CZECH UNIVERSITY OF LIFE SCIENCES PRAGUE

Faculty of Environmental Sciences
Department of Environmental Modeling

The Effect of Longitudinal Slope on Solids Transport and Friction in Mixture Flow above Stationary Deposit in Pipe

Master Thesis

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DIPLOMA THESIS ASSIGNMENT

Milica Jovanovic

Landscape Engineering
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Thesis title

The Effect of Longitudinal Slope on Solids Transport and Friction in Flow above Eroded Bed

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Objectives of thesis

The subject of the presented thesis is an investigation of two-phase flow composed of water and solid particles in a pipe. The objective is an analysis of how the angle of pipe inclination affects the frictional pressure drop in the flow above a stationary deposit, while special attention is paid to friction at a highly eroded surface of bed and transport of bed load. The aim is to validate the formulas for inclined stratified flows with a stationary bed used in a layered model by Matoušek et al. (2018).

Methodology

Experiments were carried out in the laboratory of the Institute of Hydrodynamics in Prague. Experimental pipe loop used for this purposes is composed of two sections: horizontal and inclined pipe section. Tests were carried out with the 0.55mm narrow graded sand mixture flow. The electronic data acquisition system produced a large time-series of recorded data of all pressure differences, slurry mean velocity and slurry temperature, pump speed and engine power supply, concentration profiles and delivered concentrations. The predicted Cvd will be compared with the measured Cvd with the aim to validate formulas for transport and friction of bed for inclined pipe flow with a stationary bed used in the layered model by Matoušek et al. (2018).

The proposed extent of the thesis

40 pages

Keywords

solids transport; sheet flow; bed load; pipe experiment;

Recommended information sources

Krupička, J., 2014. Mathematical and Physical Modelling of Pipe Flow of Settling Slurries. Doctoral thesis, Prague

Matoušek, V., Krupička, J., Chára, Z., 2014. Stationary- and sliding beds in pipe flows of settling slurry. Proc. 15th Int. Freight Pipeline Soc. Symposium, Prague, Czech Republic, 24-26 June 2014

Matoušek, V., Krupička, J., Kesely, M., 2018. A layered model for inclined pipe flow of settling slurry. Powder Technol., 333, 317-326.

Meyer-Peter, E., Müller, R., 1948. Formulas for bed-load transport, Proceedings of the International Association for Hydraulic Research, Third Annual Conference, Stockholm, Sweden, pp. 39-64.

Pugh, F. J., Wilson, K. C., 1999. Velocity and concentration distributions in sheet flow above plane beds. ASCE J. Hydraul. Eng, 125(2), 117-125.

Sumer, B. M., Kozakiewicz, A., Fredsøe, J., Deigaard, R., 1996. Velocity and concentration profiles in sheet flow layer of movable bed. ASCE J. Hydraul. Eng, 122(10), 549-558.

Wilson, K.C., 1976. A unified physically-based analysis of solid-liquid pipeline flow, Proc.Hydrotransport 4, BHRA, Cranfield, UK, pp.1-16.

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Declaration

I declare that I have worked on my diploma thesis titled "The Effect of Longitudinal Slope on Solids Transport and Friction in Mixture Flow above Stationary Deposit in Pipe" by myself and I have used only the sources mentioned at the end of the thesis. As the author of the diploma thesis, I declare that the thesis does not break copyrights of any other person.

This thesis was carried out at Czech University of Life Sciences in cooperation with the Institute of Hydrodynamics of Czech Academy of Sciences.

In Prague on____________________ ____________________________

Abstract

This thesis presents results of the work analyzing the effect of the longitudinal slope of the top of a stationary deposit on solids transport and solids-based friction in flow carrying solid particles above the bed deposit in a pressurized pipe. The analysis is based on results of the experiments carried out in a 100-mm pipe loop with an inclinable U-pipe section in the laboratory of Institute of Hydrodynamics of Czech Academy of Science in Prague. Integral flow parameters and concentration profiles were measured in flows above deposit in the pipe installed to a broad range of inclination angles between -35 and +35 degree from horizontal.

In the literature, various versions of formulas can be found for solids (bed load) transport. However, all of them take the effect of the longitudinal slope insufficiently into account as they inherently expect an application to conditions where a variation in the flow slope is marginal. In this work, I employ the classical Meyer-Peter and Müller (MPM) transport formula for bed load and the bed friction formula derived recently for the intense transport condition. The aim is to use the measurement results for the observed flows including those at very steep inclinations to examine how the effect of the longitudinal slope should be implemented in the formulae to correctly predict the transport and friction of bed load. In addition, sensitivity analyses are done to show a detailed representation of the influence of changing input parameters on the output results.

Applications of the presented research results include sediment transport and morphology of mountain streams as well as settling slurry flows in inclined pressurized pipes where the transport and friction formulae are used in a layered model predicting the energy head loss and internal structure of the stratified slurry flow.

KEY WORDS: solids transport, sheet flow, bed load, pipe experiment

Abstrakt

Obsahem této diplomové práce je analýza vlivu podélného sklonu erodované sedliny na chod splavenin a odpor povrchu sedliny při proudění nesoucím pevné částice nad sedlinou v tlakovém potrubí.

Analýza je založena na výsledcích experimentů prováděných na trubní lince (vnitřní průměr potrubí 100 mm)s náklonou sekcí Ústavu pro hydrodynamiku AV ČR v Praze. Předmětem experimentů bylo měření integrálních veličin a koncentračních profilů v proudění nad sedlinou v potrubí nastaveném do různých náklonů v širokém rozmezí úhlů mezi -35 až +35 stupni od vodorovné polohy.

Literatura nabízí různé vzorce pro výpočet průtoku dnových splavenin nad erodovaným dnem. Všechny však berou nedostatečně v úvahu účinek podélného sklonu toku, protože předpokládají použití v případech, kdy jsou sklon a jeho změny relativně malé. V této práci byla použita klasická transportní rovnice Meyer-Petera a Müllera (MPM) pro chod dnových splavenin a nedávno odvozená třecí rovnice pro povrch dna , obě upravené pro podmínky intenzivního chodu dnových splavenin. Cílem je použít výsledky měření pro pozorovaná proudění, včetně těch při velmi strmých sklonech, k prozkoumání, jak by měl být účinek podélného sklonu zohledněn ve výše zmíněných vzorcích pro předpověď transportu a tření při intenzivním chodu dnových splavenin. Dalším cílem je provedení citlivostní analýzy vlivu měnících se vstupních parametrů na výsledky předpovědí.

Výsledky prezentovaného výzkumu lze použít při určování transportu sedimentů a vývoje morfologie horských toků, zejména za povodňových průtoků. Rovněž je lze použít při výpočtu proudění směsí v nakloněných tlakových potrubích, kde se transportní a třecí rovnice uplatňují v předpovědním modelu pro energetickou ztrátu třením a vnitřní strukturu stratifikovaného proudění směsi.

KLÍČOVÁ SLOVA: hydraulická doprava sypanin, chod splavenin, dnové splaveniny, trubní experiment

Content

List of Symbols

A cross-sectional area of pipe [m²]

B_s constant in friction law for hydraulically rough boundary

c local volumetric concentration

$C_{b,}$ mean spatial volumetric concentration in the bed

C_a mean spatial volumetric concentration in area above bed

C_{vd} mean delivered volumetric concentration

C_{vi} mean spatial volumetric concentration in entire pipe cross-section

d_{50} mass-median diameter of particle [m]

D inner diameter of pipe [m]

g gravitational acceleration [m/s²]

i hydraulic gradient

k_s equivalent roughness of the top of the bed [m]

L length of pipe [m]

O perimeter [m]

p pressure [Pa]

q_s volumetric transport rate for unit width [m²/s]

Q_s volumetric flow rate of solids [m³/s]

Q_m volumetric flow rate of mixture [m³/s]

Re Reynolds number

Re^* particle stress Reynolds number

Re_p particle Reynolds number

R_h hydraulic radius (m)

S relative density

S_m, S_s specific gravity of mixture, solids respectively [-]

u_b* shear velocity

U_s volume fraction of solids in mixture [m^3]

U_m total volume of mixture [m^3]

ΔU_s volume of solids delivered by a pipe during time period Δt [m^3]

ΔU_m volume of mixture delivered by a pipe during time period Δt [m^3].

V_m mean velocity of flow [m/s]

v_t dimensionless settling velocity of grain

w_t terminal settling velocity of solid particle [m/s]

y vertical position above bottom of pipe [m]

α empirical coefficient in solids transport formula

β empirical coefficient in solids transport formula

θ Shields parameter (dimensionless shear stress) for bed

κ Karman constant

λ Darcy-Weisbach friction coefficient

ν_f kinematic viscosity of fluid (m^2/s)

ρ density of fluid (kg/m^3)

τ shear stress at flow boundary (Pa)

Φ Einstein transport parameter

ω angle of pipe inclination

Subscripts:

a, b	area above bed, bed respectively
f, m, s	fluid, mixture, solids
fric, man	frictional, manometric
w	wall
ω	inclined

1. Introduction

1.1. Thesis Objective

Slurry transportation has big complexity due to mutual interactions between phases (liquid and solid), interactions between the phases and the flow boundaries (e.g. pipe wall) and other variables which can influence flow friction and solids deposition. Slurry flows, especially those exerting a certain degree of stratification, are very sensitive to changes in their inclination. The subject of the presented thesis is an investigation of two-phase flow composed of water and solid particles in a pipe. The objective is an analysis of how the angle of pipe inclination affects the frictional pressure drop in the flow above a stationary deposit, while special attention is paid to friction at a highly eroded surface of bed and transport of bed load. The aim is to validate the formulas for inclined stratified flows with a stationary bed used in a layered model by Matoušek et al. (2018).

The application is primarily in hydraulic transportation of solid materials by pipelines. This type of transport often contains inclined sections of various lengths and slopes which produce slurry flows of different characteristics. In order to improve this type of transportation, it is required to use reliable predictive models. Existing models for stratified slurry flows are not sufficiently validated. The effect of really steep inclination it is not taken into account and there is a lack of the experimental database.

The other application can be found in the appearance of highly concentrated two-phase flow above the eroded bed in open channels at steep slopes and large discharges. The bed load transport as one of the essential problems in sediment transport and morphology of mountain streams can be further analysed in more detail by adopting the results obtained in this work.

1.2. Aims of the Thesis

- Expansion of available database of the inner structure of settling slurry flows in inclined pipes.

- Contribution to the development of predictive models for settling slurry flows in pipes by validating existing formulas for calculation of bed load transport and bed friction.

- Evaluation and possible refinement of ways to determine the amount of particles contributing to the static pressure due to inclination in pipe flow with a stationary deposit.

1.3. Research Questions

1. Are the formulas used in the layered model for inclined settling slurry flows (Matoušek et al.2018) good enough and can they correctly predict transport and friction of bed load in inclined pipe flow with a stationary bed?

2. What is the right way of subtracting the static part of the total pressure drop in inclined slurry flow when a stationary deposit is present?

3. Can we rely on the experimentally determined thickness of the stationary bed as an input parameter for flow analyses and calculations although its determination is usually associated with a high degree of uncertainty?

2. Literature Review

The experimental results of characteristic parameters of the flow in inclined pressurized pipes have a big impact on the further development of the models used for predictions in engineering practice. In order to make good description and modelling of the effects of pipeline inclination on a slurry flow it is necessary to understand the development of the internal structure of slurry flow such as velocity and concentration distributions in a cross-section of a pipeline.

This thesis is focused on effects of the inclination on the behaviour of stationary bed at the bottom of the pressurized pipe and on the flow above the bed in order to validate formulas used in a layered model for transport and roughness of the bed and to make the analysis of sensitivity to input parameters. For this reason, the literature used for the thesis is based on previous work related to formulas obtained for this purpose.

Different studies showed that transport of solids (particularly transport of bed load) and bed friction are interrelated in flows at high bed shear. In order to analyze the mutual relation, it is essential to have information on the distribution of concentration (and if possible also velocity) in mixture flow. Such information is scarce.

Experiments made by Sumer et al. (1996) show that flow resistance in the bed-flow layer can be expressed in terms of the ratio of Nikuradse's equivalent sand roughness to the grain diameter, k_s/d with dependence on other parameters as shields parameter and parameter related to the fall velocity of sediment grains. The evidence that sediment transport is influenced by the turbulent process is also shown in the work.

Pugh and Wilson in their paper from 1999 gave results of experiments on particle-water flow over stationary beds. Analyzing the measured data of hydraulic gradient, discharge, concentration profiles, velocity profiles and delivered concentration they made the evaluation of the thickness of the transport layer and found its variation in proportion to the shear stress. Their analysis pointed out that the equivalent roughness of the top of the deposit, k_s, is not proportional to the particle size as that had been assumed in the past but it is proportional to the dimensionless shear stress, θ_b. Also, their tests showed a linear distribution of concentration across a shear layer and proposed an analytical solution of the general equation for the volumetric solids flow rate, for the flows with linear profiles of solid concentrations. They did not proceed to formulate a model for a prediction of the flow structure.

However, the results of flow experiments (Sumer et al. 1996., Pugh and Wilson 1999.) provide information about the distribution of solid concentrations, velocity of solids across a flow, as well as the thickness of a shear layer and enabled further determination of empirical coefficients α and β of solid transport formula of the Meyer-Peter and Müller (MPM) type.

A range of predictive models for slurry flows was developed during the years, but most of them do not take the presence of a deposit below the flow into account even such flows exist in practice.

According to Matoušek (2009), the transport formula of the MPM type proposed in that paper is validated by experimental data for different fractions and seems to be appropriate for a prediction the solids flow rate of contact load and combined-load flows at high shear stresses.

In order to provide a good predictive model for a case with inclined settling partially stratified flow, it is important to take important aspects of the inclination into account. The effect of pipe inclination on the degree of stratification into the layers of settling-slurry flow, the effect on the pressure drop and the minimum safe velocity have to be considered during the mathematical and physical modelling. A layered model is one of the modelling approaches for accounting the effect of pipe inclination, based on principles originally by Wilson (1976), further developed to different versions for fully- or partially stratified flow and adapted for inclined pipes.

Going back to the past, it is necessary to look at a widely used formula by Worster and Denny (1955). The formula by Worster and Denny quantifies the effect of an angle of pipe inclination on the total pressure drop using the parameter called solids effect, which is a difference in pressure gradients for slurry and carrying liquid at the same velocity in the same pipe. The concept of inclination effect by Worster and Denny is that inclined flow is seen as a combination of horizontal flow and vertical flow and the solids effect of slurry flow in the inclined pipe is suggested to be equal to the sum of the solids effects in the horizontal and vertical legs of the hypothetical triangle. Worster and Denny did not specify the range of formula application and did not verify it by experiments. Even this formula is widely used there were many disagreements about applicability. First problem because correlation assumes the same behaviour of flow in ascending pipe and in a descending pipe including the same distribution of solids in a pipe cross-section, which cannot be the case as it is already proven strong influence of inclination angles on shapes of measured distribution of solids. Another problem is that Worster and Denny neglected the difference between the average spatial volumetric concentration of solids, C_{vi}, and the averaged delivered concentration of solids, C_{vd}, but this difference is not negligible in stratified slurry flows and layered model recognize the difference between them. Apparently, the formula is based on the assumption not perfectly appropriate for partially stratified flows and needs to be modified.

Shook and Roco (1991) formed the force balance equation of a two-layered model for the inclined flow of partially stratified slurry many years later. Due to the density of slurry in each layer (slurry density because of the spatial volumetric concentrations of solids in the layer), the static pressure drop was developed in inclined flow. The model computational outputs were not verified by experiments.

Doron et al. (1997) proposed the extended model of the three-layer model of Doron and Barnea (1993) with account for the angle of inclination. The aim was to test the ability of a layered model to express the effect of inclination on pressure drop and the limit deposit velocity on stratified flow. Tests were done for such a narrow range of pipe inclinations (−7 deg. to +7 deg.) and did not include measurements of the distribution of solids in a pipe cross-section.

Matoušek (2011) proposed a predictive model for flow above the plane bed at high bed shear in a pipe. It predicts the slope of the energy grade line, i, and the thickness of the deposit, y_b, for flow carrying certain solids at certain V_m and C_{vd} in the pipe of D.

The model employs suitable formulae for solids transport above deposit (Matoušek 2009) and friction at the top of the deposit (Matoušek and Krupička 2014).

Recently, the formulae were applied in a modified form in a layered model proposed for inclined flows of settling slurries where solids distributions and corresponding flow parameters considerably vary with a pipe inclination angle (Matoušek et al. 2018). Usually, the model is employed to predict inclined stratified slurry flows with sliding beds and it was validated for such flows just for outputs (hydraulic gradients, concentration profiles) with experimental data from two laboratory loops (Institute of Hydrodynamics in Prague and Delft University of Technology). The particular formulas used in the model were not specifically validated and therefore that is the focus of this thesis.

3. Theoretical Background

3.1. Basic Properties of Mixture Flow

The flow of carrier liquid containing solid particles is labelled as a slurry flow. The slurry flow in a pipe can be characterized by several parameters whatever regime appears and regardless of the mechanism of particle motion. The average velocity of the mixture,V_m representing the velocity within the cross-section area, including stationary deposit at the bottom of pipe if exist and it is defined as Q_m/A. Volumetric flow rate, Q_m, is a sum of flow of solids, Q_s, and flow of water, Q_w. Presence of the solids in the liquid is influencing the density of the mixture, ρ_m, see Eq. (1). The fraction of solids in the mixture is usually determined by volumetric concentration, C_v. The place where is the fraction of solids determined in mixture flow can be different and hereof we need to differ spatial volumetric concentration of solids in the pipe, C_{vi} (2), when solids resident in an isolated part of a horizontal pipe and delivered volumetric concentration, C_{vd} (3), collected the discharged mixture at the outlet.

$$\rho_m = C_{vi} \cdot \rho_s + (1 - C_{vi}) \cdot \rho_f \tag{1}$$

$$C_{vi} = \frac{U_s}{U_m} \tag{2}$$

$$C_{vd} = \frac{Q_s}{Q_m} = \frac{\Delta \mathrm{U}_s}{\Delta t} = \frac{\Delta t}{\Delta U_m} \tag{3}$$

The energy dissipation in a steady slurry flow is characterized by the pressure difference along a pipeline section of constant diameter. When we consider a horizontal pipeline with a flowing mixture a total pressure drop over the pipeline section is equal to the pressure drop due to internal friction in flowing mixture. The hydraulic gradient or frictional head loss (4) is the head that is lost owing to friction and divided by the length of a pipeline section L:

$$i = \frac{\Delta P}{\rho f g L} \tag{4}$$

3.1.1. Flow Regimes

It is known that laminar and turbulent regime can appear in a carrying liquid in a pipeline. Laminar flow is flow composed of layers that move over each other with different velocities. Reynolds number defines a threshold when laminar regime tends to become turbulent and its value is 2300. When flowing carrying liquid is water it is hardly possible to have laminar regime. In a dredging pipeline due to kinematic viscosity about 10^{-6} m²/s and the diameter of a dredging pipeline with a typical value of 1m, velocity of a carrier can be around 2.3 mm/s at maximum to maintain a laminar regime of flow. In practice, in mixture dredging flows velocity is usually much higher than the threshold velocity for a laminar flow even for high concentrated non-settling mixtures and results are disturbances between neighbouring layers, which leads to flow with the turbulent regime which is typical for dredging pipelines and other industrial slurry pipelines.

3.1.2. Flow Patterns

Depending on the dispersive mechanism that keeps solid particles within the fluid flow, solid particles can be transported as a bed-load or as a suspended load. The mixture flow can be fully stratified, fully suspended and partially-stratified flow according to the degree of solid distribution across the pipe section.

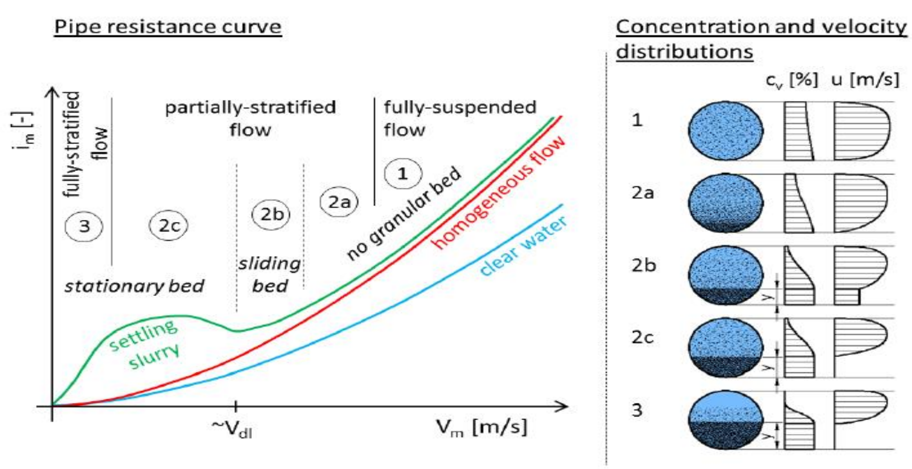


Figure 1: Concentration and velocity distributions on the right side correspond to regimes of settling slurry flow indicated above resistance curve. (Krupička 2014).

Fully stratified flow is the case where the solid particles form a contact load which can slide or be stationary at the bottom of the pipe and the turbulence of transport flow is not able to suspend any solid particle in a pipeline. The flow pattern completely opposite from previous one is fully suspended type of flow in which stationary or sliding bed does not exist and all solid particles are suspended within a transport flow. Third type and most common for the dredging operations are the partially-stratified flow which represents intermediate flow pattern. In this type of mixture flow, an accumulation of solid particles appears near the bottom of a pipeline while the rest of solids are non-uniformly distributed in the flow above the deposition bed. Slurry flow which is the subject of the investigation belongs to the partially-stratified type of flow.

3.1.3. Forces Governing Inclined Flow, Flow Friction

The driving force for flow in an inclined pressurized pipeline is a result of a gravitational force component in the flow direction and a force based on the pressure difference over a pipe length. As a result of pressure difference and gravitational force to the flow direction, we have a driving force which is balanced by the friction at a flow boundary.

The bed shear stress, i.e. the mean shear stress at the top of the stationary bed, is related to the frictional part of the total pressure drop in the flow. There, this pressure drop must be determined from the measured manometric pressure drop, $\Delta p_{\omega}^{fric} = \Delta p_{\omega}^{man} - \Delta p_{\omega}^{static} = \Delta p_{\omega}^{man} - C_a\left(\rho_s - \rho_f\right) g \sin\omega$, where C_a represents the concentration of solids contributing to the static part of the manometric pressure drop. It can be associated with C_{vi}, the mean concentration of the entire pipe cross-section area, as it is the case in inclined flow with sliding beds (inclined flow with a sliding bed in e.g. Matoušek et al. 2018), or associated with C_{via}, the mean concentration of the flow are above the bed only.

Calculation of shear stress at the flow boundary can be done using the Eq. (5), where $R_h = \frac{A_a}{O_a}$ is a hydraulic radius of the flow area:

$$\tau = \rho_f \cdot g \cdot R_h \cdot i \tag{5}$$

Evaluation of friction factor can be hard due to the presence of solids particles. The relation between shear stress and kinetic energy of flow via the friction factor, λ, enables calculation of this factor:

$$\lambda = \frac{8 \cdot \tau}{\rho_{m \cdot V_a^2}} \tag{6}$$

where V_a is the mean velocity of slurry flow above the stationary bed, $V_a = V_m \cdot \frac{A}{A_a}$.

<u>3.1.4. Forces Acting on Particles</u>

As long as acting forces (particle gravity force, drag and lift forces induces by the flow of fluid, buoyancy and reaction from grains in contact) are in equilibrium, particles in a granular bed will be stabilized (not moving). Shields parameter is a non-dimensional number for calculation of the initiation of motion of sediment in a fluid flow, see Eq. (7). Shields (1936) in his work derived a general criterion for incipient motion, where the critical value of the Shields parameter, θ_{cr} is defined by Eq. (8) and Re* is particle shear Reynolds number defined by Eq. (9).

$$\theta = \frac{\tau_b}{(\rho_s - \rho_f) \cdot g \cdot cos\omega \cdot d} \tag{7}$$

$$\theta_{cr} = f({R_e}^*) \tag{8}$$

$$Re_* = \frac{u_{b*} \cdot d}{\nu_f} \tag{9}$$

Shields explained a different behaviour of solid-liquid flow in four distinct regions. In the fully developed turbulent regime, the critical Shields parameter becomes constant with a value of approximately 0.03–0.06.

3.2. Transport of Bed Load

The relation between solids flow rate and bed shear stress is visible from transport formula. The dimensionless solids transport formula of the Meyer-Peter and Müller (1948) is often used for sediments transported as contact load:

$$\Phi = \alpha \cdot (\theta - \theta_c)^{\beta} \tag{10}$$

Expressed using the Einstein transport parameter Φ, Einstein, (1950) modified for the inclination angle ω:

$$q_s = \Phi \cdot \sqrt{(\mathrm{S} - 1) \cdot \mathrm{g} \cdot \cos\omega \cdot \mathrm{d}^3} \tag{11}$$

During the years the original formula was rewritten so many times in order to be more precise and include high bed-shear-stress conditions. Ribberink (1998) and Cheng (2002) gave results of their measurements for 0.7mm sand from Wilson (1966) as well and proposed respectively $\alpha = 12$, $\beta = 1.5$, and $\alpha = 13$, $\beta = 1.5$. The value of 1.5 for β has been broadly accepted as appropriate for the power-law exponent in a bed-load transport formula of the MPM type, while different formulas for high bed shear vary in a suggested value of the coefficient α.

The version of the MPM-formula used in this work has been obtained by integrating of a product of local velocities and concentrations of solids over the discharge area of a shear layer (Matoušek 2011) and it has been calibrated by a large number of experimental data for horizontal flows (Matoušek and Krupička 2014)

$$\Phi = \left(5.2 + \frac{39}{Re_p^{0.62}}\right) \cdot \theta^{\left(1.2 + \frac{2.6}{Re_p^{0.39}}\right)} \tag{12}$$

The particle Reynolds number:

$$Re_p = w_t \cdot cos\omega \cdot d_{50}/\nu_f \tag{13}$$

The sediment volumetric discharge per unit width of bed obtained from the above formulae can be also interpreted as:

$$q_s = C_a \cdot A_a \cdot V_a / O_b \tag{14}$$

3.2.1. Delivered Concentration - Definition and Determination

The flow rate of solids transported through a pipeline is an important parameter, as it gives the amount of dry solids (in volume or mass) delivered at the pipeline outlet over a certain time period. Delivered concentration is defined as the flow rate (either volumetric, Q_s, in m^3/s or mass in kg/s) of solids at the outlet of a slurry pipeline

$$C_{vd} = \frac{Q_s}{Q_m} \tag{15}$$

Determination of delivered concentration of solids in flowing slurry is possible by calculation from the pressure difference over the vertical measuring section. The method is based on simultaneous measurement of pressure drops in ascending and descending limb of vertical U-tube, as corresponding hydraulic gradients differ due to the effect of solids weight. The most simplified relation (16) was introduced by Clift & Clift (1981). Further on the system of equation was derived under the assumption that friction loses are not the same in the ascending and descending limb. Volumetric delivered concentration for the vertical section is calculated from pressure differences by Eq. (17). A more detailed explanation can be found in Krupička (2014).

$$C_{vd} = \frac{i^{\uparrow} - i^{\downarrow}}{2 \cdot (S_s - 1)} \tag{16}$$

$$C_{vd} = C_v^{\uparrow} \cdot \left[1 - \frac{V_t}{V_m} \cdot \left(1 - C_v^{\uparrow}\right)^m\right] = C_v^{\downarrow} \cdot \left[1 + \frac{V_t}{V_m} \cdot \left(1 - C_v^{\downarrow}\right)^m\right] \tag{17}$$

3.3. Bed Roughness Formula

Top of the deposit can be seen as a hydraulically rough boundary what therefore Nikuradse equation was applied for determining the k_s, equivalent roughness of the top of the bed:

$$\sqrt{\frac{8}{\lambda_b}} = \frac{1}{k} \cdot \ln(\frac{B_s \cdot R_{hb}}{k_s}) \tag{18}$$

According to theoretical considerations and empirical testing from the past, the provisional formula was proposed for the Eq. (19) as a function of the Shields parameter:

$$\frac{k_s}{d_{50}} = 1.7 \cdot \frac{W_s^{1.1}}{Fr_b^{2.3}} \cdot (\frac{R_{hb}}{d_{50}})^{0.32} \cdot \theta^{1.4} \tag{19}$$

in which W_{s*} is the dimensionless settling velocity of particles

$$W_s = \sqrt[3]{\frac{(\rho_s - \rho_f)}{\rho_f \cdot g \cdot cos\omega \cdot \nu_f}} \cdot w_t \cdot cos\omega \tag{20}$$

and Fr_b is bed Froude number

$$Fr_b = \frac{V_a}{\sqrt{R_{hb} \cdot g}} \tag{21}$$

A determination of the bed roughness, k_s, in layered models requires an appropriate representation of bed-shear parameters and at the same time sufficient simplicity of the k_s -formula to avoid problems with the model numerical solution. Based on extensive testing, a formula was suggested (Matoušek and Krupička 2014) that satisfied the contradicting requirements:

$$\frac{k_s}{d_{50}} = 1.35 \cdot W_{s*}^{0.5} \cdot \theta^{1.58} \tag{22}$$

4. Experimental Work

4.1. Materials

4.1.1. Particle Size and Its Distribution in Solids Fraction

A solid particle can be characterized by its size and shape. Size is given by a particle diameter and usually median particle size, d_{50} , is used as a characteristic dimension of non-uniformly sized particles. The particle size distribution of material (solid fraction) is defined by the method used for this purpose. Before starting usage of the material and starting the experiments, the grain size distribution was determined. First, the distribution was produced by the sieve method and then the sedimentation method was applied for particles which were not too coarse for this test.

4.1.2. Sieve Analysis

For our experiment, sieves were used as the most easily understood method. The process implies passing the material through a number of sieves of different mesh size in order to separate fine particles from larger ones. The set of sequentially decreasing sieves is posted on the vibrating machine. Under the motion, the particles with smaller dimensions to the sieve mesh opening are passing to the next sieve. The results are presented in a graph by distribution curve which shows the relation between the percentage of material and the corresponding diameters. The set of sieves for the sand used in our experiments was in range 0.063–0.9 mm as it is shown in Table 1. below.

Table 1: Results of sieve analysis of narrow-graded medium sand.

	Sieve opening [mm]	Sample I -weight [g]	Sample II -weight [g]	Sample I -cumulative mass	Sample II -cumulative mass	Sample 1 -cumulative %	Sample 2 -cumulative %
1	0.9	29	59	29	59	97.11	94.09
2	0.63	222	275	251	334	74.95	66.57
3	0.5	371	326	622	660	37.92	33.93
4	0.355	331	296	953	956	4.89	4.30
5	0.3	31	22	984	978	1.80	2.10
6	0.25	9	7	993	985	0.90	1.40
7	0.2	4	3	997	988	0.50	1.10
8	0.15	1	2	998	990	0.40	0.90
9	0.1	1	3	999	993	0.30	0.60
10	0.063	1	3	1000	996	0.20	0.30
11	<0.063	2	3	1002	999	0.00	0.00
		1002	999				

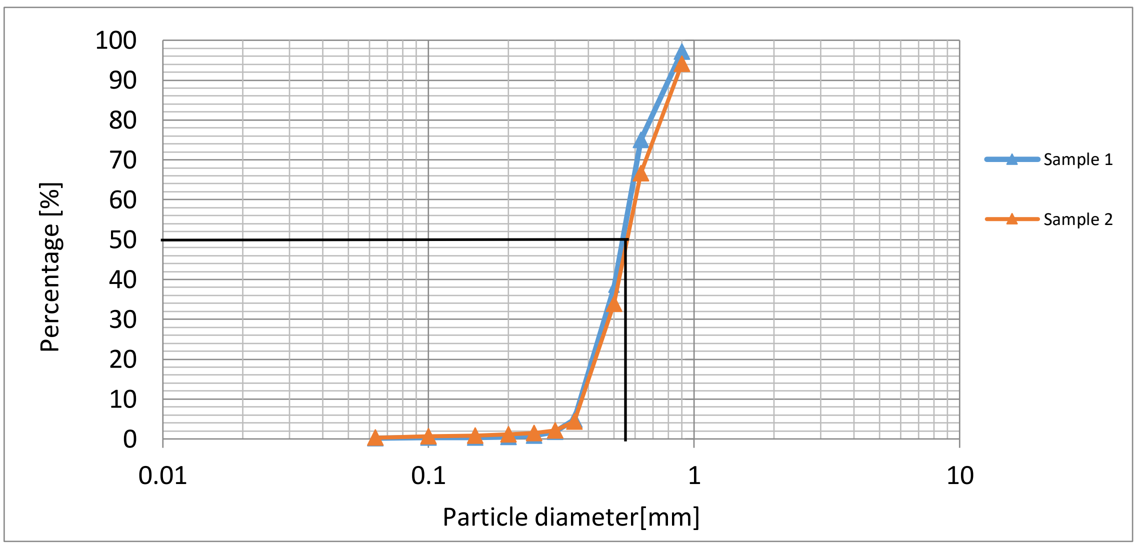


Figure 2: Particle size distribution by sieve method and determination of median particle size d_{50}.

The mass-median diameter d_{50} is the diameter which shows that half of the particles in a sample are larger and half of all particles are smaller than the size d_{50}.

4.1.3. Sedimentation Method

The sedimentation method is based upon a study of terminal velocity acquired by solid particles suspended in a liquid. Interaction between fluid and particle during its sedimentation is shown by settling velocity. When the particles were already sorted by the sieve method, sedimentation method was done. From each group, a cup of grains was poured into the water column and time period was measured until the particles reached the distance of 2m. Time was measured for the first and last particle of a dispersed cloud. A more detailed explanation can be found in Appendix I.

4.2. Description of Experimental Loop

Experiments were carried out in the laboratory of the Institute of Hydrodynamics in Prague. Experimental pipe loop used for this purposes is built of stainless steel, pipes with an inner diameter of 100 mm, composed of two sections: horizontal and inclined pipe section. The total length of the circuit was 52 m (basic loop), 93 m (extended loop). A schematic illustration of this loop appears in Figure 2. The most important part (Figures 2–3) of the loop was a section of pipe that could be inclined from 0 (horizontal) to 90 (vertical).

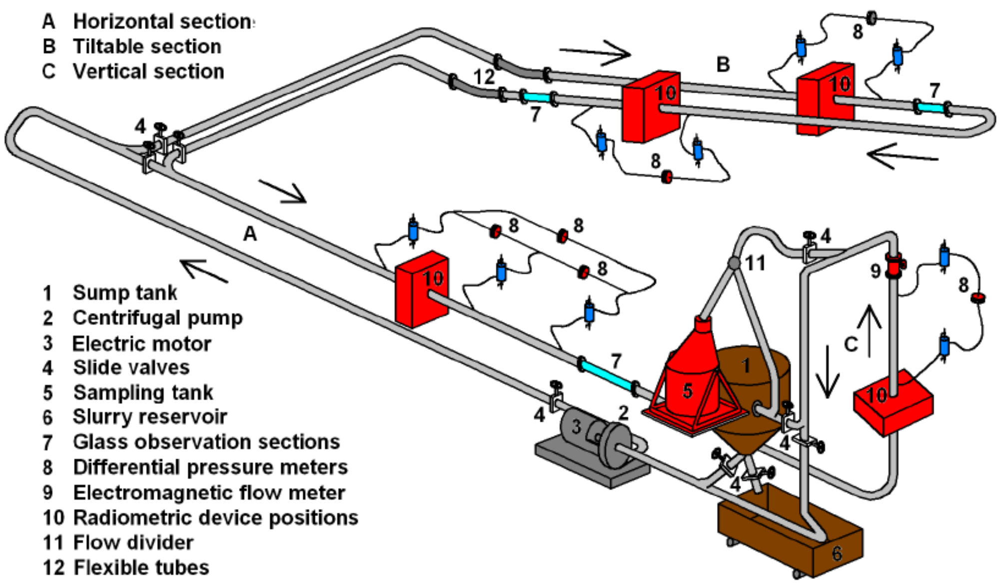


Figure 3: The test loop at the Institute of Hydrodynamics of Academy of Sciences of the Czech Republic (Krupička, 2014).

A mixture of the required concentration of solids with water used for these experiments was prepared in a sump tank. The flow was produced with a (GIW LCC-M 80-300) centrifugal pump powered by an electric motor (Siemens 1LG4283-2AB60- Z A11) with the maximum power of 90kW and a variable speed option.

Figures 4–5: The inclinable section of the experimental loop in the horizontal position (left) and inclined position (right).

In the ascending and descending legs of the U-tube, which are fully inclinable from horizontal to vertical, were located two-meter long measuring sections for differential pressure measurements. Pressure differences over the measuring sections were measured by the Rosemount differential pressure transmitters (model 1151DP).

In the same part of the pipe were positioned radiometric devices for measuring concentration profiles. Radiometric density meters composed of a gamma-ray source (Cesium 137Cs, activity 740 MBq) on one side and a detector with a NaI(Tl) scintillator on the opposite side of the pipe. One vertical measuring section 1.5m long was used for evaluation of delivered concentration of solid material.

The pipe had transparent sections made of glass for visual observation of flow. These sections were positioned behind the measuring sections. In order to prevent any movement on the glass part since else it will break, they were placed in a very sturdy metal frame and for additional safety protected with foil over the top.

Video cameras were mounted on the transparent section as it showed in Figure 5. The cameras were connected to a monitor so that flow through the section could be observed by the loop operator and the formation of a settled bed of stationary particles could be detected. The cameras were special Area Scan Cameras (colour, full HD (1920*1080) @ 25fps, the CMOS chip has a pixel size of 2.2*2.2 µm). They have a detachable lens with standardized C or CS-mount. The lens is a fixed focal at 8 mm, iris 1, 8-22, max. (Image circle 1/2.5").

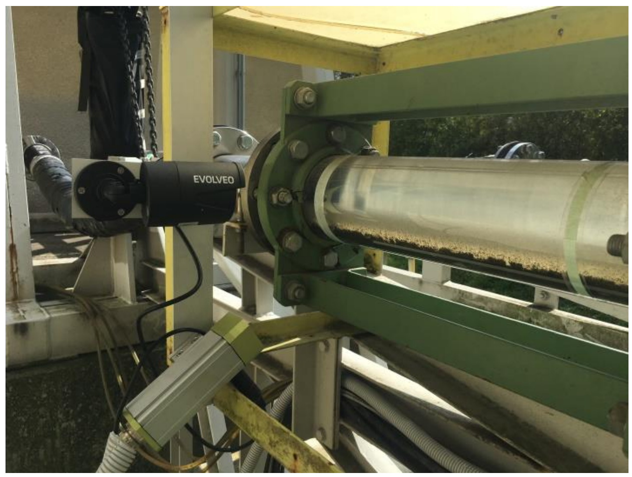


Figure 6: Video cameras for observation of a stationary bed mounted on the transparent section of the pipe.

Using the Krohne magnetic flow meter (model OPTIFLUX 5000 with the converter IFC 300 W) posted to the vertical section of the circuit the mean velocity was measured.

In order to maintain pressure taps free of bubbles and particles and to control calibration of pressure transmitters, the system of valves and tubes was used. Slide valves enable to bypass the inclinable U tube. The flow divider provides an accumulation of slurry samples in the sampling tank of the calibrated volume.

4.3. Description of Procedure

Tests were carried out with the 0.55 mm narrow graded sand mixture flow. The amount of narrow graded sand added to the water was 325kg for the first and 620kg for later tests. The slurry was let into the experimental loop by lifting the shaft which is closing the sump outlet. During the experiment the tank was by-passed. Using the shaft again to close the outlet at the end of the experiment all the solids were collected in the tank.

Flow rate data were collected and processed to database from the Krohne electromagnetic flow meter. Sampling tank was used periodically for the purpose of measuring errors estimation.

Transparent sections of pipe were used for visual observations of the position of the surface of a stationary deposit. The height of deposit, y_b, was read using the scale fixed to the pipe rim. Except from using visual observation through the transparent section, the height was possible to deduce from the shape of vertical concentration profiles measured using the radiometric devices.

Rosemount DP transmitters recorded time series of pressure differences for measuring sections where they were placed. To obtain hydraulic gradient in inclinable ascending and descending pipe of the U-tube, those differences were averaged and divided by the lengths of corresponding sections.

Calibration was made before and after each experiment session. Each time prior to the usage of the pipe system and pressure transmitters, the water column was used and, if necessary, valves were flashed in order to adjust the measuring devices. A detailed explanation of the standard procedure for clear water calibration which was performed can be found in Krupička et al. 2014.

4.3.1. Radiometric Device for Density Measurements

The radiometric method for sensing local densities is non-invasive and based on good experience (Matoušek, 1997). This method can be classified as a traditional one and because of this was selected for use in the test loop at the Institute of Hydrodynamics.

Constructions of both radiometric devices (Figure 5.) installed in the loop were the same. The main components are a gamma-ray source (Cesium Cs-137, activity) on one side and a detector on the opposite side of the pipe, which is converting photons of gamma-ray to showers of photons of lower energy. A digital analyzer counts detected photons, which invoke an electrical response in adjoining photomultiplier. Sufficient spatial resolution of the measurement is ensured by a lead plate with a wide opening used to collimate gamma-ray beam at the detector. Position of the source-detector can be different and shifted from below position to above pipe centre by a linear position drive providing in that way parallel projection data.

Results are local chord averaged densities (or concentrations) and concentration profiles composed of a set of local measurements at different vertical positions. The linear positioning drive is managed using the computer according to the laser sensor signal which gives the information about the position. Rotation around the pipe and acquiring parallel projections at different inclinations are possible thanks to radial positioning drive which support linear positioning drive to rotate. Radial positioning drive has to be handled manually.

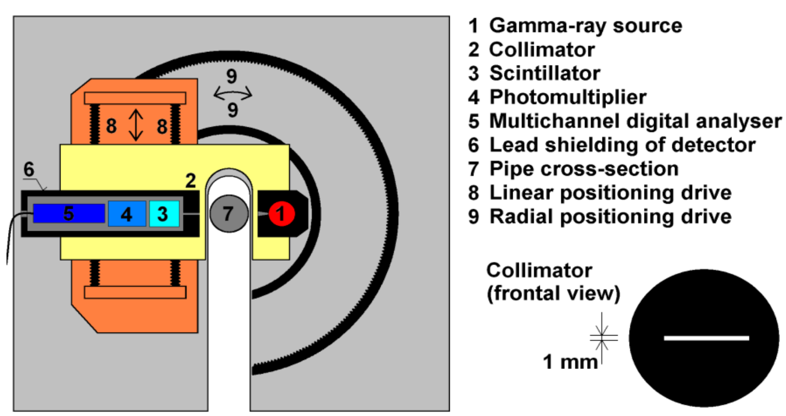


Figure 7: Scheme of radiometric device and main components (Krupička, 2014).

4.3.2. C_{vd} Determination

The method used for the experimental determination of the delivered concentration in the previous experiments was based on measurements of pressure drops in ascending and descending pipe of U-tube posted to the vertical position. In our experiments, the delivered concentration could not be based on measurements from the U-tube in the vertical position like in previous cases due to our focus on the influence of different inclinations on flow parameters. For this reason, another vertical part of the pipe loop had to be used. The results from these sections rely on appropriate calibrated equations as described below.

Pressure difference measured over some length in a vertical pipe is a sum of a friction loss and a hydrostatic part due to mixture column. Adopting the Eq. (23) for the equivalent-liquid model (where the hydraulic gradient of mixture, i_m, mixture relative density, S_m, and $C_v = C_{vd}$), Eq. (24) was derived for the volumetric concentrations. Further on using Eq. (25) delivered concentration was calculated from the volumetric one. A more detailed explanation can be found in Krupička (2014).

$$i_m - i_f = i_f \cdot A' \cdot (S_m - 1) = i_f \cdot A' \cdot C_{vd} \cdot (S_s - 1) \tag{23}$$

$$C_v^{\uparrow} = \frac{i^{\uparrow} - i_f}{\left(1 + A' \cdot i_f\right) \cdot (S_s - 1)} \tag{24}$$

$$C_{vd} = C_v^{\uparrow} \cdot \left[1 - \frac{V_t}{V_m} \cdot \left(1 - C_v^{\uparrow}\right)^m\right] = C_v^{\downarrow} \cdot \left[1 + \frac{V_t}{V_m} \cdot \left(1 - C_v^{\downarrow}\right)^m\right] \tag{25}$$

5. Analysis

5.1. Collection of Experimental Results

The electronic data acquisition system produced a large time-series of recorded data of all pressure differences, slurry mean velocity and slurry temperature, pump speed and engine power supply, concentration profiles and delivered concentrations. During the experiment, it was much easier to produce stationary bed condition in the ascending leg of the inverted U-tube than in the descending leg.

5.1.1. Integral Quantities

For each test run, integral flow quantities were obtained. Slurry flow mean velocity, V_m, and frictional pressure drop over the measuring section interpreted as the hydraulic gradient, i_m, were sensed electronically and recorded as a time series when the flow became stable. The slurry density as the average delivered concentration of solids was calculated from measured pressure differentials in the vertical part of the loop, as it is already explained. All data obtained directly during experiments were noted in a diary. In the table below its shown how experimental output data were written in the diary for flow on the inclination of 35 degrees:

Table 2: Diary with experiment output data: Hz—the power of the motor pump, Vm-mean flow velocity, T- temperature, Reg- regime in ascending (UP) and descending (DOWN) pipe of U-tube section.

SP3031_20180815_Inc35						
	Hz	Vm	T	Reg.UP	Reg.DOWN	Remark
1	21.8	3.49	28.3	SF	SF	vse v pohybu
2	21.2	3.28	28.7	SF	SF	vse v pohybu
3	20.5	3	29.4	SF,US	SF	kraticke zastavani,PROFIL
4	19.9	2.76	30.3	SF,US	SF	kraticke zastavani,PROFIL
5	19.4	2.51	30.6	SF,US	SF	UP 95/5,PROFIL
6	19.1	2.34	31.5	SF,US	SF	UP 80/20, DOWN vse v pohybu, PROFIL
7	18.8	2.22	32	SF,US	SF	UP 50/50, DOWN vse v pohybu,
8	18.6	2.15	32.2	SF,US	SF	UP 30/70 , DOWN vse v pohybu
9	18.4	2.01	33	UPB	SF	UP … DOWN SF
10	18.1	1.84	33.1	UPB	SF	UP … DOWN SF
11	17.8	1.7	33.2	UPB	SF	UP … DOWN SF
12	17.3	1.51	33.3	UPB	SF	UP … DOWN SF
13	16.6	1.27	33.4	UPB	SF	UP … DOWN SF
14	15.5	1.01	33.4	UPB	SF	PROFIL

5.1.2. Local Quantities

The radiometric device for density measurements provides data of chord averaged density and concentration profiles. The thickness of stationary deposit, y_b, was either observed visually in transparent sections of a pipe, by reading the scale glued to the pipe wall or deduced from the shape of a measured concentration profile (Figure 8).

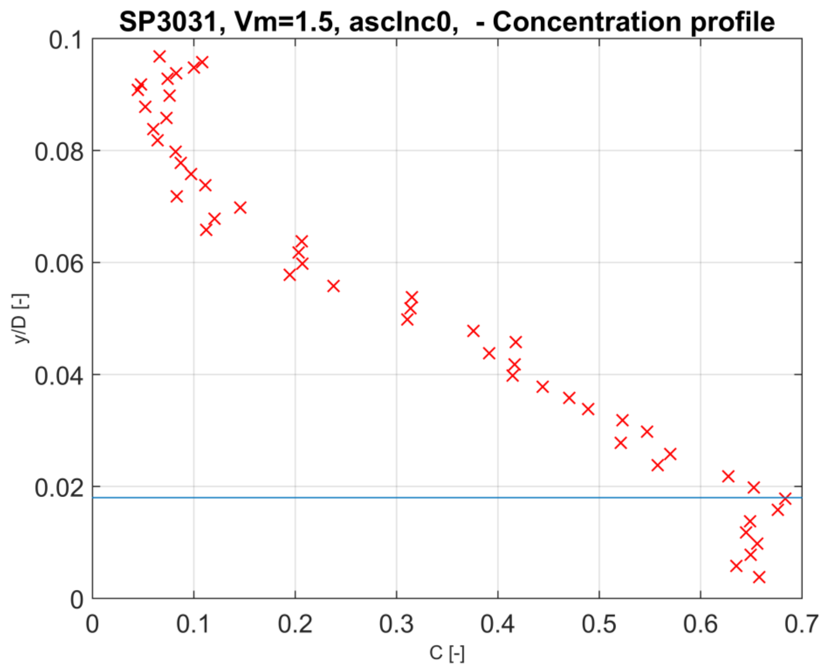


Figure 8: Concentration profile of slurry flow for at V_m = 1.5 m/s in horizontal position of the U-tube.

5.2. Processing Procedure for Experimental Results

To answer our research questions and to reach the goals of this thesis will be possible if we first get familiar with a layered model for inclined pipe flow of settling slurry (Matoušek et al. 2018). In order to validate formulas for the transport of bed load and for bed roughness used in this layered model, we need to look at the procedure for processing all data obtained from experimental runs for flow with stationary bed on the bottom of the pipe.

The inputs to the model are pipe parameters (the internal diameter, the angle of inclination of the pipe, the wall roughness), properties of the liquid and solids, the velocity of slurry flow, V_m, measured pressure gradient, i_m, the spatial volumetric concentration, C_{vi} and solids concentration in the contact layer.

It is important to mention that experiments were done with the steady-state uniform fully developed flow, which means that the longitudinal angle of the top of the bed was equal to the inclination angle of the pipe itself. Simply by introducing the gravitational acceleration component normal to the surface of the inclined bed, formulas were modified for conditions in inclined flow. The effect of inclination was included in all parts where the gravitational component is present. In the following paragraphs, you will be able to see how this is employed in the Einstein formula, the Shields parameter formula and in the formula for particle Reynolds number.

Concentration profiles enabled us to choose the position of the top of the bed. This gave us a possibility to determine the shear condition at the top of the bed and Shields parameter which was further used in the transport formula in order to predict C_{vd}.

The predicted C_{vd} will be compared with the measured C_{vd} with the aim to validate formulas for transport and friction of bed for inclined pipe flow with a stationary bed. The purpose of the next paragraphs is to give an overview of the used model. All formulas are already given in the previous chapter on the theoretical background.

5.2.1. Flow Pattern

Slurry flow which is the subject of our investigation belongs to the partially-stratified type of flow, with a stationary bed at the bottom of the pipe and the rest of solids non-uniformly distributed in the flow above. Contrary to standard open channel flow, pipe flow above a mobile bed is significantly affected by the 'side-wall effect', i.e. by the presence of the pipe wall. The cross-sectional area of the flow above the bed must be split into two parts, one associated with the top of the bed and the other with the pipe wall (Figure 2). The boundary between the two sub-areas crosses the position of the maximum local velocities in the central vertical of the flow cross-section.

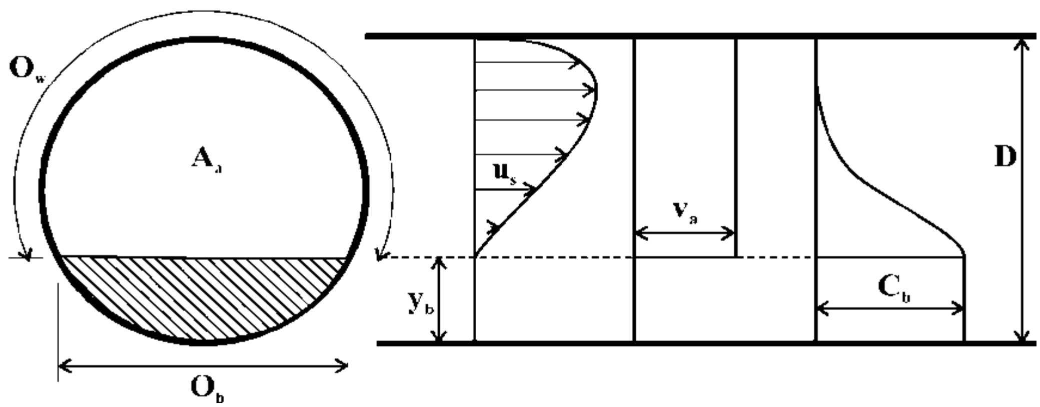


Figure 9: Schematic distribution of velocity and concentration in the flow above the stationary bed (Matoušek 2011).

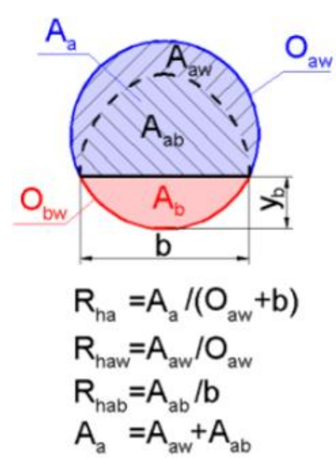


Figure 10: Schematic geometry of pipe cross section (Matoušek 2011).

5.2.2. Frictional Hydraulic Gradient

The angle of inclination affects the bed velocity especially due to an action of the longitudinal component of the bed submerged weight induced by the pipe incline. In positively inclined flow this force for the bed acts as an additional resisting force and tends to decrease the bed velocity compared with the horizontal flow, while in the negatively sloped flow, the weights have power as an additional driving force which makes bed slides faster than horizontal flow.

Attention needs to be paid to the process of including the effect of the bed submerged weight in the calculations. With concerns that in our experiments we are looking at a stationary deposit and since particles in the bed are not moving, they are bound to the pipe wall and so they should not contribute to the static part of the total hydraulic gradient. Hence, calculation of the frictional hydraulic gradient, i, was done by subtracting only the weight of particles contributing to the spatial volumetric concentration, C_{via}, just in the area above the stationary deposit (26) instead of the mean spatial volumetric concentration for the entire pipe cross-section, C_{vi}. The process of calculation of the hydraulic gradient considering a contribution of all particles and the process of calculation of the hydraulic gradient by considering just particles above the stationary bed is shown below in formulas 27–30:

$$C_{via} = \frac{Cvi \cdot A - Cvb \cdot A_b}{A_a} \tag{26}$$

$$\rho_{cvi} = \rho_f + C_{vi} \cdot (\rho_s - \rho_f) \tag{27}$$

$$\rho_m = \rho_f + C_{via} \cdot (\rho_s - \rho_f) \tag{28}$$

$$i_{cvi} = i_{man} - \frac{\rho_{cvi} - \rho_f}{\rho_f} \cdot sin\omega \tag{29}$$

$$i = i_{man} - \frac{\rho_m - \rho_f}{\rho_f} \cdot sin\omega \tag{30}$$

5.2.3. Friction at The Surface Of The Stationary Deposit

The degree of solids stratification and the deposition velocity are both affected by the pipe inclination. Distributions of solid particles in a slurring flow together with a distribution of velocity across the flow cross-sectional area transform an internal structure of the flow and have a big influence on the flow friction. Two different velocities appearing in the flow above the bed and in the bed itself produce shearing of the top of the bed and development of the shear layer. Further intense development of the transport layer above the plane bed and washing out of possible bed forms appears due to high bed shear stress.

Two different values of hydraulic roughness can be recognized due to the existence of two different boundaries in the cross-sectional area of a partially-stratified flow. Hence, there is a difference between the hydraulic roughness of the pipe wall and the roughness of the top of the bed.

Assumptions were made for the calculation procedures:

-the same average velocity in sub-areas of A_a ($V_{aw} = V_{ab}$)

-the same hydraulic gradient in both sub-areas of A_a (i_{aa}==i_{ab})

-no solids effect on wall friction

-no interfacial interaction at the boundary between the two sub-areas

5.2.4. Calculation Procedure for Determination of Bed Shear Stress from Experiments

This chapter gives a short overview of a calculation procedure for the shear stress at the top of the bed as employed in the model. The procedure starts with a determination of τ_a, shear stress for the area above the bed layer (31), where R_{ha} is hydraulic radius also just for the area above stationary deposit calculated as $R_{ha} = A_a/O_a$. Next step was a calculation of the hydraulic radius of subarea associated with pipe wall, where formula (32) is for a smooth pipe and x and y are calibrated as in the Blasius equation, x = 0.316, y = 0.25. The further calculation produces the friction coefficient for the pipe wall, λ_w (33) and for the top of the bed, λ_b (34). Moreover, the bed shear stress τ_b, (35) is determined from the calculated friction coefficient for the top of the bed.

$$\tau_a = \rho_f \cdot g \cdot R_{ha} \cdot i, \qquad \lambda_a = \frac{8 \cdot \tau_a}{\rho_m \cdot V_a^2} \tag{31}$$

$$R_{hw} = \frac{1}{4} \cdot \left(\frac{x \cdot \upsilon_f^y \cdot V_a^{2-y}}{2 \cdot g \cdot i}\right)^{\frac{1}{y+1}} \tag{32}$$

$$\lambda_w = \frac{8 \cdot g \cdot i}{V_a^2} \cdot R_{hw} \tag{33}$$

$$\lambda_b = \frac{\lambda_a \cdot O_a - \lambda_w \cdot O_w}{O_b} \tag{34}$$

$$\tau_b = \frac{\lambda_b}{8} \cdot \rho_m \cdot V_a^2 \tag{35}$$

5.2.4.1. Shields parameter

Particle mobility is evaluated by a value of the bed Shields parameter determined using Eq. (7). The effect of inclination is taken into account for the Shields parameter through bed shear stress and the cosine term that represents the frictional contribution to the total solids effect.

$$\theta = \frac{\tau_b}{(\rho_s - \rho_f) \cdot g \cdot cos\omega \cdot d} \tag{7}$$

5.2.4.2. Validation of bed friction formula

Eq. (22) was used for validating the bed roughness (18) as it was explained before in the chapter on the theoretical background:

$$\sqrt{\frac{8}{\lambda_b}} = \frac{1}{k} \cdot \ln(\frac{B_s \cdot R_{hb}}{k_s}) \tag{18}$$

$$\frac{k_s}{d_{50}} = 1.35 \cdot W_{s*}^{0.5} \cdot \theta^{1.58} \tag{22}$$

5.2.4.3. Transport of Bed Load

For our calculations for intense transport of solids above the bed used equation was given by Matoušek (2009) as it is already explained in the theoretical background chapter:

$$\Phi = \left(5.2 + \frac{58}{Re_p^{0.62}}\right) \cdot (\theta - \theta_c)^{1.2 + \frac{1.3}{Re_p^{0.39}}} \tag{12}$$

In the formula for Particle Reynolds number, the gravitational acceleration component normal to the surface of the inclined bed is included as an influence on settling velocity.

$$Re_p = w_t \cdot cos\omega \cdot d_{50}/\nu_f \tag{36}$$

A value of C_{vd} can be predicted from a predicted value of Φ by using Eq. (11) and considering $C_{vd} = q_s O_b / Q_m$.

6. Discussion of Analytical Results

After explaining how all data were obtained from experimental work and after explaining the procedure for processing the data using formulas of the layered model, now we can look at the results we have got systematically applying the above-mentioned procedures. Before starting any calculation, an additional check was made to prove that our procedure is physically correct. For each test, the ratio C_{via} over C_{vd} was verified to be bigger than 1. When this step has been completed, we are now ready to look carefully and do a closer inspection of all gained results.

As it is already stated, the thickness of the stationary deposit, y_b, is an important input parameter to an evaluation of flow conditions from our measurements. It is usually determined visually in the transparent section of pipe and also, the top of the deposit can be deduced from a shape of a concentration profile.

I applied both ways of determination of y_b to the results of our test runs and noticed that a predicted value of C_{vd} is extremely sensitive to the chosen position of the top of the bed, y_b. An experimental determination of y_b is associated with the highest uncertainty from all measured parameters. Earlier observations revealed that there might be a considerable difference between the position determined from a shape of the measured concentration profile and from the position observed visually in the clear section of a pipe (Matoušek et al. 2014). Therefore, I decided to drop the use of experimental y_b as an input parameter to the procedure. Instead, I employed the transport formula in an inverse procedure serving to set an appropriate position of y_b and shear conditions at the interface.

For each run, I found a combination of suitable values of y_b and of the corresponding mean concentration of solids determining the static part of the pressure drop (based on C_{via} resulted from the shape of the profile and y_b) for which the transport formula predicted a C_{vd} value very similar to the measured one.

6.1. Horizontal Pipe Measurements and Predictions

The use of the previously explained procedure was justified by a very good agreement between the measured and predicted C_{vd} in flow at 0 degrees, where no static part had to be subtracted (see Figure 11). The y_b-position at which the satisfactory agreement was reached corresponded with the position detected at the shape of the profile (see Figure 12).

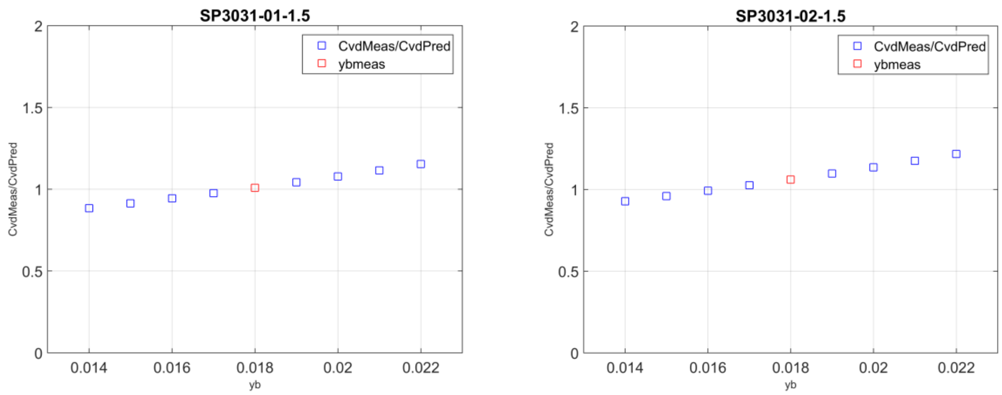


Figure 11: Ratio between $CvdMeas$ and $CvdPred$ for the flow at V_m = 1.5 m/s in ascending (left) and descending (right) section of the U-tube in horizontal position and estimated position of top of stationary bed, y_{meas}.

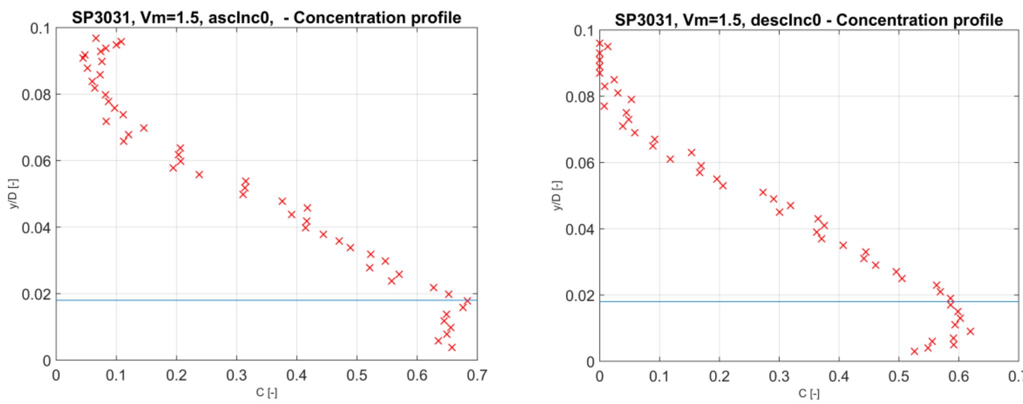


Figure 12: Concentration profiles for the flow at V_m = 1.5m/s in ascending (left) and descending (right) section of the U-tube in horizontal position and estimated position of top of stationary bed, y_{meas}.

6.2. Inclined Pipe Measurements and Predictions

As a result of a change of a pipe elevation, the hydrostatic pressure develops and the total pressure drop changes. Measured manometric pressure differential enables us to obtain the pressure differential due to friction in order to determine the shear condition at the top of the bed. The pressure differential due to friction can be obtained from the measured manometric pressure drop by subtracting the static pressure differential as it was explained previously. To find out which part of particles is actually included in the static part, first tests were conducted with the contribution of all particles, C_{vi}. It was much easier to produce a stationary bed condition in the ascending leg of the inverted U-tube. At inclination angles steeper than 20 degrees, it was virtually impossible to reach the stationary bed condition in the descending pipe. Not only because of the weight of grains and gravitation but also due to more intense accumulation of grains in the ascending leg of the U-tube.

6.2.1. Delivered Concentration Predictions

Results for predictions of delivered concentration when all particles in the pipe cross-section, C_{vi}, (particles occupying stationary bed and particles transported in the area above) are contributing to the static pressure due to inclination, showed that for the slurry mean velocity 1 m/s, friction values are negative for all test inclinations. Prediction of C_{vd} for the velocity of 1.5 m/s gave a quite different value from the measured one. Graphs below present predicted and measured value of C_{vd} for a range of thickness deposit bed in order to show a difference in results and to prove what was stated before. If we observe results for flow in a descending pipe, then it can be noticed that the model overestimates, while for the flow in the ascending pipe the results show that the model underestimates the value of the delivered concentration:

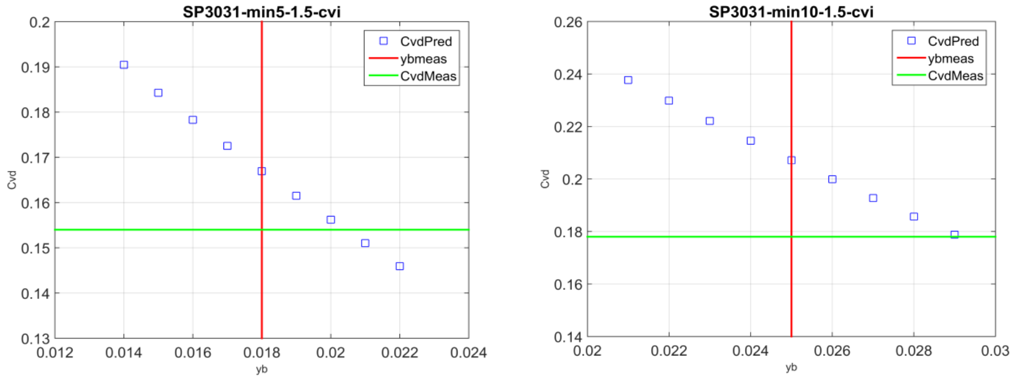


Figure 13: Predicted Cvd for the flow at V_m = 1.5 m/s in descending pipe inclined 5 degrees (left) and 10 degrees (right) with estimated position of top of stationary bed, y_{meas} and measured Cvd.

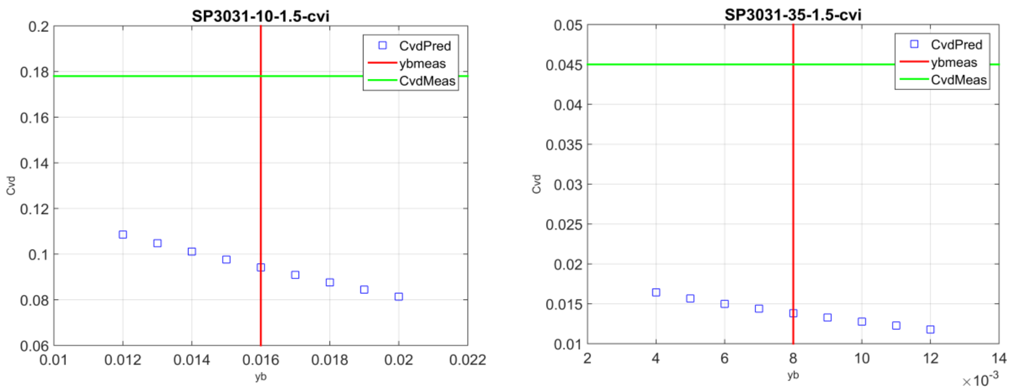


Figure 14: Predicted Cvd for the flow at V_m = 1.5 m/s in ascending pipe inclined 10 degrees (left) and 35 degrees (right) with estimated position of the top of the stationary bed, y_{meas} and measured Cvd.

With concerns that in our experiments we are looking at a stationary deposit and since particles in the bed are not moving, it was decided to exclude the submerged weight of the stationary bed. The procedure of getting pressure differential due to friction was done using obtained spatial volumetric concentration just for the area above the stationary deposit (C_{via}) instead of mean spatial volumetric concentration for the entire pipe cross-section, C_{vi} (case for sliding bed).

When calculations were done with the participation of particles above the stationary bed only, it became obvious from the results that it helped to avoid negative values for some inclinations of flow with the velocity of 1 m/s, what is shown on Figure 15. Predictions of C_{vd} for mean velocity of 1.5 m/s were closer to the measured value compared with the previous procedure, especially for the flow in descending pipe, while ascending pipe predictions were still not satisfactory.

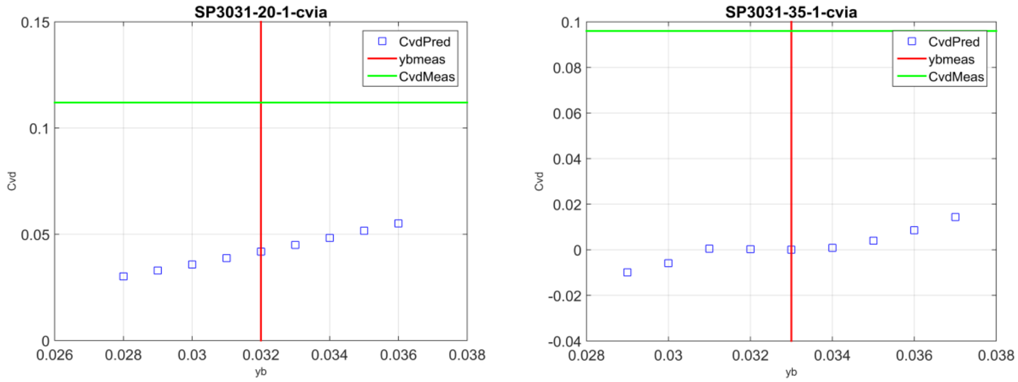


Figure 15: Predicted Cvd for the flow at V_m = 1 m/s in ascending pipe inclined 20 degrees (left) and 35 degrees (right) with estimated position of the top of the stationary bed, y_{meas} and measured Cvd.

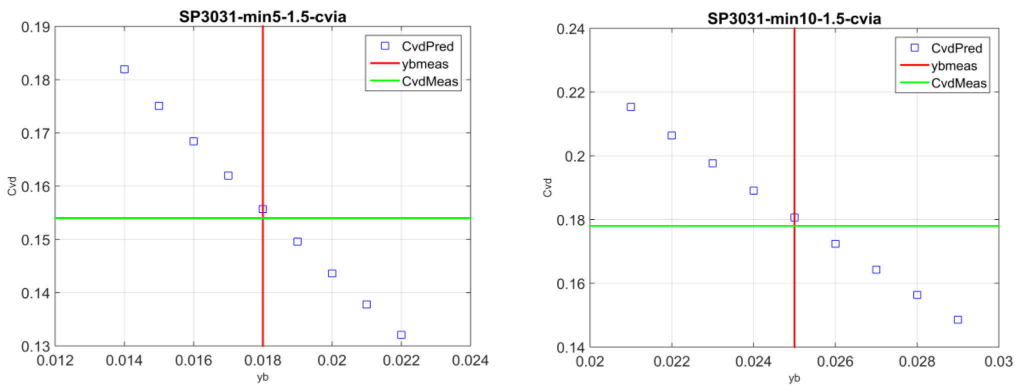


Figure 16: Predicted Cvd for the flow at V_m = 1.5 m/s in descending pipe inclined 5 degrees (left) and 10 degrees (right) with estimated position of the top of the stationary bed, y_{meas} and measured Cvd.

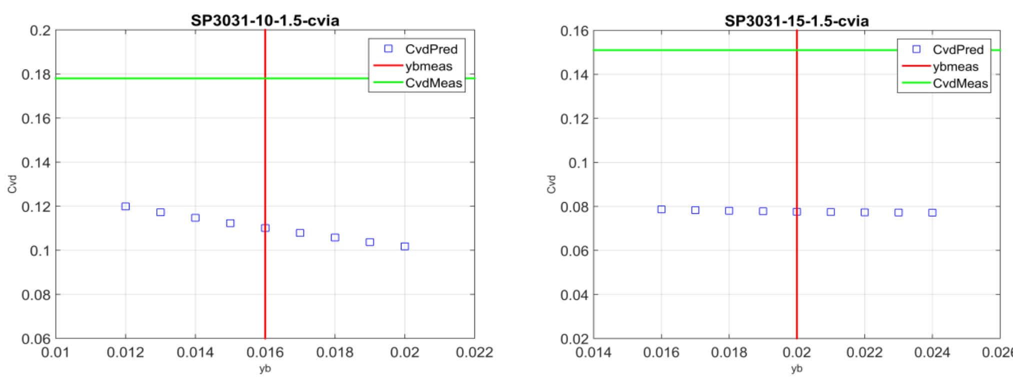


Figure 17: Predicted Cvd for the flow at V_m = 1.5 m/s in ascending pipe inclined 10 degrees (left) and 15 degrees (right) with estimated position of the top of the stationary bed, y_{meas} and measured Cvd.

The obtained results indicated that the problem remained as we did not get the result that we expected. Actually, the procedure revealed that concentration smaller than C_{via} should be used to determine the static pressure drop.

As our procedure required that we found a combination of suitable values of y_b and of the mean concentration of solids determining the static part of the pressure drop for each run, firstly values of the bed thickness had to be fixed. The values of the bed thickness were estimated from shapes of concentration profiles. The mean concentration could not be fixed, and it was decided to alter its value with a correction factor for the mean concentration.

With a goal to use a smaller concentration than C_{via} for the static pressure drop, the empirical correction factor, C_{corr} was introduced. A value of C_{corr} was applied to modify the concentration in the process of calculation of the hydraulic gradient in Eq. (30) with as a modification criterion very similar values of predicted and measured C_{vd}. An implementation of the factor can be seen in formula below, Eq. (37).

$$\rho_m = \rho_f + \boldsymbol{C_{corr}} \cdot C_{via} \cdot (\rho_s - \rho_f) \tag{37}$$

$$i = i_{man} - \frac{\rho_m - \rho_f}{\rho_f} \cdot sin\omega \tag{30}$$

Values of the correction factor were found for the range of bed thicknesses (estimated y_b) for the applied criterion of C_{vd}-predicted very close to C_{vd}-measured.

Unfortunately, this set of data was not enough to establish a trend in choosing the correction factor or to provide some table with fixed values of the factor. The values of the correction factor produced a considerable scatter and did not indicate any clear trend (see Figures 18–19). On the other hand, the values remained close to unity showing that a small correction already led to satisfactory results.

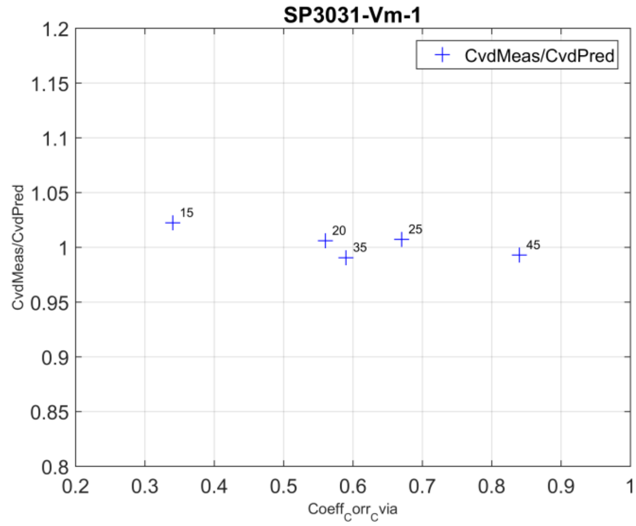


Figure 18: Correction factor for the flow at V_m = 1 m/s for all inclinations tested in the experiments

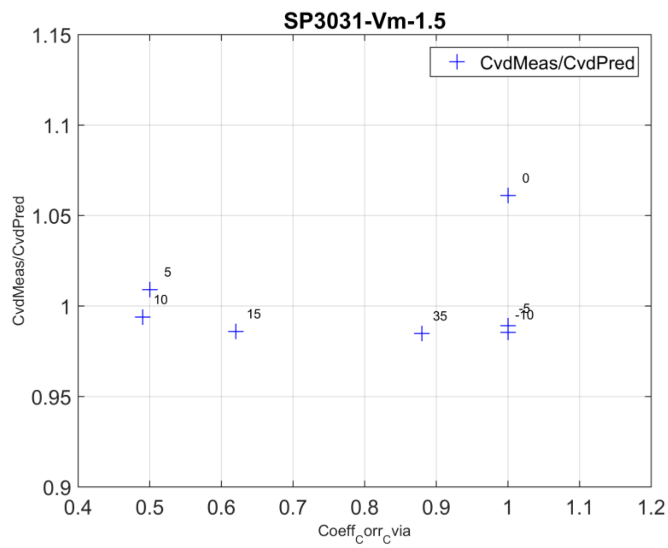


Figure 19: Correction factor for the flow at V_m = 1.5 m/s for all inclinations tested in the experiments.

6.2.2. Roughness Measurements and Predictions

After going through prediction of delivered concentrations for all test inclinations, choosing the position of the top of the bed, y_b, and a value of the correction factor, the next step is a validation of the roughness formula used in the model. The experimentally determined equivalent roughness for the log law in Eq. (37) is compared with the predicted roughness from Eq. (38) in Figures 21 and 22:

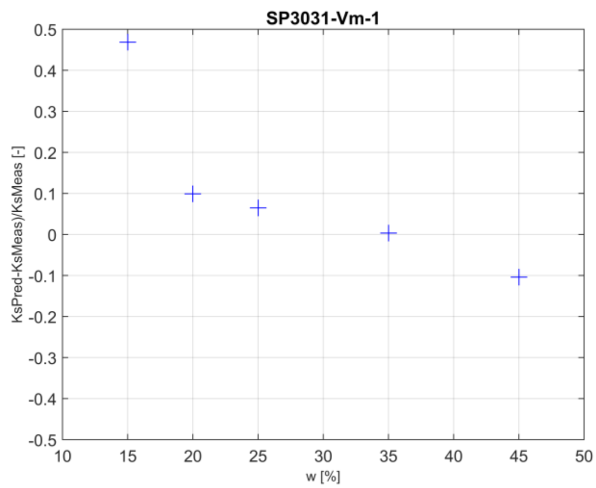


Figure 20: Relative deviation of bed roughness predicted ($KsPred$) from bed roughness obtained by experiment ($KsMeas$), flow at V_m = 1 m/s.

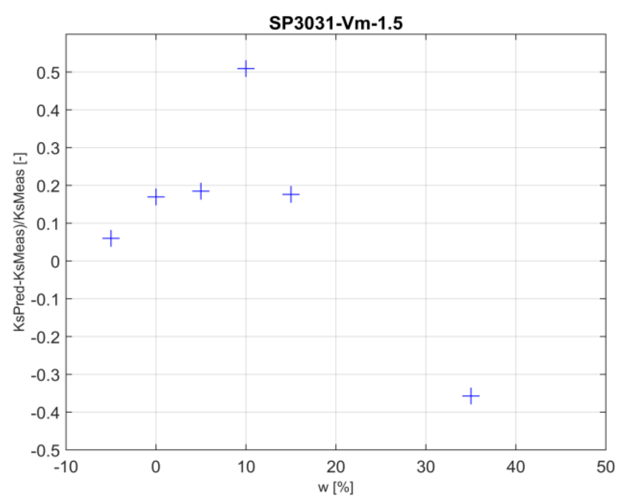


Figure 21: Relative deviation of bed roughness predicted ($KsPred$) from bed roughness obtained by experiment ($KsMeas$), flow at V_m = 1.5 m/s.

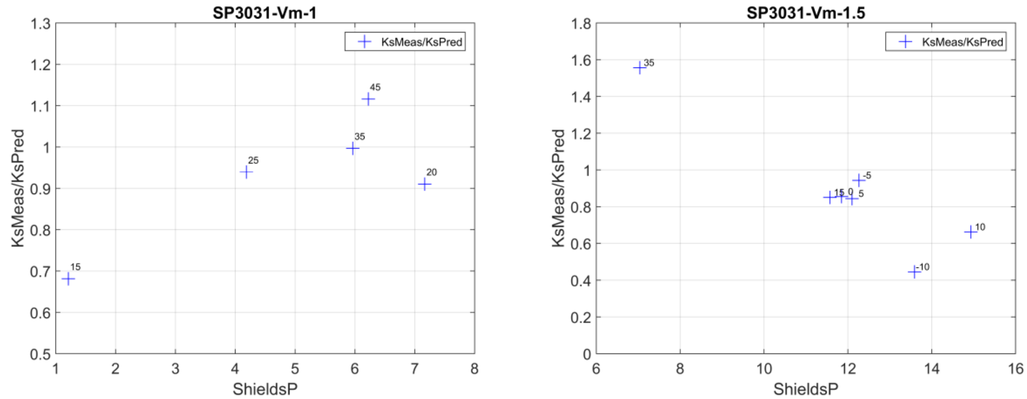


Figure 22: Ratio of bed roughness predicted and measured for the flow at V_m = 1m/s (left) and V_m = 1.5 m/s (right) with changes of Shields parameter for the all inclination tested in the experiments.

The agreement is very reasonable for the entire range of available flow conditions, including a broad range of angles of flow inclination from −5 to +45 degree. Based on this comparison, it can be concluded that for the tested flows the effect of flow inclination is sufficiently captured in the formulae used in the layered model for inclined flows.

A more general conclusion must be based on a broader data set including flows of different delivered concentrations and different solids fractions. A collection of such a data set is future work currently.

6.2.2.1. Comparison of Roughness Predictions Using Different Formulas

In the chapter on the theoretical background, we were discussing the bed roughness formulas and indicated that we will use the formula of Eq. (19): $\frac{k_s}{d_{50}} = 1.35 \cdot W_{s*}^{0.5} \cdot \theta^{1.58}$ (Matoušek and Krupička 2014) instead of Eq. (22): $\frac{k_s}{d_{50}} = 1.7 \cdot \frac{W_s^{1.1}}{Fr_b^{2.3}} \cdot (\frac{R_{hb}}{d_{50}})^{0.32} \cdot \theta^{1.4}$, because it is a simpler form helping to avoid problems with the model numerical solution. In this subsection, we will compare results gained with both formulas, proving the validation of our choice.

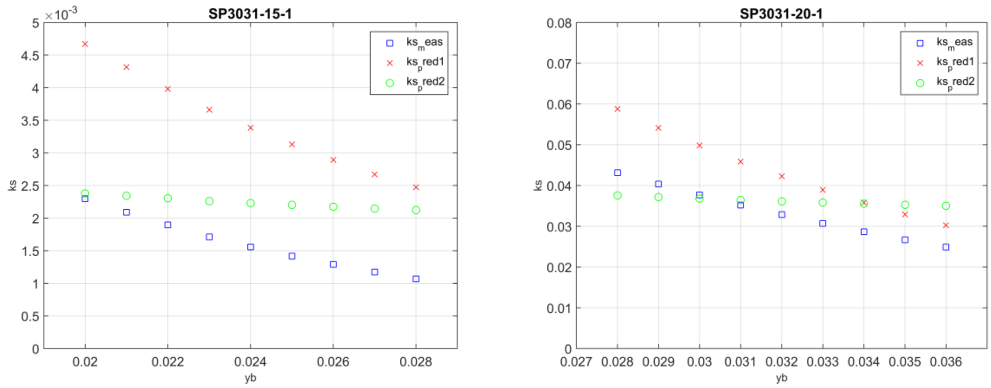


Figure 23: Bed roughness-measured (blue □), prediction formula 19 (red x) and prediction formula 22 (green o) for the flow at V_m = 1 m/s with a range values of bed thickness for inclination 15 degrees (left) and 20 degrees (right).

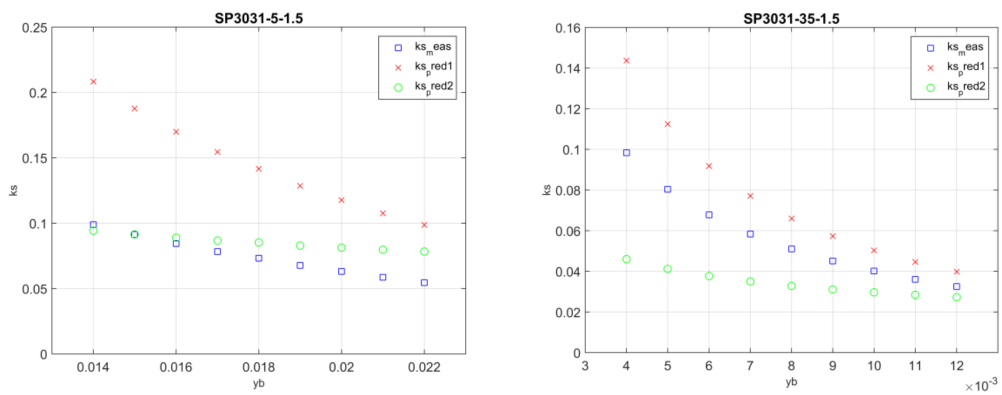


Figure 24: Bed roughness-measured (blue □), prediction formula 19 (red x) and prediction formula 22 (green o) for the flow at V_m = 1.5 m/s with a range values of bed thickness for inclination 5 degrees (left) and 35 degrees (right).

As we can see from the presented figures, almost in all different inclinations of pipe for two mean velocities, formula Eq. (22) is giving values closer to the experimentally acquired values than formula Eq. (19). More figures can be found in Appendix V.

6.2.3. Sensitivity Analysis

The posted question at the beginning of this research was if we could rely on the thickness of the stationary bed as an input parameter as it is usually determined with high uncertainty. In previous chapters, the procedures were given for validating formulas used in the layered model for inclined flow, including the procedure for finding the best way to determine the position of the top of the stationary bed. In this chapter, we will try to look deeper into this problem and to give some advice for future work.

In order to find at which condition all particles above stationary bed contribute to the static pressure in our experimental runs, a fixed value of correction factor to 1 and changed the thickness of the bed in order to get C_{vd}-predicted very close to C_{vd}-measured. The next step was to compare results obtained with chosen and fixed correction factor in order to see how big the change in the position of the stationary deposit is.

On the left side of Figures 25 and 26, the graphs show the thickness of bed obtained with the variable correction factor in order to get C_{vd}-predicted = C_{vd} measured, while right graphs show the results for the fixed correction factor equal to 1 and the position of the bed selected to get the same C_{vd} predicted and measured.

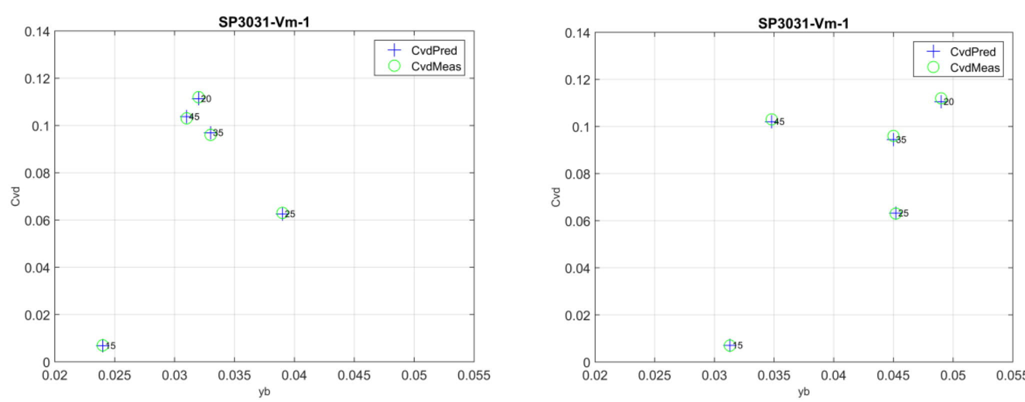


Figure 25: Range of bed thickness values with variable correction factor (left) and fixed correction factor to 1 (right) at mean velocity V_m = 1 m/s.

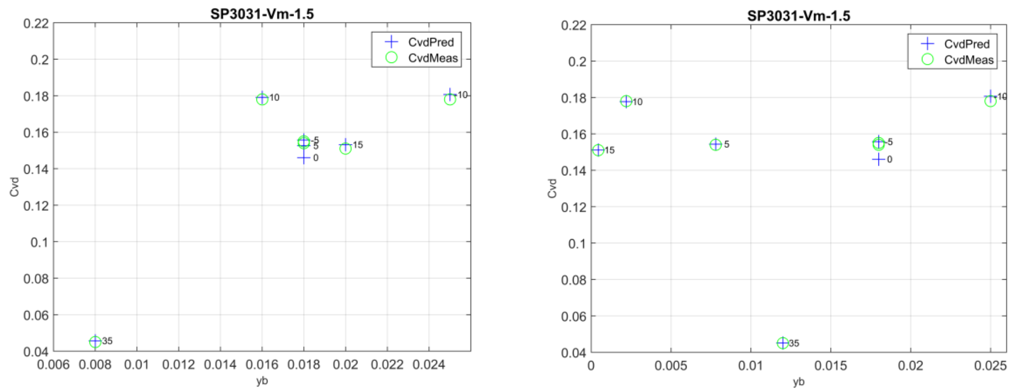


Figure 26: Range of bed thickness values with variable correction factor (left) and fixed correction factor to 1 (right) at mean velocity V_m = 1.5 m/s.

From the graphs for the mean velocity V_m = 1 m/s, it is obvious that the obtained thickness of the bed is bigger with the fixed value of the correction factor. Comparison of values of the bed thickness is given in Table 3.

Table 3: Bed thickness values for different inclinations related to correction factor.

V_m = 1 [m/s]	Variable Corr.Factor	Fixed Corr.Factor
Inclination[deg°]	y_b [m]	y_b [m]
15	0.024	0.0313
20	0.032	0.0490
25	0.039	0.0452
35	0.033	0.0450

The situation is different for the mean velocity V_m = 1.5 m/s. The thickness of the bed is changing as well, but not strictly increasing. The thickness of the bed is smaller when the correction factor is equal 1 for inclinations of 5/10/15 degrees in ascending pipe. With higher velocity, more particles are in movement and therefore more particles are contributing to the static pressure due to inclination. Attention should be paid on this in future researches, as the position of the stationary deposit plays an important role in the whole procedure. Values of the bed thickness changes are presented in the table below.

Table 4: Bed thickness values for different inclinations related to correction factor.

V_m= 1.5 [m/s]	Variable Corr.Factor	Fixed Corr.Factor
Inclination[deg°]	y_b [m]	y_b [m]
-10	0.025	0.025
-5	0.018	0.018
0	0.018	0.018
5	0.018	0.008
10	0.016	0.002
15	0.020	0.0004
35	0.008	0.012

7. Conclusion and Recommendations

The experimental results for slurry flow of the 0.55 mm sand above the stationary bed in a 100 mm pipe inclined to various angles between −35 to +35 degrees were used to test the combination of the transport and friction formulae for their ability to capture the effect of flow inclination and to validate the formulae for such flows. The further goal was to find the right way of subtracting the static pressure drop from the measured manometric pressure drop to get the frictional drop in the inclined flows. The final goal was to evaluate the importance of the position of the top of the stationary bed as an input parameter to the calculation procedure for bed transport and friction through sensitivity analysis.

Previously, the layered model for settling slurry flow in an inclined pipe (Matoušek et al. 2018) was validated for flows with sliding beds and only the entire model overall outputs (hydraulic gradients, concentration profiles) was validated. This work validates the combination of the formulas for bed transport and friction used in the model by employing collected experimental data for inclined flows with stationary deposit.

Our data indicate that in the case of flow with stationary deposit grains occupying the stationary bed do not contribute to the static pressure drop developed due to pipe inclination. This is, different from grains in sliding beds where all grains contribute to the static pressure drop. Moreover, the experiments suggest that only a certain portion of grains in the shear layer above the stationary bed actually contributes to the static pressure drop.

As it turned out that it could not be relied on the bed thickness as an input parameter in the validation procedure, the other way of validation of the model formulas had to be chosen. It included the use of the bed transport formula in the validation procedure and an introduction of the appropriate correction factor for a determination of particles contributing to the static pressure drop. In this way, the bed transport formula served as a tool used to validate the formula for the bed roughness. The validation procedure revealed that the combination of the two formulae seems to satisfactorily describe the shear conditions at the top of the inclined eroded bed and hence their use in the layered model for inclined flows is justified.

The sensitivity analysis highlighted and quantified the uncertainty of the experimental determination of the bed thickness, y_b. It occurred that even a small variation in millimetres could significantly affect the accuracy of the other related results.

The obtained results provide a suitable basis for a further investigation of inclined flows above a stationary bed in a pipe.

The overall recommendation based on this work is that care must be taken when it comes to an evaluation of the effect of grains contributing to the static pressure drop in stratified inclined flows. A more specific recommendation is to produce a broader data set including flows of different delivered concentrations and different types of solids fractions.

8. References

1. Clift R. and Clift D. H. M., 1981: Continuous measurement of the density of flowing slurries. International Journal of Multiphase Flow 75: 555 - 561
2. Cheng N. S., 2002: Exponential formula for bedload transport. Journal of Hydraulic Engineering 128: 942 - 946.
3. Doron P. and Barnea D., 1993: A three-layer model for solid-liquid flow in horizontal pipes. International Journal of Multiphase Flow 19: 1029 - 1043.
4. Doron M., Simkhis M. and Barnea D., 1997: Flow of solid-liquid mixtures in inclined pipes. International Journal of Multiphase Flow 23: 313 - 323
5. Einstein H. A., 1950: The bed-load function for sediment transportation in open channel flows. United States Department of Agriculture Technical Bulletin 1026, Washington, D.C.
6. Krupička J., 2014: Mathematical and Physical Modelling of Pipe Flow of Settling Slurries. Doctoral Thesis, Prague, Czech Republic.
7. Meyer-Peter E. and Müller R., 1948: Formulas for bed-load transport. In Proceedings of the International Association for Hydraulic Research, Third Annual Conference, Stockholm, Sweden, pp. 39 - 64.
8. Matoušek V., 2004: Erosion of Plane Bed by Sand Slurry current in Pipe. Journal of Hydrology and Hydromechanics 52: 156 - 161. Delft University of Technology, Mekelweg 2, 2628 CD Delft, The Netherlands.
9. Matoušek V., 2009: Concentration profiles and solids transport above stationary deposit in enclosed conduit. Journal of Hydraulic Engineering 135: 1101 - 1006
10. Matoušek V., 2011: Solids transport formula in predictive model for pipe flow of slurry above deposit. Particulate Science and Technology 29: 89 - 106.
11. Matoušek V., Krupička J. and Chára Z., 2014: Stationary- and sliding beds in pipe flows of settling slurry. Proc. 15th Int. Freight Pipeline Soc. Symposium, Prague, Czech Republic, 24 - 26 June 2014.
12. Matoušek V. and Krupička J., 2014: Interfacial friction and transport in stratified flows. P I Civil Eng-Marit. Eng. 167: 125 - 134
13. Matoušek V., Krupička J. and Pěník V., 2014: Distribution of Medium-to-Coarse Glass Beads in Slurry Pipe Flow: Evaluation of Measured Concentration Profiles. Particulate Science and Technology: An International Journal 32: 186 - 196
14. Matoušek V., Krupička J. and Kesely M., 2018: A layered model for inclined pipe flow of settling slurry. Powder Technology 333: 317 - 326.

15. Matoušek V., Kesely M. and Zdeněk C., 2019: Effect of pipe inclination on internal structure of settling slurry flow at and close to deposition limit. Journal Powder Technology 343: 533 - 541.

16. Matoušek V., Kesely M., Konfršt J. and Vlasák P., 2019: Anomalous pressure drop in settling slurry flow through pipe of mild negative slope. Proc. 19th Int. Conference on Sedimentation and Transportation of Solid Particles, Cape Town, South Africa.

17. Pugh F. J. and Wilson K. C., 1999: Velocity and concentration distributions in sheet flow above plane beds. Journal of Hydraulic Engineering 125: 117 - 125.

18. Ribberink J. S. 1998: Bed-load transport for steady flows and unsteady oscillatory flows. Coastal Engineering 34: 59 - 82.

19. Shields A., 1936: Anwendung der Aehnlichkeitsmechanik und der Turbulenzforschung auf die Geschiebebewegung. Mitteilungen der Preußischen Versuchsanstalt für Wasserbau und Schiffbau, Berlin, Nr 26.

20. Shook C.A. and Roco M.C., 1991: Slurry Flow. Principles and Practice, Butterworth-Heinemann, Stoneham, MA.

21. Sumer B. M., Kozakiewicz A., Fredsøe J. and Deigaard R., 1996: Velocity and concentration profiles in sheet-flow layer of movable bed. Journal of Hydraulic Engineering 122: 549 - 558.

22. Worster R.C. and Denny D.F., 1955: Hydraullic transport of solid materials in pipelines. Proceedings of the Institution of Mechanical Engineers 169: 563 - 586

23. Wilson K. C., 1966: Bed-load transport at high shear stress. Journal of the Hydraulics Division 92: 49 - 59

24. Wilson K.C., 1976: A unified physically-based analysis of solid-liquid pipeline flow. Proc. Hydrotransport 4, BHRA, Cranfield, UK, pp. 1 - 16.

9. Appendices

I Particle-size-distribution curves and curves of settling velocities

II Profiles of chord-averaged concentrations in inclined pipe

III Performance of predictive formulae for C_{vd} calculations

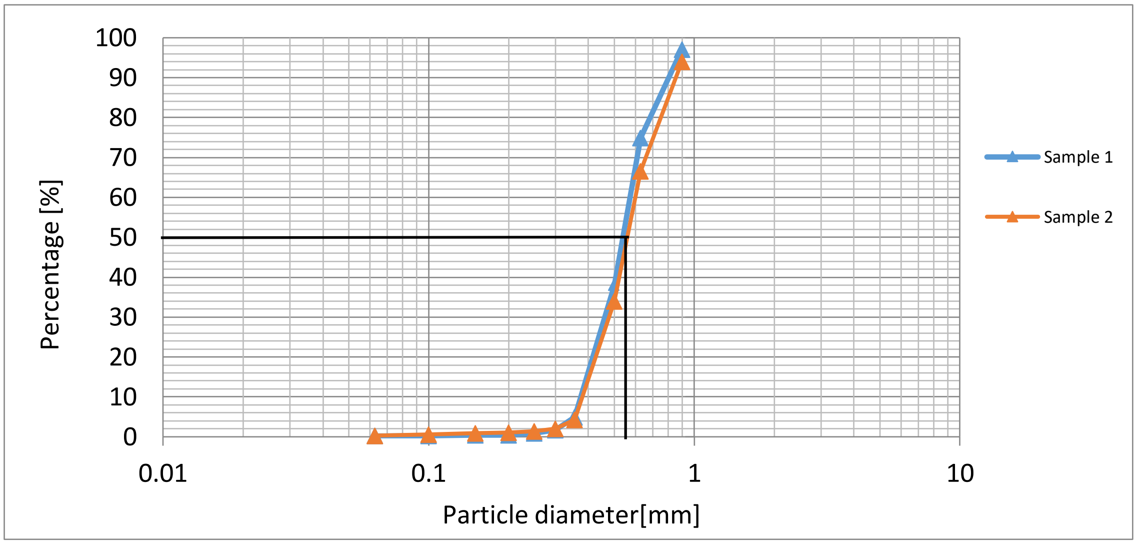
IV Performance of predictive formulae for hydraulic roughness

V Comparing performance of two predictive formulae for hydraulic roughness

VI Sensitivity of model for inclined stratified flow to model inputs

Appendix I: Particle-size-distribution curves and curves of settling velocities

Measured particle size distribution and settling velocities of tested narrow graded sand are presented. Sample 1 represents sand before usage while Sample 2 represents sand already used in the experiments.



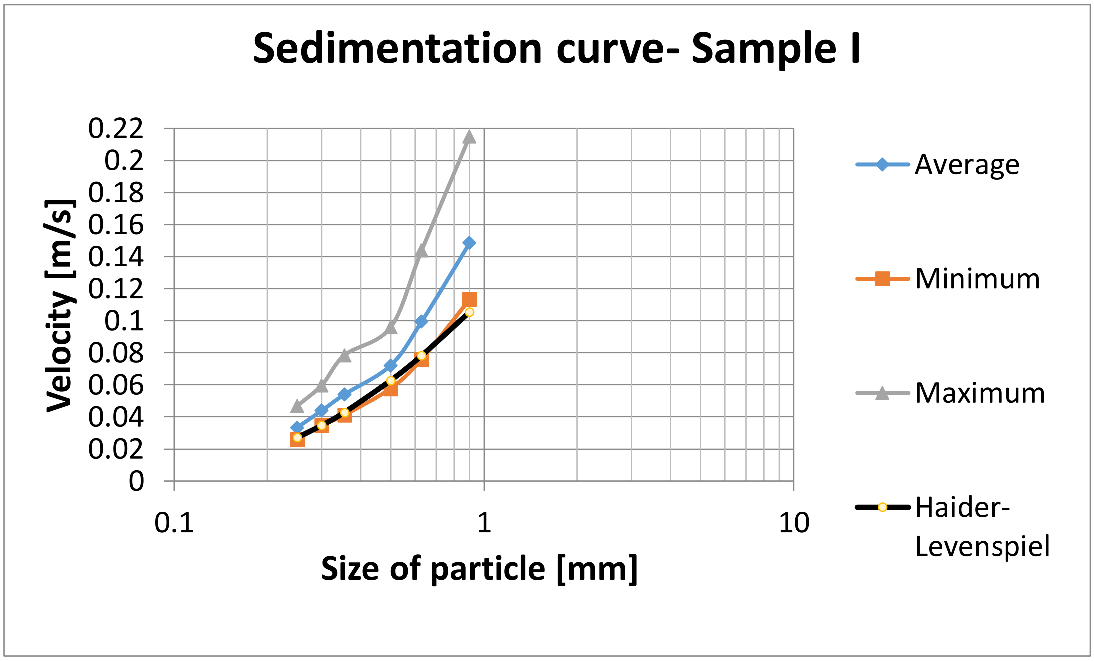
$$d_{50} = 0.55mm$$

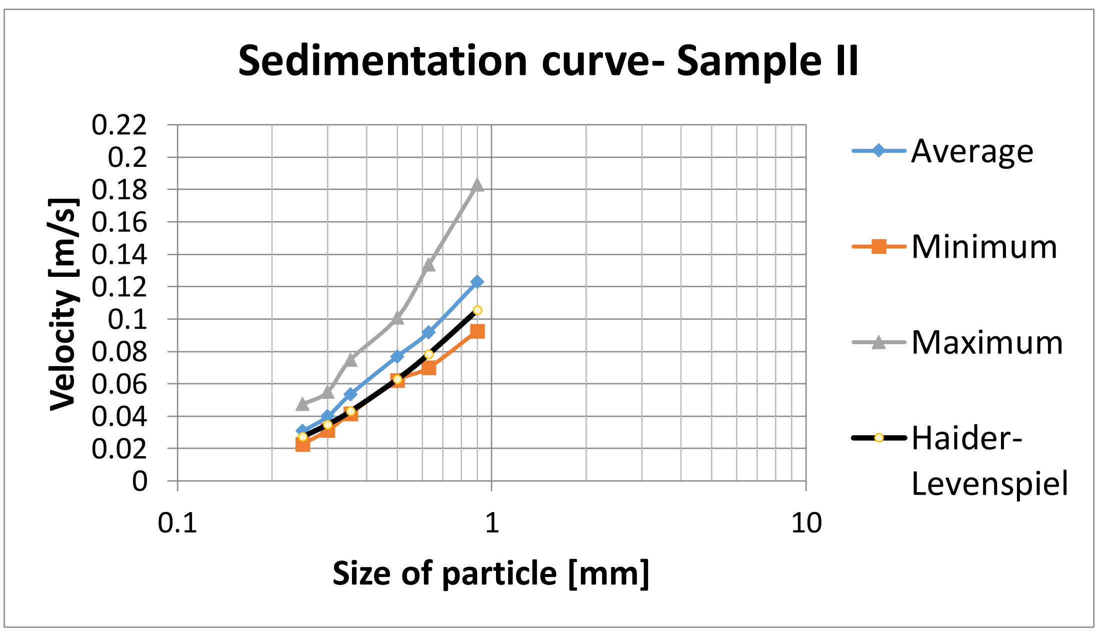
Settling velocity of non-spherical particle calculations (method by Haider-Levenspiel):

$$v_t^* = \left(\frac{18}{d^{*2}} + \frac{2.3348 - 1.7439 \cdot \Psi}{d^{*0.5}}\right)^{-1}$$

$$d^* = d \cdot \sqrt[3]{\frac{\rho_v \cdot (\rho_s - \rho_v) \cdot g}{\mu_v^2}}$$

$$v_t^* = v_t \cdot \sqrt[3]{\frac{\rho_v^2}{\mu_v \cdot (\rho_s - \rho_v) \cdot g}}$$


Sedimentation curve- Sample I
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
Velocity [m/s]
0.1
1
10
Size of particle [mm]
Average
Minimum
Maximum
Haider-Levenspiel


Sedimentation curve- Sample II
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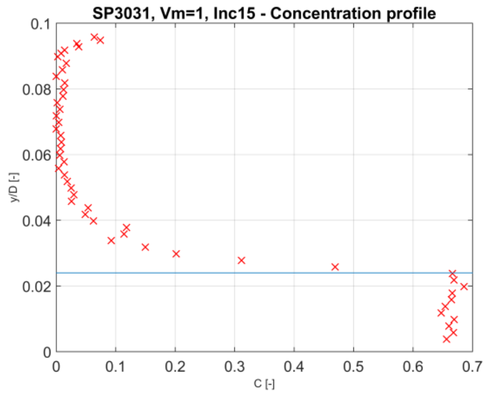
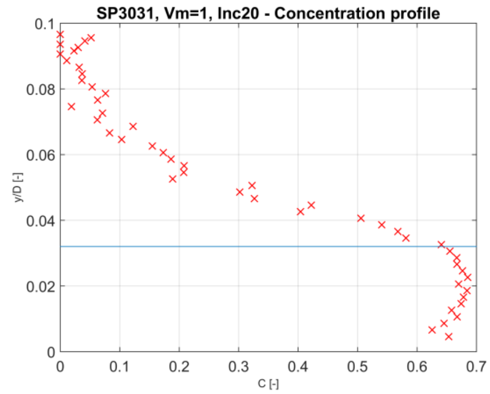
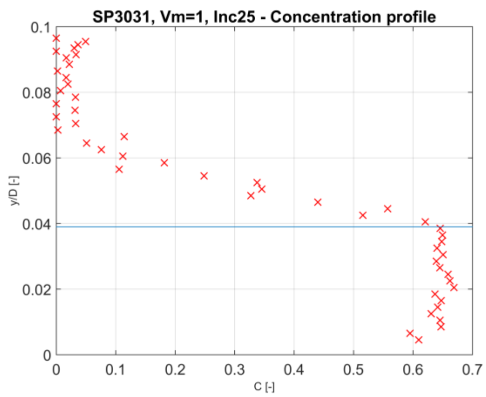
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
Velocity [m/s]
0.1
1
10
Size of particle [mm]
Average
Minimum
Maximum
Haider-Levenspiel

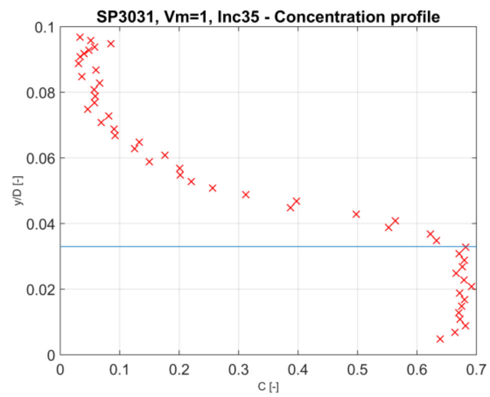
Appendix II: Profiles of Chord-Averaged Concentrations in Inclined Pipe

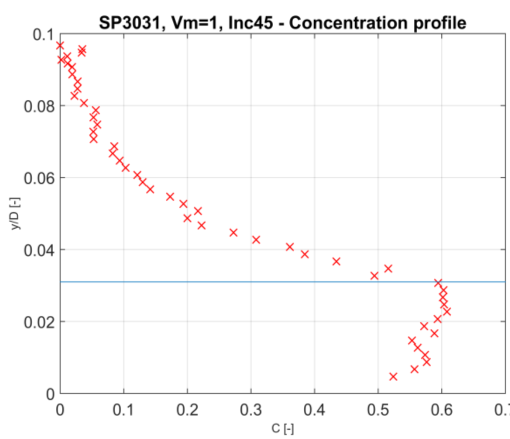
Mean velocity $V_m = 1$ m/s



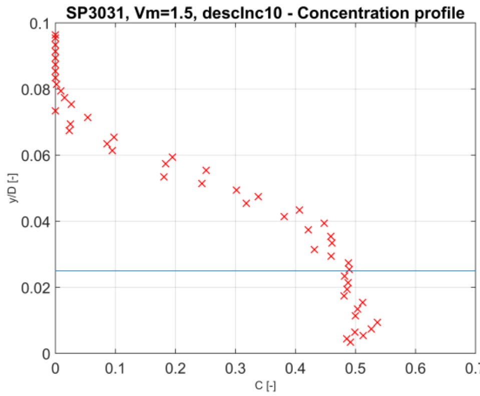


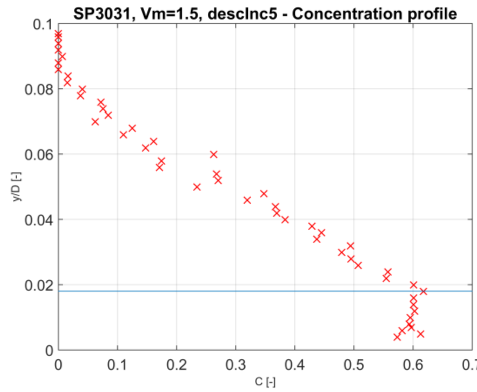


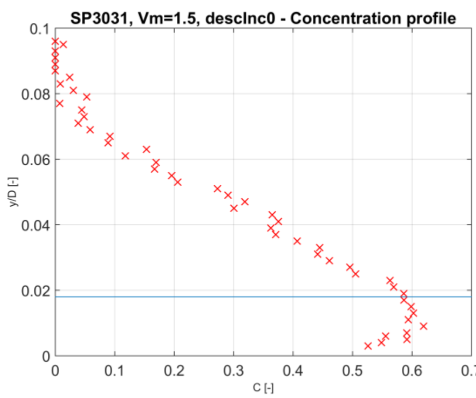


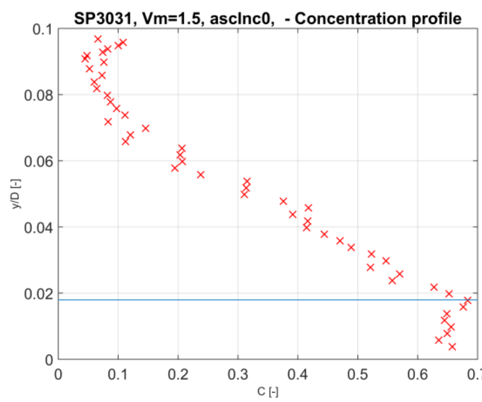


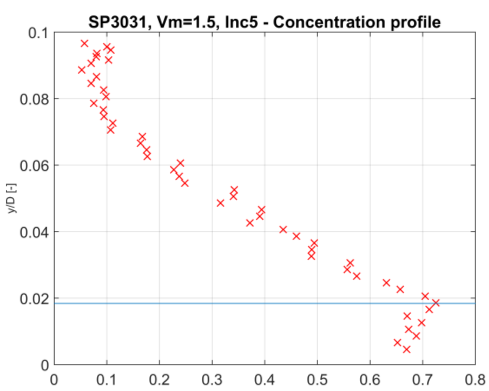
Mean velocity V_m = 1.5 m/s

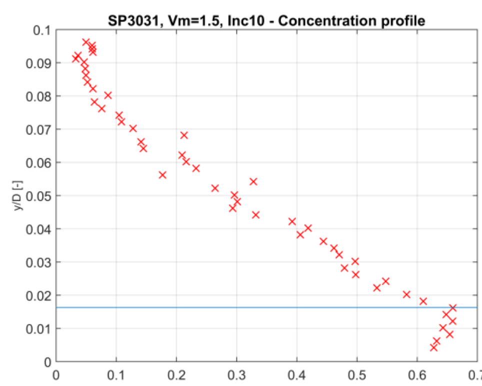


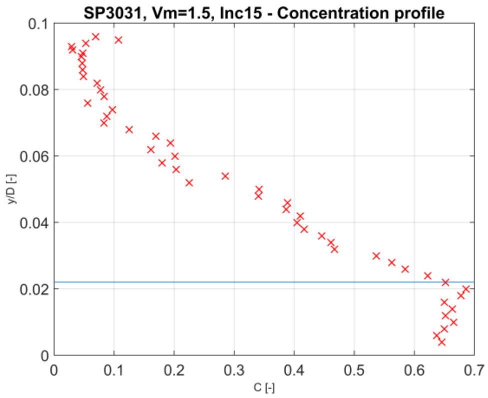











SP3031, Vm=1.5, Inc15 - Concentration profile
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C [-]
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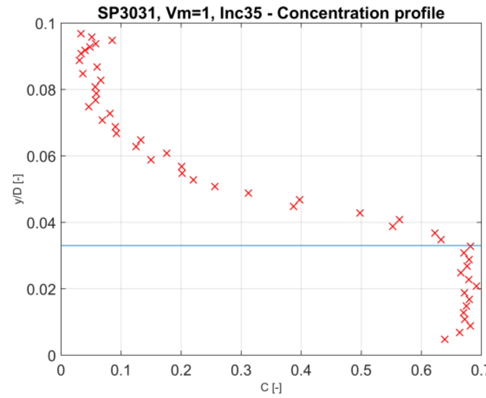
SP3031, Vm=1, Inc35 - Concentration profile
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C [-]
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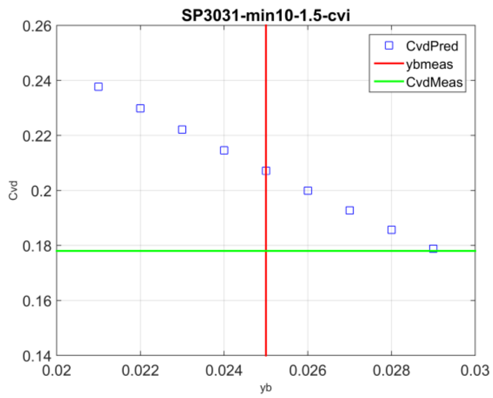
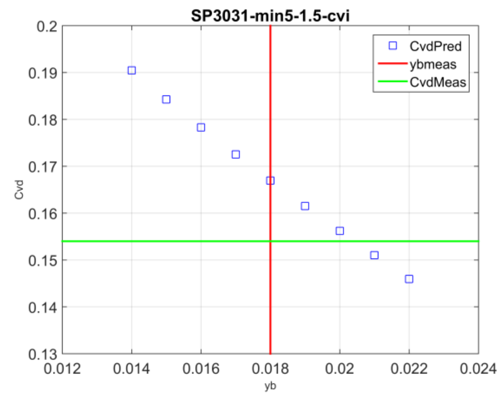
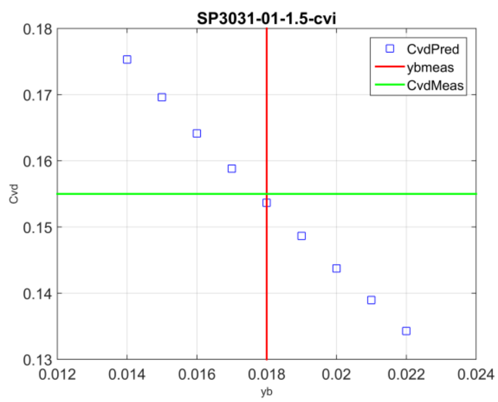
0.06
0.04
0.02
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7

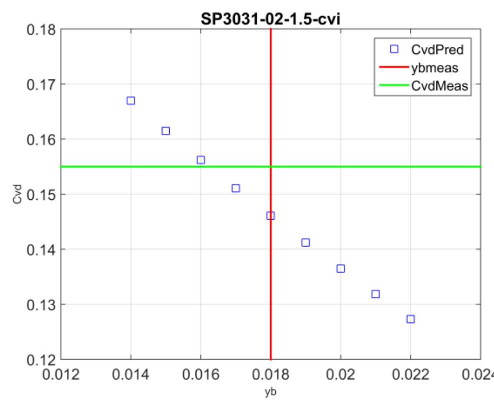
Appendix III: Performance of Predictive Formulae for C_{vd} Calculations

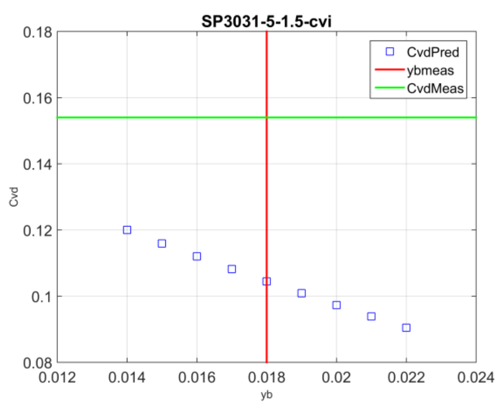
Calculations based on calculations of frictional drop subtracting static part using mean spatial volumetric concentration for the entire pipe cross section, C_{vi}.

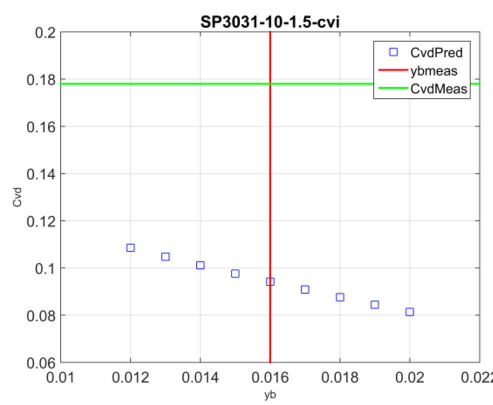


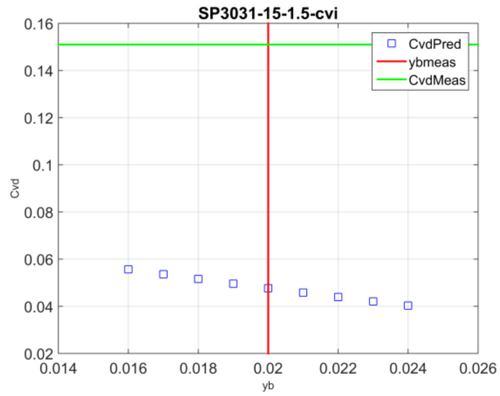


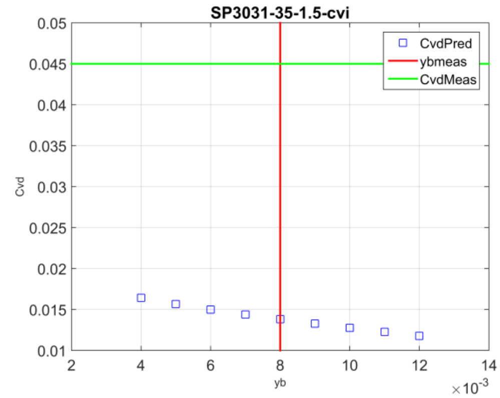






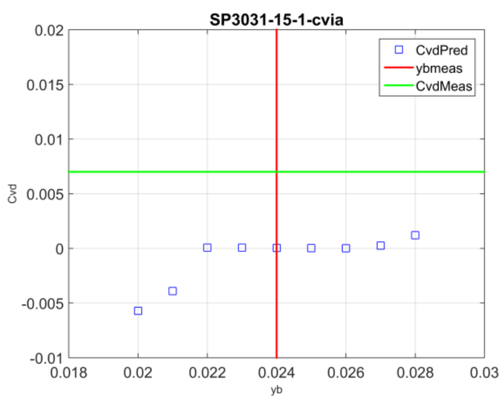


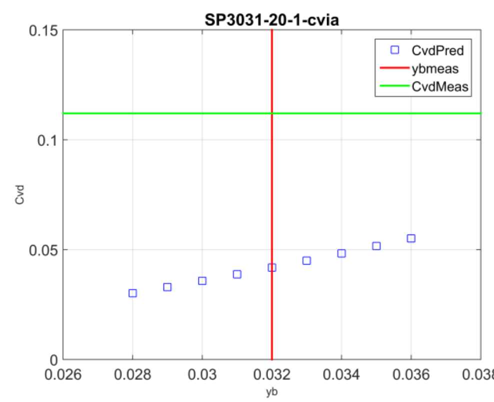


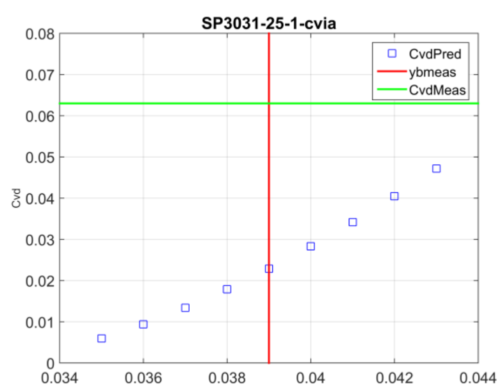


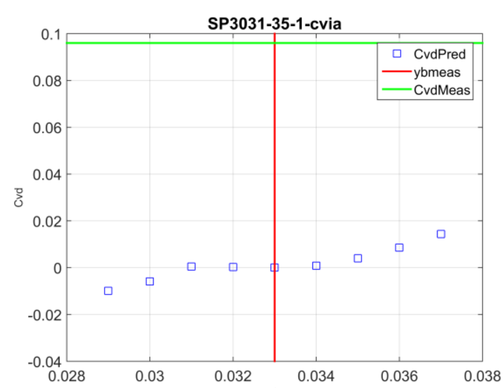
Calculations based on calculations of frictional drop subtracting static part considering contribution of all grains present in the flow above the stationary bed, C_{via}.

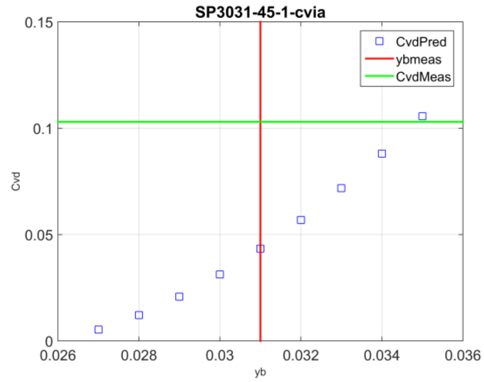
Mean velocity $V_m = 1$ m/s



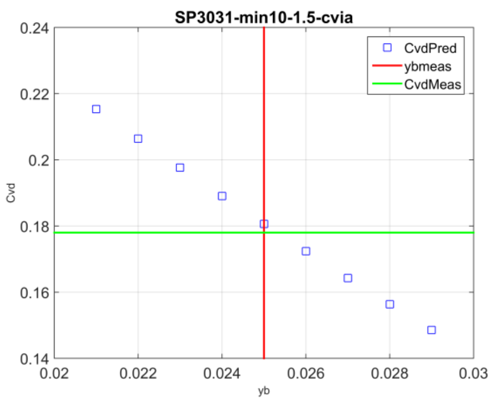


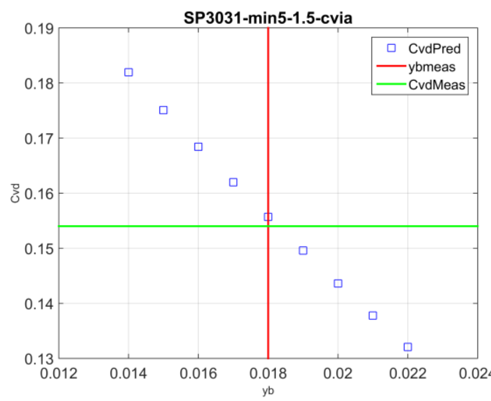


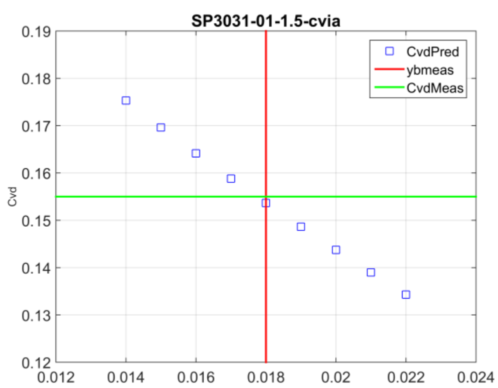


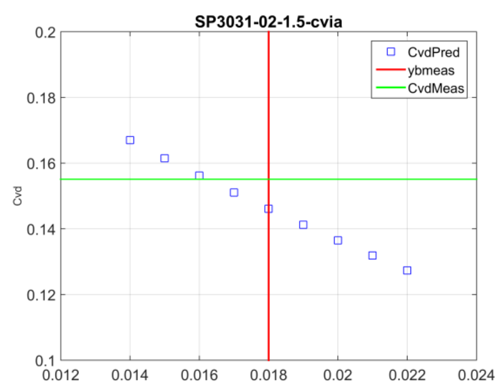


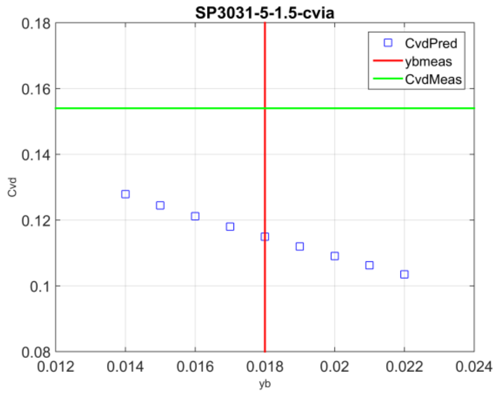
<u>Mean velocity V_m = 1.5 m/s</u>

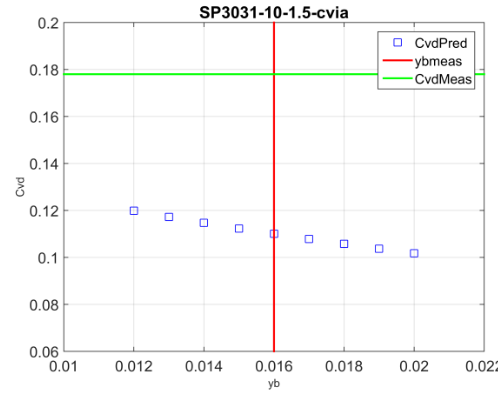


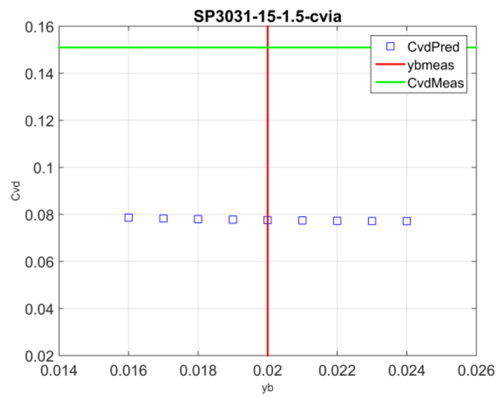


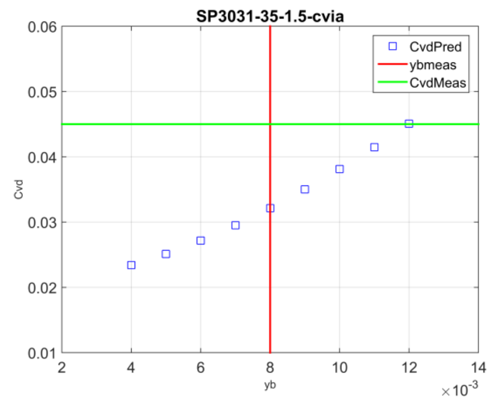






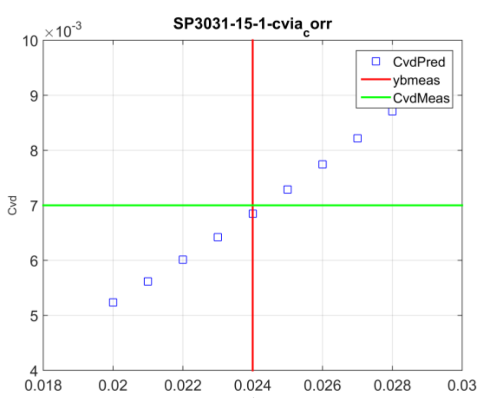


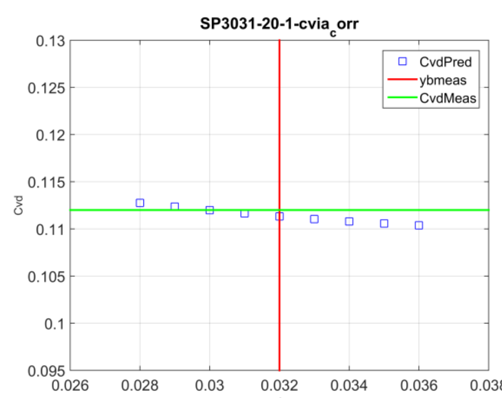


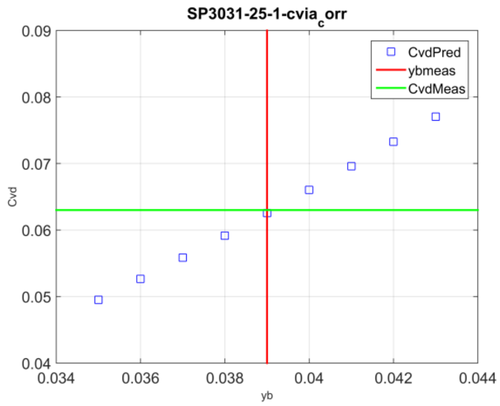


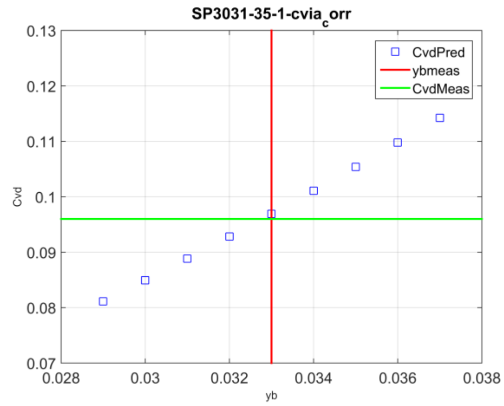
Calculations based on calculations of frictional drop subtracting static part considering contribution of part of the grains present in the flow above the stationary bed, C_{via} with correction factor.

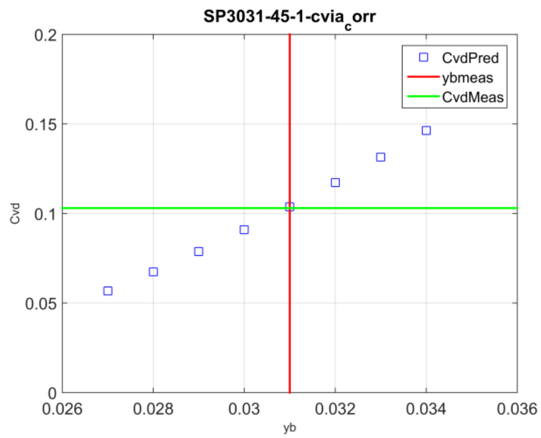
Mean velocity $V_m = 1$ m/s



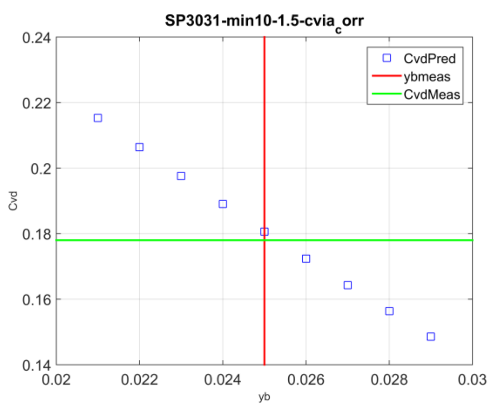


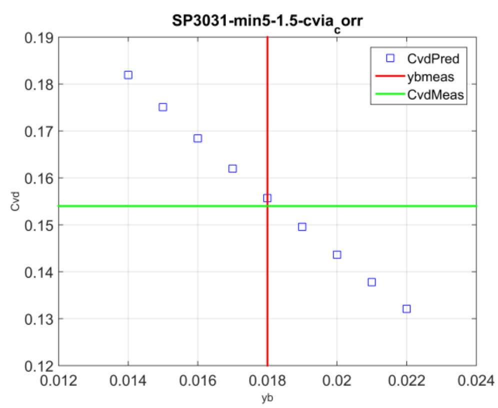


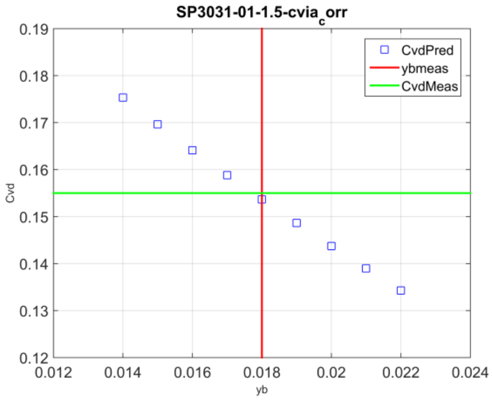




Mean velocity $V_m = 1.5$ m/s



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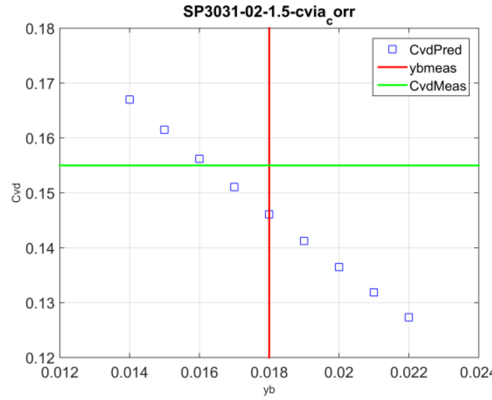
ybmeas
CvdMeas
Cvd
yb
0.19
0.18
0.17
0.16
0.15
0.14
0.13
0.12
0.012
0.014
0.016
0.018
0.02
0.022
0.024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ybmeas
CvdMeas
Cvd
yb
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0.17
0.16
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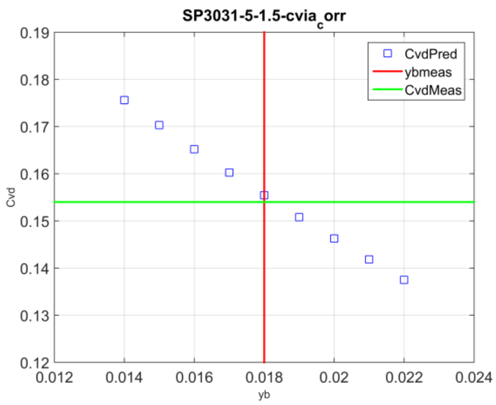
0.14
0.13
0.12
0.012
0.014
0.016
0.018
0.02
0.022
0.024


SP3031-5-1.5-cvia_corr
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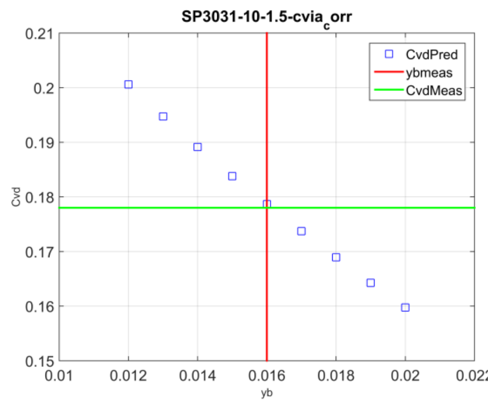
ybmeas
CvdMeas
Cvd
yb
0.19
0.18
0.17
0.16
0.15
0.14
0.13
0.12
0.012
0.014
0.016
0.018
0.02
0.022
0.024


SP3031-10-1.5-cvia_corr
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ybmeas
CvdMeas
Cvd
yb
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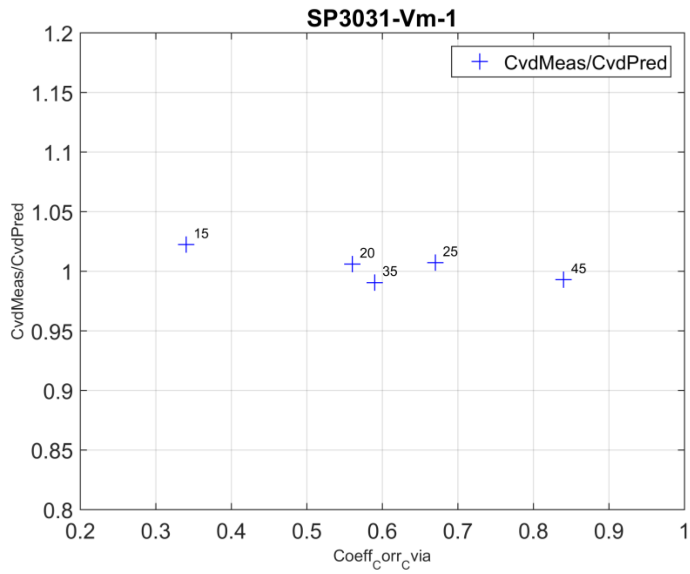
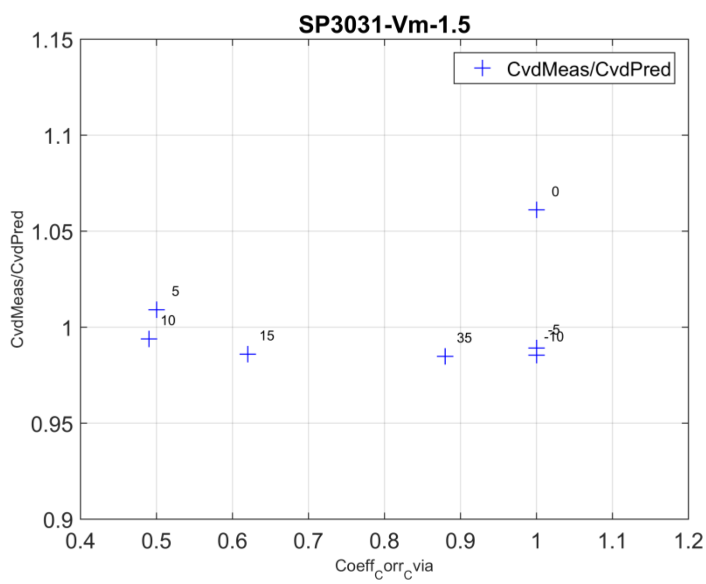
0.2
0.19
0.18
0.17
0.16
0.15
0.01
0.012
0.014
0.016
0.018
0.02
0.022

Changing of value for chosen correction factor (x axis) of particles contributing to static pressure for mean velocity 1 m/s.

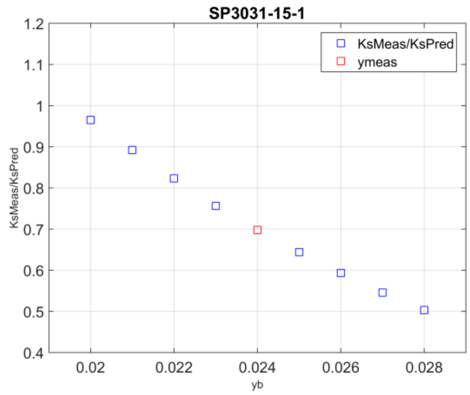


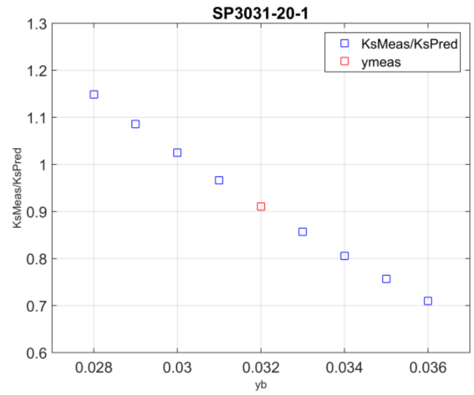
Changing of value for chosen correction factor (x axis) of particles contributing to static pressure for mean velocity 1.5 m/s

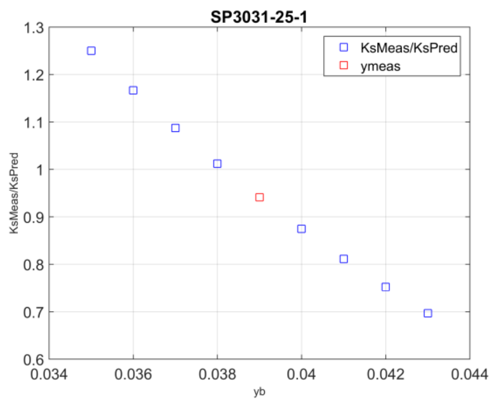


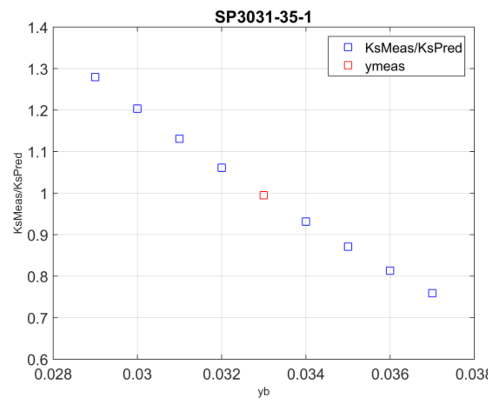
Appendix IV: Performance of Predictive Formulae for Hydraulic Roughness

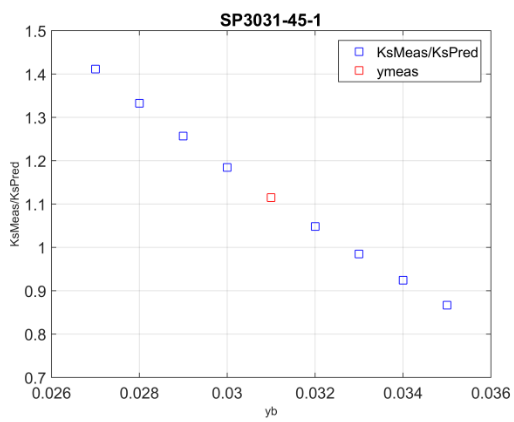
Mean velocity $V_m = 1$ m/s



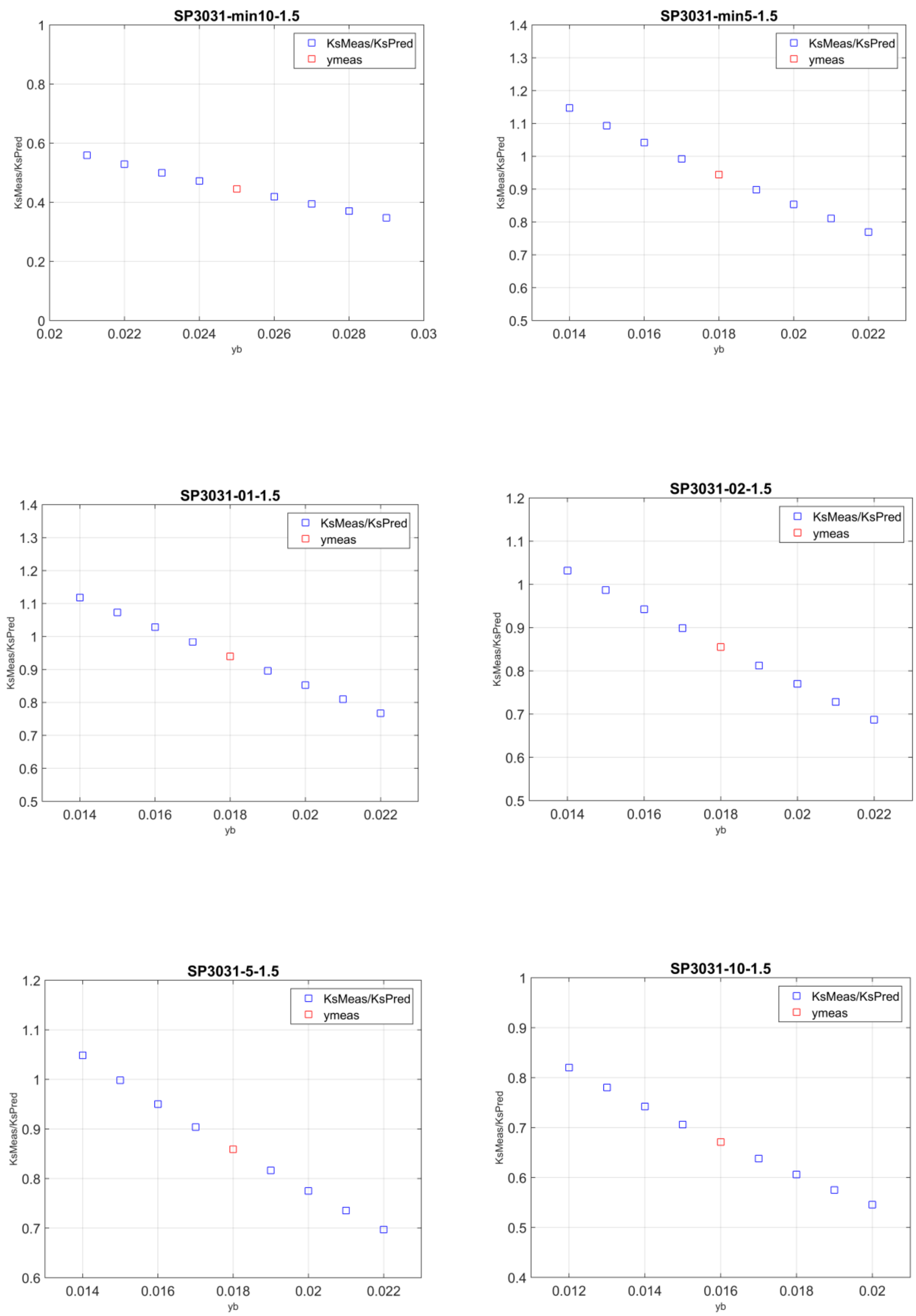


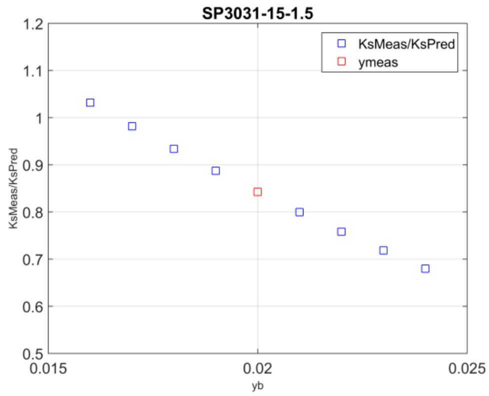


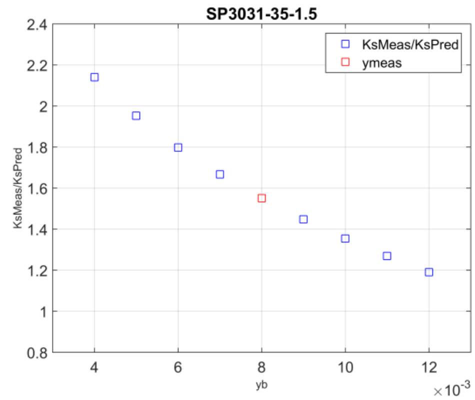




Mean velocity V_m = 1.5 m/s

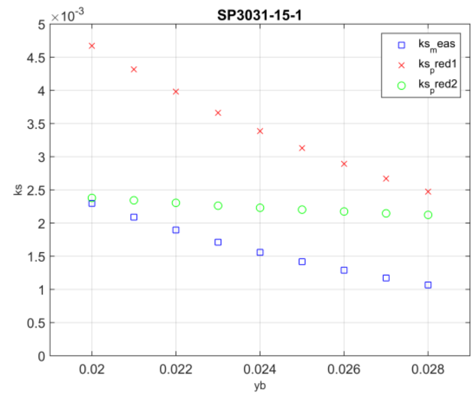


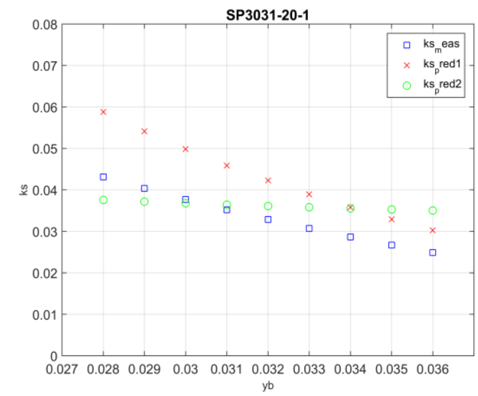

SP3031-15-1.5
KsMeas/KsPred
ymeas
KsMeas/KsPred
yb
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.015
0.02
0.025

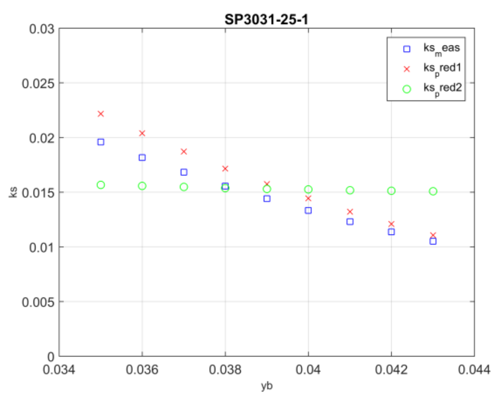

SP3031-35-1.5
KsMeas/KsPred
ymeas
KsMeas/KsPred
yb
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
4
6
8
10
12
×10^-3

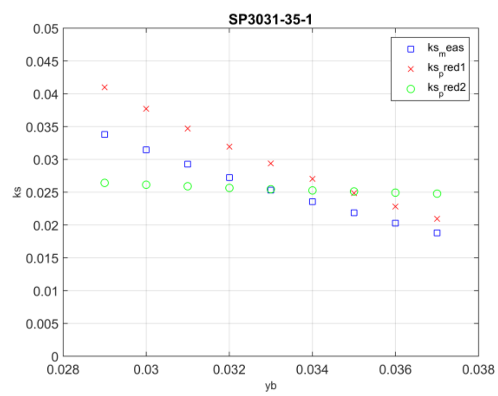
Appendix V: Comparing Performance of Two Predictive Formulae for Hydraulic Roughness

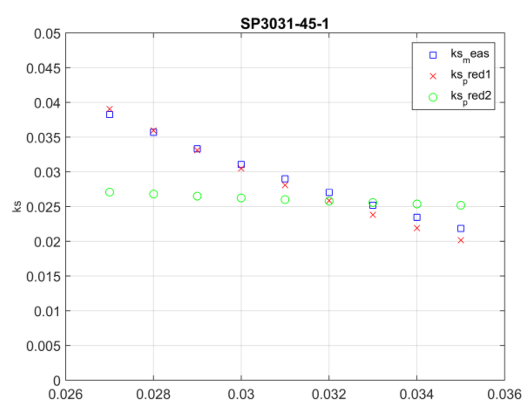
Mean velocity V_m = 1 m/s



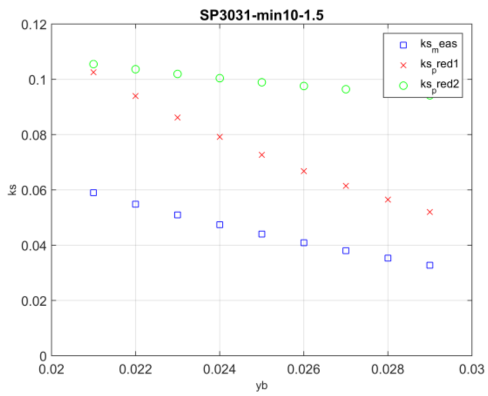


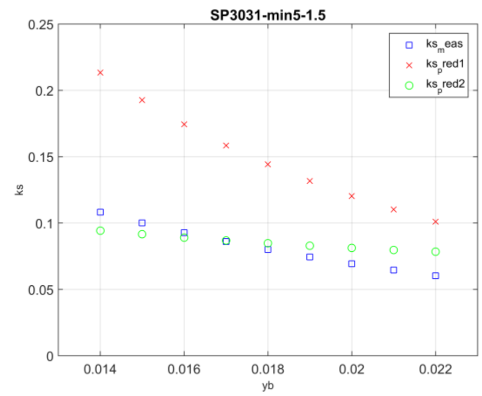


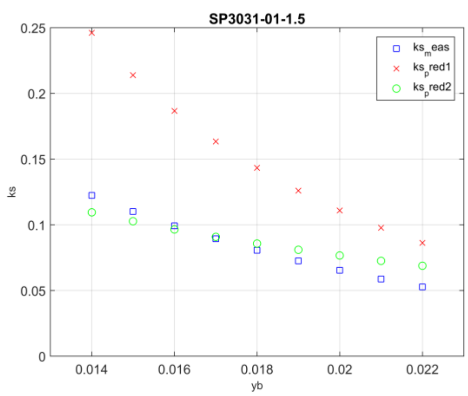


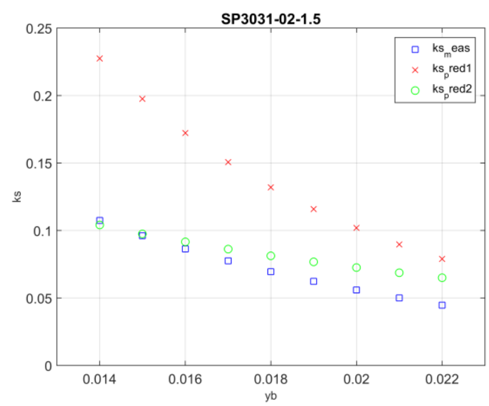


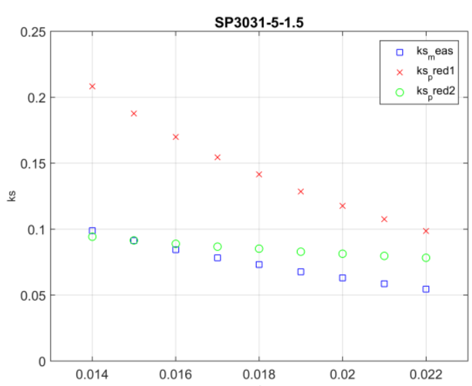
<u>Mean velocity V_m = 1.5 m/s</u>

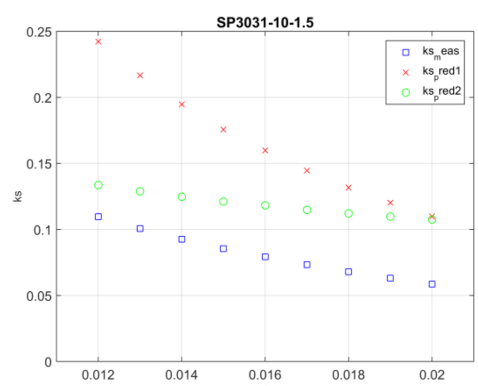


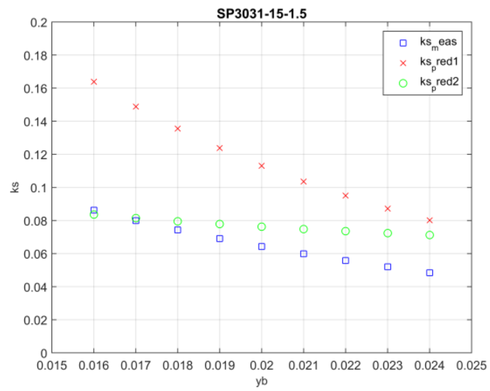


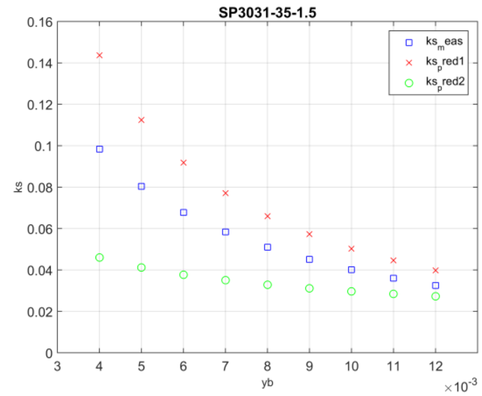






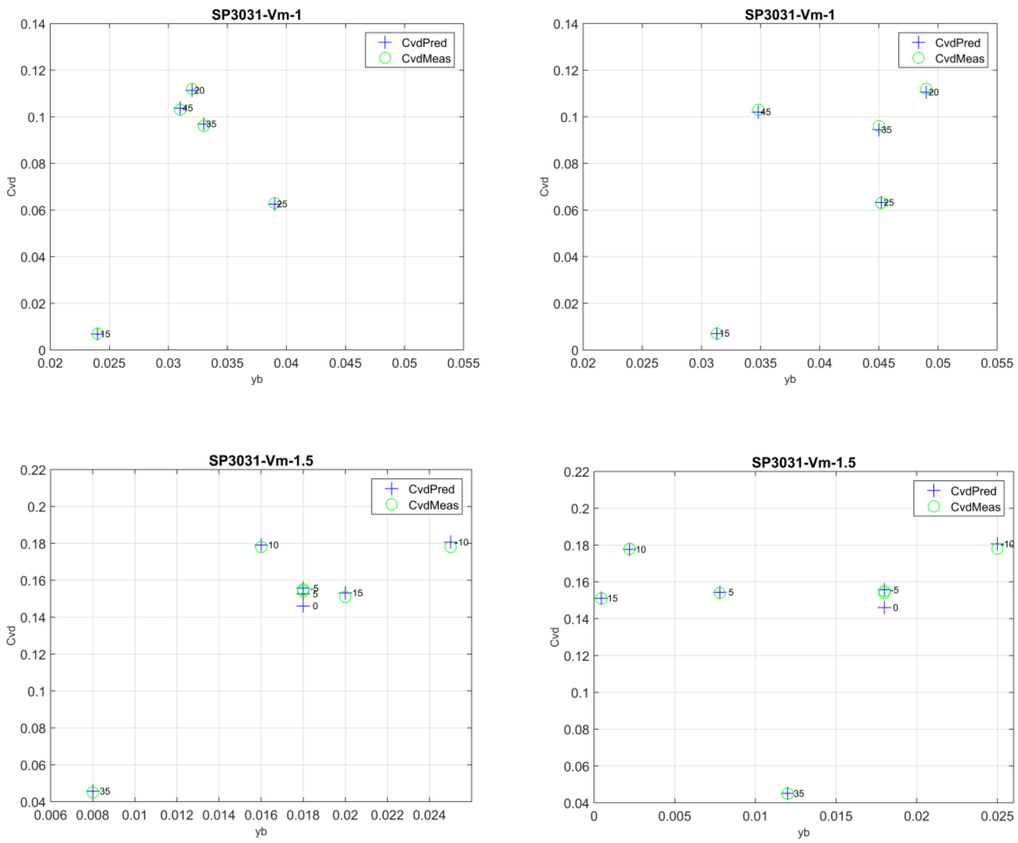



SP3031-15-1.5
ks
yb
ks_meas
ks_pred1
ks_pred2


SP3031-35-1.5
ks
yb
×10-3
ks_meas
ks_pred1
ks_pred2

Appendix VI: Sensitivity of Model for Inclined Stratified Flow to Model Input

On the left side are shown graphs with thickness of bed when correction factor is chosen in order to C_{vd} predicted = C_{vd} measured, while right graphs shows fixed correction factor to 1 and changed position of the bed to have the same C_{vd} predicted and measured.



10. Acknowledgements

Undertaking this Master thesis would not have been possible to do without the support and guidance that I received from many people.

I would like first to say a big thank you to my professor and supervisor prof. Dr Ing. Václav Matoušek for all the support, guidance and constant feedback. Many thanks for the encouragement he gave me, especially for participation in 19th International Conference on Transport and Sedimentation of Solid Particles, which was a great experience. It was a pleasure to work with him. Without him, this thesis would not have been achievable.

My thanks also go out to the support I received from Czech University of Life Sciences, Faculty of Environmental Sciences for the previously mentioned conference that I attended presenting part of the results obtained for master thesis purposes.

I gratefully acknowledge Visegrad fund for the scholarship that support me during my second year of master studies.

I would also like to say thank you to my family and friends for always believing in me and encouraging me.

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