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AN EMPIRICAL TESTING OF CAPM MODEL ON CRYPTO CURRENCY MARKET

EMPIRICKÉ TESTOVÁNÍ CAPM MODELU NA TRHU KRYPTOMĚN

MASTER'S THESIS

DIPLOMOVÁ PRÁCE

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Objectives of the work, methods, and procedures of processing
Theoretical background of the thesis
Analysis of the current state
Own proposals for solutions
Conclusion
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Attachments

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Abstract

This master's thesis explores the application of the Capital Asset Pricing Model (CAPM) to the cryptocurrency market. It investigates the feasibility of CAPM in this emerging market, focusing on constructing the efficient frontier, tangency portfolio, minimum variance portfolio, and beta coefficient testing. Utilising quantitative methods, the thesis evaluates the linearity of beta coefficients and the influence of non-systematic risk on returns and assesses the validity of CAPM assumptions. The evidence is tested using a market portfolio and benchmark portfolio for the construction of the model and compare the findings. This empirical analysis contributes to understanding traditional financial models' adaptability to digital asset markets.

Keywords

Bitcoin, cryptocurrency, modern portfolio theory, CAPM

Abstrakt

Tato diplomová práce zkoumá aplikaci Modelu ocenění kapitálových aktiv (CAPM) na trhu s kryptoměnami. Práce zkoumá proveditelnost CAPM na tomto nově vznikajícím trhu, s důrazem na konstrukci efektivní hranice, tangenciálního portfolio, portfolia s minimální variancí a testování beta koeficientů. Pomocí kvantitativních metod tato práce hodnotí lineárnost beta koeficientů a vliv nesystematického risku na návratnost a posuzuje platnost předpokladů CAPM. Aplikace modelu je testována na tržním portfoliu a srovnávacího portfolia a výsledky jsou porovnány. Tato empirická analýza přispívá k pochopení aplikace tradičních finančních modelů na trzích s digitálními aktivy.

Klíčová slova

Bitcoin, kryptoměny, moderní teorie portfolio, CAPM

Rozšířený abstrakt

Bitcoin se od svého uvedení na trh v roce 2009 stal předmětem zájmu mezi investory, akademiky a veřejností. Jedinečné vlastnosti jako decentralizace a známá budoucí inflace, způsobili masivní nárust na jeho ceně. Od uvedení v roce 2009 šla cena Bitcoinu z nuly na 73 000 USD v březnu 2024 a dosáhl nejvyšší tržní kapitalizace přes miliardu dolarů. Kolem Bitcoinu následně vznikl nový trh digitálních aktiv, kryptoměn. Nedávné schválení Bitcoinových spotových ETF nyní přitahuje nejen spekulanty, ale také profesionální investory, veřejnost a firmy, k investici do kryptoměn. Proto je třeba porozumět tomuto novému typu finančních aktiv a způsobům, jak začlenit kryptoměny do investičních portfolií.

V teoretické části je nejdříve popsáno, proč investoři považují Bitcoin a kryptoměny za finanční aktiva. Navíc z teorie efektivních trhů navíc vyplývá, že v budoucnu budou investoři k těmto aktivům přistupovat více standartně a méně spekulovat na jejich cenně. Proto je třeba porozumět rizikům kryptoměn a způsobu, jak je zařadit do diverzifikovaných investičních portfolií.

Diverzifikovaná portfolia kombinují aktiva, aby dosáhla nižšího rizika a vyššího očekávaného zisku. Taková portfolia vycházejí z moderní teorie portfolia a stěžejního modelu ocenění kapitálových aktiv (CAPM), a využívají konceptu beta koeficientu. Beta je míra, která vyjadřuje citlivost výnosu investice na změny výnosů na trhu jako celku. K výpočtu bety se běžně používá tržní portfolio.

Tato práce tedy popisuje ekonometrickou a statistickou teorii, na které jsou tyto modely postaveny. Následně popisuje analytický způsob řešení těchto modelů, a to včetně efektivní hranice portfolia, přímku kapitálového trhu a dvou pro investory nejatraktivnějších portfolií: tangenciálního portfolia a portfolia s minimální variancí.

Efektivní hranice portfolia představuje grafickou reprezentací všech možných portfolií, které lze vytvořit z daných aktiv, a označuje ty, které jsou "efektivní" nebo optimální. Z hlediska investora je cílem při vytváření portfolia dosáhnout bodu na efektivní hranici, který nejlépe odpovídá jeho preferencím v oblasti rizika a výnosu. Tím se optimalizuje návratnost portfolia a minimalizuje se riziko.

Přímka kapitálového trhu představuje vztah mezi rizikem a očekávaným výnosem portfolia, která kombinují bezrizikové aktivum jako státní dluhopis.

Tangenciální portfolio se nachází na tečně efektivní hranice portfolia a přímky kapitálového trhu. Toto poskytuje investorům optimální poměr mezi rizikem a výnosem a mělo by odpovídat tržnímu portfoliu, ve kterém jsou aktiva rozdělená na základě váženého průměru tržní kapitalizace.

Portfolio s minimální variací minimalizuje riziko portfolia a je schopno pomocí správné kombinace aktiv dosáhnout nižší riziko než jakékoliv samotné aktivum.

Na závěr teoretické části je popsán test předpokladů modelu ocenění kapitálových aktiv, pomocí kterého se otestují čtyři hlavní předpoklady:

- Přesnost očekávané návratnosti na časové řadě
- Přesnost očekávané návratnosti aktiv
- Linearita beta koeficientů a očekávané návratnosti
- Vliv nesystematického rizika na návratnost

V praktické části jsem vybral pět kryptoměn na základě jejich tržní kapitalizace, doby na trhu, a zdali jsou určeny k investování. Jako tržní portfolio jsem vybral kryptoměnové tržní portfolio založené na váženém průměru tržní kapitalizace a indexu S&P 500 jako benchmarku. Díky tomu jsou modely otestovány i proti běžně uznávanému benchmarku a výsledky jsou mezi sebou porovnány. Testovací období je zvoleno po dobu pěti let od 31.12.2018 do 31.12.2023.

Pomocí metodiky z teoretické části jsem vypočítal očekávaný výnos a beta koeficienty pro vybrané kryptoměny na základě obou vybraných tržních portfolií. Beta koeficienty byly testovány pomocí standardních statistických metod, F-testu variance a T-statistiky sklonu regresní přímky. Tyto testy potvrdily statistickou významnost všech beta koeficientů, což podporuje jejich použití pro odhad rizika aktiv. Efektivní hranice portfolia byla vytvořena, abych identifikoval možná portfolia s optimálním poměrem očekávaného výnosu a rizika. To zahrnovalo sestavení portfolia s minimální variancí a nalezení tangenciálního portfolia, které nabízí nejlepší poměr očekávaného výnosu k riziku.

Testování předpokladů modelu ocenění kapitálových aktiv ukázalo, že kryptoměnové tržní portfolio poskytuje přesnější odhad očekávaných výnosů než S&P 500. Pro obě tržní portfolia testy také prokázali lineární vztah mezi beta koeficienty a očekávanými výnosy. A nakonec že riziko není významně ovlivněno nesystémovými faktory.

Kryptoměnové tržní portfolio se shodovalo s tangenciální portfoliem, což odpovídá teorii a podporuje jeho použití jako optimálního portfolia.

Na základě těchto zjištěních podporuji využitelnost modelu ocenění kapitálových aktiv na trhu s kryptoměnami, protože teorie je v souladu s empirickými daty. V případě zařazení kryptoměn do složeného portfolia z jiných typů aktiv je možné využít indexu S&P 500 pro výpočet beta koeficientů kryptoměn i přes menší přesnost očekávaných výnosů. Pro pasivní investory doporučuji tržní kryptoměnové portfolio jako optimální portfolio pro investování, výpočet beta koeficientů kryptoměn a jejich očekávaných výnosů.

Bibliographic citation

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Affidavit

I declare that the present master project is an original work that I have written myself. I declare that the citations of the sources used are complete, that I have not infringed upon any copyright (pursuant to Act. no 121/2000 Coll.).

Brno dated 29th Apr 2024

Bc. Petr-Lev Bařina

author’s signature

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I want to express my gratitude to my family for supporting me during my long university years.

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INTRODUCTION

Bitcoin has gained immense popularity and fascination among investors and academics since its launch in 2009. Its unique non-inflationary properties and decentralisation have sparked massive speculation on its price, resulting in the price going from virtually zero in 2008 to an all-time high price of over 73 000 US dollars in March 2024 and having an all-time high market capitalisation of over one trillion US dollars. This price action followed Bitcoin´s spot ETF approval in the US, attracting, for example, the biggest ETF issuer on the planet, BlackRock, to join the world of cryptocurrency. The three biggest Bitcoin ETFs have collectively attracted over 45 million US dollars in investments. This attracts not only speculators but also professional investors, the general public and companies, resulting in a growing need to understand this financial asset and how to incorporate cryptocurrencies into investment portfolios.

Due to market restraints, modern portfolio theory tries to create the best investment portfolio using quantitative methods. This approach can help investors know how much risk they are exposed to, given their expected return. This thesis tries to conclude how to successfully use this theory in the cryptocurrency market. The critical model used is the Capital Asset Pricing Model. This model uses the concept of a market portfolio for its construction. To validate the findings, the thesis uses a cryptocurrency market portfolio and, as a benchmark portfolio, uses the index S&P 500. The findings of the cryptocurrency market portfolio and benchmark market portfolio are compared.

The first part discusses theoretical points about money, assets, and cryptocurrencies. Modern portfolio theory and CAPM are discussed, and a detailed overview of methods is provided to calculate and test this model.

The second part constructs and tests the model using Microsoft Excel software by deriving Excel functions from the theory described in the first part.

In the third part, all the findings are discussed and evaluated.

The last part is suggestions for improvement.

OBJECTIVES, METHODS AND PROCEDURES OF PROCESSING

Objectives

This thesis aims to empirically investigate the applicability of the Capital Asset Pricing Model (CAPM) in the cryptocurrency market and provide insights into the effectiveness of the CAPM in capturing systematic risk in this asset type.

Solution methodology

This thesis uses a deductive approach. Academic literature is used to research the theory behind the approaches used and formulate hypotheses. The hypothesis will be tested and analysed. The analysis will be based on statistics and econometrics methods. The primary literature used is focused on applying modern portfolio theory in the traditional financial market. This is done to ensure that the utilised approaches stem from original and validated sources due to the relative youth of the cryptocurrency market.

The critical part is computing CAPM, efficient portfolio frontier and two key portfolios using Microsoft Excel. The CAPM model calculates all the model coefficients using the market portfolio concept. The model is constructed twice using the cryptocurrency market portfolio and benchmark portfolio to validate and compare the findings. The benchmark portfolio used is the S&P 500 index.

Secondary data from the cryptocurrency market are used to construct the investment portfolios. Five cryptocurrencies were selected for the study based on three criteria: market capitalisation, having at least five years of market data, and not being stablecoins. The thesis will rely on quantitative methods to analyse historical price data of various cryptocurrencies, aiming to uncover patterns and relationships that inform portfolio construction strategies.

By analysing historical market data and employing statistical techniques, the study aims to identify optimal portfolio compositions that maximise expected returns for a given level of risk. The quantitative approach will involve data analysis, statistical modelling, and hypothesis testing to assess the applicability of CAPM in the cryptocurrency market context, providing valuable insights into portfolio management strategies in this emerging asset class.

The key model used in this thesis is the Capital Asset Pricing Model developed by William Sharpe in the 1960s. This model operates under the assumption of market equilibrium, where markets are efficient, and the price reflects all the available information. CAPM allows for the analytical calculation of efficient frontier and two of the most exciting portfolios for investors: the minimum variance portfolio, which offers the lowest risk in a given asset class, and the tangency portfolio, which offers the best risk-to-reward ratio.

To test the CAPM, the key literature used is "Testing the CAPM model: A study of the Chinese Stock Market." (Yang, Xu, 2006), which offers a methodological solution to testing the CAPM assumptions. Based on the test proposed in this literature, I will conclude whether the CAPM applies to the cryptocurrency market. The test is done for both market portfolios.

1 THEORETICAL BACKGROUND

1.1 Money

Money serves as a universally accepted medium of exchange. It has three main roles:

- Store of value
- Unit of Accounting
- Medium of exchange

For money to meet this criterion, its value must be preserved over time. Money comes in public and private forms. Public money is primarily in the form of physical cash issued by central banks. Private money consists mostly of balances in commercial banks denominated in public money, such as US dollars. Most of the liabilities of banks and other institutions that are donated in public money and can be effectively used as transfers can be viewed as money. In today's world, most money people interact with is private money. Private money can also be easily exchanged for public money (United States Department Of The Treasury, 2022).

1.2 Real and financial assets

Asset refers to the material wealth within an economy. We can distinguish between real and financial assets. Real assets are comprised of land, buildings, and machinery that directly contribute to producing goods and services, and financial assets, such as stocks and bonds, represent claims on real assets. While real assets generate income, financial assets facilitate the distribution of that income among investors.

Financial assets include debt securities, equity securities and financial derivatives.

Debt securities, also called fixed income or bonds, promise fixed or formula-based income streams. Equity securities offer ownership in firms, and while equity holders do not have fixed payments, they can receive dividends, a share of the company's profit, using their real assets. Lastly, derivatives are securities whose value is derived from an underlying asset price the derivative was derived from, such as stock or bonds.

Each type of financial asset serves distinct purposes in investment portfolios, ranging from providing stable income to offering the potential for capital appreciation or risk management (Bodie, Kane, Marcus, 2008).

1.2.1 Financial markets

As described before, while real assets create wealth in the economy, financial assets allow us to use them to their fullest potential. By utilising investors in a capitalistic system, we can ensure the effective allocation of capital resources that would otherwise be impossible.

Financial markets serve the purpose of capital allocation through financial assets. For example, when a company demonstrates the possibility for future profitability, its stock price rises, making it easier for the company to raise funds for expansion and innovation. On the other hand, if a company's chances to be profitable are low, investors drive down its stock price, leading to fewer opportunities to raise new capital or even downsize. Despite occasional inefficiencies, such as short-time market trends, the stock market efficiently allocates capital to firms with perceived growth potential based on the collective judgment of analysts and investors.

Risk allocation is the next fundamental aspect of financial markets, with various investment instruments created to hedge risk or change the risk profile. Real assets inherently involve uncertainty, and financial markets enable investors to select securities, enabling them to change the risk they are exposed to. For instance, investors seeking higher returns may choose stocks with greater business risk, while those preferring stability may prefer bonds that offer fixed payments. This risk allocation mechanism accommodates investors' preferences and supports firms in raising capital for their investments, as securities can be priced optimally to attract investors with differing risk tolerances (Bodie, Kane, Marcus, 2008).

1.2.2 Efficient market hypothesis

The efficient market hypothesis (EMH) states that security prices rapidly reflect all available information, challenging the effectiveness of active investment strategies. It suggests that passive management, such as having diversified portfolios, may offer better returns without the costly security analysis. Thus, an investor can only obtain above-average market returns by investing in riskier assets.

This form of EMH is called strong EMH, and there is not much empirical evidence to support its claims. On the other hand, weak EMH states that current asset prices reflect all historical information, and future prices will exhibit a random walk, meaning that while the markets are not perfectly efficient, investors can get above-average market returns by pure chance.

This is a controversial statement, and since the 1960s, when this hypothesis started to become popular in mainstream economics, there has been much empirical evidence confirming and rejecting this hypothesis (Titan, 2015).

1.2.3 Adaptive Market Hypothesis

The Adaptive Market Hypothesis (AMH) merges the principles of the Efficient Market Hypothesis (EMH) with insights from behavioural finance. The AMH proposed by Andrew Lo (2004) acknowledges market participants' rationality while recognising that market efficiency can vary over time and across different market conditions. Unlike the EMH, which assumes market participants are fully rational and markets are always efficient, the AMH acknowledges that investors may behave irrationally at times, leading to market inefficiencies.

The adaptive market hypothesis seeks to merge the principles of the efficient market hypothesis and behavioural finance. It argues that markets and people are predominantly rational, but there are periods when investors may overreact during increasing market volatility (Lo, 2004). Based on the adaptive market hypothesis, market efficiency will rise over time and with growing interest in Bitcoin, allowing standard financial theory to be applied to it.

1.3 Cryptocurrency

Cryptocurrency is a digital or virtual form of currency. It can be considered a private form of money in a private monetary system. However, it should not be confused with digital money in the form of bank deposits. Unlike conventional currencies, cryptocurrencies operate on decentralised networks based on blockchain technology, which securely records all transactions or individual account balances across a distributed ledger (Narayanan, Bonneau, Felten, Miller, Goldfeder, 2016).

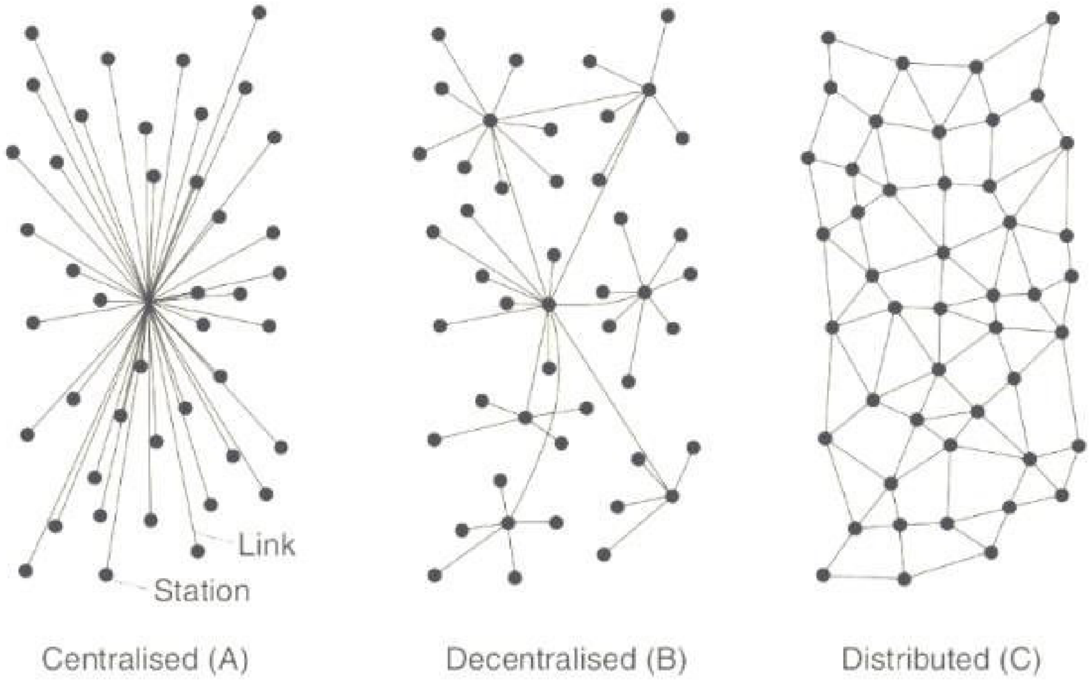


Figure 1: **Types of networks**

(Source: Nguyen, Upul, Tai-Won, Gyu Myoung, 2016)

Bitcoin, introduced in 2009 by an unknown person or group using the pseudonym Satoshi Nakamoto, was the first cryptocurrency (Nakamoto, 2008). Since then, numerous other cryptocurrencies, often called altcoins, have emerged, each with unique features and purposes.

The blockchain technology upon which the cryptocurrencies are made ensures transparency, security, and immutability of transactions by storing them in blocks linked together cryptographically. This eliminates the need for intermediaries such as banks or governments, allowing for peer-to-peer transactions across the globe without the need for traditional financial institutions. Moreover, cryptocurrencies offer anonymity and privacy, as transactions are settled using cryptographic keys, not personal information. However, because all transactions are transparent and visible on the blockchain, they can be traced to their origin.

Cryptocurrencies serve various purposes beyond facilitating peer-to-peer transactions. Some altcoins focus on utility through decentralised applications or smart contracts, some focus on decentralized finance (DeFi) like supply chain management and digital identity verification, and there are many more classes of altcoins (Narayanan, Bonneau, Felten, Miller, Goldfeder, 2016).

Bitcoin is the first and biggest cryptocurrency, the trailblazer of this new technology.

1.4 Bitcoin

Bitcoin is a decentralised peer-to-peer payment network that uses blockchain technology to store transactions on a distributed, decentralised ledger. The token used to settle transactions on the Bitcoin network is also Bitcoin (Segendorf, 2014).

The Bitcoin system utilises cryptographic proof to record transactions into the ledger, ensuring trust between network parties without needing third parties like traditional financial institutions.

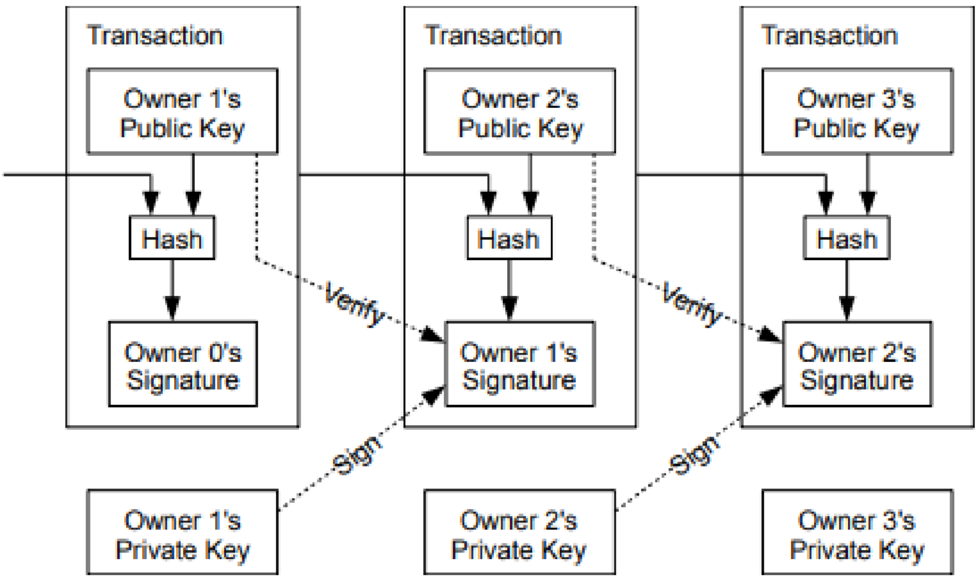


Figure 2: Bitcoin's transaction process

(Source: Nakamoto, 2008, p. 2)

In this proposed system, each transaction is recorded as a chain of digital signatures, with each owner transferring the coin to the next by digitally signing the transaction details. To prevent double spending without relying on a central authority, transactions are publicly announced, encoded with timestamps, and participants collectively agree on the chronological order of transactions. This mechanism ensures that the majority of network nodes validate the transaction order.

Participants earn Bitcoin tokens by validating transactions as a reward for their computational power and electricity costs, earning tokens to participants and distributing them organically into the system without any central authority. Moreover, participants pay transaction fees to the network distributed between validators. This ensures that once the transaction rewards in newly issued Bitcoin tokens are not high enough, the fees will keep the incentive to participate as a validator. Once the predetermined number of tokens enter the system, the fees will be the only incentive, and the Bitcoin token will be completely inflation-free (Nakamoto, 2008).

For the purpose of this thesis, I will be addressing the Bitcoin token as Bitcoin and will focus on analysing the price behaviour of this token in relation to real-world currencies such as the US dollar.

1.4.1 The role of Bitcoin for investors

Initially, Bitcoin was proposed as a digital currency used in a private monetary system. Digital currency should not be confused with electronic money. Electronic money is digitally stored, most often private money, which can be used as public money and freely exchanged for public money. As discussed above, there are three primary roles of money:

- Store of value
- Unit of Accounting
- Medium of exchange

Over time, Bitcoin has proven to be a great medium of exchange, offering low transaction fees and not being able to be blocked by a central authority. However, its high volatility makes it unpractical as a unit of accounting and store of value, suggesting that Bitcoin cannot be used as a form of money (Lengyel-Almos, Demmler, 2021).

Moreover, data suggest that less than 50% of Bitcoins in circulation are used in transactions. Investors use the rest of the Bitcoins for diversification and hedging of other financial instruments. It has also been used for speculation; at times, the price action displays characteristics of a speculative asset bubble. This suggests that investors consider Bitcoin an asset (Kurihara, Fukushima, 2017).

1.4.2 Bitcoin halving

Halving is arguably the most important technical aspect of Bitcoin in terms of price prediction. Halving is an event where miners' rewards are slashed in half – so the rewards of Bitcoins to participants in transaction validation decrease by 50%. This event occurs approximately every four years and signals a new Bitcoin market cycle. Because the reward is smaller after halving, the return on investments for validators is smaller, but Bitcoin inflation is also more negligible. In the months following the halving, the price of Bitcoin surged dramatically (Meynkhard, 2019).

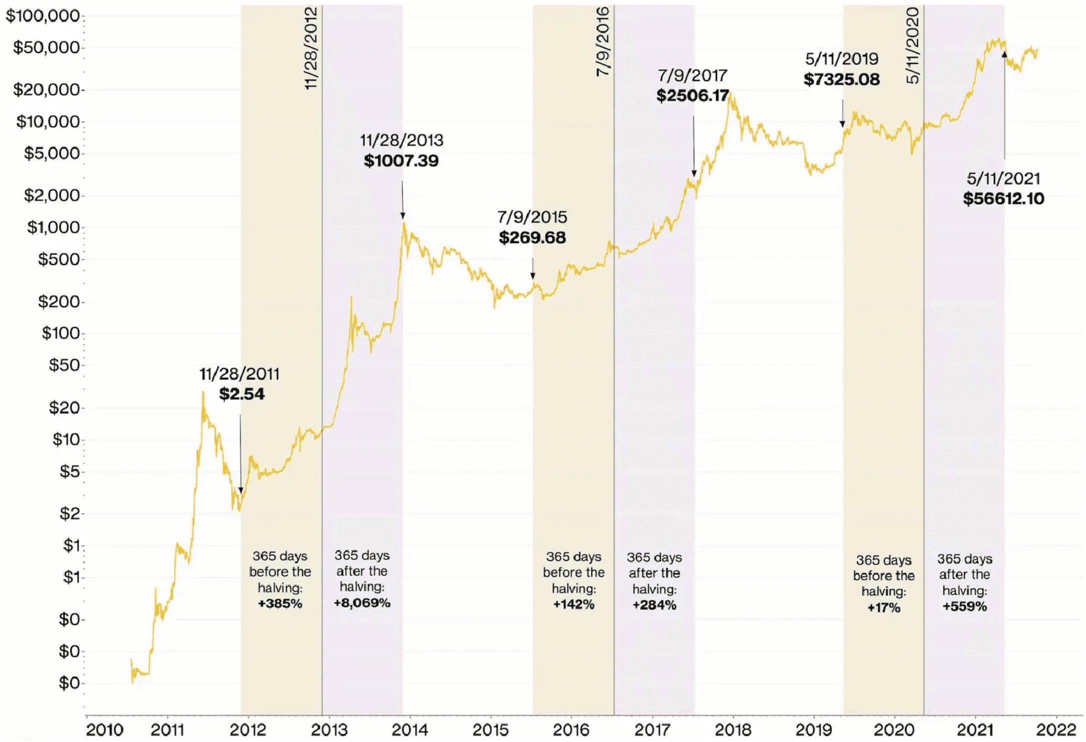


Figure 3: Bitcoin price on a logarithmic scale and effects of halving event

(Source: Hertig, @2024)

Halving events are attractive to investors, not only because of short-term price increases and changes in supply but also because of known future inflation. There are going to always be only 21 million Bitcoins available for investors. And while some literature suggests that this price action has characteristics of an asset bubble, some empirical evidence suggests a correlation between miners' rewards and the price of Bitcoin (Meynkhard, 2019).

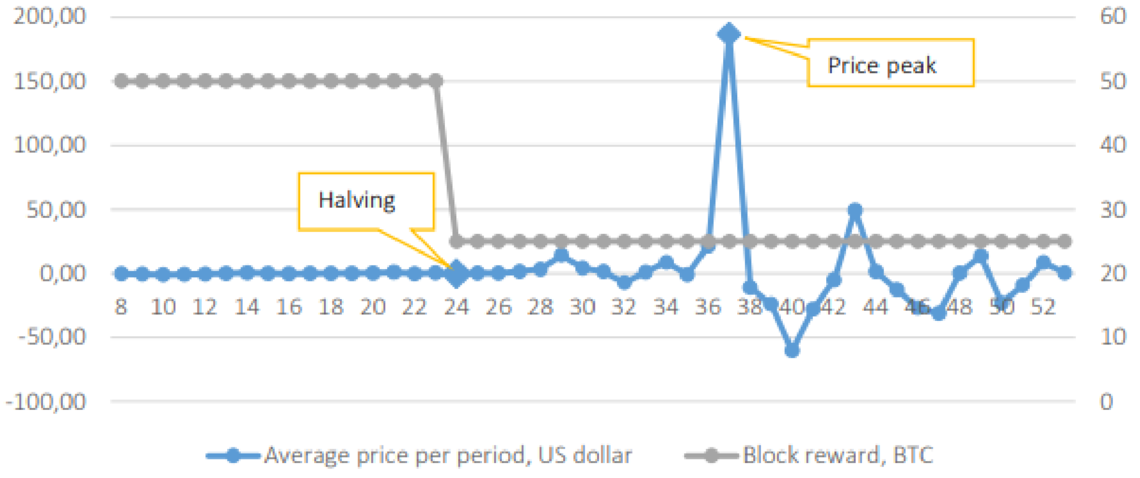


Figure 4: **The effect of halving on Bitcoin price, July 2011 – March 2015**
(Source: Meynkhard, 2019, p. 82)

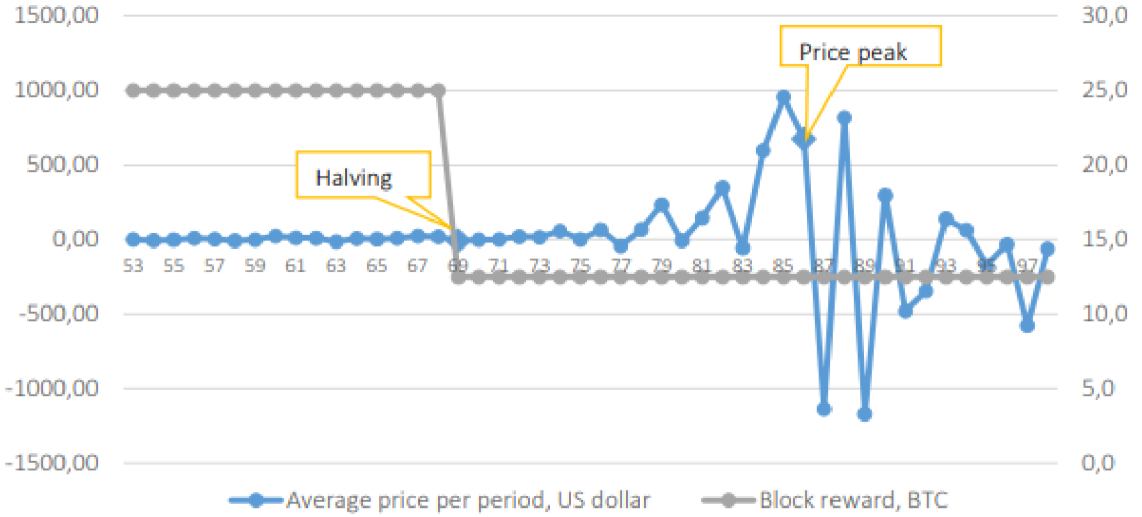


Figure 5: **The effect of halving on Bitcoin price, April 2015 – November 2018**
(Source: Meynkhard, 2019, p. 82)

The above pictures show price fluctuation after halving events.

1.4.3 Efficiency of Bitcoin

Several pieces of literature have tested the efficient market hypothesis (EMH) on Bitcoin. One qualitative analysis, Lengyel-Almos and Demmler (2021), looked at 25 high-rated papers that tested EMH on Bitcoin markets using empirical data covering different periods and different models to test EMH. Twenty of these studies rejected EMH, while five accepted it. The five studies accepted weak or semi-strong hypotheses, suggesting that no exploitable opportunities in the Bitcoin market can lead to above-market returns.

However, most of the twenty studies rejecting EMH also suggest that there are times of efficiency in the market and when the price exhibits signs of an asset price bubble, which rejects the rational behaviour of investors in efficient markets. For example, Kurihara and Fukushima, 2017, tested the period from 2010 to 2016 and found that for the whole tested period, the EMH was rejected. However, the market was efficient in the second part of the testing period. This means that the period after the halving event at the end of 2012 was efficient. They also suggest that the market might become more efficient in the future as it becomes more liquid. This claim is supported by a number of the twenty studies that initially rejected the EMH in their tested periods.

It is worth noting that while some of the works used models such as GARCH to capture the changing periods of high and low volatility, none of them specifically accounted for the halving event and only talked about the price increase in these periods as an asset price bubble (Lengyel-Almos, Demmler, 2021).

These findings support the adaptive market hypothesis, which combines EMH and behavioural finance. AMH could explain investors' overreaction to halving events and increasing volatility. Khuntia and Pattanayak (2018) tested AMH on the Bitcoin market, and their findings are consistent with EMH works. Thus, the hypothesis of Bitcoin having an adaptively efficient market was confirmed. There are periods of greater and lesser efficiency, but over time, Bitcoin volatility decreases while efficiency increases (Khuntia, Pattanayak, 2018).

While investors may have above-market returns when speculation on Bitcoin's price is based on halving events, as the market's efficiency may increase over time, investors need to have a diversified portfolio of investments.

1.5 Mathematical modelling, statistics and econometrics

Mathematical modelling represents real-world phenomena through mathematical equations. It enables researchers to analyse, predict, and understand complex relationships. Mathematical models can help us make decisions when solving real-life problems using empirical data.

Statistics is a science field built upon a probability theory. Using mathematical methods, statistics helps collect, analyse and interpret data.

Econometrics applies mathematical and statistical tools to economics. Economists or financial institutions use econometrics to simplify, understand and predict economic phenomena (Boyle, 2020).

1.5.1 Expected value

The expected value is the average outcome of a random variable, providing a way to quantify the long-term behaviour of random events. For random variable *X* the expected value is denoted *E(X)* (Skrondal, Everitt, 2010).

1.5.2 Arithmetic mean

The arithmetic mean is the average value of a sample of observations. For random variable *X,* the sample arithmetic mean is calculated as the sum of all values, $x_1, \dots, x_n$, divided by the count of the values.

Equation 1: Arithmetic mean
(Source: Skrondal, Everitt, 2010, p. 27)

$$\bar{x} = \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)$$

The sample mean for *X* is denoted as $\bar{x}$ while the population mean is denoted μ_x (Skrondal, Everitt, 2010).

1.5.3 Geometric mean return

The geometric mean return measures the average return rate on investment over multiple periods, considering compounding effects (Bodie, Kane, Marcus, 2008).

Equation 2: Geometric mean return
(Source: Processed according to Skrondal, Everitt, 2010, p. 186)

$$R = \left(\prod_{i=1}^{n}(1 + R_i)\right)^{\frac{1}{n}}$$

Where *n* is the number of observations in the data set.

1.5.4 Conditional probability

Conditional probability represents the likelihood of an event occurring, given that another event has already occurred. It is adjusting the odds based on new information. The conditional probability of event A happening given event B is expressed and calculated as:

Equation 3: The conditional probability of A given B
(Source: Boyle, 2020, p. 63)

$$P = (A|B) = \frac{P(A)P(B|A)}{P(B)}$$

However, not all events are conditional. If the probability of event A given B is the same as the probability of A, then the events are independent (Skrondal, Everitt, 2010).

Equation 4: The conditional probability of A if A and B are independent
(Source: Skrondal, Everitt, 2010, p. 98)

$$P = (A|B) = (A)$$

1.5.5 Data distribution

Data distribution is a pattern of the probability of each value in the data set. Sometimes referred to as probability distribution, it shows the probability density of observing values of a given variable. The most important aspect of the data distribution is its shape, which can vary depending on the nature of the data set used (Skrondal, Everitt, 2010).

1.5.6 Normal distribution

Normal distribution, also known as the Gaussian distribution, is characterized by a symmetrical, bell-shaped curve. In a normal distribution, the majority of data values cluster around the mean, with fewer values occurring as one moves farther away from the mean in either direction. The curve is defined by two parameters: the mean (μ) and the standard deviation (σ). The standard deviation determines the spread or dispersion of the data points around the mean, with larger standard deviations indicating greater variability (Skrondall, Everitt, 2010). For random variable X which follows normal distribution, the distribution function is:

Equation 5: **Normal distribution function**

(Source: Processed according to Skrondal, Everitt, 2010, p. 305)

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} exp\left[-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right]$$

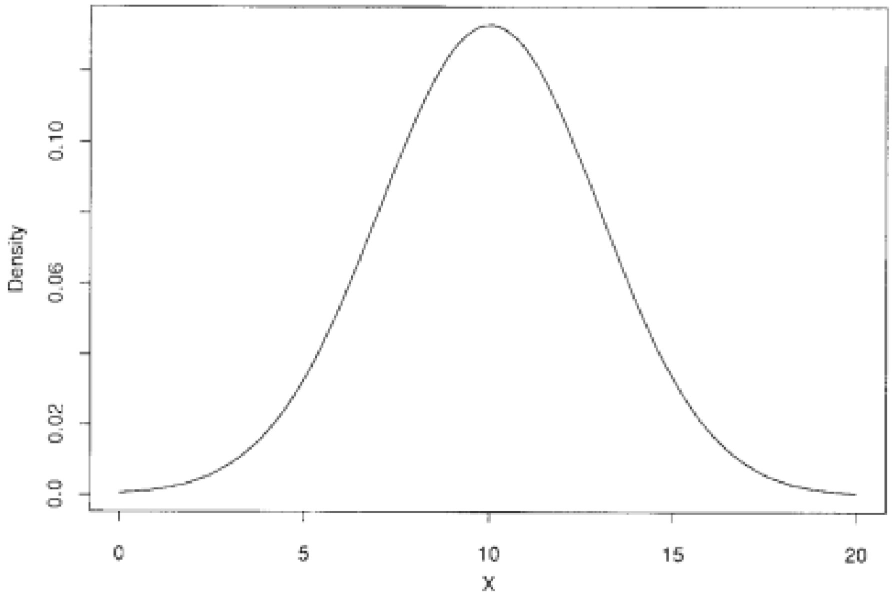


Figure 6: **Normal distribution**

(Source: Skrondal, Everitt, 2010, p. 306)

1.5.7 Students's distribution

This distribution has thicker tails than normal distribution and is used in testing conditional probability and hypothesis. The thicker tails allow for testing samples of smaller sizes or with unknown standard deviations. It is also characterised by the "degrees of freedom" (v) parameter, which influences the distribution shape. With an increasing number of degrees of freedom, v, approximating the standard normal distribution (Skrondall, Everitt, 2010).

Equation 6: **Student´s t-distribution**

(Source: Processed according to Skrondal, Everitt, 2010, p. 419)

$$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$$

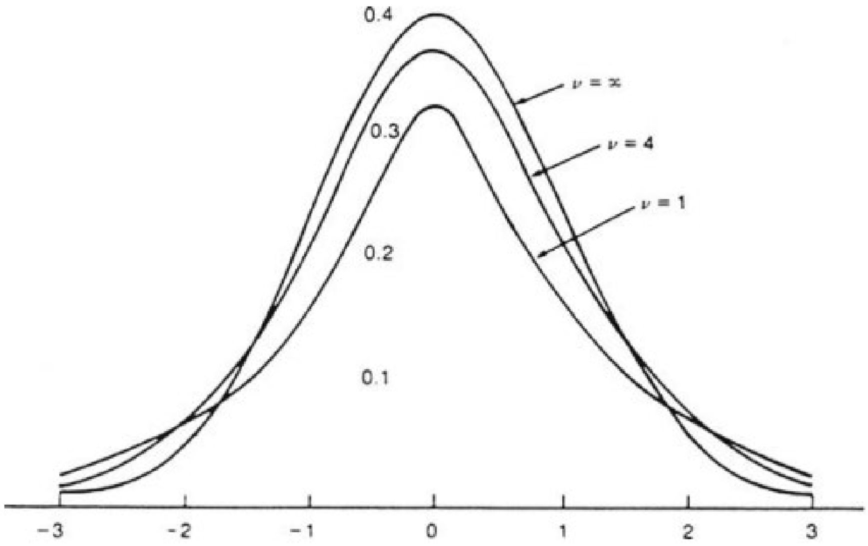


Figure 7: Student's t-distribution for various v
(Source: Skrondal, Everitt, 2010, p. 419)

1.5.8 Variance

Variance quantifies the difference of data points in a given set from their mean. Variance is calculated as the sum of differences from the mean squared. By squaring the differences, we ensure only positive values are used in the calculation, so the positive and negative variances from the mean do not cancel each other in the final number, showing us lower variance. On the other hand, squaring the variance returns our units of the calculation squared, making it challenging for interpretation. Variance is used for other calculations but usually not as a stand-alone statistic.

Equation 7: Population variance for *X*
(Source: Boyle, 2020, p. 117)

$$\sigma_X^2 = \frac{\sum_{i=1}^{n}(x_t - \mu_X)^2}{n}$$

Equation 8: Sample variance for *X*
(Source: Boyle, 2020, p. 118)

$$s_X^2 = \frac{\sum_{i=1}^{n}(x_t - \bar{x})^2}{n - 1}$$

For sample variance calculation, Bessel's correction is used in the denominator to correct population estimation bias (Boyle, 2020).

1.5.9 Standard deviation

Standard deviation is the most often used tool for comparing deviations between more data sets. It is calculated as the square root of variance, and as stated before, variance squares not only deviation in the data set but also units, making it unsuitable for easy, intuitive comparisons.

Equation 9: Population and sample standard deviation
(Source: Processed according to Boyle, 2020, p. 119)

$$\sigma_X = \sqrt{\sigma_X^2}$$

$$s_X = \sqrt{s_X^2}$$

If the data set follows a normal distribution, then the so-called empirical rule applies to standard deviation. The empirical rule states that within 1 standard deviation lies 68% of the data set; within 2 standard deviations, 95% of the data set; and within 3 standard deviations, 99,7% of the data set (Boyle, 2020).

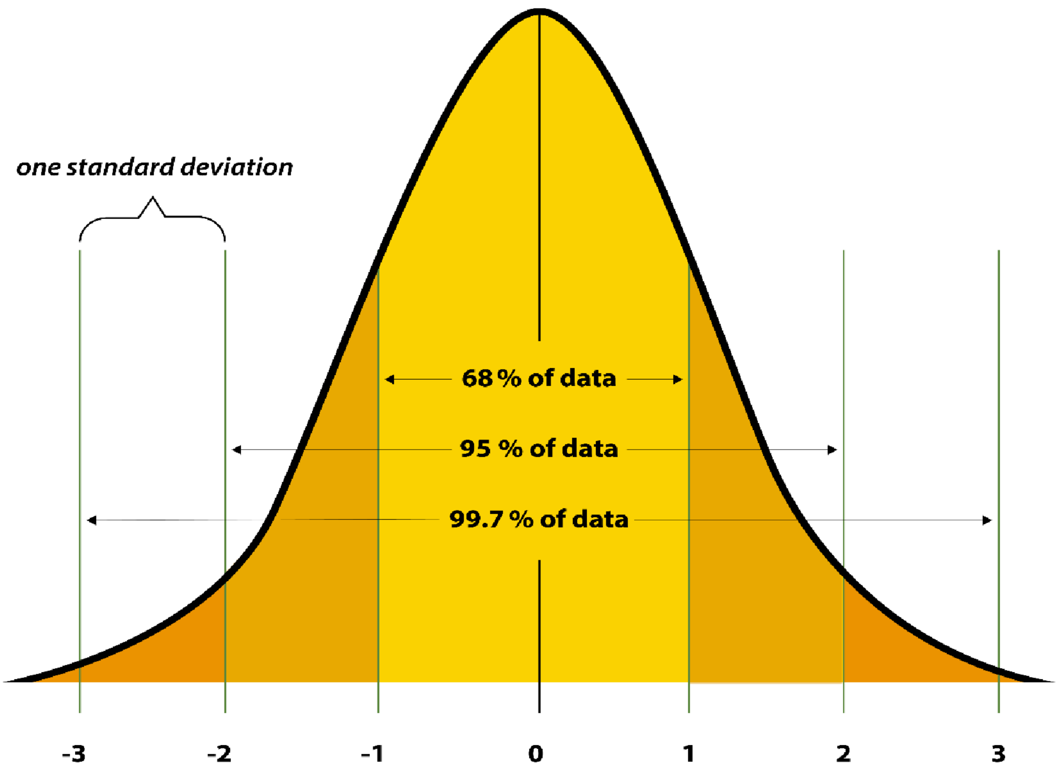


Figure 8: Empirical rule
(Source: National Library of Medicine, @2024)

For sample standard deviation, a standard error quantifies the precision with which the sample standard deviation estimates the population standard deviation. A smaller standard error suggests that the sample standard deviation is a more reliable estimate of the population standard deviation (Skrondal, Everitt, 2010).

Equation 10: Standard error
(Source: Processed according to Skrondal, Everitt, 2010, p. 409)

$$SE = \frac{\sigma}{\sqrt{n}}$$

1.5.10 Covariance

Covariance is a statistical measure that quantifies the degree to which two random variables change together. It measures the directional relationship between two variables, with positive covariance indicating an increasing linear relationship and a negative decreasing relationship. Covariance is sensitive to changes in scale, making it difficult to interpret and compare across different datasets, but it is a key part of many further calculations (Boyle, 2020).

Equation 11: Population covariance as expected value
(Source: Processed according to Skrondal, Everitt, 2010, p. 110)

$$cov(X, Y) = E[(X - \mu_X)(Y - \mu_Y)]$$

Or:

Equation 12: Population covariance
(Source: Processed according to Boyle, 2020, p. 171)

$$cov(X, Y) = \frac{\sum_{i=1}^{n}(X_i - \mu_X)(Y_t - \mu_Y)}{n}$$

Equation 13: Sample covariance
(Source: Processed according to Skrondal, Everitt, 2010, p. 110)

$$c_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{n - 1}$$

1.5.11 Linear regression

Linear regression is one of the most important mathematical models used in econometrics. It allows the relationship between the dependent variable Y and independent variable X to be found and quantified. We can estimate the average value of Y as a conditional expectation E(Y|X), and the relationship may be linear or non-linear.

Equation 14: Simple linear regression
(Source: Skrondal, Everitt, 2010, p. 253)

$$E(Y|X) = \beta_0 + \beta_1 Y$$

Where:

β_0 is the *Y*-intercept with the y-axis, the predicted value of *Y* when *X*=0

β_1 is the slope of the regression line

(Skrondal, Everitt, 2010)

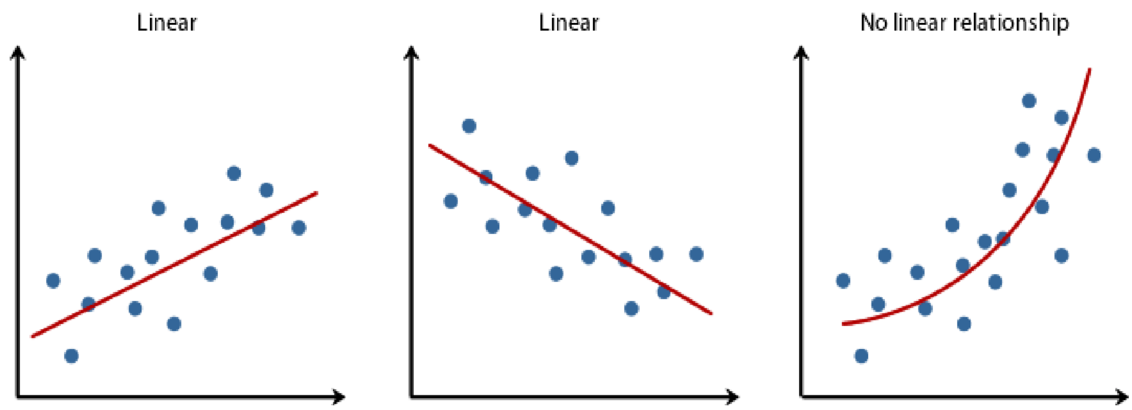


Figure 9: Linear relationship in a data set
(Source: Lund Research Ltd, @2018)

The coefficients β_0 and β_1 are commonly estimated using the sum of squared error (SSE) method. Once the equation is solved, we can predict the value of *y* at unobserved *x* by substitution in the regression function (Boyle, 2020).

1.5.12 F-statistic

F-statics is used to assess the overall significance of a group of explanatory variables or factors in explaining the variability in a response variable when doing regression analysis. A high F-statistic indicates that regression coefficients are significantly different from each other, suggesting that the model explains the variance between the two. Low F-statics suggest the regression does not provide a good fit to the data.

It is calculated by dividing the explained variance, the sum of squares regression, by the unexplained variance, the sum of squares residual.

Equation 15: F-statistics
(Source: Processed according to Skrondal, Everitt, 2010, p. 286)

$$F = \frac{RSS_m - RSS_{m+1}}{RSS_{m+1}/(n - m - 2)}$$

Where RSS is the residual sum of squares with m explanatory variables (Skrondal, Everitt, 2010).

1.5.13 Coefficient of Determination

The coefficient of determination, often denoted as R^2, assesses the correlation between two variables used in regression. It is a simple tool to quickly measure how well the regression model can predict the value of the dependent variable and allows us to compare regression models. It ranges from 0 to 1, where 0 indicates that the independent variables do not explain any of the variability in the dependent variable, and 1 indicates that they explain all the variability (Boyle, 2020).

Equation 16: F-statistics
(Source: Processed according to Skrondal, Everitt, 2010, p. 286)

$$R_{XY}^2 = \frac{cov(X, Y)}{\sigma_X \sigma_Y}$$

1.5.14 Null hypothesis

The null hypothesis tests whether any observed difference between two possibilities arises from chance. Statistical testing involves constructing a model or distribution representing the data under the assumption of pure chance or randomness to assess this hypothesis. The observed results are compared against these expected outcomes, enabling the determination of whether the null hypothesis is to be rejected or accepted (Skrondal, Everitt, 2010).

1.5.15 T-test

The t-test tests hypotheses about population means under student's t-distribution.

T-test encompasses two primary versions: the single sample t-test, applied to evaluate if the mean of a population aligns with a specified value, and the independent samples t-test, utilised to scrutinise the equality of means between two populations.

Equation 17: T-test statistics

(Source: Processed according to Skrondal, Everitt, 2010, p. 420)

$$t = \frac{\overline{x_1} - \overline{x_2}}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

The test statistic t is calculated as the difference between the sample means, normalised by the standard error. This standard error is estimated using estimated sample standard deviation *s*.

Equation 18: Estimated sample standard deviation

(Source: Processed according to Skrondal, Everitt, 2010, p. 420)

$$s = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_1 - 2}}$$

If the null hypothesis of equal population is true, the t-test follows a Student's t-distribution with degrees of freedom $n_1 + n_1 - 2$ equal to the total sample sizes. This allows for the p-value to be calculated (Skrondal, Everitt, 2010).

1.5.16 P-value

The p-value indicates the likelihood of obtaining results as extreme as the sample outcome if the null hypothesis holds true. A lower p-value corresponds to a reduced probability of observing the result, thereby providing more robust evidence to reject the null hypothesis. As the p-value focuses on extreme cases, it computes the probability within the upper or lower tail of the distribution (Boyle, 2020).

1.6 Modern Portfolio Theory

An investment portfolio is a collection of investments. Investors are trying to allocate their resources across a spectrum of asset types to generate returns and diversify risk. By diversifying across multiple asset classes and securities, investors seek to mitigate the impact of individual asset volatility on overall portfolio performance, thereby enhancing the risk-return profile of their investments (Bodie, Kane, Marcus, 2008).

Portfolio theory, pioneered by Harry Markowitz in the 1950s, begins with the fundamental premise that investors are inherently risk-averse, seeking high returns while minimising uncertainty. Markowitz's groundbreaking work, for which he was awarded the Nobel Prize in Economics in 1990, provided a rigorous mathematical framework for portfolio construction and risk management.

Markowitz's seminal book, "Portfolio Selection," emerged from his PhD dissertation at the University of Chicago and remains a cornerstone of modern finance. His research revealed that by strategically combining assets in a portfolio, investors could achieve a lower risk level than any individual asset, a concept known as diversification (Malkiel, 2016).

1.6.1 Diversification according to Modern Portfolio Theory

Modern Portfolio Theory (MPT) provides a framework for understanding the principles of diversification and portfolio optimisation, grounded in mathematical analysis. At its core, MPT emphasises the importance of constructing portfolios that balance risk and return by strategically allocating investments across a diverse set of assets (Malkiel, 2016).

Analytically, suppose there are *N* securities in the market. Let r_i denote the anticipated return per dollar invested in security *i* at time *t,* and let *d* represent the discount rate for the return on security *i* at time *t*. The relative amount invested in security *i* is denoted as X_i, where $X_i \geq 0$ for all *I*, reflecting the exclusion of short sales. Additionally, $\sum_{i=1}^{N} x_i = 1$, ensuring all capital is used in investing and no cash is left in the portfolio. The discounted anticipated return of the portfolio (*R*) is expressed as:

Equation 19: Expected return of portfolio under MPT
(Source: Markowitz, 1952, p. 78)

$$R = \sum_{t=1}^{\infty} \sum_{i=1}^{N} d_i \times r_{i,t} \times x_i = \sum_{i=1}^{N} x_i \times E(d_i \times r_{i,t})$$

Where R_i represents the discounter return of security i. The portfolio return *R* is thus a weighted average of the discounted returns of individual securities, with the weights x_i representing the allocation of capital across assets (Markowitz, 1952).

1.7 Capital Asset Pricing Model (CAPM)

The Capital Asset Pricing Model (CAPM) was developed in the 1960s by William Sharpe, John Lintner, and Jan Mossin, extending earlier work by Harry Markowitz on portfolio theory. William Sharpe's seminal paper, "Capital Asset Prices: A Theory of

Market Equilibrium under Conditions of Risk", published in 1964, laid the foundation for CAPM by introducing the concept of beta as a measure of systematic risk.

CAPM operates under the assumption of market equilibrium, where asset prices adjust to reflect all available information and investor expectations. In equilibrium, the expected return on an asset compensates investors for the time value of money (risk-free rate) and the additional risk associated with holding that asset (market risk premium).

The CAPM considers how imperfect correlation among asset returns influences the investor's risk-return trade-off. While risk in a portfolio combines nonlinearly due to diversification, expected returns combine linearly. This means that the expected return of a portfolio is simply the weighted average of the expected returns of its constituent assets. By holding assets with similar expected returns and standard deviations in a portfolio, diversification lowers its risk without compromising its expected return. (Perold, 2004).

There are four key assumptions underlying the CAPM model:

1. Taxes, transaction costs, and other practical factors can be ignored.
2. All investors employ mean-variance optimisation (MVO) principles outlined by Markowitz to formulate their investment portfolios.
3. Investors share identical capital market assumptions regarding expected returns, standard deviations, and correlations when constructing portfolios.
4. All investors have unrestricted access to borrowing and lending at the same risk-free rate (Idzorek, Xiong, Kaplan, Ibbotson, 2015).

1.7.1 Components of CAPM

CAPM incorporates a risk-free rate R_f representing the return on a hypothetical investment with zero risk, such as a government bond. The risk-free rate serves as the baseline return against which investors compare the expected returns of risky assets.

The beta coefficient quantifies the concept of systematic risk β_i of an asset. Beta measures the sensitivity of an asset's returns to the returns of the overall market portfolio. Mathematically, beta is calculated as the covariance of the asset's returns with the market returns divided by the variance of the market returns:

Equation 20: The beta calculation for the CAPM model
(Source: Processed according to Idzorek, Xiong, Kaplan, Ibbotson, 2015, p. 75)

$$\beta_i = \frac{Cov(R_i, R_m)}{\sigma^2_{R_m}}$$

Where R_i is a return of asset i, and R_m return of a market portfolio.

The beta of the portfolio can be easily calculated as the weighted average of individual assets beta.

Equation 21: The beta of a portfolio
(Source: Processed according to Lee, Su, 2015, p. 75)

$$\beta_p = \sum_{i=1}^{N} \beta_i \times w_i$$

Where w_i is a weight of an asset i, in a portfolio p.

The expected return $E(R_i)$ of an asset is determined by its systematic risk expressed as β_i and the market risk premium.

Equation 22: Expected return of individual assets under CAPM
(Source: Processed according to Idzorek, Xiong, Kaplan, Ibbotson, 2015, p. 75)

$$E(R_i) = R_f + \beta_i \times MRP$$

Where $MRP = E(R_m) - R_f$ expressing the expected excess return of the market portfolio above the risk-free rate R_f(Idzorek, Xiong, Kaplan, Ibbotson, 2015).

Five years of data are usually considered for the most accurate Beta estimations when applying the CAPM model (Syed Jawad Hussain, Zakaria, Naveed, 2014). In order to smooth out huge volatility spikes in the estimation, weekly data frequency is calculated as an arithmetic average of daily data (Yang, Xu, 2007).

The most accurate risk-free rate for the CAPM model is the mean of US short-term treasury bills (Mukherji, 2011).

1.7.2 Efficient Portfolio Frontier

The efficient frontier illustrates the optimal trade-off between risk and return for a given set of investment opportunities. It is a graph curve showing all possible portfolio combinations and their risk-to-return trade-offs.

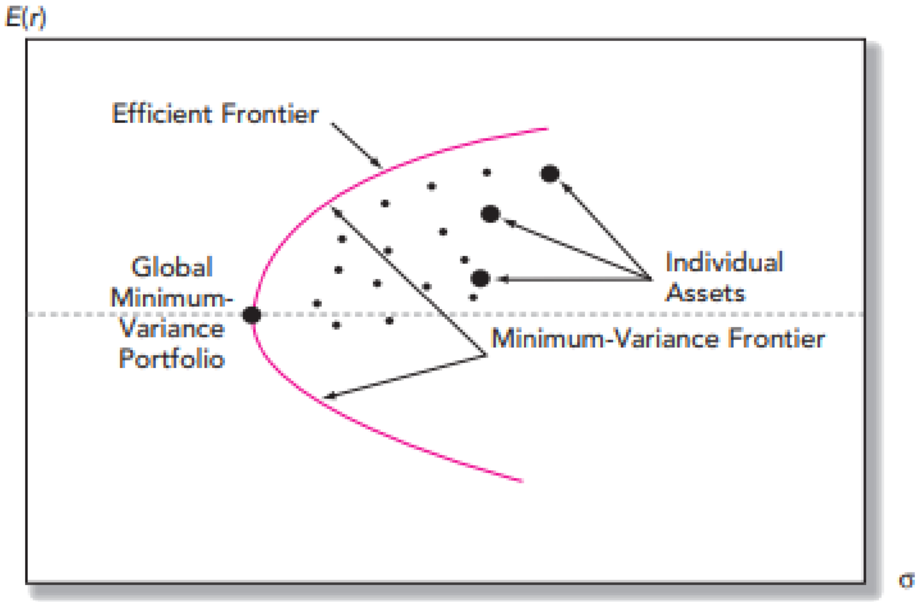


Figure 10: Efficient Frontier
(Source: Bodie, Kane, Marcus, 2008, p. 210)

The efficient frontier concept arises from mean-variance optimisation, which aims to construct portfolios that maximise expected return while minimising portfolio variance (or standard deviation). The efficient frontier is derived by systematically varying the allocation of assets in a portfolio to achieve different risk-return combinations, resulting in a curve representing the set of optimal portfolios.

Portfolios lying below the efficient frontier are suboptimal, as they offer lower expected returns for the same level of risk or higher risk for the same expected return compared to portfolios on the efficient frontier. Conversely, portfolios lying above the efficient frontier are unattainable or infeasible, as they represent risk-return combinations that cannot be achieved using available assets (Markowitz, 1952).

1.7.3 Analytical derivation of Efficient Frontier

As introduced by Markowitz in 1952, the efficient frontier has been constructed only qualitatively as a graphical solution for more than three assets. In 1972, Merton introduced an analytical derivation of the efficient frontier to solve this problem.

Like for a portfolio under MPT, for a portfolio of N assets with their expected return $E(E_i)$ and their standard deviation σ_i, the portfolio return is given by the weighted sum of individual asset returns:

Equation 23: The expected return of the portfolio by Merton
(Source: Processed according to Merton, 1972, p. 1852)

$$E = \sum_{i=1}^{N} x_i E_i$$

Where w_i represent the weight of asset i, with constrain $\sum_{i=1}^{N} x_i = 1$, so all the capital will be invested with no leverage. The portfolio variance is:

Equation 24: Portfolio variance
(Source: Processed according to Merton, 1972, p. 1852)

$$\sigma^2 = \sum_{j=1}^{N} \sum_{i=1}^{N} x_i x_j \sigma_{ji}$$

The covariance and inverse covariance matrix are calculated from portfolio returns (Merton, 1972). The covariance matrix is a symmetric matrix in which diagonal values represent the variances of individual assets. In the literature, it is often called a variance-covariance matrix (Skrondal, Everitt, 2010).

Equation 25: Covariance matrix
(Source: Processed according to Skrondal, Everitt, 2010, p. 445)

$$\sum \begin{bmatrix} \sigma_1^2 & \cdots & Cov(R_1, R_N) \\ \vdots & \ddots & \vdots \\ Cov(R_N, R_1) & \cdots & \sigma_N^2 \end{bmatrix}$$

An inverse covariance matrix is calculated the same way as a standard inverse matrix.

For 2x2 matrix Σ where:

Equation 26: Inverse matrix calculation
(Source: Stover, Weisstein, @2023)

$$\Sigma = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

The matrix inverse is:

$$\Sigma^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Four vectors are needed to construct the efficient frontier.

- Vector of ones *(1)*.
- Vector of expected returns of individual assets *(R)*.
- Vector of asset weights *(X)*.
- Portfolio mean-variance vector *(G)*.

Vector *(X)* represents the weights of individual portfolio assets and is calculated as a matrix multiplication of *(1)* vector and inverse covariance matrix.

Equation 27: *X* vector
(Source: Processed according to Merton, 1972, p. 1853)

$$X = (1) \times \Sigma^{-1}$$

Portfolio mean-variance vector *(G)* calculated as matrix multiplication of *(R)* vector and inverse covariance matrix. It represents both the portfolio's expected returns and the risk (measured by variance). By multiplying the vector of expected returns by the inverse covariance matrix, investors obtain a measure that combines the expected performance of the portfolio with its risk characteristics (Merton, 1972).

Equation 28: *G* vector
(Source: Processed according to Merton, 1972, p. 1853)

$$G = R \times \Sigma^{-1}$$

These four vectors are then used to calculate four additional coefficients, allowing us to construct an efficient frontier and minimum variance portfolio. Denotation *T* implies that the vectors are transposed in the calculation.

Equation 29: Efficient frontier constants
(Source: Processed according to Lee, Su, 2014, p. 71)

$$A = (1) \times X^T$$

$$B = (1) \times G^T$$

$$C = R \times X^T$$

$$D = A \times C - B^2$$

Efficient frontier function of risk σ is now given by:

Equation 30: Efficient frontier risk function
(Source: Processed according to Merton, 1972, p. 1854)

$$\sigma = \sqrt{\frac{CE^2 - 2AE + B}{D}}$$

Where *E* is the desired expected return.

The minimum variance portfolio (MVP) is now constructed by calculating the weights of individual assets using their value in *(X)* vector divided by *A*. Its expected return is given by dividing coefficient *B* by coefficient *A* or the product of *MVP* vector and *(R)* vector. Risk is given by the square root of *1/A* or the square root of matrix multiplication of *MVP* and Covariance matrix times transposed *MVP* (Lee, Su, 2014).

Equation 31: MVP function
(Source: Processed according to Lee, Su, 2014, p. 75 and 76)

$$MVP = \frac{X}{A}$$

$$R_{MVP} = \frac{B}{A} = MVP \times R$$

$$\sigma_{MVP} = \sqrt{\frac{1}{A}} = \sqrt{MVP \times (\Sigma \times MVP^T)}$$

1.7.4 Capital Market Line and Tangency Portfolio

The Capital Market Line (CML) describes the relationship between risk and return in the context of efficient portfolio construction. The CML is constructed based on the principles of mean-variance analysis and the efficient frontier, which defines the set of portfolios that offer the highest expected return for each level of risk or the lowest risk for each level of expected return. The CML extends this concept by introducing the risk-free asset, typically represented by short-term government securities, as a vital portfolio component.

Mathematically, the CML is expressed as the linear combination of the risk-free rate and the efficient portfolio of risky assets.

The tangency portfolio lies at the point of tangency between the efficient frontier and the CML. It is considered optimal because it offers the best risk-to-return ratio. Investors can then move the optimal tangency portfolio along the CML by incorporating risk-free assets into the portfolio.

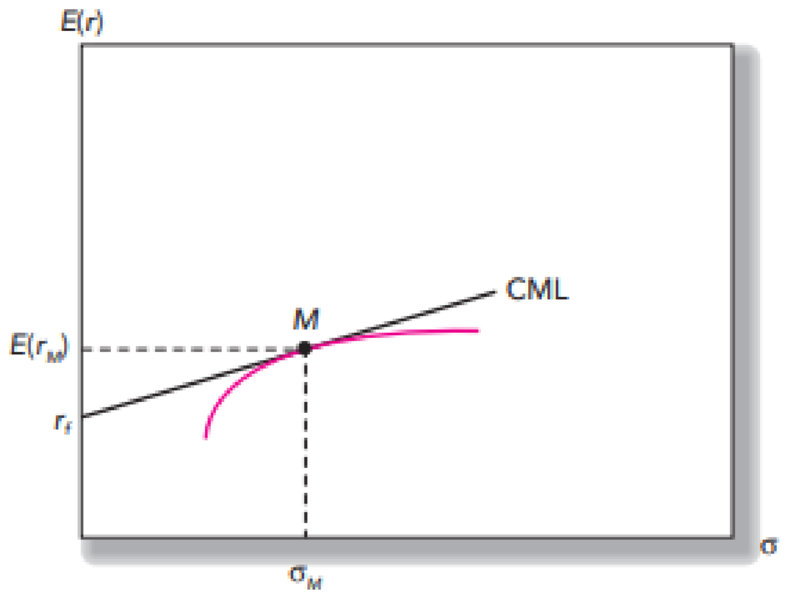


Figure 11: Capital Market Line
(Source: Bodie, Kane, Marcus, 2008, p. 282)

For calculation, we will use the efficient frontier coefficients *A*, *B*, *C* and *D* in combination with the risk-free rate R_f (Lee, Su, 2014).

Equation 32: CML coefficients
(Source: Processed according to Lee, Su, 2014, p. 73 and 74)

$$R_T = \frac{C + B \times R_f}{B - A \times R_f}$$

$$\sigma_T = \sqrt{\frac{A \times R_T^2 - 2 \times B \times R_T + C}{D}}$$

$$\beta_T = \frac{R_T - R_f}{\sigma_T}$$

$$\lambda_T = \frac{C - B \times R_f}{D}$$

$$\mu_T = \frac{D \times R_f - B}{D}$$

Where:

R_T is the return of tangency portfolio, σ_T risk of tangency portfolio, β_T the slope of the tangency portfolio, and λ_T and μ_T are additional constants used in the calculation of portfolio weights.

Equation 33: Tangency portfolio weights of individual assets
(Source: Processed according to Lee, Su, 2014, p. 74)

$$Tangency\ portfolio = \lambda_T \times X + \mu_T \times G$$

Equation 34: Capital market line
(Source: Processed according to Lee, Su, 2014, p. 74)

$$CML = R_f + \sigma_T \times \beta_T$$

However, CML also explains why investors should hold a simple market portfolio composed of a weighted average of all assets available in the market. According to CAPM, investors are rational and seek to maximise their returns while minimising risk. The tangency portfolio, where the risk-return trade-off reaches an optimal balance, coincides with the market portfolio in CAPM. This alignment occurs because the market portfolio represents the most efficient combination of risk and return available to investors. As a result, under CAPM, all investors should seek to hold the market portfolio to achieve the highest possible return for a given level of risk. Holding the market portfolio ensures that investors are exposed to systematic risk, which is compensated with the market risk premium.

If the market portfolio is used in the calculation, the tangency portfolio should have the same weights for each asset as the market portfolio (Elton, Gruber, Goetzmann Brown, 2014).

1.7.5 Security market line

The security market line is used in the CAPM model to represent the trade-off between risk (beta) and expected return for individual securities or portfolios within a well-diversified market (Lee, Su, 2014).

1.8 Testing the CAPM

In this section, possible tests are outlined to test assumptions underlying the CAPM model. The testing is usually done on portfolios comprising around ten assets. This is done to ensure that there is no selection and measurement bias, which could affect the equilibrium model of CAPM (Elton, Gruber, Goetzmann, Brown, 2014).

However, because the cryptocurrency market is a new emerging market, it would be a challenge to gather five years of data for hundreds of cryptocurrencies. Mainly because many of the biggest cryptocurrencies have not been on the market for five years. For the purpose of this thesis, I have decided to test individual cryptocurrencies, which is still possible based on the literature.

1.8.1 Time series test

This test is based on the regression of the time series of excess portfolio or asset returns on excess market return.

Equation 35: CAPM time series test
(Source: Processed according to Yang, Xi, 2007, p. 16)

$$r_{eit} = \alpha_i + \beta_i \times r_{emt}$$

$$r_{eit} = r_{it} - r_{ft}$$

$$r_{emt} = r_{mt} - r_{ft}$$

Where:

r_{eit} is the excess return of asset *i* at time *t*

r_{emt} is the excess return of market portfolio at time *t*

r_{it} is the return of asset *i* at time *t*

r_{it} is the return of the market portfolio at time *t*

r_{ft} is the risk-free rate at time *t*

β_i is estimated beta

α_i is regression intercept

If CAPM is true, then there is no difference between the expected return based on the time series and the expected return based on CAPM. That means α_i should be zero for all assets. The results are interpreted using a T-test with a 95% confidence interval (Yang, Xi, 2007).

1.8.2 General equilibrium test

If the CAPM as a general equilibrium model is true, then it follows three main assumptions:

- The higher the risk of an investment the higher its expected return
- The expected return is linearly related to the beta
- There is no added return for non-market risk exposure

This would ensure that deviations of securities or portfolios from equilibrium are purely random and do not offer opportunities for above-market returns (Elton, Gruber, Goetzmann, Brown, 2014).

The excess return of portfolio or asset i within a specified period is determined as the mean of its excess returns, while the beta β_i was derived from a time series regression of the asset's excess return on the market's excess return.

Equation 36: CAPM excess return testing
(Source: Processed according to Yang, Xi, 2007, p. 19)

$$R_{ei} = \gamma_0 + \gamma_1 \times \beta_i$$

$$R_{ei} = \frac{1}{n}\sum_{i=1}^{n} r_{it} - r_{ft}$$

Where R_{ei} is average excess return above the risk-free rate.

If CAPM is valid, the excess return R_{ei} is equal to regression $\gamma_0 + \gamma_1\beta_i$, implying $\gamma_0 = 0$ and $\gamma_1 = R_i - R_f$.

To test the non-linearity of returns and beta, the following regression equation is used:

Equation 37: CAPM beta testing
(Source: Processed according to Yang, Xi, 2007, p. 19)

$$r_p = \gamma_0 + \gamma_1 \times \beta_i + \gamma_2 \times \beta_i^2$$

If the assumptions of CAPM are true, the coefficient γ_2 should equal to 0. That would mean that returns and betas are linearly dependent.

Lastly, we test if the excess returns are determined by systematic risk.

Equation 38: CAPM systematic risk testing
(Source: Processed according to Yang, Xi, 2007, p. 19)

$$r_p = \gamma_0 + \gamma_1 \times \beta_i + \gamma_2 \times \beta_i^2 + \gamma_3 \times RSS$$

Additionally, it incorporates the impact of non-systematic risk, captured by the term RSS, residual sum of squares, which denotes the residual variance of portfolio return, reflecting the portion of risk that is specific to individual assets and not explained by systematic factors.

If the hypothesis holds true, γ_3 would be expected to be equal to zero, indicating that non-systematic risk does not play a significant role in determining expected excess returns beyond what is already captured by systematic risk.

T-test is used to statistically test the CAPM testing coefficients with a significance level of 95% (Yang, Xu, 2007).

1.8.3 Benchmark market portfolio S&P 500

The S&P 500, often considered a benchmark index for the U.S. stock market, comprises 500 large-cap companies listed on stock exchanges in the United States. Established in 1957 by Standard & Poor's, the index aims to represent the broader market's performance by including companies from various sectors such as technology, healthcare, finance, and consumer goods. The weight of each asset within the index is determined by market capitalisation and in total the index covers approximately 80% of the U.s. market capitalization. The S&P 500 is widely used by investors, analysts, and financial professionals as a benchmark for measuring the performance of investment portfolios and mutual funds (Laney, 2024). This index is used as a market portfolio in the construction of CAPM on the cryptocurrency market.

2 ANALYSIS OF THE CURRENT STATE

In this section of the thesis, econometrics models are constructed and evaluated.

For model construction, I have selected five cryptocurrencies for a year from 31.12.2018 to 31.12.2023. The five cryptocurrencies were selected based on the following conditions:

- Market capitalisation as of 31.12.2023
- Being traded before 31.12.2018
- Not being stablecoin

The below table shows the biggest cryptocurrencies as of 31.12.2023.

Table 1: Cryptocurrencies based on market capitalisation
(Source: Own processing based on CoinMarketCap, @2023)

Cryptocurrency	Symbol	Market capitalisation (in mil. USD)	Price (USD)	Type
Bitcoin	BTC	827 811	42 265.19	Cryptocurrency
Ethereum	ETH	274 194	2 281.47	Cryptocurrency
Tether USDT	USDT	91 675	0.99	Stablecoin
Binance coin	BNB	47 394	312.44	Cryptocurrency
Solana	SOL	43 576	101.51	Cryptocurrency
XRP	XRP	33 284	0.61	Cryptocurrency
USDC	USDC	24 520	1.00	Stablecoin
Cardano	ADA	21 015	0.59	Cryptocurrency

The data in the table are from Coin Market Cap, a price-tracking website for cryptocurrencies owned by Binance and operating since 2013. Companies and even the US government trust Coin Market Cap for its real-life price information, making it a reliable source of price data for cryptocurrency analysis.

Based on the criteria for portfolio selection, Tether USDT and USDC are not considered because, as a stablecoin, their price is pegged to the US dollar and is not supposed to be held as investments. Also, Solana is not considered because it was introduced in 2020, thus not having five years of market data (CoinMarketCap, @2024).

The selected five cryptocurrencies for this thesis are:

- Bitcoin (BTC)
- Ethereum (ETH)

- Binance Coin (BNB)
- XRP (XRP)
- Cardano (ADA)

The cryptocurrency market portfolio (MP) is based on a weighted average of market capitalisation.

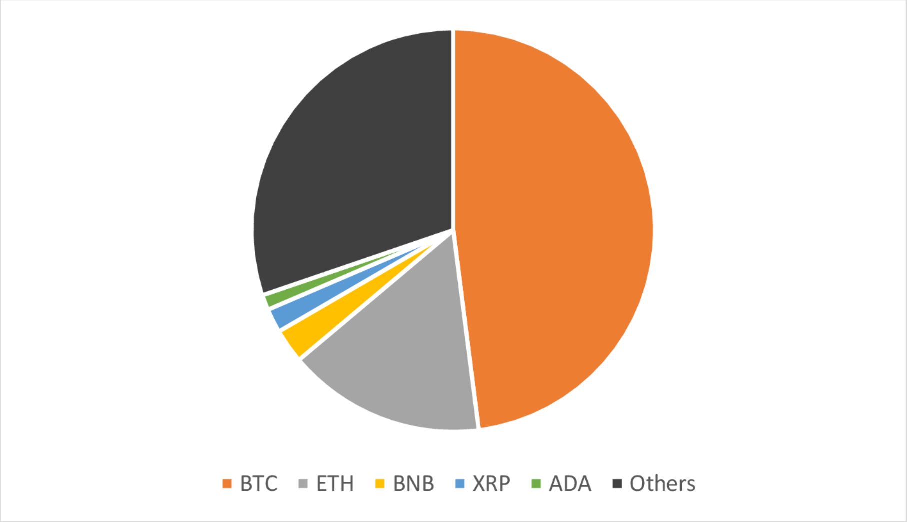
Table 2: Weights of the Cryptocurrency Market Portfolio
(Source: Own processing based on CoinMarketCap, @2024)

Symbol	Market capitalisation	w_i
BTC	827 811	69%
ETH	274 194	23%
BNB	91 675	4%
XRP	47 394	3%
ADA	43 576	2%
SUM	1 284 650	100%

As of 31.12.2023, the entire cryptocurrency market capitalisation is 1 725 302 million USD, consisting of over 14 000 individual cryptocurrencies (CoinGecko, @2024). The selected cryptocurrency market portfolio covers 70% of the whole cryptocurrency market, which is sufficient coverage to represent the market as a whole.

Table 3: Cryptocurrency market capitalisation
(Source: Own processing based on table n. 2 and CoinGecko, @2024)

Symbol	Market capitalisation	w_i
BTC	827 811	48%
ETH	274 194	16%
BNB	47 394	3%
XRP	33 284	2%
ADA	21 015	1%
Others	521 604	30%
SUM	1 725 302	100%



Graph 1: Cryptocurrency market capitalisation

(Source: Own processing based on table n. 3)

I have chosen the S&P 500 index (ticker SPX), which covers approximately 80% of the U.S. equity market, as a benchmark and alternative market portfolio. The alternative market portfolio is used to compare results against the cryptocurrency market portfolio. Historical data are obtained from Yahoo Finance. Yahoo Finance is a financial news and data network that has been operating since 1997. It is one of the biggest finance websites in the world and a reliable source of market data (Wikipedia, @2024).

I have chosen the US three-month treasury bills as a risk-free rate. Market data are obtained from the Federal Reserve Economic Data database, operated by the Federal Reserve Bank of St. Louis (FRED). As a government body, FRED is a reliable source of market data for treasury bills (FRED, @2024).

2.1 Data

Data for all cryptocurrencies are obtained from CoinMarketCap for a period of five years, from 31.12.2018 to 31.12.2023, denoted in US dollars.

S&P 500 index market data are obtained for the same period from Yahoo Finance.

Table 4: **Table of daily prices for selected portfolio and benchmark index**
(Source: Own processing in software Microsoft Excel 365)

Date	BTC	ETH	BNB	XRP	ADA	S&P500
31/12/2018	3742.7	133.3683	6.164732	0.352706	0.041063	2506.85
01/01/2019	3843.52	140.8194	6.075273	0.364771	0.042547	n.a.
02/01/2019	3943.409	155.0477	6.188613	0.375243	0.045258	2510.03
03/01/2019	3836.741	149.135	5.903535	0.360224	0.042682	2447.89
04/01/2019	3857.718	154.5819	6.065138	0.356747	0.043812	2531.94
05/01/2019	3845.195	155.6386	6.065543	0.355275	0.044701	n.a.
06/01/2019	4076.633	157.7462	6.395979	0.368395	0.049261	n.a.
07/01/2019	4025.248	151.6992	6.291411	0.364347	0.047996	2549.69
08/01/2019	4030.848	150.3596	6.640054	0.365315	0.048525	2574.41
09/01/2019	4035.296	150.8031	6.64155	0.37089	0.052169	2584.96
10/01/2019	3678.925	128.6252	5.915919	0.332652	0.044529	2596.64
11/01/2019	3687.365	127.5483	6.075342	0.332904	0.044147	2596.26
12/01/2019	3661.301	125.9665	5.976965	0.3288	0.043704	n.a.
13/01/2019	3552.953	116.8978	5.568757	0.317863	0.040241	n.a.

In order to obtain the most accurate beta estimation and smooth out huge volatility spikes, weekly data frequency is calculated as an arithmetic average of daily. Moreover, weekly data frequency allows data from crypto markets that are open non-stop seven days a week to be matched to the SP&500 index, which is traded only during business days.

Table 5: **Table of average weekly prices**
(Source: Own processing in software Microsoft Excel 365)

Week	BTC	ETH	BNB	XRP	ADA	S&P500
1	3877.99	149.477	6.12269	0.36191	0.04419	2499.18
2	3810.28	135.986	6.15857	0.34468	0.0459	2580.39
3	3665.39	123.342	6.33922	0.32806	0.04436	2623.14
4	3593.26	116.713	6.70793	0.31646	0.04271	2644.67

Weekly returns are now calculated. The first week of data is lost because there is no previous week against which the return can be calculated. After calculating the returns, we have 260 data points in total. Returns of the cryptocurrency market portfolio (MP) are calculated as the weighted average of weekly returns based on the market capitalisation.

Table 6: Table of weekly returns
(Source: Own processing in software Microsoft Excel 365)

Week	BTC	ETH	BNB	XRP	ADA	SP&500	MP
1	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
2	-0.017	-0.090	0.006	-0.048	0.039	0.032	-0.033
3	-0.038	-0.093	0.029	-0.048	-0.034	0.017	-0.048
4	-0.020	-0.054	0.058	-0.035	-0.037	0.008	-0.025

The risk-free rate is calculated as a mean of short-term US treasury bills. Daily data for three-month treasury bills from the secondary market from 31.12.2018 to 31.12.2023 were obtained from the Federal Reserve Economic Data database, operated by the Federal Reserve Bank of St. Louis. A simple mean of 1.91% was calculated from these data points, which is used as the risk-free rate in the model.

2.2 Statistics calculation

Annualized geometric mean return is used to calculate the return of individual cryptocurrencies.

In Excel, we can express this using the formula:

Equation 39: Annualized geometric mean return in Excel
(Source: Processed according to equation number 2)

$$= \left(1 + \left(PRODUCT(1 + i_1 : i_n)^{\frac{1}{n}} - 1\right)\right) \hat{}\, p - 1$$

Where $i_1 : i_n$ is a selection of weekly returns of cryptocurrency *i,* and p is the number of periods, in this case, 52, reflecting 52 weeks in a year to get annualised statistics from weekly data (Bodie, Kane, Marcus, 2008).

The -1 on the end of the equation is used to calculate the return in percentage.

Annualised standard deviation from weekly data is obtained using the STDEV.S function in Microsoft Excel with p=52 as well to get the annualised value.

Equation 40: Annualised standard deviation in Excel
(Source: Processed according to equation number 9)

$$= STDEV.S(\, i_1 : i_n) * SQRT(p)$$

The beta coefficient is the sensitivity of an asset's returns to the returns of the overall market portfolio. In this case, the S&P500 and the cryptocurrency market portfolio will be considered.

Equation 41: Beta coefficient calculation in Excel
(Source: Processed according to equation number 20)

$$\beta_i = COVARIANCE.S(i_1{:}\,i_n, m_1{:}\,m_n))/VAR.S(m_1{:}\,m_n)$$

Where $i_1{:}\,i_n$ is a selection of weekly returns of cryptocurrency i and $m_1{:}\,m_n$ is a selection of weekly returns of the market portfolio.

The expected return of individual cryptocurrencies can be calculated directly:

Equation 42: Expected return of individual assets under CAPM
(Source: Processed according to equation number 22)

$$E(R_i) = R_f + \beta_i \times (R_m - R_f)$$

Where R_i is the return of cryptocurrency *i*, R_f is a risk-free rate, R_m is the return of the market portfolio and β_i is the beta of cryptocurrency *i*.

Table 7: Table of return, risk, Beta and expected return based on S&P500
(Source: Own processing in software Microsoft Excel 365)

	BTC	**ETH**	**BNB**	**XRP**	**ADA**	**S&P500**	**Risk-free**
Returns	61.55%	72.76%	118.93%	11.67%	69.20%	13.84%	1.91%
Risk (σ)	57.53%	72.85%	89.10%	92.73%	88.22%	15.30%	0%
$\boldsymbol{\beta_i}$	1.31	2.05	1.88	1.61	2.20	1.00	
E(R$_i$)	17.49%	26.34%	24.37%	21.07%	28.11%	13.84%	

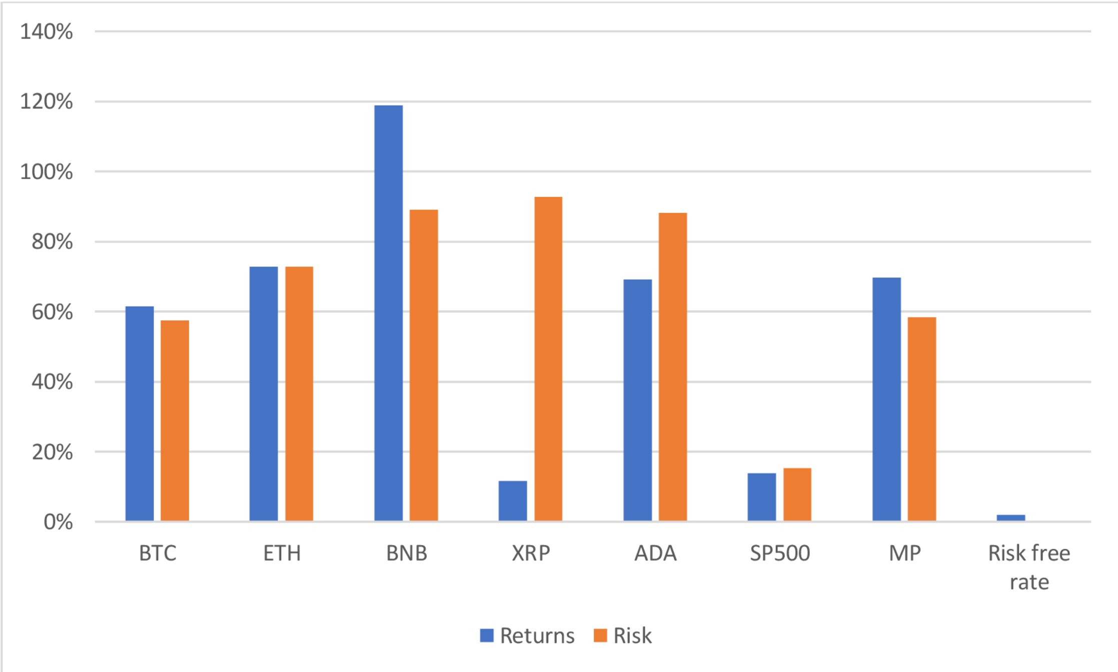
Table 8: Table of return, risk, Beta and expected return based on MP
(Source: Own processing in software Microsoft Excel 365)

	BTC	**ETH**	**BNB**	**XRP**	**ADA**	**MP**	**Risk-free**
Returns	61.55%	72.76%	118.93%	11.67%	69.20%	69.71%	1.91%
Risk (σ)	57.53%	72.85%	89.10%	92.73%	88.22%	58.46%	0%
$\boldsymbol{\beta_i}$	0.96	1.12	1.02	0.87	1.08	1.00	
E(R$_i$)	67.09%	78.06%	70.89%	60.96%	75.12%	69.71%	

These tables outline basic metrics for our five picked cryptocurrencies, the risk-free rate and a comparison of beta and expected return if we use the S&P500 index or cryptocurrency market portfolio for calculation.

The returns row indicates an annual cumulative return investors may expect from a given cryptocurrency based on five years of data. Notably, BNB has the highest return at 118.93%, experiencing the highest growth among the assets listed, while XRP had the lowest return at 11.67%. Besides XRP, all cryptocurrencies outperformed the market index of the S&P500.

The row "Risk (σ)" displays the annualised standard deviation, which measures the volatility or risk associated with each asset's returns. Assets with higher standard deviations are generally considered riskier investments. In this case, XRP has the highest risk, with a standard deviation of 92.73%, while the S&P 500 index exhibits the lowest risk, at 15.30% and MP 58.46%.

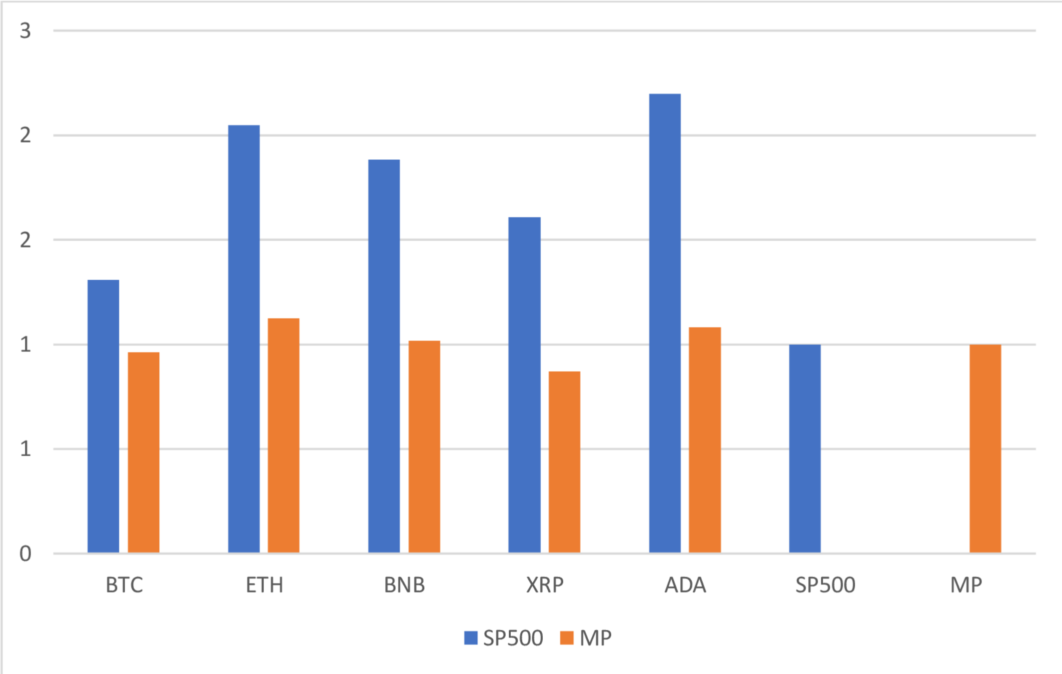


Graph 2: Return and risk of picked assets

(Source: Own processing in software Microsoft Excel 365)

Based on historical return and risk, we can see that BTC has the lowest risk of 57.53% out of the picked cryptocurrencies, which is still higher than the risk of the S&P500 by over 40 percentage points. BNB experienced the highest growth but also has the second highest risk. The only cryptocurrency that did not outperform the S&P500, XRP, also has the highest risk.

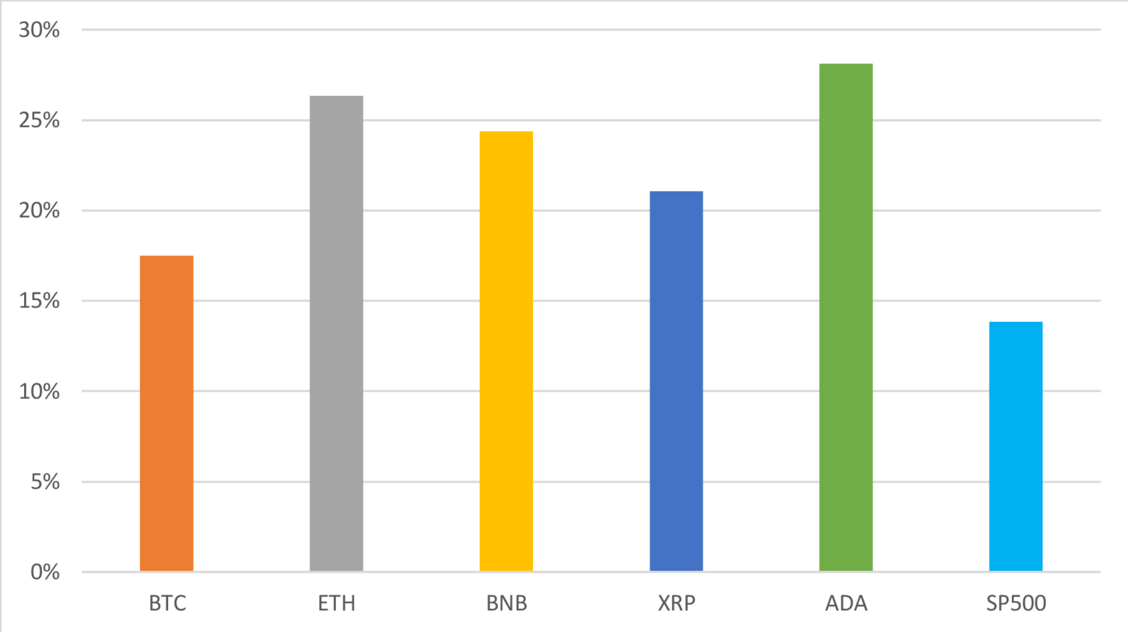
The beta coefficient β_i measures each asset's systematic risk or sensitivity to market movements compared to the broader market. A beta greater than 1 indicates higher volatility relative to the market, while a beta less than 1 suggests lower volatility. We can compare all asset betas to the selected two market portfolios. For S&P500 as a market portfolio, ADA stands out with the highest beta of 2.20, indicating it is more volatile than the market, while the S&P 500 itself has a beta of 1.00 as the benchmark. On the other hand, if we calculate beta against the cryptocurrency market portfolio, the highest beta is 1.12 for ETH.



Graph 3: Beta coefficients

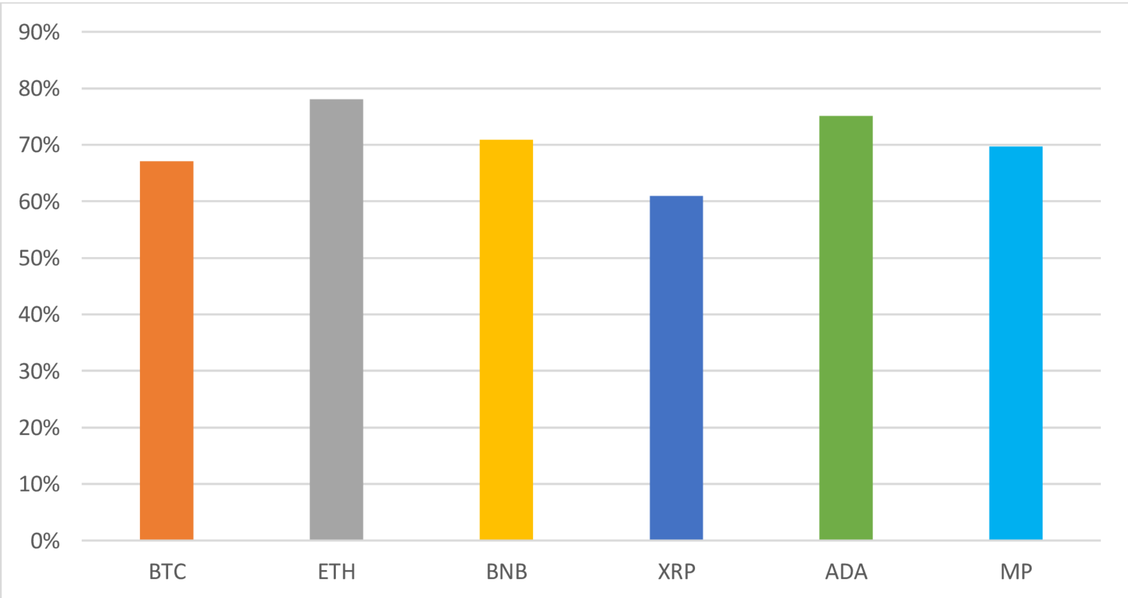
(Source: Own processing in software Microsoft Excel 365)

Finally, the $E(R_i)$ represents the expected return for each asset, calculated based on the Capital Asset Pricing Model (CAPM) using the beta coefficient and the market's expected return.



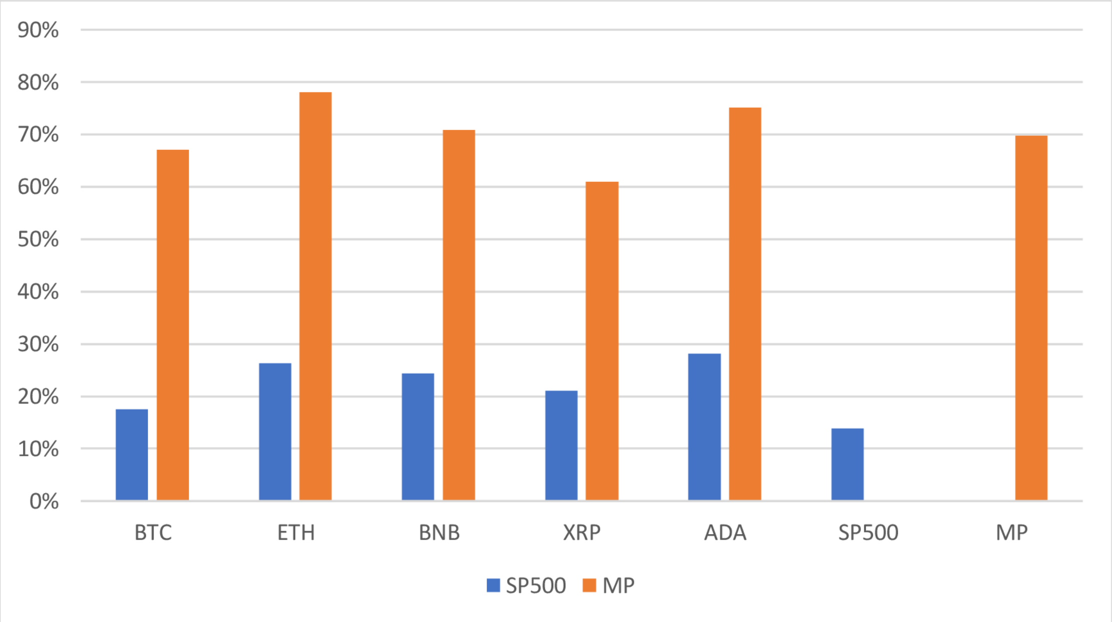
Graph 4: The expected return of assets based on the S&P500
(Source: Own processing in software Microsoft Excel 365)

While BNB recorded the highest returns among the assets listed, its expected return of 24.37%, as derived from the S&P500, is surpassed by ADA's expected return of 28.11%. This discrepancy between actual and expected returns suggests that ADA's performance may have exceeded what would be predicted by its systematic risk, as measured by its beta coefficient. While XRP had the lowest return out of the picked cryptocurrencies, its expected return is higher than BTC's expected return.



Graph 5: The expected return of assets based on the MP
(Source: Own processing in software Microsoft Excel 365)

When using a cryptocurrency market portfolio, the expected returns are notably higher for all cryptocurrencies. The market portfolio has an expected return of 69.71% compared to the expected return of the S&P500 of 13.84%. The highest expected return has, in this case, ETH of 78.06%, over 50 percentage points higher than ETH's expected return based on S&P500. XRP has the lowest expected return of 60.96% which is still higher than any expected return based on S&P500.



Graph 6: Expected return of assets comparison of market portfolios
(Source: Own processing in software Microsoft Excel 365)

These findings show the importance of selecting an appropriate market portfolio when estimating expected returns under the CAPM. Using a cryptocurrency market portfolio leads to higher expected returns for all assets, highlighting the differences between the cryptocurrency market and traditional equity markets represented by the S&P500. Investors and analysts should consider this when selecting a market portfolio used to calculate the beta coefficient and expected return.

In order to decide which approach is better for the cryptocurrency market, we have to test both approaches individually and compare the results.

2.3 Variance testing

Variance and its statistical significance is tested using one-tail and two-tail f-tests. This is done to ensure the accuracy of the t-test done in testing the regression in the next step.

An F-test was done on individual cryptocurrencies against market portfolios using the "Data Analysis" pack in Microsoft Excel. See screenshot of Excel attached in Annex I. on page number 95. Variable 1 is the historical weekly returns of individual cryptocurrencies, and Variable 2 is the historical weekly returns of market portfolios. Alpha 0.05 is the significance level for p-values.

For the two-tail f-test, the null hypothesis H_0 is that there are no differences in variation in the two samples, the alternative hypothesis H_1 is a difference in variance between the two samples.

Table 9: F-Test Two-Sample for Variances for S&P500

(Source: Own processing in software Microsoft Excel 365)

	BTC	ETH	BNB	XRP	ADA	S&P500
μ	0.012	0.016	0.022	0.009	0.017	0.003
$\boldsymbol{\sigma^2}$	0.006	0.010	0.015	0.017	0.015	0.000
Observations	260	260	260	260	260	260
Degrees of freedom	259	259	259	259	259	259
F-statistics	14.131	22.657	33.895	36.714	33.226	
Two-tail F-test p-value	0.000	0.000	0.000	0.000	0.000	

Above table is F-test for S&P 500 and individual cryptocurrencies.

The highest F-statistic of 36.714 for XRP indicates a strong relationship between its mean return and variance compared to the other assets. Despite having the lowest return and highest risk among the assets in the previous table, the high F-statistic suggests that XRP's returns are statistically significant relative to its risk. This implies that although XRP may have exhibited high volatility and lower returns compared to other assets, there is still a significant relationship between its mean returns and variances.

On the other hand, Bitcoin (BTC) and Ethereum (ETH) show lower F-statistics of 14.131 and 22.657, respectively, indicating less significant relationships between their mean returns and variances. While these assets may still offer investment opportunities, their returns are less tied to their volatility.

We can reject the null hypothesis for two-tail F-tests, it was concluded that the variance of all tested cryptocurrencies was higher than that of the S&P500. This means that the accuracy of T-statistics used in testing β_i is limited.

Table 10: F-Test Two-Sample for Variances for MP
(Source: Own processing in software Microsoft Excel 365)

	BTC	ETH	BNB	XRP	ADA	MP
μ	0.012	0.016	0.022	0.009	0.017	0.013
σ^2	0.006	0.010	0.015	0.017	0.015	0.007
Observations	260	260	260	260	260	260
Degrees of freedom	259	259	259	259	259	259
F-statistics	1.033	1.552	2.323	2.516	2.277	
Two-tail F-test p-value	0.796	0.000	0.000	0.000	0.000	

The above table is an F-test for the cryptocurrency market portfolio and individual cryptocurrencies.

The variance observed is understandably the same as in the table for the S&P500 because it is calculated from the same data. However, the variance for the market portfolio is 0.007, compared to 0 for the S&P500.

For BTC, we can accept the null hypothesis that the variance is the same as that of the cryptocurrency market portfolio. Given that the cryptocurrency market portfolio consists mostly of BTC based on its highest market capitalisation. Based on this result, the beta coefficient of BTC based on the cryptocurrency market will probably be the most accurate.

2.4 Regression testing

Regression testing is done using the LINEST Excel function. This function calculates regression coefficients, standard error, T-stat, two-tail p-value based on T-stat, coefficient of determination R^2, F-stat and degrees of freedom.

Regression is calculated between cryptocurrency week returns and S&P 500 returns.

Equation 43: Regression in Excel
(Source: Processed according to equation n. 14)

$$= LINEST(i_1 : i_n, m_1 : m_n, TRUE, TRUE)$$

Where $m_1 : m_n$ are the weekly returns of the market portfolio, and TRUE signals to Excel to calculate coefficients and all the testing statistics.

The null hypothesis is the coefficient being equal to zero, alternative hypothesis is the coefficient not being zero. The significance level selected for the p-value is 0.05.

Table 11: **Regression based on S&P 500**
(Source: Own processing in software Microsoft Excel 365)

	BTC	ETH	BNB	XRP	ADA
$\boldsymbol{\beta_i}$	1.307	2.048	1.883	1.606	2.197
Standard error	0.219	0.267	0.343	0.364	0.332
T-stat	5.955	7.657	5.490	4.416	6.623
Two tail p-value	0.000	0.000	0.000	0.000	0.000
$\boldsymbol{R^2}$	12.1%	18.5%	10.5%	7.0%	14.5%
F-Stat	35.457	58.632	30.144	19.504	43.860
Degrees of freedom	258	258	258	258	258

Table 12: **Regression based on MP**
(Source: Own processing in software Microsoft Excel 365)

	BTC	ETH	BNB	XRP	ADA
$\boldsymbol{\beta_i}$	0.961	1.123	1.017	0.871	1.080
Standard error	0.013	0.034	0.071	0.083	0.066
T-stat	73.567	33.443	14.404	10.555	16.460
Two tail p-value	0.000	0.000	0.000	0.000	0.000
$\boldsymbol{R^2}$	95.4%	81.3%	44.6%	30.2%	51.2%
F-Stat	5412.106	1118.410	207.472	111.406	270.926
Degrees of freedom	258	258	258	258	258

The Standard error provides the beta coefficient estimates' standard deviation, indicating the estimated betas' precision. The T-stat assesses the significance of the beta coefficient. Higher t-statistic values indicate greater significance. The P-value represents the probability of observing a t-statistic as extreme as the one computed. Based on p-values being zero, we reject the null hypothesis, meaning all of our coefficients are different than zero.

We can see that the coefficient of determination R^2 for S&P 500 regression is between 7% for XRP and 18.5% for ETH, below the customarily accepted threshold in the literature of 70%. R^2 indicates the proportion of variation in the cryptocurrency's returns that changes in the returns of the S&P 500 can explain.

The R^2 for betas calculated from the cryptocurrency market portfolio have considerably higher amounts between 30.2% for XRP and 95.4% for BTC.

Based on these finding we concluded that all our beta coefficients for both market portfolios are statistically significant. For S&P 500 as market portfolio the most accurate beta estimation is for ETH while for cryptocurrency market portfolio the BTC has the most accurate beta estimation.

2.5 Efficient Frontier

The covariance matrix, Σ, for the whole portfolio of cryptocurrencies is calculated using the COVARIANCE.S Microsoft Excel function. This function takes the weekly return of one cryptocurrency, i_X, against the weekly returns of the next cryptocurrency, i_Y. This calculation is repeated for the whole matrix.

Equation 44: Covariance matrix in Excel
(Source: processed according to equation number 25)

$$\Sigma = COVARIANCE.S(i_{X_1}:i_{X_n}, i_{Y_1}:i_{Y_n})$$

Table 13: Covariance matrix
(Source: Own processing in software Microsoft Excel 365)

	BTC	ETH	BNB	XRP	ADA
BTC	0.33	0.34	0.31	0.24	0.32
ETH	0.34	0.53	0.37	0.36	0.45
BNB	0.31	0.37	0.79	0.40	0.49
XRP	0.24	0.36	0.40	0.86	0.45
ADA	0.32	0.45	0.49	0.45	0.78

The inverse covariance matrix is calculated using the MINVERSE function, which returns the inverted matrix.

Equation 45: Inverse covariance matrix in Excel
(Source: processed according to equation number 26)

$$\Sigma^{-1} = MINVERSE(\Sigma)$$

Table 14: Inverse covariance matrix
(Source: Own processing in software Microsoft Excel 365)

	BTC	ETH	BNB	XRP	ADA
BTC	9.29	-4.76	-1.19	0.13	-0.44
ETH	-4.76	6.49	0.01	-0.64	-1.39
BNB	-1.19	0.01	2.41	-0.34	-0.84
XRP	0.13	-0.64	-0.34	1.83	-0.53
ADA	-0.44	-1.39	-0.84	-0.53	3.10

The first two vectors for efficient frontier are already given as a vector of ones and a vector of the previously calculated expected return of individual assets.

Two remaining vectors are calculated using the MMULT function, which multiplies matrixes.

Equation 46: Vector (X) and (G)

(Source: processed according to equations number 27 and 28)

$$X = MMULT\big((1);\Sigma^{-1}\big)$$

$$G = MMULT\big((R);\Sigma^{-1}\big)$$

Table 15: Vector (X) and (G)

(Source: Own processing in software Microsoft Excel 365)

Vector					
(1)	1	1	1	1	1
(X)	3.03	-0.29	0.05	0.45	-0.10
$(\mathbf{R})_{\boldsymbol{S\&P\ 500}}$	0.17	0.26	0.24	0.21	0.28
$(\mathbf{R})_{\boldsymbol{MP}}$	0.67	0.78	0.71	0.61	0.75
$(\mathbf{G})_{\boldsymbol{S\&P\ 500}}$	-0.02	0.35	0.07	0.01	0.11
$(\mathbf{G})_{\boldsymbol{MP}}$	1.42	0.45	0.08	0.06	0.03

Vectors (1) and (X) are the same for both market portfolios. Only vectors (R) and (G) differ because they are calculated from expected returns.

Coefficients A, B and C are calculated using the MMULT function and the function TRANSPOSE, which transposes a vector.

Equation 47: Efficient frontier coefficients

(Source: processed according to equation number 29)

$$A = MMULT((1);TRANSPOSE(X))$$

$$B = MMULT((1);TRANSPOSE(G))$$

$$C = MMULT((R);TRANSPOSE(X))$$

$$D = A \times C - B\verb|^|2$$

Table 16: Efficient frontier coefficients based on S&P 500

(Source: Own processing in software Microsoft Excel 365)

Coefficient	
A	3.14
B	0.53
C	0.14
D	0.16

Table 17: Efficient frontier coefficients based on MP
(Source: Own processing in software Microsoft Excel 365)

Coefficient	
A	3.14
B	2.04
C	1.42
D	0.29

An efficient portfolio function can be plotted by creating a row of expected returns *(E)* starting at 0% and adding 1% point and for each point of expected return, calculating risk given by the risk function of the efficient frontier. Function SQRT is used to calculate the square root.

Equation 48: Efficient frontier risk calculation in Excel
(Source: processed according to equation number 30)

$$\sigma = SQRT((CE^2 - 2AE + B)/D)$$

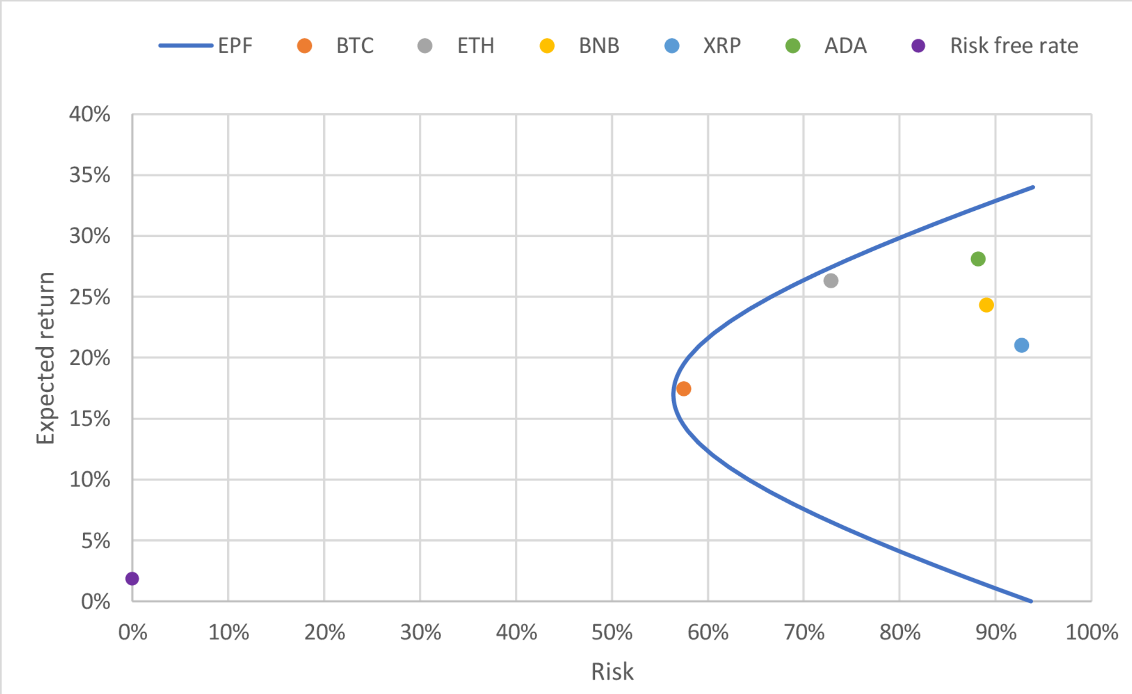
Table 18: Efficient frontier function based on S&P 500
(Source: Own processing in software Microsoft Excel 365)

σ	$\mathbf{E(R_i)}$
93.69%	0.00%
90.21%	1.00%
86.81%	2.00%
83.51%	3.00%
80.31%	4.00%
77.24%	5.00%

Table 19: Efficient frontier function based on MP
(Source: Own processing in software Microsoft Excel 365)

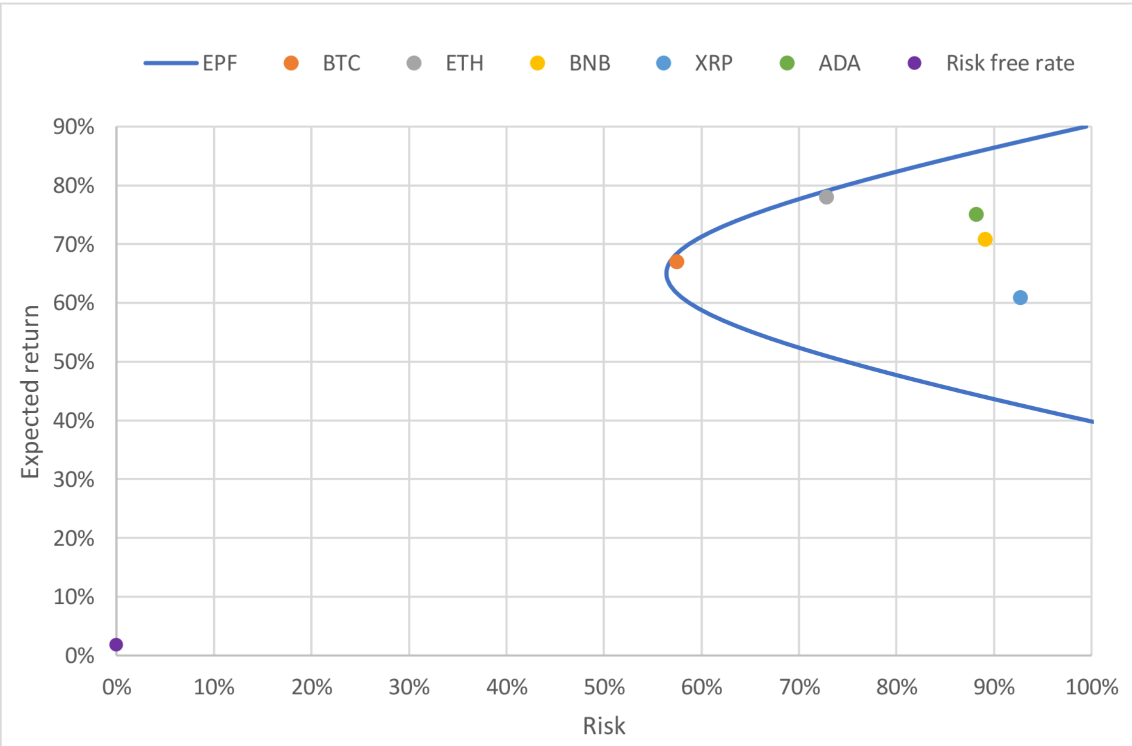
σ	$\mathbf{E(R_i)}$
220.48%	0.00%
217.31%	1.00%
214.14%	2.00%
210.98%	3.00%
207.83%	4.00%
204.67%	5.00%

By calculating the risk for a large number of expected returns, we get the efficient frontier, which can be plotted into a graph with the expected returns of individual assets.



Graph 7: Efficient frontier function for S&P 500

(Source: Own processing in software Microsoft Excel 365)



Graph 8: Efficient frontier function based on MP

(Source: Own processing in software Microsoft Excel 365)

This graph shows the risk and expected return relationship for our selected cryptocurrencies. The function of the efficient frontier signals all possible investment portfolios created out of these five cryptocurrencies, where all possible investments lie under the function graph. No combination of selected cryptocurrencies could offer a combination of expected return and risk that lies outside of the function graph.

We can now see the impact of different market portfolios selected for the construction of the model.

2.6 Minimum variance portfolio

With coefficients and vectors used in the efficient frontier calculation, we can also calculate the minimum variance portfolio (*MVP*), which minimises risk. The weights of *MVP* are given by vector (*X*) and divided by coefficient *A*. The sum of individual cryptocurrencies weights in the portfolio has to equal 100%, as investing all of our capital is one of the constraints.

Equation 49: Weights of MVP in Excel
(Source: processed according to equation number 31)

$$MVP = (X)/A$$

Table 20: Weights of MVP based on S&P 500
(Source: Own processing in software Microsoft Excel 365)

	BTC	**ETH**	**BNB**	**XRP**	**ADA**	**SUM**
MVP	96.43%	-9.19%	1.55%	14.29%	-3.09%	100.00%

Table 21: Weights of MVP based on MP
(Source: Own processing in software Microsoft Excel 365)

	BTC	**ETH**	**BNB**	**XRP**	**ADA**	**SUM**
MVP	96.43%	-9.19%	1.55%	14.29%	-3.09%	100.00%

Because the weights are calculated using vector *(X)* and coefficient *A*, that have the same values for both market portfolios, the MVP weights are the same for both market portfolios. What will differ is the expected return, while the risk is going to be the same as well.

The expected return of MVP can be calculated using two approaches: by dividing the *B* coefficient by the *A* coefficient or by multiplying the weights of the *MVP* with the *(R)* vector.

Equation 50: Expected return of MVP in Excel
(Source: processed according to equation number 30)

$$R_{MVP} = B/A = MVP \times R$$

The risk of *MVP* can be calculated using two approaches as well.

Equation 51: Risk of MVP in Excel

(Source: processed according to equation number 30)

$$\sigma_{MVP} = SQRT(1/A) = SQRT(MVP \times (\Sigma \times MVP\hat{}T))$$

For precision purposes, the expected return R_{MVP} and risk σ_{MVP} is calculated using both approaches to verify the model.

Table 22: Expected return of MVP based on S&P 500

(Source: Own processing in software Microsoft Excel 365)

	B/A	$MVP \times R$
R_{MVP}	16.97%	16.97%

Table 23: Expected return of MVP based on MP

(Source: Own processing in software Microsoft Excel 365)

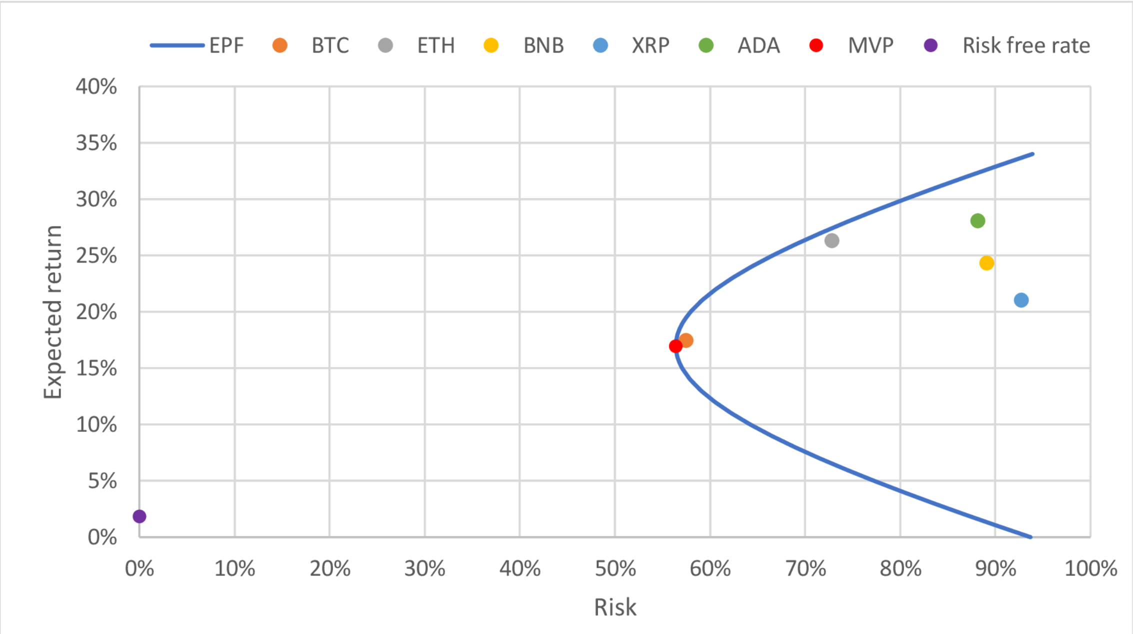
	B/A	$MVP \times R$
R_{MVP}	65.02%	65.02%

Table 24: Risk of MVP based on S&P 500 and MP

(Source: Own processing in software Microsoft Excel 365)

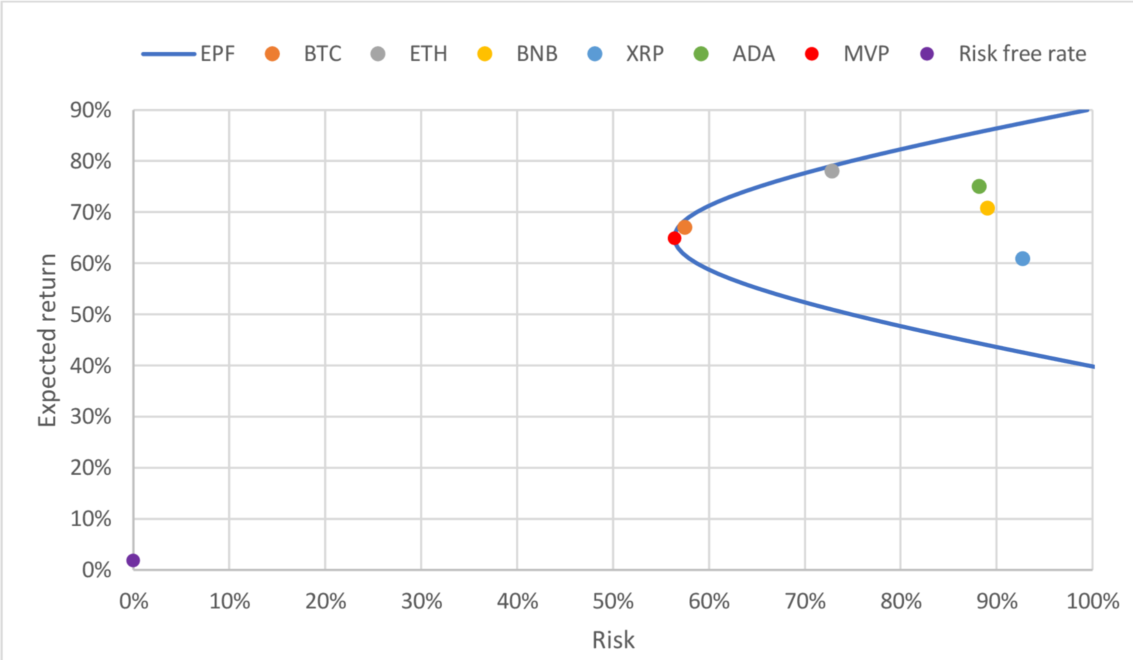
	$SQRT(1/A)$	$SQRT(MVP \times (\Sigma \times MVP\hat{}T))$
σ_{MVP}	56.41%	56.41%

Both calculations have the same result, meaning the minimum variance portfolio and the efficient frontier are calculated correctly.



Graph 9: Minimum variance portfolio based on the S&P 500

(Source: Own processing in software Microsoft Excel 365)



Graph 10: Minimum variance portfolio based on MP
(Source: Own processing in software Microsoft Excel 365)

We can see that both calculations returned the exact numbers, and the *MVP* sits on the least risky part of the efficient frontier for both market portfolios. In theory, this portfolio consisting of these five cryptocurrencies should minimize the variance of our portfolio. It is worth noting that the portfolio lies very close to Bitcoin on the graph and consists of 96.43% of Bitcoin. This suggests that Bitcoin is the least risky of the selected cryptocurrencies.

2.7 Capital market line and tangency portfolio

Capital market line and tangency portfolio are calculated using the same coefficients utilizing the covariance matrix and vectors calculated before.

Equation 52: CML coefficients in Excel
(Source: Processed according to equation number 32)

$$R_T = (C + B \times R_f)/(B - A \times R_f)$$

$$\sigma_T = SQRT((A \times R_T{}^\wedge 2 - 2 \times B \times R_T + C)/D)$$

$$\beta_T = (R_T - R_f)/\sigma_T$$

$$\lambda_T = (C - B \times R_f)/D$$

$$\mu_T = (D \times R_f - B)/D$$

Table 25: Tangency portfolio and CML coefficients based on S&P 500
(Source: Own processing in software Microsoft Excel 365)

$\mathbf{R_T}$	27.84%
$\boldsymbol{\sigma_T}$	74.02%
$\boldsymbol{\beta_T}$	0.35
$\boldsymbol{\lambda_T}$	-0.04
$\boldsymbol{\mu_T}$	2.11

Table 26: Tangency portfolio and CML coefficients based on MP
(Source: Own processing in software Microsoft Excel 365)

$\mathbf{R_T}$	69.71%
$\boldsymbol{\sigma_T}$	58.46%
$\boldsymbol{\beta_T}$	1.16
$\boldsymbol{\lambda_T}$	-0.01
$\boldsymbol{\mu_T}$	0.50

Because the tangency portfolio and the cryptocurrency market portfolio are the same portfolios, the expected return and risk of the tangency portfolio are the same as the expected return and risk of the market portfolio, as calculated initially in table number 8.

The CML line function and Tangency portfolio can be constructed using these coefficients.

Equation 53: Capital market line function
(Source: Processed according to equation number 34)

$$CML = R_f + \sigma_T \times \beta_T$$

CML function can be plotted by creating a vector of risk σ_T starting at 0% and adding 1% point, and calculating the return given by the CML function for each point of risk.

Table 27: CML based on S&P 500
(Source: Own processing in software Microsoft Excel 365)

$\boldsymbol{\sigma_T}$	**CML**
0.00%	1.91%
1.00%	2.26%
2.00%	2.61%
3.00%	2.96%
4.00%	3.31%
5.00%	3.66%
6.00%	4.01%
7.00%	4.36%
8.00%	4.71%

Table 28: CML based on MP
(Source: Own processing in software Microsoft Excel 365)

σ_T	CML
0.00%	1.91%
1.00%	3.07%
2.00%	4.23%
3.00%	5.39%
4.00%	6.55%
5.00%	7.71%
6.00%	8.87%
7.00%	10.03%
8.00%	11.19%

The weight of individual assets in the tangency portfolio is given by:

Equation 54: Tangency portfolio weights of individual assets
(Source: Processed according to equation number 33)

$$Tangency\ portfolio = \lambda_T \times X + \mu_T \times G$$

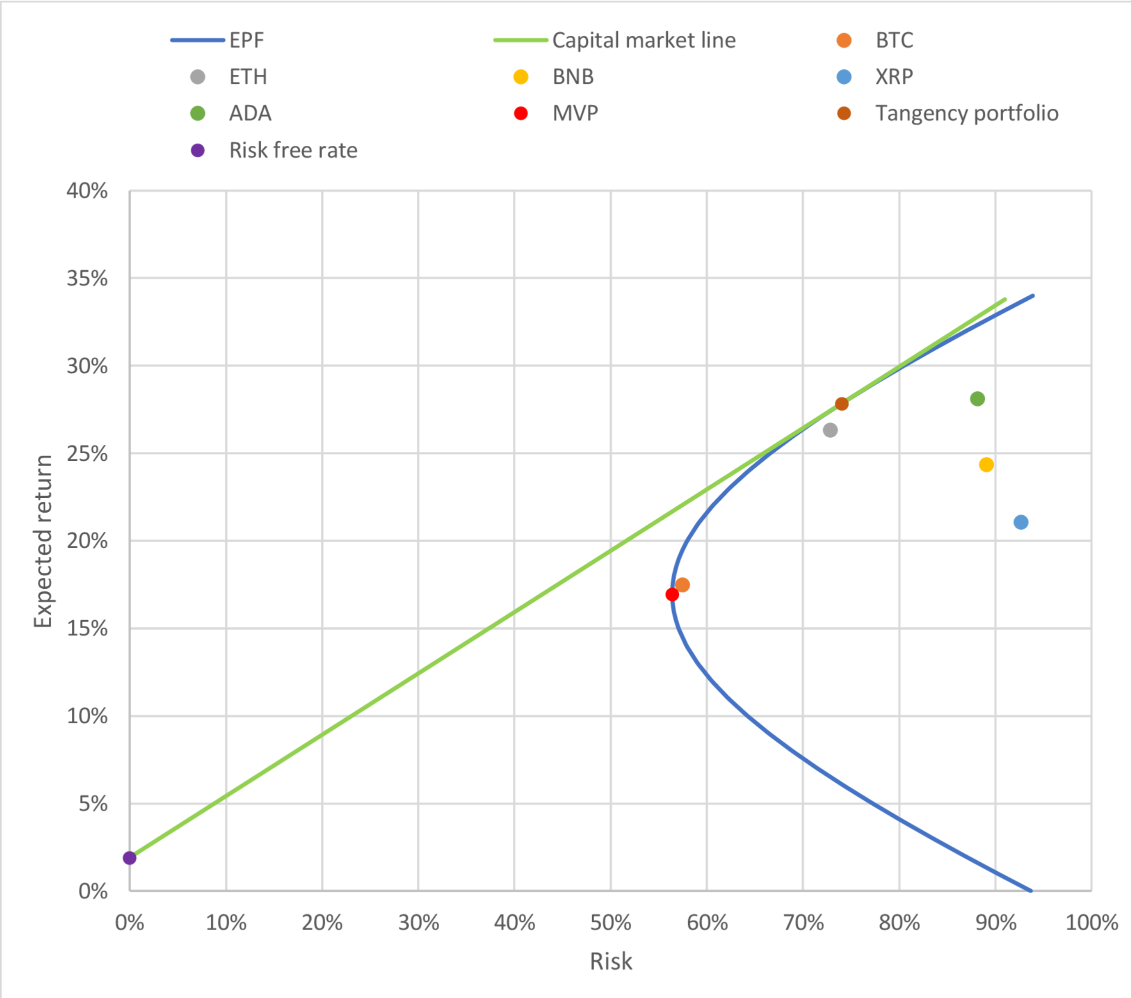
Table 29: Tangency portfolio based on S&P 500
(Source: Own processing in software Microsoft Excel 365)

BTC	ETH	BNB	XRP	ADA	SUM
-15.44%	75.96%	15.37%	-0.23%	24.34%	100.00%

Table 30: Tangency portfolio based on S&P 500
(Source: Own processing in software Microsoft Excel 365)

BTC	ETH	BNB	XRP	ADA	SUM
68.77%	22.78%	3.94%	2.77%	1.75%	100.00%

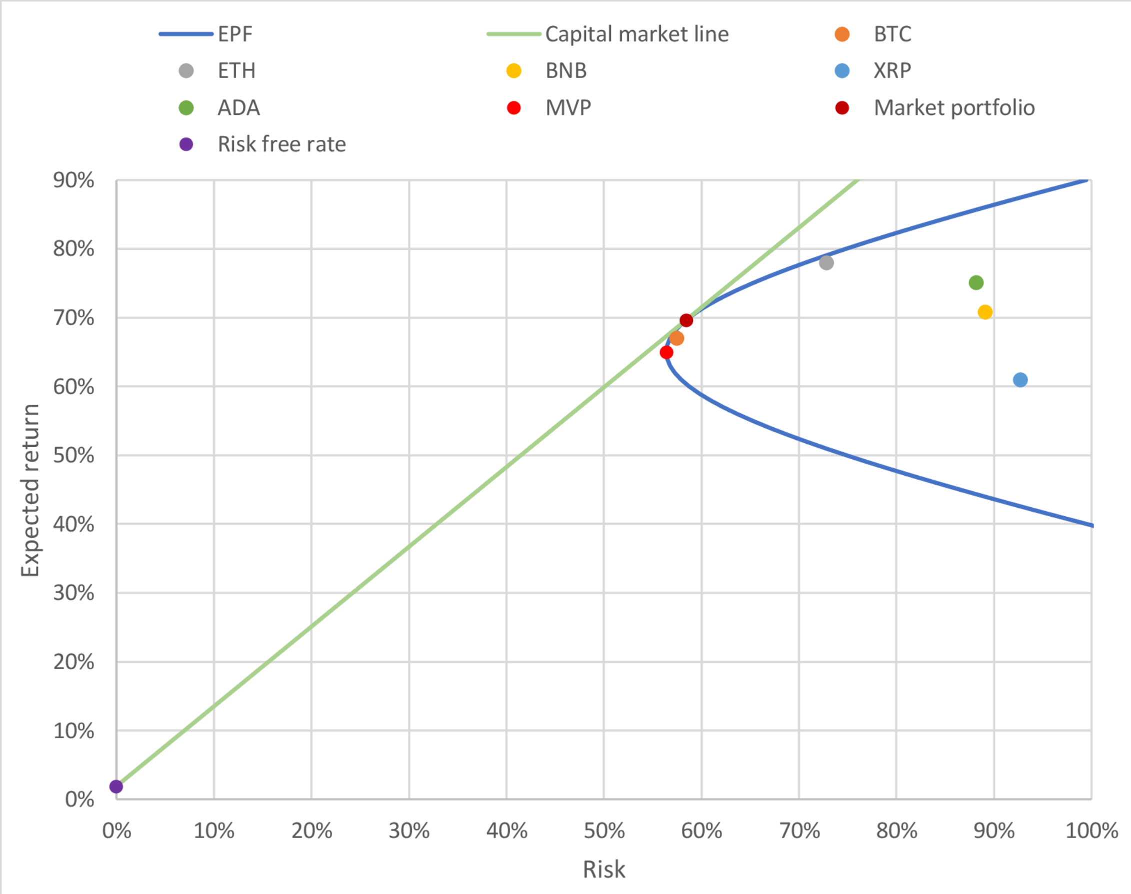
Adding CML and tangency portfolios to the efficient frontier graph shows that the CML function is tangent to the efficient frontier function and intercepts the y-axis at the risk-free rate of a three-month treasury note. The tangency portfolio is at the tangent point of CML and the efficient frontier. This portfolio should, in theory, give investors the best risk-to-reward ratio for cryptocurrencies.



Graph 11: CML and tangency portfolio based on S&P 500

(Source: Own processing in software Microsoft Excel 365)

Compared to the minimum variance portfolio, the tangency portfolio lies higher on the efficient frontier curve, which is close to Ethereum. This is reflected in the weight of Ethereum in the portfolio, representing 75.96% of the portfolio. Interestingly, this combination of risk and return is achieved by shorting the least risky cryptocurrency, Bitcoin. If investors use the S&P 500 as a market portfolio, the CAPM suggests shorting some cryptocurrencies to achieve an optimal portfolio. However, the model does not take into consideration possible costs related to shorting assets.



Graph 12: CML and tangency portfolio based on MP

(Source: Own processing in software Microsoft Excel 365)

In the model using the cryptocurrency market portfolio, the tangency portfolio is the same portfolio as the market portfolio, as expected based on the literature. This portfolio has an expected return of 69.71% and a risk of 58.46%, compared to the tangency portfolio based on the S&P 500, with an expected return of 27.84% and a risk of 74.02%. So, the cryptocurrency market portfolio has a higher expected return by 42% points while having a lower risk of 16% points. Also, the cryptocurrency market portfolio does not short-sell any assets.

If investors seek to create an optimal cryptocurrency portfolio, this result suggests that a cryptocurrency market portfolio is more optimal than a tangency portfolio created using the S&P 500 as a market portfolio in CAPM.

2.8 Testing CAPM

After the construction of the models we can test underlying CAPM assumptions to validate the findings.

2.8.1 Time series test

In this test, we are testing the intercept of regression between excess returns of individual assets at time *t*, r_{eit}, and excess returns of the market portfolio at time *t*, r_{emt}. Excess return is calculated by subtracting the risk-free rate at time *t*.

Equation 55: Time series test in Excel
(Source: Processed according to equation number 35)

$$= LINEST(r_{ei1}: r_{ein}, r_{em1}: r_{emn}, TRUE, TRUE)$$

Where:

$$r_{eit} = r_{it} - r_{ft}$$

$$r_{emt} = r_{mt} - r_{ft}$$

and TRUE signals to Excel to calculate coefficients and all the testing statistics.

The null hypothesis is the intercepts α_i being zero, the alternative hypothesis is the intercepts are different than zero. The significance level used for the p-value is 0.05.

Table 31: Time series test based on S&P 500
(Source: Own processing in software Microsoft Excel 365)

	BTC	ETH	BNB	XRP	ADA
α_i	0.016	0.035	0.038	0.020	0.040
Standard error	0.007	0.008	0.010	0.011	0.010
F-stat	35.457	58.632	30.144	19.504	43.860
Degrees of freedom	258	258	258	258	258
RSS	1.449	2.154	3.540	3.982	3.313
T-stat	2.461	4.376	3.665	1.789	3.985
p-value	0.015	0.000	0.000	0.075	0.000

For the time series test based on the S&P 500, we reject the null hypothesis of intercept being zero for four of the cryptocurrencies; we accept the null hypothesis only for XRP based on a p-value of 0.075. This suggests that the expected returns predicted by the CAPM using the S&P 500 are not accurate; only the expected return for XRP might be statistically accurate.

Table 32: **Time series test based on MP**
(Source: Own processing in software Microsoft Excel 365)

	BTC	ETH	BNB	XRP	ADA
α_i	-0.001	0.003	0.008	-0.005	0.004
Standard error	0.001	0.003	0.006	0.007	0.005
F-stat	5412.106	1118.410	207.472	111.406	270.926
Degrees of freedom	258	258	258	258	258
RSS	0.075	0.495	2.192	2.991	1.891
T-stat	-1.392	1.229	1.452	-0.812	0.804
p-value	0.165	0.220	0.148	0.417	0.422

For the time series test based on the cryptocurrency market portfolio, we accept the null hypothesis for all five of our tested cryptocurrencies, meaning the CAPM assumptions are correct and all intercepts α_i are zero. This makes the cryptocurrency market portfolio the correct market portfolio for CAPM construction, and the expected returns calculated using this market portfolio are statistically accurate.

Based on the time-series test of CAPM, the cryptocurrency market portfolio is more accurate than the S&P 500 index for the calculation of expected returns.

2.8.2 General Equilibrium Testing

The general equilibrium test tests the assumptions of the CAPM model in three separate tests. For these tests, we need a new table that is used in regressions. The table consists of average excess returns, beta estimation, beta squared and residual sum of squares for each tested cryptocurrency.

Equation 56: **Average excess return in Excel**
(Source: Processed according to equation number 36)

$$R_{ei} = AVERAGEA(r_{ei1}: r_{ein})$$

The average excess return R_{ei} is calculated based on individual excess returns calculated in the time-series test. Beta coefficient β_i have been calculated before. The β_i^2 is squared beta coefficient. The residual sum of squares, *RSS*, has also been calculated in the previous time-series test.

Table 33: **General equilibrium table based on S&P 500**
(Source: Own processing in software Microsoft Excel 365)

	BTC	ETH	BNB	XRP	ADA
R_{ei}	-0.012	-0.008	-0.002	-0.015	-0.007
β_i	1.307	2.048	1.883	1.606	2.197
β_i^2	1.707	4.195	3.546	2.580	4.828
RSS	1.449	2.154	3.540	3.982	3.313

Table 34: **General equilibrium table based on MP**
(Source: Own processing in software Microsoft Excel 365)

	BTC	ETH	BNB	XRP	ADA
R_{ei}	-0.012	-0.008	-0.002	-0.015	-0.007
β_i	0.961	1.123	1.017	0.871	1.080
β_i^2	0.924	1.261	1.035	0.759	1.166
RSS	0.075	0.495	2.192	2.991	1.891

The first test is an extension of the time series test. It involves regressing the average excess returns above the risk-free rate for all assets against their beta coefficients.

Equation 57: **CAPM excess return testing in Excel**
(Source: Processed according to equation number 36)

$$= LINEST(R_{ei1}\colon R_{ein}, \beta_{i1}\colon \beta_{in}, TRUE, TRUE)$$

This test if the CAPM assumption about expected returns. The null hypothesis is regression intercept γ_0 being zero with a significance level of 0.05.

Table 35: **CAPM excess return test based on S&P 500**
(Source: Own processing in software Microsoft Excel 365)

	γ1	γ0
Coefficient	0.008	-0.022
Standard error	0.006	0.011
T-stat	1.225	-1.979
p-value	0.308	0.142
R^2	0.333	
F-stat	1.500	
Degree of freedom	3	

Because the p-value of the intercept γ_0 is 0.142, we accept the null hypothesis of the intercept being zero. This means that the expected returns of assets based on the S&P 500 are accurate. However, when we combine this finding with previous time-series test of expected returns, where we rejected the same null hypothesis for four out of five cryptocurrencies, we cannot comfortably reject nor accept that expected returns estimated based on the S&P 500 are accurate.

Table 36: CAPM excess return test based on MP
(Source: Own processing in software Microsoft Excel 365)

	γ1	γ0
Coefficient	0.029	-0.038
Standard error	0.021	0.021
T-stat	1.390	-1.793
p-value	0.259	0.171
R^2	0.392	
F-stat	1.932	
Degree of freedom	3	

For the test of excess returns based on the cryptocurrency market, we accept the null hypothesis with a p-value of 0.171. This is in line with the result of the previous time series test, and we can comfortably say that the expected returns of cryptocurrencies are accurately estimated from the cryptocurrency market portfolio.

Second, is the test of non-linearity of returns and beta. The test is done by regressing the excess returns of assets with their beta and beta squared. If the CAPM assumption of linearity is true, the regression coefficient γ_2 is equal to zero. This is our null hypothesis; the significance level is set to 0.05.

Equation 58: CAPM beta testing in Excel
(Source: Processed according to equation number 37)

$$= LINEST(R_{ei1}: R_{ein}, \beta_{i1}: \beta_{in}, \beta_{i1}^2: \beta_{in}^2, TRUE, TRUE)$$

Table 37: CAPM beta testing based on S&P 500
(Source: Own processing in software Microsoft Excel 365)

	γ2	γ1	γ0
Coefficient	-0.005	0.024	-0.036
Standard error	0.03	0.104	0.089
T-stat	-0.156	0.228	-0.407
p-value	0.89	0.841	0.723
R^2	0.341		
F-stat	0.518		
Degree of freedom	2		

We can accept the null hypothesis of the linearity of beta coefficients and excess returns based on a p-value of 0.89 for coefficient γ_2. Beta linearity is one of the most important assumptions of the CAPM model. This means that while the S&P 500 as a market portfolio might not accurately estimate expected returns, its beta estimation is statistically significant. Investors can use the S&P 500 index to estimate the beta coefficients of cryptocurrencies.

Table 38: CAPM beta testing based on MP
(Source: Own processing in software Microsoft Excel 365)

	γ2	γ1	γ0
Coefficient	-0.328	0.684	-0.362
Standard error	0.248	0.496	0.246
T-stat	-1.322	1.381	-1.475
p-value	0.317	0.301	0.278
R^2	0.675		
F-stat	2.081		
Degree of freedom	2		

For the beta estimations based on the cryptocurrency market portfolio, we accept the null hypothesis as well, with a p-value of 0.317. This proves that expected returns and betas are linear, and investors can use the cryptocurrency market portfolio to estimate the beta of individual cryptocurrencies.

The last test relates to the systematic risk of returns. If CAPM is true, then excess returns are not influenced by non-systematic risk outside of the cryptocurrency market, and investors cannot achieve above-market returns by exposure to this type of risk. The non-systematic risk is captured by the residual sum of squares, *RSS*, and the null hypothesis is that its regression coefficient γ_2 equals zero.

Equation 59: CAPM systematic risk testing in Excel
(Source: Processed according to equation number 38)

$$= LINEST(R_{ei1}: R_{ein}, \beta_{i1}: \beta_{in}, \beta_{i1}^2: \beta_{in}^2, RSS_1: RSS_n, TRUE, TRUE)$$

Table 39: CAPM systematic risk testing based on S&P 500
(Source: Own processing in software Microsoft Excel 365)

	γ3	γ2	γ1	γ0
Coefficient	-0.001	-0.013	0.055	-0.06
Standard error	0.005	0.053	0.189	0.155
T-stat	-0.253	-0.25	0.292	-0.39
p-value	0.842	0.844	0.819	0.763
R^2	0.381			
F-stat	0.205			
Degree of freedom	1			

We accept the null hypothesis based on a p-value of 0.842 for the coefficient γ_3. This proves the CAPM assumption that non-systematic risk does not influence cryptocurrency returns when the model is constructed with the S&P 500 as a market portfolio.

Table 40: CAPM systematic risk testing based on MP
(Source: Own processing in software Microsoft Excel 365)

	γ3	γ2	γ1	γ0
Coefficient	-0.001	-0.013	0.055	-0.06
Standard error	0.005	0.053	0.189	0.155
T-stat	-0.253	-0.25	0.292	-0.39
p-value	0.400	0.313	0.301	0.284
R^2	0.381			
F-stat	0.205			
Degree of freedom	1			

For CAPM constructed based on cryptocurrency market portfolio we also accept null hypothesis of non-systematic risks not influencing the cryptocurrencies returns.

2.9 Evaluation Of The Practical Part

In the practical part, I have selected five cryptocurrencies based on their market capitalisation, their time on the market and whether or not they are supposed to be investments or stablecoins. As a market portfolio, I have selected the cryptocurrency market portfolio (MP) based on a weighted average of market capitalisation and index S&P 500 as a benchmark market portfolio. I have gathered daily price prices for all selected cryptocurrencies and the S&P 500 for a period of five years from 31.12.2018 to 31.12.2023. Their weekly returns have been calculated using a simple average of their daily price change and their annualised returns using geometric mean. The risk has been calculated as an annualised standard deviation. Lastly, I have selected 3-month treasury bills as risk-free rates and obtained their average rate of return in the same time period.

Using the CAPM methodology, I calculated the expected return and beta coefficients for selected cryptocurrencies based on both selected market portfolios. Beta coefficients were tested using a standard statistical approach. First was the F-test of variance, followed by the T-statistic of regression slope. The coefficient of determination was also used to support the findings.

Overall, I rejected that cryptocurrencies have the same variation as both market portfolios, making the T-statistics used in testing the regression less accurate. T-statistics has determined that all beta coefficients are different than zero, making them statistically significant. Lastly, the coefficient of determination allowed me to compare the beta coefficients between both market portfolios and with higher coefficient of determination based on the cryptocurrency market portfolio I supported the idea that the cryptocurrency market portfolio estimated betas are more accurate then the S&P 500 estimated betas.

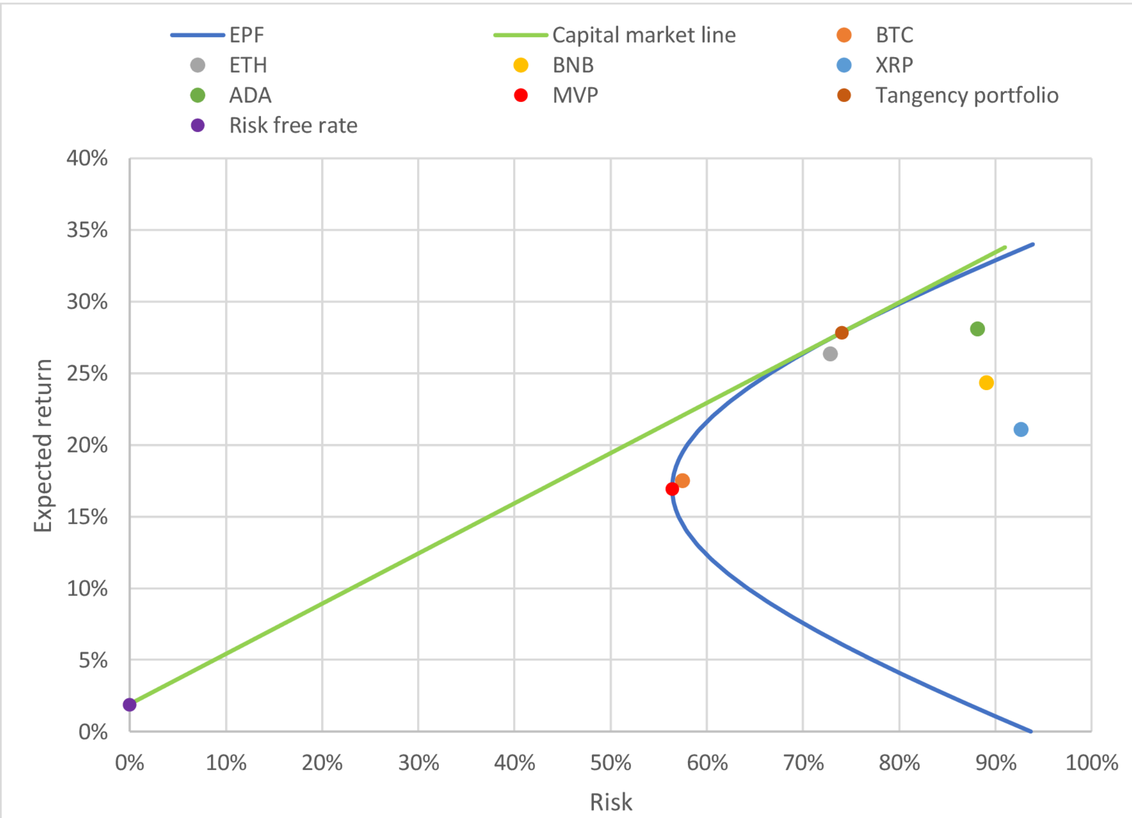
Table 41: **Beta estimation results**
(Source: Own processing in software Microsoft Excel 365)

Test	S&P 500	MP
F-test	Rejected	Rejected
T-stat	Accepted	Accepted
$\mathbf{R^2}$	Lower	Higher

This allowed me to construct the efficient frontier function that outlines all possible portfolios and their expected return-to-risk ratios investors are able to achieve using these cryptocurrencies. The minimum variance portfolio has been constructed, which minimises variance in the investment portfolio. This portfolio, consisting of 96.43% from Bitcoin, has a risk of 56.41%. While the weights of individual cryptocurrencies and the risk of the minimum variance portfolio are the same when using both market portfolios, an expected return of the minimum variance portfolio is 16.97% based on the S&P 500 and an expected return of 65.02%. This shows investors' different return expectations when using different market portfolios. However, the minimum variance portfolio also has a lower risk than any individual cryptocurrency may offer. This demonstrates the power of diversification in the modern portfolio theory.

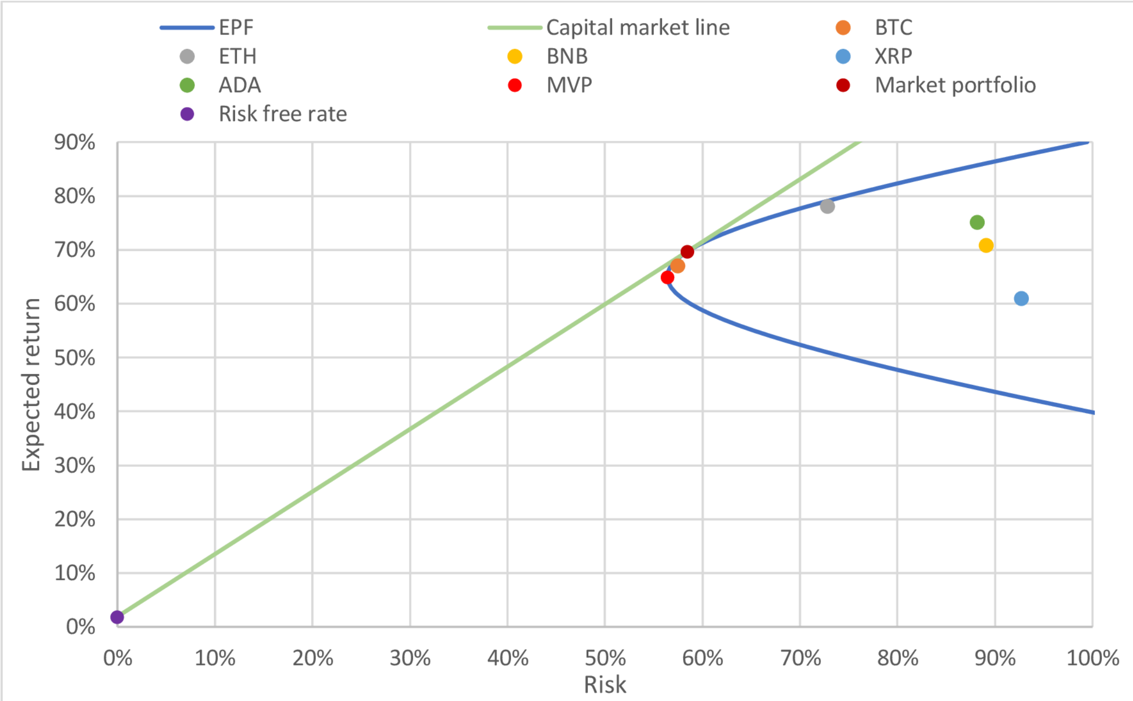
The capital market line was calculated in order to find the tangency portfolio. In theory, this portfolio should offer the best expected return-to-risk ratio. When constructed based on the S&P 500, this portfolio, consisting mainly of Ethereum with a weight of 75.96%, has an expected return of 27.84% and a risk of 74.02%, which is a higher expected return than any single cryptocurrency as estimated based on the S&P 500.

When constructing the tangency portfolio based on the cryptocurrency market portfolio, I discovered that the tangency portfolio is exactly the same as the cryptocurrency market portfolio itself, as predicted by the literature. This means that if CAPM assumptions are true, the market portfolio is the most optimal portfolio. This portfolio has an expected return of 69.71% and a risk of 58.46%. This is not the highest expected return; three cryptocurrencies have higher expected returns as estimated by the cryptocurrency market portfolio. However, this portfolio has a higher expected return than the tangency portfolio based on the S&P 500 by 42% points and does not require any short selling to achieve this expected return. Because there might be additional costs related to short selling, I have concluded that a cryptocurrency market portfolio created as a weighted average of the overall market is the most optimal portfolio if the CAPM assumptions are correct.



Graph 13: CML and tangency portfolio based on S&P 500

(Source: Own processing in software Microsoft Excel 365)



Graph 14: CML and tangency portfolio based on MP
(Source: Own processing in software Microsoft Excel 365)

To test the CAPM assumptions, I have implemented four tests:

1. Time series test of excessive returns
2. Test of average excess returns
3. Test of beta linearity
4. Test of non-systematic risk

All of these tests were done for both market portfolios to compare the results.

Table 42: CAPM testing results
(Source: Own processing in software Microsoft Excel 365)

Test	S&P 500	MP
1	Rejected	Accepted
2	Accepted	Accepted
3	Accepted	Accepted
4	Accepted	Accepted

While the S&P 500 index failed to accurately estimate expected returns for most cryptocurrencies, the cryptocurrency market portfolio expected returns are statistically significant, suggesting its superiority for CAPM construction. The test of non-linearity of returns and beta indicated that beta estimation based on both market portfolios index remains statistically significant, allowing investors to use it to estimate beta coefficients of cryptocurrencies. Furthermore, the analysis revealed that non-systematic risk does not significantly influence cryptocurrency returns in either CAPM model. Overall, the results support the use of a cryptocurrency market portfolio over the S&P 500 index for more reliable CAPM estimations in cryptocurrency markets.

I conclude that CAPM theory is consistent with empirical test of selected five cryptocurrencies for period of five years between 31.12.20218 and 31.12.2023 and market portfolio of cryptocurrency market can be used by investors to estimate beta of cryptocurrencies.

3 PROPOSAL

3.1 Recommendation for investors

For passive investors looking to incorporate exposure to cryptocurrency, I recommend using a weighted average cryptocurrency market portfolio for the best risk-to-reward ratio. Passive investors may be willing to expose themselves to the fast-growing cryptocurrency market but may not be interested in active analysis and trading cryptocurrencies because they believe the markets are efficient and cannot generate above-average returns or because they find analysing cryptocurrencies too costly.

Based on the findings, I recommend the cryptocurrency market portfolio as the optimal way to incorporate cryptocurrencies into investment portfolios. Furthermore, I believe that using a cryptocurrency market portfolio for beta calculations and expected returns yields the most accurate results, but investors can still use the index S&P 500 to calculate beta coefficients of cryptocurrencies in case they want to calculate beta coefficients for all of their investments based on the same benchmark. The calculated beta coefficients will still be statistically significant.

3.2 Limitation

While the most robust empirical tests of CAPM do so on high number of assets sorted into market portfolios that are tested to eliminate selection bias, I have carried the testing only on five individual cryptocurrencies. This was due to the relative youth and dynamics of the market, because currently many of the biggest cryptocurrencies, such as Solana, do not have five years' worth of data. Also, as of May 2024 Cardano (ADA) is no longer fifth largest coin fitting our criteria. The biggest limitation of this study is lower number of cryptocurrencies tested, even though they covered 70% of the market at time of the selection.

Selected time period of five years, that is standard in literature, is also limitation due to the high volatility of the market and market cycles based on halving events. Some studies on traditional markets split the testing period into smaller section and carry CAPM testing over more periods.

Lastly some studies tried to test some aspects of CAPM on cryptocurrencies, but I have not been able to find a study conducting empirical test of CAPM on the cryptocurrency market following similar methodology. The results of this study cannot be verified against another source at the time of writing.

For these reasons I believe the findings do apply for five selected cryptocurrencies in the selected period, but a general application to the whole market is limited.

3.3 Future studies

Future studies on the issue could take two directions. First is elimination of described limitation and expanding the study across more cryptocurrencies and using time periods split into smaller testing periods. Second is linking the cryptocurrency returns to market cycles and creating adjusted model for cryptocurrency market.

CONCLUSION

In conclusion, this thesis has aimed to investigate the applicability of the Capital Asset Pricing Model (CAPM) in the cryptocurrency effectiveness of the CAPM in capturing systematic risk in this asset class. Through a deductive approach, utilising academic literature and statistical methods, the study has tested various CAPM assumptions using two market portfolios: the S&P 500 index and a cryptocurrency market portfolio.

The findings suggest that while the S&P 500 index struggled to estimate expected returns for most cryptocurrencies accurately, the cryptocurrency market portfolio exhibited statistically significant expected returns, indicating its superiority for CAPM construction. Moreover, beta estimation based on both market portfolios remained statistically significant, enabling investors to effectively estimate cryptocurrencies' beta coefficients. Additionally, the analysis revealed that non-systematic risk does not significantly influence cryptocurrency returns in either CAPM model.

Overall, the results support using a cryptocurrency market portfolio over the S&P 500 index for more reliable CAPM estimations in cryptocurrency markets. Moreover the study accept the assumption of CAPM in cryptocurrency market and find that CAPM theory is consistent with empirical data from the market.

This study contributes to the understanding of portfolio management strategies in the emerging cryptocurrency asset class, providing valuable insights for investors seeking to apply traditional financial models on cryptocurrency markets.

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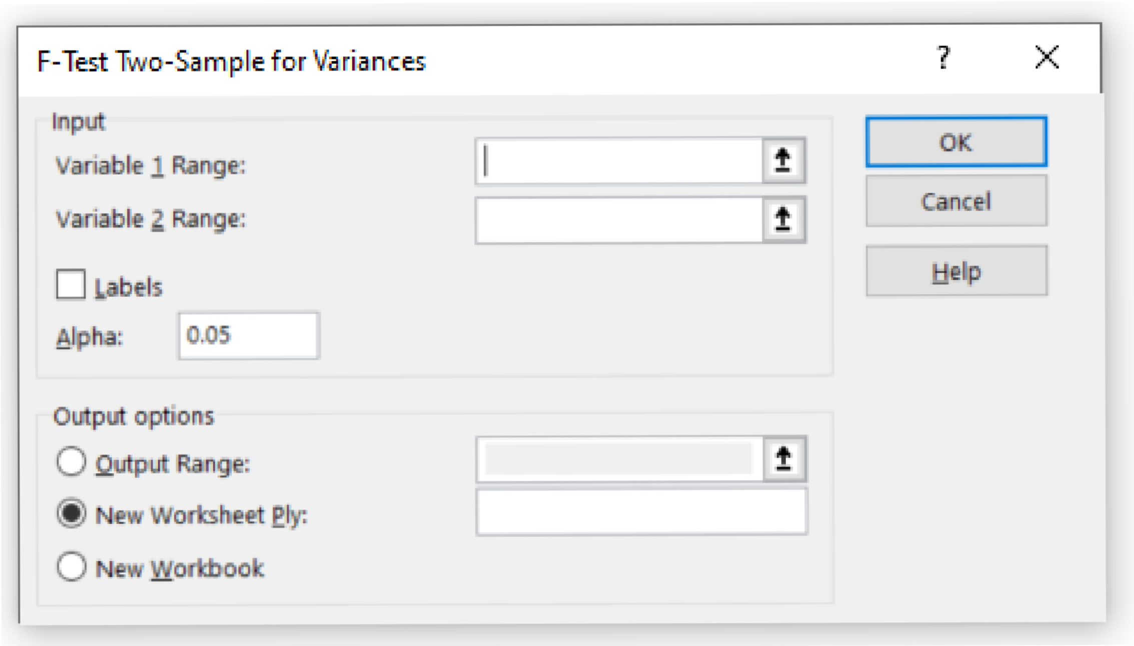
ANNEX I.

F-test in Data analysis pack in Microsoft Excel.



(Source: Screenshot from Microsoft Excel)