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Counterfactual Quantum Communication and Decoherence

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Declaration

I hereby declare that the thesis entitled “ Counterfactual Quantum Communication and Decoherence ” has been composed solely by myself under the guidance of doc. Mgr. Petr Marek, Ph.D. by using resources, which are referred to in the list of literature. I agree with the further usage of this document according to the requirements of the Department of Optics.

In Olomouc April 20, 2023

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Bibliographical identification

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Abstract	Quantum nonlocality is one of the most exciting features of quantum mechanics. Counterfactual communication utilizes it to transmit information between two parties seemingly without any physical interaction. The goal of this bachelor thesis is to simulate a protocol for quantum counterfactual communication, analyze its behavior with regards to losses and analyze different errors in the communication process.
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Abstrakt	Kvantová nelokalita je jednou z nejzajímavějších vlastností kvantové mechaniky. Kontrafaktuální komunikace ji využívá k přenosu informací mezi dvěma stranami zdánlivě bez jakékoliv fyzikální interakce. Cílem této bakalářské práce je simulovat protokol pro kvantovou kontrafaktuální komunikaci, analyzovat jeho chování s ohledem na ztráty a analyzovat různé chyby v komunikačním procesu.
Klíčová slova	Kontrafaktuální komunikace, měření bez interakce, ztráta informace
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Contents

Introduction

What is a paradox?

‘*The "paradox" is only a conflict between reality and your feeling of what reality "ought to be."*’ - Richard Feynman.

Direct counterfactual communication is a concept that has intrigued physicists for years. The idea of exchanging information without the need for physical particles to travel between two parties is not only fascinating but also has the potential to revolutionize the field of communication [19], [20]. In recent years, researchers have explored the possibility of achieving direct counterfactual communication using a chained version of the Zeno effect [1]. By increasing the number of inner and outer Mach-Zehnder interferometer cycles and enhancing the transmissivity of the inner and outer beam splitters with each step, Alice and Bob can communicate without particles moving between them.

Decoherence is the process by which a quantum system loses its coherence due to interactions with its environment, leading to information loss [18]. In the context of direct counterfactual communication, decoherence can result in miscommunication or information loss when Alice and Bob attempt to communicate.

In counterfactual communication we will see that the transmitter can send a physical particle to the reciever but transmits the informative message without this ever happening [1], [4]. So the reciever never recieves a physical particle which came from the transmitter, but it receives the information.

To understant this protocol let us imagine a scenario in which Alice and Bob want to communicate and Bob wants to let Alice know of his choice between $|\mathrm{O}\rangle$ and $|\mathrm{C}\rangle$ (They represent whether Bob keeps the channel Open or Closed). So we will have

$$|R\rangle_A|\mathrm{C}\rangle_B \xrightarrow{After\ the\ communication} |\mathrm{C}\rangle_A|\mathrm{C}\rangle_B, \tag{1}$$

$$|R\rangle_A|O\rangle_B \xrightarrow{After\ the\ communication} |O\rangle_A|O\rangle_B, \tag{2}$$

Here R stands for the result of the communication which is unknown for Alice before the communication.

Bob could make any decision between $|O\rangle$ and $|C\rangle$, therefore, we can write the state in the form of superposition between two states as follows

$$|\psi\rangle_B = \alpha|C\rangle_B + \beta|O\rangle_B, \tag{3}$$

where α and β are some coefficients that are used to show the separate probability of each two states occurring. Finally, we obtain

$$|R\rangle_A|\psi\rangle_B \xrightarrow{After\ the\ communication} \alpha|C\rangle_A|C\rangle_B + \beta|O\rangle_A|O\rangle_B, \tag{4}$$

This communication is called counterfactual communication if no physical particles are exchanged between Alice and Bob. If a physical particle is present in the communication channel then our protocol is non-counterfactual.

There are still several challenges that need to be addressed. Miscommunication or information loss when Alice and Bob attempt to communicate are the main parts of our investigation and the probability of miscommunication resulting from the unexpected clicking of a certain detector cannot be overlooked.

In this context, the objective of this research is to investigate the feasibility of achieving direct counterfactual communication and the probability of error. Specifically, we aim to explore the extent to which the Zeno effect can be exploited to achieve information transmission without particle movement and examine the influence of environmental factors on the probability of error.

If successful, direct counterfactual communication could provide a new and secure method of transmitting information [19], [20].

While the concept of direct counterfactual communication is fascinating, there are still many unanswered questions that need to be explored. This study provides valuable insights into the feasibility of achieving direct counterfactual communication and the probability of errors, laying the groundwork for future research in this exciting and rapidly evolving field.

Chapter 1

Basic Information and used Methods

In this chapter we are going to introduce all the concepts and methods that are necessary to tackle this paradox.

In the present thesis, we shall provide a brief overview of the formulations employed. It is to be noted that there exist numerous variations of the postulates of quantum mechanics; however, for the sake of consistency, we shall adhere to the following formulations as outlined in references [17] and [4].

- To study single-photon systems, we can express the states by the following vectors

$$\hat{a}^{\dagger}{}_{k}\, |00 \ldots 000 \ldots 0\rangle = |00 \ldots 010 \ldots 0\rangle, \tag{1.1}$$

  Where k corresponds to a specific mode for a single photon and we use creation an annihilation operators $\hat{a}^{\dagger}$ and $\hat{a}$, which are defined as

$$\hat{a}^{\dagger}|n\rangle = \sqrt{n+1}|n+1\rangle,$$

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle, \tag{1.2}$$

  And we will use subscripts ($\hat{a}_j$) to assign these operators to specific modes.

- The initial state of the photon will be represented by numerical vector throughout this thesis. Where the position of 1 corresponds to the mode.

$$|\psi_{in}\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix}_{m\times 1}, \tag{1.3}$$

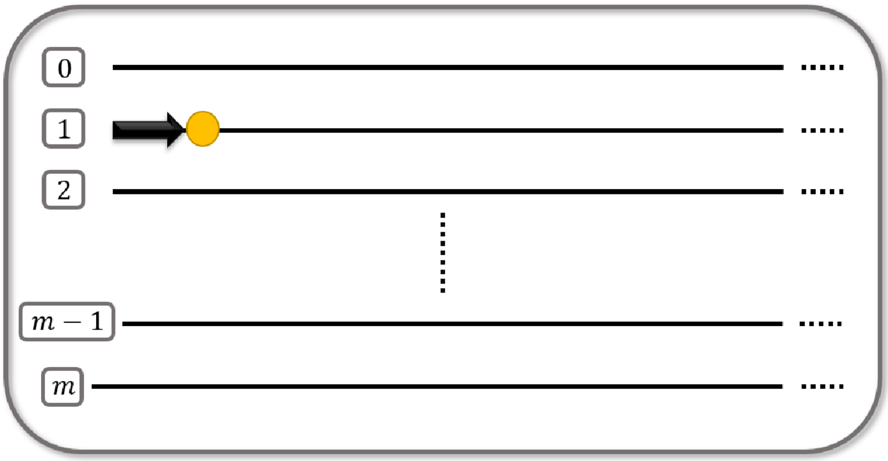

Figure 1.1: Rows correspond to possible paths of the photon

- The beam splitters in our interferometers, can be expressed generally by $|r|^2 + |t|^2 = 1$, where r and t represent the reflection and transmission coefficients, respectively. As operation with $\hat{H} = (\hat{a}^{\dagger}{}_k\hat{a}_z - \hat{a}^{\dagger}{}_z\hat{a}_k)$, it acts on the state (1.2), where k and z correspond to a specific modes.
- The evolution of a closed quantum state is a unitary process. The evolution is described by the time-dependent Schrödinger equation: $i\hbar\partial_t|\psi\rangle = \hat{H}|\psi\rangle$, where $\hat{H}$ is the Hamiltonian operator of the system and ∂_t is the partial time-derivative. Solution of this equation leads to : $|\psi, t\rangle = \hat{U}|\psi, 0\rangle$, where $\hat{U} = e^{-i\hat{H}t}$.
- The phases of the matrix elements are adjustable as long as they satisfy the unitary property, $\hat{U}^{\dagger}\hat{U} = 1$.
- We can define the output wave function after going through n beam splitter by $|\psi_{out}\rangle = \hat{U}_n \ ... \ \hat{U}_2\,\hat{U}_1\,|\psi_{in}\rangle$.

Chapter 2

Quantum Zeno Effect

The quantum Zeno effect, also known as the quantum Zeno paradox, is a quantum mechanical phenomenon that describes the behavior of an unstable particle that is being continuously observed. The effect is named after the Greek philosopher Zeno of Elea, who postulated that an arrow in flight could never reach its target because an infinite number of observations would need to occur in order to measure its position, thus preventing it from ever moving.

In the quantum mechanical context, the Zeno effect is a consequence of the wave-particle duality, which states that particles can exhibit both wave-like and particle-like behavior. When a particle is observed, its wave function collapses into a definite position, effectively freezing its motion. If a particle is observed continuously, its wave function will remain collapsed, and it will not change its position [17].

The effect was initially proposed by Misra and Sudarshan [2] through the decay of an unstable state. To demonstrate the fundamental characteristics of the quantum Zeno effect, we shall begin with a straightforward model: We have a certain observable $\hat{O}$ of a system $\mathcal{S}$ that is being observed repeatedly and the initial state of which is the n-th eigenstate $|\psi_n\rangle$ of $\hat{O}$. We will treat the repeated measurements as a limiting scenario of discrete measurements separated by very small time intervals τ. We know that between two measurements (in the interval τ) the evolution of $\mathcal{S}$ is governed by the Schrödinger equation $i\hbar\partial_t|\psi\rangle = \hat{H}|\psi\rangle$ where $\hat{H}$ is the Hamiltonian operator of the system and ∂_t is the partial time-derivative.

This evolution causes $\mathcal{S}$ with some probability to have a transition from the eigenstate $|\psi_n\rangle$ to some other state. The first interval begins at *t* = 0, and for sufficiently small times τ we may expand the Schrödinger equation into a power series, arriving, before the first measurement,

$$|\psi(\tau)\rangle \simeq \left[1 + \frac{\hat{H}\tau}{i\hbar} + \frac{1}{2}\left(\frac{\hat{H}\tau}{i\hbar}\right)^2 + \dots\right]|\psi_n\rangle, \quad (2.1)$$

$\hat{H}$ is the Hamiltonian of the evolution. The probability that the system is still in the *n*-th eigenstate, which means that no transition has occurred is given by

$$P_{nn} = |\langle\psi_n|\psi(\tau)\rangle|^2 \simeq \left|1 + \frac{\tau}{i\hbar}\langle\psi_n|\hat{H}|\psi_n\rangle - \frac{\tau^2}{2\hbar^2}\langle\psi_n|\hat{H}^2|\psi_n\rangle\right|^2$$

$$\simeq 1 - \frac{\tau^2}{\hbar^2}\left[\langle\psi_n|\hat{H}^2|\psi_n\rangle - \langle\psi_n|\hat{H}|\psi_n\rangle^2\right], \quad (2.2)$$

until the second order in τ . The quantity in square brackets is the variance of the energy in the initial state $|\psi_n\rangle$, we can rewrite the whole expression

$$P_{nn} = 1 - \frac{\tau^2}{\hbar^2}(\Delta E)^2, \quad (2.3)$$

We can repeat the same procedure *j* times. At the end we will obtain

$$P_{nn}\ (j\tau) = P_{nn}{}^{j}(\tau) = \left[1 - \frac{\tau^2}{\hbar^2}(\Delta E)^2\right]^j, \quad (2.4)$$

where $j\tau$ is the total time for the j steps.

For very small τ . In the limit of vanishing τ relative to $(\frac{\Delta E}{\hbar})$, the probability that no transition happens is

$$P_{nn}(j\tau) = 1, \quad (2.5)$$

So basically the system is "frozen" in the initial state $|\psi_n\rangle$. In conclusion, when any observable with discrete spectrum is continuously monitored with infinite accuracy, the system is "forced" to remain in the initial state. [17]

Just to make it easier to understand how this effect is used in the counterfactual protocol, we will use an example. Let us consider a quantum state consisting of a single qubit. The qubit starts at $|0\rangle$, and at every single step we are going to rotate it toward $|1\rangle$ by $\vartheta = \pi/2N$. After one rotation, we will have

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \tag{2.6}$$

Where $\alpha = \cos\vartheta$ and $\beta = \sin\vartheta$. After second step we will get

$$|\psi\rangle = \cos(2\vartheta)\,|0\rangle + \sin(2\vartheta)\,|1\rangle, \tag{2.7}$$

And finally after *N* steps we obtain

$$|\psi\rangle = \cos(N\vartheta)\,|0\rangle + \sin(N\vartheta)\,|1\rangle, \tag{2.8}$$

Then the final state after *N* steps is goin to be $|1\rangle$.

Now if we decide to measure the state after each rotation, we will get different results. For example after the first rotation, we measure $|0\rangle$ with very high probability, but this measurement will collapse the state back to $|0\rangle$. Due to the fact that $\left[\cos^2\left(\frac{\pi}{2N}\right)\right]^N \simeq \left[1 - \left(\frac{\pi}{2N}\right)^2\right]^N$, we see that after each measurement we are left with $|0\rangle$.

In conclusion, the Quantum Zeno Effect claimes that making continuous measurments will delay the transition of a state to another by repeatedly collapsing the qubit back to the original state.

Chapter 3

Interaction-free measurement

In this chapter we will analyze a experimental set up which takes us one step further at comprehending counterfactual communication, by introducing the interaction-free measurement.

3.1 The Mach-Zehnder interferometer

The Mach-Zender interferometer [12], which we will be examining, is comprised of two 50:50 beam splitters, two mirrors, and two detectors, as depicted in figure 3.1. To begin with, we will provide a description of this setup using classical optics. Essentially, the input light beam is partially reflected and partially transmitted by the beam splitters.

The source emits a beam of light that is divided into two segments, the upper and lower, by the first beam splitter. These segments are reflected by the mirrors and then reunited at the second beam splitter, which directs them to detectors D1 and D2. In order to generate a relative phase difference φ between the two constituent light beams, we introduce a phase shifter into the upper path At the second beam splitter the two beams interfere and such interference can be destructive if $\varphi = \pi$ or constructive if $\varphi = 0$ (or a multiple of 2π). The transmission and reflection coefficients *t* and *r* of the beam splitters can vary between 0 and 1, with the condition $r^2 + t^2 = 1$. All the devices in this setup are linear. And the output is proportional to the input. [10], [12], [17].

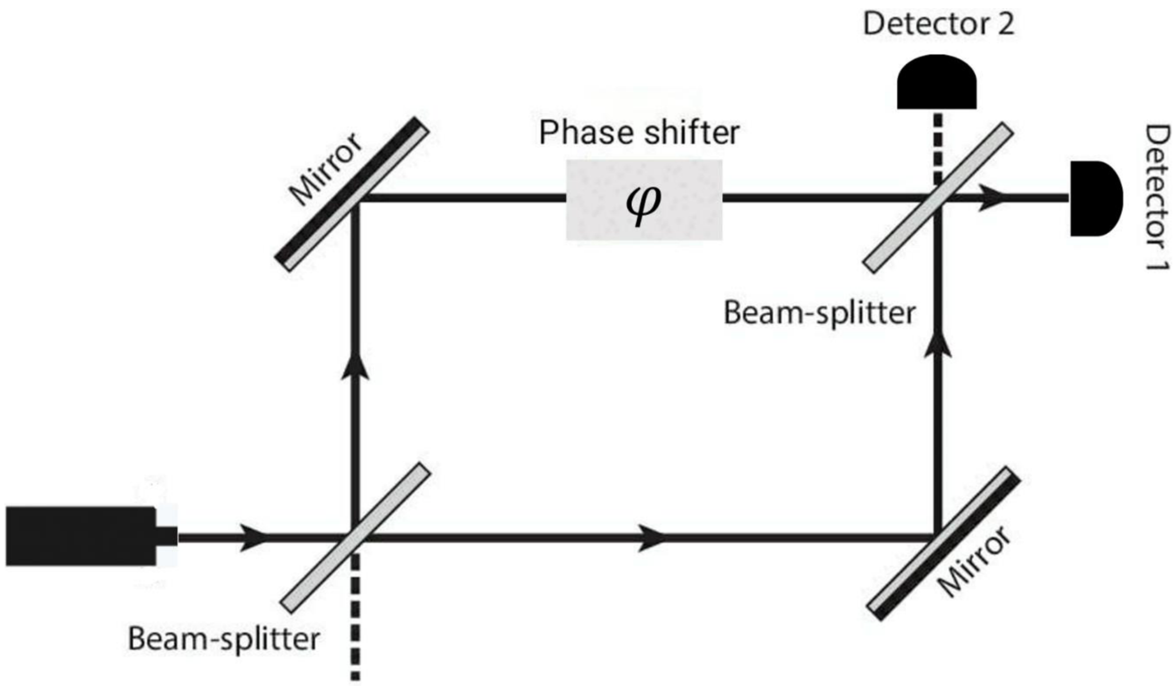

Figure 3.1: The Mach-Zehnder interferometer optical set up with a phase shifter

3.2 The Elitzur–Vaidman Bomb (EVB)

Elitzur and Vaidman proposed interaction-free measurements as a means of locating infinitely delicate objects without causing harm to them. Our objective is to ascertain whether a Bomb (B) is situated in the upper path [3], [15] as shown in figure 3.2. The Mach-Zehnder interferometer serves as the foundation for the EV method. Symmetric beam splitters, BS1 and BS2, are employed in this method. If the phase difference φ is zero beyond BS2, constructive interference ensures that all photons are detected by detector D1, while D2 remains unresponsive due to destructive interference. Only one photon is present in the interferometer at any given time. Let us suppose that B is present in path. Then, we have three possible outcomes of the experiment:

1. No detector clicks

2. D1 clicks

3. D2 clicks

There are three possible outcomes in this scenario. In the first scenario, with a 50% probability, the photon is absorbed by the Bomb and subsequently triggers an explosion. In the second scenario, which has a 25% probability of occurrence, the photon arrives at detector D1 (it could have also arrived at D1 if the Bomb was not present, with a probability of 1). The third scenario has a probability of 25% corresponding to click of D2. If we conduct this test and detector D2 registers a click, we deduce that the Bomb is located in the upper path without any interaction with it. Detector D2 will only click if one of the arms is obstructed by the Bomb.

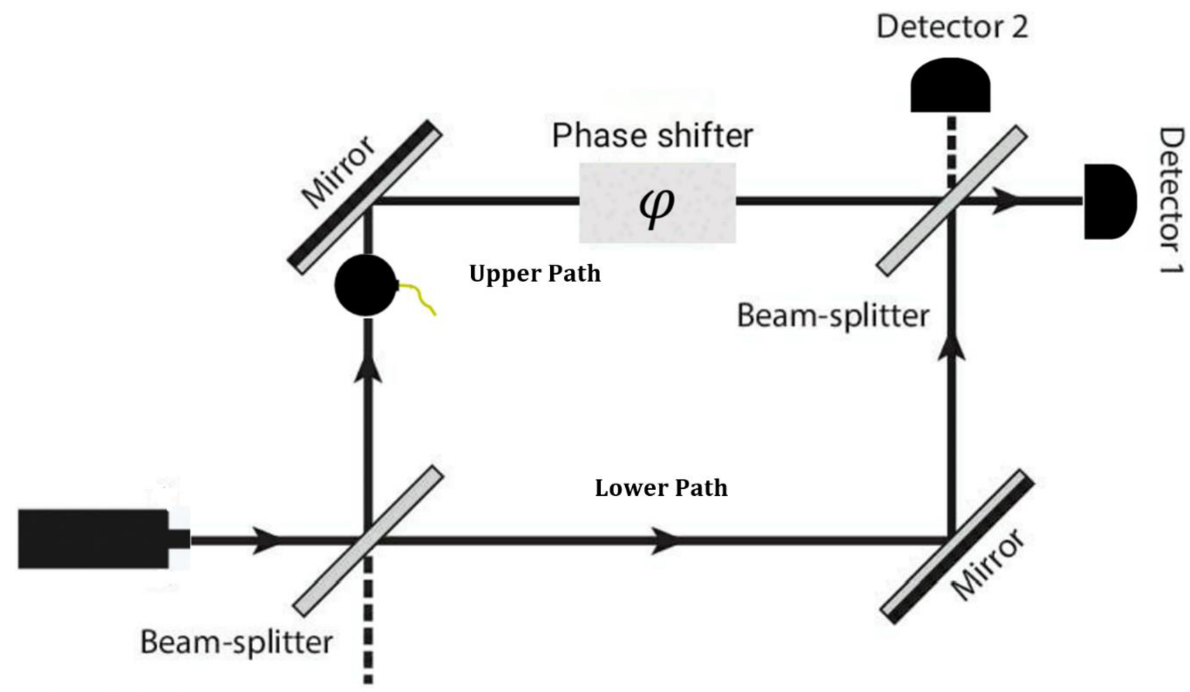

Figure 3.2: The Mach-Zehnder interferometer optical set up with a phase shifter and a Bomb

A new notation can be introduced here also as an alternative method to determine which detector will click in this situation.

When a photon passes through the MZI, it can be viewed as a quantum system with two states. If we select a basis where the upper path represents the photon's state, then it can be denoted as $|\uparrow\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ and the state of the photon in the lower path is denoted by $|\downarrow\rangle = \begin{pmatrix}0\\1\end{pmatrix}$. And we arbitrarily denote the initial state of the photon emitted from the source as $|I\rangle = \begin{pmatrix}1\\0\end{pmatrix}$. The state of the photon propagating towards detector D1 as path state is $|\mathrm{D1}\rangle = \begin{pmatrix}0\\1\end{pmatrix}$ and the state of the photon propagating towards detector D2 as the path state is $|\mathrm{D2}\rangle = \begin{pmatrix}1\\0\end{pmatrix}$.

The matrix representations of the quantum mechanical operators that correspond to beam-splitter 1 (U_1), beam-splitter 2 (U_2), Bomb (*B*) in the upper path and a phase shifter in the upper path (U_φ) when the basis vectors are chosen in the order $|\uparrow\rangle$, $|\downarrow\rangle$ are:

$$U_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}, \qquad U_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix},$$

$$B = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \qquad U_\varphi = \begin{pmatrix} e^{i\varphi} & 0 \\ 0 & 1 \end{pmatrix}, \qquad (3.2.1)$$

If $\varphi = 0$ then the phase shifter is simply the unitary matrix and has no effect. To see the probability of detectors clicking when bomb is/is not in the channel we use the following method by taking the final state of the photon as $|\mathrm{F}\rangle$:

$$\mathrm{P_{D1}} = |\langle \mathrm{D1}|\mathrm{F}\rangle|^2 = \left|\langle \mathrm{D1}\middle|U_2 U_\varphi B U_1\middle|I\rangle\right|^2 = \frac{1}{4}, \qquad (3.2.2)$$

$$\mathrm{P_{D2}} = |\langle \mathrm{D2}|\mathrm{F}\rangle|^2 = \left|\langle \mathrm{D2}\middle|U_2 U_\varphi B U_1\middle|I\rangle\right|^2 = \frac{1}{4}, \qquad (3.2.3)$$

$$\mathrm{P_{D1}} = |\langle \mathrm{D1}|\mathrm{F}\rangle|^2 = \left|\langle \mathrm{D1}\middle|U_2 U_\varphi U_1\middle|I\rangle\right|^2 = 1, \qquad (3.2.4)$$

$$\mathrm{P_{D2}} = |\langle \mathrm{D2}|\mathrm{F}\rangle|^2 = \left|\langle \mathrm{D2}\middle|U_2 U_\varphi U_1\middle|I\rangle\right|^2 = 0, \qquad (3.2.5)$$

As shown in Figure 3.3, if our set up consists of a series of *N* interferometers, with *N* being large, then we get better results. The reflectivity $|r|^2$ of each of the *N* BSs is chosen to be $\cos^2(\pi/2N)$ and the relative phases between corresponding paths in the upper and lower halves to be zero. [16]

The outcome is that the amplitude of the particle transfers from the lower to the upper halves of the interferometers.

Upon the conclusion of all N cycles, the photon will exit via the upper outlet. Subsequently, we install a sequence of Bombs in the top portion of the equipment, which hinders the interference. At each beam splitter, there is a very small probability that the photon takes the upper path and triggers a Bomb, and a large probability $\cos^2(\pi/2N) \simeq 1 - \frac{\pi^2}{4N^2}$ that it continues to travel on the lower path. The final probabilities of the detectors clicking after *N* cycles are

$$\mathrm{P_{D2}} = |\langle \mathrm{D2}|\mathrm{F}\rangle|^2 = \left|\left\langle \mathrm{D2}\left|\left[U_2 U_\varphi B U_1\right]^N\right|I\right\rangle\right|^2 = \left(\cos^2(\pi/2N)\right)^N \simeq \left(1-\frac{\pi^2}{4N^2}\right)^N$$

$$\simeq 1 \tag{3.2.6}$$

$$\mathrm{P_{D1}} = |\langle \mathrm{D1}|\mathrm{F}\rangle|^2 = \left|\left\langle \mathrm{D1}\left|\left[U_2 U_\varphi U_1\right]^N\right|I\right\rangle\right|^2 = 1, \tag{3.2.7}$$

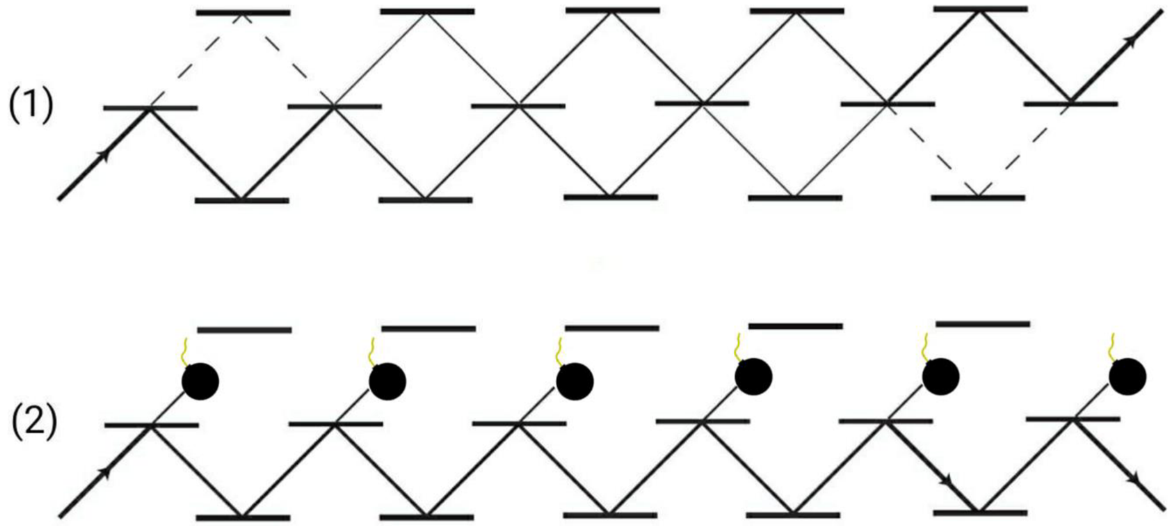

Figure 3.3: (1) The set up consists of a series of *N* interferometers. (2) Inserted *N* Bombs

This tells us the probability that we can detect the presence of the Bomb without interacting with it which for large *N* is almost equal to 1. The probability that the photon will be absorbed by the Bomb is very low and is given by $1 - P$.

Chapter 4

Counterfactual Quantum Communication (CQC)

In this chapter, we will explore the counterfactual quantum communication protocol, which allows Alice and Bob to communicate without using any physical particle. By increasing the number of cycles, the transmissivity of the beam splitters also increases. By leveraging the bomb principle and creating a specific setup, we can attain counterfactual communication.

4.1 CQC setup with open channel

Initially, we introduce the protocol proposed by Salih [1]. We will proceed with a situation where Bob keeps his end of the channel unobstructed. This will facilitate the setup for the photon particle. Our setup will include a sequence of N interferometers, as illustrated in chapter 3.

Alice initiates the process by transmitting a photon in the input state of $|10\rangle$. The alteration of the state that occurs at the beam splitters is outlined as follows

$$|10\rangle \rightarrow \cos(\theta)\,|10\rangle + \sin(\theta)\,|01\rangle, \tag{4.1.1}$$

And

$$|01\rangle \rightarrow \cos(\theta)\,|01\rangle - \sin(\theta)\,|10\rangle, \tag{4.1.2}$$

Where $\cos\theta = \sqrt{R}$ with R being the reflectivity of the BS and $\theta = {}^{\pi}/_{2N}$.

By leaving his end of the channel unobstructed, Bob enables Alice's photon to propagate coherently. After n cycles, the photon's state can be expressed as

$$|10\rangle \rightarrow \cos(n\theta)\,|10\rangle + \sin(n\theta)\,|01\rangle, \tag{4.1.3}$$

Thus, at the end of N cycles ($n = N$), the final state is $|01\rangle$ and the detector D2 clicks. In this scenario when Bob keeps the channel open, the photon's final state is $|01\rangle$.

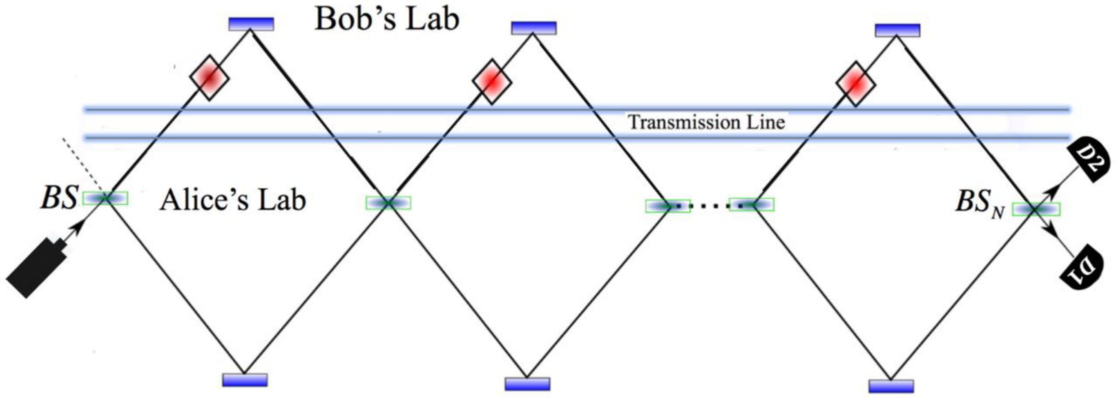

Figure 4.1: A series of N interferometers, while Bob switches ON/OFF the Block

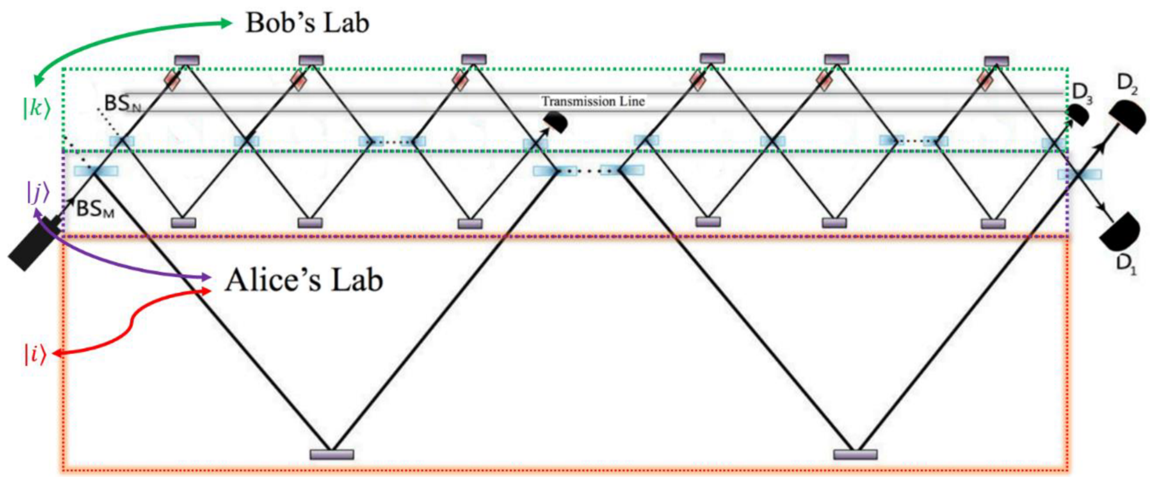

Figure 4.2: Fully counterfactual communication, while Bob switches ON/OFF the Block

Salih suggested a protocol that will lead not only to direct communication between Alice and Bob but is also counterfactual. Using the chained version of the quantum Zeno effect, as shown in figure 4.2 The signal photon passes through M big cycles separated by BS_Ms with $\theta_M = {}^{\pi}/_{2M}$. For the *m*th cycle $(m \leq M)$, there are *N* beam splitters BS_Ns with $\theta_N = {}^{\pi}/_{2N}$.

In Figure 4.1, Alice sends a solitary photon, with all unused ports in the vacuum state. As a result of beam splitter transformations, we will have three photon states $|i, j, k\rangle$ where $|i\rangle$, $|j\rangle$, and $|k\rangle$ correspond to the photon states at different sides of the chains as shown in figure 4.2, respectively. So if Bob passes Alice's photon, for the *m*-th cycle, we obtain

$$|010\rangle \rightarrow \cos(n\theta_N)\,|010\rangle + \sin(n\theta_N)\,|001\rangle \xrightarrow{n=N} |001\rangle, \qquad (4.1.4)$$

The initial state of the total system is $|100\rangle$. When Bob does not block the channel (logic 0). After the *m*-th cycle, the resulting photon state is

$$|100\rangle \rightarrow cos^{m-1}\theta_M\,(\cos(\theta_M)\,|100\rangle + \sin(\theta_M)\,|010\rangle) \xrightarrow{m=M} |100\rangle, \qquad (4.1.5)$$

After M big cycles and *N* small cycles, detector D1 clicks. A click at the detector D1 ensures counterfactuality as any photon in the transmission channel would lead to a click at one of the detectors after the transmission channel.

The probability of a click at D1 is obtained by collecting all the contributions that are reflected from all the beam splitters BS_Ms and is given by

$$P_1 = \cos^{2M}\theta_M, \qquad (4.1.6)$$

We can aproach the calculations of the probability of a certain detector clicking using the following equations and matrices to understand how exactly quantum states change, which shows the effects of all the beam splitters and blocks and mirrors on the initial state of our photon. The following equation are what we used in our Python/Matlab codes to create our figurs and results.

We will start by introducing the matrix representation of beam splitters in which each basis represents the evolution of a certain mode. We start with the outer beam splitters

$$U_{M_n} = \begin{pmatrix} 1 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & t_n & \cdots & r_n & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & -r_n & \cdots & t_n & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 \end{pmatrix}_{m\times m} , \qquad (4.1.7)$$

Where n is the number of the outer cycles and $m = n + 3$ is the dimension of the matrices (number 3 represents the number of initial measurable modes starting with one cycle and everytime we move on to the next cycle we add the new measurable mode. Although this pattern will change in the following chapters) with $|r|^2 + |t|^2 = 1$, where $r = \sin(\theta)$ and $t = \cos(\theta)$ represent the reflection and transmission coefficients, respectively. And also if the elements of the matix is represented by $U_{M_{n_{ij}}}$ and $i, j \in \mathbb{N}$ then we have $U_{M_{n_{22}}} = t_n$, $U_{M_{n_{23}}} = r_n$, $U_{M_{n_{32}}} = -r_n$ and $U_{M_{n_{33}}} = t_n$. And the matric representaion of the inner beam splitters is

$$U_{N_l} = \begin{pmatrix} 1 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & t_l & \cdots & r_l & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & -r_l & \cdots & t_l & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 \end{pmatrix}_{m\times m} , \qquad (4.1.8)$$

Where l is the number of the inner beam splitters. And also if the elements of the matix is represented by $U_{N_{l_{ij}}}$ and $i, j \in \mathbb{N}$ then we have $U_{N_{l_{33}}} = t_l$, $U_{N_{l_{3,n+3}}} = r_l$, $U_{N_{l_{n+3,3}}} = -r_l$ and $U_{N_{l_{n+3,n+3}}} = t_l$. And using the matric representaion of the Block (Bomb),

$$B_n = \begin{pmatrix} 1 & \cdots & 0 & \cdots & 1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 1 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 \end{pmatrix}_{m\times m} , \qquad (4.1.9)$$

where based on the number of outer beam splitters, it will block a certain quantum state. And also if the elements of the matix is represented by $B_{n_{ij}}$ and $i, j \in \mathbb{N}$ then we have $B_{n_{n+3,n+3}} = 0$ and $B_{n_{1,n+3}} = 1$.

Now to calculate the probabilities of the detectors clicking as shown in figure 4.5, we will use the following equation and equation (1.3) by following $m = n + 3$ to calculate the final state of the photon.

$$|\psi_{out}\rangle = U_{M_n} U_{N_l} \dots U_{N_2} U_{N_1} U_{M_1} |\psi_{in}\rangle, \quad (4.1.10)$$

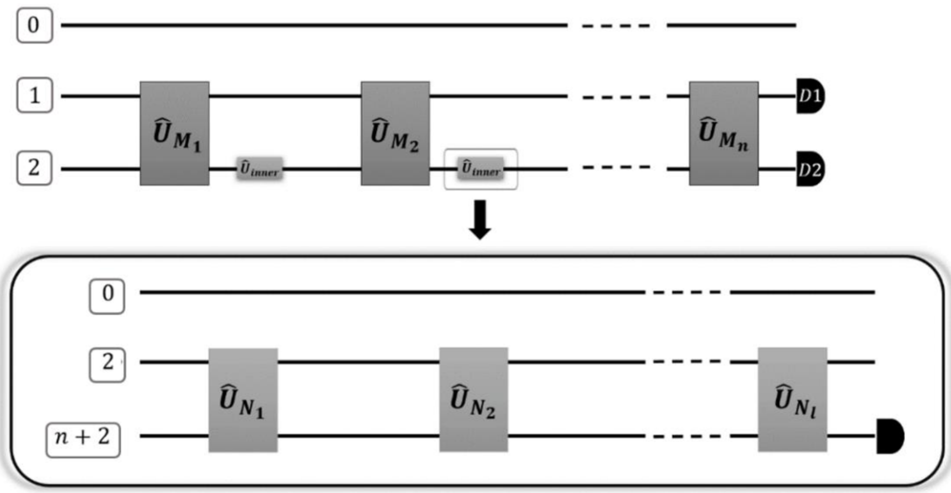

Figure 4.3: Counterfactual setup with open channel (Logic 0)

4.2 CQC setup with blocked channel

Again we start with Salih's approach. If Bob blocks the photon in the scenario with series of *N* interferometers [1], the photonic state after *n* cycles is

$$|10\rangle \rightarrow cos^{n-1}\theta\, (\cos(\theta)\, |10\rangle + \sin(\theta)\, |01\rangle) \approx |10\rangle, \quad (4.2.1)$$

where we assumed *N* to be large and $\cos^N \approx 1$. Here the square of the overall factor ($\cos^{2(n-1)}\theta$) represents the probability of having the state $|10\rangle$ after $n - 1$ cycles. In this case the photon is reflected and the detector D1 clicks as shown in figure 4.1.

The Mach-Zehnder arrangement illustrated in figure 4.3 facilitates counterfactual communication. However, it is important to note that the protocol's counterfactuality is only partial. If Bob chooses not to block the channel, the counterfactual nature of the communication will be lost.

In the new protocol if Bob blocks the channel (logic 1), we have (for the m-th cycle)

$$|010\rangle \rightarrow \cos^{n-1}\theta_N \left(\cos(\theta_N)\,|010\rangle + \sin(\theta_N)\,|001\rangle\right) \xrightarrow{n=N} |010\rangle, \qquad (4.2.2)$$

where we assume $N \gg 1$. After the m-th cycle, the photon state is

$$|100\rangle \rightarrow \cos(m\theta_M)\,|100\rangle + \sin(m\theta_M)\,|010\rangle \xrightarrow{m=M} |010\rangle, \qquad (4.2.3)$$

Here also the signal photon passes through M big cycles separated by BS_Ms with $\theta_M = \pi/_{2M}$. For the mth cycle $(m \leq M)$, there are N beam splitters BS_Ns with $\theta_N = \pi/_{2N}$.

Thus after M big cycles and N small cycles, detector D2 clicks as shown in figure 4.4. Again counterfactuality is ensured by a click at D2 as any photon in the transmission channel would be absorbed by the blocking device and would not be available for detection at D2.

But the method we used in this thesis for our calculations using matrices (1.3), (4.1.7), (4.1.8) and (4.1.9) is

$$|\psi_{out}\rangle = U_{M_n} U_{N_l} B_n \dots U_{N_2} B_1 U_{N_1} U_{M_1} |\psi_{in}\rangle, \qquad (4.2.4)$$

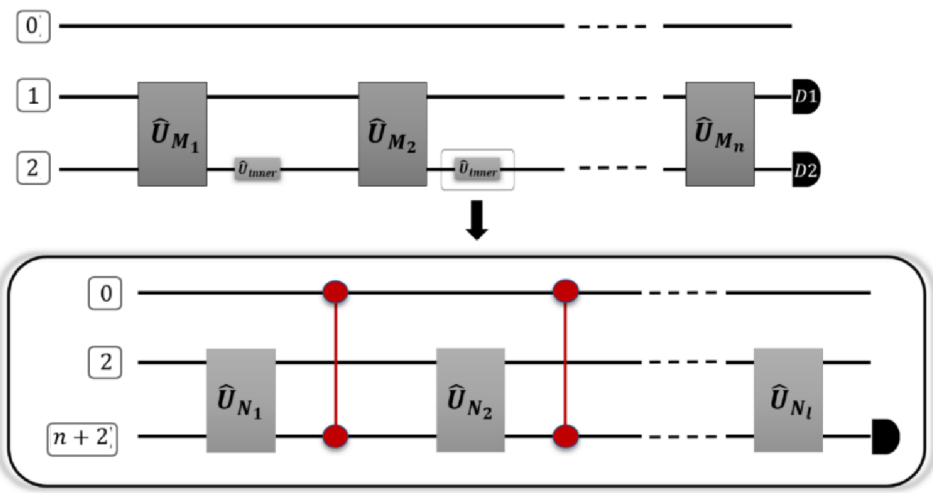

Figure 4.4: Counterfactual setup with closed channel (Logic 1)

And as for the probabilities of the detection of the photon , whether Bob is blocking the channel or not, we have the following figure

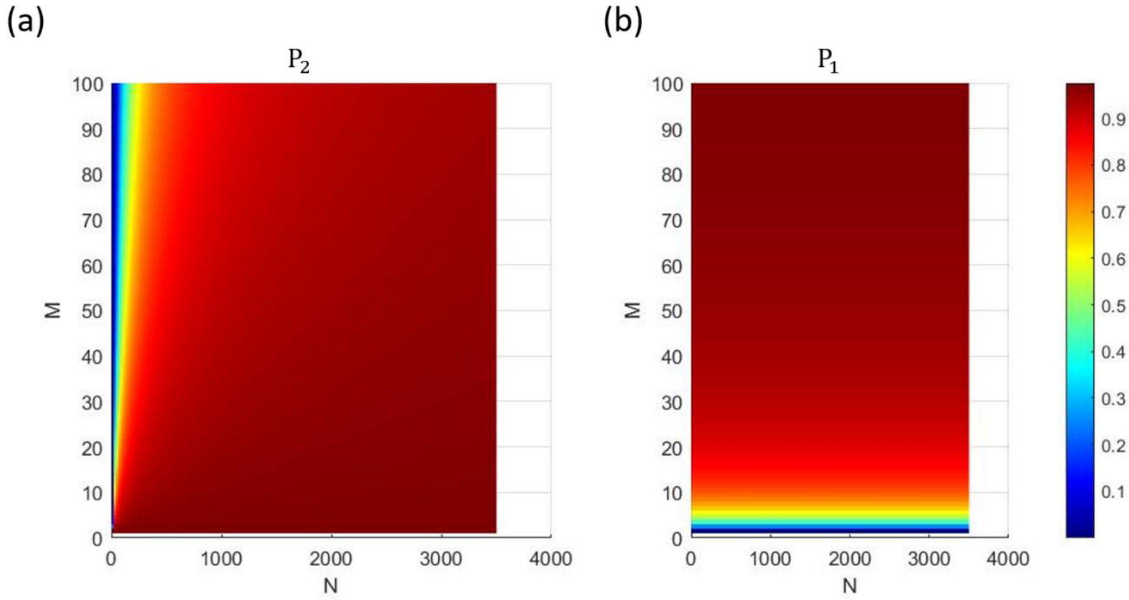

Figure 4.5: The success probability of the (a) logical 1 process and (b) logical 0

Chapter 5

Decoherence and Information loss

In this particular chapter, the primary focus will be on the analysis of information loss which occurs as a result of coupling the system with additional beam splitters that are inserted into the transmission channel. However, before delving into the crux of the matter, it is essential to discuss the concept of decoherence. In simple terms, decoherence refers to the transition that takes place from quantum to classical. The notion of decoherence came about as a solution to the interpretation problem in quantum mechanics. It provides a mechanism to impose an effective classicality on quantum systems.

In quantum information, decoherence plays a critical role in two aspects. The role of measurement is to convert quantum states and quantum correlations into classical, definite outcomes. Decoherence results in the environment-induced superselection (einselection) that validates the existence of preferred pointer states. Consequently, it enables the effective establishment of a boundary between the quantum and classical in simple terms, which does not require the appeal of the "collapse of the wave packet." The formal tool that quantum theory provides for this occasion is the density matrix, which can be employed to describe the probability distribution over alternative outcomes. [18]

Within this chapter, we shall utilize beam splitters as a mechanism for the representation of losses. The phenomena of loss can be attributed to several factors, including absorption, scattering, and other similar occurrences.

5.1 Error of information loss

The subsequent subchapter will delve into the analysis of the photon loss probability, which serves as a crucial metric in assessing the effectiveness and reliability of a communication system. This analysis provides insights into the likelihood of photons carrying valuable information being lost or absorbed during transmission, thereby impacting the overall efficiency and success of the communication process.

Photon loss probability refers to the possibility that the photons carrying the critical information may not reach their intended destination, leading to a loss of data. This phenomenon can occur due to various factors, such as the quality of the transmission channel, external interference, and the use of specific components, such as beam splitters, that may cause photon absorption or loss.

It is essential to understand the significance of photon loss probability and its implications on the overall effectiveness of the communication system. High photon loss probability can lead to an increase in errors and data loss, resulting in the need for retransmission or resending of information, thereby leading to a decrease in the efficiency of the system. Additionally, high photon loss probability can impact the security of the communication system, making it vulnerable to potential attacks and breaches.

Therefore, the analysis of photon loss probability plays a vital role in evaluating the efficiency and reliability of a communication system, allowing for the identification of potential weaknesses and areas of improvement. By understanding the factors that contribute to photon loss probability and implementing measures to mitigate them, it is possible to enhance the effectiveness and reliability of communication systems.

First we study Information loss with blocked channel. Now we will add extra beam splitters to the setup as shown in figure 5.1 and 5.2. These extra beam splitters are place in the transmission channel. Our goal for doing this is to analyze the rate of information loss of the protocol.

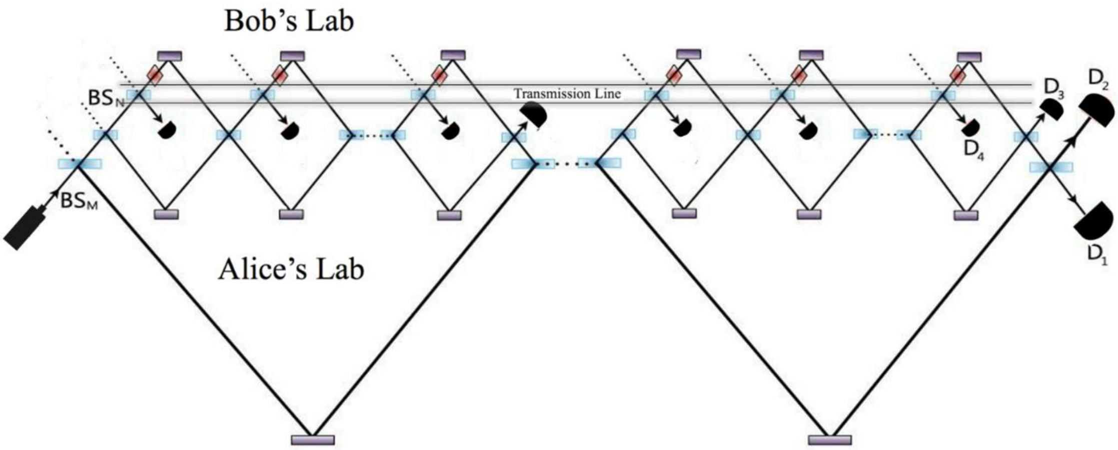

Figure 5.1: Added extra beam splitters to the CQC setup with (Blocked/Open) channel

First we will start by introducing the matric representation of the outer beam splitters again

$$U_{M_n} = \begin{pmatrix} 1 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & t_n & \cdots & r_n & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & -r_n & \cdots & t_n & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 \end{pmatrix}_{m\times m}, \quad (5.1.1)$$

Where n is the number of the outer cycles and $m = (l \times n) + (n-1) + 3$ is the dimension of the matrices with $|r|^2 + |t|^2 = 1$, where $r = \sin(\theta)$ and $t = \cos(\theta)$ represent the reflection and transmission coefficients, respectively. And also if the elements of the matix is represented by $U_{M_{n_{ij}}}$ and $i, j \in \mathbb{N}$ then we have $U_{M_{n_{22}}} = t_n$, $U_{M_{n_{23}}} = r_n$, $U_{M_{n_{32}}} = -r_n$ and $U_{M_{n_{33}}} = t_n$. And the matric representaion of the inner beam splitters is

$$U_{N_l} = \begin{pmatrix} 1 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & t_l & \cdots & r_l & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & -r_l & \cdots & t_l & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 \end{pmatrix}_{m\times m}, \quad (5.1.2)$$

Where l is the number of the inner cycles per outer cycle. And also if the elements of the matix is represented by $U_{N_{l_{ij}}}$ and $i, j \in \mathbb{N}$ then we have $U_{N_{l_{33}}} = t_l$, $U_{N_{l_{3,q}}} = r_l$, $U_{N_{l_{q,3}}} =$

$-r_l$ and $U_{N_{l_{qq}}} = t_l$ where $q = \big((l+1) \times n - (l+1)\big) + 4$ which we use to find the desired matrix element here. And using the matric representaion of the Block (Bomb)

$$B_n = \begin{pmatrix} 1 & \cdots & 0 & \cdots & 1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 1 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 \end{pmatrix}_{m\times m} , \tag{5.1.3}$$

Where based on the number of outer beam splitters, it will block a certain quantum state. And also if the elements of the matix is represented by $B_{n_{ij}}$ and $i,j \in \mathbb{N}$ then we have $B_{n_{n+3,n+3}} = 0$ and $B_{n_{1,n+3}} = 1$. And the matric representation of the extra beam splitters is

$$U_E = \begin{pmatrix} 1 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & \eta & \cdots & \delta & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & -\delta & \cdots & \eta & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 \end{pmatrix}_{m\times m} , \tag{5.1.4}$$

Where $|\eta|^2 + |\delta|^2 = 1$, and δ and η represent the reflection and transmission coefficients. And also if the elements of the matix is represented by $U_{E_{n_{ij}}}$ and $i,j \in \mathbb{N}$ then we have $U_{E_{qq}} = \eta$, $U_{E_{q,q+l}} = \delta$, $U_{E_{q+l,q}} = -\delta$ and $U_{E_{q+l,q+l}} = \eta$.

Now to calculate the probabilities of the detectors clicking as shown in figure 5.2 and we will see the dependance between the detector D1 and D2 and η and δ, we will use the following equation to calculate the final state of the photon with the initial state $|\psi_{in}\rangle$ (1.3).

$$|\psi_{out}\rangle = U_{M_n} U_{N_l} B_n U_{E_s} \ldots U_{N_2} B_1 U_{E_1} U_{N_1} U_{M_1} |\psi_{in}\rangle, \tag{5.1.5}$$

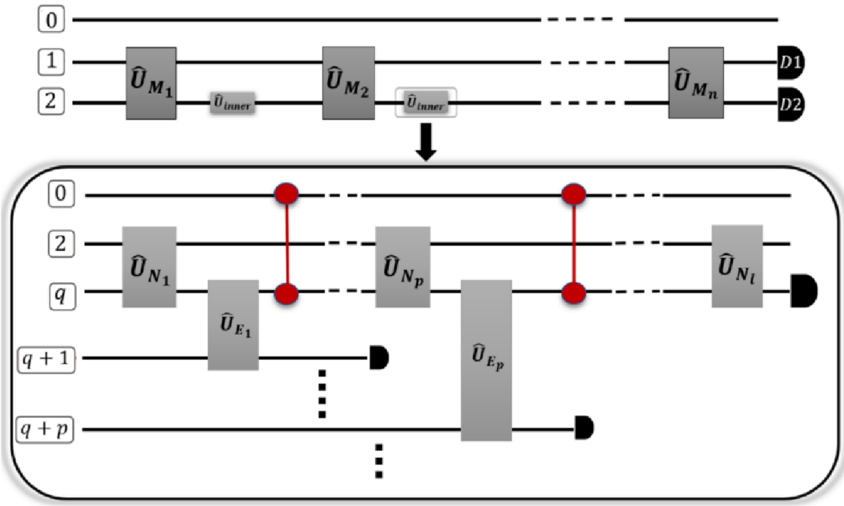

Figure 5.2: Analysis of the losses with closed channel (Logic 1), where $q = \big((l+1)\times n-(l+1)\big)+3$

In the case of information loss with open channel also shown in figure 5.3 and using the same matrices that were introduced in the privious sub chapter we can calculate the probabilities of the detectors clicking as shown in figure 5.4 when the channel is open and we will see the dependance between the detector D1 and D2 and η and δ, we will use the following equation to calculate the final state of the photon.

$$|\psi_{out}\rangle = U_{M_n}U_{N_l}U_{E_s}\dots U_{N_2}U_{E_1}U_{N_1}U_{M_1}|\psi_{in}\rangle, \tag{5.1.6}$$

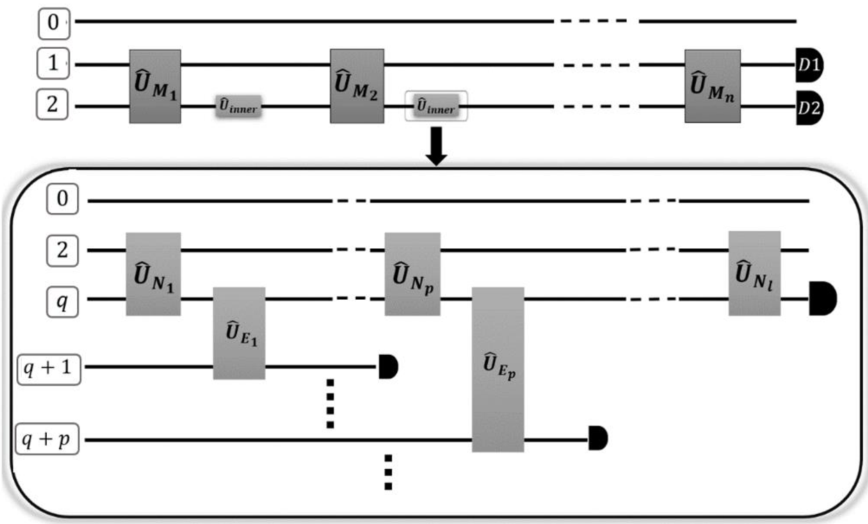

Figure 5.3: Analysis of the losses with open channel (Logic 0), where $q = \big((l+1)\times n-(l+1)\big)+3$

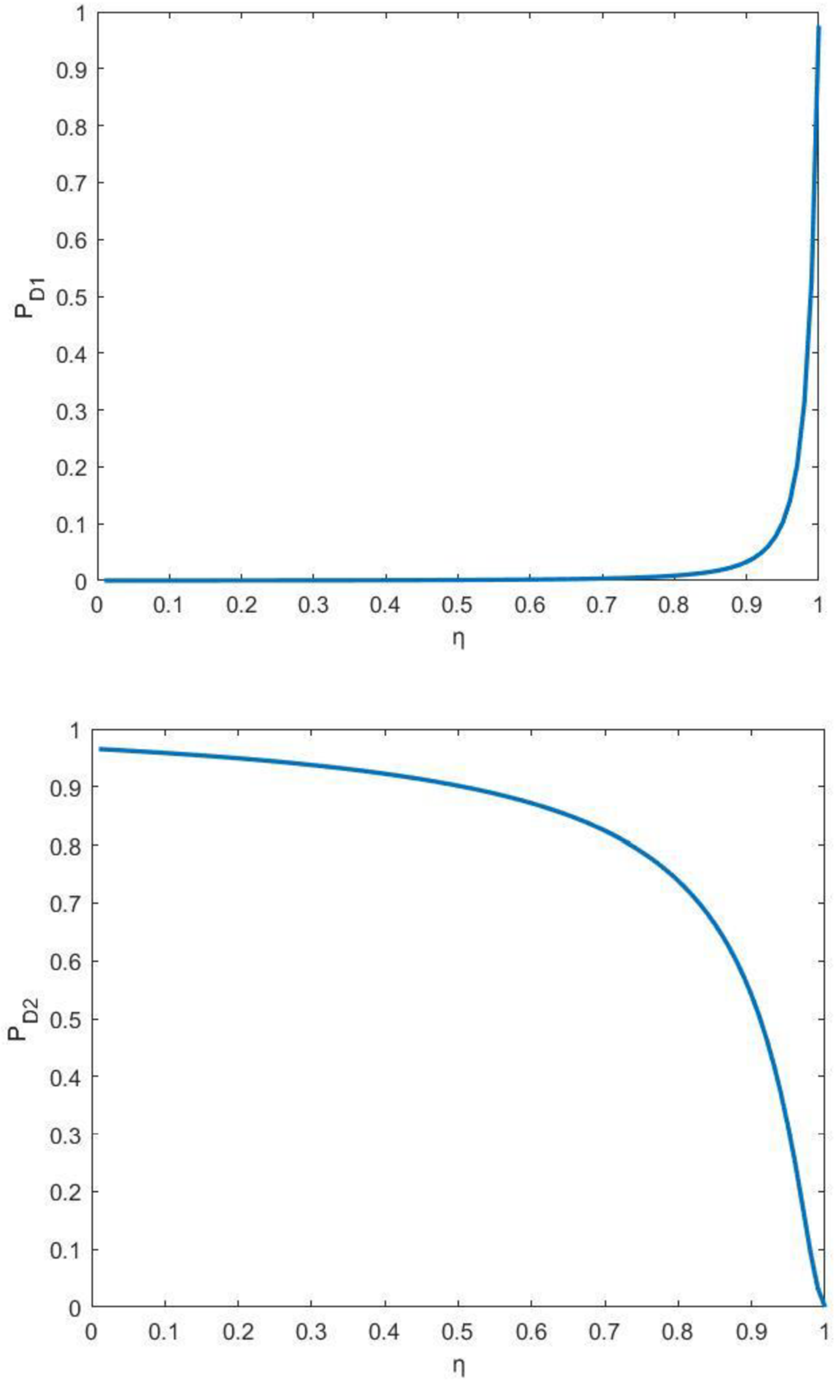

Figure 5.4: Change in photon detection for D1 and D2 as η changes for M = 100 and N = 3500.

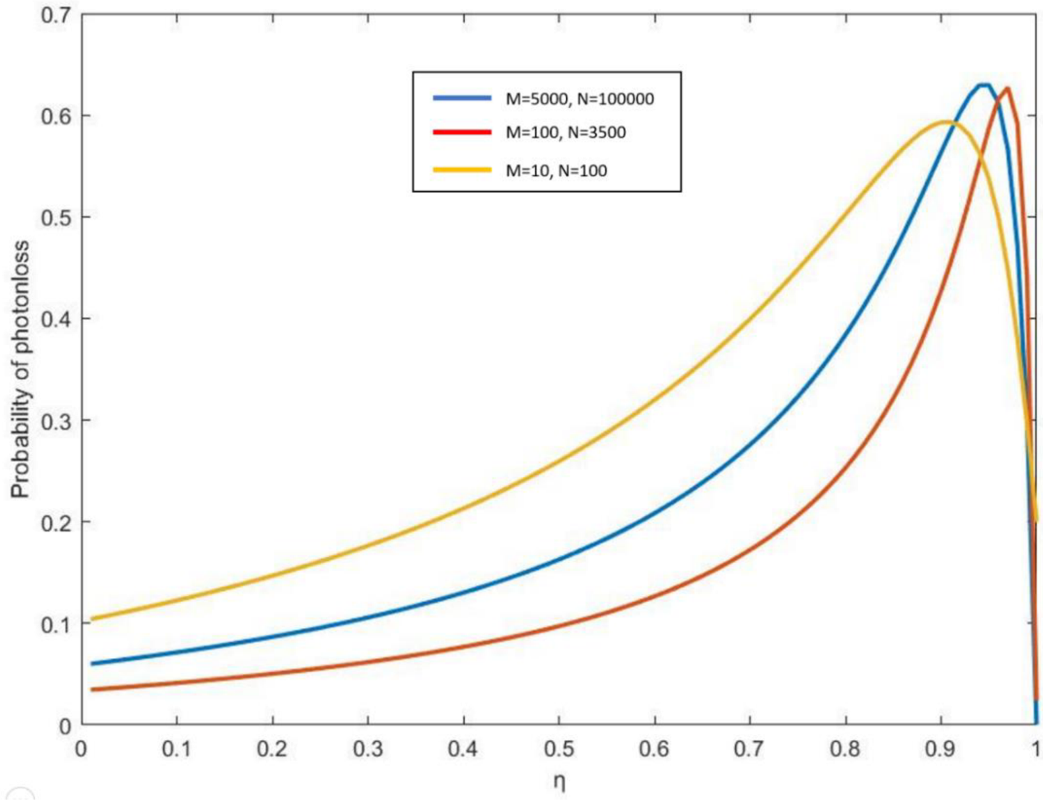

Figure 5.5: Change in photonloss probability as η changes for different values of M and N.

The information presented in Figure 5.5 is of great significance in understanding the behavior of the communication channel in relation to the probability of photon loss. The graph illustrates the probability of photons being lost during transmission based on the value of η. As we can see by increasing the number of cycles, we might even have higher probability of photon loss.

$$\begin{cases} M = 10\,, N = 100 \rightarrow \eta < 0.03 \\ M = 100\,, N = 3500 \rightarrow \eta < 0.54 \\ M = 5000\,, N = 100000 \rightarrow \eta < 0.31 \end{cases}$$

are the most tolerable η for the communication with approximately 10% and less chance of photon loss.

When η is equal to 1, the communication channel experiences no loss, and the transmission of photons carrying information occurs without any errors or data loss. On the other hand, when η approaches 0, the entire channel effectively acts as a bomb or block for the outer interferometer, leading to a complete halt in the transmission process. In this scenario, there is no loss of photons since no data is being transmitted through the channel.

It is essential to consider the factors that contribute to photon loss probability and implement measures to mitigate them to ensure the smooth and secure transmission of information. Factors such as the quality of the transmission channel, external interference, and the use of specific components can significantly impact the probability of photon loss.

5.2 Error of miscommunication

The results of the analysis of the miscommunication probability are of great significance in evaluating the reliability and effectiveness of communication systems. Miscommunication probability refers to the likelihood of receiving a message different from what was originally sent. The analysis of miscommunication probability can help in assessing the impact of factors and transmission errors on the accuracy and reliability of the communication system.

In this sub chapter, using the same equations introduced at the beginning of this chapter, we will present the results of the analysis of miscommunication probability, which will show the odds of Alice receiving a message different from what Bob has sent.

To calculate the probability of the miscommunication error we take the average of the normalized probabilities of wrong detectors clicking. As we know the information content is equal to entropy. So, the used equation means that we are assuming equal probabilities of zeros and ones which means that we are assuming maximal entropy of the message.

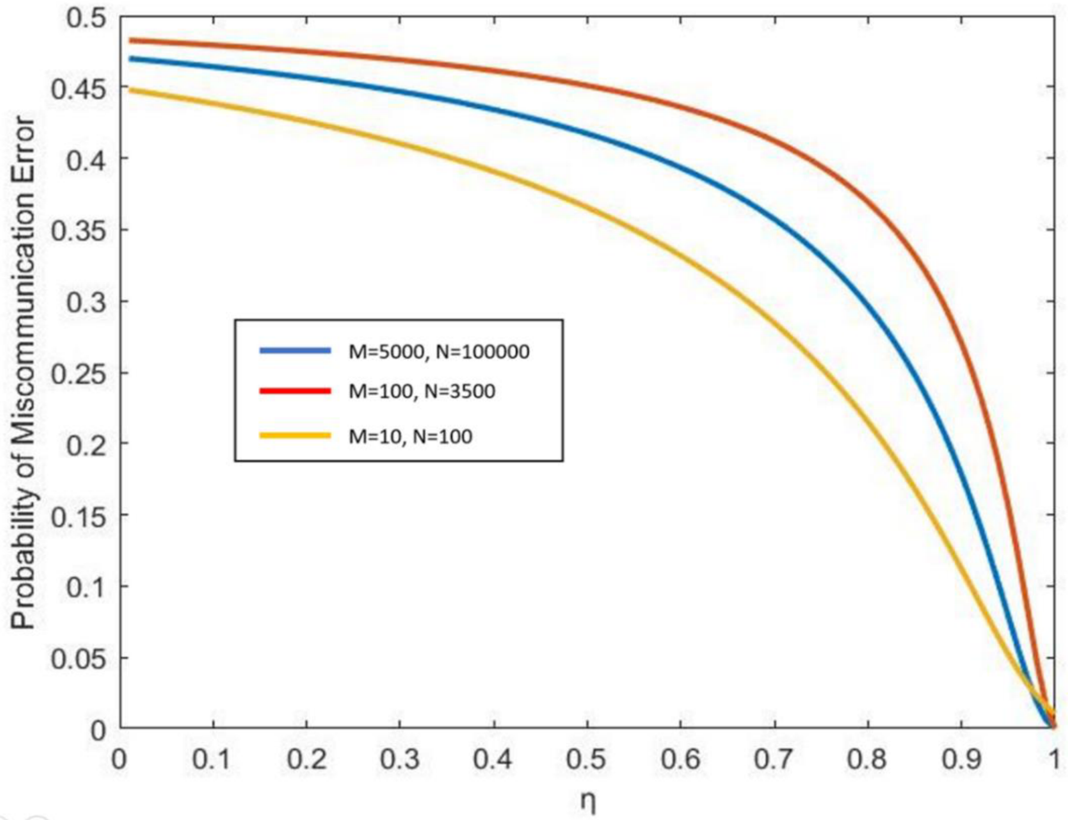

Figure 5.6: Change in Miscommuncation probability as η changes for different values of M and N.

Here as shown in Figure 5.6 the average probability of miscommunication in communication systems across different ranges of values for η. As shown in the figure 5.6,

$$\begin{cases} M = 10\,, N = 100 \rightarrow \eta > 0.95 \\ M = 100\,, N = 3500 \rightarrow \eta > 0.96 \\ M = 5000\,, N = 100000 \rightarrow \eta > 0.97 \end{cases}$$

are the most tolerable η for the communication with approximately 5% and less chance of miscommunication. As we can also see, by increasing the number of cycles we do not necessarily get better results.

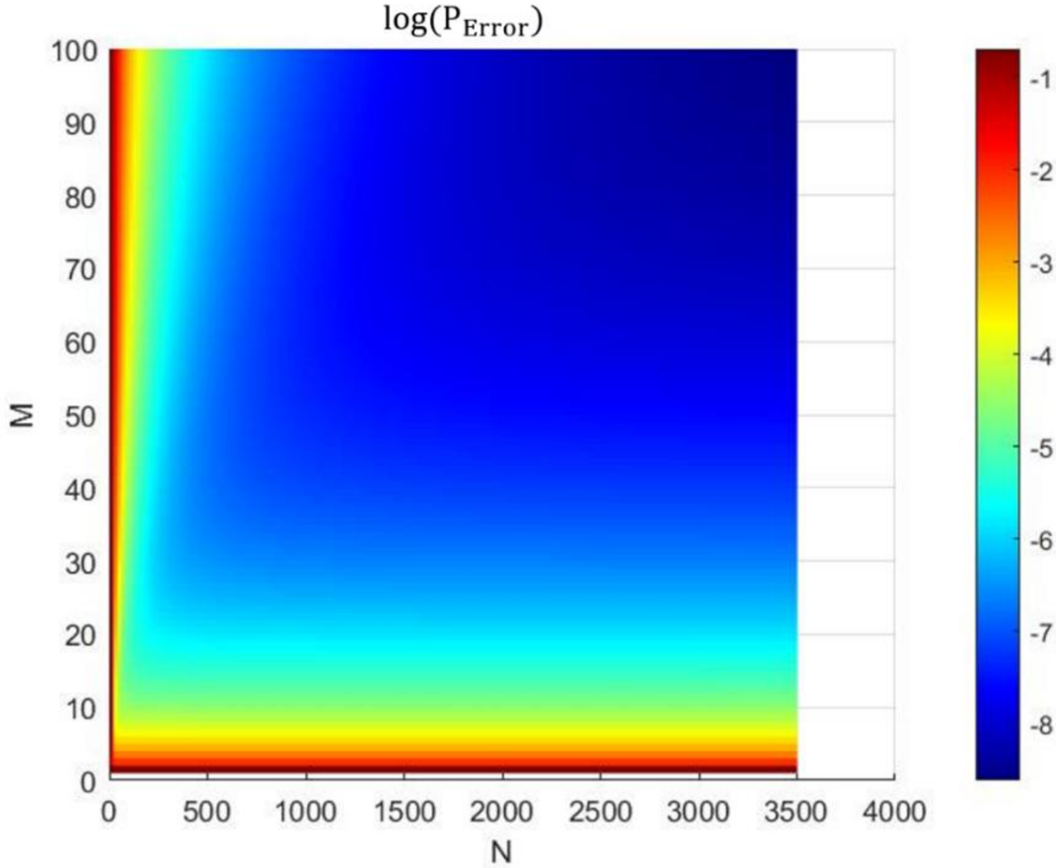

Figure 5.7: The average probability of the the miscommunication on logarithmic scale as N and M change for $\eta = 1$.

Figure 5.7 is a significant graphical representation that provides valuable insights into the average probability of miscommunication in communication systems. This graph is displayed on a logarithmic scale, which helps to highlight the variations in miscommunication probability across different ranges of values for N and M. As shown when the number of inner and outer cycles increases equally, this probability approaches zero. And with that we see the effects of decoherence on the protocol.

Conclusion

Throughout the preceding chapters, we have seen that direct counterfactual communication can be achieved by utilizing a chained version of the Zeno effect. By increasing the number of inner and outer Mach-Zehnder interferometer cycles, as well as enhancing the transmissivity of the inner and outer beam splitters with each step, Alice and Bob can exchange information without the need for physical particles to travel between them [1]-[3], [9], [11]. However, our error analysis has revealed that when Alice and Bob attempt to communicate, miscommunication or information loss is inevitable due to decoherence, which occurs with a certain probability.

Our results show that the probability of information loss is highest when the transmissivity of the added extra beam splitters (η) is close to 0.9, and this probability is approximately 0.6 and the information loss is lowest for $\eta = 1$. And $\eta = 0$ has no photon loss and no information transfer. Even when the number of inner and outer cycles is significantly altered, these probabilities will only slightly change.

Despite the fact that Bob and Alice have established communication, there is still a possibility that the message Bob sends to Alice will not be received correctly for different values of η , resulting in miscommunication. The probability of miscommunication resulting from the unexpected clicking of a certain detector cannot be overlooked. However, as the number of inner and outer cycles increases equally, this probability approaches zero, depending on the value of η.

Also, part of our future investigation lies in finding out what actually is in the channel, if not the particle. The concept of information transmission without the need for particles remains an open question that has been explored in various ways, including the many-worlds theory [5], [8], and the notion that a locally conserved, massless current of modular angular momentum, $L_z \bmod 2\hbar$, carries the one bit of information between two parties [7], [14], among others. Further investigation is required to fully understand how information can travel between two parties independently of particles.

In conclusion, while our analysis has shed light on the potential of direct counterfactual communication, there is still much to be explored and discovered. The study of information transmission without the need for particles is a fascinating and rapidly evolving field, with numerous exciting avenues for future research.

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