, "Definitions, Rules and Axioms," in A. N. Prior, *Papers in Logic and Ethics*, ed. P. T. Geach and A. J. P. Kenny (London: Duckworth, 1976), 47–48; Zuzana Rybaříková, "Prior's definition of creative definitions: Sobociński-Prior-Lejewski's discussion on the Leśniewskian definitions," *Organon F* (forthcoming).

3. 2. 3 Prior's Non-nominal Quantification

As was mentioned earlier, Prior was aware of the fact, that a non-nominal quantification was not atypical at the beginning of modern logic, even though it was not used by every logician. According to Prior, even Russell and Whitehead admitted its appearance in their *Principia Mathematica*.[187] In addition, logicians such as Peirce or Leśniewski dealt with it in an even more natural way.

If the temporal operators *F*, *P*, *G* and *H* are considered quantifiers, then Prior was an advocate of the non-temporal quantification in even his *Diodoran Modalities*.[188] It is difficult to determine exactly, however, where he began to argue for non-nominal quantifications, while he had been even prior to the publication of *Diodoran Modalities* a proponent of Leśniewski's Prototethic.[189]

From the point of view of traditional quantifiers, he clearly adhered to the non-nominal quantification when he introduced world-propositions in his and Meredith's *Modal Logic with Functorial Variables and Contingent Constant*, which was discussed in the previous part of my dissertation. The formal definition included here a proposition which is bound by a quantifier:

$$Wp \overset{def}{\equiv} (p \wedge \forall q\,(q \rightarrow \Box(p \rightarrow q))$$ [190]

Additionally, Prior's description in accordance with Meredith's and Wittgenstein's ideas distinctly demonstrates a non-nominal quantification.[191] The usefulness of a non-nominal quantification was further demonstrated by Prior when he hybridized his logic. It was not propositions which were values of bound variables in Prior's hybrid logic but possible worlds.[192] He initially introduced, however, an idea concerning quantification over times even in his *Time and Modality*.[193]

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[187] Prior, *Object of Thought*, 39.
[188] Prior, "Diodoran Modalities," 205–213.
[189] Prior, *Formal Logic*, 91–103.
[190] Meredith and Prior, "Modal Logic,"101.
[191] Ibid., 102.
[192] Prior, *Papers on Time and Tense*, 117–138.
[193] Prior, *Time and Modality*, 112.

Prior did not intend to postulate the existence of these entities. This was the reason why he could not accept Quine's ontological commitment of values of bound variables. In addition, as was mentioned in the first chapter of this part, Prior used different variables in order to distinguish between existent and non-existent entities. Therefore, he did not have to assume Quine's condition, even though he considered it to be justifiable in the nominal quantification.

The first part of Prior's book *Object of Thought* is devoted to the denial of the propositions existence in reality. It is therefore not surprising that his major attack on Quine's theory of quantification is formulated here. While he constructed his argument, Prior used weapons of both mentioned precursors Ramsey and Leśniewski. He maintained that if a propositional-variable is bound by a quantifier, it stands for the proposition which this variable represented. However, since propositions, according to Prior, are not genuine objects, it did not refer to the object "proposition" or to some sole object.[194] This feature and the object of reference in this case will be discussed in more detail in the next section.

In conclusion, non-nominal quantification of traditional quantifiers played a significant role in Prior's ontology. Prior based it on Leśniewski's Prototethic and Ramsey's ideas and claimed that it had no ontological commitment. He primarily expressed these ideas in his posthumously published books *Object of Thought* and *World, Times and Selves*, although it also appeared in *Past, Present and Future* and *Papers on Time and Tense*. In addition, this type of quantification is essential for Prior's concept of possible worlds, his hybridisation of logic and his epistemic logic.

3. 3 Other Types of Quantifiers

Apart from two traditional quantifiers, Prior also regarded certain other operators as quantifiers. These quantifiers will be presented in this chapter.

[194] Prior, *Object of Thought*, 33–40.

First of all, modal quantifiers will be dealt with, followed by temporal quantifiers. Prior finally invented quantifiers which contain a mixture of objectual, modal and temporal functions, with this being the focus of the last part of this chapter.

Although, there are also, according to Prior, quantifiers, intensional logicians who deal with them usually do not require the actual existence of entities which are bound by them.[195] Prior also claimed that they have no ontological commitment. From the ontological point of view, they are therefore not as controversial as traditional quantifiers. Nevertheless, the chapter concerning Prior's quantification cannot be complete without at least a brief introduction to them.

3. 3. 1 Modal Quantifiers

Prior even identified the modal operators $\Box$ and $\Diamond$ with quantifiers in his book *Time and Modality*.[196] His reasons were primarily practical, however. The similarity between modal logic and theory of quantification enabled him to use some rules of theory of quantification in order to obtain theorems in Ł-system of logic. Namely, when Prior introduced his system of tense logic, he defined these axioms: [197]

1. $Fn\neg p \rightarrow \neg(Fnp)$
2. $\neg(Fnp) \rightarrow Fn\neg p$
3. $Fn(p \rightarrow q) \rightarrow (Fnp \rightarrow Fnq)$
4. $Fop \rightarrow p$
5. $FmFnp \rightarrow FSmnp$
6. $(Fm\exists nFnp) \rightarrow (\exists nFmFnp)$

Due to the quantification theory, Prior obtained the theorems:

[195] Prior, *World, Times and Selves*, 52–53.
[196] Prior, *Time and Modality,* 6.
[197] The operator *S* means "It is the case that", the variables *n* and *m* stand for the number of days and similarly the symbol *o*. It means 0 days i.e. now. (Prior, *Past, Present and Future,* 10–12.)

7. $\Diamond p \rightarrow \neg\Box\neg p$	($\exists n(Fnp) \rightarrow (\neg\forall n\neg(Fnp))$ by quantification theory; and $(\neg\forall n\neg Fnp) \rightarrow (\neg\forall n(Fn\neg p))$ by transposition from $(\forall n(Fn\neg p)) \rightarrow (\forall n(\neg Fnp)$, this coming from axiom I by $\forall$I and $\forall$2)
8. $\neg\Box\neg p \rightarrow \Diamond p$	(similarly from axiom 2).
9. $\Box p \rightarrow p$	($\forall n(Fnp) \rightarrow (Fop)$ by quantification theory, and $\forall n(Fnp) \rightarrow p$ from this by axiom 4 and syllogism).
10. $(\Box p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)$	(by axiom 3 and quantification theory).
11. $\Box p \rightarrow \Box\Box p$	($(\exists m\exists n(FmFnp) \rightarrow (\exists nFnp)$ from 5 by quantification theory; $(\exists m(Fm\exists n(Fnp))) \rightarrow (\exists n(Fnp))$ from this by 6 and quantification theory; $(\exists n(Fn\exists n(Fnp))) \rightarrow (\exists n(Fnp))$, or $\Diamond\Diamond p \rightarrow \Diamond p$, from this by quantification theory; and $\Box p \rightarrow \Box\Box p$ from this by the usual transpositions, etc.).[198]

He was fairly skeptical to make a link between quantification theory and the modality which is contained in the Barcan formula:

$$\Diamond\exists x\,(Fx) \rightarrow \exists x(\Diamond Fx)$$

He demonstrated in *Time and Modality* that this formula is based on ontological conditions which are unacceptable for him. It in particular contradicts his presentism, since it presupposes the existence of all individuals who have existed, exist now and will exist in the future.[199] This problem will be more profoundly discussed in the last section of my dissertation.

This connection was a key motif throughout Prior's further work. Even one of his last papers *The Parallel between Modal Logic and Quantification Theory*, which appeared in his posthumously published book *Worlds, Times and Selves*,[200] contained this comparison. This entire book also dealt more or less with the parallel between modality and the theory of quantification.

[198] Prior, *Time and Modality*, 13–14.
[199] Ibid., 26–40.
[200] Prior, *World, Times and Selves*, 9–27.

3. 3. 2 Temporal Quantifiers

The invention of temporal quantifiers might have been one of the most important of Prior's contributions to modern logic. It provided his followers with an occasion to call him "the founding father of modern tense logic."[201] The first temporal operator introduced by Prior was *F*. It has the meaning: "It will be the case that…"[202] and appeared first in Prior's *Diodoran Modalities*. The same paper also contained additional temporal operators. The operator *G* is introduced first which means "It always will be the case that…" or "It will not be the case that not p", which is defined as:

$$Gp \stackrel{def}{=} \neg F \neg p$$ [203]

Secondly, Prior handled the operator *P* which is understand as "It was the case that…" and finally *H*, which stands for "It was always the case that…" or "It has not been the case that not p", also defined as:

$$Hp \stackrel{def}{=} \neg P \neg p$$ [204]

These operators were presented as part of the logic of propositions although Prior later also used them in predicate logic.[205]

3. 3. 3 The Quantifiers W, Q and T

The operators which Prior also identified with quantifiers are linked with world-propositions and instant-propositions. *W* was the first of them presented. *Wp* means, as was presented in the previous part of this dissertation, "p

[201] Per Hasle, *The Founder of Temporal Logic*, Prior Studies, accessed March 31. 2015, http://research.prior.aau.dk/anp/?anp=About_Prior.
[202] Prior, "Diodoran Modalities," 205.
[203] Ibid., 206.
[204] Ibid., 211.
[205] E.g. Prior, *Time and Modality*, 85.

comprehends all truths".[206] It was described more profoundly in a previous part of my dissertation as it is closely linked with Prior's concept of possible worlds.

When Prior discussed world-propositions and instant-propositions in *Past, Present and Future*, he added the quantifier *Q* which simulates world-propositions even more precisely. *Qp* stands for "*p* is totality of the truth at some time," or from the point of view of world-propositions "*p* is a possible world."[207] This operator is defined as:

$$Qp \stackrel{def}{=} \Diamond p \wedge \forall q\, [\Box(p \rightarrow q) \vee \Box(p \rightarrow \neg q)]$$[208]

Prior was not the only one who considered this operator a quantifier. Blackburn also identified it with a quantifier when he declared a parallel between Prior's hybridisation of his logical systems and modern hybrid logic.[209]

The last Priorean quantifier T is closely connected with Q and can be defined from it.[210] Namely:

$$Tpq \stackrel{def}{=} (Qp \wedge \Box (p \rightarrow q)$$[211]

In contrast with previously mentioned operators, the operator *T* is an operator which binds two propositions. *Tpq* means "In the state of affairs in which *p*, it is the case that *q*."[212] The axioms of the system where the operator *T* appears are as follows:

1, $Tpq \rightarrow \neg \Box \neg p$ [i.e. $Tpq \rightarrow \Diamond p$]

2, $Tpq \rightarrow (Tpr \vee Tp\neg r)$

3, $Tpq \rightarrow \Box Tpq$

4, $Tpq \rightarrow (p \rightarrow q)$

[206] Meredith and Prior. "Modal Logic," 101.
[207] Prior. *Past, Present and Future*, 80.
[208] Prior, *Papers on Time and Tense*, 237.
[209] Blackburn, "Arthur Prior and Hybrid Logic," 354.
[210] Prior, *Papers on Time and Tense*, 249–252.
[211] Ibid., 252.
[212] Ibid., 250.

5, □(p ≡ q) → (Tpp → Tqp)[213]

In addition, Prior used this quantifier in his hybridisation, where *Tap* means "It is the case in world *a* that *p*" and is therefore, an essential part of Prior's hybridisation of logic.

In conclusion, Prior dealt with several types of quantifiers. In general, he did not agree with Quine's concept of ontological commitment and criticised it in his papers. Although he admitted its usefulness in the case of nominal quantification at a later point, his concept of propositional quantification lacked it. In addition, modal and temporal operators, which Prior also viewed as quantifiers, did not require the existence of the entity their variables stand for. If Prior permitted Quine's ontological commitment, then he would be committed to acknowledging the existence of propositions. As was emphasized elsewhere in my dissertation it would have violated his nominalism. Prior, however, had a certain concept of propositions, as will be shown in the following section.

[213] Ibid., 250.

4, Propositions

> We do not fear, hope, desire or think sentences – we must stick fast to that. And fearing, hoping, desiring and thinking do not *consist in* relations between people and sentences…[214]

The affirmation of proposition's existence has a long history in philosophy. According to McGrath, it dates back to the stoics' discussion on "lecta".[215] Hints of it also appeared in the celebrated period of medieval philosophy where *complexe significabilia* were postulated by George of Rimini and Adam Wodeham, as it was discussed by Spade. *Complexe significabilia* were the entities outside humans' minds which served as the object of significance for an entire sentence. This significance was different than the significance of each name which built the sentence. Since these objects were independent and constant, they were adopted into their theory of truth.[216]

The most influential theory of propositions was suggested, however, by Gottlob Frege. He assumed that sentential objects of reference are situated outside the human mind. This place was called by him "a third realm" and made Frege the founding father of platonism in analytic philosophy.[217] Frege's motivation was similar to Rimini's and Wodeham's: he needed propositions as stable objects to which every sentence could refer.

There are valid reasons for accepting the existence of propositions. Frege, as was previously mentioned, first postulated their existence in order to create a firm basis for logic. Their existence was important for him since they provide a

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[214] Prior, *Object of Thought*, 16.
[215] Matthew McGrath, "Propositions," in *The Stanford Encyclopedia of Philosophy.* Spring 2014 Edition, ed. Edward N. Zalta, accessed April 15, 2015 http://plato.stanford.edu/archives/spr2014/entries/propositions/.
[216] P. V. Spade, *Thoughts, Words and Things: An Introduction to Late Medieval Logic and Semantic Theory,* accessed October 7, 2010, http://pvspade.com/Logic/docs/Thoughts,%20Words%20and%20Things1_2.pdf, 168–172.
[217] Gottlob Frege, "The Thought: A Logical Inquiry," *Mind* 65 (1956): 301–302.

constant reference for every single sentence, which implied the steadiness of the system of logic.[218] Secondly, Frege dealt in his paper *On Sense and Reference* with expressions of propositional attitude, in which the existence of propositions is also important.[219] Specifically, Frege claimed that there is a problem with sense and reference and that the reference might not be a truth-value as in ordinary sentences.

These ideas were not appreciated by Prior. As a nominalist, he could not approve of the theory, since it increased the number of entities in the universe. He therefore rejected Frege's concept of propositions.[220] Prior's theory of propositions did not consist, however, of only a rejection of propositions' being actual objects. It also contained a keen adoption of those ideas which could support his view.

It was above all Quine's thoughts as he denied the existence of propositions in his book *Word and Object*. Quine's approach was affected by his approval of desert landscapes in ontology, which made him reject a considerable amount of abstract entities, inter alia propositions.[221] The second logician who was important for Prior's concept of propositions was Frank P. Ramsey. His theory was independent of Quine's and differs substantially as it is based on Wittgenstein's *Tractatus*. Wittgenstein, who was Ramsey's friend, inspired him to understand propositions in a different way than Quine.[222]

It should be emphasized that Prior used the term proposition in the description of two different contexts. On the one hand, he identified propositions as logical constructions which are bearers of truth-values. This interpretation appears primarily in his book *Object of Thought* and will be presented in this chapter. On the other hand, Prior used this term in a medieval understanding of it. This was namely when he claimed that the truth-value of a proposition could change.[223]

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[218] Frege, "The Thought," 289–311.
[219] Gottlob Frege, "On Sense and Reference," in *Translations from the Philosophical Writings of Gottlob Frege*, eds. and trans. P. Geach and M. Black (Oxford: Blackwell, 1980), 65–69; Frege, "Über Sinn und Bedeutung," 34–40.
[220] Prior, *Object of Thought*, 15–16.
[221] Quine, *Word and Object*, 208–209.
[222] Ramsey, *The Foundation of Mathematics*, 140–143.
[223] Prior, *Papers on Time and Tense*, 213.

As Uckelman points out, this is a feature of medieval "*propositio"* the meaning of which is"… not an abstract entity, always existing and eternally true or eternally false, but rather a specific mental, spoken, or written token declarative sentence."[224] These two concepts seem to be mutually exclusive since the former serves as the bearer of truth-values, while truth-values of the latter could change. However, since Prior considered propositions to be merely logical constructions, the difference is not as sharp as it appears to be. This part of my dissertation is only focused on the former meaning of the term, since the latter is not significant from the ontological point of view.

This part of the dissertation is divided into five chapters. Frege's theory of propositions and its consequences will be presented in the first chapter. This theory establishes a foundation for the acceptance of the propositions' existence in analytic philosophy. Frege's theory of propositional attitudes' reference is consequently discussed. Quine's denial of the existence of propositions is the topic of the third chapter. Since Prior was also influenced by Frank P. Ramsey, his theory of propositions is presented in the fourth chapter. Finally, the last chapter deals with Prior's theory of propositions. It is a denial of the existence of propositions but also an approval of Quine's and Ramsey's ideas. In addition, since Prior's approach to the existence of propositions developed through his life, this chapter also contains a brief history of this development.

4. 1 Gottlob Frege I. – Propositions as Genuine Objects

Matthew McGrath points out that Gottlob Frege was not the only one who postulated propositions, although he was without a doubt the most influential proponent of this theory.[225] The Bohemian philosopher Bernard Bolzano also dealt with propositions. They are called "Sätze an sich" in his work and are, according to him, neither a mental nor a linguistic phenomenon.[226] This attempt illustrates Bolzano's diversion from psychological tradition. Frege's theory of

[224] Uckelman, "Prior on an Insolubilium of Jean Buridan," 487–488.
[225] McGrath, "Propositions."
[226] Bernard Bolzano, *Wissenschaftslehre*, §19.

proposition was, however, considerably more influential from the historical point of view. Frege's work was more renowned than Bolzano's and Frege's followers were also more numerous and celebrated than Bolzano's. When Prior therefore opposed the widespread theory of propositions he did not argue against Bolzano but against Frege, even though he was aware of Bolzano and his theory.[227]

Gottlob Frege in his preciseness differentiated between the sense (*Sinn*) and the reference (*Bedeutung*)[228] not only in the case of words but also in the case of sentences. The sentential meaning, which is entitled "a proposition", is, according to him, substantial for the truth-value of each proposition. As was mentioned earlier, these propositions are independent from the human mind. In addition, they are not reliant on the empirical world. As Frege pointed out, the predicate "to be true" differs from the predicates "to be bitter", "to be red" or "to smell like lilac" which describe something understandable by our experience.

In contrast to this, the predicate "to be true" cannot be so easily found in the empirical world. Frege demonstrated this using the example of the sun. If the sunrise is observed, the sentence "the sun rose" is true, even though, the truthfulness of this sentence is nothing empirical. No empirical thing which represents this can be found in that situation. The truthfulness of the sentence is not determined due to senses but is deduced.[229]

Furthermore, Frege claimed that even properties which are distinguished in the empirical world are preceded by their abstract ancestors, which guarantee their truthfulness. It should be found at first that the fact that "an apple is red" is true, then it is possible to assert "an apple is red".[230] This analysis makes propositions an essential part of Frege's theory. Propositions are linked with truth and

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[227] Prior discussed it in his *The Craft* (quoted from *The Doctrine of Propositions and Terms*, 19–23) and in *Object of Thought*, 6–7.
[228] The term "Bedeutung" is also translated as "a meaning" for instance in the alternative translation of this article (Gottlob Frege, "On Sense and Meaning," in *Collected Papers on Mathematics, Logic, and Philosophy*, ed. B. McGuinness, trans. M. Black (Oxford: Basil Blackwell, 1984), 157–177). I chose the translation "a reference", since the term "Sinn" is also sometimes entitled as meaning (e.g. Quine, "On What There Is.")
[229] Frege, "The Thought," 292–293.
[230] Frege, "The Thought," 295 and 307–308.

therefore crucial for sciences. Frege seemed in that point as if to be continuing in an ancient philosophical tradition of prioritizing philosophy as the queen of all sciences. It is no longer, however, a philosophy which foregoes every science but logic.

Frege argued carefully in his paper *The Thought: A Logical Inquiry* for the view that propositions belonged to neither the empirical world nor the psychological world. There is therefore some other kind of reality in which propositions are situated. He calls this world the third realm. It stands somewhere between the empirical and the psychological world. It is specifically independent from human minds as the empirical world and does not rely on the senses as the psychological world.[231]

There is another feature of Frege's theory of propositions which distinguished his theory of propositions from Prior's theory. Frege claimed that each sentence contained within itself the place, the time and other circumstances of its utterance. This is called "indexical" in modern analytic philosophy.[232] This means that each sentence is unique and refers to unique propositions. The sentence "today is a beautiful day", asserted by me on 11 April 2015, is different from the same utterance communicated by someone else or even by the utterance which I claimed at some other time. Prior, in contrast, held the view that this sentence is identical in each situation in which it is uttered no matter by whom, when or where it is uttered.[233]

Although, Frege's postulation of propositions as abstract objects could be found debatable, his claim that the reference of each sentence is its truth-value is even more controversial.[234] This can be demonstrated with a quotation from Prior's *What Do General Statements Refer to?* as to which type of reaction it gave birth to among logicians:

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[231] Ibid.,298–302 and 309.
[232] David Braun, "Indexicals," in *The Stanford Encyclopedia of Philosophy*, Spring 2015 Edition, ed. Edward N. Zalta, accessed July 9, 2015 http://plato.stanford.edu/archives/spr2015/entries/indexicals/.
[233] Prior, *Papers on Time and Tense*, 257.
[234] Frege, "On Sense and Reference," 63–64; Frege, "Über Sinn und Bedeutung," 33–34; Kevin C. Klement, *Frege and the Logic of Sense and Reference* (London: Routledge, 2002), 62.

> Frege, as is well known, believed that statements, or at all events sentences, refer to or denote (bedeuten) one or the other of a pair of objects called the True and the False. I have never thought that this particular technicality was a very happy one, and I'm glad to drop it.[235]

Frege found an eminent follower of his theory in Bertrand Russell. Beaney claims that Russell remade Frege's propositions in his book *The Principles of Mathematics*. They are viewed, however, as concrete objects by Russell and therefore did not quite correspond to Fregean propositions. Furthermore, Russell began to be doubtful about the propositions in his later work. He postulated another theory in which a designation to propositions was replaced by a complicated way of designation. It was Russell, however, who made Frege's theory well-known and widespread, even though, he did not agree with it later.[236]

4. 2 Gottlob Frege II. – Expressions of the Propositional Attitude

Frege also opened the question of expressions of propositional attitude in his paper *On Sense and Reference*. He demonstrated that it is another field of logic in which the existence of propositions could be crucial. Furthermore, if Frege's theory of proposition as objects of the third realm proved useful, it was in the context of propositional attitudes. Frege asserted that words with a similar reference could be replaced by one other without a change in the truth-value in his paper *On Sense and Reference*.[237] However, this rule could not be applied in the case which is called by Frege "indirect quotation". He therefore examined carefully indirect quotations in order to explain its rules.[238]

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[235] Arthur Prior, "What Do General Statements Refer to?", 178.
[236] At least for the English-speaking world. Beaney maintains that Frege was well-known among German-speaking mathematicians and philosophers. (Michael Beaney, "Russell and Frege," in *The Cambridge Companion to Bertrand Russell*, ed. N. Griffin (Cambridge: Cambridge University Press, 2003), 129–131.
[237] Frege, "On Sense and Reference," 62–64; Frege, "Über Sinn und Bedeutung," 32–35.
[238] Frege, "On Sense and Reference," 65–66; Frege, "Über Sinn und Bedeutung," 35.

These sentences consist of two clauses, the first part identifies who believes in (or knows) something, followed by a proposition describing the object of belief (or knowledge). Frege called the former a main clause and the latter a subordinate clause. The query which he had to solve lied in the substitution of the subordinate clause. He observed that its replacement by a clause with an identical meaning could cause a change in the entire sentence's truth-value. For instance, the sentence "Darwin knew that evolution is based on natural selection" is true but the sentence "Darwin knew that evolution runs at the level of genes" is not, even though, the subordinate clauses have a similar meaning, i.e. a similar truth-value.

Frege had to therefore explain how the substitution runs in the indirect quotation. Frege suggested that the valid replacement in these cases is not based on the customary meaning of subordinate clauses. It could be established with the indirect meaning of subordinate clauses, which Frege identified with the customary senses of them. The sense of each subordinate clause is a proposition.[239]

Vickers has pointed out that this step, which is once again directed against psychologism, differentiates from logical tradition and its understanding of syllogism. On the one hand, the truth is described as extensional, can be based on rules and is independent from the empirical world. On the other hand, expressions of a propositional attitude are not extensional, but are still independent from the human mind or mental acts. The reference is the case of language and the world.[240]

Frege's concept of indirect quotation raised several objections against logicians and philosophers. One of the most influential was Quine's, which will be introduced in the following chapter. These objections were primarily motivated by the attitudes of both extensionalists and physicists. They were not held by Frege whose motivation was different. The philosophers who shared Frege's

[239] Frege, "On Sense and Reference," 66–70; Frege, "Über Sinn und Bedeutung," 35–40; Hugly and Sayward, *Intesionality and Truth*, 67–69.
[240] John M. Vickers, "Ramsey on Judgment: The Theory of "Facts and Propositions"," *Dialectica* 58 (2004): 501–502.

enthusiasm for logic and mathematics and were not afraid of platonism therefore appreciated his theory.

4. 3 Quine's Approach

In contrast to previously mentioned logicians such as Frege or Prior, Quine rejected intensionality and tried to avoid it.[241] He claimed:

> Intensions are creatures of darkness, and I shall rejoice with the reader when they are exorcised...[242]

His efforts for ontological minimalism were not motivated by minimalism itself. At least in the case of propositions and expressions of propositional attitudes, Quine claimed that his main motivation was to prevent any opacity. He argued in particular against inaccuracy of the quantification, which played an important role in all of Quine's ontology. In order to be the rule of existence it required clarity in its reference, which was difficult to preserve in the belief context. As he wrote in *Word and Object:*

> There was little no banning of locutions without benefit of passable paraphrase. The nearest we came to that was perhaps the banning of quantification into opaque constructions, but even there no clear loss was sustained, no loss that would be felt as such from any plausible point of view; useful cases of apparent quantification into opaque contexts were generally salvaged by paraphrase. There was no banning of abstract objects on scruples of nominalism; no banning of intensional objects on scruples of extensionalism; nor any banning of indicator words on scruples of absolutism.[243]

Quine pointed out that belief can be formulated transparently or opaquely and that both formulations make sense.[244] A representative of the opaque construction could be the sentence "Frida believes that someone drank

[241] Quine, *Word and Object*, 168–169.
[242] W. V. O. Quine, "Quantifiers and Propositional Attitudes," *The Journal of Philosophy* 53 (1956): 179.
[243] Quine, *Word and Object*, 191.
[244] Ibid., 147.

hemlock" while the sentence "Frida believes that he drank hemlock." is constructed in a clearer way. In the first case the subject of the subordinate clause can be referential but not necessarily. This is not the case, however, in the second clause. It has to refer to someone, presumably to Socrates. Quine emphasized that all opaque sentences which stated propositional attitudes should be expressed transparently and hence should be reformulated to do so.[245]

In addition, Quine's reformulation of sentences cannot be a universal cure for all ambiguous formulations. As Crawford points out, Quine differentiated between two senses of the expressions of propositional attitudes. They are called rational and notional and cannot be replaced by one other. The rational sense describes the relationship between the person who holds a certain propositional attitude and the object of the propositional attitude. This relationship is not present in the notional sense of the propositional attitude.[246]

The first sense is expressed in the sentence "Ernst hunts lions", where lions are a specific group of animals where Ernst is located. The second sense represents the sentence "I need a sloop", since the sloop is not a specific sloop or a group of sloops in the harbour where I am. It instead expresses my need. Therefore, if there is not a certain bed I am looking for a group of sloops which can be taken into account, the sense of a propositional attitude is only notional. In addition, the rational sense indicates that there exists a certain object which somebody has a relationship with but it is not maintained in the notional sense.[247] Since there is no guarantee that in the notional sense the individual which is the object of the sentence exists, Quine banned the quantification in these contexts.

The specification of each sense lies, however, in the circumstances of the utterance and is not contained in the sentence itself. The utterance "I want a sloop" can have the rational sense if I am in a harbour and seeking for a sloop between those which are present there. In contrast, it can also have the notional

[245] Ibid., 156.
[246] Sean Crawford, "Quantifiers and Propositional Attitudes: Quine Revised," *Synthese* 160 (2008): 76–78.
[247] Quine, "Quantifiers and Propositional Attitudes," 177–178.

sense if I am only expressing my need for a sloop without a certain group of sloops. Concerning these two senses and the circumstances of utterance Quine claimed:

> … I suggest that the question how far we can rephrase a belief, and not lose the right to impute it, depends on our purpose in imputing it. Correspondingly for propositional attitudes other than belief.[248]

Quine added that even the rational sense in which there is an object of reference contained opacity. He discussed the case of Ralph who believed that somebody is a spy. But Ralph met the same man on the beach, where he recognised him as a pillar of society. Since Ralph was not aware of the fact that the one who was in his view a spy and the man he met on beach were one and the same person, he could not believe that the man he met on the beach was a spy. Although Ralph could be accused of inconsistency, the example Quine used is neither unnatural nor impossible.[249]

In summary, Quine endeavoured to find a clear formulation for expression of propositional attitudes but was forced to argue at the end of the discussion that it is impossible to avoid opacity in these contexts. They are used and could be useful in everyday communication, but for the sake of their ambiguity they should be excluded from every scientific theory.[250]

Although Quine claimed that ontology was not the reason why he discussed expressions of propositional attitude, he also mentioned it. He was aware of the fact that propositions were defined as the vehicles for truth-values and the objects of reference in these contexts.[251] They can be introduced to his ontology as posits.[252] Quine had no objections to useful abstract entities. He replaced them, however, in his *Word and Object* with eternal sentences.

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[248] W. V. O. Quine, "Propositional Objects," *Crítica: Revista Hispanoamericana de Filosofía* 2 (1968): 9.
[249] Quine, "Quantifiers and Propositional Attitudes," 179.
[250] Quine, "Propositional Objects," 9; André Leclerc, "Quine on Logic, Propositional Attitudes, and the Unity of Knowledge," *Principia* 7(2003): 138.
[251] Quine, *Word and Object*, 192
[252] Posits are objects of an ontic commitment in Quine's theory. Everything which is a posit can be a value of a bound variable of a true statement. The only criterion is the theory of

Concerning truth-values, Quine maintained that the declarative sentence is not true or false. Sentences such as "It is raining" can be true at a certain moment in some specified place but false at another place or moment. The sentence can be evaluated, however, by only one truth-value when the data which specifies these circumstances of utterances are added. It creates an eternal sentence from an unspecified declarative sentence. The propositions can be preserved as meanings or eternal sentences but since Quine denied the existence of meanings, they have no place in Quine's theory. Additionally, they contained the problem of individuation according to Quine.[253]

In conclusion, Quine dealt with propositions and expressions of propositional attitude for the sake of clarity. He suggested a paraphrasing in certain cases in order to make propositions less ambiguous. However, he was aware of the fact that they could not always be paraphrased. He was also convinced that they could not be clear to such an extent in order to be used in scientific theories. He therefore argued for their exclusion from science. From the ontological point of view, Quine prioritized eternal sentences from propositions. Propositions are, according to Quine, neither necessary nor unproblematic.

The failure of Quine's efforts would seem to invalidate any other attempt to handle the belief context. Atlas claims, in contrast, that Quine appears to not be successful in his attempt to convincingly maintain that these sentences are referentially opaque.[254] It provides logicians who incline more to these contexts with an occasion to deal with them and demonstrate that the opposite is true. In addition, since Prior did not connect quantification with ontology in these contexts, hints of ambiguity which could occur here might have not bothered him.

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quantification. It determines which entity is a posit (i.e. belongs to one's theory) and which is not. (Quine, *Word and Object*, 22).

[253] Quine, "Propositional Objects," 3–5; Quine, *Word and Object*, 208–209.

[254] Jay David Atlas, "Aboutness, Fiction, and Quantifying into Intentional Contexts: A Linguistic Analysis of Prior, Quine, and Searle on Propositional Attitudes, Martinich on Fictional Reference, Taglicht on the Active/Passive Mood Distinction in English, etc.," accessed June 13, 2015, http://pages.pomona.edu/~jda14747/Atlas_Aboutness_QuantifyingIn_March1991dup2dup-pdf.pdf, 11.

4. 4 Ramsey's Approach

The previous part of my dissertation, where I discuss propositional quantification, makes mention that Ramsey suggested a theory of proposition which differed from Fregean's and Russell's. He analysed propositions in predicate logic, where the proposition was formalized as ϕa. The a referred to an individual and ϕ to the property. Vickers claimed that this formalisation approximates Ramsey to a traditional subject-predicate interpretation of the sentence.[255] It is also close, however, to Russell's interpretation in predicate logic. Ramsey placed an emphasis on the fact that these two categories cannot be commuted, especially, that each entity could belong to just one of them. He claimed that the confusion of these two categories, which appeared in Russell's theory, resulted in a misinterpretation of propositions.[256] Russell specifically made a kind of predicate from both subject and predicate.

Although Ramsey dealt also with propositional variables p, q, r, etc., he preferred a predicate analysis of propositions. This enabled him to formulate his theory of propositional variables, which was discussed in a previous section of this work. Namely, when universal and particular sentences were analysed as $x(\phi x)$ or $\exists x(\phi x)$, it was not difficult for Ramsey to claim that the quantifier did not only bind the variable but the entire sentence. Ramsey understood the universal sentence as a conjunction of all the sentences where ϕ is ascribed to the individual. The existential quantifier maintained, according to his interpretation, that there is at least one sentence which claims "a is ϕ", "b is ϕ" , "c is ϕ"... i.e. it is a disjunction of all sentences where ϕ is asserted about an individual. It also demonstrated its usefulness in the analysis of propositional attitudes.[257]

From the propositional point of view, this analysis is crucial. The sentences are transcribed here as basically constructed from an individual and a certain predicate. It corresponded with Ramsey's view of propositions. He did not deny

[255] Vickers, "Ramsey on Judgment," 512.
[256] Ramsey, *The Foundation of Mathematics*, 5–6, 22–27 and 24.
[257] Ramsey, *The Foundation of Mathematics*, 8–9.

them, but maintained that they were not genuine objects.[258] The propositions played the role of the bearer of truth-values in Ramsey's theory. Ramsey admitted that he was inspired by Wittgenstein in this step. They do not exist as ideal objects,[259] but they are logical constructions similarly as the sentences assigned to them built from individuals and their predicates.[260]

This suggestion influenced Prior and his theory of proposition. He also considered propositions as only logical constructions.[261] The emphasis which both authors placed on the logical constructions is not only important from an ontological point of view. They can deny propositions as genuine objects due to it. It also allowed them, however, to use the reformulation of sentences in cases in which sentences seemed to have demanding ontological commitments, in order to deny these consequences.[262]

The second important feature which affected Prior was Ramsey's concept of facts. Firstly, Ramsey claimed that facts were logical constructions as were propositions. This means that facts are also not genuine objects. Vickers claims that the world consists of individuals and their properties and relations according to Ramsey. Facts are therefore only our mental constructions of it.[263] Secondly, he asserted that when somebody believed (wanted, knew, etc.) in something, it was not a relation with this individual and proposition as it was in Frege's concept. Propositions served in his theory only as a bearer of the truth values. According to Ramsey, when somebody believes in something, it is a relationship between this individual and a fact, which is merely a logical construction.[264]

[258] Ibid., 12.
[259] Ibid., 70–75.
[260] Ibid., 120.
[261] Prior, *Object of Thought*, 9–13.
[262] Ramsey, *The Foundation of Mathematics*, 141–143; Prior, *Object of Thought*, 9–13.
[263] Vickers, "Ramsey on Judgment,"512–513.
[264] Ramsey, *The Foundation of Mathematics*, 140.

4. 5 Prior's Approach

4. 5. 1 The Development of Prior's Ideas

Prior began with the rejection of propositions as far back as in his early book *The Craft*. Prior demonstrated a deep understanding of the history of logic in this book. He offered here a historical introduction which was mostly based on Aristotle's logic but also mentioned Bolzano and Frege. He followed here Ryle's criticism of the proposition's existence and provided several reasons as to why it could not be accepted.[265] Although he criticised propositions, his objections were against the inconsistency which this theory included. He did not mention their ontological redundancy.

The ontological aspects of this problem were actually discussed in his book *Object of Thought*, even though, Prior also dealt with them in some of his previously published papers e.g. *Oration Obliqua*.[266] Prior differentiated between two ways of understanding the title phrase of *Object of Thought*. Firstly, it could mean what we think, i.e. it includes our thoughts, beliefs and knowledge. This takes place on the level of propositions as was shown in the chapter devoted to expressions of propositional attitudes. It could even be propositions in the Fregean sense, even though, Prior's favoured solution was different. Secondly, it could have the meaning of what we think about. Namely, it is the object (or the objects) which is represented by the subject of the proposition. Prior claimed that the second meaning is more common and that both concepts included problems. The propositions might actually have been false and the entity which is referred to by the subject non-existent.[267]

[265] Prior, *Propositions and Terms*, 19–23.
[266] A. N. Prior, "Oratio Obliqua," in A. Prior, *Papers in Logic and Ethics*, ed. P. T. Geach and A. J. P. Kenny (London: Duckworth, 1976), 147–158.
[267] Prior also added a third meaning, "what we think about it", but he did not work with this characteristic in the further part of his book, Prior, *Object of Thought*, 3–4.

4. 5. 2 Prior's Relationship to Frege

Beginning with propositions, it should be emphasized that there is a certain common ground between Prior and Frege. Prior agreed with Frege that the extensions of sentences are their truth-values. He considered both extensions, however, to be only logical constructions, and therefore, not the objects. Prior also had a preference for intensional logic as Frege did, since it is able to express the sentences in their entirety. Hugly and Sayward claimed that Prior's and Frege's theory also have a common thread in the adoption of a non-extensional context and the truth-value evaluation of the expressions of a propositional attitude.[268]

Although Prior approved these features of Frege's theory, this did not mean that the propositions were unproblematic for Prior. It was completely otherwise. As was mentioned earlier, Prior tried to deny them as abstract entities. In contrast to his effort, the propositions, according to Frege's theories, seem to always be abstract.

Another problem which arises here is that false propositions such as "the snow is black" are similar objects as the true one is "the snow is white". From the ontological point of view, they seem to be equally real according to Frege, which is unacceptable for Prior.[269] He explained his relationship to Frege's theory as follows:

> Criticizing Frege is a thing one does 'more in sorrow than in anger', or in anger just because it is in sorrow; for there has perhaps been no greater philosophical logician, certainly none who has better appreciated the importance of carving up sentences in right places if we are to see clearly what they are conveying–the technique of Chapter 2, though not its application, is wholly his.[270]

[268] Hugly and Sayward, *Intensionality and Truth*, 94.
[269] Prior, *Object of Thought*, 4–6.
[270] Ibid.,52.

4. 5. 3 Prior's Concept of Propositions

As was mentioned previously, Prior rejected Frege's view that propositions are real objects. Prior argued that to say that "Anna fears that Ebola disease will spread to Europe" is true, meaning exactly that Anna fears so. However, the sentence "Anna fears that Ebola disease will spread to Europe and it is true" signifies a situation in which Anna fears that certain facts will happen.

Similarly, the sentence "Anna fears that Ebola disease will spread to Europe and it is false" describes a situation in which Anna's fears are unjustified. The truth-values of the sentences are dependent on the facts and reality not on the propositions. The statement "Anna fears that Ebola disease will spread to Europe" is true in the case when Anna fears so, regardless of the reality. In contrast, the statement "Anna fears that Ebola disease will spread to Europe and it is true" is only true if what Anna fears will be the case.[271]

It is not possible to refer to Ebola or to the situation in Europe, since it is not the case. It has to be referred to as "Ebola disease will spread to Europe", but it is not an abstract object according to Prior. He claimed that it is a proposition but following Ramsey he asserted that it is only a logical construction.[272] There is no doubt that the truth-value's evaluation of the sentence is impossible without it. However, it is not a genuine object and is dependent on the human mind. The last remark was not acceptable for Frege and neither Prior or Ramsey had any objections to it.

Formalizing this view, Prior used Leśniewski's quantification, which does not require an ontological commitment, as was presented in the previous part of my dissertation. The lack of ontological commitment and the fact that the propositions are logical constructions meant that Prior preferred the logic of the propositions in a temporal and modal context. There is no need to postulate intensional objects in the logic of propositions, according to Prior.[273] It is not the

[271] Ibid.,15–16.
[272] Ramsey, *The Foundation of Mathematics*, 141–143; Prior, *Object of Thought*, 9–13.
[273] Prior, *Papers on Time and Tense*, 93–94.

case, however, in the predicate temporal or modal logic, as will be discussed in the following part of the dissertation.

4. 5. 4 Expressions of Propositional Attitudes

Prior was aware of the fact that sentences which expressed propositional attitudes could also have required the actual existence of propositions as objects.[274] In order to deny their real existence and reference to them in these cases, Prior claimed that human thoughts are not sentences. By doing so, Prior pointed out that there is a difference between thinking in sentences and think sentences.[275] He approved the former but denied the latter.

He specifically argued that human beings did not fear, hope, or desire sentences but in sentences, which meant that objects of human passions are generally not sentences but the objects which the sentences handled. There is no doubt that the sentence itself could also be the object of passions e.g. if it is written by someone we love, or occurs as graffiti on some historical building. It is usually the content of the sentence, however, which makes someone fear, desire, love or hate.

If an individual fears the sentence "Ebola disease will spread to Europe," they are not the letters of this sentence which are frightening and not even the words themselves. An individual who does not understand English might not find this sentence fearful. It is the content of this sentence, the possibility of such a horrible disease as Ebola, which can affect the European population, which can be an object of fear for someone. The content could be the object of fear even in the case when it will not be formulated in this sentence.[276]

Prior only asserted that the names referred to objects. He denied the designation of sentences or verbs. Sentences, according to him, stated human thoughts but names have reference to "what we think about". The sentence

[274] Prior, "Intentionality and Intensionality,"189; Prior, *Object of Thought*, 14.
[275] Prior, *Object of Thought*, 14.
[276] Ibid., 14–17.

"Ebola disease will spread to Europe" is about the disease and its appearance in Europe. However, the sentence "Anna fears that Ebola disease will spread to Europe" seems to be more complicated, since it did not consist of names bound by verbs but names bounded with sentences. The question arises as to how to designate the second part of the sentence. Prior to suggesting an analysis in which he was inspired by Ramsey:

> So we eliminate the apparent name 'that there will be a nuclear war' and the suggestion it carries that the complete sentence expresses the relation between X and the 'proposition' designated by this name, simply by ceasing to parse the whole as 'X fears / that there will be a nuclear war', and parsing it instead as 'X fears that / there will be a nuclear war'.[277]

These two possible analyses are discussed here by Prior as the second one, as "the apparent name" implies the ontological commitments which Prior was not prepared to accept. Namely, names, according to Prior, referred to objects and Prior did not intend to postulate such an object. This analysis requires the genuine existence of the proposition "Ebola disease will spread to Europe" as the reference. It harms two of the previously mentioned Prior principles: that human beings do not think sentences, but in sentences and that there are no propositions as an abstract object of their reference.

Prior's suggested solution did not contain these requirements. They disappear, if the sentence "Anna fears that Ebola disease will spread to Europe", is analysed as "Anna fears that / Ebola disease will spread to Europe".[278] Prior, inspired by Ramsey, claimed that due to elimination of superfluous parts of a sentence, it can be clearer that the reference is not an eternal object but a logical construction. Ramsey originally dealt with the sentence "The proposition that *p* is false", which can be reduced to a sentence with the similar meaning "Non-p", e.g. "The proposition that snow is violet is false", is equivalent to "The snow is not violet".

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[277] Ibid., 19.

[278] Hugly and Sayward point out that it was also Frege's analysis of this type of sentence, even though, his conclusion differed from Prior's. (Hugly and Sayward, *Intensionality and Truth*, 78–79.)

Similarly, the sentence "Anna fears that Ebola disease will spread to Europe" can be replaced with "Ebola disease is feared by Anna to spread to Europe". An analysis of this sentence is slightly more complicated but is not opposed to Prior's conditions. In addition, it is not Prior's invention. Prior acknowledged Quine as the founder of this method of analysis. Although Prior disagreed with Quine in a number of cases here he admitted: "… it is one of the two points in the philosophy of logic on which Quine seems to me dead right."[279]

Prior also discussed types of sentences which could not be so easily analysed. Firstly, there are sentences in which somebody has a sum of beliefs, fears, etc. e.g. "Everything Barbara believes is true" Secondly, there are sentences the references to which are somehow opaque such as the sentences "Some things Carla believes I do not believe" or "Daniel and I believe the same thing". Prior used Ramsey's theory here again as he claimed that these types of propositions had to be reformulated into language more distant from ordinary English, where propositional quantification is brought in. The first sentence can be rewritten as "For any *p*, if Barbara believes that *p*, then it is the case that *p*", the second "For some *p*, Carla believes *p*, and I do not believe that *p*", and the third "For some *p*, Daniel believes *p*, and I also believe that *p*".

Another query could also be found, however. The identity of the propositions is questionable in the cases of the expressions of propositional attitudes. Since the sentence "Estelle knows that all unmarried men are unmarried" is analytically true, it is not the case of the sentence "Estelle knows that all bachelors are unmarried", even though, both sentences describe a similar situation. Prior was aware of this problem, but maintained that it is more of a sophistic inquiry within a propositional identity. The sentence "Estelle knows that all bachelors are unmarried" is not the same as the sentence "Estelle knows that all unmarried men are unmarried". At least different words are used, "bachelors" in the former sentence and "unmarried men" in the latter.[280]

[279] Prior, *Object of Thought*, 19–20.
[280] Ibid., 52–56.

As was mentioned in the previous part of the dissertation, Prior advocated a propositional quantification which had no ontological commitment. He had to prove that this step did not introduce platonism, which was rejected by him. As was stated previously, however, he appeared to succeed in this attempt. There are certain difficulties with the form of the record of the propositional quantification as Prior pointed out that "For some *p*, *p*" is not a correct formulation in English. He asserted, however, that correct English formulation can be transformed to it, using adverbs such as "somewhere", "somehow", or "wherever". The formula "For any *p*, if *p* the *p*" can be transformed into "If anywhether than thether."[281] He was inspired in this point by Leśniewski's Protothetic.

Prior also found in English parts where propositional variables are used explicitly such as in an excerpt from *Enquire Within upon Everything*:

> *Here, there* and *where*, originally denoting place, may now, by common consent, be used to denote other meanings; such as, '*There* I agree with you', '*Where* we differ', 'We find pain *where* we expected pleasure', '*Here* you mistake me'.[282]

The examples which appeared in the quotation above contained, according to Prior, propositional variables in the Wittgensteinian sense.[283] It also occurred in the solution proposed by Ramsey. He claimed that the quantified formula "Something is red" meant "This is red, or that is red, or that other is red, etc.," and similarly with a propositional quantification. "For some *p*, *p*" can be analysed as "Either snow is white or grass is green, or sky is black, or apples are blue, etc."[284]

In conclusion, although Prior denied that propositions are objects he admitted that they had some place in his theory. They are logical constructions according to him and as such are bearers of truth and bearers of reference in the case of

[281] Ibid., 37.
[282] Ibid., 39.
[283] Ibid., 38–39.
[284] Prior, *Object of Thought*, 42; Ramsey, *The Foundation of Mathematics*, 8–9.

propositional attitudes. It helped Prior handle with the intensional context without a postulation of intensional objects. This approach is a sort of "psychologization" and would not have likely been approved by Frege. Prior nevertheless found a precursor for this view in Quine and Ramsey.

The clash regarding propositions involves more than just a question concerning their nature. It reflects the struggle between extensionality and nominalism on the one hand and intensionality and platonism on the other. It is a case of all of the concepts of science and the scientific approach for each philosopher. Hence, the view of science of each philosopher has affected his theory of propositions. Prior in this case inclined more to the extensional and nominalistic side of Quine and Ramsey, the theories of which influenced him a great deal. He did not want, however, to abandon intensional logic, even though, he did not involve the existence of intensional entities. He therefore also approved of Frege's inventions to some extent. Once again, in the case of propositions, Prior acted as a proponent of nominalism and intensionality.

In contrast, propositions are an essential part of Prior's concept of possible worlds and time instances as was maintained previously. Van Cleve points out that the reduction of propositions to logical constructions is not compatible with this view. He claims: "You cannot identify worlds with propositions unless you have propositions to identify them with."[285] The question is consequently whether Prior's nominalism did not reach its limits in the problem of propositions.

[285] van Cleve, "Objectivity without Objects."

5, Names and Individuals

> To sum up, this is still the untidiest and the most obscure part of tense-logic, though even here the alternatives that are open to us are beginning to emerge some clarity.[286]

Arthur Prior was aware, that his approach to ontology and his intensional systems of logic could be problematic as he claimed in the opening quotation. This was the reason why he preferred propositional logic for his tense and modal logic.[287] He could not completely exclude individuals and predicate logic. This part of my dissertation focuses on this query.

One of the major sources of Prior's, from the point of individuals and tense logic, seems to be *Time and Modality*. Although Prior discussed individuals and names in his previous publications such as *The Craft* or *Formal Logic*,[288] *Time and Modality* introduced the concise theory of tense predicate logic for the first time. It is significant that there are chapters in which Prior handled Russellian predicate logic and at the same time systems of logic which instead adopted Leśniewski's characteristics. Prior later abandoned both of these theories and developed his system of logic. Despite having certain features of both previously mentioned theories, it considerably differs from them, even though, Prior never presented it concisely.

If Prior had a concise theory of names, he might have intended to introduce it in more detail in his book *Object of Thought*, in which a considerable amount of his ontological thoughts are presented. His premature death, however, affected this publication a great deal. Prior planned to deal with names in the two last chapters, which had not been completed when he died. Geach and Kenny completed these chapters with the help of Prior's published and unpublished

[286] Prior, *Past, Present and Future*, 172–173.
[287] Prior, *Papers on Time and* Tense, 93.
[288] E.g. Prior, *The Doctrine of Propositions and Terms*, 85; Prior, *Formal Logic*, 158–159.

papers, although Prior's own writing might have been different.[289] The chapters were not written, however, and Prior's view had to be therefore reconstructed in accordance with the sources which were published.

Prior claimed that there were two theories of names, which are considerably elaborated, Russell's theory and Leśniewski's theory.[290] He discussed both theories in his papers and books and both theories had a certain impact on his own view. Both theories and Prior's approach to them will consequently be introduced in the first two chapters of this part of the dissertation.

The third chapter is focused on Prior's theory of names and its various aspects. It is divided into sections which correspond with Prior's thought development, since, as Prior claimed in the opening quotation, his concept of names was developing gradually. The last three subchapters, however, deal with specific features of Pror's theory, identifiable individuals, theory of reference and identity.

Russell's concept of names was prevalent and the most discussed concept of names when Prior formulated his theory. Prior also handled, however, Leśniewskian names, which he knew from correspondence with Leśniewski's students and colleagues and from their papers, considering carefully the advantages and disadvantages of each theory. Both theories had a considerable impact on Prior's own theory, even though, Prior could not agree entirely with either of them. Russell and Leśniewski tended toward extensional logic, while Prior was a wholehearted intensional logician.

5. 1 Russell's Concepts of Names

Russell had two types of opponents, traditional logicians and Alexuis Meinong, when he introduced his theory. Firstly, modern logicians generally denied that existence is a predicate. This was, however, not the case in traditional logic, in

[289] Geach and Kenny, "Editorial note," viii–ix.
[290] Prior, *Object of Thought*, 167.

which logicians claimed that the assertion "Leo Sachse is a German" did not imply "Leo Sachse exists".[291] Consequently, empty terms could be the subjects of a true statement in traditional logic. This feature was not approved of by the founding fathers of modern logic such as Frege or Russell. Frege maintained his view in the discussion with Pünjer and demonstrated to his opponent that the approval of empty terms in theory could lead to a contradiction. Russell consequently formulated a precise theory as to how to handle empty terms in a predicate logic. He asserted that they were not names but descriptions.

Secondly, Russell denied Meinong's theory of object.[292] Meinong claimed that the exclusion of non-existent entities from ontology could lead to the deprivation of his theory of object. He famously maintained that we should not have prejudices in favour of existence. The non-existent entities are according to him objects as well as the existent ones, and therefore, some way of being should be ascribed to them as well. If the statements as "The golden mountain is a mountain", "The round square is the round square" or "I met a unicorn" would have a truth-value, then there has to be something such as the golden mountain or a unicorn.[293]

Russell did not agree either with the claim that existence is a predicate or with Meinong's conclusions. He denied both of them by formalisations of statements in Frege's predicate logic.[294] Namely, he pointed out that the subject predicate structure in previously mentioned statements is only illusive. A number of statements do not have names such as subjects or predicates but their subjects and predicates are descriptions. He further differentiated between definite and indefinite description.

The indefinite description is true, when variables refer at least to one individual. It is called indefinite since the number of individuals is not settled and neither individual which it refers to is identified. According to Russell, the predicate is

[291] G. Frege, "Dialog s Pünjerem," *Studia Neoaristotelica* 5 (2008): 59.
[292] Bertrand Russell, *Introduction to Mathematical Philosophy* (London: George Allen and Unwin, 1920), 169–172.
[293] Alexius Meinong, "On the Theory of Objects," in *Realism and the Background of Phenomenology*, ed. Roderick Chisholm (Glencoe, Ill.: Free Press, 1960), 76–81.
[294] Russell, *Introduction to Mathematical Philosophy*, 167–178.

an indefinite description in statements such as "I meet a unicorn". Russell formalized this statement as:

$$\exists x\,[M(x) \wedge U(x)]$$[295]

This can be transcribed as: "There exists at least one individual, which I met and which is a unicorn". The predicate *M* stands for "to be met by me (the speaker of the statement)" and the predicate *U* stands for "to be a unicorn". In the present state of the world there are no unicorns, hence, there is no individual which the variable can refer to and the statement cannot be true. The advantage of Russell's system is that the truth-value of the statement could be stated without postulation of some being of unicorns.

Secondly, the definite description is true if the variable stands for precisely one individual. If there are more or no individuals to which the variable refers, the statement is not true. Examples of definite descriptions are "the golden mountain", "the round square" or Pegasus. Consequently, the statement "The golden mountain is a mountain", is formalized as:

$$\exists x\,\{G(x) \wedge \forall y[G(y) \supset (y = x)] \wedge M(x)\}$$[296]

i.e. "There is a golden mountain and everything which is a golden mountain is that individual and it is a mountain." The predicate *G* stands for "to be a golden mountain" and the predicate *M* stands for "to be a mountain". The conjunct in the middle guarantees that there is just one golden mountain.[297] Since there is no golden mountain in the present state of the world, the statement is false.

Apart from descriptions, Russell admitted that there are also proper names such as "Scott" or "Socrates" in his paper *On Denoting* and also in the book

[295] The formal structures of formulas were taken from Scott Soames, *Philosophy of Language* (Princeton and Oxford: Princeton University Press, 2010). Russell himself used notation, which did not correspond entirely with the present form of predicate logic.
[296] The formula was rewritten in accordance with Peter Ludlow, "Descriptions," in *The Stanford Encyclopedia of Philosophy.* Fall 2013 Edition, ed. Edward N. Zalta, accessed January 14, 2016, http://plato.stanford.edu/archives/fall2013/entries/descriptions/.
[297] The golden mountain can also be analysed as $G(x) \wedge M(x)$, where *G* means "to be golden" and *M* has the same meaning as in the previous analysis. Nonetheless, the result will be the same, since there is no individual that is golden and at the same time a mountain. For the sake of simplicity, the former solution was chosen.

Introduction to Mathematical Philosophy. He claimed that descriptions describe entities, while names name them.[298] He also formalised proper names. They are, according to Russell, constants. Therefore, the statement "Socrates is a man", is formalized as:

$$M(a)$$

where the predicate *M* stands for "to be a man" and the constant *a* for Socrates. In his epistemological paper *Knowledge by Acquaintance and Knowledge by Description* Russell pointed out that nearly every noun is a description. Namely, we, as a user of language, are acquainted with a limited group of name-bearers. "Bismark" or "Scott" are therefore for us who never met them, merely sums of characteristics, i.e. descriptions. Russell then maintained that there are only two genuine proper names "I" and "this".[299]

It was also mentioned that Russell denied that existence is predicate. He reduces existence to a quantifier. Thus the statement "The Pegasus does not exist" could not be analysed as:

$$\exists x\,(P(x) \wedge \neg\, E(x))$$

where *P* stands for "to be Pegasus" and *E* for "to exist". Quine, whose theory is built on Russell's ideas, provides the following formalization:

$$\exists x\,(a = x)$$[300]

and where the existence is linked not with the predicate but with the quantification.

In order to specify the reference, the semantic of Russell's theory has to be introduced. The semantic of predicate logic is based on set theory. The predicates refer to sets, while constants refer to individuals. Quantifiers and operators specify the scope of reference. The theory was originally founded on

[298] Russell, "On Denoting," 480–491; Russell, *Introduction to Mathematical Philosophy*, 179.
[299] Bertrand Russell, "Knowledge by Acquaintance and Knowledge by Description," *Proceedings of the Aristotelian Society* 11 (1910 - 1911): 121.
[300] Quine, "Existence and Quantification", 94.

naïve set theory, but as Irvine stresses, it had to be restricted later, when Russell's paradox was discovered.[301] Russell solved this problem by postulating a hierarchy among sets, which is currently known as the theory of types. Certain other queries were lessened by Russell's introduction of the axiom of reducibility, even though this axiom was criticised by several authors e.g. Frank P. Ramsey,[302] as was mentioned in the previous parts of the dissertation.

Although Russell's concept of names and descriptions was highly influential, there are several objections to it. Firstly, as Soames stresses, the logical structure of Russell's analysis did not correspond with the grammatical structure of the statements.[303] Where there is a grammatically just categorical statement (i.e. subject predicate statement), Russell created a more sophisticated hypothetical statement (i.e. a statement composed of more categorical statements).

Secondly, Ludlow points out that Russell's theory is a great deal dependent on the difference between definite and indefinite articles, although a considerable amount of languages (Latin, some Slavic and Asian languages) do not require any article with nouns.[304] Finally, the link between existence and quantification which is a distinctive feature of Russell's theory was not approved by several logicians as was mentioned previously.

5.2 Leśniewski's Ontology and His Understanding of Names

Several features of Leśniewski's theory were introduced previously. In this chapter his theory of names, which appears primarily in the system called Ontology, is added. It also includes the theory of quantification. Since this theory was discussed in the fourth chapter of my dissertation, I will not discuss it here.

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[301] A. D. Irvine, "Bertrand Russell," In *The Stanford Encyclopedia of Philosophy*. Winter 2015 edition. Edited by E. N. Zalta, Accessed December 12, 2015, http://plato.stanford.edu/archives/win2015/entries/russell/.
[302] Ramsey, *The Foundation of Mathematics*, 11–12.
[303] Soames, *Philosophy of Language*, 25.
[304] Ludlow, "Descriptions."

Names are the basic semantical category for Leśniewski's Ontology. Leśniewski's concept of names was based on his Polish intuitions. Since Polish lacks articles before nouns, constants which stand for names in true statements do not have to refer exclusively to one individual as in Russell's case. The terms "człowiek", "Socrates" and "jednorożec" are all viewed as names in Leśniewski's Ontology, even though, "człowiek" refers to more than one individual and "jednorożec" refers to none.[305]

Moreover, it implies that there is no difference between the subject and predicate in Leśniewski's system of logic. They are formalized by the same constants and both belong to the same semantical category, the category of names. As Zuber points out the statements "Jacek jest żołnierziem" and "Żołnierziem jest Jacek" are both grammatically correct in Polish, even though their meaning is slightly different.[306] Since the subject and predicate belong to the same semantic category, the formalizations of the statements "Socrates est homo" and "Mark Twain est Samuel Langhorne Clemens" are similar. Namely:

[ab]: a ε b[307]

and

[cd]: c ε d

Apart from constants, which stand for names, Ontology consists of operators. Among them the operator *ε*, which occurs in the formulas above, is the most important. This operator is also based on Leśniewski's Polish intuitions. He described it as Polish "jest". [308] He further pointed out that in order to fit the system "jest" had to have a timeless meaning and did not have to include the

[305] Simons, "Stanisław Leśniewski."
[306] R. Zuber, "Polish Logic, Language and Philosophy," in: *Lvov Warsaw School and Contemporary Philosophy*, ed. K. Kijania-Placek and J. Woleński (Dordrecht: Kluwer Academic Publisher, 1998), 230–233.
[307] The meaning of this formula is for every *a* and *b*, *a* is *b* (in the meaning of the verb "be", which will be explained). Leśniewski sometimes differentiated between names, which have precisely one reference such as Socrates, by using the capital letter for it. As Urbaniak pointed out, this was not a settled rule. (Urbaniak, *Leśniewski's Systems*, 90).
[308] It is the verb "is", but as will be explained further, the meaning is due to a lack of articles slightly different in Polish. In contrast, it has a similar meaning such as Latin "est".

meaning "is existent". Nevertheless, if a constant stands for a non-existent individual as in the statements ‘‘Hamlet is the Prince of the Danes" or "Barack Obama is a vampire", the statements are false.[309] However, Słupecki has maintained that the interpretation of Leśniewski's operator is difficult especially when it is interpreted in languages which differentiate between definite and indefinite articles.[310]

Due to a lack of articles certain English statements do not have make sense in Polish without further interpretation (i.e. "Żołnierz jest odważny" [A soldier is brave]).[311] There are also statements which are meaningful in Polish but which still do not suit Leśniewski's Ontology. General statements such as "The whale is a mammal" have to be reformulated according to Leśniewski as "Whatever is a whale is a mammal", in order to be formalized with the operator ε.[312]

The difference between Russell and Leśniewski's theory which is caused by the use respectively lack of definite and indefinite articles was demonstrated by Słupecki with the example of three statements:

Socrates is a man.	Socrates est homo.
[Every] dog is an animal.	[Omnis] canis est animal.
Socrates is the husband of Xantippe.	Socrates est coniunx Xantippae.[313]

Słupecki has pointed out that in the right column the verb "is" has three different meanings. It was caused by the fact that English uses definite and indefinite articles. The meanings of the verb can be grasped by the formalization of it. Woleński has maintained that, the verb "is" in the statement "Socrates is a man" could be formalized by $\in$, the symbol of the set's membership. The meaning of the verb in the second statement is $\subset$, which is the set theory's

[309] Leśniewski, "Foundation of Mathematic," 376–382.
[310] J. Słupecki "S. Leśniewski's Calculus of Names," in *Leśniewski's Systems: Ontology and Mereology*, ed. J. T. J. Srzednicki and V. F. Rickey (The Hague: Nijhoff, 1984), 65.
[311] Zuber, "Polish Logic," 230–233.
[312] Leśniewski, "Foundation of Mathematic," 376–382.
[313] The words "every" and "omnis" were added to Słupecki's original text, in order to make the discussion less complicated. Originally there were "The dog is an animal" and "Canis est animal". As was mentioned previously, these sentences have to be rewritten in order to enable their formalization in Leśniewski's Ontology. (Słupecki, "Calculus of Names," 65.)

symbol for the subset. This means that the set of dogs is a subset of the set of animals. The verb "is" can be replaced by = in the last statement.[314] Additionally, Russell was also aware of the fact that the verb "is" has more than one meaning in English.[315]

In contrast, the left column possesses only one meaning of the verb, if it is not considered to be merely a translation of the right column. Słupecki claimed that in all these statements the verb can be formalized as ε. In addition, all names are considered names from the point of view of Leśniewski's Ontology.

The difference between Polish and English led Słupecki to maintain:

> The difference which in Leśniewski's view exists between any of the meanings attributed to the English word "is" and the meaning of the Polish "jest" or Latin "est" and thus also of the meaning of the primitive term "ε" in Leśniewski's ontology makes it impossible to illustrate the schema "$A\ \varepsilon\ B$" by means of examples taken from the English language.[316]

Rickey and Woleński were convinced, however, that philosophers who are speakers of languages where articles are used, can understand Leśniewski's system of logic and use it in English. Rickey suggests that the operator ε should be used in the technical way which is defined in Ontology,[317] i.e. in accordance with the definition of the operator:

> [Aa]: A ε a.≡.[∃B]. A ε B. B ε a.[318]

Woleński agrees with Słupecki, that the proper understanding of the operatorε is difficult. Apart from the difference between Polish and English, Woleński stresses that even native speakers of Polish could misinterpret it. He points out that the difference in the meaning of the verb "est" in Russell's and Leśniewski's

[314] Jan Woleński, "Reism and Leśniewski's Ontology," in Woleński, J.: *Essays in the History of Logic and Logical Philosophy* (Cracow: Jagiellonian University Press, 1999), 18.
[315] Russell, *Introduction to Mathematical Philosophy*, 172.
[316] Słupecki, "Calculus of Names," 66.
[317] Rickey, "A Survey of Leśniewski's Logic," 31–32.
[318] Roughly speaking, the formula means: All *A* is *a*, if and only if, for some *B*, *A* is *B* and *B* is *a*. The verb "is" has the meaning of the Polish "jest".

theory is caused by the analysis and semantics, which each of them uses.[319] Thus, it is a case of interpretation. The semantics of set theory which is used by Russell cannot provide semantics for Ontology, but another semantics could be more successful.

In addition to the problem of interpretation, Leśniewski's Ontology possesses certain controversial features as well as Russell's theory. Specially, Leśniewski's system, in contrast to Russell's theory, does not contain settled semantics. As Urbaniak emphasizes, Leśniewski did not claim convincingly which entities were the values of his constants and variables. Various authors have interpreted Leśniewski's semantics in accordance with their preferences although Leśniewski did not provide any hints of an appropriate semantic for his system of logic.[320] In contrast, the semantics for Russell's names and theory of descriptions is clearly based on the set theory.

In conclusion, since Leśniewski grounded his system on article-free Polish, logicians who dealt with the Latin or Greek logical tradition found his concept of names more suitable than Russell's system of logic. Leśniewski did not differentiate between names and descriptions such as Aristotle and medieval logicians. His analysis of sentences also corresponds more than Russell's with the subject-predicate form of sentences. Additionally, certain philosophers favoured his theory of quantification which did not include the ontological commitment.[321] There were, however, also good reasons for adopting at least some of Russell's inventions. Hence, Arthur Prior tried to combine both systems of logic.[322]

[319] Woleński, "Reism and Leśniewski's Ontology," 18–21.
[320] Urbaniak, *Leśniewski's Systems*, 191–192.
[321] E.g. Lejewski, "Logic and Existence," 104–119.
[322] E.g. Prior, *Time and Modality*, 63–75.

5. 3 Prior's Concepts of Names

5. 3. 1 A Comparison of the Two Concepts of Names

Although Prior in his final works differed from both presented concepts of names, his ideas were formulated in a comparison with them. Prior discussed the differences between Russell's and Leśniewski's concept of logic, initially his paper *Definitions, Rules and Axioms*. He compared Russell's and Leśniewski's theories of definitions here.[323] Prior later focused chiefly, however, on a comparison of their theories of names. He introduced the discussion in his paper *English and Ontology*, where he focused on the operator ε.[324] Prior agreed with Lejewski that the operator ε could be replaced by $\subset$, which best corresponds to its Polish meaning. Consequently, Prior claimed that Leśniewski's names are from this point of view common nouns. He also mentioned that Leśniewski's quantification lacked an ontological commitment, which Prior appreciated, as was mentioned previously.

5. 3. 1. 1 Time and Modality

Prior's systems of logic were based chiefly on Russellian ideas, as were the majority of the systems of his contemporaries. One important ontological fragment of modal logic was the Barcan formula which Prior rejected[325]. The Barcan formula, which was postulated by Ruth Marcus Barcan, is presented in Prior's book as $\Diamond\exists x(\phi x) \rightarrow \exists x\Diamond(\phi x)$.

This means that if it is possible that something would ϕ, then there is something which could possibly ϕ, which requires the existence of the individual that will ϕ. These requirements are even clearer if the modal operator is replaced by the temporal operator *F*. Then the formula has a form $F\exists x(\phi x) \rightarrow \exists xF(\phi x)$ and means "If it will be the case that some *x* will ϕ, then there is *x* of whom will be the case

[323] Prior, "Definitions, Rules and Axioms," 199–216.
[324] Prior, "English and Ontology," 64–65.
[325] Prior, *Time and Modality*, 29–32.

that it ϕ. For instance, "If it will be the case that someone will fly to Mars, then there is someone, who will fly to Mars."

Consequently, if the formula is true, then the individual who will fly to Mars has to be somehow present in the ontology. It will not be an actual problem in the case of someone who will fly to Mars. This individual could exist at present, even though, it is not known who it will be. The point is that this formula should be applicable to every future statement, which means that every entity of which the future statement could be formed has some way of being. Prior called this position "a permanent pool of objects" and was not prepared to admit this ontological requirement. He did not include, however, the Barcan formula into his system of logic.

Although the existence of "a permanent pool of objects" is unacceptable for Prior from the ontological point of view, he demonstrated that it is also paradoxical. If there are individuals who do not exist at present, could some of them be blue-eyed now? We could deny that since the individual is non-existent, and only existent entities could be or could not be blue-eyed, but this solution implies that there are properties, which could only be ascribed to existent individuals. Consequently, there are properties, which could also be ascribed to non-existent individuals, in order to allow formulating statements about them. Prior emphasized, however, that there is no solid line between these two kind of properties.[326]

In a similar way he also rejected certain theorems of predicate logic, since they did not suit his presentism. He claimed that the formula $A(y) \rightarrow \exists x\, A(x)$, could not be a theorem in his system of logic. It implied that if the statement "Alexander rode Bucephalus" is true, then there has to exist (presently) some creature which was ridden by Alexander. If there is no such creature the implication is false. The present non-existence of Bucephalus is not an issue for

[326] Ibid., 31–32.

Russell or Quine who were eternalists but it is not acceptable for Prior who was a presentist.[327]

Another troublesome formula is $\neg\Box p \rightarrow \Diamond\neg p$, which is one of the basic theorems of modal logic. Prior maintained, however, that when this formula is transcribed to tense logic the implication did not hold. He demonstrated it with the following example. The formula $\neg Hp \rightarrow P\neg p$ interpreted as "If it is not true that it had always been the case that there were some facts about me, then it has been the case that there were no facts about me" has a true antecedent but a false consequent. The antecedent could be true since 1000 years before my birth there were no facts about me. In contrast, the consequent is false, since if there are no facts about me, then there is at least one fact, namely that there is no fact about me. Prior rejected similarly the formula $\neg\Box\neg p \rightarrow \Diamond p$. None of these formulas are possible in his presentist interpretation.[328]

Prior's own position consisted in the claim that individuals are sempiternal,[329] i.e. they began to exist at a certain moment and ceased to exist sooner or later afterwards. He admitted that there were also facts about entities which had not begun to exist yet. Prior suggested a system of logic which is sufficient for these sempiternal entities, the system Q. The system has six truth-values and deals with two time instances, today and yesterday. The truth-values are:

(1) true today and also yesterday,
(2) true today and unstatable yesterday,
(3) true today and false yesterday,
(4) false today and true yesterday,
(5) false today and unstatable yesterday, and
(6) false today and also yesterday.[330]

[327] Ibid., 32–34.
[328] Ibid., 34–35.
[329] As Jakobsen points out there were a great discussion between Prior and Jack Smart on sempiternal individuals. (Jakobsen, "Arthur Norman Priors bidrag til metafysikken," 35–39). They appeared to be highly controversial to Smart and consequently to the tradition which he represented.
[330] Prior, *Time and Modality*, 41.

It used modal operators $\Diamond$ and $\Box$, but their meaning is different. $\Box$ means "in both times", while $\Diamond$ means "at some time". In addition, Prior intended to enlarge the number of values of the system including more tense operators to it.[331]

Prior pointed out that the system Q is not only sufficient for tense logic, where it could be a tool for grasping sempiternal individuals, but it could be also interesting tool for modal ontology. He claimed:

> For if tense-logic is haunted by the myth that whatever exists at any time exists at all times, ordinary modal logic is haunted by the myth that whatever exists, exists necessarily….,[332]

Additionally, Prior emphasized that the system Q, in contrast to Łukasiewicz's system of logic, did not claim that there were no necessary entities at all. Prior claimed that there are individuals as God or numbers, which might have been considered eternal and necessary and his system Q allowed it. However, at the same time, he had to admit that the precise reference of the variables of the system had not been settled.[333]

Prior presented a detailed comparison between Russell's and Leśniewski's concept of names in the chapter *Proper-name Logic and Common-noun Logic*. These two kinds of logic were formalized by him to the two systems of logic: ΣT_1 and ΣT_2. He seemed to favour ΣT_2 (i.e. common-noun logic), in which constants do not have to refer to any entity. This feature suits better his temporal ontology than Russell's requirements. As he argued:

> The immense advantage of all this is that we can now regard the range of values for our bound variables as being fixed once and for all without being thereby committed to the view that all individuals are sempiternal, or-in modal contexts-that all individuals exist necessarily. For in this system to be a value of a variable is *not* the same thing as to be.[334]

[331] Ibid., 41–43.
[332] Ibid., 48.
[333] Ibid., 51–52.
[334] Ibid., 65.

Prior was also aware, however, of the weak points of Leśniewski's Ontology. The main disadvantage is according to Prior the limited possibility of differentiation between nouns and the limited possibility of their specification. This is not a problem in Leśniewski's extensional logic, but Prior demonstrated that it caused an inaccuracy in intensional logic. In addition, the extensionality is another feature which excluded Leśniewski's Ontology, from Prior's wholehearted approval. Therefore, Prior concluded that these two systems should be combined. Prior suggested the system ΣT_3, which was obtained from ΣT_2.[335]

The system of logic ΣT_3 is built on one important feature and Prior differentiated in it between two meanings of the definite article the strong "the" and the weak "the". Since Prior used there Leśniewski's operator ε he also postulated three meanings of it.[336] The difference is based on his concept of propositions and tense ontology. Neither Russell nor Leśniewski agreed with these features of Prior's logic, therefore, neither of them would agree with this differentiation.

If the statement contains the weak "the", its truth-value depends on the time frame in which it is stated. The weak "the" occurs for instance as the subject of the statement "The Prime Minister of the Czech Republic is a Socialist." It is true only temporally. The statement is true, when it is asserted at the moment, when I write this dissertation, but it was not true several years before (where the Prime Minister was from the right side of Czech politics). Accordingly, this state of affairs could change in the future and the statement will not be true then. Prior maintained that by the weak "the" are specified the names in the system ΣT_2, in which the operator ε is used.

In contrast, the strong "the" fix the reference that it is not influenced by time or conditions of utterance. For example, in the statement "The author of *Star Wars* is from the United States" the reference is the same, regardless of the speaker or the time of speech. George Lucas is the only individual to which the subject can refer. The strong "the" appears in the system ΣT_3 and the operator ε is replaced by ε'. In addition, the strong "the" and also the weak "the" can be defined in the

[335] Ibid., 66–75.
[336] Ibid., 76–83.

Russellian system of logic ΣT_1, but then the Russellian names have to also be admitted. Prior demonstrated that ΣT_3 did not include the weaknesses of Leśniewski's Ontology and that individuals could be identified in them even in intensional ontology.

Prior deduced from the ε' the operator ε'' in order to obtain a system, which suited his concept of sempiternal individuals. He emphasized that this further step is necessary since the formula $a\varepsilon'b$ is always false when a does not exist at present. Hence he postulated ε'', which also formed the true statements from constants which stint for individuals which ceased to exist or have not begun to exist yet. Prior called them "identifiable individuals" and their features will be described in the following chapters.

5. 3. 1. 2 Past, Present and Future and Further Papers

Prior also discussed the Q system in his book *Past, Present and Future*. He had to admit, however, that this system had not been axiomatized yet, even though, there was a matrix which characterized truth-values and the evaluation of operators. Its axiomatization was in progress, when Prior published *Past, Present and Future*. It was not only Prior, who developed it, but certain improvements were suggested by Bull, Mackie and Lemmon.[337]

Prior focused in the book primarily on the improvement of his systems of temporal and modal logic. Although the last chapter is entitled *Time and Existence* he had to admit that there was not as much progress as in the formal systems and it turned out to be "the untidiest and the most obscure part of tense logic".[338] He still considered, however, the combination of Russell's predicate logic with Leśniewski's Ontology as one of the possible alternatives for tense ontology.[339]

Prior also dealt with the comparison in the books and also published a paper

[337] Prior, *Past, Present and Future*, 154–158.
[338] Ibid., 172.
[339] Ibid., 162–167; 173–174.

Existence in Leśniewski and in Russell which is entirely focused on it. He clearly presented Ontology in contrast with Russell's predicate logic. Prior claimed here that Leśniewski's names could be described as class names.[340] This was later criticised by Simons, who pointed out that Leśniewski as a nominalist would not have approved any postulation of classes.[341] Notwithstanding, this seems to be a misunderstanding of Prior's interpretation, who was himself also a nominalist, as I have discussed in my paper *The Reception of Stanisław Leśniewski's Ontology in Arthur Prior's Logic*.[342] Prior did not intend to introduce classes into Leśniewski's system of logic, but only claimed that Leśniewski's names behave like class names in his system of logic.[343] From the ontological point of view, they are, according to Prior, common nouns.[344]

In his paper *Tense Logic for Non-Permanent Existents* Prior differentiated between modal calculus Q and tense calculus QT. He postulated a new operator *S* in the modal system Q. *Sp* means "in all possible worlds there is a proposition *p*" This operator only binds propositions, even though, if the proposition is reformulated in Ramsey's manner as ϕx, it can form the formula e.g. $S \forall x \phi x$ or $S(x = x)$. In the tense-logical system QK_T, Prior included two operators: *T* for tomorrow and *Y* for yesterday. The system is axiomatized and the rule of detachment, which was lacking in *Time and Modality's* version of it, as Łukasiewicz pointed out, is fulfilled.[345] Prior was still aware of the fact that Leśniewski's analysis of names could be a further alternative solution.[346]

Finally, Prior also developed his system Q in *Worlds, Times and Selves*. A number of the papers which had been previously published in this book were included in the new edition of *Paper of Time and Tense*. This was namely the paper *Modal Logic and the Logic of Applicability*. Prior focused on variables which stand for

[340] A. N. Prior, "Existence in Leśniewski and in Russell," in *Formal Systems and Recursive Functions*, ed. J. N. Crossley and M. A. E. Dummett (Amsterdam: North-Holland Publishing Company, 1965), 150–151.
[341] Simons, "On Understanding Leśniewski," 165.
[342] Rybaříková, "The Reception," 243–262.
[343] However, this feature of Prior's interpretation was criticised by Sagal (P. T. Sagal, "On How Best to Make Sense of Lesniewski's Ontology," *Notre Dame Journal of Formal Logic* 14 (1973): 259–262).
[344] Prior, "Existence in Lesniewski and in Russell," 153.
[345] Prior, *Papers on Time and Tense*, 255–273.
[346] Ibid., 274.

possible worlds i.e. *a*, *b*, *c* etc., which he bounded with the operator *T*. It is no longer the operator for "tomorrow", *Tap* means "proposition *p* is true in world *a*". Consequently, the *T* could also bind the variables of predicate logic. If there is no individual which *x* refers to in the possible world *a*, the statement "x-is-eating-chocolate" is inapplicable in world *a*.[347]

5. 3. 1. 3 Object of Thought

Prior analysed Russell and Leśniewski's concepts of names in his unpublished paper *Names*, which was later included as an appendix to *Object of Thought*. The entire book is focused on ontology of intensional logics, especially epistemic logic, as was presented in the previous part of my dissertation. Prior pointed out that Russell's theory of names, even though, more precise than Leśniewski's theory, is not sufficient to grasp the difference between the statements "X said that Y is bald" and "X said about Y that he is bald" here. He further demonstrated that these statements are not equivalent since there are cases in which the speaker could say the former, but is not aware of the latter. For instance, if he or she is not aware of the fact that the individual he or she described is *Y*. Prior also suggested, however, certain Russellian procedures which could overcome this inaccuracy.[348]

This analysis was a starting point for further discussion in which he asked: do names refer to individuals or do they only apply to individuals? He claimed that common nouns, adjectives and verbs were merely applied to individuals whereas names referred to them. He demonstrated this with the example of Sherlock Holmes and Winston Churchill that even names referred indirectly.[349] When he analysed further the indirect quotation he claimed:

> Without Russellian individual names it seems to me that there is no plausible definition of the name-forming functor 'thing thought to be a —' in terms of the sentence-forming functor 'it is thought that —'; the introduction of such

[347] Ibid., 282.
[348] Prior, *Object of Thought*, 155–158.
[349] Ibid., 159–161.

> functors therefore seems bound to complicate ontology. However, philosophers who are not extensionalists, and who believe that there are not and cannot be any such expressions as Russellian individual names, may well find the complications worth accepting.

Prior consequently favoured Russellian names for this analysis, although he still used a rather Leśniewskian quantification in it. Namely, he transcribed the existential quantifier as "For some x…", which was an interpretation suggested by Lejewski.[350] Prior discusses the theories of Lejewski, Kenny and Kripke further in the text but did not agree with any of them. He claimed that his own theory could deal better with individuals in intensional logic.[351]

In addition, Lambert identifies points which differentiate Prior's theory from Russell's. Firstly, Prior called Russellian names descriptions which would have not been acceptable for Russell. Namely, Prior in accordance with Frege and Ramsey, maintained that the statements "The dog which I met was not black" and "It was not the case that the dog which I met was black" cannot be distinguished from the extensional point of view. Secondly, Russellian names in Prior's understanding are also context-relative. Hence, the statement "The man over there is clever" could be true, according to Prior, even if the speaker of the statement is confused and the person is in fact a women or a robot, who is nonetheless clever.[352]

In summary, Prior formulated his systems primarily in the Russellian predicate logic but some features of Leśniewskian Ontology appealed to him and hence he also formulated systems which were influenced by Leśniewski's Ontology. He seemed to approve of Leśniewskian names for the flexibility of their reference and the Russellian names for their accuracy. Since his logic was intensional, whereas Russell and Leśniewski preferred extensional logic, he later abandoned both systems.

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[350] Lejewski, "Logic and Existence," 113.
[351] Prior, *Object of Thought*, 167–170.
[352] Lambert, "Russellian Names", 411–412.

5. 3. 2 Identifiable individuals

Considering the fact that in his view individuals began to exist at a certain moment, Prior asked whether the individuals were elsewhere before this moment. He claimed, following Buridan and Geach, that they were not. They do not have any kind of being before. Prior argued for essentialism in both the past and the future. He argued that which does not exist could not be identified or named, but whenever it began to exist, it had to have its properties. Prior agreed with Ryle, who claimed that before he was born, there were no facts about him either true or false and that there is no singular but only general truths about the future.[353]

This problem also has a different side. Prior maintained that the past is settled and whatever an individual possessed in the past is its necessary property. In his paper *Identifiable Individuals*, Prior argued against N. L. Wilson's view which appeared in Wilson's *Substances without Substrata*.[354] Wilson dealt here with the question as to what the world would be like if Caesar had all the properties of Antonius and vice versa. Prior claimed that even the question itself is dubious. Namely, it is not certain what would be the references of the names "Julius Caesar" and "Marcus Antonius".

Additionally, Prior did not agree with Wilson's solution that the possible world in which such an event happened is exactly same as our actual world. Firstly, he was not convinced that in discussing Caesar and Antonius we had to describe all possible worlds and everything which is in it. It could be even impossible as Prior maintained: "no one sees everything"[355] Secondly, Prior emphasized the fact that everything looking like our actual world did not guarantee that it was identical to our actual world. Finally, Prior argued that since the properties of Caesar and Antonius would be switched, the possible world in which this happened, cannot be exactly same as the actual world. It could look the same but it included this difference, which had further consequences.

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[353] Prior, *Past, Present and Future*, 140–143.
[354] N. L. Wilson, "Substances without Substrata," *The Review of Metaphysics* 12 (1959): 522–523.
[355] Prior, *Papers on Time and Tense*, 81.

This brought Prior to a more important question, i.e. what makes individuals a certain individual. Who or what is the individual to which we refer to as "Antonius". Wilson suggested that it is just the individual, which the majority of people described as Antonius. Another widespread solution was that Antonius was characterised by his distinct property or by the sum of properties. Prior did not agree, however, with any of these solutions. He did not want to separate Caesar from his properties or claimed that it is possible. He did not favour calling individuals in a possible world, which he considered "merely an imaginary" Caesar, Antonius or by any other names.

Focusing on properties, Prior denied Wilson's idea that Caesar could have all Antonius' properties. He started with the very beginning of Caesar's life, and claimed that it was not possible that Caesar would have had Antonius' parents and vice versa. As before Caesar's birth there was no individual who could be identified as Caesar and after his birth it was quite late for him to have different parents than he had.[356] Prior's claim depends on his ontological position. Since he did not intend to postulate any way of being of individuals before their existence began, nothing could be matched as Caesar before Caesar's birth.

In conclusion, from the ontological point of view, Caesar cannot be Antonius and Antonius Caesar, since they were both already themselves, according to Prior. Time eliminates all ontological possibilities, they cannot lose or win other battles than they win or lose, they cannot love different women or die differently. This view seems to exclude any possibility, and Prior really did not admit any possibility of changing the past in his ontology. He stated that it is logically possible that Antonius was born to Caesar's parents, but it is not the case.[357]

[356] Prior, *Papers on Time and Tense*, 81–85.
[357] Ibid., 90–92. This Prior's remark might have led van Cleve to claim that Prior was haecceitists and consequently that Prior allowed the twist of properties between Antonius and Caesar. As follows from my previous analysis of Prior's view, I do not agree with this interpretation of Prior's theory. (van Cleve, "Objectivity without Objects.")

5. 3. 3 The Reference in Tense Ontology

Prior pointed out that tense logic could be formed on a base of propositions but it could also be predicate logic. As was mentioned previously, he preferred the former but did not exclude the latter. He therefore struggled with the question as to if there are statable facts about individuals which do not exist presently and with the question as to what type of entities the variables of predicate logic stood for.

The question as to what variables in tense predicate logic stand for is difficult to answer for Prior. He argued that the traditional approach was that they stand for individuals (or substances). He continued that according to Russell they stand for sets, at least variables and the names also stand for individuals. Prior pointed out that Johnson suggested a solution that corresponds better with tense ontology. He claimed that the variables could stand for "continuants", those objects whose properties change over time. Specifically, my childhood is not part of, it belongs to my personal history. At the same time, I am still the same individual, even though, living in a different place and having a different weight, height and hair colour.

Johnson's approach could solve the problems with changes within individuals, however, the variables in this case could only stand for individuals who are changing but exist presently. There are cases when we attempt to handle entities who have ceased to exist or who have not started to exist yet. Prior inquired into several possible solutions. He initially claimed that if the variables do not refer to objects but simple particles which are eternal, the problem would have not appeared. Consequently, if the entire universe is considered to be only one individual, the reference of tense predicate logic is not troublesome. Prior objected to those suggestions that none of them was how the things were, it was just how they were better suited to tense predicate logic.

Other possible solutions could be that the statements would be verified in the time about which they are formulated. For instance, the subject of the statement "Someone will fly to Mars" will be verified, if there is or will be someone who

will once fly to the Mars at the time when this individual (or individuals) do this journey.[358] This solution would violate Prior's presentism, however, since it requires a certain way of existence of past and future at present. Jakobsen discusses this query more profoundly in his dissertation.[359]

In addition, Prior's presentism is problematic when one tries to refer to individuals who do not currently exist e.g. "My grandmother was an excellent cook". The reference is also opaque when a statement contains a reference to some former state of an individual e.g. "I am fatter than I was". Namely, Prior did not incline to an introduction of pseudo-entities such as "me-at-t" and "me-at-t'", nor was he interested in claiming a certain way of existence for dead people and individuals who had passed away. Prior coped with the change of weight with the reformulation:

> For some girths *G* and *G'*, it was the case that my girth is *G* and it is the case that my girth is *G'*, and *G* is (i.e. is-always) less than *G'*.[360]

The problem of individuals who have passed away is more serious. He claimed, however, that as the statement with the change of weight is solved by using the relation between two different girths, the existence of my grandmother (or ancestors in general) is also related to me (or the descendants of the dead ancestors). Namely, if there had been nobody as my grandmother I would have never been born.[361]

It is easier, however, to identify this way for past individuals than for the future ones. This could be caused by the indeterminacy of the future, which was a corner stone of Prior's philosophy. However, as could be seen from the previous paragraphs, Prior also did not have a concise theory of reference for past individuals. The presented ideas seem to be only suggestions.

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[358] Ibid., 93–95.
[359] Jakobsen, "Arthur Norman Priors bidrag til metafysikken,", 43–45 and 96–98.
[360] Prior, *Past, Present and Future*, 170.
[361] Ibid., 169–171.

5. 3. 4 Identity

Postulating sempiternal individuals, Prior had to cope with the problem of identity. There are cases in which one individual turned to be two individuals. When Prior discussed this problem in his paper *Opposite Number* he chose a quite unusual example:

> Suppose people reproduced like amoebae, and suppose you and I are the two products of such a fission, each of us having a perfect memory of having been the one original person, though now the two of us are both being and doing quite different things, say me reading Plato and you not. This would put us in a situation not unlike that envisaged by John Wyndham in the story 'Opposite Number' in his *Seeds of Time*.[362]

It seems to be an improbable situation, even though, there are situations where something similar really happened, e.g. when the prokaryote or bacteria split into two organisms or when a fetus split into two when twins began their separate life. The question is what happened with the original individual. Does it cease to exist when the two other started? Is it one of these two? Is it both, even though, they are not identical anymore? It is obvious that these two individuals were one before the fission, but they live their separate life after it. This situation could challenge the traditional concept of identity in logic.[363]

Prior found two solutions to this query. First, he stressed that all the problems were caused by sempiternal individuals. He suggested a similar solution as in the problem of reference. If eternal individuals were chosen, the predicate logic would have been less troublesome. Prior proposed atoms,[364] even though the physics indicated that they were not as eternal as seemed to be at present. There are, however, smaller units which atoms consist of which might be eternal. In addition, Prior claimed that the predicate logic, which handled atoms as the values of variables was not developed when he questioned it.[365]

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[362] Prior, *Papers in Logic and Ethics*, 64.
[363] Prior, *Papers in Logic and Ethics*, 64–65; Prior, *Papers on Time and Tense*, 96–101.
[364] Prior, *Past, Present and Future*, 174; Prior, *Papers on Time and Tense*, 94.
[365] Prior, *Papers on Time and Tense*, 95–96.

Second, Prior pointed out that there are also systems of logic, which described the relation between the whole and the parts, that could deal with this query more effectively. He mentioned Carnap's system, which Carnap presented in the paper *Introduction to Symbolic Logic* and Mereology, which Leśniewski developed and Lejewski discussed.[366]

The fact that he did not find a plausible solution to this query did not trouble Prior all that much. He argued:

> This enterprise has been carried so far now that it is worth at least exploring the consequences of carrying it through to the end. But we cannot do this satisfactorily until much more work has been done, of the sort that is already beginning to be done, on the whole notion of an individual thing, and of 'the same individual thing'. If, even at this early stage, it appears that we shall be faced with the abandonment or at least the modification of Leibniz's law, we may reflect that this has been contemplated in recent years for quite trivial reasons, so we need not be too dismayed if we now have to contemplate it for serious ones.[367]

Consequently, it did not lead to a plausible solution in predicate logic, but it called into doubt Leibnitz's law, which could also be beneficial.

Taking everything into account, Prior was inspired by Russell and Leśniewski. His theory was formulated through a comparison with these two concepts of names, despite the fact that it also differed from each of them. Due to his premature death, this theory is difficult to reconstruct, even if certain hints of it were presented in his writings. He claimed that individuals are sempiternal and that all their properties, once they began to exist, are essential. The past was linear for him and only the future was open. Although Prior suggested several solutions to solving the problem of reference of these sempiternal individuals, none of them seemed to satisfy him. Additionally, he did not solve the problem

[366] Ibid., 100–101.
[367] Ibid., 101.

of identity, which was discussed in the last sub-chapter. Notwithstanding, as could be seen from Prior's comment on this failure, it could also be useful.

6, Conclusion

> But we cannot do this satisfactorily until much more work has been done, of the sort that is already beginning to be done, on the whole notion of an individual thing, and of 'the same individual thing'. If, even at this early stage, it appears that we shall be faced with the abandonment or at least the modification of Leibniz's law, we may reflect that this has been contemplated in recent years for quite trivial reasons, so we need not be too dismayed if we now have to contemplate it for serious ones. [368]

I am aware of the fact that the last chapter was not truly conclusive. Although various aspects of Prior's ontology were presented, several questions remained unresolved and therefore the entire project of the reconstruction of Prior's ontology seems to be unsuccessful. Prior's premature death could be blamed, since as was elsewhere previously mentioned, it interrupted him in the middle of his work. His concept of names in particular was not settled when he passed away. In contrast, his concept of possible worlds, even though, questioned and criticised is part of the history of analytic philosophy.

There are two key concepts which appear to allow Prior's combination of nominalism, presentism and intensional logic. These are the theories of quantification and his concept of propositions. It was claimed that at least the former theories were to a certain extent enabled by Prior's adoption of Leśniewski's ideas. Prior's theory of propositions was then primarily affected by Ramsey's ideas, even though Prior also approved of Quine's contribution to these topics. Since propositions were only logical constructions to him and his propositional quantification lacked an ontological commitment, he could handle non-present contexts. He was additionally aware that in certain queries he had to deal with names and individuals, but as was mentioned, he did not solve this problem convincingly.

[368] Prior, *Papers on Time and Tense*, 101.

In addition, there are also questions which belong to a more elaborate part of Prior's ontology but which are still problematic. First, if propositions are so important for Prior's concept of possible worlds, could they be only logical constructions? Van Cleve's objection was presented here which appears to be justified. Second, what are the semantics of Prior's system of logic? The question is whether the features which he adopted from Leśniewskian's system allowed for the remaining of a certain possibility of having Russellian semantics for predicate logic. Additionally, as Cresswell points out Prior did not advocate a standard semantics.[369] Lastly, Prior postulated sempiternal individuals, but the names which should represent them were not convincingly defined as was similarly the case with their reference and the theory of identity.

Although Prior did not present one consistent concept of ontology, when formulating his views he questioned several theories such as the Leibniz law of identity or Quine's ontological commitment and to a certain extent provided an alternative to them. His in many respects controversial views encouraged debates which lasted long after his death, even up until the present. I would therefore like to paraphrase Prior's concluding remarks from *Time, Existence and Identity*, which was used as an introductory quotation for this conclusion: Prior's ontology cannot be reconstructed satisfactorily since there is much more work which could have been done. However, even in the unfinished state of Prior's work, it seems that he highlighted problems which seriously questioned certain important theories of philosophy of language.

[369] Max Cresswell, "Prior on the semantics of modal and tense logic," *Synthese* (forthcoming).

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