

Faculty of Economics and Management

Department of Systems Engineering



Bachelor thesis

Travelling salesman problem in selected company

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CZECH UNIVERSITY OF LIFE SCIENCES PRAGUE

Faculty of Economics and Management

BACHELOR THESIS ASSIGNMENT

Mira Mauleshova

Business Administration

Thesis title

Travelling salesman problem in selected company

Objectives of thesis

The aim of this bachelor thesis is to improve the current situation of route scheduling between the selected company and its customers.

Using the selected methods for travelling salesman problem the route scheduling will be analysed and the shorter and more efficient path will be identified.

Methodology

Bachelor thesis consists of theoretical and practical parts. The theoretical part includes review of specialized literature and selection of proper methods of solution. The practical part contains collection of real data and application of appropriate selected methods. After analysis and comparison of real and alternative situations the conclusion will be made based on obtained results.

The proposed extent of the thesis

30-40

Keywords

Logistics, distribution of food production, operations research, Travelling salesman problem, approximation methods.

Recommended information sources

APPLEGATE, D.L., BIXBY, R.E., CHVÁTAL, V. and COOK, W.J. The Traveling Salesman Problem – a Computational Study. Oxford: Princeton University Press. 2006. ISBN 978-0-691-12993-8
DANTZIG, G., FULKERSON, R. and JOHNSON, S. Solution of a Large-Scale Traveling-Salesman Problem. Journal of the Operations Research Society of America. 1954, 2(4), 393-410. ISSN 0096-3984.
GUTIN, G. and PUNNEN, A.P. The Traveling Salesman Problem and Its Variations. New York: Springer. 2007. ISBN 0-387-44459-9

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Declaration

I declare that I have worked on my bachelor thesis titled "Travelling salesman problem in selected company" by myself and I have used only the sources mentioned at the end of the thesis. As the author of the bachelor thesis, I declare that the thesis does not break copyrights of any person.

In Prague on the 7th of November 2018

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Travelling salesman problem in selected company

Abstract

The particular Bachelor Thesis is dedicated to the branch of applied mathematics – operations research, to be more precise, to the approximation methods of Travelling Salesman Problem. In the selected distribution company distance of the routes was intended to decrease by means of the approximation methods in order to reduce distribution expenses. The Bachelor Thesis is divided into theoretical as well as practical part. Theoretical part is arranged as detailed description of logistics in general, its history, objectives and decisions, together with overview of operations research, what is more, the intersection of logistics and operations research, which leads to comprehensive representation of operations research in logistics, such as Travelling Salesman problem, its graphical representation and the methods of solution.

Practical part explains the topic more profoundly, due to the fact, that practical part is tightly connected to the theoretical part, what is more, illustrates how theoretical knowledge can be applied in the real situation of the distribution company, which is described in an anonymous way with respect of professional confidentiality. The features of the company and the way of its route construction are described thank to the real data, which has been provided by the company. Also, by the means of the same real data, the square matrices were constructed, which served as a basis for alternative routes construction. The results were achieved by the means of Nearest Neighbour Algorithm, Vogel Approximation Method and Savings Method, which are introduced by the software TSPkosa. Results were analyzed and compared numerically and graphically. Based on the results of the total distance of the route, the most appropriate one was chosen, and the recommendation was made.

Keywords: Approximation methods, distribution of food production, logistics, operations research, Travelling Salesman Problem.

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1 Introduction

The competitiveness influences the result in the highest possible way. There is always a fierce competitiveness between a substantial number of companies in the market area. According to the Malthus theory – ‘Survival of the fittest’, the aim of all systems is to provide the prolongation of own existence (Darwin, 1859, p. 34). The main aim of a firm is, therefore, to achieve as high profit as possible to survive. When products are interchangeable, a company wills to increase sales volume, so it starts to lure other company’s customer by own products price reduction (Dixit, et al., 2009, p. 139). Price reduction leads to increasing of quantity demanded according to left-to-right decreasing demand curve (Sojka & Konečný, 2006, p. 106). This fact provokes companies to resort into price wars when one competitor underpricing another, that leads to more undercharge prices from other competitors. There are always limits, so this race often results in the insolvency of competitors, what is more, everyone loses profit, which is in opposite to the initial goal (Meadows, 2009, p. 142).

$$\textit{Profit} = \textit{Revenues} - \textit{Expenses}$$

But what if we look at this quotation from a different angle. Focusing only on incoming money is not the only way to be more successful, profit can be increased by the reduction of leaving money. The efficiently constructed system allows to function and develop, not only to survive. Definitively, profit can be increased by reducing expenses at manufacture, inventory keeping, distribution stages etc.

This Bachelor Thesis is dedicated to logistics strategy of expenses reduction by minimization of transportation costs. It is understandable that with reduction of travelled kilometers, fuel consumption and time of delivery go down, so then transportation expenses reduce. Nevertheless, nowadays a lot of companies do not use any methods to become more efficient. The main aim, reduction of the route distance, is achieved by means of operations research, to be more precise, by the approximation methods of transportation problems.

The reason for choosing this theme is an inspiration by the idea of optimality and improvement of the systems’ performance by efficient decisions and rational approach. The idea that everything in this world can be not only described, but even be improved by the mathematics, intrigues consciousness.

2 Objectives and Methodology

2.1 Objectives

The aim of the Bachelor Thesis is to improve the current situation of route scheduling between the selected company and its customers. The route schedule will be constructed using the selected approximation methods for Travelling Salesman Problem, the route scheduling will be analyzed, and the shorter and more efficient path will be identified, which will positively effect on distribution expenses as a result. The appropriate improvement will be recommended.

2.2 Methodology

The Bachelor Thesis consists of theoretical and practical parts:

1. The theoretical part includes a review of the specialized literature, which is also divided into 2 parts:
 - literature concerning logistics area of business, logistics history, the current situation of logistics, its objectives and decisions.
 - literature regarding the quantitative approach side of particular Thesis about operations research as general and logistics view of the application, which is preferentially focused on Travelling Salesman Problem part, its representation and related proper methods of its solution.
2. The practical part contains:
 - detailed, the same time anonymous, description of the selected company, its features, the way of routes construction, current position in respect of professional confidentiality.
 - data obtained from the company provide background for matrix construction by Yandex. Maps., which is used to obtain the most appropriate result by the TSPkosa program.
 - specified usage of appropriate methods for the selected day, comparison of results, numeric and graphical representation by Mapy.cz. of the best method.
 - an indication of the best method for the whole week and interpretation in distribution expenses, depending on the result, the recommendation is given.

3 Literature Review

3.1 Supply Chain & Logistics

The Council of Supply Chain Management Professionals defines logistics as “the process of planning, implementing, and controlling procedures for the efficient and effective transportation and storage of goods including services, and related information from the point of origin to the point of consumption for the purpose of conforming to customer requirements”. (Vitasek, 2013, p. 117)

“Broadly speaking, its mission is to get the right materials to the right place at the right time...”. (Rushton, et al., 2010, p. 4)

Following equation of logistics helps to describe one of key relationship:

$$\textit{Logistics} = \textit{Materials Management} + \textit{Distribution}$$

By extension of this idea, the supply chain is illustrated as a combination of the supply of raw materials, logistics, as well as the delivery of products to the final customer. The supply chain is illustrated as a broader system of business. Thus:

$$\textit{Supply Chain} = \textit{Suppliers} + \textit{Logistics} + \textit{Customers}$$

(Rushton, et al., 2010, p. 4)

In other words, logistics activities are the operational components of supply chain management. Logistics aims at more specific tasks, meanwhile, the supply chain includes more level and function global activities. (The USAID | DELIVER PROJECT, 2011, p. 1)

3.1.1 Logistics history

The evolutionary beginning of logistics dates to ancient times: Ancient Greece, Roman Empire. However, as more early origins could be considered the formation and extension of the Great Silk Road. The Great Silk Road is a caravan route system, connecting countries of Europe with China from the II century BC to the XV century AD. Clearly, during the III-II centuries BC the term “Logistics” had not been used yet, but already existed control

movement of material flows over time and space, today it defines the term logistics. (Аникин & Родкина, 2013, p. 7)

The origin of the term can be considered a Greek word - *logistikos* means 'skilled in calculating'. The creation of term logistics dates to late XVIII and early XIX centuries in the context of military activities. Military logistics deals with getting soldiers and munitions to the battlefield in time for the fight. (Tseng, et al., 2005, p. 3)

3.1.2 Logistics today

Nowadays there is high competitiveness on the market thus business organizations do its best to improve its position on the market against competitors. This leads to the creation of many innovative ideas for improvement. (Rushton, et al., 2010, p. 9)

In recent years, several strategic and technological changes have had a marked impact on logistics.

1. **Globalization.** In order to take advantages of lower manufacturing costs or cheap raw materials available in some countries, more and more companies start to act at the global world level. As a result of these actions, the importance of supply chain management was increased including transportation needs. (Ghiani, et al., 2004, pp. 16-17)

2. **Information technologies.** Information technologies are a crucial tool at the operational level of companies, which are involved in delivery operations. As an example can be considered geographic information systems (GIS), a global positioning system (GPS) and onboard computers, which allow dispatchers to keep track of vehicles. (Ghiani, et al., 2004, pp. 16-17)

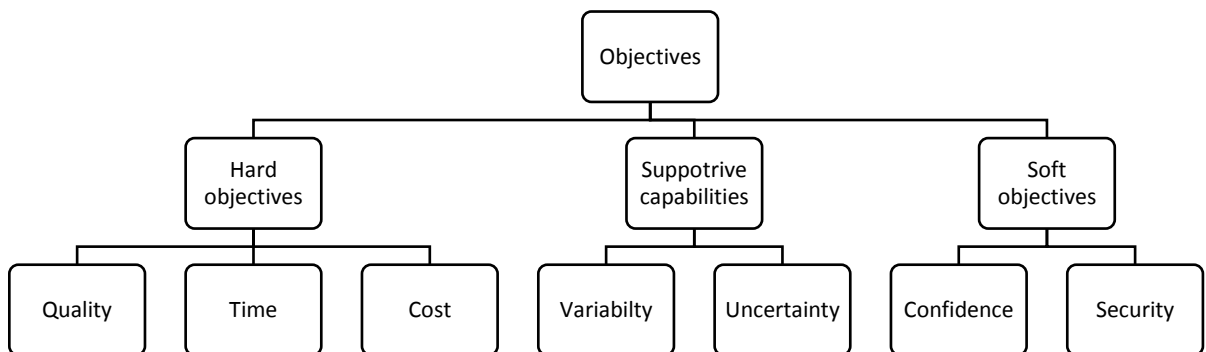
3. **E-commerce.** E-commerce goes hand in hand with globalization and information technologies. Today increasingly grow commercial transactions through the internet in the business sphere. Because of e-commerce, more direct deliveries are expected between manufacturers and end-users. (Ghiani, et al., 2004, pp. 16-17)

3.1.3 Logistics objectives

To take advantages over other participants on the market a company, in the first place, must clearly define its goals. Goals need to be clear, measurable and quantifiable.

Objectives for creating logistics advantage are divided into ‘hard objectives’, ‘supportive capabilities’ and ‘soft objectives’. Each category of objectives includes a few elements. Quality, time and cost are elements of ‘hard objectives’ because they are measurable and obvious to the end-customer. ‘Supportive capabilities’ include such elements as variability and uncertainty. In logistics processes variability must be controlled. When controlling variability, real and identifiable differences within a population are controlled. Dealing with uncertainty means dealing with events that are not known in advance. These elements refer to ‘supportive capabilities’ because they have almost the same importance as ‘hard objectives’. ‘Soft objectives’ are different from previous objectives by the ways of measuring. It is easier to measure ‘hard objectives’ rather than ‘soft objectives’. Examples of ‘soft objectives’ are confidence and security. Customers need to be sure about the safety of their personal information, they must be treated in a secure manner. Furthermore, service needs to be at the high level, so that customers are confident that queries are answered promptly and efficiently. (Harrison & Van Hoek, 2008, pp. 16-20)

Following diagram shows divisions properly:



Source: own processing of data based on the source - Harrison & Van Hoek, 2008, pp. 16-20

After the goals of taking advantages are defined, a logistics strategy needs to be devised. It is always a compromise between three main objectives: capital reduction, cost reduction and service level improvement. Under the term ‘capital reduction’ is implied the reduction of the level of investment in the system as much as possible. Cost reduction is about minimization of the transportation and storage costs. Service level improvement is a tool for customer satisfaction that has a major impact on revenues. (Ghiani, et al., 2004, p. 14)

Considering background above, for the actual Bachelor Thesis, the objective of cost reduction by minimization of the transportation costs has been chosen as a part of a logistics strategy.

3.1.4 Logistics decisions

According to supply chain management, all processes must be planned and controlled. Planning means making a plan, for instance, what amount of product should be distributed and sold each month. These plans must be controlled. Controlling means keeping to a plan against all problems that may get in the way. The main idea is to handle each process by means of planning and control to achieve customer needs correctly. (Harrison & Van Hoek, 2008, p. 7)

Certain hierarchy of planning reflects various planning time horizon. (Rushton, et al., 2010, p. 18)

According to the planning horizon, which is reflected by a hierarchy of planning, logistics decisions are divided into strategic, tactical and operational. (Ghiani, et al., 2004, p. 18)

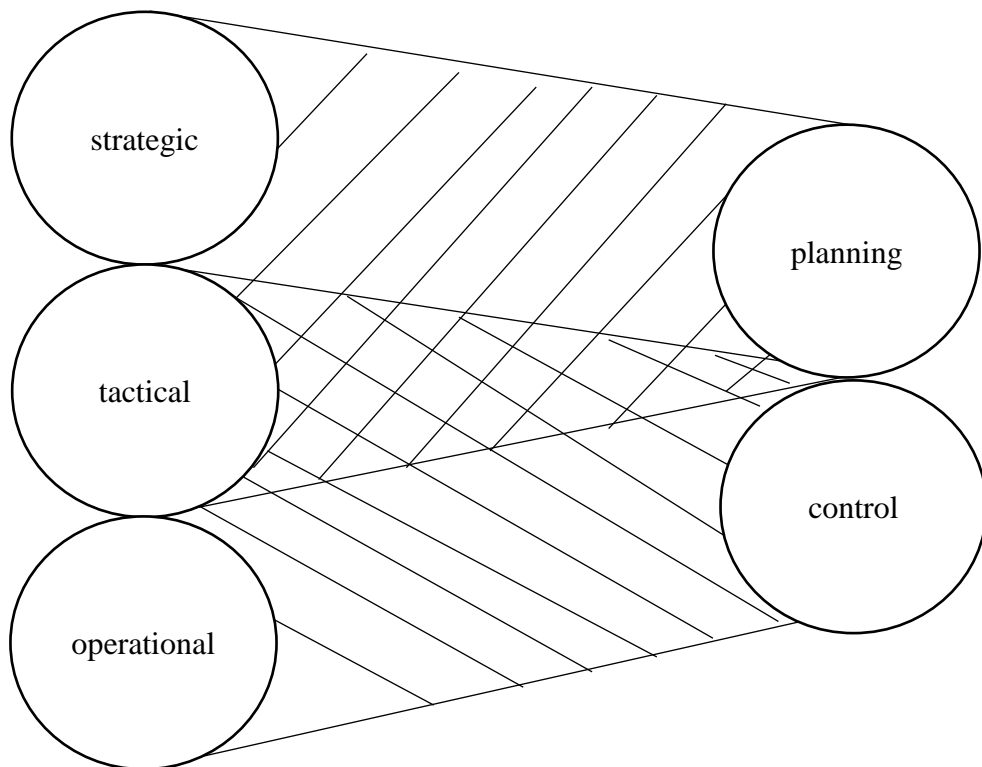
1. **Strategic decisions.** Such activities as logistics systems design and the acquisition of costly resources refer to strategic decisions because it has long-lasting effects. (Ghiani, et al., 2004, p. 18)

2. **Tactical decisions.** Acceptance on a medium-term basis is the main feature of tactical decisions, that include production and distribution planning, resource allocation. (Ghiani, et al., 2004, p. 18)

3. **Operational decisions.** Operational decision refers to a short-term planning, which includes a day-to-day decision (Misni & Lee, 2017, p. 94). For instance, warehouse order picking and shipment and vehicle dispatching (Ghiani, et al., 2004, p. 18).

Figure 2 indicates the interrelationship of planning and control within this hierarchy:

Figure 2: Logistics planning hierarchy



Source: (Rushton, et al., 2010, p. 18)

These two kinds of different elements used for effective logistics operations. The elements must be planned properly in the first place. Does not matter how strict the process is controlled if it had been planned in the wrong way in the beginning. Once the process planned properly, it is needed to be controlled to ensure that the operation is running as well as it should be. (Rushton, et al., 2010, p. 18)

3.2 Quantitative approach

“Quantitative approach to management problems requires that decision problems be defined, analyzed, and solved in a conscious, rational, logical and systematic and scientific manner”. (Murthy, 2007, p. 1)

There are two important conditions of quantitative methods. Models and methods of solving the problems must be logically structured as the first condition. The second and more crucial aspect of quantitative approaches is the attitude of search, which is performed on a scientific basis. Once accepted this attitude, the quantitative approach applications are included in those spheres where, at first glance, quantitative data are difficult to come by. (Murthy, 2007, p. 2)

3.2.1 Operations research

“Operations research is a branch of mathematics – specially applied mathematics, used to provide a scientific base for management to take timely and effective decisions to their problems”. (Murthy, 2007, p. 1)

The aim of operations research is to improve the performance of systems by the advancement of a rational and systematic approach to the solution of problems. Logical decisions need to be made to achieve the best solutions with the information that can effectively and profitably be used. (Siemens, et al., 1973, pp. 3-4)

“Operations research is concerned with scientifically deciding how to best design and operate man-machine systems, usually under conditions requiring the allocation of scarce resources.” (Brans & Gallo, 2007, p. 2) When decision makers deal with complex real-world problems, those methods seem abstract, but its goal is a useful application. Sometimes it is harder to construct the model that is maximally related to the reality, and only after that, the appropriate method can be applied.

Operations research is a ‘baby of war’ because, the problem related to the question, how to set the time fuse bomb to be dropped from an aircraft on to a submarine, was the first attempt to solve in a systematic way. In fact, the primary origin of Operations research dates to the Second World War. (Murthy, 2007, p. 3)

According to the name, the word Operations is used for the problems of military and the word Research is used for inventing a new method. As this method of solving the problem was originated during the war, the name given is ‘OPERATIONS RESEARCH’ and abbreviated as ‘O.R.’. (Murthy, 2007, p. 4)

There are eight general categories that can be solved efficiently with the help of operations research: allocation, competition, inventory, queueing, replacement, routing, search and sequencing. (Siemens, et al., 1973, p. 4)

3.2.2 Operations research in logistics

Operations research, as a part of quantitative analysis, offers a variety of planning tools, which are essential for intelligent logistics decision-making. (Ghiani, et al., 2004, p. 18)

3.2.2.1 Optimization

Optimization is about maximizing or minimizing some objective function with conditions and constraints that exist in the physical environment. According to the conditions of a quantitative approach, if an appropriate model is used to express the problem, the realistic solution can be reached. If there is a significant difference in the relationship between the model and the problem, obviously the model’s optimal solution is not to be the optimal solution to the problem that the model was intended to represent. (Siemens, et al., 1973, pp. 8-9)

“When using an optimization model, a key aspect is to keep model size as small as possible” (Ghiani, et al., 2004, p. 20). Optimization is divided into ‘easy’ and NP-hard problems. It is important to note that, for ‘easy’ optimization problems no matter how large instance size is, they can be solved within a reasonable amount of time. Meanwhile, NP-hard optimization problems can be solved consistently within a reasonable amount of time only if instance size is sufficiently small. Moreover, most integer programming (IP), mixed-integer programming (MIP), and nonlinear programming (NLP) problems are hard to optimize. (Ghiani, et al., 2004, p. 19)

“Even when optimization is the goal, optimality cannot always be proved or even assured” (Siemens, et al., 1973, p. 9). If, however, the model and the problem are not significantly different, the model solutions should achieve realistic approximations to the optimal solutions of the problem. (Siemens, et al., 1973, p. 9)

3.2.2.2 Continuous approximation methods

A few classes of logistics decisions such as production planning, location decisions, vehicle routing and scheduling can only be modeled as IP or MIP problems. It encourages the expansion of fast heuristic algorithms; its solutions are not necessarily the best but 'good enough'. (Ghiani, et al., 2004, p. 19)

Finding near-optimal solutions became a goal after the discovery of NP-hard completeness problems (Gutin & Punnen, 2007, p. 208). It is the most common approach when a 'good enough' solution can be reached in seconds by relaxing the requirements of obtaining an optimal solution. (Williamson & Shmoys, 2011, p. 14)

For instance, continuous approximation methods can be used when number customers are so large that demand is seen as a continuous spatial function. Approximation often gives closed-form solutions and can be used as a simple heuristic. (Ghiani, et al., 2004, p. 20)

3.2.3 Graph theory

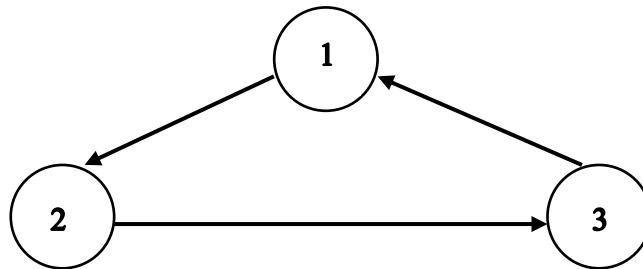
Set of vertices and edges, forming a graph, can illustrate the substantial number of situations. (Šubrt, et al., 2015, p. 251)

A graph $G = (V, E)$ is described as a mathematical structure consisting of two sets V and E . The elements of V are called vertices (or nodes), and the elements of E are called edges. Each edge has a set of one or two vertices associated with it, which are called endpoints. (Gross & Yellen, 1999, p. 2)

Vertices can represent a distribution center and retail outlets, as in the practical part, or project milestones in project management meanwhile edges define routes between vertices or project tasks. (Šubrt, et al., 2015, p. 251)

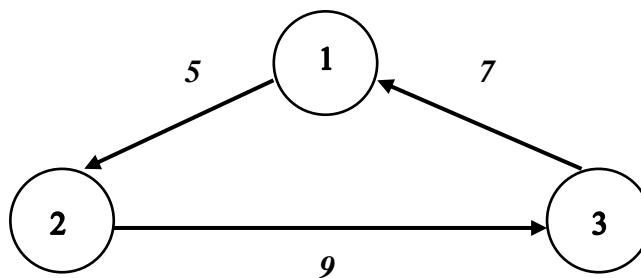
For a directed graph each edge is directed. One endpoint of a directed edge is designated as the tail, and another endpoint is designated as the head. (Gross & Yellen, 1999, p. 3)

Figure 3: Directed graph



Once a number is assigned to each edge, the graph is called a weighted graph. (Gross & Yellen, 1999, p. 40)

Figure 4: Weighted graph



Oriented path is a graph, which has vertices v_1, v_2, \dots, v_n and directed edges $e_1=(v_1,v_2), e_2=(v_2,v_3), \dots, e_{n-1}=(v_{n-1},v_n)$. The difference between cycle and path is in additional edge, which is created by path end-vertex v_n and initial one v_1 , i. e. has vertices v_1, v_2, \dots, v_n and directed edges $e_1=(v_1,v_2), e_2=(v_2,v_3), \dots, e_{n-1}=(v_{n-1},v_n), e_n=(v_n, v_1)$. There is neither initial nor end-vertex. (Šubrt, et al., 2015, p. 255)

A cycle containing all the vertices of a graph is said to be a Hamilton cycle of the graph. The term of Hamilton cycle originates from a game, which was invented in 1857 by Sir William Rowan Hamilton based on the construction of cycles containing all the vertices in the graph of the dodecahedron. (Bollobás, 1990, p. 12)

The graphical representation is more elegant and illustrative rather than ‘classical’ mathematical models. What is more, graphical view of the model is more comprehensible for those, who do not deal with mathematical modeling. (Šubrt, et al., 2015, p. 251)

3.2.4 Travelling Salesman Problem

The Travelling Salesman Problem (TSP) refers to ‘hard’ optimization problem, to be precise, to a nondeterministic polynomial-time (NP) class of problem. As was mentioned above, solving procedure of NP problems is very difficult if not impossible when instance number is large. (Gutin & Punnen, 2007, p. 1)

The origin of the TSP is the work of Dantzig, Fulkerson, and Johnson (Gutin & Punnen, 2007, p. 2). According to the work, the Travelling Salesman Problem is described as follows: “Find the shortest route for a salesman starting from a given city, visiting each of a specified group of cities, and then returning to the original point of departure”. (Dantzig, et al., 1954, p. 11)

From the mathematical point of view, the TSP is described as a graph $G = (V, E)$ and F be the family of Hamiltonian cycles in that graph G . For each edge $e \in E$ a cost c_e is assigned. Then the aim is to find the Hamiltonian cycle in graph G in such a way that the sum of edges costs of the tour is as small as possible. (Gutin & Punnen, 2007, p. 3)

Using geometric graph theory TSP can be accessible not only for mathematicians but for everyone with a pencil and a clean sheet of paper. For instance, cities are represented as locations and the costs as distances between pairs. (Applegate, et al., 2006, p. 15)

It is important to note that the concept of the ‘shortest way’ relates to distance, time, or cost or combinations of these ‘distances’. Furthermore, these ‘distances’ are not associated with one vertex, but with getting from one vertex to another. No matter whether distance, time or cost is included to ‘distance’ measurement, one must decide on way to quantify these variables. (Siemens, et al., 1973, p. 239)

The next important fact to be noted is triangle inequality. Definition of triangle inequality says: “Let G be a weighted simple graph with vertices $1, 2, \dots, n$, such that ij has weight c_{ij} .

Then G is said to satisfy the triangle inequality if $c_{ij} \leq c_{ik} + c_{kj}$ for all i, j , and k ” (Gross & Yellen, 1999, p. 230). The TSP obeying the triangle inequality. There must be realistic arcs and roads between the warehouse and customers in two-dimension space. The distances must not be chosen randomly. Rounding errors also influence triangle inequality violations. Rounding to the nearest integer or decimal point can overstate an arc value, so triangle inequality violation exists. What is more, triangle inequality effects on solution time more rather than asymmetry. In a situation when violations are encountered, either route with violations are excluded or values of the matrix are directly changed to satisfy the inequality. Realism must be preserved to appropriately reflect the reality of the routing network. When triangle inequality violations are ignored, the result will be inaccurate and optimistic. (Fleming, et al., 2012, pp. 1-2)

TSP is divided into two classes: the symmetric travelling salesman problem (STSP) and the asymmetric travelling salesman problem (ATSP). This splitting depends on the nature of the cost matrix. The problem is called symmetric when cost C is symmetric, in other words, it is when a graph G is undirected. Concerning the asymmetric travelling salesman problem, once the graph G is directed, cost C is not necessarily symmetric (Gutin & Punnen, 2007, pp. 3-4). The situation of the ATSP is different from the STSP in a few aspects. The ATSP heuristics is to cope with a much broader range of instance structures. The ATSP is a more difficult problem that is why none of heuristics of the ATSP can get as close to optimal as in the STSP. Finally, heuristics for the ATSP is much less experimentally studied. (Gutin & Punnen, 2007, p. 446)

There is no effective algorithm to find exact mathematical optimum, the time of solving larger instance number would be incomparably longer even than the Universe time existence, due to the fact, that limiting constraint number of a mathematical model of this problem exponentially grows with increasing number of place. (Šubrt, et al., 2015, p. 99)

The heuristics are applied in situations when it is difficult to evaluate each possibility. On the one hand it gives a result quickly, but on the other hand, there is the risk of missing the best result. Heuristics drives a heuristic algorithm. A definition of a heuristic algorithm says: “It is forfeiting the guarantee of finding the best solution so that it can be terminate quickly”. (Gross & Yellen, 1999, p. 229)

There are series of approximation methods, which solution may be considered as economical optimum. (Šubrt, et al., 2015, p. 99)

3.2.4.1 Savings Method

The base of Savings Method is savings procedure to create Savings matrix (Pelikán, 1993, p. 46). The procedure of Savings Method is based on distance comparison of the straight routes between the route through the central city and other two cities. The straight routes between pairs of cities are called process edges.

Algorithm:

1. Compute the savings for all pairs of cities according to the formula: $s_{ij} = c_{i0} + c_{0j} - c_{ij}$ to create matrix S .
2. Build the descending order of straight routes (process edges) between pairs of cities according to the value of savings s_{ij} .
3. Based on the conditions that all edges with added process edge form the set of vertex-disjoint paths and the city's capacity sum of each path does not overpass the vehicle capacity, the solution must be added. This step with relevant conditions must be repeated until each city is located on some of the paths.
4. Finally, add the initial city to all the paths to create cyclic routes.
(Kučera & Krejčí, 2013, p. 3)

3.2.4.2 Vogel Approximation Method

Vogel Approximation Method uses differences between the most favorable distances in the set of distribution tables. (Šubrt, et al., 2015, p. 83)

Algorithm:

1. Fill the matrix by distances between each pair of the cities. On the main diagonal are always zeros.
2. Create a new row and column which is named 'difference' by finding the differences between two the most favorable distances in each row and column. Logically, the most favorable are the lowest values for minimization.

3. Choose the highest difference from the whole matrix, in other words, the highest difference can be chosen either from the row or from the column. Important to note that in the beginning there are same values difference since it has mirror feature, so it is up to a solver from where to choose.
 4. Find in a matrix the lowest distance which is related to the highest difference.
 5. Exclude the impossible routes. A row and a column related to the chosen lowest distance in a matrix are crossed out.
 6. Exclude a mirror cell which is located across the main diagonal.
 7. Recalculate 'difference' once again considering procedure above.
 8. Repeat the procedure from step 3 to step 7 until all cities are connected by the path.
- (Šubrt, et al., 2015, p. 102)

3.2.4.3 Nearest Neighbour Algorithm

Nearest Neighbor Algorithm is the simplest TSP heuristic. It is very fast and easy to implement. The main idea is one of short-sighted greed: start with whatever initial point, pick the cheapest way to go somewhere else. After passing all points go back to the initial one. (Gross & Yellen, 1999, p. 229)

Algorithm:

1. Fill the matrix by distances between each pair of the cities. On the main diagonal are always zeros.
 2. Set an initial city and cross off column related to this city. It means that a traveler returns to the initial city only at the end.
 3. Find a cell with the lowest value of distances in the row related to the city and mark this cell in a matrix. By this distance, a traveler moves to a city in a column.
 4. Cross off a column related to the chosen cell. A traveler does not go back to a city in a column anymore.
 5. Repeat the procedure from step 3 to step 4 until all cities are connected by the path.
 6. Set the next city as an initial and follow the same procedure until the number of paths is equal the number of given cities.
 7. Find the most favorable path, e. i. the shortest path.
- (Šubrt, et al., 2015, p. 101)

3.2.4.4 Branch and Bound

Branch and Bound method is not polynomial, that is why calculation for large-size problems cannot be implemented. (Pelikán, 1993, p. 52)

Algorithm:

1. Fill the matrix by distances between each pair of the cities. On the main diagonal are infinity, means there is no direct path from c_i to c_j .
2. Find the minimum value in each row and subtract this minimum value from each element in that row to get reduced matrix.
3. Find the minimum value in each column and subtract this minimum value from each element in that column to get reduced matrix.
4. Sum up all reductions were made to find lower bound of the starting node.
5. In the reduced matrix of parent node add a new edge by set outgoing edges for parent city i and all incoming edges to city j to infinity. Also set $(j; i)$ to infinity too.
6. Calculate lower bound of the path starting at new node j using resulting cost matrix. The lower bound equals to zero when is in reduced form.
7. Calculate cost for node j via formula: cost = cost of parent node i + cost of edge $(i; j)$ + lower bound of the path starting at node j .
8. Repeat the same procedure for all adding edges from step 5 to step 7.
9. Compare all paths and choose the shortest one. It becomes parent node for next set of nodes.
10. Continue until a tour is completed and then return back to the root node.

(Rai, 2017)

All these methods are used in the software TSPkosa.

3.2.5 TSPkosa

The program TSPkosa was created in Microsoft Visual Basic 6.5. by the Department of Systems Engineering and Department of Statistics of Czech University of Life Sciences in 2010. The aim of the program is to find the most preferable paths for Travelling Salesman Problem by four methods. Three of them are approximation: Nearest Neighbour Algorithm, Vogel Approximation Method and Savings Method. There is also one optimization method – Branch and Bound. The program allows the entering of problems with up to 250 nodes. Nearest Neighbour Algorithm, Vogel Approximation Method and Savings Method will be

used in the practical part, only Branch and Bound will not be used due to the fact that the problem includes more than 15 nodes, in other words, it is the large-size problem, so calculation in TSPkosa cannot be implemented. (Krejčí, et al., 2010, p. 1)

4 Practical Part

4.1 Practical usage in a selected company

The practical part is tightly related to the theoretical part, what is more, illustrates the practical usage of scientific knowledge of operations research in logistics.

A distribution company was selected as illustration of Travelling Salesman Problem in selected business case, due to the fact that distribution company is the most appropriate example in logistics sphere, which faces everyday activities of products transportation to customers and then, at the end of the working day, returns back to the depot, which creates the Hamiltonian cycle.

Then the aim is to find the Hamiltonian cycle in such a way that the sum of edges costs of the tour is as small as possible, as was described in 3.2.3 paragraph. To achieve the objective, appropriate methods will be used, which are proposed by the software TSPkosa.

4.1.1 Company features

The company in Kazakhstan is described in an anonymous way to the respect of professional confidentiality. It is the distribution company, which has been successfully competing on the market for more than 5 years. The structure of the company consists of 4 components: administrative-managerial department, sales department, warehouse and department of distribution.

It deals with Russian manufacturer of foodstuffs such as candies, snacks, beverage etc. All foodstuffs locate in the depot, which corresponds to the standards of safe-keeping and treatment. Then the company distributes the foodstuff shipments to mini- and supermarkets in town and some regions behind by rented car type – GAZelle, which uses propane gas as combustible matter.

The author of this Bachelor Thesis had an opportunity to complete an internship in the described company to see how it works in real life, so there is an ability of detailed description of working processes and routes construction.

4.1.2 Company's routes construction

Routes construction is started with a collection of orders by sales representatives, each of them has own fixed district. The collection of orders carried out every week, and the order is usually registered every second week. During a day a sales representative attends approximately 30 trade points. Orders are accepted by a personal digital assistant after that are sent to the operator's PC. The operator registers a list of a total number of products and a shipping list with addresses. All documentation is sent to the warehouse for the successive collection of products and shipments for the next day. Freight forwarders deliver products from the warehouse to trade points and return back. Each freight forwarders are received the shipping list with addresses, according to the given information, construct the route relying on own experience and knowledge.

The information about 5 working days was provided by the company. According to the provided data, the table below is constructed. Addresses are denoted by denotation in respect of the confidential customer information.

Table 1: Sequence and distance of the initial route on Monday

sequence denotation	distances (km)
A ₁	0
A ₂	5.4
A ₃	2.5
A ₄	1.5
A ₅	0.4
A ₆	0.1
A ₇	0.9
A ₈	0.4
A ₉	0.1
A ₁₀	0.6
A ₁₁	0.3
A ₁₂	3.1
A ₁₃	0.1
A ₁₄	0.2
A ₁₅	0.7
A ₁₆	1.3
A ₁₇	1
A ₁₈	1.4
A ₁	2.9
Total	22.9

- The denotation A₁ refers to the warehouse, others relate to the trade points.
- The information includes the list of addresses for each day that are ordered in sequence form, the address of the warehouse always in the beginning and at the end.
- The table shows how many kilometers were covered by freight forwarders during Monday.

Source: own processing, based on real provided data by the company

The real distance for Monday is 22.9 kilometers. The aim is to decrease this value as much as possible. Monday data will be used for a comparison between real and alternative situations.

The second way is a graphical representation, which is widely used in operations research, due to the fact, which was mentioned in 3.2.3 paragraph, that everything is more visible and understandable for everyone, once it is illustrated. The following map represents the current situation of Monday.

Figure 5: Graphical representation of the initial (really used) route on Monday



Source: own processing by means of web page Mapy.cz

According to the map, A_1 , which is a denotation of the warehouse, is the 'END' of the route, also, nodes on the map represents a sequence of nodes A_n, A_{n+1}, \dots .

The route was created by the help of web Mapy.cz because neither Yandex Maps nor Google Maps allow creating route more than 10 nodes. The aim is to transform the figure so that it approximately appears less as a circle.

4.2 Alternative way of routes construction

To show the alternative way of routes construction, the model is designed, which is as close as possible to the real situation, due to the fact of paragraph 3.2, that carefully constructed model is the most important condition of quantitative research on the one hand. On the other hand, the represented model, as each mathematical model, is a simplification of a real world, that is why the condition of time windows and delivery time by agreement are not counted.

The matrix of Monday is constructed to show how the situation of covered distances can be changed. The matrix features are:

1. Headings of the table in rows and columns are named by the denotations A_n, A_{n+1}, \dots , which are addresses of trade points and warehouse;
2. The size of the matrix is 18x18; because a freight forwarder makes a circle, no node must be forgotten by a freight forwarder;
3. There are empty cells on the main diagonal to illustrate zero value distances between same nodes;
4. Filled cells demonstrate distances in kilometers, which are covered by a freight forwarder, between pairs of nodes. The values are obtained owing to the web – Yandex. Maps;
5. The matrix is asymmetric because back and forth routes are different on account of road condition;
6. Values of distances are rounded to decimal numbers to avoid inaccuracy;

Table 2: The Monday coefficient matrix

Monday	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	A ₇	A ₈	A ₉	A ₁₀	A ₁₁	A ₁₂	A ₁₃	A ₁₄	A ₁₅	A ₁₆	A ₁₇	A ₁₈
A ₁		5.4	3	4.3	4.4	4.5	5	5.1	5.1	5.4	5.4	3.1	3.1	3.1	3.7	4.3	4.2	3.2
A ₂	5.4		2.5	1.6	1.7	1.8	1.4	1.6	1.6	1.9	2	2.4	2.3	2.4	2	2.7	1.6	2.5
A ₃	3	2.7		1.5	1.7	1.7	2.3	2.4	2.4	2.6	2.7	0.7	0.7	0.7	0.8	2.1	1.5	1.1
A ₄	4.2	1.7	1.5		0.4	0.4	1	1.1	1.1	1.4	1.4	1.9	1.9	1.9	1.6	2.9	1.8	2.1
A ₅	4.4	1.3	1.7	0.4		0.1	0.9	1	1	1.2	1.2	2.1	2	2.1	1.9	3.2	2.2	2.4
A ₆	4.4	1.3	1.7	0.4	0.1		0.9	1	1	1.3	1.3	2.1	2.1	2.1	2	3.3	2.2	2.5
A ₇	5	1.6	2.3	1	0.8	0.8		0.4	0.5	1	0.7	2.7	2.7	2.7	2.6	3.9	2.8	3
A ₈	5.1	1.7	2.4	1.1	1	0.9	0.4		0.1	0.6	0.6	2.8	2.8	2.8	2.6	3.9	2.9	3.1
A ₉	5.1	1.7	2.4	1.1	1	0.9	0.5	0.1		0.6	0.6	2.8	2.8	2.8	2.6	3.9	2.9	3.1
A ₁₀	5.4	2	2.6	1.4	1.2	1.2	1	0.6	0.6		0.3	3.1	3	3	2.9	4.2	3.2	3.4
A ₁₁	5.4	2.1	2.7	1.4	1.2	1.2	0.7	0.6	0.6	0.3		3.1	3.1	3.1	3	4.3	3.2	3.5
A ₁₂	3	2.4	0.7	1.9	2.1	2.1	2.7	2.8	2.8	3.1	3.1		0.1	0.1	0.7	1.6	1.4	0.6
A ₁₃	3.1	2.3	0.7	1.9	2	2.1	2.7	2.9	2.9	3	3.1	0.1		0.2	0.6	1.7	1.3	0.5
A ₁₄	3.1	2.5	0.7	2	2.2	2.2	2.8	2.9	2.9	3.2	3.2	0.1	0.2		0.7	1.7	1.4	0.6
A ₁₅	3.9	1.9	1	2.1	2.3	2.3	2.8	3	3	3.3	3.3	0.9	0.8	0.9		1.3	0.7	1
A ₁₆	4.3	2.3	1.4	2.6	2.8	2.8	3.4	3.5	3.5	3.7	3.8	1.3	1.2	1.4	1.1		1	1.4
A ₁₇	4.2	1.7	1.6	1.8	2.2	2.2	2.8	2.9	2.9	3.2	3.2	1.4	1.3	1.3	1	1.1		1.4
A ₁₈	2.9	2.6	0.9	2.1	2.2	2.3	2.9	3	3	3.2	3.3	0.2	0.3	0.3	0.9	1.6	1.5	

Source: own processing matrix constructed by means of the real provided addresses of warehouse and trade points

In the same way, the rest 4 of matrices are constructed, which are attached to the Appendix.

4.2.1 Proposed solution.

To achieve the objective, minimization of the distance, the program TSPkosa was used. The program is described in 3.2.5 paragraph. The software offers 4 methods for the Travelling Salesman Problem: Nearest Neighbour Algorithm, Vogel Approximation Method, Savings Method, Branch and Bound. The first three of them are applied for each working day in the selected business case. The method Branch and Bound is not applied, because it is the large-size problem, so calculation cannot be implemented. The methods have different approaches, so it offers diverse alternative results. The most desirable, the lowest, the result of each day will be chosen as the best, and corresponding methods will be used for graphical and numeric comparison between the real and alternative position.

4.2.2 Special case of Monday

As was mentioned above, Monday matrix is used to demonstrate the appropriate methods of solution, it is not only due to the fact that Monday is the first day of the week, what is more, the Monday matrix illustrates the special case of a solution, which is important to mention.

4.2.2.1 Vogel Approximation Method

For Monday matrix Vogel Approximation Method was used, which based on a calculation of differences between two lowest numbers in each row and column. After that, the most preferable difference is chosen. A detailed description can be found in 3.2.4.2 of theoretical part.

The Travelling Salesman Problem, as was mentioned in 3.2.4 paragraph, refers to nondeterministic polynomial-time (NP) class of problem. When instance number is large, it is complicated to obtain solution if not impossible. However, the number of nodes is not the only one complexity of this type of problem, also the same differences influence problem outcome. During more complex algorithms the problem of overflow may occur.

This situation took a place during the procedure of Vogel Approximation Method working with Monday matrix. The procedure was implemented according to the manual, the program TSPkosa started to process matrix data. The time-consuming process at the end gave the

message “out of memory”. The situation can be solved by reduction of nodes, but our aim is to maintain as close as possible to the real situation, so the Vogel Approximation Method is not applied in case of Monday. It is not a problem, it happens when there are too many same differences in the matrix, it leads to branching. So, for the comparison will we used the mean value between Nearest Neighbour Algorithm and Savings Method.

4.2.2.2 Nearest Neighbour Algorithm

Nearest Neighbour Algorithm has also been used to find the shortest route in Monday matrix. The main feature of Nearest Neighbour Algorithm is repeating of procedure, which was described in theoretical part 3.2.4.3, until the number of routes equals the number of nodes, after that the shortest route need to be chosen. The software TSPkosa proposed 4 minimal cycles with the length of 18 kilometers. The difference in sequence between 4 minimal cycles was insignificant, in some cases, it was pairs of switched nodes. The most important is the length of routes, for all it is equally 18 kilometers, so it does not matter which route of those 4 to choose. The result of Nearest Neighbour Algorithm for Monday is shown in the following table:

Table 3: Sequence and distance of the route constructed by Nearest Neighbour Algorithm on Monday

sequence denotation	distances (km)
A_1	0
A_{18}	3.2
A_{12}	0.2
A_{13}	0.1
A_{14}	0.2
A_3	0.7
A_{15}	0.8
A_{17}	0.7
A_{16}	1.1
A_2	2.3
A_7	1.4
A_8	0.4
A_9	0.1

A_{11}	0.6
A_{10}	0.3
A_6	1.2
A_5	0.1
A_4	0.4
A_1	4.2
Total	18

Source: own processing of data, results are obtained by the program TSPkosa

The table shows that the longness of covered kilometers is 18, it means that using Nearest Neighbour Algorithm, length of route will decrease by 4.9 kilometers comparing to initial route, which is essential, because length of initial route was reduced by 21.4%, this factor will have result in future on fuel consumption and, logically, on distribution expenses, which will be calculated at the end of this Bachelor Thesis after summation results of methods for each day.

4.2.2.3 Savings Method

As was mentioned in 3.2.4.1 paragraph, Savings Method is based on savings procedure, which is calculated by the quotation $s_{ij} = c_{i0} + c_{0j} - c_{ij}$ to create matrix S . Then the procedure is implemented according to the Savings Method algorithm represented in the paragraph.

According to the result of the software TSPkosa, 12 minimal cycles were proposed with the length of 17.3 kilometers. The difference in a sequence was insignificant as well. The result Savings Method for Monday is shown in the following table:

Table 4: Sequence and distance of the route constructed by Savings Method on Monday

sequence denotation	distances (km)
A_1	0
A_{18}	3.2
A_{12}	0.2
A_{14}	0.1
A_{13}	0.2
A_{15}	0.6

A ₁₆	1.3
A ₁₇	1
A ₂	1.7
A ₇	1.4
A ₁₁	0.7
A ₁₀	0.3
A ₈	0.6
A ₉	0.1
A ₆	0.9
A ₅	0.1
A ₄	0.4
A ₃	1.5
A ₁	3
Total	17.3

Source: own processing of data, results are obtained by the program TSPkosa

According to the table above, the number of covered kilometers is 17.3 by means of Savings Method, which shows improvement of length in comparison with the initial route by 5.6 kilometers, which is almost 25% progress of the situation. This will also have an impact on the future condition of distribution expenses. The next part will show which the comparison between each method.

4.2.3 Comparison

All things are known by comparison. For comparison contains within itself a power which immediately demonstrates in objects which is more, less or equal.

Leon Battista Alberti, "On Painting".

Comparison is divided into numeric and graphical in this Bachelor Thesis. Monday numeric comparison will show distance and percentage difference between the initial route and all proposed methods, after that the best will be identified. Graphical comparison will show how the most appropriate method changes the initial route.

4.2.3.1 Numeric comparison

Numeric comparison is a tool to summarize all result for the Monday matrix. The numeric comparison of route length is illustrated via the table, which represents results of the initial route and all methods mentioned before. For Nearest Neighbour Algorithm and Savings Method carried out by the program TSPkosa, only for Vogel Approximation Method calculation was not generated, so the value was calculated as mean value between the results of Nearest Neighbour Algorithm and Savings Method, but for the matrices of other days Vogel Approximation Method perfectly worked, the reason to choose the Monday matrix is to show special case. According to the table, it is seen that all of the methods show improvement of the initial route more than 20%, it means that even during the Monday matrix procedure, it is evident, that the proposed idea and solution by the TSPkosa program is a better alternative of the current position.

Table 5: Numeric comparison of the initial and alternative routes

	Initial		Nearest Neighbour Algorithm		Vogel Approximation Method		Savings Method		
	distance (km)	% from initial	distance (km)	% from initial	distance (km)	% from initial	distance (km)	% from initial	
Monday	22.9	100%	↓ 18	78.6%	↓ 17.65	77.07%	↓ 17.3	75.55%	
Difference from initial	-	100%	4.9	21.4%	5.25	22.93%	5.6	24.45%	
								THE BEST	

Source: own processing of data, results are obtained by the program TSPkosa

What is more, the table indicates the most appropriate method for Monday case, in the first row we are looking for the lowest number of distance and the lowest percentage, because this ‘% from initial’ illustrates the proportion of new distance comparing to the initial one. And in the second row, logically, the search target is the highest difference between the proposed methods and the initial route. Those values are related to Savings Method, so it is the best one for Monday case. This case will be used for calculating liters of fuel usage and related expenses in results, 5.1 paragraph.

4.2.3.2 Graphical representation of the best method

The second tool to represent the transformation is a graphical representation because it word for word shows how a figure transforms from one to another. The figures 6 and 7 are dedicated to the comparison between the real and the best alternative route on Monday.

Figure 6: Graphical representation of the initial route

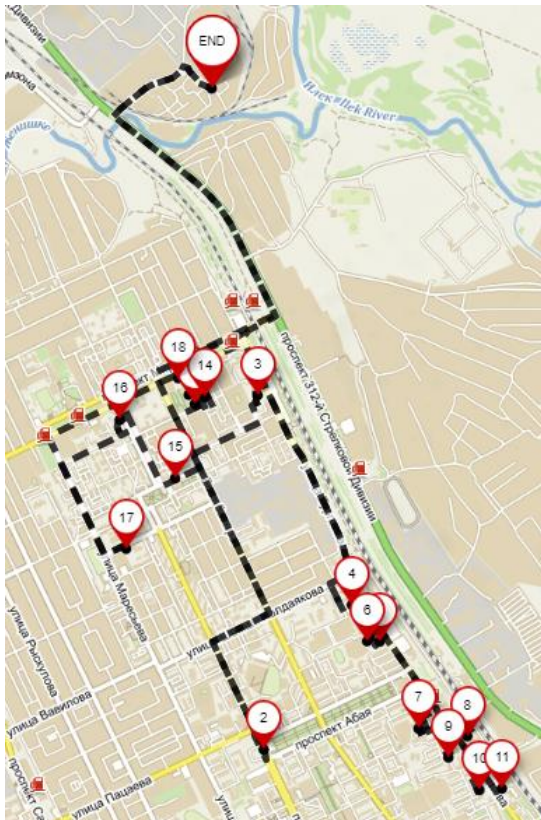
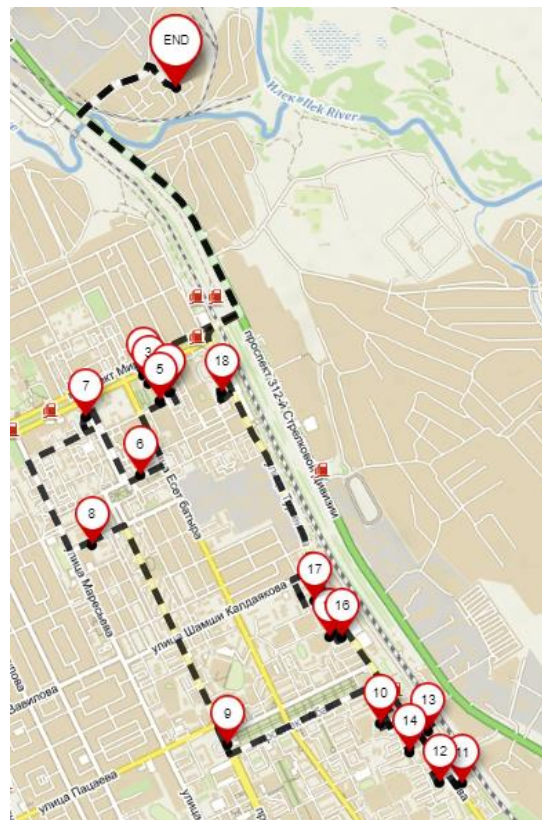


Figure 7: Graphical representation of the alternative route by Savings Method



Source: own processing by means of web page mapy.cz

The first thing to be noticed during comparison is that nodes on the figure 6 represents sequence of nodes A_n, A_{n+1} and the same time the denotation of addresses, meanwhile nodes on the figure 7 represents sequence of nodes A_n, A_{n+1} only, the numbers on the map are not the denotation of addresses A_n , to not be confused.

Comparing two maps it is evident that majority connections between nodes are extremely expanded during the real route construction, for instance, the path from warehouse to node 2

and back to node 3 seems too long, meantime the route proposed by Savings Method is distinctly organized, so the length of connections goes to minimum.

Looking at the provided figures above, the cautious reader will notice that the figure not only changed but now it more reminds circle.

5 Results and Discussion

5.1 Results

After scrupulous data processing of each working day, daily results of Nearest Neighbour Algorithm, Vogel Approximation Method, Savings Method have been reached by the TSPkosa software. Only during Monday Vogel Approximation Method showed no result, the special case has been described in 4.2.2.1 paragraph, so the value of Vogel Approximation Method for Monday is calculated as a mean value between results of Nearest Neighbour Method and Savings Method. The following table illustrates reached results:

Table 6: Distances comparison of the initial and alternative routes for the whole week

	Initial (km)	Nearest Neighbour Algorithm (km)	Vogel Approximation Method (km)	Savings Method (km)
Monday	22.9	18	17.65	17.3
Tuesday	36.1	29.8	28.7	26.5
Wednesday	48.2	34.3	36.8	34.2
Thursday	41.2	36.4	35.4	34.7
Friday	27.8	28.8	28.2	27.7
Total for a week (km)	176.2	147.3	146.75	140.4
				THE BEST

Source: own processing of data, results are obtained by the program TSPkosa

According to the table, the aim of this Bachelor Thesis is achieved, because there is a reduction of total route distance of all methods comparing to the initial route. The yellow cells show the best, minimum values, which are related to the method for each day. In each case, the most appropriate method is Savings Method. The lowest difference between the real and most preferable alternative routes is 0.1 kilometer on Friday. There is the highest difference - 14 kilometers on Wednesday. On average, the difference between the initial route and the routes proposed by Savings Algorithm is essential, 7.16 kilometers. What is more, during the Friday procedure Nearest Neighbour Algorithm and Vogel Approximation Method show even worse results rather than initial route, it can happen, because the methods are approximation ones, as was mentioned in paragraph 3.2.2.2, its solutions are not necessarily the best but 'good enough'. It means that on Friday a freight forwarder chose the route, which is optimal or at least a little better rather than routes proposed by Nearest Neighbour Algorithm and Vogel Approximation Method.

The total routes distance for the whole week is 140.4 kilometers, which is by 20.32%, is better than the initial route. The aim is achieved, the next step is the calculation of distribution savings for the best method.

Due to the fact that Savings Method has shown the best result for the chosen week in the distribution company, this method is used to represent the saving, which is related to distribution expenses of the company. As was mentioned in 4.1.1 paragraph, the company rents car type – GAZelle, which uses propane gas as a combustible matter. The consumption is 0.2 liters per kilometer. The cost of 1 propane gas liter is 51 KZT. According to the exchange rate on 09.06.2018 1 EUR equals to 392.91 KZT. This information helps with saving calculation.

The total distance for real and alternative way of route construction and following saving calculation is illustrated in the table below:

Table 7: Expenses and saving calculation of the initial and the best alternative route *per one car*

	Initial	Savings Method
Monday	22.9 km	17.3 km
Tuesday	36.1 km	26.5 km
Wednesday	48.2 km	34.2 km
Thursday	41.2 km	34.7 km
Friday	27.8 km	27.7 km
Total for a week (km)	176.2	140.4
Consumption (liter/km)	0.2	0.2
Propane gas (liters)	35.24	28.08
Cost of propane gas (EUR)	0.1298	0.1298
Expenses per week (EUR)	4.57	3.64
Expenses per year (EUR)	237.86	189.53
Saving (EUR)	0	48.33
% of saving for the best (%)		20.32

Source: own processing of data

According to the table, the usage of propane gas is 28.08 liters for Savings Method, which is logically less than the usage of propane gas of initial route, because the distance of route by Savings Method is shorter than the real one. Therefore, expenses per week for the route of Savings Method is less than initial one, but the difference is not that big, for the first glance, only 0.93 EUR, but if calculate expenses for the whole year, the difference looks more significant, 48.33 EUR, this difference between expenses per year of the initial and

route proposed by Savings Method represents savings per year. The alternative route expenses are by 20.32% better than the initial ones. Although distances for each day during a year may differ depending on number of nodes, so the expenses and saving will vary too, the year expenses and saving are not precise number, but usage of the TSPkosa software shows certain improvement to the way of 'good enough' rather than do not use methods at all. What is more, calculation is done only *per one car*, there are 5 cars in the company's arsenal, so the annual savings for the 5 cars are 241.65 EUR and will grow if the company increase the size of the fleet or when the price of fuel grows, the savings will bring a big opportunity to the company in today's fierce competitiveness.

6 Conclusion and Recommendation

There is always a fierce competitiveness between a substantial number of companies on the market area. In this competitiveness, each chance of advantage over others counts, due to the fact that once success is achieved, it will lead to the next one, which creates more and more opportunities (Meadows, 2009, pp. 126-131). A great number of methods to get those opportunities are offered by the means of operations research. One of them is the implementation of software technology, such as the TSPkosa, which offers improvement of route scheduling by the approximation methods, namely Nearest Neighbour Algorithm, Vogel Approximation Method and Savings Method for Travelling Salesman Problem. Technology is an important force of change, it's better to pay attention to it. When technology used right, it becomes an accelerator of development (Collins, 2011, pp. 165-167).

The aim of the Bachelor Thesis is a reduction of route distance to decrease distribution expenses was achieved. The TSPkosa allowed for the Bachelor Thesis to show that in the selected company the situation can be positively shifted. According to the procedure, the most appropriate approximation method for the selected case is Savings Method. The comparison of real and the best alternative route distance illustrates, that usage of the approximation methods decreases the distance by 35.8 kilometers. That result reflects positively on distribution expenses as well as on savings. The alternative route expenses per year are by 20.32% better than the initial ones. It means that expenses are decreased, so profit value will be higher, so the main aim of firms to get as highest profit as possible has been achieved.

The fact of continued and consecutive improvement originates tremendous power (Collins, 2011, p. 187). The recommendation is that rational investment in technology improvements, such as the purchase of optimization or approximation software, will decrease distribution expenses in future, and save money, which will give the other opportunity and increase capital. This rational strategic decision will lead to prolongation of existence and success over other competitors. It is something more than just making money. Undoubtedly, profit is essential for business activities, but "growth for growth's sake" has crucial importance (Collins, 2011, pp. 203-207).

7 Bibliography

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Table 1: The Monday matrix

Monday	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	A ₇	A ₈	A ₉	A ₁₀	A ₁₁	A ₁₂	A ₁₃	A ₁₄	A ₁₅	A ₁₆	A ₁₇	A ₁₈	nodes	
A ₁		5.4	3	4.3	4.4	4.5	5	5.1	5.1	5.4	5.4	3.1	3.1	3.1	3.7	4.3	4.2	3.2	A ₁	
A ₂	5.4		2.5	1.6	1.7	1.8	1.4	1.6	1.6	1.9	2	2.4	2.3	2.4	2	2.7	1.6	2.5	A ₂	
A ₃	3	2.7		1.5	1.7	1.7	2.3	2.4	2.4	2.6	2.7	0.7	0.7	0.7	0.8	2.1	1.5	1.1	A ₃	
A ₄	4.2	1.7	1.5		0.4	0.4	1	1.1	1.1	1.4	1.4	1.9	1.9	1.9	1.6	2.9	1.8	2.1	A ₄	
A ₅	4.4	1.3	1.7	0.4		0.1	0.9	1	1	1.2	1.2	2.1	2	2.1	1.9	3.2	2.2	2.4	A ₅	
A ₆	4.4	1.3	1.7	0.4	0.1		0.9	1	1	1.3	1.3	2.1	2.1	2.1	2	3.3	2.2	2.5	A ₆	
A ₇	5	1.6	2.3	1	0.8	0.8		0.4	0.5	1	0.7	2.7	2.7	2.7	2.6	3.9	2.8	3	A ₇	
A ₈	5.1	1.7	2.4	1.1	1	0.9	0.4		0.1	0.6	0.6	2.8	2.8	2.8	2.6	3.9	2.9	3.1	A ₈	
A ₉	5.1	1.7	2.4	1.1	1	0.9	0.5	0.1		0.6	0.6	2.8	2.8	2.8	2.6	3.9	2.9	3.1	A ₉	
A ₁₀	5.4	2	2.6	1.4	1.2	1.2	1	0.6	0.6		0.3	3.1	3	3	2.9	4.2	3.2	3.4	A ₁₀	
A ₁₁	5.4	2.1	2.7	1.4	1.2	1.2	0.7	0.6	0.6	0.3		3.1	3.1	3.1	3	4.3	3.2	3.5	A ₁₁	
A ₁₂	3	2.4	0.7	1.9	2.1	2.1	2.7	2.8	2.8	3.1	3.1		0.1	0.1	0.7	1.6	1.4	0.6	A ₁₂	
A ₁₃	3.1	2.3	0.7	1.9	2	2.1	2.7	2.9	2.9	3	3.1	0.1		0.2	0.6	1.7	1.3	0.5	A ₁₃	
A ₁₄	3.1	2.5	0.7	2	2.2	2.2	2.8	2.9	2.9	3.2	3.2	0.1	0.2		0.7	1.7	1.4	0.6	A ₁₄	
A ₁₅	3.9	1.9	1	2.1	2.3	2.3	2.8	3	3	3.3	3.3	0.9	0.8	0.9		1.3	0.7	1	A ₁₅	
A ₁₆	4.3	2.3	1.4	2.6	2.8	2.8	3.4	3.5	3.5	3.7	3.8	1.3	1.2	1.4	1.1		1	1.4	A ₁₆	
A ₁₇	4.2	1.7	1.6	1.8	2.2	2.2	2.8	2.9	2.9	3.2	3.2	1.4	1.3	1.3	1	1.1		1.4	A ₁₇	
A ₁₈	2.9	2.6	0.9	2.1	2.2	2.3	2.9	3	3	3.2	3.3	0.2	0.3	0.3	0.9	1.6	1.5		A ₁₈	
																			total distance	22.9

Table 2: The Tuesday matrix

Tuesday	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	A ₇	A ₈	A ₉	A ₁₀	A ₁₁	A ₁₂	A ₁₃	A ₁₄	A ₁₅	A ₁₆	A ₁₇	A ₁₈	nodes	
A ₁		4.2	4.4	4.2	4.1	4	4.5	8.3	8.2	6.2	8	9.9	8.7	8.8	8.9	7.6	5.8	5.6	A ₁	
A ₂	4.2		1.7	2.1	2.2	2.1	2.4	4.5	4	2	3.8	6.6	5.7	5.7	5.7	3.6	1.6	1.4	A ₂	
A ₃	4.4	1.7		0.4	0.5	0.9	0.7	5.4	4.8	2.3	4.1	7.3	7.4	7.4	7.4	5.3	3.3	3.1	A ₃	
A ₄	4.2	2.1	0.4		0.1	0.5	0.4	5.8	5.2	2.7	4.5	7.7	7.7	7.8	7.8	5.7	3.7	3.5	A ₄	
A ₅	4.1	2.2	0.5	0.1		0.4	0.5	5.9	5.3	2.8	4.6	7.8	7.8	7.9	7.9	5.8	3.8	3.6	A ₅	
A ₆	4	2.1	0.8	0.5	0.4		0.8	6.1	5.6	3	4.9	8.1	7.8	7.8	7.8	5.7	3.7	3.5	A ₆	
A ₇	4.5	2.4	0.7	0.4	0.5	0.7		6	5.5	2.9	4.8	8	8	8	8.1	5.9	4	3.8	A ₇	
A ₈	8.2	4.5	5.4	5.8	5.9	5.8	6		0.8	3.3	1.6	2.1	3.1	3.2	3.2	1.3	3.2	3.2	A ₈	
A ₉	8.4	4.7	5.4	5.7	5.8	5.7	6	0.8		3.3	1.2	2.6	3.3	3.4	3.4	1.5	3.4	3.4	A ₉	
A ₁₀	5.9	1.8	2.1	2.5	2.6	2.5	2.7	4	3.3		2.5	5.8	5.4	5.4	5.4	3.3	1.3	1.1	A ₁₀	
A ₁₁	7.9	3.8	4.2	4.5	4.6	4.5	4.8	1.5	1	2.1		3.3	4.1	4	4	1.9	2.5	2.5	A ₁₁	
A ₁₂	9.7	6.6	7.3	7.7	7.8	7.7	8	2.1	2.6	5.3	3.2		1.5	1.4	1.2	3.4	5.3	5.3	A ₁₂	
A ₁₃	8.8	5.6	7.3	7.7	7.8	7.7	7.9	3.2	3.4	5.2	4.3	1.5		0.1	0.8	2.4	4.3	4.3	A ₁₃	
A ₁₄	8.9	5.7	7.4	7.8	7.9	7.8	8	3.2	3.5	5.3	4.4	1.4	0.1		0.9	2.5	4.4	4.4	A ₁₄	
A ₁₅	8.9	5.7	7.4	7.8	7.9	7.8	8	3.2	3.5	5.3	4.4	1.2	0.75	0.8		2.5	4.4	4.4	A ₁₅	
A ₁₆	7.3	3.2	4.9	5.3	5.4	5.3	5.5	1.7	1.7	2.8	1.9	3.7	2.9	2.9	2.9		1.9	1.9	A ₁₆	
A ₁₇	5.7	1.5	3.2	3.6	3.7	3.6	3.9	3.1	2.6	1.2	2.4	5.2	4.3	4.3	4.3	2.2		0.3	A ₁₇	
A ₁₈	5.4	1.3	3	3.4	3.5	3.4	3.7	3.6	2.9	1	2.7	5.5	4.7	4.6	4.6	2.5	0.5		A ₁₈	
																			total distance	36.1

Table 3: The Wednesday matrix

Wednesday	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	A ₇	A ₈	A ₉	A ₁₀	A ₁₁	A ₁₂	A ₁₃	A ₁₄	A ₁₅	A ₁₆	A ₁₇	A ₁₈	nodes	
A ₁		6.8	8.1	9.1	9.3	9	9.7	5.6	7.6	7.4	7.4	7.7	8.3	8.1	7.9	7.4	7.7	6.2	A ₁	
A ₂	6.8		7.8	2.3	2.5	2.5	4	2.5	4.6	3.7	3.5	3.7	4.3	4.4	4	3.9	4.9	2.5	A ₂	
A ₃	8.4	7.8		7.6	7.8	6.7	7.3	5.7	5.1	4.9	5.1	5.4	5.9	5.6	5.5	4.9	5.2	5.7	A ₃	
A ₄	8.7	2.2	7.2		0.2	1.1	2.8	3.7	4.1	3.1	3	3.1	3.7	3.8	3.4	3.4	4.4	3.1	A ₄	
A ₅	8.9	2.3	7.4	0.2		1.3	2.9	3.8	4.3	3.3	3.2	3.3	3.9	4	3.6	3.6	4.6	3.3	A ₅	
A ₆	8.7	2.1	6.3	1.1	1.2		1.9	3.5	3.2	2.3	2.1	2.2	2.8	2.9	2.5	2.5	3.5	2.8	A ₆	
A ₇	9.7	3.8	7.1	2.8	2.9	1.9		4.3	3.2	2.8	2.9	2.4	2.1	2.2	1.8	2.5	2.8	3.9	A ₇	
A ₈	5.4	2.6	5.5	3.7	3.9	3.6	4.3		2.5	1.9	2.1	2.3	2.8	3	2.5	1.9	2.8	1	A ₈	
A ₉	7.3	4	4.6	3.8	3.9	2.9	3	1.9		1.1	1.2	1.6	1.5	1.4	1.2	0.9	0.9	1.9	A ₉	
A ₁₀	7.2	3.4	4.4	3.2	3.4	2.3	2.6	1.8	1.1		0.2	0.6	1.2	1.4	0.9	0.3	1.4	1.5	A ₁₀	
A ₁₁	7	3.3	4.4	3.1	3.2	2.2	2.8	1.6	1.3	0.2		0.7	1.3	1.5	1	0.4	1.5	1.6	A ₁₁	
A ₁₂	7.7	3	5	2.8	3	2	2.6	2.3	1.7	0.6	0.7		1.1	1.2	0.8	0.9	1.8	1.7	A ₁₂	
A ₁₃	7.9	4.2	5.1	4	4.2	3.2	2.7	2.5	1.2	1	1	1.2		0.7	0.9	0.7	0.8	2.4	A ₁₃	
A ₁₄	7.9	4.3	5.1	4.1	4.3	3.2	2.5	2.9	1	1.3	1.5	1.4	0.7		0.9	1.1	0.6	2.8	A ₁₄	
A ₁₅	8	3.5	5.4	3.3	3.5	2.5	1.8	2.6	1.4	1	1.2	0.7	0.5	0.6		0.9	1.2	2.2	A ₁₅	
A ₁₆	7.2	3.7	4.6	3.5	3.6	2.6	2.4	1.8	1.1	0.3	0.4	0.9	1	1.1	0.6		1.4	1.8	A ₁₆	
A ₁₇	7.6	4.6	4.8	4.4	4.6	3.5	2.9	2.6	0.7	1.2	1.4	1.6	0.8	0.6	1.2	1		2.6	A ₁₇	
A ₁₈	6.2	2.2	5.3	3.7	3.9	2.8	3.6	1	2.3	1.6	1.4	1.5	2.6	2.7	2.2	1.7	2.6		A ₁₈	
																			total distance	48.2

Table 4: The Thursday matrix

Thursday	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	A ₇	A ₈	A ₉	A ₁₀	A ₁₁	A ₁₂	A ₁₃	A ₁₄	A ₁₅	A ₁₆	A ₁₇	A ₁₈	A ₁₉	nodes	
A ₁		3.1	2.8	3	3	4.3	3.2	6.8	6.9	6.4	6.8	7.1	6.7	7.2	7.8	8.2	9.7	9.4	9.8	A ₁	
A ₂	3.2		0.8	1.1	1.1	2.3	0.8	5.6	4.9	4.4	4.8	5.2	4.7	5.3	5.8	6.3	9	9.7	10.2	A ₂	
A ₃	2.8	0.8		0.4	0.7	2	1.6	4.8	4.1	3.8	4.2	4.5	4.1	4.6	5.1	5.7	9.8	10	10	A ₃	
A ₄	3	1.1	0.4		1	2.3	1.9	4.8	3.9	3.6	4	4.3	3.9	4.4	4.9	5.5	10	10	10	A ₄	
A ₅	3.1	1.1	0.7	1		1.6	0.9	4.9	4	3.7	4.2	4.4	4	4.5	5.1	5.7	9.1	9.8	10	A ₅	
A ₆	4.4	2.4	1.6	1.4	1.7		2.1	4.2	3.6	3	3.4	3.7	3.4	3.8	4.3	5	10	9.9	9.6	A ₆	
A ₇	3.2	0.8	1.6	1.9	0.9	1.7		5	4.1	3.8	4.4	4.5	4.2	4.6	5.1	5.9	8.2	8.9	9.4	A ₇	
A ₈	7.5	5.6	5	4.8	4.9	4.3	5		1.2	1.3	0.9	0.5	1	0.7	1	1.4	8.1	7.5	7.8	A ₈	
A ₉	6.7	4.7	4.1	4.2	4	3.5	4.5	1.2		1.3	1	0.7	1.1	0.8	1.2	2.3	7	6.4	6.7	A ₉	
A ₁₀	6.4	4.4	3.9	3.6	3.7	3.2	4.1	1.3	1.3		0.6	0.8	0.5	1	1.5	2.1	8.1	7.5	7.8	A ₁₀	
A ₁₁	6.8	4.8	4.3	4	4.3	3.8	4.8	0.9	1	0.7		0.4	0.3	0.5	1	1.7	8	7.4	7.7	A ₁₁	
A ₁₂	7.1	5.2	4.5	4.3	4.4	3.9	4.5	0.5	0.7	0.9	0.4		0.5	0.2	0.8	1.8	7.7	7.1	7.4	A ₁₂	
A ₁₃	6.7	4.7	4.1	3.9	4	3.5	4.5	1	1.1	0.5	0.3	0.5		0.6	1.1	1.8	8.1	7.4	7.8	A ₁₃	
A ₁₄	7.2	5.2	4.6	4.4	4.5	4	4.6	0.7	0.8	1	0.5	0.2	0.6		0.6	1.8	7.5	6.9	7.2	A ₁₄	
A ₁₅	7.7	5.8	5.2	5	5.1	4.4	5.1	1.2	1.2	1.5	1	0.8	1.1	0.6		1.9	7.6	6.9	7.5	A ₁₅	
A ₁₆	8.4	6.5	5.9	5.6	5.7	5.2	6.1	1.4	2.3	2.2	1.7	1.6	1.8	1.8	1.9		9.3	8.7	9	A ₁₆	
A ₁₇	9.7	9	9.8	10	9.1	9	8.2	8.7	7.6	8.7	8.5	8.3	8.6	8.1	7.6	7.6		0.7	1.2	A ₁₇	
A ₁₈	9.4	8.8	9.6	9.9	8.9	9.7	8	8.1	7	8.1	7.9	7.6	8	7.5	6.9	8.3	0.7		0.6	A ₁₈	
A ₁₉	9.5	8.7	9.5	9.8	8.8	9.6	7.9	8.4	7.3	8.4	8.2	8	8.3	7.8	7.5	8.8	1.2	0.6		A ₁₉	
																				total distance	41.2

Table 5: The Friday matrix

Friday	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	A ₇	A ₈	A ₉	A ₁₀	A ₁₁	A ₁₂	A ₁₃	nodes
A ₁		5.5	5.4	5.9	5.8	6	6.1	6.2	8.9	8.4	7.9	8.1	9.9	A ₁
A ₂	5.3		0.1	0.6	1.1	1.3	1.3	1.5	4.4	4.1	4.2	4.4	6.2	A ₂
A ₃	5.4	0.1		0.6	1.1	1.3	1.4	1.5	4.4	4.1	4.1	4.3	6.1	A ₃
A ₄	5.9	0.6	0.6		1.3	1.5	1.6	1.7	3.8	3.5	3.9	4.1	5.9	A ₄
A ₅	6	1.1	1.1	1.1		0.4	0.5	0.6	3.9	3.6	3.8	4	6.1	A ₅
A ₆	6.4	1.5	1.5	0.9	0.4		0.1	0.3	3.7	3.3	3.8	3.8	5.8	A ₆
A ₇	6.4	1.5	1.5	0.9	0.5	0.1		0.4	3.6	3.3	3.8	3.7	5.8	A ₇
A ₈	6.7	1.4	1.4	0.8	0.9	0.5	0.5		3.5	3.2	3.6	3.8	5.9	A ₈
A ₉	8.9	4.3	4.3	3.7	4.1	3.8	3.7	4		1.1	1.4	1.7	5.2	A ₉
A ₁₀	8.4	3.9	3.9	3.3	3.8	3.4	3.4	3.7	1.1		0.7	0.7	4.5	A ₁₀
A ₁₁	7.9	4.3	4.2	3.7	4	3.9	3.8	4.2	1.4	0.7		0.3	4.1	A ₁₁
A ₁₂	8.1	4.2	4.2	3.6	4.2	4.1	4.1	4.4	1.7	0.7	0.3		3.8	A ₁₂
A ₁₃	10	6.7	6.6	6.3	6.8	6.5	6.4	6.8	4.6	4.1	4.4	4.1		A ₁₃
													total distance	27.8

Table 6: The sequence of initial route and the best alternative route by Savings Method with distances on Monday

Monday			
Initial		Savings Method	
sequence	distances	sequence	Distances
A ₁	0	A ₁	0
A ₂	5.4	A ₁₈	3.2
A ₃	2.5	A ₁₂	0.2
A ₄	1.5	A ₁₄	0.1
A ₅	0.4	A ₁₃	0.2
A ₆	0.1	A ₁₅	0.6
A ₇	0.9	A ₁₆	1.3
A ₈	0.4	A ₁₇	1
A ₉	0.1	A ₂	1.7
A ₁₀	0.6	A ₇	1.4
A ₁₁	0.3	A ₁₁	0.7
A ₁₂	3.1	A ₁₀	0.3
A ₁₃	0.1	A ₈	0.6
A ₁₄	0.2	A ₉	0.1
A ₁₅	0.7	A ₆	0.9
A ₁₆	1.3	A ₅	0.1
A ₁₇	1	A ₄	0.4
A ₁₈	1.4	A ₃	1.5
A ₁	2.9	A ₁	3
Total	22.9	Total	17.3

Table 7: The sequence of initial route and the best alternative route by Savings Method with distances on Tuesday

Tuesday			
Initial		Savings Method	
sequence	distances	sequence	Distances
A ₁	0	A ₁	0
A ₂	4.2	A ₆	4
A ₃	1.7	A ₅	0.4
A ₄	0.4	A ₄	0.1
A ₅	0.1	A ₇	0.4
A ₆	0.4	A ₃	0.7
A ₇	0.8	A ₁₀	2.3
A ₈	6	A ₁₁	2.5
A ₉	0.8	A ₉	1
A ₁₀	3.3	A ₈	0.8
A ₁₁	2.5	A ₁₂	2.1
A ₁₂	3.3	A ₁₅	1.2
A ₁₃	1.5	A ₁₄	0.8
A ₁₄	0.1	A ₁₃	0.1
A ₁₅	0.9	A ₁₆	2.4
A ₁₆	2.5	A ₁₇	1.9
A ₁₇	1.9	A ₁₈	0.3
A ₁₈	0.3	A ₂	1.3
A ₁	5.4	A ₁	4.2
Total	36.1	Total	26.5

Table 8: The sequence of initial route and the best alternative route by Savings Method with distances on Wednesday

Wednesday			
Initial		Savings Method	
sequence	distances	sequence	Distances
A ₁	0	A ₁	0
A ₂	6.8	A ₈	5.6
A ₃	7.8	A ₁₈	1
A ₄	7.6	A ₂	2.2
A ₅	0.2	A ₄	2.3
A ₆	1.3	A ₅	0.2
A ₇	1.9	A ₆	1.3
A ₈	4.3	A ₇	1.9
A ₉	2.5	A ₁₂	2.4
A ₁₀	1.1	A ₁₁	0.7
A ₁₁	0.2	A ₁₀	0.2
A ₁₂	0.7	A ₁₆	0.3
A ₁₃	1.1	A ₁₅	0.6
A ₁₄	0.7	A ₁₃	0.5
A ₁₅	0.9	A ₁₄	0.7
A ₁₆	0.9	A ₁₇	0.6
A ₁₇	1.4	A ₉	0.7
A ₁₈	2.6	A ₃	4.6
A ₁	6.2	A ₁	8.4
Total	48.2	Total	34.2

Table 9: The sequence of initial route and the best alternative route by Savings Method with distances on Thursday

Thursday			
Initial		Savings Method	
sequence	distances	sequence	Distances
A ₁	0	A ₁	0
A ₂	3.1	A ₄	3
A ₃	0.8	A ₃	0.4
A ₄	0.4	A ₂	0.8
A ₅	1	A ₇	0.8
A ₆	1.6	A ₅	0.9
A ₇	2.1	A ₆	1.6
A ₈	5	A ₁₀	3
A ₉	1.2	A ₁₃	0.5
A ₁₀	1.3	A ₁₁	0.3
A ₁₁	0.6	A ₁₆	1.7
A ₁₂	0.4	A ₈	1.4
A ₁₃	0.5	A ₁₅	1
A ₁₄	0.6	A ₁₄	0.6
A ₁₅	0.6	A ₁₂	0.2
A ₁₆	1.9	A ₉	0.7
A ₁₇	9.3	A ₁₇	7
A ₁₈	0.7	A ₁₈	0.7
A ₁₉	0.6	A ₁₉	0.6
A ₁	9.5	A ₁	9.5
Total	41.2	Total	34.7

Table 10: The sequence of initial route and the best alternative route by Savings Method with distances on Friday

Friday			
Initial		Savings Method	
sequence	distances	sequence	Distances
A ₁	0	A ₁	0
A ₂	5.5	A ₁₃	9.9
A ₃	0.1	A ₁₂	4.1
A ₄	0.6	A ₁₁	0.3
A ₅	1.3	A ₁₀	0.7
A ₆	0.4	A ₉	1.1
A ₇	0.1	A ₇	3.7
A ₈	0.4	A ₆	0.1
A ₉	3.5	A ₅	0.4
A ₁₀	1.1	A ₈	0.6
A ₁₁	0.7	A ₄	0.8
A ₁₂	0.3	A ₃	0.6
A ₁₃	3.8	A ₂	0.1
A ₁	10	A ₁	5.3
Total	27.8	Total	27.7