VYSOKÉ UČENÍ TECHNICKÉ V BRNĚ
BRNO UNIVERSITY OF TECHNOLOGY

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MATHEMATICAL MODEL OF SOLAR DRYING OF SEWAGE SLUDGE

MATEMATICKÝ MODEL SOLÁRNÍHO SUŠENÍ KALU Z ČISTÍČKY ODPADNÍCH VOD

DIPLOMOVÁ PRÁCE
MASTER'S THESIS

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BRNO 2010

Abstrakt
Solární sušení je mimo jiné energeticky nenáročná a tudíž vysoce ekologická cesta pro zpracovní kalu z malých čistíren odpadních vod. Výsledkem tohoto procesu je produkt, který je velmi dobře použitelný v zemedelství jako přírodní hnojivo. V rámci této práce je sestaven zjednodušený dynamický model sušení vrstvy kalu s jednou prostorovou souřadnicí. Model bude diskretizován pomocí vhodné numerické metody a bude provedena jeho implementace do výpočtového programu v prostředí MATLAB.

Summary
Solar drying of sludge is a very enviromental-friendly way of sludge treatment. This work deals with modeling of sludge drying. Different models are introduced, based on experiments, like empirical models or detailed models. The one-dimensional mathematical model is implemented by appropriate numerical method in MATLAB and results are presented in this thesis.

Klíčová slova
kal, solární sušení, matematický model, rychlost odpařování

Keywords
sewage sludge, solar drying, mathematical model, drying rate

FICZA, I.*Mathematical model of solar drying of sewage sludge*. Brno: Vysoké učení technické v Brně, Fakulta strojního inženýrství, 2010. 57 s. Vedoucí diplomové práce Doc. Ing. Jiří Hájek, Ph.D.

Prohlašuji, že jsem diplomovou práci *Matematický model solárního sušení kalu z čističky odpadních vod* vypracovala samostatně pod vedením Doc.Ing Jiřího Hájka, Ph.D., s použitím materiálu uvedených v seznamu literatury.

Ildikó Ficza

Děkuji svému školitelvi Doc. Ing. Jiřímu Hájkovi, Ph.D. za vedení mé diplomové práce. Dále bych chtěla poděkovat Ing. Tomášovi Juřenovi za pomoc při ladění výsledného programu. Také bych chtěla poděkovat rodičom a blízkým za jejich podporu během studia.

Ildikó Ficza

Contents

1. Introduction

1.1. Motivation

Drying is one of the most common, diverse and oldest process engineering unit operations. It is a process which can be understood as removing volatile substance (moisture) to yield a solid product. Drying including solids is much more difficult to model than fluid-phase processes, since their physical properties are highly dependent on their structure. Solar drying of sewage sludge is an attractive and at the same time particularly environment-friendly way of minimising the mass of sewage sludge. There exist different approaches to modeling of drying sewage sludge. Several empirical models are already existing. On the other hand, detailed models especially for solar drying of sewage sludge are not so widespread.

1.2. Objective of the thesis

The objectives of the thesis give an overview of wastewater treatment possibilities and drying technology, to construct a one-dimensional mathematical model and to verify it with a numerical method.

1.3. Scope of the thesis

The second chapter deals at the beginning with wastewater treatment plants and with the treatment steps the wastewater undergoes. Then review of drying follows. Finally, the last part of the chapter is about sewage drying, the moisture distribution in sludge, mechanical dewatering and thermal, respectively solar drying of the sewage sludge is described.

The third chapter is a review of approaches to modeling of the sludge. Classification of the existing dryer models is provided and the empirical models are introduced.

The fourth chapter includes scoping of a solar dryer plant, description of the empirical statistical models and calculations of the evaporation rate and water loss in the sludge during drying.

Detailed description of a one-dimesional model is given in the last chapter. Main assumptions are stated, then processes occuring during drying are introduced, i.e. the moisture evaporation at the surface of the bed and internal moisture transport towards the surface. The governing partial differential equations are set and completed with initial and boundary conditions. The chapter later presents the results of the simulation which was provided with an appropriate numerical method, namely the finite volume method.

2. Review of sewage sludge treatment and its drying

First part of the chapter is devoted to research in the fields of wastewater treatment and drying of sewage sludge. A short description is given of each process the wastewater has to undergo in a wastewater treatment plant and of further sludge treatment and disposal possibilities, too. In the section about drying, different drying possibilities are discussed, emphasizing solar drying. Then an overview of sewage sludge drying follows which summarizes its most important features.

2.1. Sewage sludge and its treatment in wastewater treatment plants

In this section the functioning of a wastewater treatment plant is described, based on [21].

Sewage treatment is the process of removing contaminants from wastewater or household sewage. Households, hospitals, commercial and industrial institutions, and many others are producing sewage (for instance liquids from toilets, baths, showers, kitchens, sinks, etc.). It can be treated close where it is created or transported to a municipal treatment plant. Sewage collection and treatment is typically subject to local regulations and standards. The objective of sewage treatment is to produce a water stream and a sludge suitable for discharge into either a stream, a river, a bay, or a sea, or to reuse back into the environment. Sludge can be often contaminated with a lot of toxic organic and inorganic compounds. To remove contaminants from sewage, physical, chemical and biological processes are used. Sewage treatment consists of three main stages, namely primary, secondary and tertiary treatment.

2.1.1. Primary treatment

Primary treatment removes all the large objects (those can be sticks, rags, tampons, cans, fruit, etc.) from the wastewater. Large solids after mechanical screening are dumped to a landfill. The next step at this stage is grit removal. Sand, stones and grit are necessary to be removed at this stage to avoid damaging the equipment in latter treatment steps. There is included a so-called degritter or sand-catcher. In this sand or grid chamber or channel the velocity of the incoming wastewater is controlled, so the sand or grit can settle down, while keeping the majority of the suspended organic material in the water column. The retained materials are carried to landfills.

At times of rainstorms, the flow of sewage into the works may be too high to be accommodated by the downstream treatment stages. In these circumstances, some of the flow may be diverted at this point to storm tanks where it is stored temporarily before returning it for treatment when the flow subsides.

The last step at this stage is sedimentation. The sewage passes in to large sedimentation tanks, where the sludge settles down. This sludge is known as "primary sludge" and further it is treated separately. The goal here is also to produce a homogenous liquid which is later treated biologically.

2.1.2. Secondary treatment

The main purpose of secondary treatment is to reduce the biological content in the sewage, this stage is also called as "biological treatment".

Most of the plants are using aerobic biological processes to do so. Here the sewage comes into contact with micro-organisms which remove and oxidize most of the remaining organic pollutants. Secondary treatment systems are classified as fixed film or suspended growth.

At smaller works, the biological stage often takes the form of a packed bed of graded mineral media through which the sewage trickles and on the surfaces of which the micro-organisms grow. At most larger works, the sewage is mixed for several hours with an aerated suspension of flocs of micro-organisms. This is known as the activated sludge process. Following secondary (biological) treatment, the flow passes to final settlement tanks where most of the biological solids are deposited as sludge, called secondary sludge. In the case of the activated sludge process, some of the secondary sludge is returned to the aeration tanks for further contact with the sewage. The secondary sludge from biological treatment also requires separate treatment and disposal and may be combined with the primary sludge for this purpose.

2.1.3. Tertiary treatment

The tertiary stage of treatment can be used to remove most of the remaining suspended organic matter from the effluent before it is discharged to the receiving environment (sea, river, lake, ground, ...). More than one tertiary treatment process may be used at any treatment plant. Tertiary treatment is effected by sand filters, mechanical filtration or by passing the effluent through a constructed wetland such as a reed bed or grass plot.

Wastewater may contain high levels of the nutrients of nitrogen, like ammonia (NH_3), nitrites (NO_2^-), nitrates (NO_3^-) and phosphorus (P_4).

The removal of nitrogen is effected through the biological oxidation of nitrogen from ammonia to nitrate what is called nitrification. Nitrification is a two-step aerobic process, each step facilitated by a different type of bacteria. Denitrification requires anoxic conditions to encourage the appropriate biological communities to form. This is followed by denitrification, the reduction of nitrate to dinitrogen gas. Nitrogen gas is released to the atmosphere and thus removed from the water.

Phosphorus removal is important as it is a limiting nutrient for algae growth in many fresh water systems. Phosphorus can be removed biologically by a process called enhanced biological phosphorus removal, where specific bacteria – poly-phosphate accumulating organisms ($PAOs$) – accumulate large quantities of phosphorus within their cells. Another way of phosphorus removal is so-called chemical precipitation. Chemical phosphorus removal requires smaller equipment footprint than biological removal, is easier to operate and is often more reliable than biological one. It is also particularly important for water reuse systems where high phosphorus concentrations may lead to fouling of downstream equipment.

2.1.4. Disinfection, further treatment and disposal

Another important step in wastewater treatment is disinfection. It means, the number of microorganisms in the water must be substantially reduced. The effectiveness of the disinfection depends for example on the quality of the water being treated or on the type of disinfection being used. Ozone, ultraviolet light or chlorine are examples of disinfection methods.

The accumulated sludge in wastewater treatment processes has to be treated and disposed of in a safe and effective way. Anaerobic digestion, aerobic digestion and composting belong to the group of most common treatment options.

Anaerobic digestion, which is applied to large-scale plants, is a bacterial process that is carried out in the absence of oxygen. Opposite process to above mentioned one is aerobic digestion, which is suitable for small-scale plants. During anaerobic digestion biogas is produced, which can be used e.g. for electricity production. Aerobic digestion occurs in the presence of oxygen, under aerobic conditions the organic matter is consumed and converted to carbon dioxide by bacteria. Furthermore, composting is also an aerobic process which involves mixing the sludge with sources of carbon such as sawdust, straw or wood chips.

A liquid sludge has to undergo also dewatering to reduce its volume. This is necessary for getting an end form of the sludge suitable for final disposal. It is also possible to recycle the sludge in many ways, including from already mentioned anaerobic digestion (to produce biogas) or composting, pyrolysis (to produce syngas) and incineration (to produce heat and possibly electricity).

2.2. General overview of drying

Drying is a process which can be understood as removing volatile substance (moisture) to yield a solid product. First, some basic properties of drying are mentioned as well as general approaches to drying. Drying is one of the most common, diverse and oldest process engineering unit operations. There are different reasons for drying, e.g. preservation and storage, reduction in cost of transportation, achieving desired quality of the product, etc. Most common materials to be dried are: vegetables and fruits, grain, wood, textile products, paper, peat, bio-fuels, sludge and many others. Drying is an essential process in chemical, agricultural, pharmaceutical, etc. industry. One of its basic features is the phase change and production of a solid phase as end product. Designing or analyzing a dryer can be difficult. One has to consider the various properties of the material which is going to be dried. Some features of drying are mentioned in [17]:

- product size may range from microns to tens of centimeters (thickness or height);
- product porosity may range from 0 to 99.9 %;
- drying times range from 0.25 s (e.g. tissue paper) to 5 months (hardwood species);
- product capacities may range from 0.10 kg/h to 100 tons/h;
- product speeds may range from 0 (stationary) to 2000 m/min (tissue paper);
- drying temperatures range from below the triple point to above the critical point of the liquid;
- operating pressures range from fraction of a milibar to 25 atm.

When a wet solid is subjected to thermal drying, two processes are occurring simultaneously [17]:

1. transfer of energy (mostly heat) from the surrounding environment to evaporate the surface moisture;

2. transfer of internal moisture to the solid's surface.

The 1. process, it means removal of water as vapor from the materials surface depends on external conditions such as temperature, air humidity, rate and direction of airflow, pressure, surface area, etc.

The 2. process means the movement of moisture within the solid, it is a function of the physical nature of the solid, the temperature and its moisture content.

Drying is a complex operation involving transient transfer of heat and mass along with several rate processes, such as physical or chemical transformations, which, in turn, may cause changes in product quality as well as the mechanisms of heat and mass transfer. Physical changes that may occur include shrinkage, puffing, crystallization, and glass transitions. In some cases, desirable or undesirable chemical or biochemical reactions may occur, leading to changes in color, texture, odor, or other properties of the solid product.

Drying occurs by effecting vaporization of the liquid by supplying heat to the wet feedstock. Heat may be supplied by convection (direct dryers), by conduction (contact or indirect dryers), or by radiation. Heat is supplied at the boundaries of the drying object so that the heat must diffuse into the solid primarily by conduction. The liquid must travel to the boundary of the material. before it is transported away by the carrier gas. Transport of moisture within the solid may occur by any one or more of the following mechanisms of mass transfer [17]:

- liquid diffusion, if the wet solid is at a temperature below the boiling point of the liquid;

- vapor diffusion, if the liquid vaporizes within material;

- Knudsen diffusion, if drying takes place at very low temperatures and pressures, e.g., in freeze drying;

- surface diffusion (possible although not proven);

- hydrostatic pressure differences, when internal vaporization rates exceed the rate of vapor transport through the solid to the surroundings;

- combinations of the above mechanisms;

Since the physical structure of the drying solid is subject to change during drying, the mechanisms of moisture transfer may also change with elapsed time of drying.

2.2.1. Different dryer types

Several possible subdivisions for dryer classification are available. As in the previous section was mentioned, heat can be supplied to the solid either by convection, by conduction or by radiation. So, the first classification of dryers can be based on the type of heat transfer: convection heating, conduction heating or radiation heating. Special case of conduction drying is freeze drying.

Convection is the most common type of drying, over 85 % of industrial dryers are of this type. They are also called as direct dryers. Heat is supplied by heated air or gas flowing over the surface of the solid. Heat for evaporation is supplied by convection to the exposed surface of the material and the evaporated moisture carried away by the drying medium [17]. Convective-type dryers are for instance suspension dryers, such as fluid bed, flash, rotary and spray dryers, or packed bed.

Conduction dryers are also called indirect dryers. Heat for evaporation is supplied through heated surfaces (stationary or moving) placed within the dryer to support, convey, or confine the solids. The evaporated moisture is carried away by vacuum operation or by a stream of gas that is mainly a carrier of moisture [17]. They are suitable for thin products or very wet solids. Freeze drying is a special case where under vacuum at a temperature below the triple point of water, here water (ice) sublimes directly into water vapor [17]. Paddle dryers, rotary dryers or drum dryers are examples of indirect dryers.

For example, solar dryers belong to radiation-type dryers. Section 2.2.2. is devoted to their description.

Another classification of dryers is due to the type of drying-vessel, e.g. rotating drum, tray, fluidized or spray dryers. Dryers can be divided also according to the physical properties of the material which is going to be dryed. More facts and details of individual dryer types, as much as selection requirements are available e.g. in [17]. The next part of the chapter is focusing on solar drying.

2.2.2. Solar drying

Solar drying is a very attractive way of drying different things and materials. Since the Sun is a free, nonpolluting and renewable energy source, people were using open-air sun drying since a very long time. However, large-scale productions may have several disadvantages, like large area requirement, high labor costs, damages caused by insects and birds, difficulties due to bad weather or the lack of ability to control the drying process. Over time open-air sun drying was replaced by other drying methods, many types of dryers were developed. But nowadays the importance of solar drying is becoming more and more apparent because of the above mentioned advantages of the Sun. When designing and constructing a solar dryer it is important to consider also the specific characteristics of the place where the dryer will be located, including geographic position, the number of sunny days yearly and the intensity of incident radiation, etc.

Main part of a solar dryer is the drying space, where the material to be dried is placed and where the drying takes place. Besides, other optional parts are:

- collector to convert solar radiation into heat;
- auxiliary energy source;
- heat transfer equipment for transferring heat to the drying air or to the material;

- means to keep the drying air flowing;
- heat storage unit;
- measuring and control equipment;
- ducts, pipes, and other appliances [17].

Three main groups of solar dryers can be distinguished, these are:

1. solar natural dryers,
2. semi-artificial solar dryers,
3. solar-assisted artificial dryers.

Solar natural dryers are using ambient energy sources only, while semi-artificial dryers can have a fan driven by an electric motor for keeping continuous air flow through the drying space. Solar-assisted artificial dryers are able to operate by using a conventional energy source.

Since in this work the drying of sludge will be modeled with a natural dryer, in the following some basic tasks of solar natural dryers are shown.

We can divide solar natural dryers into 2 groups, active and passive convection solar dryers. Active dryers can have additional fan or a small wind turbine to help the air flow. Cabinet-, tent-, greenhouse- or chimney-type dryers belong to the group of passive solar dryers.

Cabinet-type dryers are the simplest and cheapest dryers. Their design is very simple, the drying material is laying on tray, the bottom plate of which is perforated. The tray is covered by a south-oriented transparent wall (glass or foil). This wall is protecting against rain and pollution. Main characteristic of this type is that the drying material is irradiated directly. There are holes at the upper part of the cabinet where the air can flow out. This type can be applied for smaller amounts of fruits and vegetables.

For drying bigger quantity it is necessary to increase the area where the material is dried. For this purpose tent-type dryers are a good possibility. Their triangular framework is covered by a thin sheet, where the south-oriented wall is transparent, while the other, the back wall may be covered by a black sheet. Chimney-type dryers are good for drying large amounts of materials.

2.3. Drying of sewage sludge

Sewage sludge, as the by-product of wastewater treatment plants, needs further treatment. One possible way for doing so is drying. Sludge consists of suspended solids, coagulation chemicals, usually an alum or polymers with a limited amount of biological materials. First, the sludge undergoes dewatering, which is followed by thermal drying. An important issue is to select the proper dryer-type for sludge drying. In [17] several types are named to be suitable, they are e.g. spray, drum or paddle dryers, but solar dryers are also a possibility.

There are many reasons for drying sludge, as it is shown e.g. in [13]:

- reduce its mass;

- improve handling;
- facilitate final disposal;
- avoid bad smells when stored.

2.3.1. Moisture distribution in sludge and sludge dewatering

The moisture in sludge can be as high as 99%. Moisture distribution takes the following form [17]:

- free moisture that is not attached to the sludge particles and can be removed by gravitational settling;
- interstitial moisture that is trapped within the flocs of solids or exists in the capillaries of the dewatered cake and can be removed by strong mechanical forces;
- surface moisture that is held on the surface particles by adsorption and adhesion;
- intracellular and chemically bound moisture.

Figure 2.1 shows the distribution of water in the sludge.

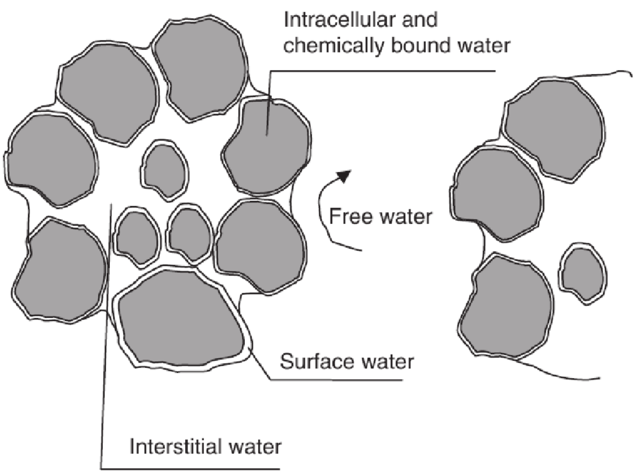

Figure 2.1: Moisture distribution in the sludge [17]

Amount of water that can be removed depends on the dewatering process and also the status of the water in the sludge. The mechanically removable water includes free, interstitial and partially surface moisture. The bound water is the remaining part. Its content is the theoretical limit of mechanical dewatering. Bound water can be determined by methods such as dilatometric determination, vacuum filtration, expression, drying, and thermal analysis. More details about these methods can be found in [17].

After the common treatment steps in a plant, dewatering is the next step. It means removing water without evaporation. The following alternatives are common for mechanical dewatering: vacuum filters, belt filter presses, centrifuges, and membrane filter presses. Again, one can find more details about these processes e.g. in [17].

2.3.2. Thermal and solar drying of sludge

For sewage sludge two methods of drying are feasible, namely thermal drying and solar energy drying.

Thermal drying of a dewatered sludge is important in cases when the required final product water content is low, or when the heating value of the sludge needs to be improved for efficient incineration, or when the transportation costs to disposal in a landfill need to be reduced.

Thermal drying of sludge will normally consist of three stages: a constant drying rate period, first falling rate period and second falling rate period. During the constant drying rate period, free water is removed. The first falling rate period is believed to remove the interstitial water and the second falling rate period removes the surface water. The final moisture content retained within the sludge is mostly chemically bound water in amounts that depend on the type of sludge and the drying conditions. Besides the variations in moisture content and drying rates, the physical status of the sludge will change from a wet zone to a sticky zone and finally granulation, depending on the solids content [17]. The pathogens contained in sludge must be properly neutralized or destroyed. Thermal sterilization is therefore desirable byproduct of thermal drying [17].

There are numerous dryers available globally that claim to be able to dry sludge. In terms of heat and mass transfer, the available dryers can be classified as:

- direct drying systems,
- indirect drying systems,
- combined systems.

Direct dryers are characterized by simple design but the vapor released from the sludge have to be separated from the drying medium, especially in the situation when the drying medium is to be recycled to save energy. Direct dryers are typically rotary-drum, flash, moving-belt dryers, or centri-dryer types.

Indirect dryers are characterized by thin-film, rotary-disc, or rotary-tray dryers. The indirect drying system has the advantage of producing minimal amounts of vapors and is therefore easy to manage.

There are systems available that use combined conduction and convection heat transfer. An example of such system is fluidized-bed dryer.

Another possible dryer type which was not mentioned yet is solar dryer. It is a good solution for sludge drying since its simple technology, low energy demand and running costs. The drying of sludge using solar energy requires a considerable amount of land and may give rise to an odor problem that is difficult to solve.

If the environmental conditions are feasible solar drying is a natural choice comparing to thermal drying of sludge which is a very energy-intensive process. A review about wastewater treatment plants was made, as well as several ways of drying were described.

At the end the chapter short description of the sewage sludge drying was made. Now different ways of classification of approaches to modeling comes, and also the empirical and detailed model will be introduced.

3. Review of approaches to modeling of solar sludge drying

3.1. Classification of approaches of modeling drying

Drying including solids is much more difficult to model than fluid-phase processes, because their physical properties are highly dependent on their structure (e.g. particle size, porosity, etc.), while physical properties of fluids can be obtained from databanks and they are uniquely defined for given pressure and temperature.

The simultaneous heat, mass and momentum transfer processes give a highly nonlinear set of governing equations. Many parameters which affect the drying processes are difficult to evaluate. Experiments have also shown that theoretical models are extremely difficult to apply in practice and that small scale-up or pilot-plants are often more reliable than a design-mode calculation using physical properties from databanks or theoretical estimates [25].

Several classification of the dryer models is introduced here. With their help for instance design and performance calculations are possible to provide. Broad type classification of dryer software is the following:

1. calculation programs, including numerical models of dryers,
2. process simulators,
3. expert systems and other decision-making tools,
4. information delivery (online libraries or knowledge bases),
5. software for dryer control systems and instrumentation.

Dryer models can be distinguished also as qualitative or quantitative. To the first subgroup belong for example expert systems and knowledge bases. Expert systems can help e.g. with the dryer selection. The latter subgroup consists of design and performance calculation, scale-up or drying kinetics calculations. For their calculation users can either write own software or to choose among math solvers or computational fluid dynamics (CFD) software.

Dryer models can be categorized in several different ways [25]:

- The purpose or mode of the calculation.
    1. Design of a new dryer to perform given duty.
    2. For an existing dryer, calculation of performance under a different set of operating conditions.
    3. Scale-up from laboratory-scale or pilot-plant experiments to a full-scale dryer.
- The level of complexity of the calculation.
    - Level 1. Simple heat and mass balances.
    - Level 2. Scoping or approximate calculations.

- Level 3. Scaling calculations.
- Level 4. Detailed methods.

Heat and mass balances of Level 1. models can give some useful information, but do not predict the plant size or performance capabilities. They are practical for continuous dryers, but less useful for batch dryers.

Level 2 model calculation are based again on heat and mass balances and on heat transfer calculations. They give accurate estimates e.g. of cross-sectional area of convective dryers, but less accurete results for other calculations. Level 2 models are used for continuous-convective, contact or batch dryers.

Level 3 scaling models present overall dimensions and performance for dryers by scaling up drying curves from small-scale or pilot-plant experiments. A simple scaling model is e.g. integral model.

Level 4 models require more complex modeling techniques with more input data, an example of such method is CFD or incremental model.

Detailed description of above mentioned models are in [25]. As a final overview, table 8.1 in Appendix A is introduced from [25] which is a summary of characteristics of available dryer types.

3.2. Empirical models

Although solar drying is recently more and more widespread, still not many models predicting the drying rate are available. Authors in [20] were developing several prediction models for drying rate as a function of outdoor environment conditions and control actions. These models seemed to be suitable for operational and optimization purposes. Data for the models were collected in a drying installation designed by [24] in the town of Füssen, in Germany.

The evaporation rate (drying rate) in [20] is proposed as a function of outdoor environment (weather) e, the state of the sludge s, and control vector c, i.e.,

$$E = E(e, s, c). \tag{3.1}$$

The weather vector may contain outdoor temperature T_o, solar irradiation R_o, wind speed U and outdoor humidity ratio is w_o. The state of the sludge may belong the dry solid content DSC, which can be calculated from the masses of solid, S, and of water, W, i.e.,

$$DSC \equiv \frac{S}{S+W}. \tag{3.2}$$

Further, the state of sludge may be described by sludge surface temperature, sludge thickness and floor temperature. The control vector may consist of ventilation rate, Q_v, and air mixing rate, Q_m.

Based on data collected during experiments there are two main types of models proposed by [20]. One physical (mechanistic) model and so-called black-box (statistical) models which are the multiplicative model and additive model. Next, description of physical model is coming, though, it was not used for calculating the evaporation rate in this work. After the mechanistic one, short overview of the statistical models is following.

3.2.1. Mechanistic model

The mechanistic (physical) model is a so-called "resistance" model, which means that this model has analogy in the field of electrotechnics. The one-dimensional model is divided into 5 horizontal layers: sub-sludge, sludge, indoor air, cover and outdoor air, where these layers are characterized by 4 vertical resistances in series. In addition, there are 2 more horizontal resistances indicating the conductance of ventilation.

Evaporation takes place at the boundary between the sub-sludge and the sludge. Condensation on the ceiling is not permitted. The following boundary conditions are assumed: outdoor solar irradiation, outdoor air temperature, outdoor humidity, and floor temperature. Unknown parameters are temperatures at evaporation level, at the surface of the sludge and in the bulk of the air. A system of equations is given:

- energy balances at the evaporation level and at the surface of the sludge;

- sensible heat fluxes across the sub-sludge, sludge and air;

- the latent heat flux;

- net radiation at the sludge surface.

From these equations the unknown parameters are determined and the evaporation rate is calculated. The disadvantage of the model is that its predictions are inferior, which means that they do not efficiently extract the information available in data. Authors of [20] also assess that the parameter values of the model are usually not of the the expected magnitude.

3.2.2. Statistical models

[20] proposed to predict the evaporation rate based on the equation of the vapor balance method

$$E = \rho(w_{out} - w_{in})Q_v \equiv \rho \Delta w Q_v \quad [20]. \tag{3.3}$$

Equation (3.3) is obtained by measuring the humidity ratio of the ventilating air at the inlet and outlet and multiplying this difference Δw by the air density ρ and ventilation rate Q_v.

To predict Δw, statistical models are introduced, namely the additive and the multiplicative model. Their exact formulation is discussed in the next chapter.

3.3. Detailed models

The detailed model is offering a macroscopic point of view. Sludge drying is a complex process involving simultaneous mass and heat transfer in the material. To model this process it is important to describe and understand the moisture transport through the material. The object of detailed models is to predict e.g. the distribution of moisture content or temperature in the solid material.

Various authors in [18], [10] and [17] offer different ways to model the drying processes in all kind of materials, like e.g. concrete, wood or granular materials, one can choose even between one- or two-dimensional models. However, due to complexity of these models or because of the very different properties of dried materials, not all of the models serve useful information to construct the detailed model of sludge drying. A description of a detailed mathematical model is the goal of Chapter 5.

In what follows, the aim is to model the processes occuring during sludge drying. Sludge will be assumed to dry by solar energy. First an estimate of dryer plant size is calculated. Then two different models are going to be described, one is an empirical model predicting the evaporation rate and water loss in the sludge. The second model is a detailed model described with governing partial differential equations. These models are the objective of the next chapters.

4. Scoping and prediction of the evaporation rate by empirical models

Different types of modeling approaches were reviewed in the previous chapter. First, approximative scoping of the footprint area of a solar dryer plant is done. Then the next part of the chapter describes the empirical statistical models developed by [20]. Calculations of evaporation rate based on these models are also presented.

4.1. Scoping design

To estimate the size of a solar plant is a complex task. Its size depends on many factors, e.g. climatic conditions in the given place (average temperature, solar irradiation, wind speed, humidity, etc.), state of the material (dry solid content) and in some cases control functions too (e.g. ventilation rate), but these functions depend on the design of the dryer. In lack of exact formula, data are adopted from [13], [3] and [24], they are compared and an approximative formula for calculating the size of a plant is constructed.

Here, considering a very general and not precise assumption, namely that the footprint area of the plant A, depends on the evaporation rate E, and the amount of treated sludge per year M. A is given in [m^2], M in [kg/a] and E in [kg/m^2/a][1].

The average evaporation rate E can be expressed as

$$E = M_{wet} - M_{out}. \tag{4.1}$$

Equation 4.1 says that E is equal to the difference between the amount of the sludge M_{wet} at the beginning of the drying period and the amount M_{out} at the end of the drying period.

Both M_{wet} and M_{out} consist of solid and water. So, they are expressed as the sum of dry solid and water for M_{wet}, the first term in the bracket stands for the solid content and the second member of the sum stands for water content in the sludge.

$$M_{wet} = M_{wet}\left[\frac{\sigma_{in}}{100} + (1 - \frac{\sigma_{in}}{100})\right], \tag{4.2}$$

and for M_{out}

$$M_{out} = M_{out}\left[\frac{\sigma_{out}}{100} + (1 - \frac{\sigma_{out}}{100})\right]. \tag{4.3}$$

In (4.2) and (4.3) the terms σ_{in} and σ_{out} in [%] are the dry solid content of the wet sludge at the beginning and at the end of drying period, respectively. The amount of dry solid is assumed to remain consant during drying, i.e.

$$M_{wet}(\frac{\sigma_{in}}{100}) = M_{out}(\frac{\sigma_{out}}{100}), \tag{4.4}$$

[1] E is given in [mm/h] in the most of the literature sources, according to [20], [mm/h] is equivalent to [kg/m^2/h], which can be easily converted to [$kg/m^2/a$].

so

$$M_{out} = M_{wet}(\frac{\sigma_{in}}{\sigma_{out}}). \tag{4.5}$$

Finally, the average evaporation rate per unit area can be expressed as

$$E = \frac{1}{A}M_{wet}(\frac{\sigma_{out} - \sigma_{in}}{\sigma_{out}}). \tag{4.6}$$

Table 4.1 sums up all important data like climate (average annual temperature, annual precipitation and irradiation) and geographical placement, but also data such as amount of sludge treated per year, the size of the plant and initial and final dry solid contents of the sludge.

Climate data are from [19], all other data are taken from [3] and [24]. In Appendix A (table (8.2)) one can find further data of other existing solar drying plants. Furthermore, Appendix A contains climate maps which show the average annual temperatures, precipitations and annual solar irradiation, both for Europe and Czech Republic.

Plant	M_{wet} [kg/a]	σ_{in} [%]	σ_{out} [%]	A [m^2]	T [°C]	R [W/m^2]	precip. [mm/a]	height [m a.s.l.]
Miltenberg (GER)	$6 \cdot 10^6$	23	90	3000	10.6	119	500	161
Ilawa (POL)	$1.575 \cdot 10^6$	20	90	1536	8.2	114	115	105

Table 4.1: Summary of properties of solar dryer plants

The goal is now to approximately estimate the size of a solar dryer near the city of Brno, Czech Republic. From the list of solar dryers mentioned above, the most similar geographical conditions to those of Brno, Czech Republic are in Miltenberg, Germany.

In comparison, Miltenberg is situated 161 m a.s.l., it's average daily temperature in year is 10.6 [°C], the annual irradiation is 119 [W/m^2], while Brno is situated 222 m a.s.l., average daily temperature in year is 9.4 [°C], and the yearly irradiation is 122 [W/m^2] [19]. The differences in average temperature and average irradiance seem to be acceptable.

The annual evaporation rate of the dryer in Miltenberg is calculated from (4.6), it is $E = 13400$ [kg/m^2/a]. The annual evaporation rate for Brno can be assumed roughly to be similar. According to data from [4] the annual amount of sludge treated in Brno is assumed to be $6.152 \cdot 10^7$ [kg/a]. The footprint area could be estimated now, however the amount of wet sludge is higher than of those in [3] or [24]. The footprint area for such an amount of sludge would be very large. Therefore, it is assumed, that just certain part of the sludge produced in [4] in Brno is going to be dried in a solar plant.

4.2. Empirical models

This section focuses on the statistical empirical models. It is important to remark that these models were developed by [20] using measured data from the solar plant in Füssen [24]. So first, the description and technical data of the dryer plant follow. In this solar dryer plant, wet sludge is uniformly spread in a drying chamber over a concrete floor under a greenhouse-like transparent cover, where the drying chamber is 10 m wide and

50 m long. The sludge is mixed mechanically several times per day by an autonomous robot, the structure is fan-ventilated horizontally, and the indoor air is mixed by electric fans. the capacity of the mixing and ventilating fans is 150 [m^3(air)/m^2(floor)/h] [20]. At the beginning the dry solid content of the sludge (DSC) is of 0.2 to 0.3 [kg (solid)/kg (sludge)], and the final DSC is between 0.6 and 0.8 kg [(solid)/kg (sludge)].

4.2.1. Multiplicative model

The multiplicative model of evaporation rate prediction was developed by [20]. It is of the form:

$$E = \rho Q_v \Delta w, \tag{4.7}$$

where

$$\Delta w = \varepsilon \prod_{j=1}^{p} (P_j + \beta_j)^{\gamma_j} \quad [20]. \tag{4.8}$$

ε, β_j, and γ_j are constants, P_js are the predictors, and p is the number of predictors. β_js add flexibility to the model and become necessary whenever some measured values of P_js are non-positive. To ensure real-number output in the preceding equation, the following condition must be satisfied

$$P_j + \beta_j \geq 0 \tag{4.9}$$

for all relevant values of P_j.

The model has $2p + 1$ parameters. Important issue is to consider which predictors will be used in the model. The choice of the parameters depends on the purpose of the model, e.g. optimization. The potential predictors can be all the weather, state or control variables. The more predictors are used, the better prediction model can be set. A correlation matrix may be set up to help analyze relationship between the potential predictors of the evaporation rate. If the correlation between two parameters is low, it means the two variables don't contain information available in the other. On the other hand, if a variable, which is already selected as a predictor, is well correlating with another parameter, then it is not necessary to add the latter one as additional predictor. For example, once the outdoor temperature is used as a predictor, the effect of outdoor humidity is expected not to be significant, despite it's physical undeniable role. For every model can be estimated an order of predictors from most to last significant.

Based on measured data an individual model was developed [20], where there are 5 predictors, and their order is the following : solar irradiation R_0, outdoor temperature T_o, ventilation rate Q_v, air mixing rate Q_m, and dry solid content DSC (denoted in the following as σ). The model with its concrete parameters determined by [20] is :

$$\begin{aligned} E = \rho Q_v \Delta w = \rho Q_v \varepsilon \prod_{j=1}^{p} (P_j + \beta_j)^{\gamma_j} = \\ = \rho Q_v 1.962 \cdot 10^{-11} \cdot (R_0 + 1100)^{2.322} (T_o + 13.0)^{1.292} \cdot \\ \cdot (Q_v)^{-0.577} (Q_m + 0.0001)^{0.0013} (\sigma + 0.26)^{-0.353}, \end{aligned} \tag{4.10}$$

where air density ρ is 1.13 [kg (air)/m^3], R_0 is in [W/m^2], T_o is in [°C], σ is in [kg (solids)]/[kg (sludge)], Q_v and Q_m are in [m^3/m^{2}h] , and E is in [mm/h].

The role of air mixing is an order of magnitude less significant than of the ventilation rate, it is clear also from the exponents. The exponents of σ and Q_v are negative, which reflect to the physical behavior of the system (i.e. Δw is reducing with increasing Q_v and σ).

4.2.2. Additive model

Another statistical model developed by [20] was the additive model. It has the following form:

$$\Delta w = \eta + \sum_{j=1}^{p} \alpha_j P_j, \tag{4.11}$$

where, again, the humidity ratio is calculated. There are no mathematical constraints necessary in this equation. The number of parameters is $p+1$. Now, the evaporation rate formula is

$$E = \rho Q_v \Delta w = \rho Q_v (\eta + \sum_{j=1}^{p} \alpha_j P_j). \tag{4.12}$$

The empirical formula to determine the evaporations rate according to [20] is:

$$E = \rho Q_v \cdot 10^{-6}(1835 + 3.65R_o + 88.7T_o - 15.4Q_v + 2.85Q_m - 1230\sigma), \tag{4.13}$$

i.e., the most important predictors are again solar irradiation, outdoor air temperature, ventilation rate, air mixing rate and DSC. The last two terms contribute an order of magnitude less. The coefficients of R_o, T_o and Q_m are positive, while those of Q_v and σ are negative, in qualitative agreement with the physics of the system [20].

As in [20] was stated, using the available data set multiplicative model has produced better estimates of the evaporation rate than the additive or the physical model.

4.2.3. Comparison of the predicting parameters of the multiplicative model

To compare the role of each parameter like temperature, irradiation, etc. in the multiplicative model, a study on the sensitivity of the parameters was provided.

First, the roles of the ventilation and mixing rates Q_v and Q_m, respectively, were investigated. Figure (4.1) shows that the role of the ventilation rate is significant. Different values of Q_v were substituted into (4.10), while the values for other parameters of the model were set up as constants. The higher value for Q_v is chosen, the higher evaporation rate E is obtained. Testing different values of the mixing rate Q_m shows that its importance is smaller. When Q_m is not included in the model, evaporation rate is smaller. On the other hand, when considering it as parameter, higher values of E can be obtained. As figure (4.2) shows setting for Q_m different values ranging from 40 to 130 [$\mathrm{m}^3/\mathrm{m}^2\mathrm{h}$] do not change E significantly. In [20] temperature T_o and solar irradiation R_o are stated as the most important predictors. In figure (4.3) the evaporation rate is plotted for different climate conditions (average values of T_o and R_o in Brno in July, September and November taken from [19]). Clearly, the highest value of E is achieved for conditions in July, then in September, and in November.

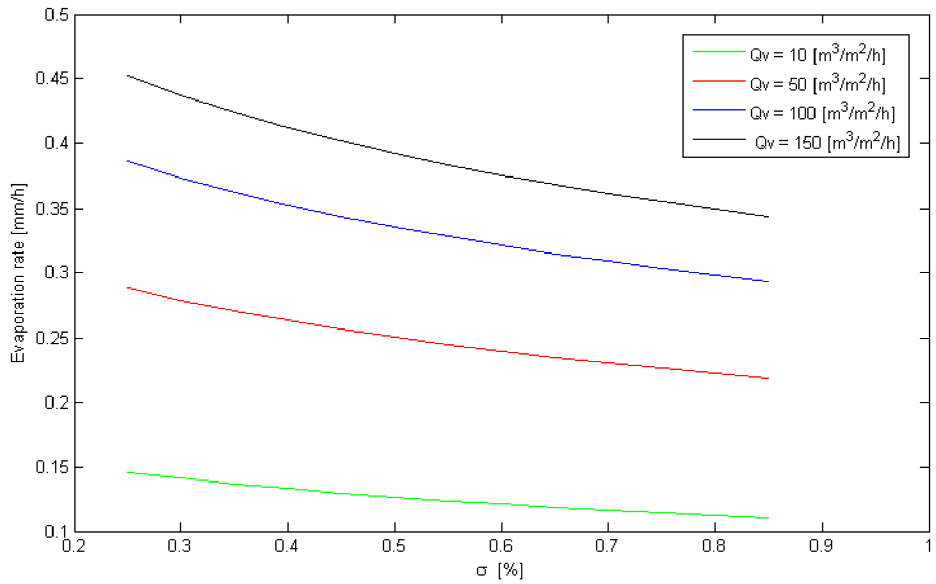

Figure 4.1: Effect of different valued ventilation rates on the evaporation rate

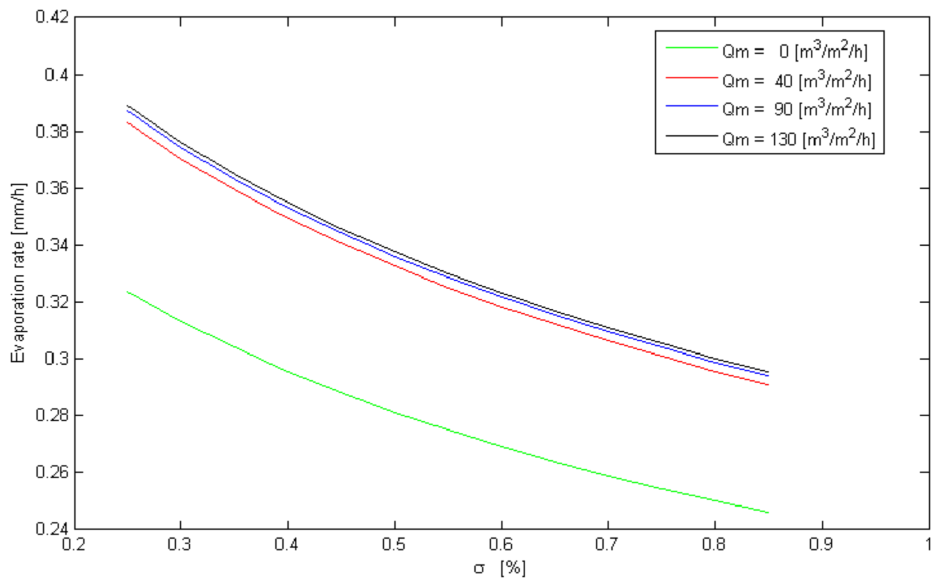

Figure 4.2: Effect of different valued air mixing rates on the evaporation rate

4.3. Estimates of evaporation rate and loss of water in the sludge

The last part of this chapter deals with estimation of the evaporation rate and the water loss in the sludge during drying.

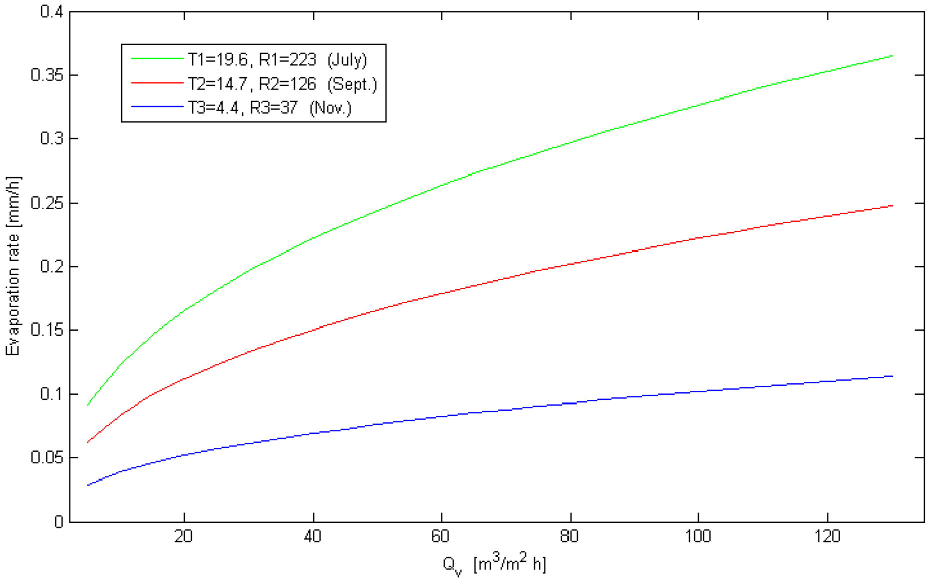

Figure 4.3: Evaporation rate for different climate data

4.3.1. Main assumptions

The multiplicative model (4.10) described in section (3.2) is used for evaporation rate calculations.

To calculate the loss of water during drying following expression was used:

$$W = W_0 - \int_{t_0}^{t} \int_A E d\tau dS, \tag{4.14}$$

where the water content in the sludge W is in [kg/m^2], time t may be given either in [day] or [month]. A is the footprint area of the plant in [m^2] and E is the evaporation rate [mm/h]. Some assumption are considered:

- At the beginning of the drying process the initial water content in the sludge is W_0.
- Footprint area A is constant.
- Average daily and monthly temperature and irradiation data are used for calculations, data are obtained from [19], [22] and [16].
- E is a function of the following parameters: temperature, irradiation, ventilation and mixing rates, initial dry solid content in the sludge.

4.3.2. Calculation and results

Following table 4.2 shows part of the results of calculations, the average daily climate data are from the year 2005 in the city of Brno. Some of the parameters of the multiplicative model are assumed as constants:

$$Q_v = 100\ [m^3/m^2h], \quad Q_m = 80\ [m^3/m^2h], \quad \sigma = 20\ [\%]. \tag{4.15}$$

Let us consider the initial value of water content per square meter in the sludge to be

$$W_0 = 100\ [kg/m^2]$$

t (day)	T_o [°C]	R_o [W/m^2]	$E(,t)$ [mm/h]	$W(t)$ [kg/m^2]
1	4.13	15	0.1012	99.90
2	4.25	19	0.1030	99.79
3	4.23	22	0.1035	99.69
		...		
365	-6.98	13	0.0261	23.07

Table 4.2: Daily estimates of evaporation rate and water loss

In the figure (4.4) one can see the daily estimates of the evaporation rate during whole year. Climate data and the value of the evaporation rate are summarized in table (4.2). The red lines in the graph stand for the average monthly values of E. Figure 4.5 shows

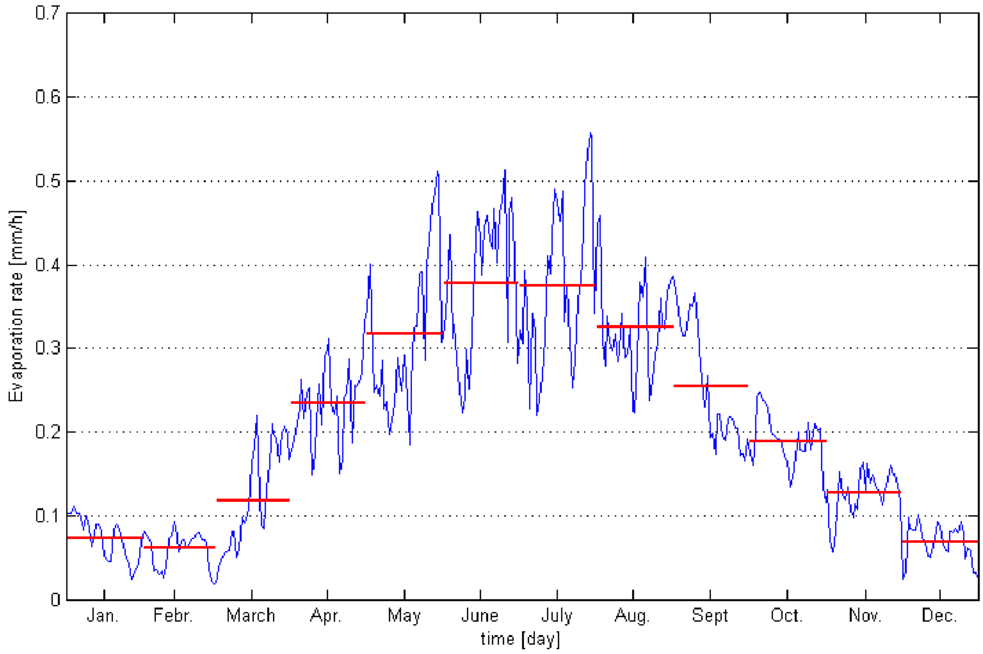

Figure 4.4: Daily estimates of evaporation rate

the loss of water during drying, the initial water content is assumed to be in this case $W_0 = 100$ [kg/m^2]. Again, the table (4.2) refers to the values of W.

Calculations of E and W were implemented in the MATLAB code. To calculate E the multiplicative model was used. The loss of water W is approximated by numerical integration [6]. The main m-file `graph.m` calls the other m-files `modelE.m` which calculates E for every day and `drying.m` which interpolates W.

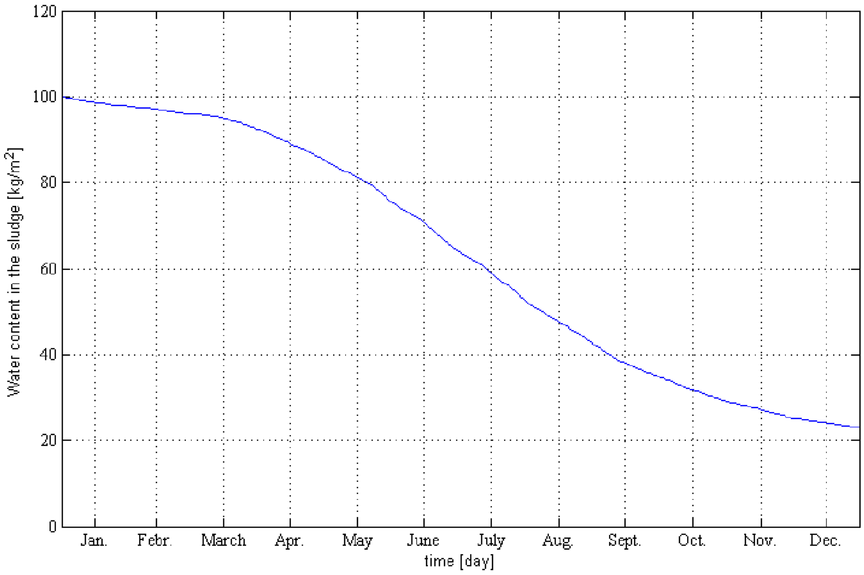

Figure 4.5: Loss of water in the sludge

As a final remark, it should be taken into account that the situation in the figure (4.5) does not reflect to a realistic situation. W is not bounded, so with lower initial value W_0 it may give negative values, what is not possible for the evaporation process. The effect of sludge mixing is also not assumed here.

The following chapter describes a detailed mathematical model which reflects more to the realistic processes occuring during drying.

5. Detailed mathematical model

Next objective of the thesis is to describe a detailed mathematical model which is applicable to the solar drying of the sludge. This model should realistically reflect the processes the sludge undergoes during drying. These processes are described here by governing partial differential equations. Since they are non-linear equations, they are solved numerically with appropriate numerical method.

Author in [11] deals with modeling of grate combustion, which is described by a mathematical model. This model is possible to apply to the solar sludge drying process. However, some changes are necessary to carry out in order to get adequate results for the drying process. The first part of the chapter deals with this mathematical model. A huge effort was made to determine all the coefficients occuring in the governing equations. In some cases, more alternatives were available for a given coefficient. On the other hand, there exist cases, when the required value of the given coefficient was not available for sludge. In these cases the values were replaced by other materials which are similar to sludge. Finally, the governing equations are completed with initial and boundary conditions.

The next part of the chapter gives a short summary about the numerical method applied to the model. It is important to note that the numerical model used for the simulation was made by [11]. Although some parts of the code were modified or replaced completely. At the end of chapter simulations are following and results are presented.

5.1. Main assumptions

The description of detailed model begins with assumptions necessary to simplify the model. On the other hand, they should reflect to the real behavior of the model.

Sludge is porous material. Its pores are filled at the beginning of drying with moisture in liquid phase and with gas at the end. Because the geometry of the porous material can be very complex, its structure is simplified. The sludge is assumed to be spread homogenously on a concrete floor and it is covered with a greenhouse-like transparent cover. Volume reduction of the solid content in the wet sludge is neglected, only the moisture content changes due to evaporation. The height of the bed where the sludge is spread is considered to be constant in the whole footprint area, so this fact lets to describe the mathematical model as one-dimensional.

Another important issues are the properties of the surrounding gas, i.e. the drying medium, which serves for carrying away the evaporating moisture above the solid's surface. Auxiliary ventilating fans are assumed to be installed in the drying plant. So the ventilation of the air is constant. The gas pressure is assumed constant. Gas-phase species included in the model are CO_2, H_2O, O_2 and N_2 [11]. No air-flow is assumed through the bed.

The only heat supplied to the sludge is the solar irradiation of the Sun. The summary of all assumptions is following:

- The system is one-dimensional.

- Drying medium is air. Its pressure is constant. Auxiliary ventilation fans are installed in the plant, and the air is convecting above the solid surface and carries away the evaporated moisture.

- Gas-phase species (of the air) included in the model are CO_2, H_2O, O_2 and N_2.

- Evaporation takes place at the surface,i.e. in the boundary layer.

- Sludge is spread on a concrete floor, its moisture distribution is uniform in the sludge. Volume reduction is due to the change of the moisture content, the mass of solid remains constant. Moisture is assumed to be liquid water.

- Solar irradiation is the only heat source involved in the drying process.

5.2. Drying processes

As mentioned before in section (2.2), during drying two processes occur simultaneously. Energy transfer from the surroundings to evaporate the surface moisture and the transfer of internal moisture towards the solid's surface. While the free water moisture evaporates in the boundary layer, the bound moisture inside the solid is transported to surface by diffusion and is subsequently evaporated.

Evaporation at the surface

Evaporation at the surface is a diffusion-limited process. It is determined as:

$$r_{H_2O} = k_m S(C_{w,s} - C_{w,f}), \tag{5.1}$$

where k_m is the mass transfer coefficient, S the particle surface area per unit volume, and $C_{w,s}$ and $C_{w,f}$ are concentrations of moisture at the sludge surface and in the air flow above the sludge, respectively.

Now, the mass transfer coefficient k_m is going to be defined. No theory is available for estimating the mass transfer coefficients using basic thermophysical properties [17]. Since the evaporation is in this case a diffusion-limited process, k_m is expressed as

$$k_m = \frac{DSh}{L}, \tag{5.2}$$

where D is the molecular diffusion coefficient of H_2O in air, L is characteristic length and Sh is the Sherwood number. For the Sherwood number Sh, different empirical formulas are available based on measurements. E.g. in [17] or [18] can be found relations for k_m for different materials (like softwood, grain, etc.) based on the Reynolds- and Schmidt-number, where both are dimensionless numbers. The Sherwood number defined for [17]

$$Sh = 2 + 0.6Re^{0.5}Sc^{1/3} \text{ [17].} \tag{5.3}$$

An interesting empirical expression of the mass transfer coefficient is introduced in [12]:

$$k_m = 0.1475v^{0.685}, \tag{5.4}$$

where v is the velocity of the convecting air. The equation (5.4) holds for convective drying of the sludge. Formula (5.4) could be useful, when e.g. values of D or Sh not available. In the simulation equation (5.3) was used for k_m.

The particle surface area per unit volume S, besides the surface area and volume of the given particle, depends on the porosity ϵ of the solid, too. The shape of a grain particle is a sphere, S is of the following form [18]:

$$S = 2(1-\epsilon)\frac{S_{particle}}{V_{particle}}. \tag{5.5}$$

The surface and volume of the particle are given as the surface area and volume of sphere, and the porosity ϵ of the solid is given by

$$\epsilon = 1 - \frac{\rho_{particle}}{\rho_s} \quad [10]. \tag{5.6}$$

So, the final form of S after simplification is

$$S = 2(1-\epsilon)\frac{4\pi r_p^2}{\frac{4}{3}\pi r_p^3} = 6r_p\frac{\rho_{particle}}{\rho_s}, \tag{5.7}$$

where r_p is the radius of the particle. For sludge the particle radius ranges from 1 to 20 mm [10].

Moisture diffusion from the solid to the surface of the bed

As a result of heat transfer to a wet solid, a temperature gradient develops within the solid while moisture evaporation occurs from the surface. This produces a migration of moisture from within the solid to the surface, which can be described e.g. by diffusion [17]

$$\frac{\partial \rho_l X}{\partial t} = \frac{\partial}{\partial x}\Big(\rho_l D\frac{\partial X}{\partial x}\Big), \tag{5.8}$$

where X is the moisture content, ρ_l the density of the liquid (water), and D the diffusion coefficient.

Similarly to the mass transfer coefficient, the diffusion coefficient D is also difficult to predict. It is possible to estimate the diffusion coefficients of gases or liquids even theoretically with big accuracy (e.g. by the Chapman - Enskog, or Stokes- and Einstein-equations [17]). But the prediction of diffusivity of liquids and gases (or vapor) in solids is a more complex task. There is no effective theory of determining the diffusivity in solids yet [17], on the other hand there are already some relations existing which have been proposed for porous materials. One of them is the Arrhenius-equation [17]

$$D = D_0 e^{\frac{-E_A}{RT}}, \tag{5.9}$$

where D_0 is the Arrhenius-factor, E_A is the activation energy for diffusion, R is the gas constant and T is temperature. In [18] there are another relations for D, e.g.

$$D = \frac{\delta\epsilon}{\tau^2}D_A, \tag{5.10}$$

where δ is constrictivity, ϵ is the porosity, τ is tortuosity and D_A is the vapor diffusivity in air in the absence of porous media, or

$$D = \frac{0.02508}{R_f}(R_f^2 - 1.6 \cdot 10^-17),\ R_f = 4.9 \cdot 10^-9 e^{0.0149 C_w}, \tag{5.11}$$

here D is given as the function of water concentration C_w in the solid. In the simulation the equation (5.9) was used.

5.3. Governing equations

A porous material consists of different phases, namely solid and fluid phase. Governing equations must be written for each phase, and these equations need to obey the assumptions stated before. The system of partial differential equations (PDE) will consist of mass and energy conservation laws for the solid phase, continuity equation for the gas and for individual gas-phase species and the the energy equations of the gas phase. The governing equations are nonlinear, thus a suitable numerical method has to be implemented to find the solution of the system. The independent variables are time $t \in \langle 0, \tau \rangle$, and one space variable $x \in \langle 0, h_b \rangle$, where is $x = 0$ is set as the bottom of the bed, and h_b is the bed height [11].

Now, detailed description of each equation is following.

Solid phase PDE's

Mass conservation equation

$$\frac{\partial \rho_s}{\partial t} = -r_{H_2O}. \tag{5.12}$$

ρ_s is the bulk density. It is defined as the mass of many particles of the material divided by the total volume they occupy. The total volume includes particle volume, inter-particle void volume and internal pore volume [5]. It is given by

$$\rho_s = \frac{m_s}{V}. \tag{5.13}$$

m_s is the mass of wet sludge, containing water and dry solid parts

$$m_s = m_{H_2O} + m_{drysolid}, \tag{5.14}$$

and V is the volume, $V = V_f + V_s$, where V_f and V_s are the parts of of volume V occupied by gas and solid, respectively.

The evaporation rate equation was already described in previous section

$$r_{H_2O} = k_m S(C_{w,s} - C_{w,f}). \tag{5.15}$$

m_s and V are constants, so the individual rate of mass change of water is given as

$$\frac{\partial m_{H_2O}}{\partial t} = -r_{H_2O} V. \tag{5.16}$$

Energy equation

$$\frac{\partial(\rho_s h_s)}{\partial t} = \frac{\partial}{\partial x}\Big(k_{eff}\frac{\partial T_s}{\partial x}\Big) + h_H S(T_a - T_s) + Q_{H_2O} + Q_r, \tag{5.17}$$

is the energy conservation law, where h_s is the specific enthalpy and T_s the solid temperature. Both of them are thermodynamic state quantities. The other coefficients in (5.17) are T_g the gas temperature, k_eff effective thermal conductivity of the sludge, h_H the heat transfer coefficient, Q_{H_2O} the heat loss due to evaporation of water and Q_r the heat received from solar irradiation.

Enthalpy can be expressed as

$$dh_s = c_{p_s} dT_s, \tag{5.18}$$

where c_{p_s} is the specific heat capacity of the solid at constant pressure.

If the gas is assumed as polytropic perfect gas, the specific heat capacity will constant. So,

$$h_s = c_{p_s} T_s. \tag{5.19}$$

Although, the gases are rarely considered as perfect, instead they are treated as ideal, i.e. that c_{p_s} won't be constant. This implies that the relation (5.18) must be integrated from a reference state to the final one, so the mean integral value [11]

$$\Delta h_s = h_{s,final} - h_{s,ref} = \int_{T_{s,ref}}^{T_{s,final}} c_{p_s} dT_s = \overline{c}_{p_s} \Delta T_s. \tag{5.20}$$

In drying calculation, it is more convenient to use the mean values of heat capacity [17]. The c_{p_s} and its mean value are

$$c_{p_{sludge}} = 1434 + 3.29 T_s \quad [2], \tag{5.21}$$

and

$$\overline{c}_{p_s} = \frac{1}{\Delta T_s}\int_{T_{s,ref}}^{T_{s,final}} c_{p_s} dT_s. \tag{5.22}$$

Another expression for specific heat capacity of wet sludge was found in [1]

$$c_{p_s} = \frac{X c_{p_{water}} + c_{p_{sludge}}}{1 + X} \tag{5.23}$$

where $c_{p_{sludge}}$ and $c_{p_{water}}$ are the specific heat capacities of dry sludge and water, respectively, and X is the moisture content of the sludge. The value of $c_{p_{water}}$ does not change significantly for different temperature values, it may be considered as constant [23]. $c_{p_{water}} = 4187\ [J \cdot K^-1 \cdot kg^-1]$.

The relation (5.20) can be used to obtain either temperature T_s or the enthalpy h_s. If the enthalpy h_{ref} is defined to be zero at the temperature state T_{ref}, then [11]

$$T_s = \frac{h_s}{\overline{c}_{p_s}} + T_{s,ref} \quad h_s = \overline{c}_{p_s} \Delta T_s. \tag{5.24}$$

Energy equation (5.17) can be now rewritten either in the terms of h_s

$$\frac{\partial(\rho_s h_s)}{\partial t} = \frac{\partial}{\partial x}\Big(\frac{k_{eff}}{c_{p_s}}\frac{\partial h_s}{\partial x}\Big) + h_H S(T_f - T_s) + Q_{H_2O} + Q_r, \tag{5.25}$$

or in the terms of T_s

$$\frac{\partial(\rho_s \overline{c_{p_s}}(T_s - T_{s,ref}))}{\partial t} = \frac{\partial}{\partial x}\Big(k_{eff}\frac{\partial h_s}{\partial x}\Big) + h_H S(T_f - T_s) + Q_{H_2O} + Q_r. \tag{5.26}$$

Now, short discussion about the effective thermal conductivity k_{eff} follows. k_{eff} depends on the geometry of the solid. Sludge is a porous material, so its structure is very complex. It is necessary to simplify it, so the following idea is considered [10]. The pore system of the solid consists of layers which are either parallel with the moisture transport or perpendicular to it. For both conductivies in parallel and perpendicular layers relationships are available in [10]. Conducitivity in "parallel" layered pore system is

$$k_{par} = (1-\epsilon)k_s + \epsilon k_l, \tag{5.27}$$

and conductivity in perpendicular layered pore system is

$$k_{perp} = \frac{1}{(1-\epsilon)k_s + \epsilon k_l}, \tag{5.28}$$

where ϵ is the solid porosity, and k_s and k_l are conductivities of pure solid (i.e. thermal conductivity of sludge) and moisture in liquid phase (i.e. thermal conductivity of the water). Both k_s and k_l are assumed to be constants in the above defined equations. Values for them are taken from [23], but since there is no available value for pure sludge, soil is considered instead. The final effective thermal conductivity k_{eff} will be

$$k_{eff} = \frac{1}{\xi k_{par} + (1-\xi)k_{perp}}. \tag{5.29}$$

It is important to note, that k_{eff} is not constant during the drying process. At the beginning the pores in the solid are filled with water, while at the end they are filled with gas. Since the liquids are better conductors then gases (here, the air), the value of k_{eff} by the end of drying will be an order of magnitude smaller than at the beginning. According [23] the values were set to $k_s = 0.2$ and $k_l = 0.6$.

The source terms Q_{H_2O} and Q_r represent the heat loss due to moisture evaporation and the heat gain due to irradiation. Q_{H_2O} is determined with the help of the latent heat of evaporation ΔH_{vap}, which equals to

$$\Delta H_{vap} = 3179 \cdot 10^3 - 2500 T_s \text{ [?Jur?]}. \tag{5.30}$$

An alternative expression to (5.30) can be found in [17]

$$\Delta H_{vap} = 352.58(T_s - 374.15)^{0.33052}. \tag{5.31}$$

In the simulations the equation (5.30) was used.

Gas phase PDE's

Continuity equation

$$\frac{\partial(\epsilon \rho_f)}{\partial t} = r_{H_2O}, \tag{5.32}$$

ρ_f stands for the gas density. To determine ρ_f, the thermodynamic state equation is used

$$\rho_f = \frac{p}{rT_f}, \tag{5.33}$$

where p is the gas pressure, r the is the specific gas constant and T_f the gas temperature. r_{H_2O} is known from (5.1).

Gas species continuity equation

$$\frac{\partial(\epsilon\rho_f X_i)}{\partial t} = \frac{\partial}{\partial x}\Big(\epsilon\rho_g D_{a,eff}\frac{\partial X_i}{\partial x}\Big) + \epsilon r_i \tag{5.34}$$

The description of the gas species continuity equation here is taken from [11]. Equation (5.34) describes the mass transfer of individual gas species subscripted by i, $i \in \{CO_2, O_2, N_2, H_2O\}$. $D_{a,eff}$ is a so-called effective axial dispersion coefficient

$$D_{a,eff} = D_i, \tag{5.35}$$

where D_i is the molecular diffusion coefficient from [15]. The species continuity equation is solved with respect to the mass fraction of the species X_i. The specific gas constant r used in (5.33) is a function of the unknowns X_i:

$$r = R\sum_i \frac{X_i}{M_i}, \tag{5.36}$$

where R is the ideal gas constant and M_i is the molar mass of the species.

Enthalpy equation

$$\frac{\partial(\epsilon\rho_f h_f)}{\partial t} = \frac{\partial}{\partial x}\Big(\epsilon k_f\frac{\partial T_f}{\partial x}\Big) + h_H S(T_s - T_f), \tag{5.37}$$

where h_f is the specific enthalpy of the gas and k_f is thermal conductivity of the gas. The specific enthalpy h_f for the gas energy equation is calculated analogously like in (5.18) and (5.20)for the solid energy equation. The specific heat capacity c_{p_f} and its mean value are given as

$$c_{p_f} = 1161.482 - 2.368819T_f + 0.01485511T_f^2 - \tag{5.38}$$

$$-5.034909 \cdot 10^{-5}T_f^3 + +9.928569 \cdot 10^{-8}T_f^4 - \tag{5.39}$$

$$-1.111097 \cdot 10^{-10}T_f^5 + 6.540196 \cdot 10^{-14}T_f^6 - 1.57353\dot{8}10^{-17}T_f^7$$

and

$$\bar{c}_{p_f} = \frac{1}{\Delta T_f}\int_{T_{f,ref}}^{T_{f,final}} c_{p_f} dT_f. \tag{5.40}$$

The thermal conductivity of gas k_f is [11]

$$k_f = 5.66 \cdot 10^{-5}T_f + 0.011 \tag{5.41}$$

Initial and boundary conditions

To complete the system of PDE's, initial and boundary conditions are required.

Initial conditions define the state of the solid and gas phases at time $t = 0$, i.e. at the beginning of the process. For $\forall x \in \langle 0, h_b \rangle = \langle A, B \rangle$
solid phase:

$$m_s(x, 0) = m_{s,0} \tag{5.42}$$

$$T_s(x, 0) = T_{s,0} \tag{5.43}$$

gas phase:

$$X_i(x, 0) = X_{i,0} \tag{5.44}$$

$$T_f(x, 0) = T_{g,0} \tag{5.45}$$

While the initial conditions are easy to obtain, this is not the case for the boundary conditions. The boundary conditions should reflect the system's behavior. At the bottom of the bed for both solid and gas temperature and mass fractions of gas species Neumann boundary condition is prescribed and is assumed to be zero. At the top of the bed the radiation heat source Q_r is taken into account, so for the solid energy equation heat influx $\dot{q}_r$ due to irradiation is considered. For gas temperature Neumann boundary condition was assumed, and for the mass fractions of gas species Dirichlet condition was assumed.

For $\forall t \in \langle 0, \tau \rangle$:
solid phase:

$$\frac{\partial T_s(A,t)}{\partial x} = 0, \quad \frac{\partial T_s(B,t)}{\partial x} = \dot{q}_r \tag{5.46}$$

gas phase:

$$\frac{\partial T_f(A,t)}{\partial x} = 0, \quad \frac{\partial T_f(B,t)}{\partial x} = 0 \tag{5.47}$$

$$\frac{\partial X_i(A,t)}{\partial x} = 0, \quad X_i(B,t) = X_{i,B} \tag{5.48}$$

As the initial and boundary conditions are set, the mathematical model is completed. The system of governing equations is non-linear. The solution to the system of PDE's is going to be found by a numerical method.

5.4. Numerical methods used used for the model

The numerical model into which the detailed model was implemented was written by [11]. In [11] the finite volume method (FVM) was used. The most important feature of the FVM is its simplicity and its straightforward compliance with the conservation laws even for the most complex equations. Therefore, FVM is chosen for discretization of the governing equations.

Here just the most important features of the FVM are summarized. More about the FVM can be found in [27], [8] or [9].

The so-called general transport equation serves for discretization of the governing equations. Its advantage is that it is applicable to all equations of the model described before. For variable Φ its one-dimensional form is introduced

$$\frac{\partial(\rho\Phi)}{\partial t} + \frac{\partial(\rho u\Phi)}{\partial x} = \frac{\partial}{\partial x}\Big(\Gamma\frac{\partial\Phi}{\partial x}\Big) + S_\Phi. \tag{5.49}$$

Equation (5.49) consists of the time, the convective, the diffusion and the source term.

The first step of the FVM is grid generation. The computational domain is divided into a finite number of discrete control volumes, also called cells. It is defined by a finite number of nodal points which are placed between the line segment of $\Omega = (A, B)$. In this work $A = 0$ and $B = h_b$ as stated in the section with initial and boundary conditions. The boundaries (or faces) of control volumes are positioned mid-way between adjacent nodes. Thus each node is surrounded by a control volume or cell [27].

Next step is the integration of the governing equation (5.49) over each control volume

$$\int_w^e \frac{\partial(\rho\Phi)}{\partial t}dx + (\rho u\Phi)_e - (\rho u\Phi)_w = \Big(\Gamma\frac{\partial\Phi}{\partial t}\Big)_e - \Big(\Gamma\frac{\partial\Phi}{\partial t}\Big)_w + \overline{S}\Delta x, \tag{5.50}$$

where Γ is the diffusion coeffiecient and $\overline{S}$ is the average value of source over the control volume. Each term of (5.50) is going to be discretized. There is no convective term in the governing equations (5.12) – (5.37). So (5.50) is simplified for the diffusion-problem. $\overline{S}$ is linearized in the form

$$\overline{S} = S_C + S_P\Phi, \tag{5.51}$$

S_C is the constant part and S_P is the coefficient part before Φ.

Using the central difference scheme for the gradients, the diffusion term of (5.50) is established as

$$(\Gamma\frac{\partial\Phi}{\partial t})_e - (\Gamma\frac{\partial\Phi}{\partial t})_w = \frac{\Gamma_e}{\Delta x}(\Phi_E - \Phi_P) - \frac{\Gamma_w}{\Delta x}(\Phi_P - \Phi_W). \tag{5.52}$$

Γ_e and Γ_w are the diffusion coefficients at eastern and western cell face, respectively. To calculate them, linear interpolation of values of the neighboring nodes is used

$$\Gamma_e = \frac{\Gamma_E + \Gamma_P}{2}, \quad \Gamma_w = \frac{\Gamma_P + \Gamma_W}{2}. \tag{5.53}$$

Let denote

$$D = \frac{\Gamma}{\Delta x}. \tag{5.54}$$

Only the spatial discretization of the time term left, it is of the form

$$\int_w^e \frac{\partial(\rho\Phi)}{\partial t}dx = \frac{\partial(\rho\Phi)}{\partial t}\Delta x. \tag{5.55}$$

After simplification and substituting (5.51), (5.52) and (5.55) into (5.50) following form of (5.50) is obtained

$$\begin{aligned}\frac{\partial(\rho\Phi)}{\partial t}\Delta x + (D_e + D_w + S_P\Delta x)\Phi_P &= \\ &= D_e\Phi_E + D_w\Phi_W + S_C\Delta x.\end{aligned} \tag{5.56}$$

The temporal discretization is carried out by Euler method [11], the time interval $(0, \tau)$ is divided into M subintervals of the same length, so time step is $\Delta t = \tau/M$. After integrating and dividing by Δt

$$\frac{\partial(\rho u)}{\partial t}\Delta x = \frac{\Delta x}{\Delta t}(\rho_P\Phi_P - \rho_P^0\Phi_P^0), \tag{5.57}$$

where the subscript 0 denotes the known value in previous time t.

Equation (5.57) becomes

$$(a_e + a_W + +S_P\Delta x + \frac{\Delta x}{\Delta t}\rho_P)\Phi_P = \\ = a_E\Phi_E + a_W\Phi_W + a_P^0\Phi_P^0 + S_C\Delta x. \tag{5.58}$$

The final general discretization equation is

$$a_P\Phi_P = a_E\Phi_E + a_W\Phi_W + a_P^0\Phi_P^0 + S_C\Delta x \tag{5.59}$$

$$a_E = D_e \tag{5.60}$$

$$a_W = D_w \tag{5.61}$$

$$a_P^0 = \frac{\Delta x}{\Delta t}\rho_P^0 \tag{5.62}$$

$$a_P = a_E + a_W + a_P^0 + (S_P + \widetilde{R}_C)\Delta x \tag{5.63}$$

The source term $\widetilde{R}_C$ is included in the equation if the continuity equation for the flow field is satisfied

$$\frac{\partial \rho}{\partial t} = \widetilde{R}, \tag{5.64}$$

where $\widetilde{R}$ is discretized as $\widetilde{R}_C\Delta x$.

Boundary conditions are completing the system again, the Dirichlet b.c.

$$\Phi(A,t) = \Phi_A \tag{5.65}$$

and the Neumann b.c.

$$\frac{\partial \Phi(A,t)}{\partial x} = 0 \quad \text{and} \quad \frac{\partial \Phi(B,t)}{\partial x} = 0. \tag{5.66}$$

Boundary conditions give the values at cell faces on the boundaries of the computational domain. Corresponding equations of the boundary conditions will be discretized to each governing equation alone.

Further details about the method and its properties are in e.g. [8] or [27]. The exact discretization of variables is carried out in

Jurena.

5.5. Simulation and results

To obtain results of the mathematical model, simulations were made. The numerical formulation of the mathematical model is implemented in MATLAB-code. The main function of the MATLAB code was made by [11]. The model of [11] is dealing with problem of grate combustion and is also implemented with FVM. However, some changes were necessary to made in order to get adequate results for the drying model. First of all, the functions of the coefficients occuring in the governing equations (e.g. solid and gas heat capacities, thermal conductivities, the volumetric surface area of the particle, etc.) were written and replaced with `m-files` on the corresponding places. The next very important issue was to rewrite the boundary conditions for all the governing equations. (While, for instance, in the model of [11] plug-flow of gas was assumed through the bed, here this isn't the case. So the mass flow rate across the sludge was assumed to be zero.) Further, a script was created where all data are initialized. These user defined data file contains the initial conditions for both solid and gas phases, the properties of the dryer plant (like the footprint area) and the sludge properties, like its initial moisture content, density, particle size, etc. Also simulation time, the time and spatial steps and convergence criteria are defined here. The main functions from which all other functions are called is `gc.m` [11].

5.5.1. Simulation

The purpose of the simulation is to examine the moisture evaporation in a solar dryer plant. To simulate the drying, following assumptions were made:

- The drying simulation was carried out in summer period, the average daily temperature and irradiation data were considered for Brno [19].
- The simulation period was one day.
- The footprint area of the plant was set to 400 m^2 (i.e. 20 × 20 m) and the bed height was 0.5 m.
- The amount of sludge to be dried was $20 \dot{1}0^3$ kg, and the initial moisture content was assumed to be 80%.
- The values of solid and gas temperatures, T_s and T_f, respectively, were considered to be the same at the beginning of the drying process.
- The only heat supply was the irradiation of the Sun.

The properties of sludge like its density or particle size, were set according to available data in [17], [10] and [18]. For the case, when some of the sludge properties were not available (e.g. efficient thermal conductivity), they were replaced by the properties of soil. The drying was considered to go on at constant pressure.

The climate data considered where the average daily temperature and irradiation in July (for the city of Brno), the reason for this choice was that the evaporating rate is the highest in the summer months.

During drying simulation the sludge was not assumed to be mixed, i.e. that after certain time the moisture content was not distributed uniformly. That is the reason the simulation was carried out just for one day. For longer simulations, the sludge mixing is recommended, so the moisture in the sludge will be again uniformly distributed. Without the mixing the upper layers of the sludge become dry while near the bottom of the bed the sludge is still full of moisture. Before presenting the results, the initial values are summarized:

- initial weight of the wet sludge $m_s = 20\dot{1}0^3$ kg, initial moisture content was 80 %;
- bed height $h_b = 0.5$ m and bed area were $A = 400\ m^2$

- there initial gas and solid temperatures were both set to $T_g = T_s = 308.15$ K, at the beginning of the drying process they were assumed to be in equilibrium
- initial mass fractions of gas species were the following: $X_{H_2O} =$, $X_{CO2} = 0$, $X_{O_2} = 0.21$, $X_{N_2} = 0.79$,
- sludge properties: $r_{part} = 15$ mm, $\rho_{particle} = 1800\ kg/m^3$ [17],
- for the irradiation the average monthly data in July was assumed $Q_r = 260\ W/m^2$
- the drying period was 24 hours.

5.5.2. Results

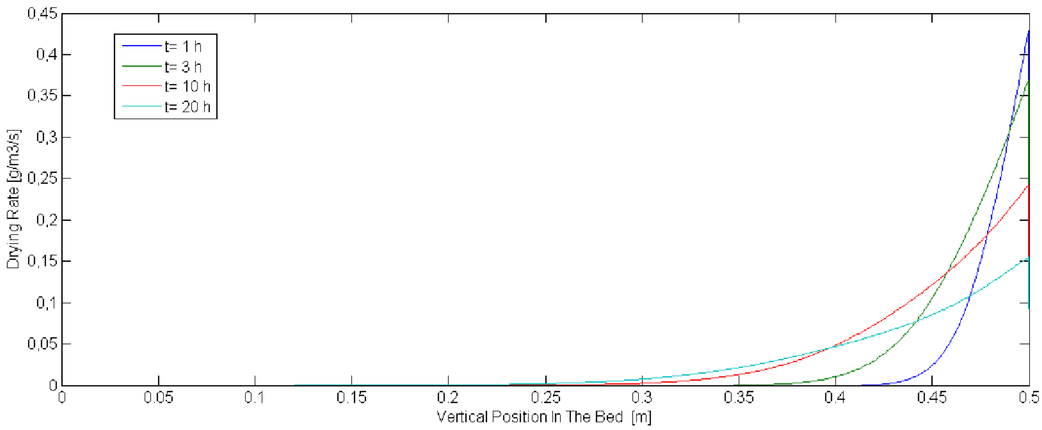

Figure 5.1: Drying rate at different times

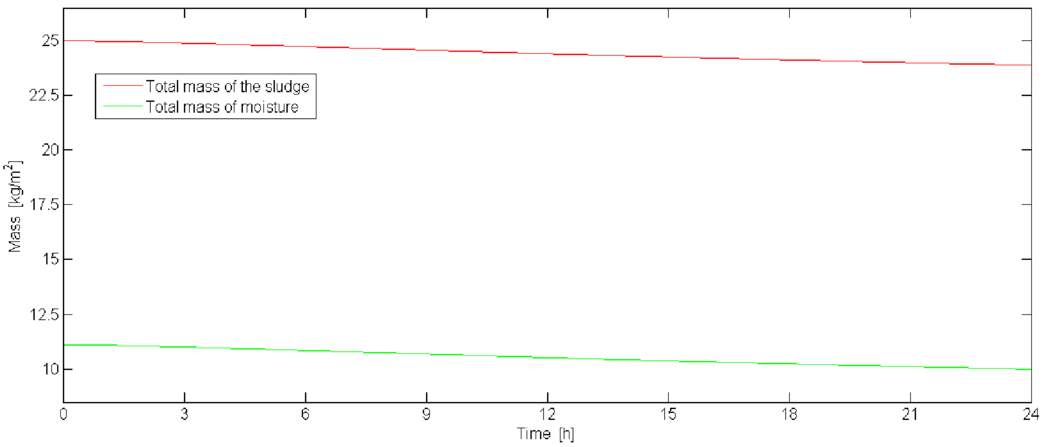

Figure 5.2: Mass of water and mass of total sludge

Figure (5.1) illustrates the drying rate at different times t=1, 3, 10 and 20 h. It can be seen from figure that the drying rate has the highest value at the top of the bed, and towards the bottom the rates are decreasing, until they will be zero. This figure gives

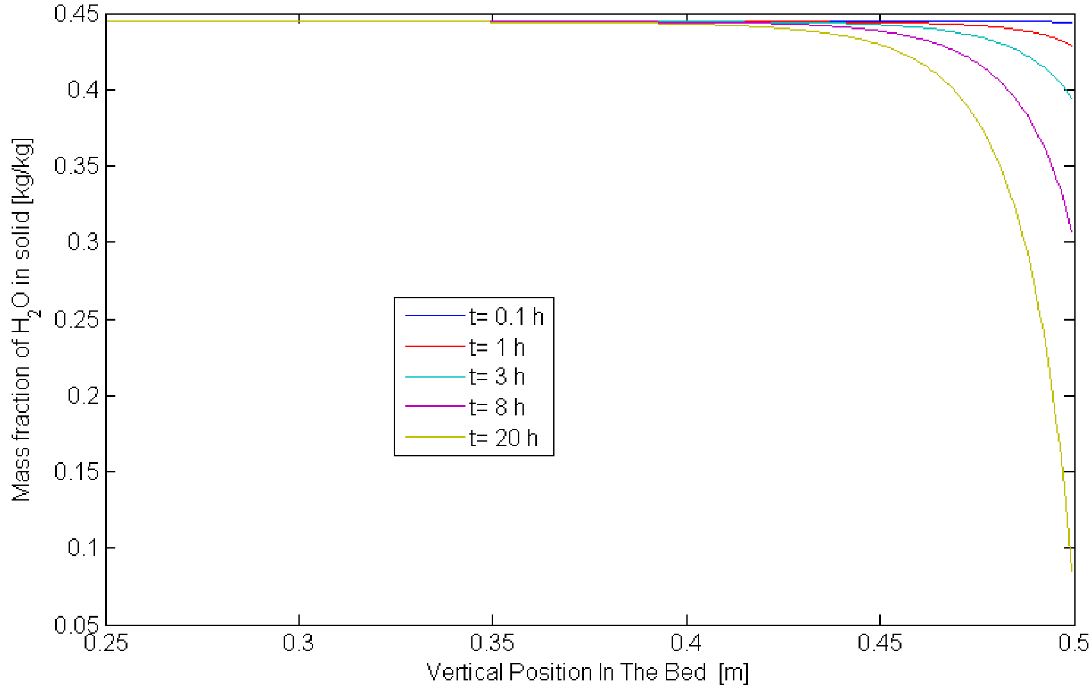

Figure 5.3: Concentration of H_2O in the solid

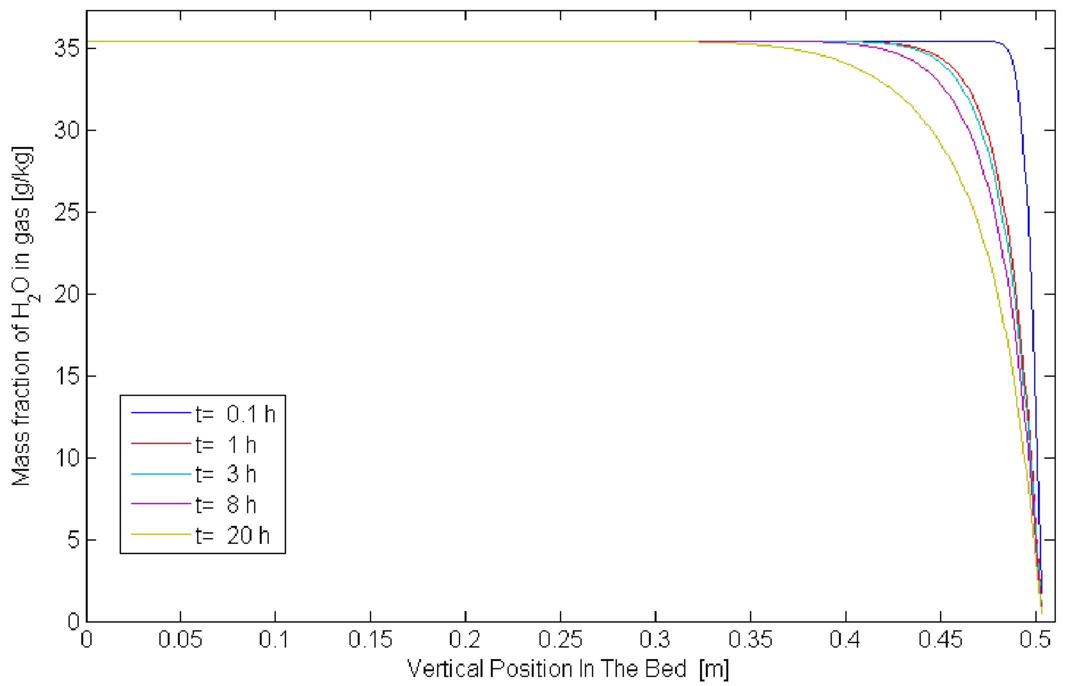

Figure 5.4: Concentration of H_2O in the gas

realistic result, higher drying rate values are expected at the surface since the evaporation

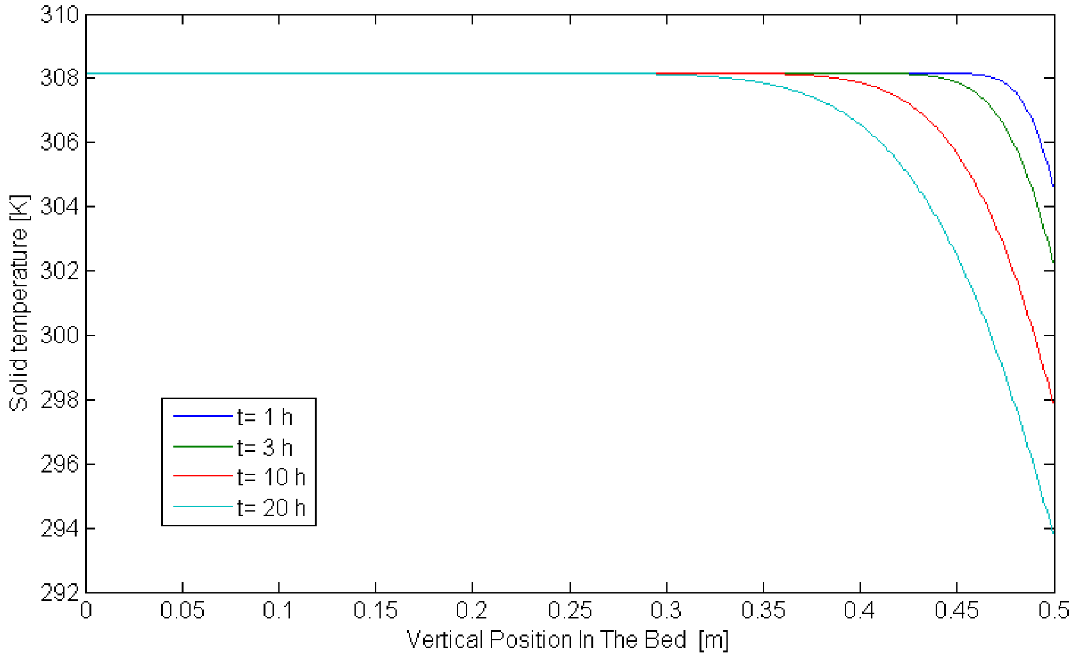

Figure 5.5: Solid temperature distribution in the bed at that different times

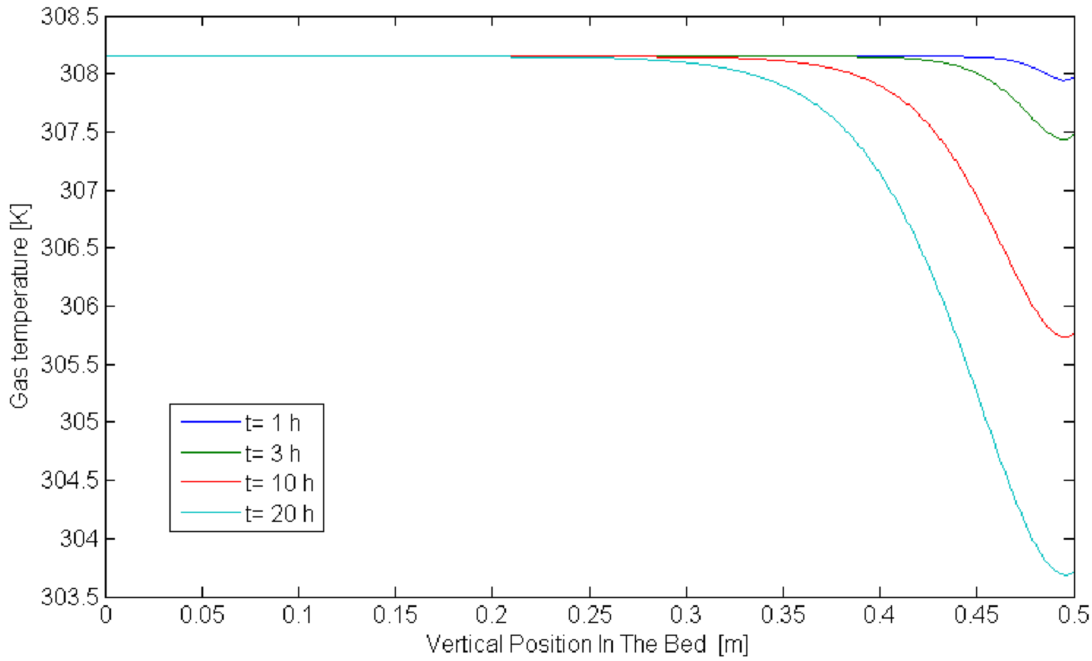

Figure 5.6: Gas temperature

takes place there. The figure (5.2) shows mass reduction of the total mass and of the water in the sludge per square meter.

Next figure(5.3) shows the mass fraction of the water in solid dependence on the bed height for different times. The representative times were chosen in 10th minute and in

1st, 3rd, 8th and 20th hour od the drying simulation. It can be seen in this figure how the mass fraction of H_2O is decreasing near the surface layer where the evaporation takes place. At time t=0.1 h the decrease of the concentration near the surface was negligible, while at time t= 20 h it was approaching zero. The second figure (5.4) shows analogously the mass fraction of water in the gas. Both figures (5.3) and (5.4) seem to be correct and these results are acceptable.

Figure (5.5) shows the evolution of the solid temperature during drying. On the figure (5.5) can be seen that the temperature in solid is decreasing. But this may be not realistic, because after the moisture is evaporated from the surface, the solid temperatures should increase due to the irradiation. The figure (5.6) shows the gas temperature dependence on the bed height with the same representative times as for the solid temperature. Comparing to the solid temperature, one can see on the figure (5.6) that the temperatures start to increase after a while. The decrease in the temperature is also much smaller then in the solid's case.

5.6. Summary

In the chapter a detailed description of mathematical model has been presented. The evaporation and the diffusion processes were also described. All coefficients occuring during drying were also determined and at the end the system of equations was completed with the initial and boundary conditions. Further, most important features of the FVM were summarized. At the end of the chapter the simulation and the results were presented.

6. Conclusion

The thesis provided a review in the field of drying and of different approaches to modeling of drying were summarized. First approximative scoping of a solar plant was also presented. Empirical models predicting the evaporation rate were described, too. Among the empirical models, the multiplicative model seemed to give the best results. The sensitivity of different parameters occuring in the multiplicative model were compared. Daily estimates of the evaporation rate and the loss of water in the sludge were calculated by numerical integration formula. The second part of the thesis described a detailed one-dimensional mathematical model. The coefficients occuring in the equations of the model were determined. The partial differential equations governing the drying process were discretized by the finite volume method. A simulation was made for a period of one day, and the results were presented. Longer simulations were not provided since the model was not considering the effect of sludge mixing. The results of the drying process may be improved by considering the mixing of the sludge every few hours or every day.

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7. Nomenclature and Acronyms

A	footprint area $[m^2]$
$c_{p,f}$	specific heat capacity of the gas phase $[J\dot{k}g^-1K^-1]$
$c_{p,s}$	specific heat capacity of the solid phase $[J\dot{k}g^-1K^-1]$
$c_{p,sludge}$	specific heat capacity of the sludge $[J\dot{k}g^-1K^-1]$
$c_{p,water}$	specific heat capacity of the water $[J\dot{k}g^-1K^-1]$
$\overline{c}_{p,f}$	mean specific heat capacity of the gas phase $[J\dot{k}g^-1K^-1]$
$\overline{c}_{p,s}$	mean specific heat capacity of the solid phase $[J\dot{k}g^-1K^-1]$
$C_{w,f}$	concentration of moisture in the gas phase $[kg/m^3]$
$C_{w,s}$	concentration of moisture at the solid surface $[kg/m^3]$
c	control vector
D	diffusion coefficient $[m^2/s]$
D_0	Arrhenius factor [-]
$D_{a,eff}$	effective axial dispersion coefficient $[m^2/s]$
D_A	vapor diffusivity in air $[m^2/s]$
D_i	molecular diffusion coefficient $[m^2/s]$
E_A	activation energy for diffusion $[s^-1]$
E	evaporation rate $[mm/h]$
e	outdoor weather vector
h_b	bed height $[m]$
h_s	solid phase specific enthalpy $[J/kg]$
h_f	gas phase specific enthalpy $[J/kg]$
h_H	heat transfer coefficient $[W/Km^2]$
ΔH_{vap}	latent heat of evaporation $[J/kg]$
k_m	mass transfer coefficient $[m/s]$
k_{eff}	effiecent thermal conductivity of solid $[W/m\dot{K}]$
k_f	thermal conductivity of gas$[W/m\dot{K}]$
k_l	thermal conductivity of liquid $[W/m\dot{K}]$

k_s	thermal conductivity soil$[W/m\dot{K}]$
k_{par}	thermal conductivity of parallel layerer pore system $[W/m\dot{K}]$
k_{perp}	thermal conductivity of perpendicular layerer pore system $[W/m\dot{K}]$

L characteristic length [m

m_{H_2O}	mass of water in sludge $[kg]$
M_out	mass of treated sludge per year at the end $[kg/a]$
m_s	total mass $[kg]$
$m_{drysolid}$	mass of solid $[kg]$
M_wet	mass of treated sludge per year at the beginning $[kg/a]$
P	predictor
p	number of predictors (empirical models)
Q_m	air mixing rate $[m^3/m^2h]$
Q_v	air ventilating rate $[m^3/m^2h]$
Q_r	solar irradiation $[W/m^2]$
Q_{H_2O}	vaporization heat loss $[W/m^2]$
R	ideal gas constant $[J/K\dot{k}g]$
r	specific gas constant $[J/K\dot{k}g]$
Re	Reynolds number
R_o	solar irradiation $[W/m^2]$
r_{H_2O}	rate of evaporation $[kg/m^3\dot{s}^-1]$
$r_{particle}$	radius of sludge particle $[m]$
S	mass of sludge solid $[kg/m^2]$
S	particle surface area $[m^2/m^3]$
Sh	Sherwood number
Sc	Schmidt number
S_Φ	source term
s	state of sludge vector
t	time $[s]$

T_o	outdoor temperature $[K]$
T_s	solid temperature $[K]$
T_f	gas temperature $[K]$
U	wind speed [m/s]
V	volume $[m^3]$
v	velocity of convecting air $[m/s]$
W	mass of sludge water $[kg/m^2]$
W_0	initial mass of sludge water $[kg/m^2]$
w_{in}	humidity ratio at inlet $[kg/kg]$
$w_i n$	humidity ratio at inlet $[kg/kg]$
X_i	gas phase species $[kg/kg]$
X	moisture content $[kg/kg]$
x	spatial variable $[m]$
α	proportionality coefficient
β	shift
γ	exponent
Γ	diffusion coefficient
δ	constrictivity
ϵ	coefficient [-]
ϵ	porosity [-]
ρ	air density $[kg/m^3]$
ρ_s	bulk density of sludge $[kg/m^3]$
ρ_f	gas density $[kg/m^3]$
ρ_l	gas density $[kg/m^3]$
σ_{in}	dry solid content at the beginning [%]
σ_{out}	dry solid content at the end [%]
σ	dry solid content of the sludge [%]
Φ	an unknown function

τ	tortuosity
CFD	Computational Fluid Dynamics
DSC	dry solid content of the sludge
FVM	finite volume method
PDE	partial different equations
CFD :	Computational Fluid Dynamics
DSC :	dry solid content of the sludge
FVM :	finite volume method

8. Appendix A

8.1. Classification of dryer models, summary of properties of existing sloar drying plants

Type of calculation	Physical basis of calculation	Dryer details calculated	Main uncertainties	Typical us
Heat and mass balance	Heat and mass balance only	None	Mass flowrate, moisture content, gas humidity	Performance
Scoping design; continuous convective dryers	(a) Heat and mass balance only, (b) heat transfer	Overall dimensions: (a) Cross-section, or diameter (b) length	(b) Heat transfer coefficients falling rate dryig kinetics	Design
Scoping design for continuous contact dryers	Heat and mass balance, heat transfer	Overall dimensions, diameter, length residence time	Heat transfer coefficients, temperature difference, falling rate kinetics	Design
Scoping design for batch dryers	Mass and volume of solids	Overall dimensions, diameter, length	Drying time and all factors influencing it	Design
Scaling (integral model)	Heat and mass balance, heat transfer, drying curve	Overall dimensions, diameter, length, residence time	Local variations inside dryer	Scale-up, performance
Detailed design, incremental model (1-D)	Heat and mass balance, heat transfer, full drying kinetics	Length (using scoping method), local conditions	3-dimensional flows, parameter measurment	Design, scale-up
Detailed design, CFD	Heat and mass balance, heat transfer, kinetics 3-D flow patterns	Local conditions throughout dryer	Measuring required parameters computing time	Design, scale-up performance

Table 8.1: Comparison of features of available models for dryers

	Plant (Country)	Amount of sludge $[t/year]$	Initial DSC [%]	Area $[m^2]$	Annual evap. rate $[kg/m^2/a]$	additional equipment
1	Iffezheim (GER)	600	20	580	5,98E+02	–
2	Burgrieden (GER)	1 100	27	1054	4,49E+02	photo-voltaic installation, glass covering
3	Allershausen (GER)	1 000	20	720	8,02E+02	–
4	Albstadt (GER)	4 200	35	1632	6,72E+02	aux.heating - waste heat, air heating
5	Sigmaringen (GER)	1 500	30	1296	4,24E+02	–
6	Main-Mud (GER)	4 000	25	3000	6,30E+02	waste heat, air heatin
7	Karlsfeld (GER)	1 560	26	1488	4,73E+02	–
8	Weil am Rhein (GER)	5 544	26	2880	8,68E+02	waste heat, air heating
9	Miltenberg (GER)	6 000	23	3000	1,03E+03	–
10	Bilten (SUI)	1 800	25	1440	5,90E+02	waste heat, floor heating
11	Sargans (SUI)	1 200	25	1200	4,72E+02	waste heat, air heating
12	Sebersdorf (AUT)	500	20	460	6,28E+02	–
13	Vils (AUT)	1 500	40	1104	2,11E+02	fermentation gas, dark radiators
14	Knittelfeld (AUT)	2 300	23	1488	7,95E+02	waste heat, air heating
15	Bad Waltersdorf (AUT)	500	20	520	5,56E+02	–
16	Passail (AUT)	750	22	800	5,02E+02	–
17	Gasselsdorf (AUT)	1 300	25	1080	5,68E+02	–
18	Ensisheim (FRA)	1 500	16	1512	6,57E+02	–
19	Brumath (FRA)	2 400	27	2000	5,16E+02	–
20	Dieuze (FRA)	870	17	960	5,81E+02	–
21	Romorantin-Lanthenay (FRA)	3 713	16	2544	9,67E+02	–
22	Vire (FRA)	5 810	20	4320	7,77E+02	–
23	Folschviller (FRA)	2 000	26	1392	6,48E+02	–
24	Ilawa (POL)	1 575	20	1536	5,92E+02	waste heat, floor heating
25	Zary (POL)	3 574	19	4176	5,13E+02	–
26	Veszprem (HUN)	7 500	28	3096	9,91E+02	waste heat, air heating
27	Wetalla (AUS)	15 100	15	4680	2204,777457	
38	Boneo (AUS)	5 860	13	4176	1018,142725	–

Table 8.2: Summary of properties of existing solar dryer plants [24], [3].

8.2. Climate maps of Europe and Czech Republic

Several maps giving information about climate data (average temperature, irradiation and precipitation maps) for Europe and Czech Republic can be found here. The maps are form [19] and [7].

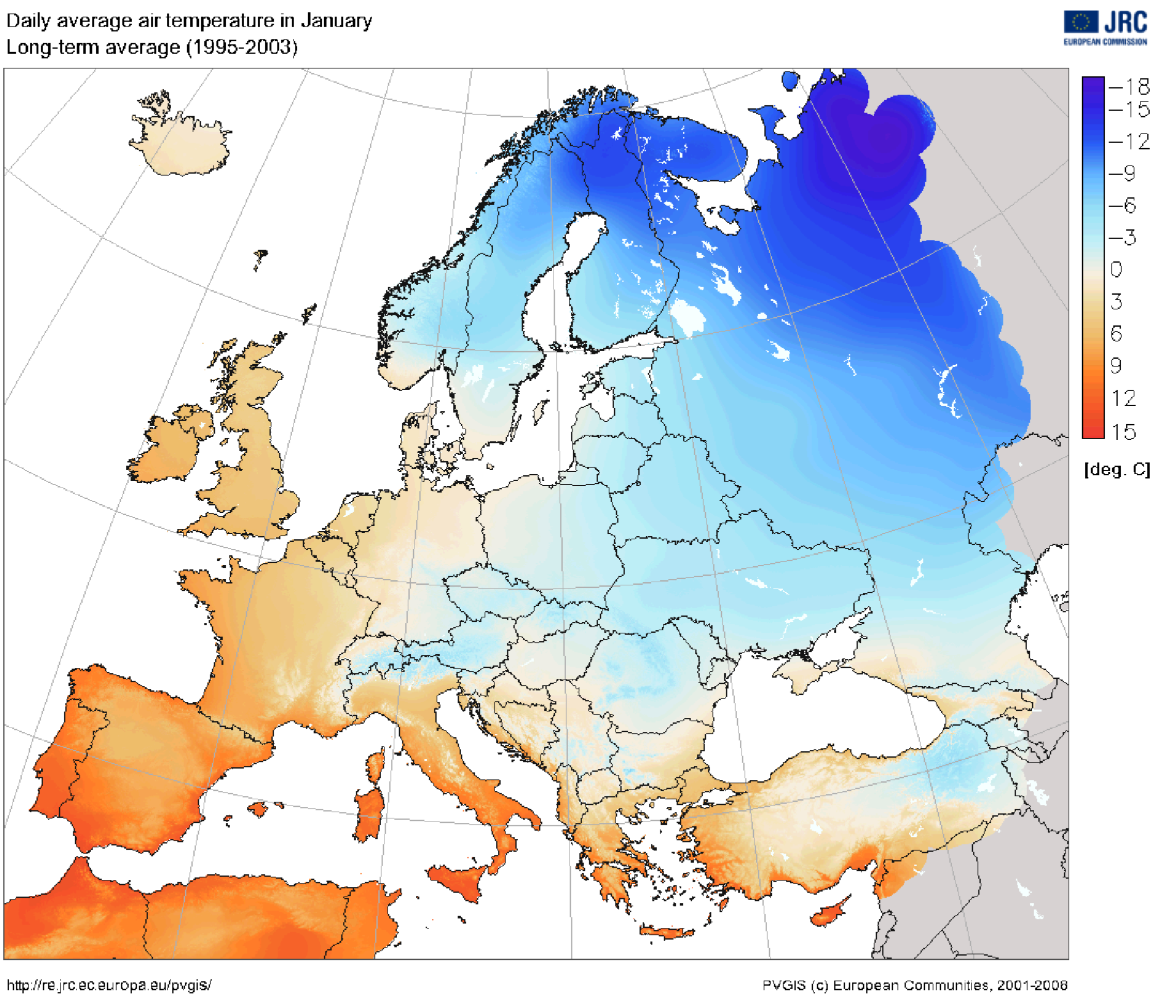

Figure 8.1: Europe - average temperature in January

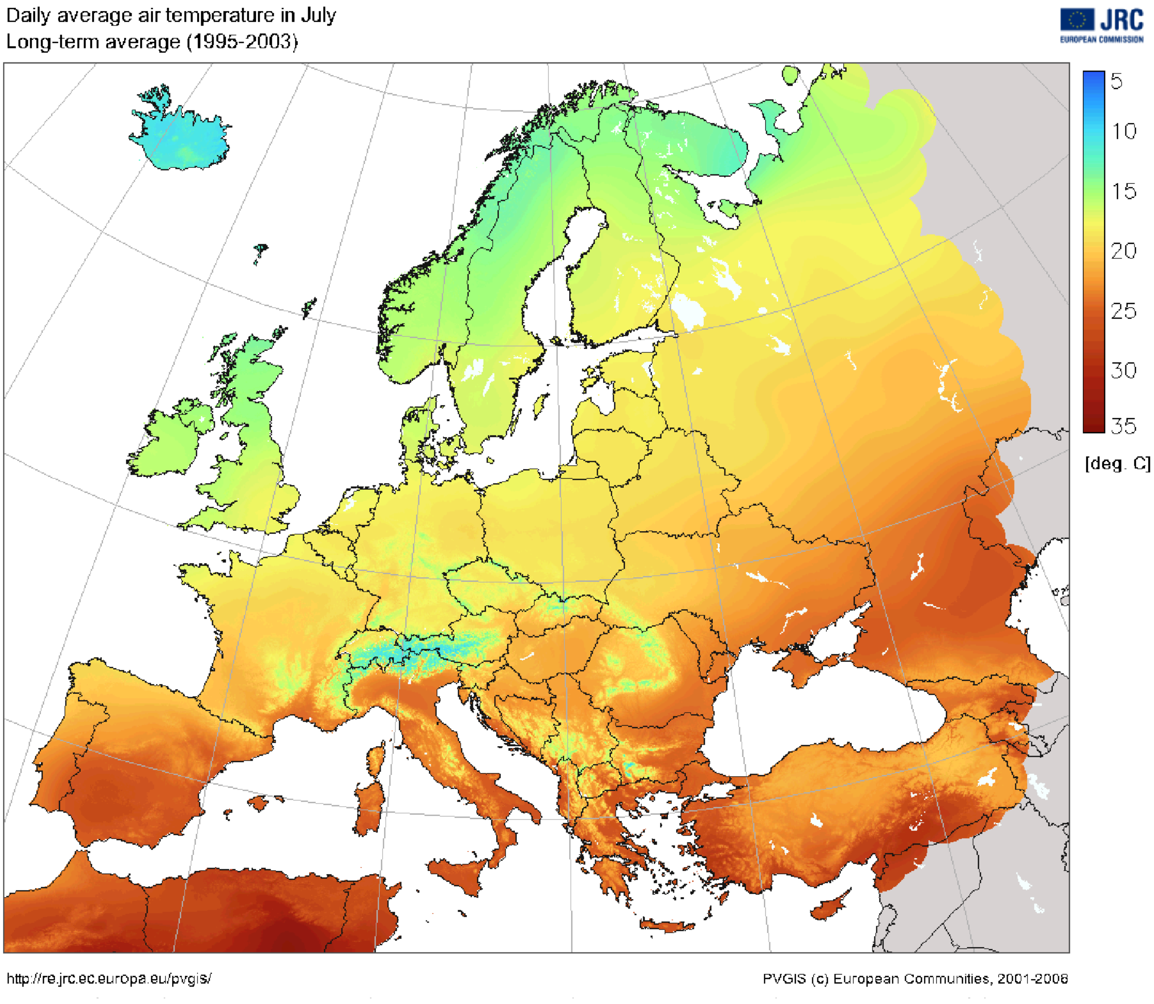

Figure 8.2: Europe - average temperature in July

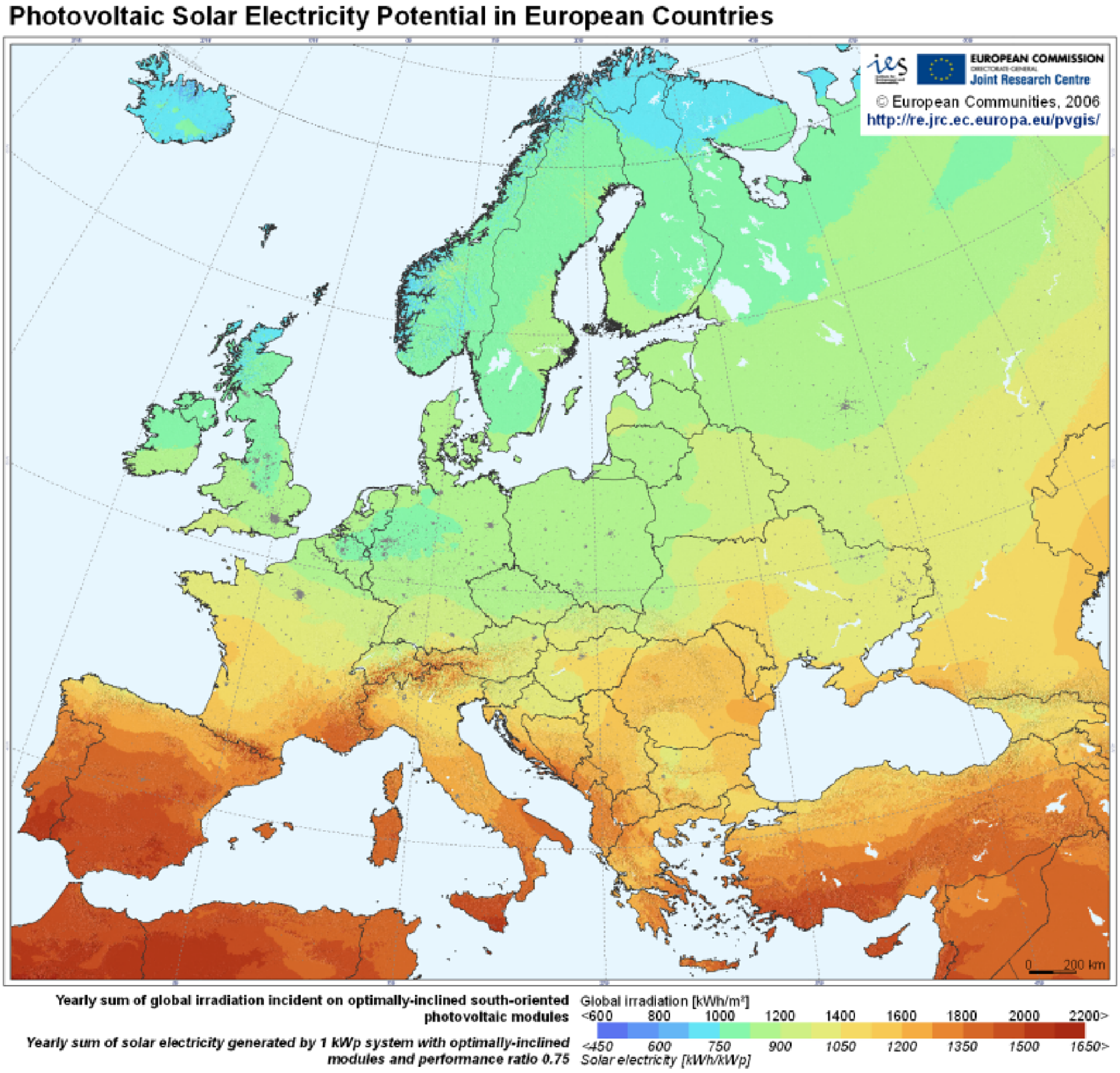

Figure 8.3: Europe irradiation map

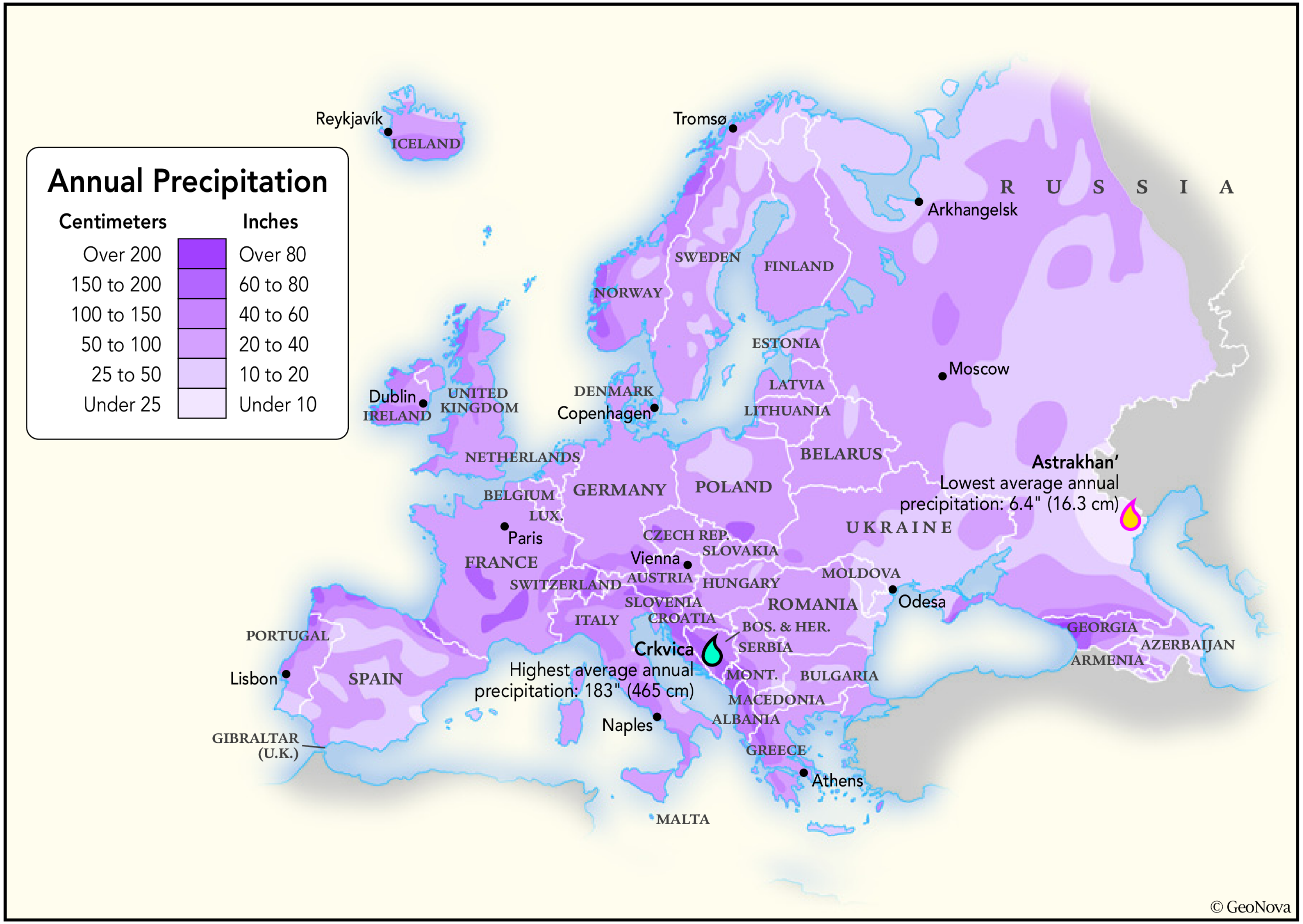

Figure 8.4: Europe - annual precipitation map

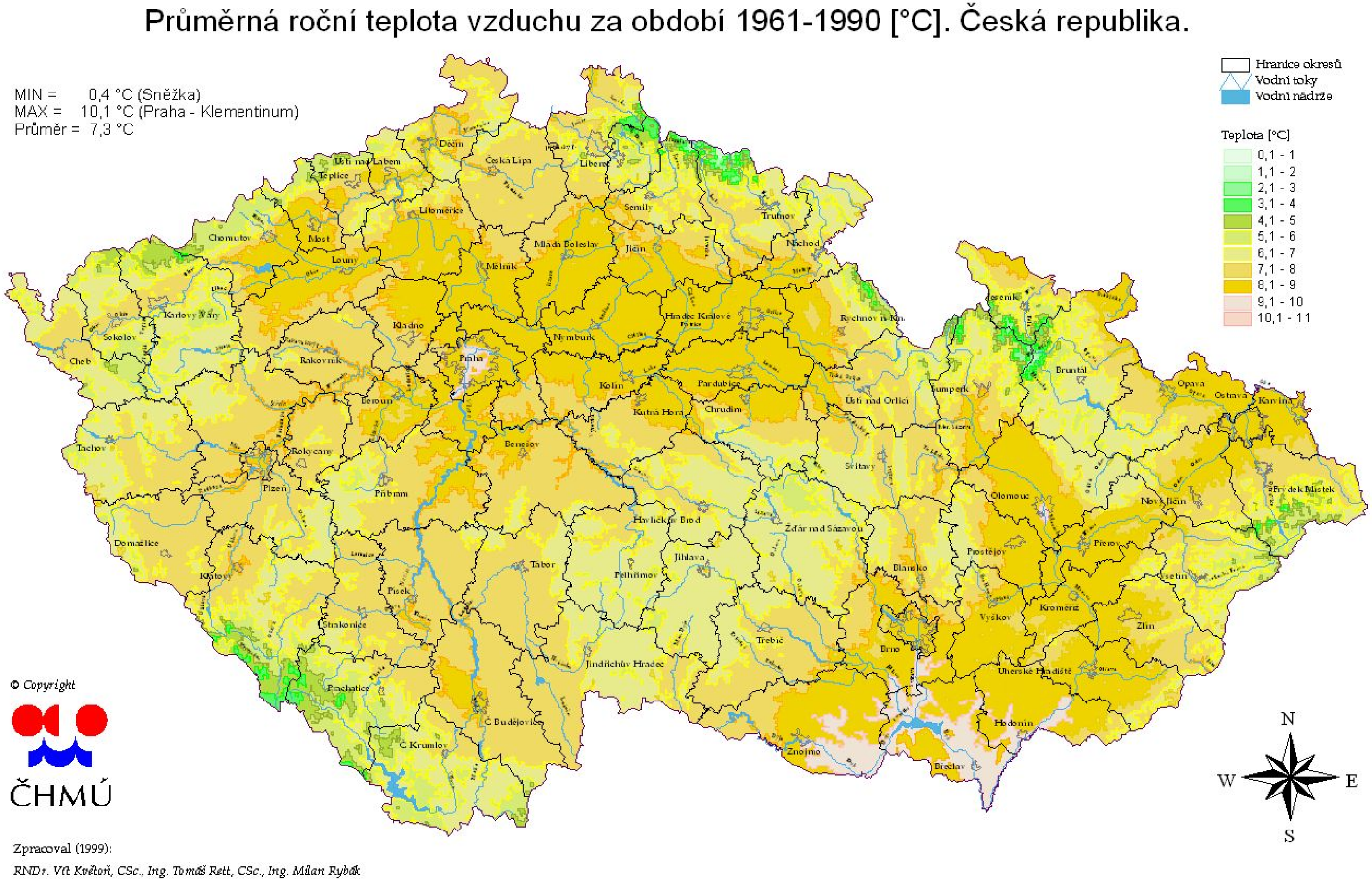

Figure 8.5: Czech Republic - annual average temperature

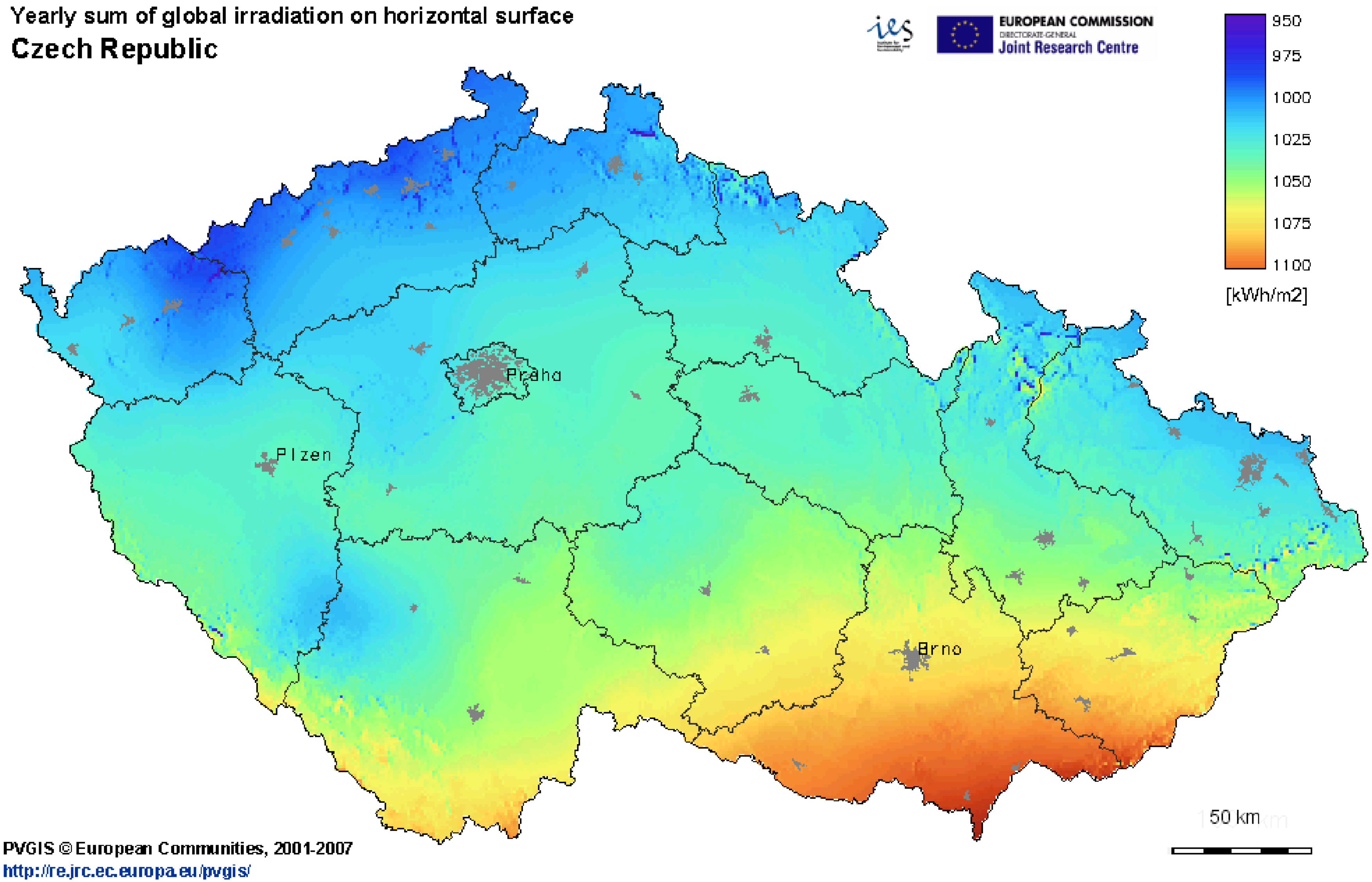

Figure 8.6: Czech Republic - annual average irradiation

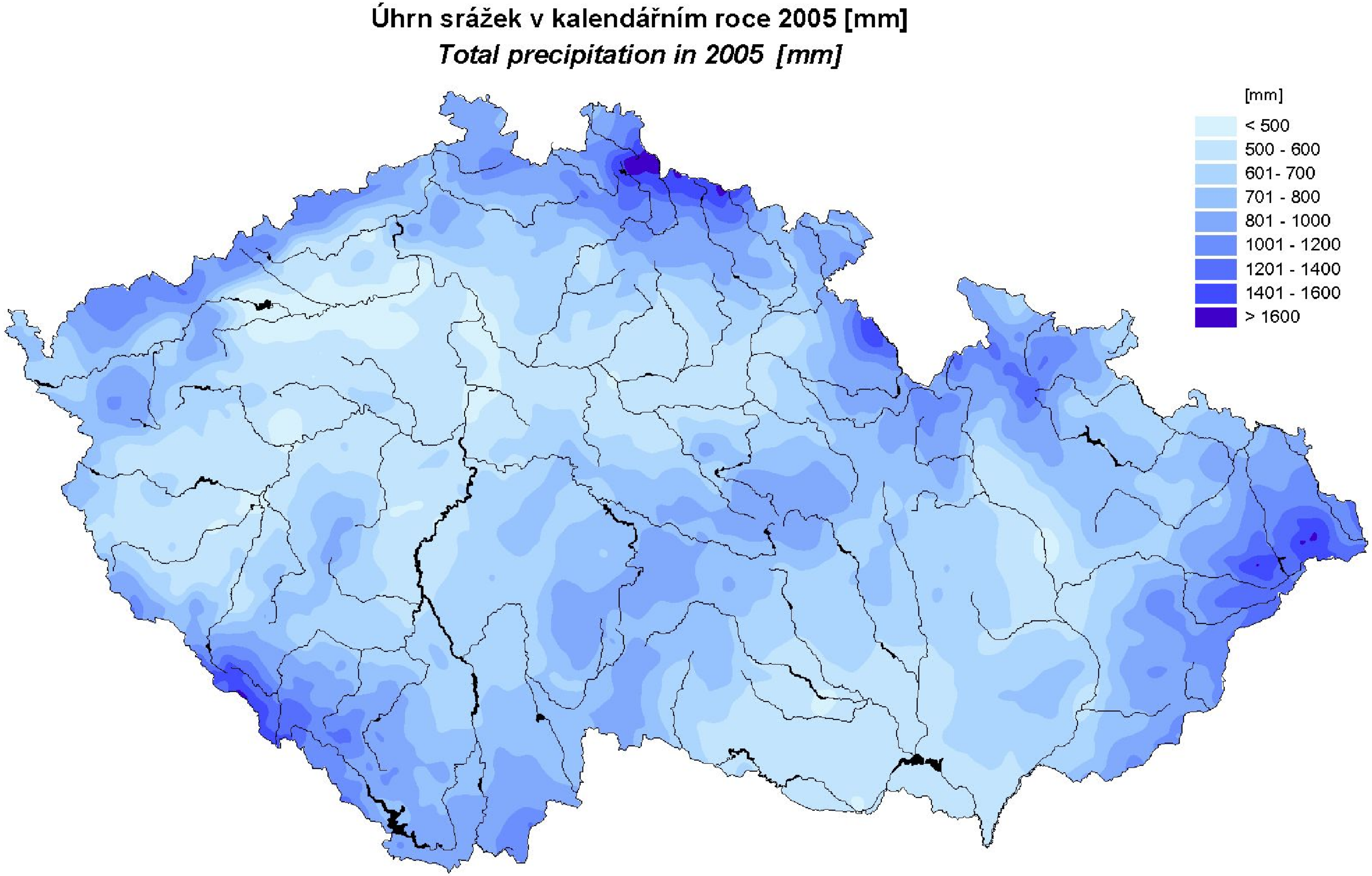

Figure 8.7: Czech Republic - annual precipitation