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ESTIMACE RYCHLOSTI VOZIDLA

VEHICLE SPEED ESTIMATION

DIPLOMOVÁ PRÁCE

MASTER'S THESIS

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Master's Thesis Assignment

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As provided for by the Act No. 111/98 Coll. on higher education institutions and the BUT Study and Examination Regulations, the director of the Institute hereby assigns the following topic of Master's Thesis:

Vehicle speed estimation

Brief description:

All active vehicle powertrain systems (such as ESP, active dampers, power steering, active antiroll bar, etc.) use the control algorithms that are parametrized by vehicle velocity at CoG (center of gravity). Knowing the vehicle's speed is an essential precondition of any such system. Taking an average of wheel speeds as a resulting vehicle velocity is generally not sufficient as one or more wheels may slip. The aim of this thesis will be devising an algorithm that fuses information available in conventional cars (wheel speeds, MEMs sensors, etc.) to calculate the estimate of vehicle speed by an estimator.

Master's Thesis goals:

1 Research usage of state observers (i.e. Kalman filter or complementary filter) in the area of vehicle speed estimation
2. Devise estimation algorithm (and implement it in Simulink) that provides estimation of real vehicle velocity based on
- measured wheel speeds
- longitudinal, lateral acceleration sensors
- yaw rate
- motor torques
Note: make sure the estimator takes into account driving situations such as driving in the corner, driving uphill, arbitrary wheel slipping, all wheels slipping, etc.
3. Test developed algorithms / observers against a vehicle model (use some model that already exists). Define the test cases with respect to the note in previous point.
4. Test implemented algorithms in real car (or on real data recorded in a car, PE will deliver the car or data)

Recommended bibliography:

LEWIS, Frank L., LIHUA Xie, and DAN Popa. Optimal and robust estimation: with an introduction to stochastic control theory. Vol. 29. CRC press, 2007.

KIENCKE, Uwe and NIELSEN Lars. "Automotive control systems: for engine, driveline, and vehicle." (2000): 1828.

Students are required to submit the thesis within the deadlines stated in the schedule of the academic year 2017/18.

In Brno, 26. 10. 2017

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Abstract
Vehicle speed is one of the crucial variables needed to be known in real-time and with high accuracy, to serve as input into vehicle dynamic control systems. Its direct measurement in the vehicle is however cost ineffective. The idea is to use the measurements from generally available on-board sensors and to consequently compute the vehicle speed. Nevertheless, the measurements are highly influenced by process noises due to complexity of motion of the vehicle. Therefore, an estimation algorithm with ability to deal with this negative influence has to be developed. The estimation algorithm presented in this thesis estimates longitudinal vehicle speed using measurements of four rotational wheel speeds, longitudinal acceleration, motor torques, yaw rate and steering wheel angle. It was tested against the numerous situations considered critical according to vehicle speed estimation, such as rapid acceleration on road with low friction coefficient, emergency braking with activation of ABS, or driving in the slope with wheels slipping, providing satisfactory results.

Abstrakt
Rýchlosť vozidla je jednou z kľúčových stavových premenných, ktorej znalosť je potrebná v reálnom čase a s vysokou presnosťou, aby mohla slúžiť ako vstupná veličina pre systémy kontroly dynamiky vozidla. Jej priame meranie vo vozidle je však finančne náročné. Riešením tohoto problému môže byť použitie meraní zo senzorov bežne dostupných na palube vozidla a ich následný prepočet na rýchlosť vozidla. Tieto merania sú však veľmi zaťažené procesným šumom, čo vyplýva z komplexnosti pohybu vozidla. Preto je nutné vyvinúť algoritmus so schopnosťou vysporiadať sa s týmito negatívnymi vplyvmi. Algoritmus prezentovaný v tejto práci odhaduje pozdĺžnu rýchlosť vozidla s použitím meraní uhlových rýchlostí štyroch kolies, pozdĺžnej akcelerácie, momentov motora, rýchlosti otáčania okolo zvislej osi a natočenia volantu. Algoritmus bol testovaný na veľkom počte situácií považovaných za kritické na odhad rýchlosti vozidla, ako napríklad prudká akcelerácia na vozovke s nízkym koeficientom trenia, núdzové brzdenie s aktiváciou ABS, či jazda v kopci s kolesami v preklze, prinášajúc uspokojujúce výsledky.

Keywords
Vehicle speed, vehicle velocity, longitudinal velocity, state estimation, state variable, state observer, state estimator, Kalman Filter, wheel speed, longitudinal acceleration, motor torque, sensor, measurement

Kľúčové slová
Rýchlosť vozidla, pozdĺžna rýchlosť, odhad stavu, stavová premenná, stavový pozorovateľ, stavový estimátor, Kalmanov filter, rýchlosť kolesa, pozdĺžne zrýchlenie, moment motora, senzor, meranie

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I declare, that I have elaborated this master's thesis and developed the attached algorithm independantly, under the supervision of academic supervisor *doc. Ing. Jiří Krejsa, Ph.D.* and professional advisor *Ing. Juraj Madarás, Ph.D.*. All used sources have been properly cited in bibliography.

Bc. Martin Roštek

I would like to thank to my academic supervisor *doc. Ing. Jiří Krejsa, Ph.D.* for his forthcoming attitude and time given, to my professional advisor *Ing. Juraj Madarás, Ph.D.* for his valued professional guidance and patience, to my collegues in *Porsche Engineering Services* for proposing this topic, and resources alocated for successful completion of this thesis and finally to my parents, close family and friends for their support during my whole studies.

Bc. Martin Roštek

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1. Introduction

1.1. Background & Motivation

In recent time went the automotive industry through meaningful transformation accompanied by considerable amount of innovations in every direction. As times have changed and vehicles have become more the network of sophisticated computational systems, than purely mechanical machines, today's most significant revolution is taking place in the field of electronics and mechatronics, bringing the biggest competition in automotive research and development. This holds especially true considering the products with a scope of autonomous driving, improvement of driving performance and the traffic safety.

Therefore, more and more variables indicating the actual state of vehicle have to be known in real-time, to serve as inputs for control systems. This also emerges the need of accurate, reliable and effective method of obtaining those data. Some of the state variables can be obtained easily by direct measurement via sensors, that are moreover cost affordable. Contrarily there are those, which require more effort to be measured or which are totally unmeasurable.

One of those state describing variables, that also suffers from lack of options to be obtained effectively, is also the vehicle velocity. It can be confidently labeled as an essential input to many electronic on-board systems, mainly the vehicle driving control systems. Probably the most well-known are ABS (Anti-lock Preventing System) or ESP (Electronic Stability Program). These could be also recognized as most important safety systems carrying out the improvement of vehicle dynamic characteristics. Furthermore, they are both compulsory elements of every series production vehicle.

So as the proper function and the appropriate intervention of such a control system can prevent vehicle and its passengers from undergoing hazardous on-road events, their malfunction can lead to life threatening situations. Therefore, it is necessary to obtain the vehicle speed with high accuracy and cost effectively at best, as the direct measurement is not the case. Solution being proposed is to obtain the measurements from instruments, that are part of general equipment of every today's vehicle. Then, by using the proper computational processing convert those to the target one - vehicle speed.

Commonly present sensors, for instance rotational wheel speed sensors, longitudinal accelerometer, motor torque sensors and several others are being offered. Unfortunately, the direct conversion to vehicle speed is not the option. The purpose is simple - the vehicle motion with all its complexity.

Extreme driving performance with pushing the vehicle into its limit conditions, whether is the purpose sport-drive, or an avoidance maneuver required to avoid a collision. Extreme weather conditions as rain, snow, ice, lowering the road surface friction. Those are the critical situations defining the challenge of the accurate vehicle speed estimation. All mentioned produce high process noise, in this particular case mostly the wheel slip, that effects the sensor measurements and also the wished result consequently. Therefore, more scientific approach is needed.

The idea is to set up the algorithm, that uses knowledge of the system and its environment, with ability to benefit from it in defined manner. Such an algorithm can be introduced as an estimator. To develop one that deals with severe situations and conditions, bringing sufficient vehicle speed estimation results is the goal of this thesis.

1.2. Research of former use of the state observers in order to estimate vehicle speed

The most primitive manner of estimating the vehicle speed, mentioned in publication [1], is to take one of four wheel speed measurements and to esteem it directly as a longitudinal velocity of the vehicle. It was usual to take for instance the one of non-driven wheels, because it should not be affected by slip during acceleration. Unfortunately, this does not hold true for the braking situation and furthermore it does not fit for vehicles with all wheels driven. Another simple strategy is to pick the sensor measurement of wheel according to driving situation, in particular, the speed of the slowest wheel during acceleration or normal drive and the fastest wheel during braking. This approach is sometimes called the best-wheel selection and although it is fast and easy to implement, the produced results are not sufficient. Major drawback is again related to the wheel over-slip, that occurs both by acceleration and braking affecting also the picked wheel which leads to highly inaccurate results.

Even though *Jiang and Gao (2000)* in [2] used the above mentioned strategy in their observer, they realized its flaw and therefore used non-linear observer besides, to deal with a possible wheel slip in braking situation. Their work is focused on providing the resultant velocity directly to ABS for further slip estimation, which is important for ABS control. As ABS operates explicitly while braking, the outcome from other driving situations is less relevant. Nevertheless, for general application the results could be insufficient.

In publications from *Jo, Chu, Kim, Sunwoo (2011)* [3] and O'Kane and Ringwood (2013) [4] the fusion of GPS and inertial sensors were used to estimate the velocity. According to authors, the wheel speed measurements captured by in-wheel sensors may greatly differ from the real velocity due to the over-slip and the longitudinal acceleration contains a bias due to the driving in slope. On the other hand, GPS has low reliability of measurement and suffers from lags as it has low sample rate. Thus, they have developed an algorithm eliminating disadvantages and combining advantages of both measurements with good results reported in the end. Nevertheless, many vehicles are not equipped with GPS device, which contradicts the idea of using generally available and affordable equipment.

Use of frontal video camera and acceleration data from drive-recorder for estimating the vehicle speed has been studied by *Osamura, Yumoto and Nakayama (2013)* [5]. Studied method is based on tracking the featured points on captured image and obtaining the vehicle velocity from their position against the vehicle changing in time. Acceleration is then used as cross-check to obtain the possible range in order to avoid the possible error created by image processing. Apart from the fact, that still very few vehicles are equipped by these devices, the output of video camera is sensitive to disturbances of environment and therefore inapplicable for subsequent image processing, for instance in certain weather conditions. Similar weakness concerns also the solution presented in [6], where the camera was mounted looking downwards, with optical axis perpendicular to the terrain.

Pettersson (2008) compared Extended Kalman Filter and Averaging observer by estimation of lateral velocity and slip angle in his master's thesis [7]. Two-track model, capturing also the chassis roll dynamics, and tire and damper models were used, with expectation of more accurate results to be provided. Besides of higher accuracy of results, demands such a complex modeling a lot effort and is susceptible to introduce errors to the system. It also entails non-linearities, so the Extended Kalman Filter needs to be used. This brings not inconsiderable increase of computational burden and probability of estimation to be biased as a result of linearization or incorrect calculation of Jacobian matrices. Based on authors' statement, the simpler Averaging observer brought similar results with less effort needed.

In [8] have *Kiencke and Nielsen (2005)* introduced the Fuzzy estimator. In this kind of estimator, the inputs are represented as linguistic variables with membership grades between 0 and 1 assigned. Separate rule-base is created for different driving states of vehicle, according to the knowledge of signal errors in corresponding state. The output comprises weighting factors, one for each wheel and one for accelerometer. Those factors serve as inputs to the weighted average equation. It is undeniable that using this method can produce good results. However, it is complicated, and it needs a lot of effort to define the rule-base and driving states, which cover all possible situations and then assign the reasonable size to corresponding weighting factors.

Kobayashi, Cheok and Watanabe (1995) [9] presented estimator with Fuzzy logic rule-based Kalman Filter, where the covariances are switched according to which sensor measurement is more reliable and therefore more useful for updating the Filter. Measurements were obtained from four wheel-speed sensors and accelerometer. In this estimator, the accelerometer offset was not taken into account introducing error to the estimate. This deficiency was complemented by *Wu (2011)* [10] and *Momeni, Moasheri, Chabok, Kheradmand (2015)* [11] by obtaining the acceleration update from Kalman Filter. Despite that, the estimate will be significantly influenced by high wheel over-slip, because even if the weight of measurement is penalized in covariance matrix, it still introduces not inconsiderable error. *Gao (2013)* used adaptive Kalman filter, measuring wheel speeds, longitudinal acceleration and motor torques in his master's thesis [12]. Main idea of his estimation algorithm is to detect wheel over-slip in time and eliminate the defective measurement from updating the state prediction. As high over-slip is considered to be the major source of negative impact influencing the result, his strategy has inspired the algorithm developed in this thesis.

2. Theoretical introduction to state estimation

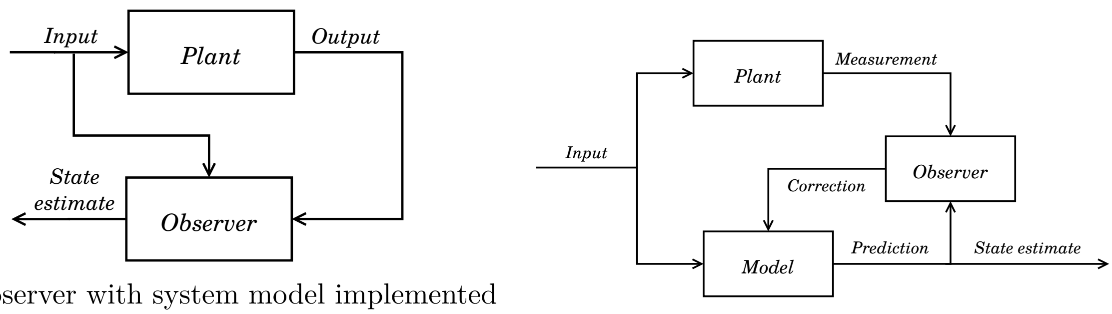
2.1. State observers

As indicated in the introduction of this thesis, the essential component for the successful state estimation is the state observer. Generally said, state observer is an algorithm, used to estimate the values of the state variables of dynamical system [13]. State observers are used in cases, when the direct measurement of the state variable is impossible, or is not effective for some specific reason. State observers employ the knowledge of the dynamical system and the measurements of other variables.

First step in understanding the motivation of using the state observer and also its function, is to realize the distinctions between the plant, how the system of our interest is being called, and its model. On one side, the plant is most typically a complex and highly non-linear object varying in time, which requires a quantum of state variables and parameters, to be represented flawlessly. On the other side, model of the plant is willing to be a simplified representation based on the knowledge of the real plant. It is usually capturing its characteristics and behavior only in narrow sphere of its operation, intentionally chosen to be sufficient to reach the defined objective, i.e. to be used for an analysis or a computation [14].

Even with putting many effort to model the plant precisely, there are still other uncertainties that tend to arise. These uncertainties may be caused by stochastic phenomenon in environment, such as wind with random force affecting the helicopter or unpredictable bumps and water on the road causing wheels of vehicle to over-slip. On top of that, also the measurement output is commonly influenced by the measurement noise.

Therefore the logic has to be developed, where the output of the model, which is feeded by the same input values as the plant, is corrected, in order to achieve the accurate value of desired state variable. Basic scheme of general state observers is shown in Figures 2.1a and 2.1b.



(a) Observer with system model implemented inside

(b) Observer with external system model

Figure 2.1: Example observers presented in diagrams

2.2. State space representation

As the majority of state observers work with the state-space representation of the dynamical system, it is considered important to be shortly introduced.

Representing linear time-invariant system of finite dimensions in state-space form, means using differential equations for its description, structured as follows [15]:

$$\dot{x} = Ax + Bu \tag{2.1}$$

$$y = Cx + Du \tag{2.2}$$

where first of equations is state equation of the system, second is known as the output equation. Vector $x \in \mathbb{R}^n$ is state vector with initial condition x_0, $u \in \mathbb{R}^s$, $y \in \mathbb{R}^m$ are system input and output vector respectively. n represents the order of the described system.

$A \in \mathbb{R}^{nxn}$ is in all cases square matrix, called state or system matrix and it determines how the previous states x_k, will be translated to new states x_{k+1};

$B \in \mathbb{R}^{nxs}$ is matrix of inputs to the system;

$C \in \mathbb{R}^{mxn}$ is output matrix which acquires only values of 0 and 1, defining the correlation between the state and output of the system;

$D \in \mathbb{R}^{mxs}$ is called feedforward or a direct transition matrix and is usually set to 0 [16],[17].

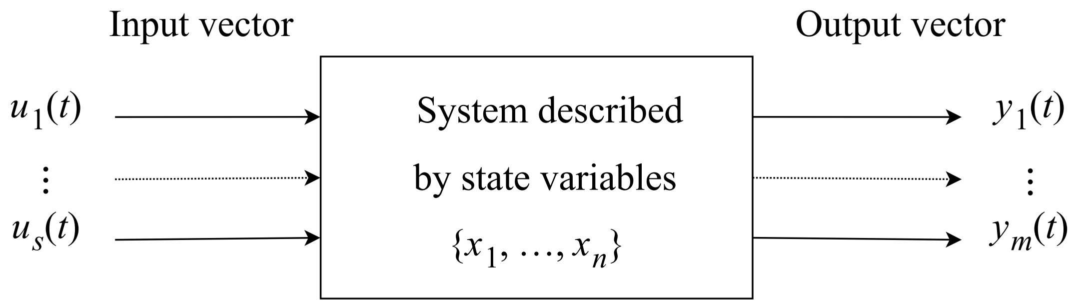


Figure 2.2: System inputs and outputs

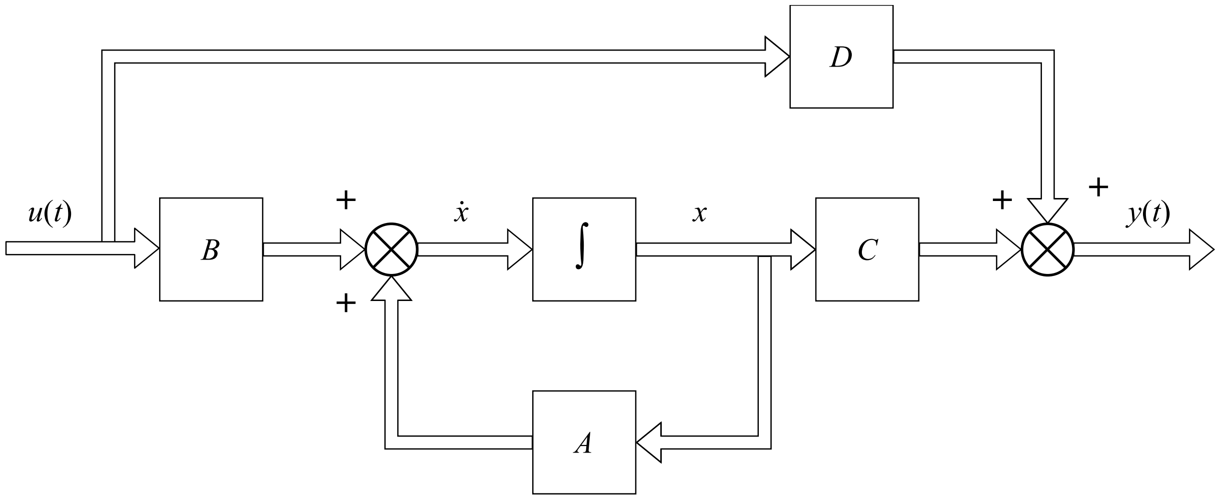


Figure 2.3: Vector block diagram of a linear system represented in state-space

2.3. System observability

Every system to be investigated by the observer has to fulfill the pre-condition of being observable.

To explain the term observability, let's consider the system:

$$\dot{x} = Ax + Bu \tag{2.3}$$

$$y = Cx \tag{2.4}$$

where all its elements are having the same meaning as described in Chapter 2.2 and initial state is set to be:

$$x(0) = x_0 \tag{2.5}$$

We find such a system observable, if its state $x(t)$ can be at any time $t \geq t_0$ uniquely determined from the inputs $u(s)$ and outputs $y(s)$, where $0 < s < t$ or if $x(t)$ can be uniquely determined from known initial state $x(0)$ and inputs [19],[20].

Investigation for observability is carried out by computing a rank of observability matrix, which has to be equal to order of the dynamical system. [19]

$$rank(Q) = rank \begin{bmatrix} C \\ CA^1 \\ \vdots \\ CA^{n-1} \end{bmatrix} \overset{!}{=} v \tag{2.6}$$

If the rank of matrix is lesser than v, what refers to number of linear independent rows of matrix, the system is unstable and therefore not observable. In spite of that, some of unstable systems can be stabilized or several methods can be followed to reach the observability.

2.4. Luenberger observer

It was D. G. Luenberger who first purposed and further developed the linear observer used for estimation of unknown state variable in publications from 1964, 1966 and 1971. [21],[22],[23]

In addition to the state space representation of dynamical system, incorporates the equation of Luenberger observer also correction term. It corrects the estimate by amount, proportional to prediction error, which is obtained from variance of estimated and measured value of the observed state variable. [20]

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y(t) - \hat{y}(t)) \tag{2.7}$$

is a system in state space form with Luenberger observer. Vectors x, u, y have the same meaning as in Equations 2.3 and 2.4. Hats above the vectors $\hat{x}$,$\hat{y}$ depict, that their values come from the estimation. Initial state is $x(0) = x_0$ and is chosen arbitrarily. t is time.

Matrices A, B, C have also the same meaning as in Chapter 2.3 and are constant in design of the Luenberger observer, contrary to the observer gain matrix L, called also Luenberger gain. Its value is arbitary and determines the identity of the observer [19].

After substituting:

$$\hat{y}(t) = C + \hat{x}(t) \tag{2.8}$$

$$y(t) = C + x(t) \tag{2.9}$$

Into the equation 2.7, we get:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + LC(x(t) - \hat{x}(t)) \tag{2.10}$$

which could be arranged to form:

$$\dot{\hat{x}}(t) = (A - LC)\hat{x}(t) + Bu(t) + Ly(t) \tag{2.11}$$

Estimation error is defined as:

$$e(t) = x(t) - \hat{x}(t) \tag{2.12}$$

and the dynamics of this error is [19]:

$$\dot{e}(t) = (A - LC)e(t) \tag{2.13}$$

To get $e(t) \to 0$ as $t \to \infty$, which in other words means to asymptotically reach accurate results without estimation error, since the estimate $x(t)$ converges to the real state $x(t)$, poles describing the behavior of the system have to be placed properly. As $(A - LC)$ is the system matrix of the observer, the mentioned poles are represented by its eigenvalues [20],[24].

Eigenvalues of the matrix can be obtained by solving its characteristic polynomial which due to [25] equals:

$$P_A = \det\big((sI - (A - LC)\big) = (s - \lambda_1), \ldots, (s - \lambda_n) \tag{2.14}$$

where λ is vector of eigenvalues of system matrix, I is identity matrix and n represents order of the observed system.

Solution of this equation brings us to relationship between poles represented by eigenvalues and elements of L matrix $l_1, \ldots, l_n$ which are the target of investigation.

To achieve suitable eigenvalues of $(A - LC)$, the general strategy of pole placement could be introduced:

- in order to prevent the system of becoming unstable, both eigenvalues must be set negative
- placing the eigenvalues too far left on the complex plane makes the observer too sensitive to noise
- by contrast placing eigenvalues close to imaginary axis makes the observer to react slow, so it will not be able to response to rapid changes [8].

2.5. Kalman filter

The Kalman filter is mathematical algorithm that employs statistical methods to provide the estimation of past, present or future states of a system by minimizing error covariance. It is prevalent tool used in wide range of applications, most commonly in engineering field, i.e. for signal processing, system control and navigation in transport or military industry, process control, but also in socio-economic tasks. For mentioned ability to solve widespread scope of problems, numerous extensions allowing solution of i.e. non-linear problems, small computational and memory requirements and relative simplicity, it has found the great favour [26],[27].

In order to estimate the state of process $x \in \mathbb{R}^n$ in discrete-time, Kalman filter uses its state-space represented model [28]:

$$x_k = Ax_{k-1} + Bu_k + w_k \tag{2.15}$$

and a measurement model, describing relationship between the state of the process and the measurements that are taking place in observed process:

$$z_k = Hx_k + v_k \tag{2.16}$$

The index k, in state vector x_k, input vector u_k and measurement vector z_k, represents the actual time step. Matrices A, B, H are system matrix, input matrix and measurement matrix respectively.

w_k is random variable, that represents the process noise and random variable v_k represents the measurement noise. Both are assumed to be white, independent of each other, and their probability distributions are:

$$P(w) \sim N(0, Q) \tag{2.17}$$

$$P(v) \sim N(0, R) \tag{2.18}$$

where Q is process noise covariance and R is measurement noise covariance. Most simply said – the lower the variances - how are the coefficients on the matrix diagonal called - of Q are, the less uncertainty is assumed and therefore the correctness of model is more trusted and similarly the lower are the variances of matrix R, the more the measurement correctness is trusted, as low uncertainty is assumed [29].

Algorithm of Kalman filter operates in two steps: prediction and update step. In prediction step, current state variables are estimated taking their actual uncertainty into account. Then, the measurement is observed and the estimate is updated using weighted average with idea, that real state should lie somewhere between the measurement and the prediction, giving more weight on observation with higher certainty presumed [15]. This certainty presumption is given by matrices Q and R.

Note, that matrices Q and R, with matrix H additionally, can be arbitrarily changed with every single time step. This is interesting feature, as they adapt the Kalman gain K according to some external conditions, making it no more constant. Therefore, this type of Kalman filter is called Adaptive [30]. Such an approach is being used i.e. in cases, when the covariance of noises is ill-conditioned, so the accuracy of the estimate would be greatly affected [31].

The above introduced is practically done by iterative solution of following equations in every time step:

Prediction

$$\hat{x}_{(k|k-1)} = A_k \hat{x}_{(k-1|k-1)} + B_k u_k \tag{2.19}$$

$$P_{(k|k-1)} = A_k P_{(k-1|k-1)} A_k^{\top} + Q_k \tag{2.20}$$

Update

$$\hat{y}_k = z_k - H_k \hat{x}_{(k|k-1)} \tag{2.21}$$

$$S_k = R_k + H_k P_{(k|k-1)} H_k^\top \tag{2.22}$$

$$K_k = P_{(k|k-1)} H_k^\top S^{(-1)} \tag{2.23}$$

$$\hat{x}_{(k|k)} = \hat{x}_{(k|k-1)} + K_k \hat{y}_k \tag{2.24}$$

$$P_{(k|k)} = (I - K_k H_k) P_{(k|k-1)} (I - K_k H_k)^\top + K_k R_k K_k^\top \tag{2.25}$$

$$\hat{y}_{(k|k)} = z_k - H_k \hat{x}_{(k|k)} \tag{2.26}$$

For better imagination, the Figure 2.4 shows how the algorithm is performed.

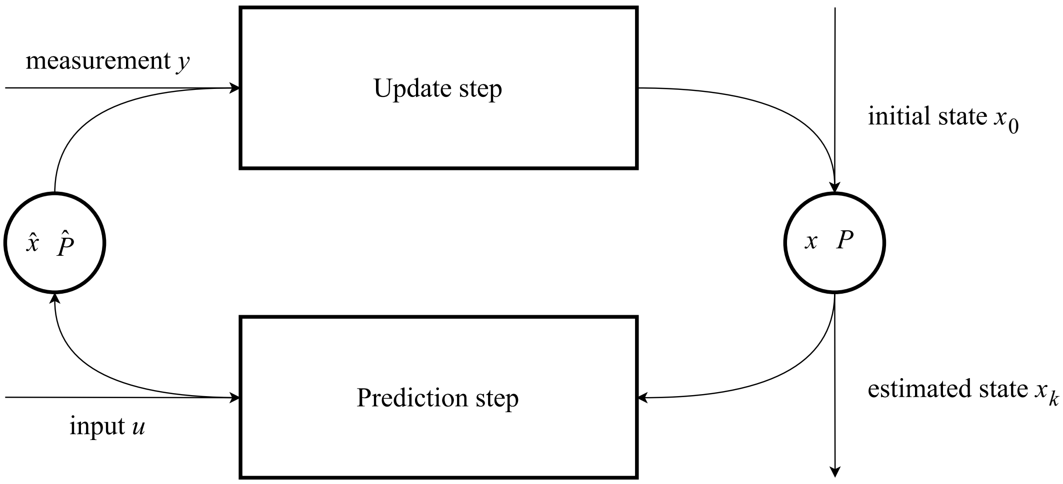


Figure 2.4: Process of Kalman filter algorithm

From the introduction section follows, that the normal Kalman filter is convenient for linear dynamic systems only. For non-linear systems is the most common application the Extended Kalman filter [15].

Assuming the all transformations being quasilinear, *Extended Kalman filter* linearizes non-linearities in every time step around the last state estimate and substitutes the matrices for linear transformation with Jacobian matrices. It is important to remark, that there are few limitations and drawbacks in using EKF. First of all, not in every case can be non-linearities approximated by the linear function well. Then, the Jacobian matrices must exist, which is not always the case. Furthermore, their calculation is nontrivial, susceptible to errors caused by human factor which cannot be easy to identifiy [15],[32].

For applications, where *Extended Kalman filter* results are considered insufficient, the improvement in form of *Unscented Kalman filter*, more detailed studied in [33], can be put in to practice.

3. Algorithm design

3.1. Algorithm overview

Announced by many studies presented in 1.2, wheel over-slip is the factor, that introduces the most significant error into the estimation. Especially high over-slip caused for instance by rapid acceleration on the road with reduced friction or due to emergency braking with wheel lock and consequent ABS activation.

The only effective way to prevent the defective measurement from affecting the estimation, is to remove that measurement from further computation. To do so, the occurrence of over-slip on each single wheel has to be detected. For this purpose, the algorithm with physically based criteria has been developed, which can be declared as the main feature of the introduced estimation algorithm. Together with other components, it carries out the successful estimation of vehicle longitudinal speed.

Algorithm operates in discrete-time with sample time of $0.01 seconds$, which corresponds to sampling period of captured input data and CPU clock. This simultaneously, after compiling to C, makes algorithm ready to be used on control unit directly.

The estimation algorithm was developed for the electric vehicle with 2 motors, one for each axle, which means that all wheels are driven.

Overview of signals from on-board sensors used for estimation can be found in Table 3.1.

Measured signal	Variable	Unit
Wheel rotational speed	$\omega_{fl}, \omega_{fr}, \omega_{rl}, \omega_{rr}$	$\mathrm{rad\,s^{-1}}$
Longitudinal acceleration	$a_{m,x}$	$\mathrm{m\,s^{-2}}$
Motor torque (front, rear)	$M_{m,f}$,$M_{m,r}$	$\mathrm{N\,m^{-1}}$
Yaw rate	$\dot{\psi}$	$\mathrm{deg\,s^{-1}}$
Steering wheel angle	δ	deg

Table 3.1: Signals used as input into the vehicle speed estimation algorithm

To introduce how the algorithm flows, the block diagram presented below was created. Sense and function of the individual components will be parsed separately in following chapters.

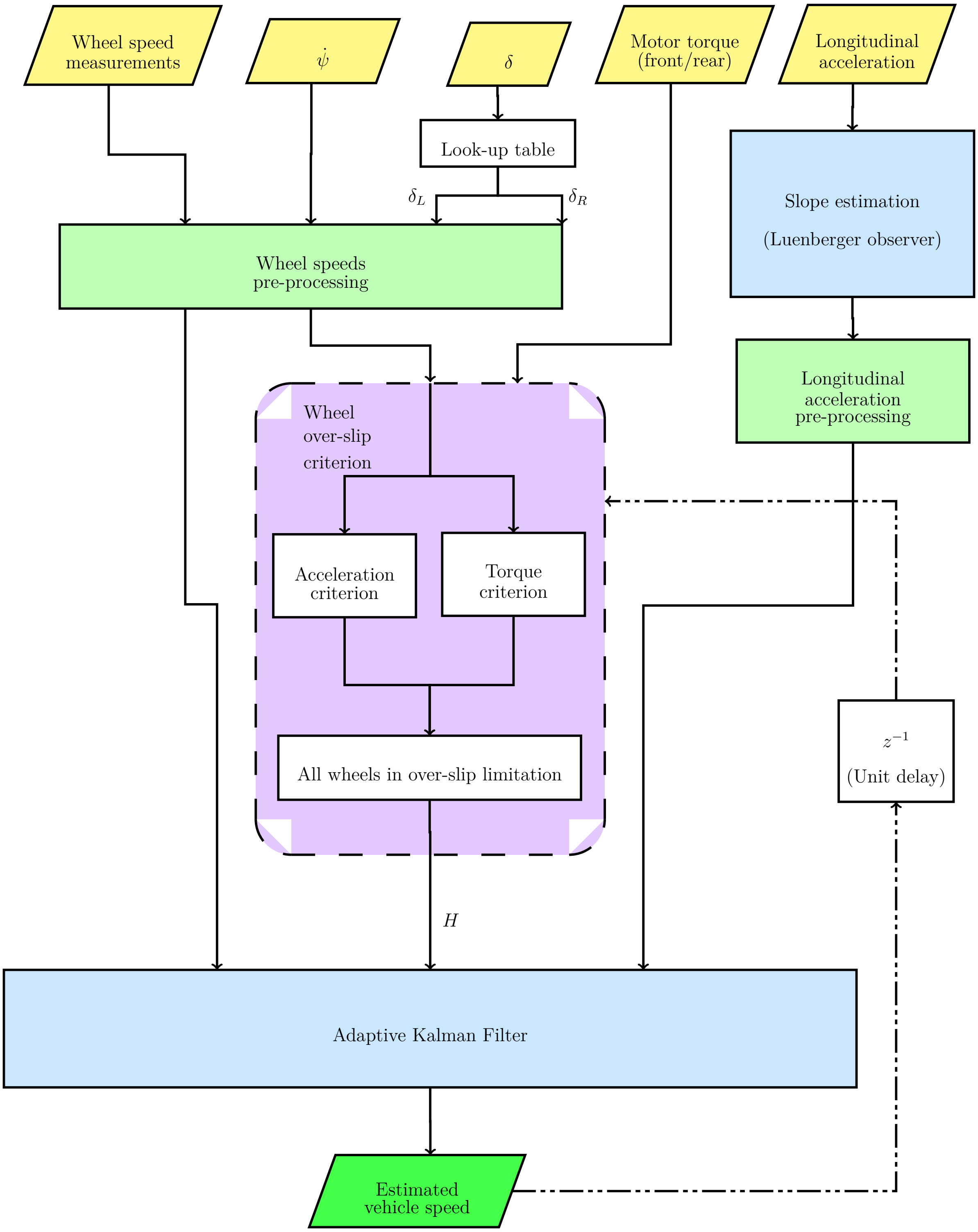


Figure 3.1: Estimation algorithm flowchart

3.2. Co-ordinate system and model reductions

Co-ordinate system used in this thesis has its origin at the vehicle center-of-gravity. It is presented in Figure 3.2.

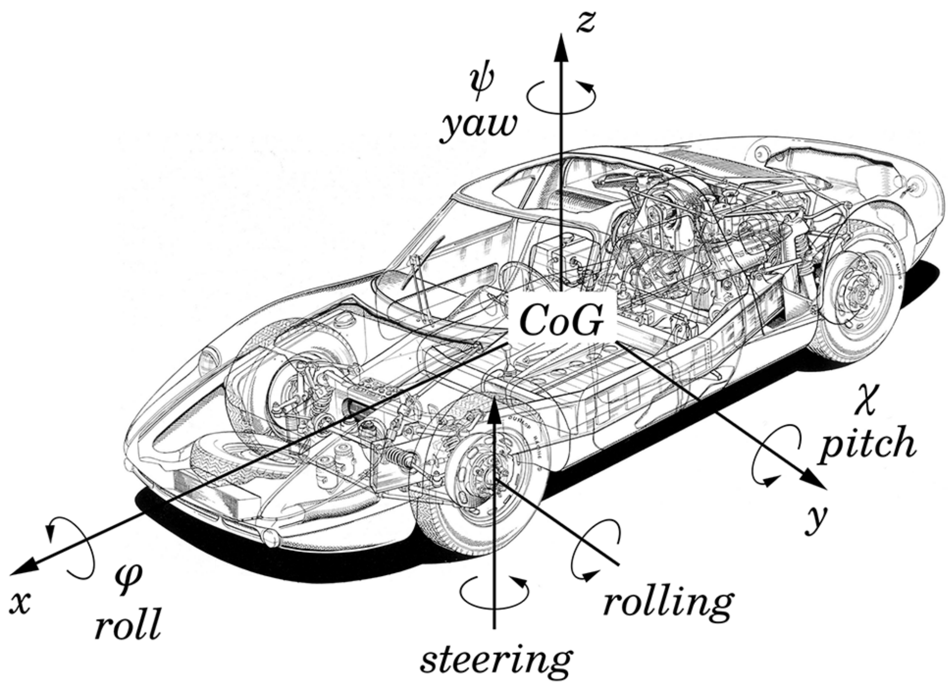


Figure 3.2: Orientation of co-ordinate system

When the vehicle dynamics has to be represented by the model for further analysis, the compromise between the perfectly accurate reproduction of the real vehicle and immoderate complexity of the model should be found. Simply said, the model has to capture the characteristics and behavior of the real vehicle good enough according to the application needs.

In this work, it is believed that in overwhelming majority of states, the commercial vehicle undergoes during its motion, is the lateral component of velocity very small and its influence on the overall velocity is negligible. Hence, from this moment, it will be only talked about the longitudinal velocity to be estimated. This allows us to dismiss rolling of the chassis and side slip angle. Such a reduction simplifies the modeling process noticeably, saves computational costs and reduces the amount of nonlinearities to deal with.

3.3. Wheel speed sensor data pre-processing

Measurement of rotational wheel speed takes place by four sensors, one installed on each wheel. Captured rotational speed ω_{ij} can be converted to wheel speed in ground contact point v_{ij} by:

$$v_{ij} = \omega_{ij} \cdot r_D \tag{3.1}$$

where i and j denote reference of measurement to *front (f)* or *rear (r)* axle and *left (l)* or *right (r)* side respectively and r_D is dynamic wheel radius.

However, every signal captured by sensor contains some systematic error and must be pre-processed [8]. Firstly are the signals filtered by 1st order low-pass filter to mitigate the sensor noise. Secondly due to driving in the curves the correction of wheel speeds by their transformation to the center-of-gravity is necessary.

Parameters and variables used to derive the correcting Equations 3.2 - 3.5 are all to be found in Figure 3.4.

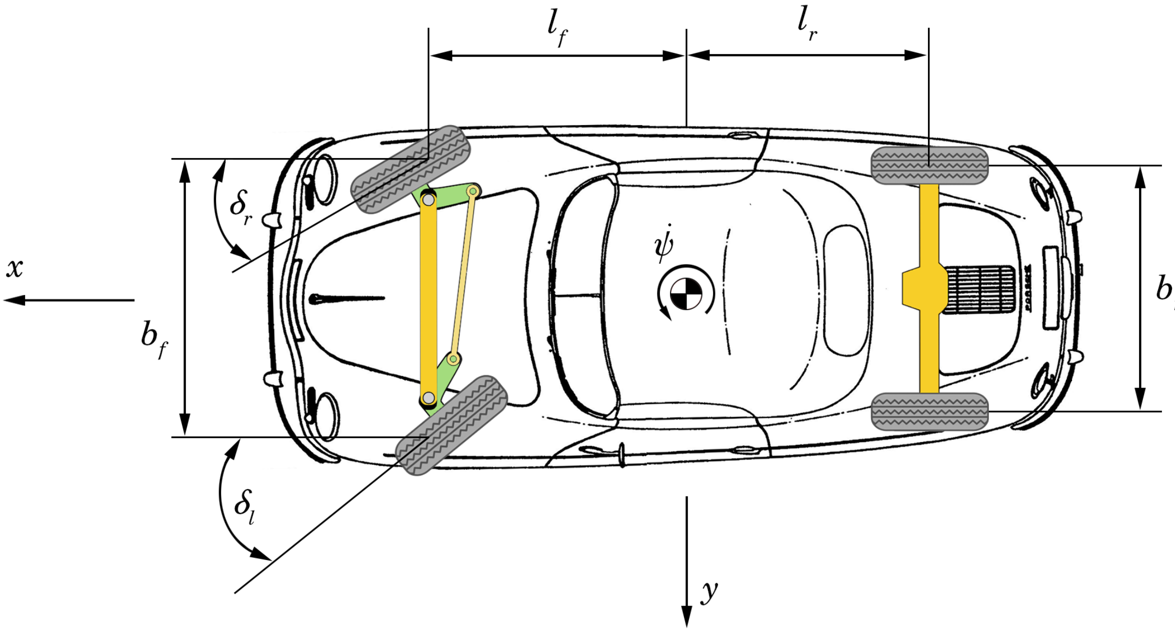


Figure 3.3: Vehicle in *xy-plane* with parameters described

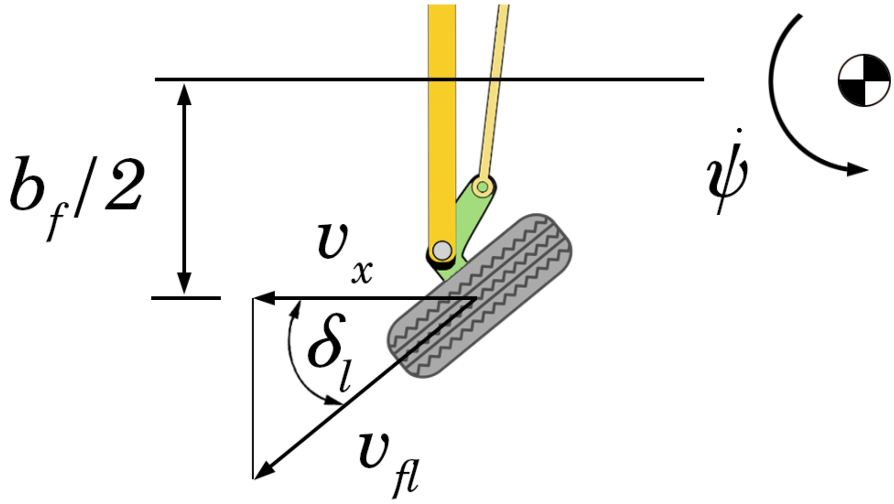


Figure 3.4: Wheel speed transformation explained by the figure

To translate the front wheels the following equations are used:

$$v_{c,fl} = v_{fl} \cos\ \delta_l + \dot{\psi}\frac{b_f}{2} \tag{3.2}$$

$$v_{c,fr} = v_{fr} \cos\ \delta_r - \dot{\psi}\frac{b_f}{2} \tag{3.3}$$

Rear wheels are translated by equations:

$$v_{c,rl} = v_{rl} + \dot{\psi}\frac{b_r}{2} \tag{3.4}$$

$$v_{c,rr} = v_{rr} - \dot{\psi}\frac{b_r}{2} \tag{3.5}$$

It may have been noticed, that depicted in the Figure 3.4 and also in translation equations, different deflection angles were used for left and right wheel. That is a consequence of the real cars steering design, where the deflection towards x-axis is for each wheel different by the same angle of lock of steering wheel. As the available signal was the signal carrying steering wheel angle, the calculation of single wheel angles δ_l and δ_r is also the part of data pre-processing in the estimation algorithm. For this calculation, where the dependence between steering wheel angle and the deflection of wheels is non-linear, the Lookup table is being used.

Importance of this pre-processing step is shown by Figures below.

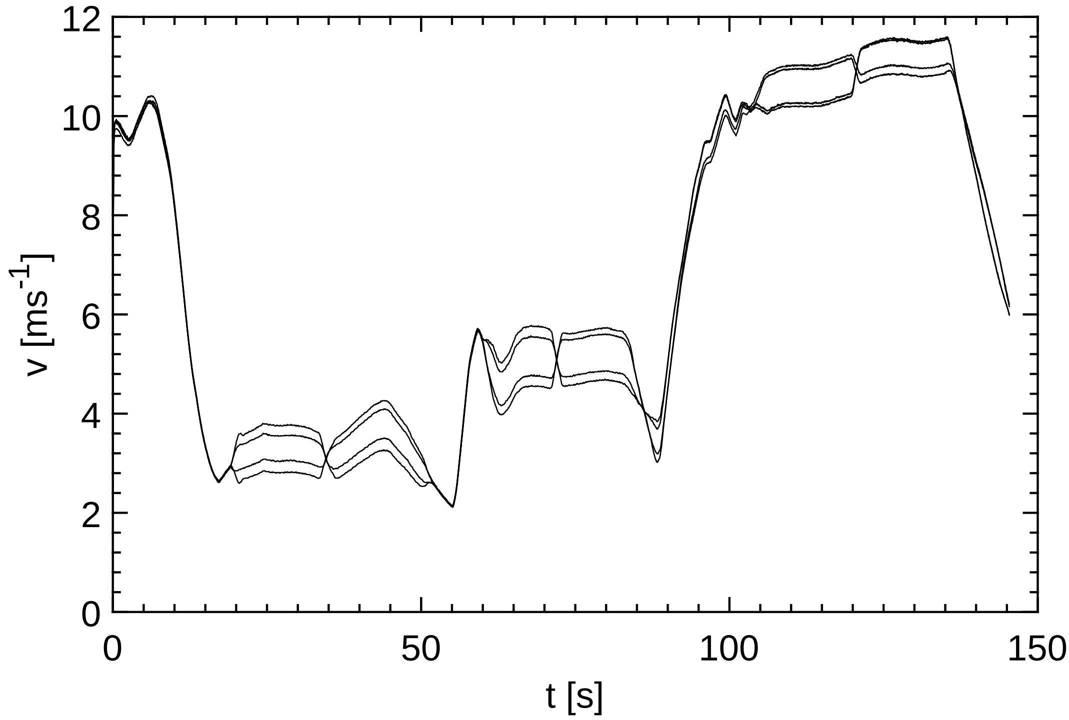


Figure 3.5: Wheel speeds at ground contact point before correction to the center-of-gravity

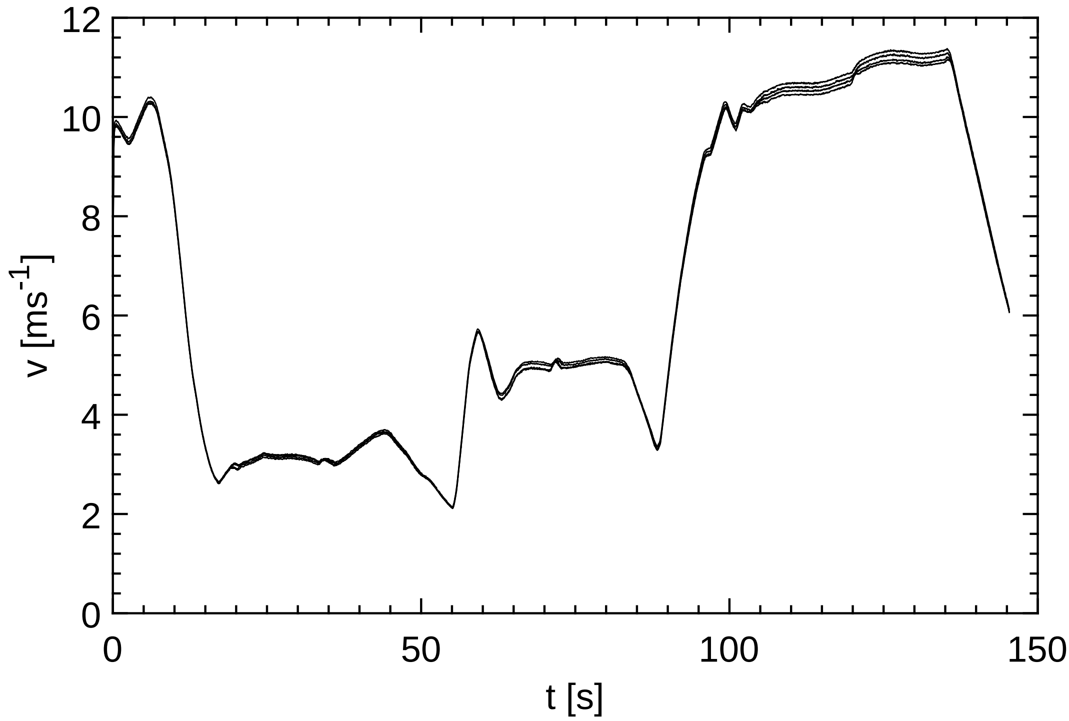


Figure 3.6: Wheel speeds at ground contact point after correction to the center-of-gravity

3.4. Longitudinal accelerometer data pre-processing

Discussed in many publications introduced in Chapter 1.2, also the measurement from longitudinal accelerator is biased by systematic error. Luckily, the part of bias created by its installation position and the effect of temperature is already corrected in the longitudinal acceleration signal provided by the CAN bus.

Still, because the sensor is fixed on the vehicle body, which deviates from xy-plane of our co-ordinate system, the additional correction is indeed mandatory. Deviation is caused by driving in the slope, where then the component of gravity acceleration is measured and also by chassis pitching during acceleration or deceleration. Impact of pitching is very small and can be therefore neglected [8].

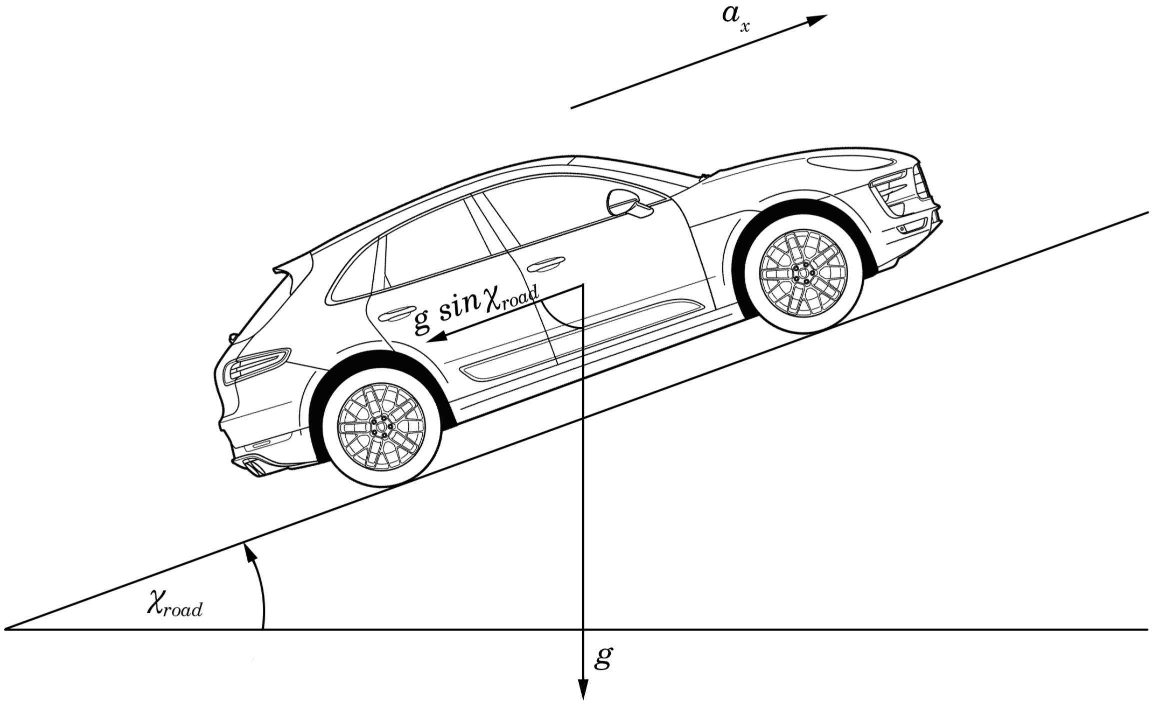


Figure 3.7: Effect of gravity acceleration on longitudinal acceleration measurement when driving in slope

In order to eliminate the gravity acceleration from measurement and thereby correct the longitudinal acceleration we use:

$$a_{m,x} = a_{c,x} + g \sin \chi_{road} \tag{3.6}$$

where $a_{m,x}$ is measured longitudinal acceleration, $a_{c,x}$ is corrected longitudinal acceleration, g in gravity acceleration and χ_{road} is road slope.

In equation 3.6 the road slope is unknown variable and must be estimated.

3.4.1. Slope estimation

Chosen state variable vector consists of longitudinal velocity and road slope:

$$x = \begin{bmatrix} v_{CoG} \\ \chi_{road} \end{bmatrix} \tag{3.7}$$

the input into the system will be longitudinal acceleration:

$$u = \begin{bmatrix} a_{m,x} \\ 0 \end{bmatrix} \tag{3.8}$$

and the current estimate of longitudinal speed $\hat{v}_{CoG}$ is output measurement:

$$y = \hat{v}_{CoG} \tag{3.9}$$

Equation 3.6 can be also written as

$$a_x = a_{m,x} - g \sin \chi_{road} \tag{3.10}$$

when driving straight:

$$a_x = \dot{v}_{CoG} \tag{3.11}$$

According to fact, that the slope of public roads is limited approximately to $\pm 12°$ [34] following linearization can be made, allowing us to employ linear observer:

$$\sin \chi_{road} \approx \chi_{road} \tag{3.12}$$

Then, if the road slope is presumed constant, the system can be written in state space form as follows:

$$\underbrace{\begin{bmatrix} \dot{v}_{CoG} \\ \dot{\chi}_{road} \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} 0 & -g \\ 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} v_{CoG} \\ \chi_{road} \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} a_{m,x} \\ 0 \end{bmatrix}}_{u} \tag{3.13}$$

$$\underbrace{\dot{v}_{CoG}}_{y} = \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}^{\top}}_{C} \underbrace{\begin{bmatrix} v_{CoG} \\ \chi_{road} \end{bmatrix}}_{x} \tag{3.14}$$

For estimation of road slope χ_{road}, the Luenberger observer will be employed. At first, the observability of given system must be analyzed.

Order of investigated system is $n = 2$, therefore:

$$Q = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & -g \\ 0 & 0 \end{bmatrix} \tag{3.15}$$

The rank will be checked by calculating the determinant. If determinant of this matrix is equal to 0, its lines are linear independent, and therefore is the system observable:

$$\det Q = \det \begin{bmatrix} 1 & -g \\ 0 & 0 \end{bmatrix} = -g \neq 0 \tag{3.16}$$

The system has been proven to be observable, so the pole placement by means of Equation 2.14 can be made. Because of order of the system, gain matrix L will have elements l_1, l_2:

$$\det \big(sI - (A - LC)\big) = \det \left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & -g \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} l_1 & 0 \\ l_2 & 0 \end{bmatrix} \right) = s^2 + s \cdot l_1 - g \cdot l_2 \tag{3.17}$$

from

$$s^2 + s \cdot l_1 - g \cdot l_2 = (s - \lambda_1)(s - \lambda_2) = s^2 - s(\lambda_1 + \lambda_2) + \lambda_1 \cdot \lambda_2 \tag{3.18}$$

we get

$$l_1 = -\lambda_1 - \lambda_2 \tag{3.19}$$

$$l_2 = \frac{-\lambda_1 \cdot \lambda_2}{g} \tag{3.20}$$

According to strategy from Chapter 2.4, we know, that l_1 has to be positive and l_2 negative. Their exact value was tuned experimentally, and it is:

$$L = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -1 \end{bmatrix} \tag{3.21}$$

In Figure 3.8 the implementation of Luenberger observer is depicted.

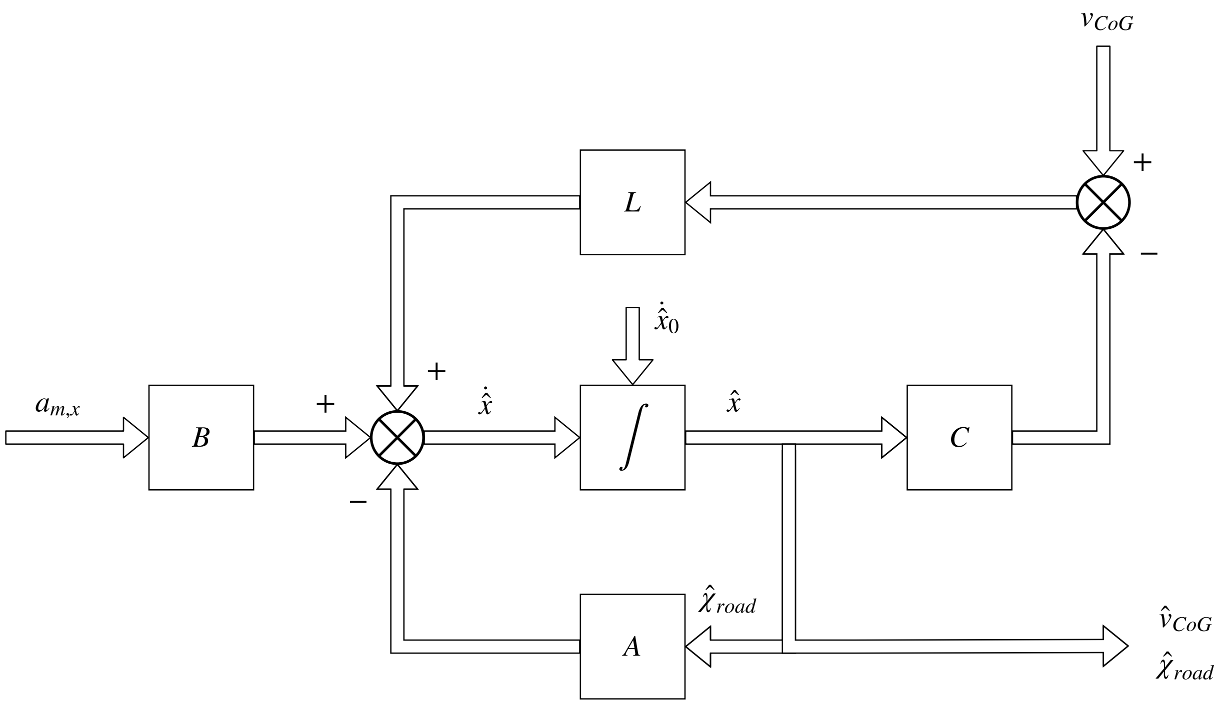


Figure 3.8: Implementation of Luenberger observer for road slope estimation

Few important measures have been made that need be remarked in the end.

The road slope estimation cannot be performed in case, when all wheels are considered in over-slip. This will introduce significant error to the estimation, as the estimated vehicle speed $\hat{v}_{CoG}$, then equals only to integration of corrected longitudinal acceleration $a_{c,x}$. Therefore, the criterion has been defined, which freezes the estimation of road slope keeping the last estimated value, when mentioned situation occurs, and holds that value until the wheel speed measurements are available again.

Additionaly, to obtain better road slope estimation results some post-processing has been implemented. The maximal road slope has been limited by saturation to benevolent value of $\sim \pm 30°$. Also the unrealistically rapid changes of incline have been mitigated by limiting the allowed change rate with calibrated constant. Comparison of results before and after is attached below.

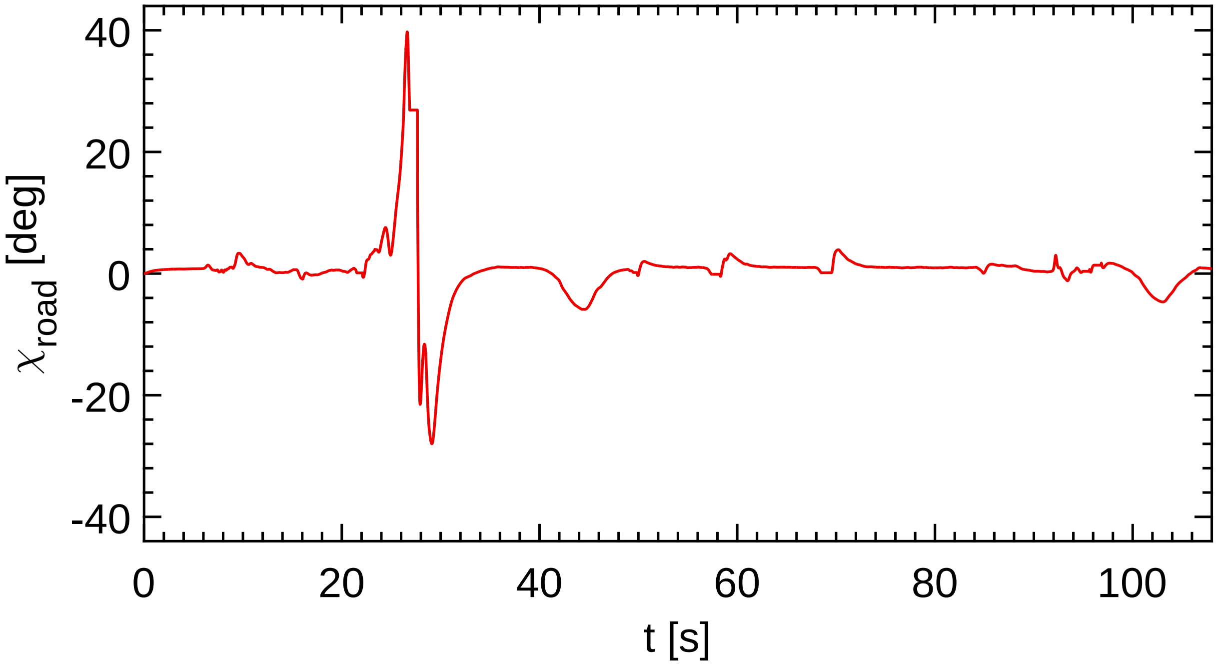


Figure 3.9: Road slope estimation result for $\sim 0°$ incline before post-processing

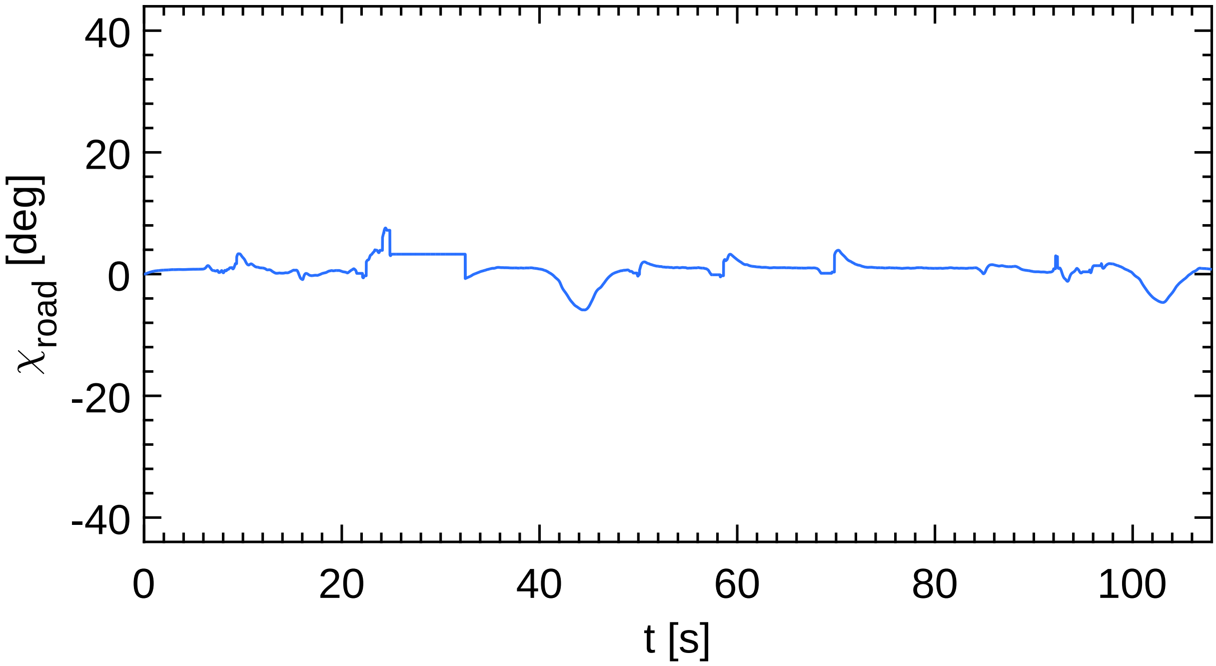


Figure 3.10: Road slope estimation result for $\sim 0°$ incline after post-processing

3.5. Kalman filter for vehicle speed estimation

To estimate the vehicle speed the adaptive Kalman filter is used. From the theory of Kalman filter, which was presented in Chapter 2.5, is known, that the trust of measurement updating the Kalman gain K is given by the covariance matrix. The problem is, that in our case, the wheel can slip extensively, so the measurement if its speed will drift away rapidly, of the real vehicle velocity. Then, even if we assign that measurement the variance of the biggest value, we can imagine, it will be still so far away from infinity. In result, the accuracy of vehicle speed estimate will be ruined. To prevent this from happening, the strategy of eliminating measurements completely, by adaptation of the measurement matrix H is proposed.

In implemented Kalman filter, vehicle speed is the state variable:

$$x = v_{CoG} \tag{3.22}$$

with initial condition set to arithmetical avarage of wheel speeds, for cases when the estimation does not start from standstill:

$$x_0 = \frac{v_{fl} \cdot v_{fr} \cdot v_{rl} \cdot v_{rr}}{4} \tag{3.23}$$

Corrected longitudinal acceleration is the input:

$$u = a_{c,x} \tag{3.24}$$

and the measurement vector consists of wheel speed measurements:

$$z_k = \begin{bmatrix} v_{fl} \\ v_{fr} \\ v_{rl} \\ v_{rr} \end{bmatrix} \tag{3.25}$$

Measurement vector is initially set to:

$$H = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{3.26}$$

but values of its elements can be changed on demand to 0 in position corresponding to the wheel in over-slip. This is carried out by over-slip criteria presented later in text.

The observed state space model is:

$$v_{CoG_k} = v_{CoG_{k-1}} + T_s a_{c,x_k} + w_k \tag{3.27}$$

where T_s is time step.

The covariance matrices Q_k and R_k were determined by the calibration as follows:

$$Q_k = 10 \tag{3.28}$$

$$R_k = \begin{bmatrix} 500 & 0 & 0 & 0 \\ 0 & 500 & 0 & 0 \\ 0 & 0 & 500 & 0 \\ 0 & 0 & 0 & 500 \end{bmatrix} \tag{3.29}$$

3.6. Wheel over-slip criteria

To detect the occurrence of the over-slip on individual wheels and to provide the information for adaptation of matrix H, the wheels-slip critetions are defined in algorithm. 2 different main criteria for the detection will be introduced in this Chapter. Both working for each wheel separately, sending further the information about detected over-slip by raising and decreasing over-slip flags, based on the state of selected variables. Then, in the last subsection, the description of subsidiary criterion is to be found.

3.6.1. Wheel over-slip

Since the term slip or over-slip is frequently mentioned in this thesis, it would be proper to define and shortly introduce what does it state for.

Wheel is said to be in over-slip, when the theoretical and the real distance traveled by wheel are not the same. This can be described by an example, where perimeter of the wheel is 2 meters. When the wheel turns 5 times, it should travel 10 meters - in ideal case. When is travels more or less, the cause is the over-slip [35]. Whether the distance traveled by wheel is longer or shorter than ideal distance, depends on origin of the slip.

The shorter distance traveled is a consequence of slip caused by motor torque (acceleration) and is defined as:

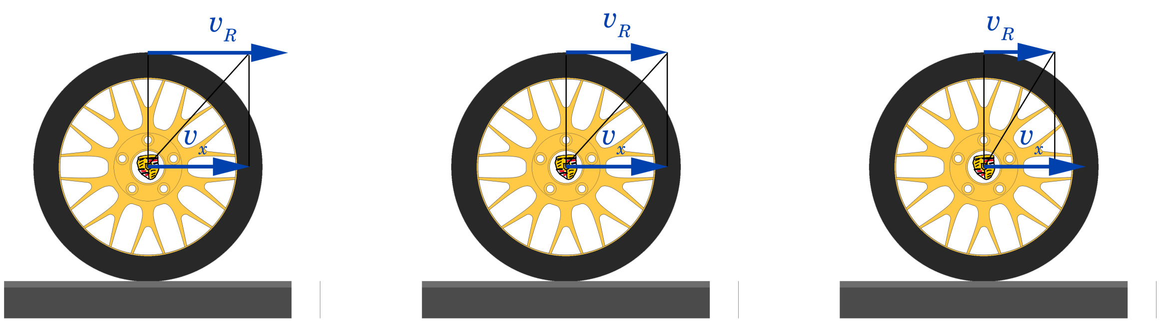
$$\lambda_x = \frac{\omega_w r_w - V_x}{\omega_w r_w} \tag{3.30}$$

and on the other hand, the longer distance is traveled when braking (deceleration):

$$\lambda_x = \frac{v_x - \omega_w r_w}{V_x} \tag{3.31}$$

where $\lambda_x =$ is wheel slip, ω_w is rotational velocity and r_w is wheel radius [36].

Figure 3.11 illustrates relationships between rotational and translational velocities of wheel in mentioned states.



(a) Wheel slip due to acceleration

(b) Wheel rolling ideally without slip

(c) Wheel slip due to deceleration

Figure 3.11: Wheel slip situations in regards to rotational and translational velocities

3.6.2. Acceleration criterion

Acceleration criterion uses the measurement of wheel speed, absolute value of their acceleration obtained from derivation of the wheel speeds and the estimate of vehicle speed v_{CoG} from previous step.

If the algorithm concludes, that the wheel may be in over-slip, it raises the over-slip flag. This is made by tick which appears in the moment, when the wheel acceleration is considered too high and exceeds the limit value. The limit value does not vary in time and was pre-calculated as follows:

$$a_{lim} = \frac{\sum F}{m_m} \tag{3.32}$$

with m_m as weight of middle loaded vehicle and sum of forces [37]:

$$\sum F = \frac{M_{max,F}}{r_D} + \frac{M_{max,R}}{r_D} - \underbrace{F_g}_{m_m g \sin \chi_{road}} - \underbrace{F_{roll}}_{m_m g f_r r} \tag{3.33}$$

where $M_{max,F}$,$M_{max,R}$ are maximal motor torques for front and rear axle respectively, r_D is dynamic wheel radius.

F_g represents the sinus component of the gravitational force when driving in slope of χ_{road}. Here, the value of slope was selected $\chi_{road} = -30°$. Road gradient of 30° is considered as the maximal to occur, when driving the conventional vehicle including extreme conditions. The negative value makes the component F_g positive, pushing the acceleration limit higher. This is wanted effect, as the slip occurs only because of that part of acceleration regarding to motor torque. Not counting with this fact will lead to unreasoning slip-flags.

F_{roll} is rolling resistance with rolling resistance coefficient f_{rr}. Value of f_{rr} lies between 0.01 and 0.04. For the commercial passenger vehicle, its typical value is ~ 0.015 [37].

Wind resistance does not act in the sum of forces. The reason is, that also the vehicle stand-still has to be considered as a potential state for over-slip to occur and then the wind resistance equals 0.

Speed of the wheel is continuously compared to the actual estimate of v_{CoG}. When the wheel speed approaches to the estimated vehicle speed it is assumed, that the wheel is turning back from the over-slip, so the tick is produced in order to decrease the over-slip flag.

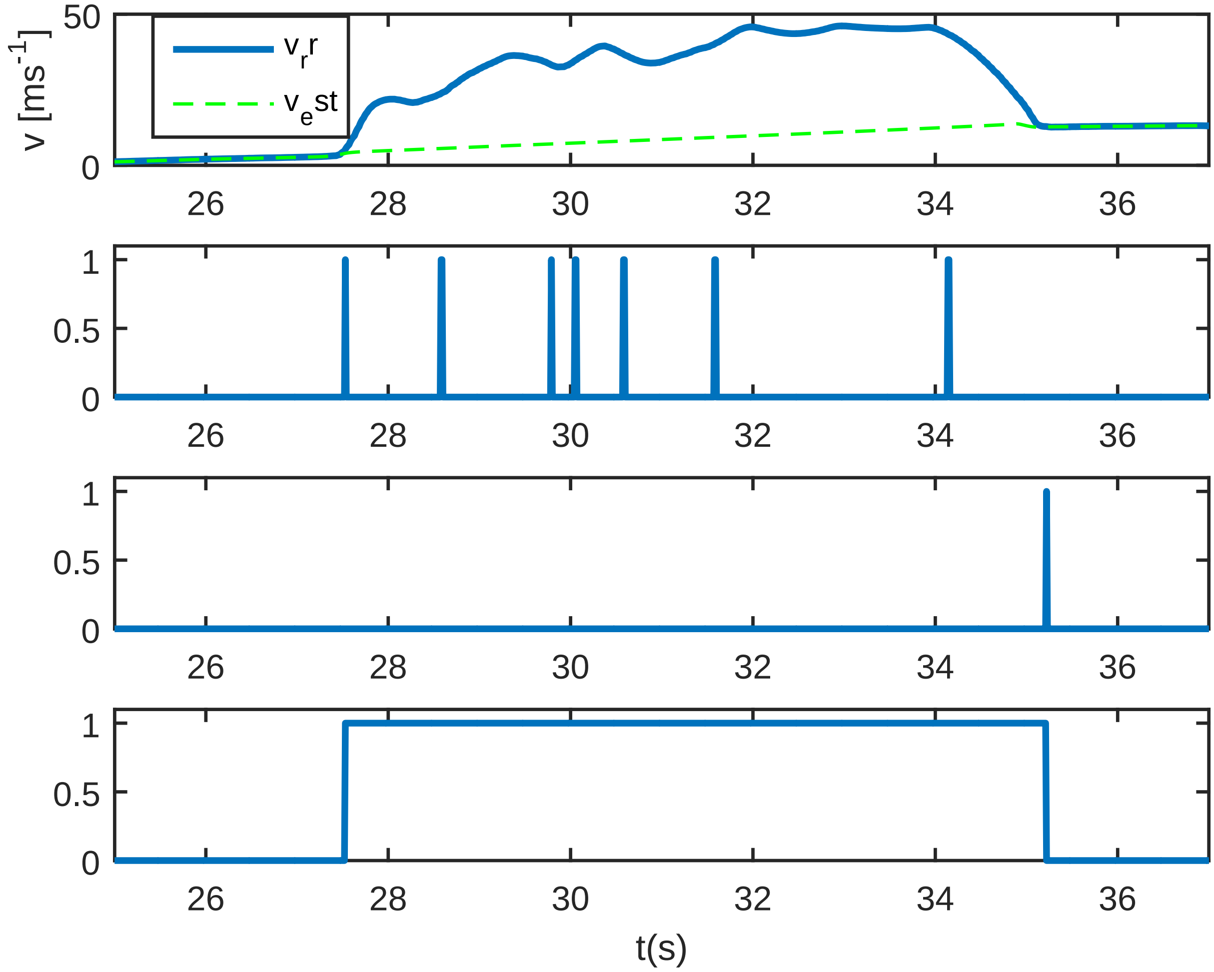


Figure 3.12: Acceleration criterion function - Raise and Decrease Flag ticks

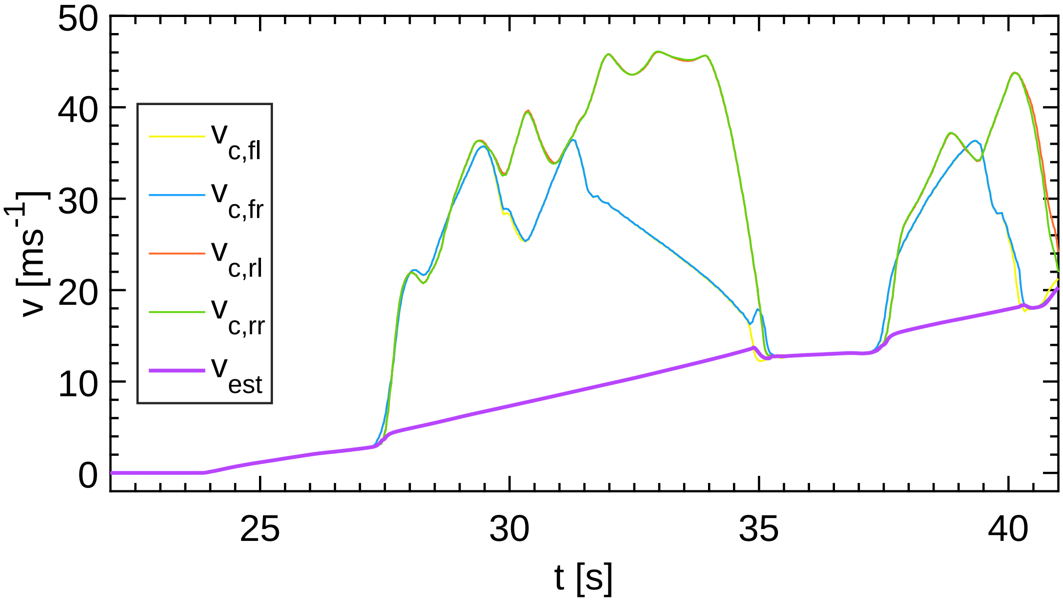


Figure 3.13: Acceleration criterion result with all wheels in over-slip

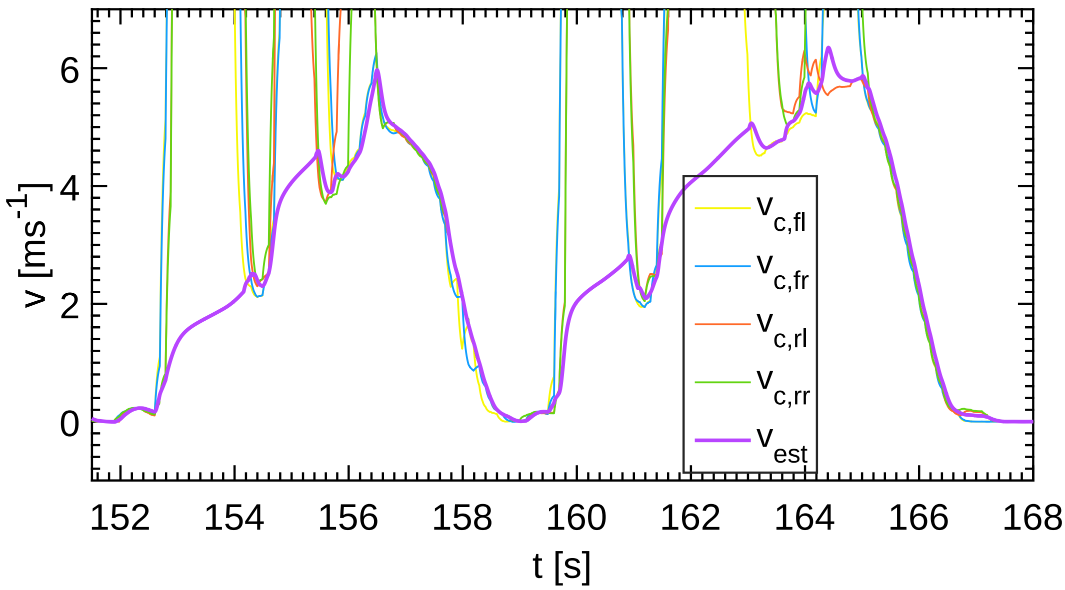


Figure 3.14: Acceleration criterion result with all wheels in over-slip with rapid return to traction and over-slip again

Results seen in the Figures 3.13 and 3.14 show, that even this criterion detects the wheel over-slip reliably, the detection is slow because of constrained dynamics of acceleration limit. This results in obvious inaccuracy of estimation. Therefore, new criterion working simultaneously has to be defined, to report the over-slip earlier.

3.6.3. Torque criterion

Use of electric vehicle allows us to define also the torque criterion, as sufficiently accurate torque can be obtained. This could be not practically done for a conventional vehicle, as the torque has to be estimated there.

The motor torque criterion is based on real-time model computation of maximal torque, that should be transmittable on the road. This is then compared to demanded motor torque applied on wheels. By normal traction, the motor torque on left and right wheel is equal. Therefore, torque for single wheel on the axle, will by obtained by dividing the motor torque on that axle by 2. Additional inputs for the criterion are wheel accelerations obtained from derivating the measured wheel speeds, corrected longitudinal acceleration and the Flag-decreasing signal from the acceleration criterion.

First the motor torque model for each wheel has to be set up. The longitudinal traction force is [38]:

$$F_{t,x} = F_N \cdot \mu \tag{3.34}$$

where road friction coefficient μ can be interpreted as:

$$\mu = \frac{a_x}{g} \tag{3.35}$$

The normal force of vehicle in dynamic state equals for front wheels:

$$F_{N,f} = F_{stat} - F_{dyn} \tag{3.36}$$

$$F_{N,f} = mg\cos\alpha\frac{l_r}{l_r + l_f} - \frac{h_{CoG}}{l_r + l_f}mg\sin\chi_{road} + a_{c,x} \tag{3.37}$$

and for rear wheels:

$$F_{N,r} = F_{stat} + F_{dyn} \tag{3.38}$$

$$F_{N,r} = mg\cos\alpha\frac{l_f}{l_r + l_f} + \frac{h_{CoG}}{l_r + l_f}mg\sin\chi_{road} + a_{c,x} \tag{3.39}$$

where all parameters are described by the Figure 3.15.

In the implementation used in estimation algorithm the equations were adjusted as follows:

$$F_{N,fl/fr} = \frac{1}{2}mg\frac{l_r}{l_r + l_f} - \frac{1}{2}\frac{h_{CoG}}{l_r + l_f}m + a_{m,x} \tag{3.40}$$

$$F_{N,rl/rr} = \frac{1}{2}mg\frac{l_f}{l_r + l_f} + \frac{1}{2}\frac{h_{CoG}}{l_r + l_f}m + a_{m,x} \tag{3.41}$$

as measured longitudinal acceleration consists of correct longitudinal acceleration and sinus component of gravity acceleration $a_{m,x} = g\sin\chi_{road} + a_{c,x}$ and this way the error introduction is eliminated. $\cos\chi_{road}$ is neglected as in incline of $\pm 12°$ it is $\cos\chi_{road} = 0.978$ and would only unnecessarily produce the risk of error being introduced due to inaccurate estimation of slope χ_{road}. As mentioned above, by division by 2, the normal force for each of wheels on the axle is obtained.

Besides the model torque compared to the actual motor torque another criterion has to be fulfilled to raise the over-slip flag on. In real-time, the algorithm checks, if the wheel with applied torque, which exceeds the model torque has accelerated enough to be said, that it is in over-slip.

For this, the difference between actual and previous value of wheel acceleration, obtained from derivation of wheel speed measurement, showed the best results in detection speed. As it is practically the $2nd$ derivation of the wheel speed, there is a lot of noise present. Therefore, calibration was made on the data capturing the driving situation with high process noise present. Additionally, the compromise between the detection speed and lowering the faulty detection rate had to be made.

Even if it comes to situation, that the flag was raised only because of the noise present, the decrease flag criterion comparing the difference between actual and previous wheel speed will intervene. The slip flag of this criterion can be also decreased by the decrease flag of the acceleration criterion, supporting the robustness again.

The estimation results with torque criterion are presented below also with comparison to acceleration criterion by itself.

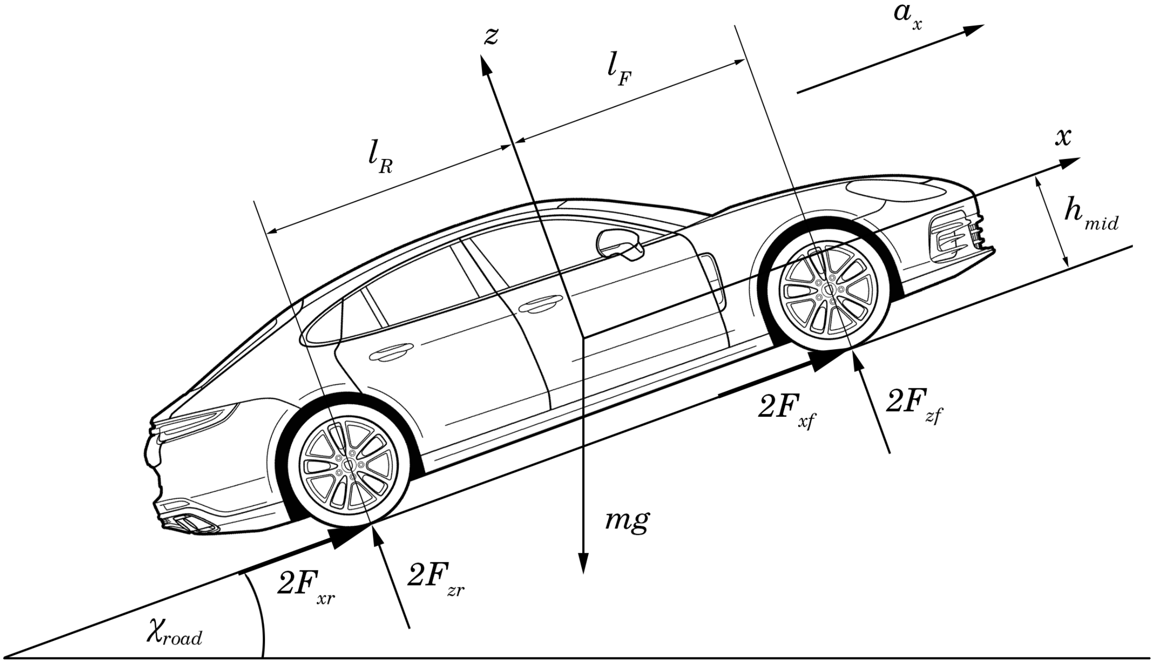


Figure 3.15: Forces acting on vehicle on inclined road, parameters from the equations

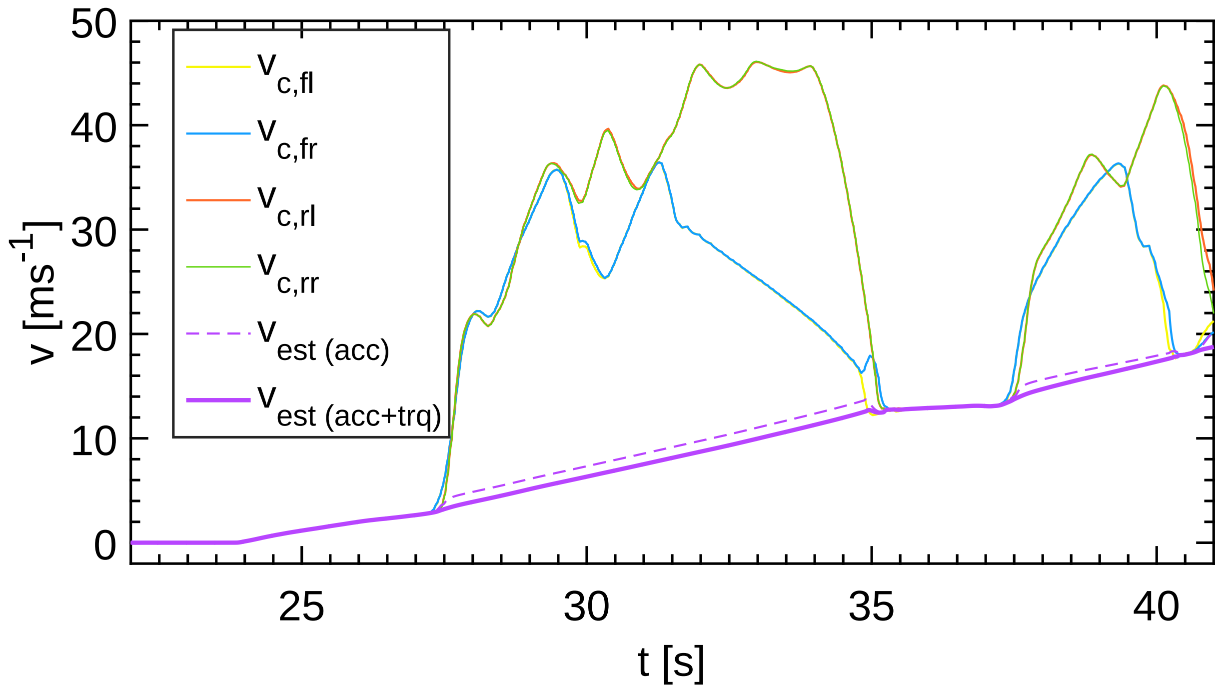


Figure 3.16: Improved estimation result with comparison to acceleration criterion only - all wheels in over-slip

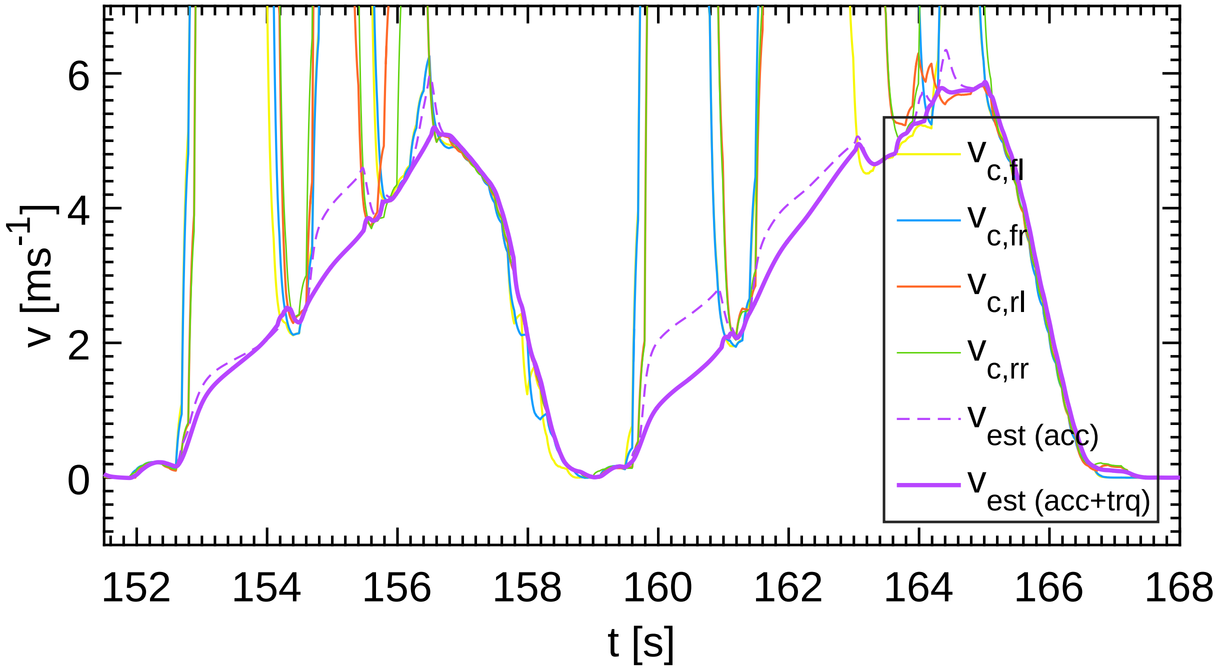


Figure 3.17: Improved estimation result with comparison to acceleration criterion only - all wheels in over-slip with rapid return to traction and over-slip again

3.6.4. Subsidiary criterion

Subsidiary criterion guards the period, during which are all the wheels considered in over-slip, so the estimation algorithm relies only on the integration of longitudinal acceleration. Criterion says, that over-slip of all wheels should not last for more than 10sec and therefore puts back to H matrix that wheel measurement, whose speed is closest to the estimated vehicle speed in that moment. This is done for very special cases and improves robustness of the algorithm by protecting it from outputting unreal values.

Even if there was a situation where all the wheels were in over-slip for more than 10sec for real, the estimate obtained from acceleration integration will be anyway most probably substantially inaccurate, so it would be no more relevant to continue without the wheel speed measurement.

4. Test against data from model vehicle

In this Chapter the test of estimation algorithm against the data obtained from vehicle model will be presented. The algorithm was tested against the model data only in very beginning of its development, as enough data captured from real vehicle had been available. Measurements obtained during real driving situations include different noises and as the algorithm has to work properly in the real vehicle, it must be also calibrated via real test cases.

Instead of that fact, the depicted tests show, how the expected result of vehicle speed should look like. This could be helpful in case, that real data are not provided by the measurement of reference real vehicle speed. All presented estimation results are compared with longitudinal velocity value provided by model.

In the first presented test, the model vehicle has been driving with initial velocity of 5ms^{-1} with constant drive torque mode. The surface had road friction coefficient $\mu = 0.8$. Only the rear wheels have been driven.

At time of 2s, the torque demand was increased to 1500Nm and the vehicle accelerated smoothly. At time 4s has the vehicle driven into the surface with low friction and the driven wheels started to slip. Algorithm has detected the over-slip and followed only the measurement of non-driven wheels which was correct. Thanks to initial condition given to Kalman Filter, there was no estimation delay present.

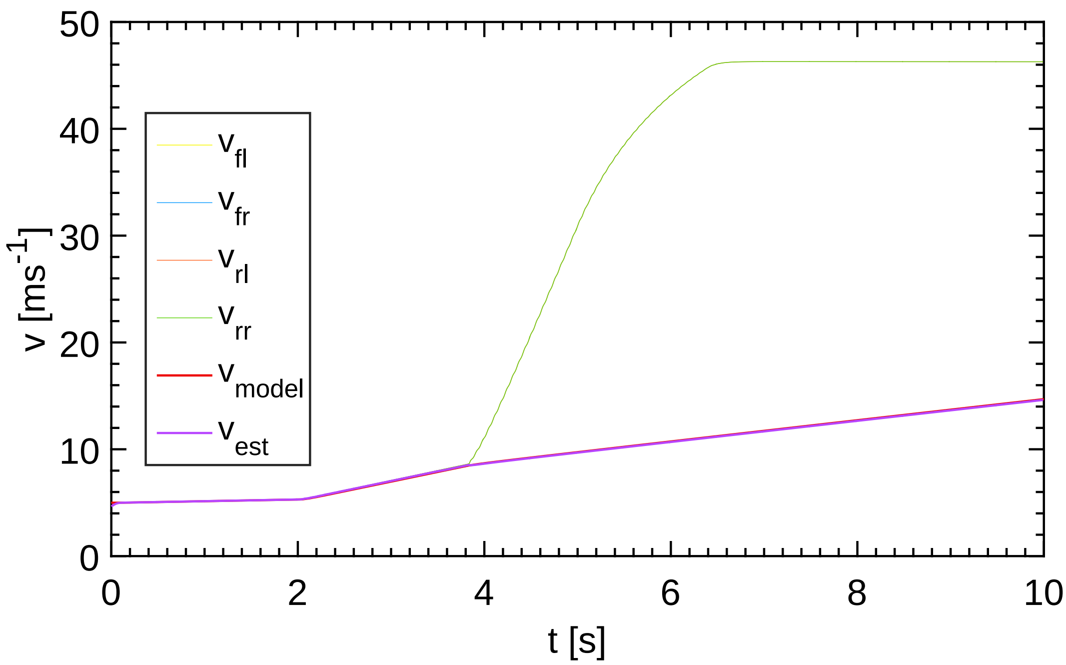


Figure 4.1: Estimation result by test of against the model - rear wheels in slip

Other modeled case had the same preconditions, but during the drive, also the torque on front wheels had been employed. Approximately at time 4s has vehicle driven into the surface with low friction and after circa 2s again to surface with prior value of road friction coefficient. The μ-jump from high to low friction was repeated three times.

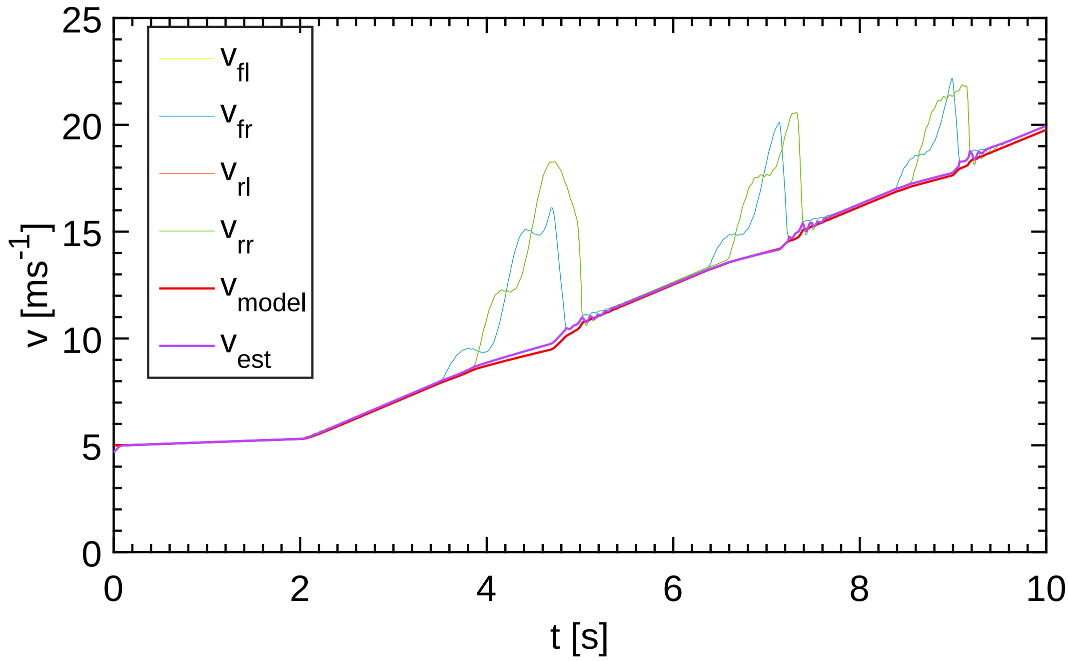


Figure 4.2: Estimation result by test of against the model - all wheels slip due to μ-jump

In this case with all wheels driven, the model vehicle has driven into road surface with low friction causing all wheels to slip. Wheels remained in over-slip till end of the sequence. Algorithm detected the wheels slipping in time so only little error has been integrated. Therefore the accurate results were obtained even the slip had last for more than 6s.

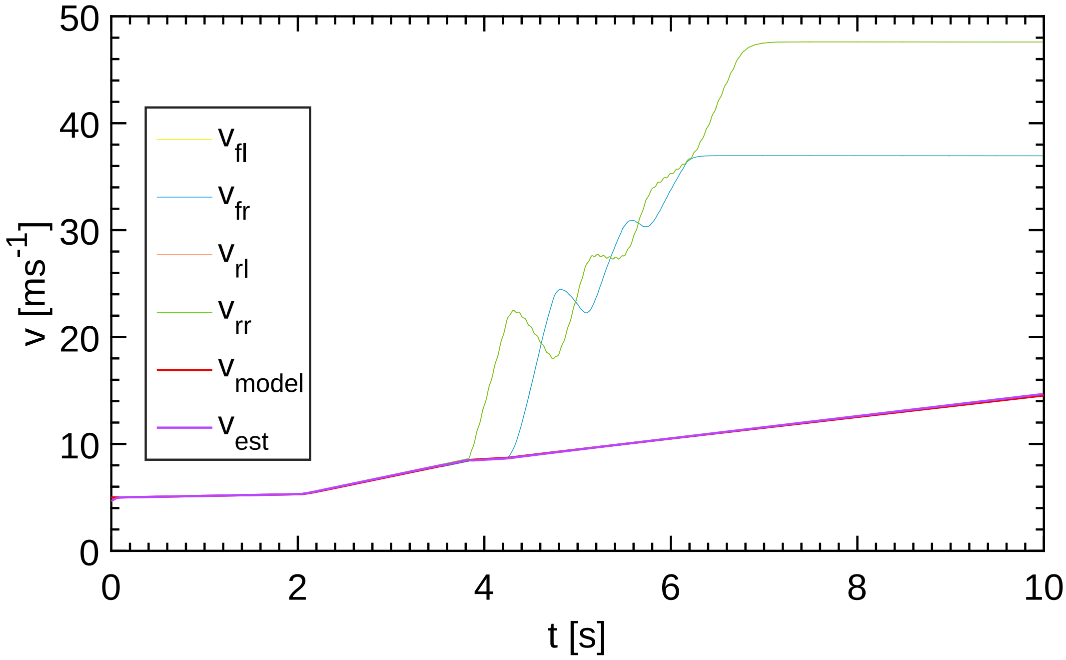


Figure 4.3: Estimation result by test of against the model - all wheels in long over-slip

5. Test against data from real vehicle

Results from testing the algorithm against the data capcured from sensor measurements of real vehicle will be introduced in this Chapter. Sample data have been chosen due to some specific situation captured, to show the function and accuracy of estimation in situation considered critical. In every Subsection, the data chacteristics will be described and the estimation results will be shown. None of presented data have the real longitudinal velocity measured, but according to experience from previous Chapter, the path of real longitudinal velocity can be easily assumed.

5.1. All wheels slip & 10% slope

This 30 seconds short sequence of measured data captures the situation of 4WD vehicle driving in winter time. During first and second acceleration, there is enough road friction offered according to acceleration request transformed into the motor torque. Then, around $21st$ second the over-slip of all 4 wheels occures 2-times. As seen from Figures 5.1 and 5.2 showing the vehicle speed estimate and the defined criteria acting in practice, represented by adapting corresponding elements of H matrix, the algorithm acts correctly. It detects wheels slipping and removes faulty wheel speed measurements, using integration of longitudinal acceleration.

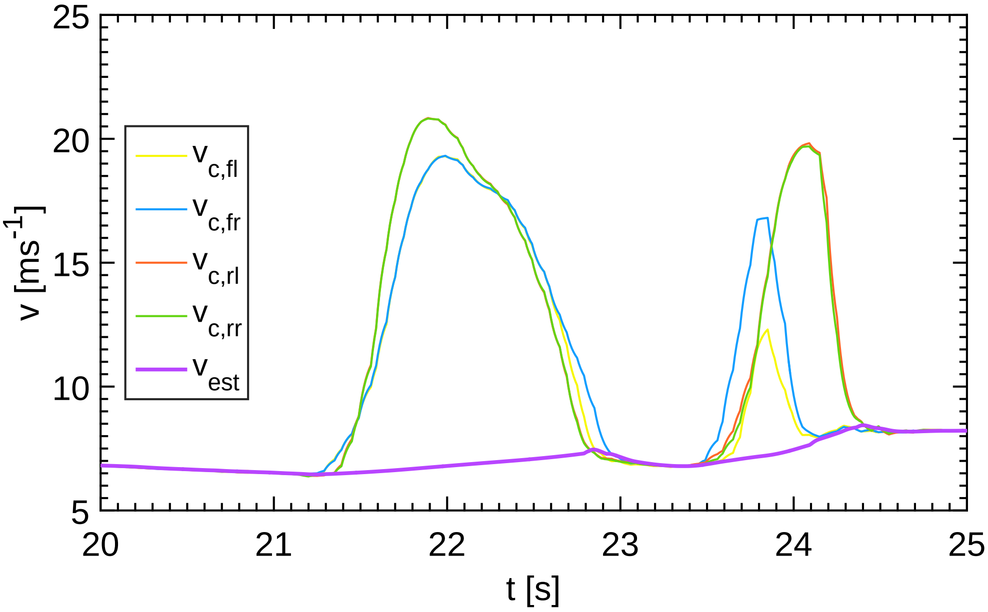


Figure 5.1: Wheel speeds and vehicle speed estimate - all wheels slip

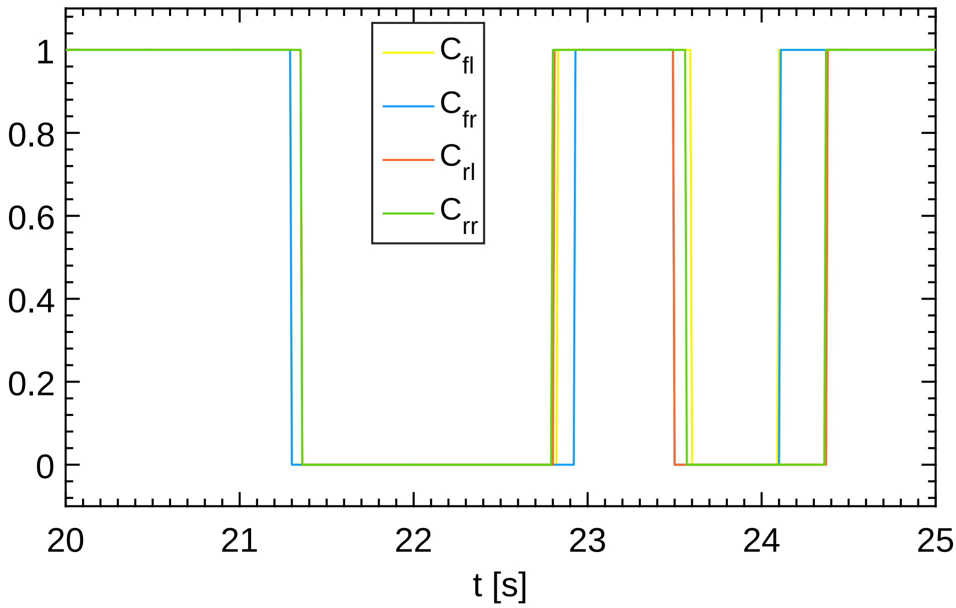


Figure 5.2: Criterion adaptation of matrix H - all wheels slip

The whole data sequence also with torques on front and rear wheels is shown here. Additionaly, it is known, that the vehicle was driving into slope with circa 10% incline, which corresponds to $\sim 5.7°$. Therefore, the result of slope estimation is also attached.

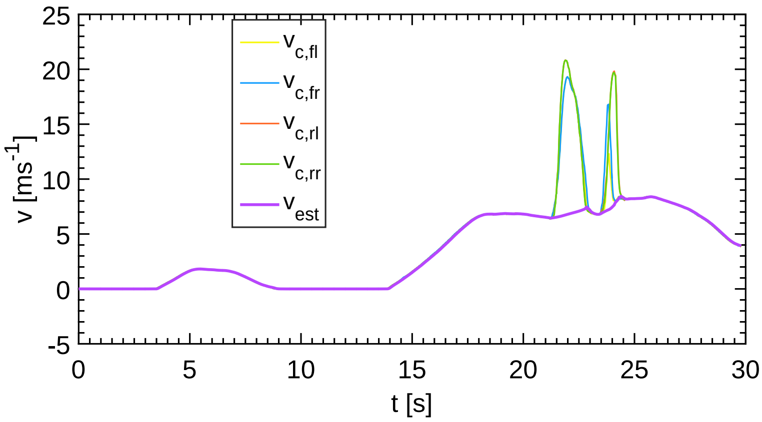


Figure 5.3: Wheel speeds and vehicle speed estimate - whole sequence all wheels slip

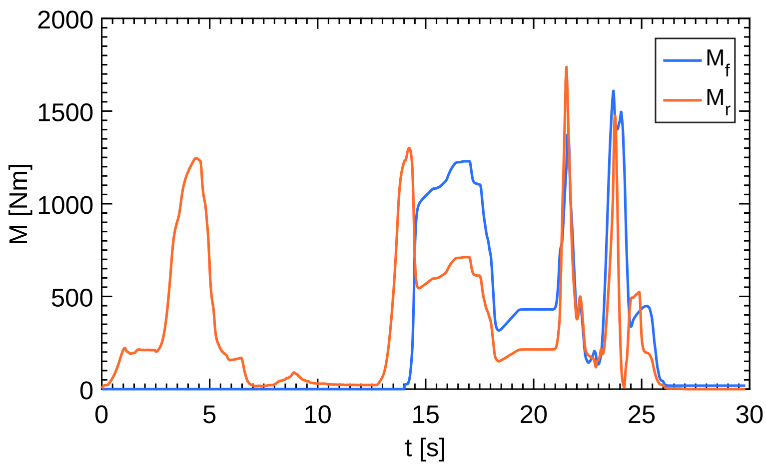


Figure 5.4: Front and rear motor torque

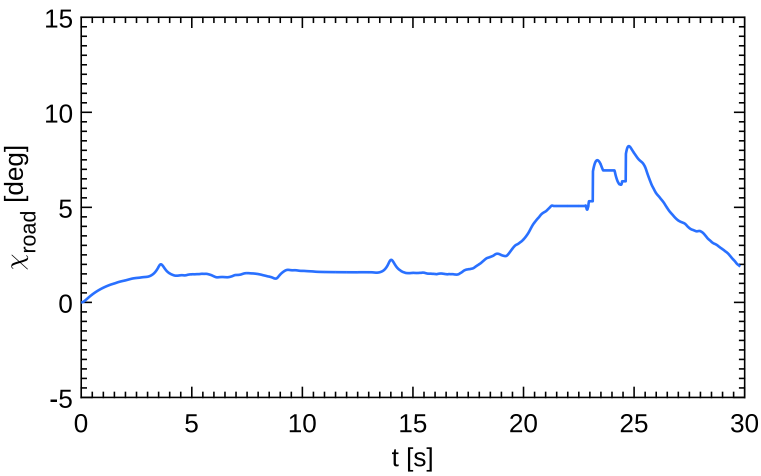


Figure 5.5: Road slope estimation

5.2. Braking with ABS & μ-split

Two braking situations with ABS activation have been captured in this test-drive. Characteristic behaviour of releasing locked wheels and reapplying the brake force in order to maintain better control of vehicle and shorten the braking distance can be recognised. The real value of vehicle speed should ”slide” on the peaks of the highest wheel speed in the moment. It is shown, that algorithm detects also the negative torque by the criterion and follows the peaks with almost none margin. The vehicle has all wheels driven and is driven on a flat road with low friction surface. Whole sequence is presented in the next page where also the slope estimate is shown.

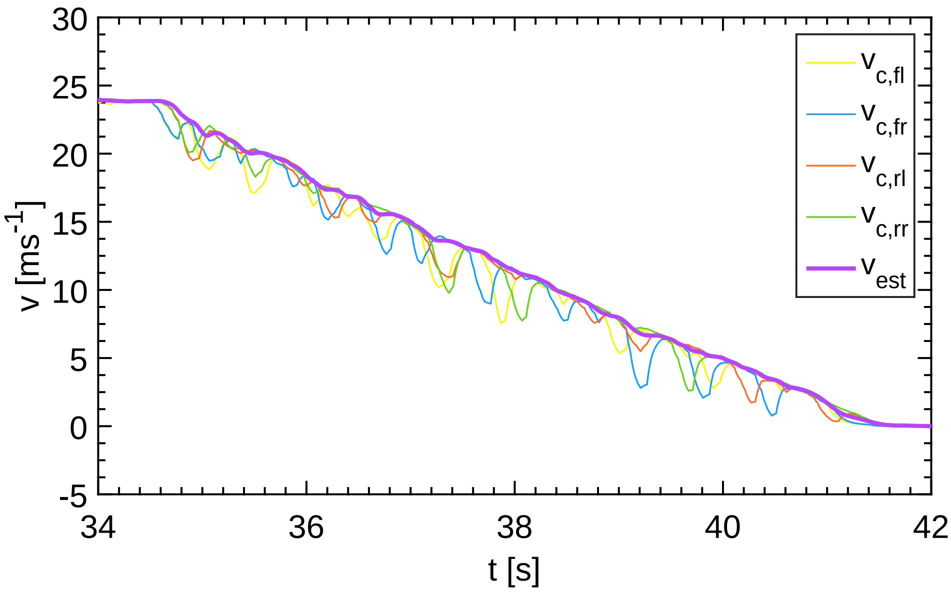


Figure 5.6: Detailed view of second ABS braking

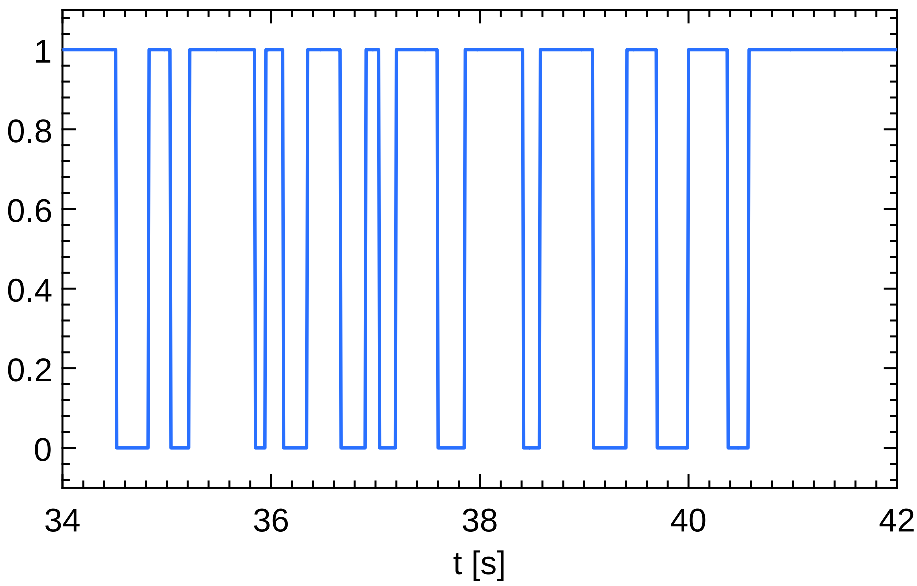


Figure 5.7: Criterion adapting the H matrix of Front Right wheel during ABS braking

Here, the μ-split situation is to be seen. μ-split names the situation, when the friction of one wheelpath side differst from the other significantly. Big positive or negative slip of wheels on right or left side is typical. The estimation algorithm had succesfully eliminated the corrupted measurements.

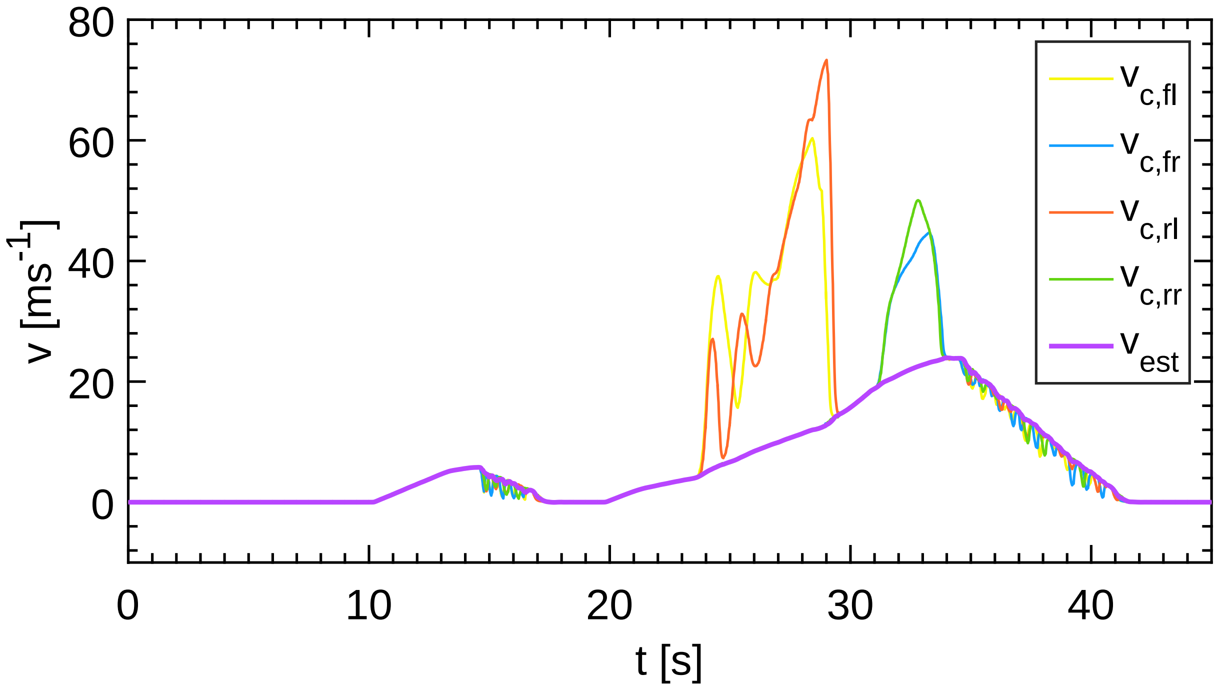


Figure 5.8: Wheel speeds and vehicle speed estimate in μ-split situation depicted

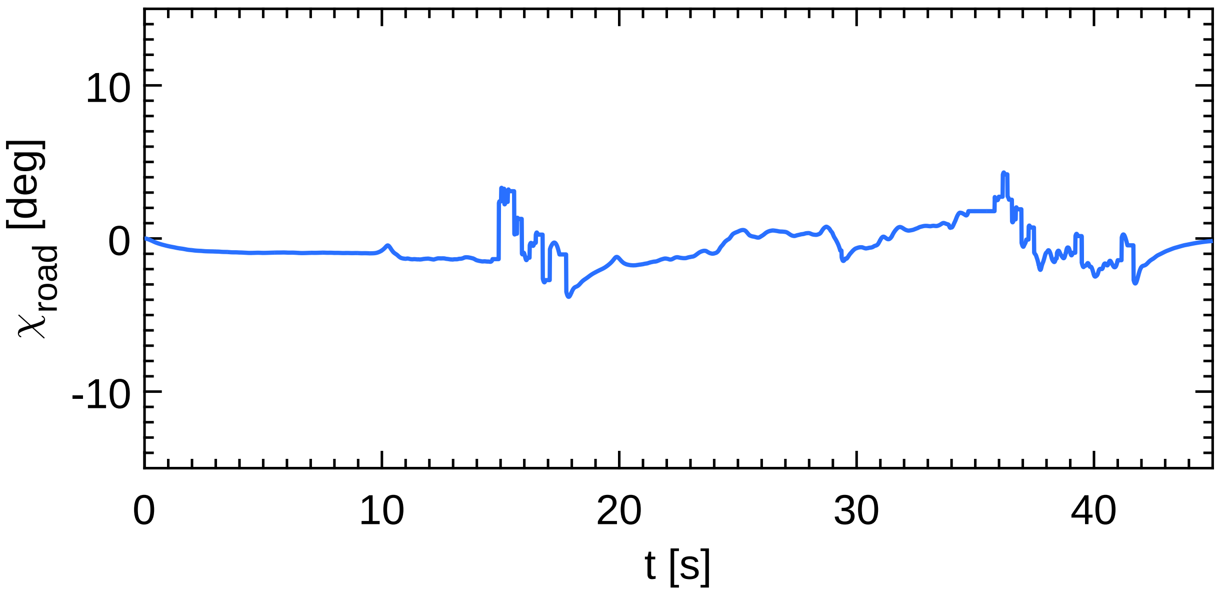


Figure 5.9: Slope estimation with assumption to be $\sim 0°$, the influence of ABS breaking on estimate can be noticed

5.3. Ice circle

Finally, the critical situation of driving in circle on ice with wheels nearly permanently slipping, steering wheel turning violently and violent acting on acceleration pedal. This is demonstrated by Figures 5.10, 5.11 and 5.12.

The algorithm produced sufficient estimation results with ability to detect the wheel in over-slip in time, even it was provided with such a noisy wheel speed measurement signals.

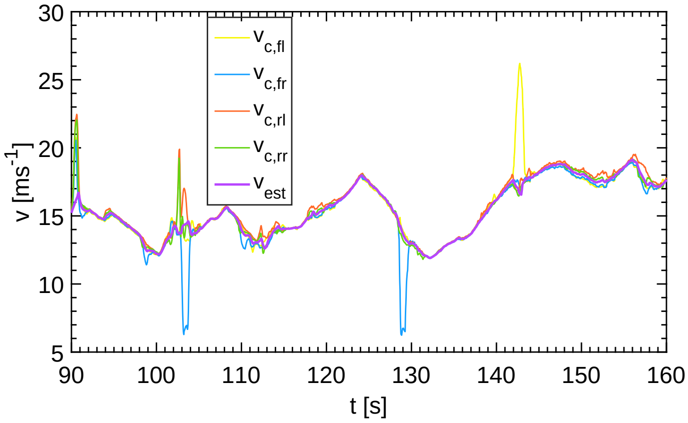


Figure 5.10: Wheel speeds and vehicle speed estimate in selected data sequence

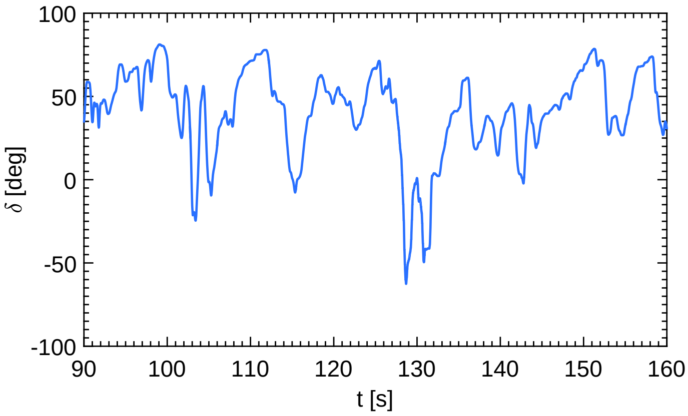


Figure 5.11: Steering wheel angle

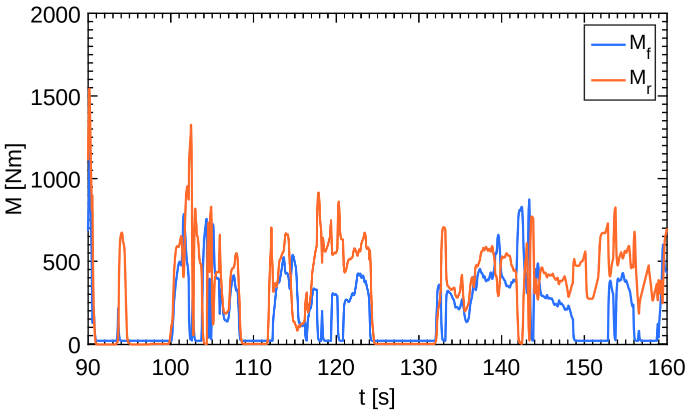


Figure 5.12: Motor torques on front and rear wheels

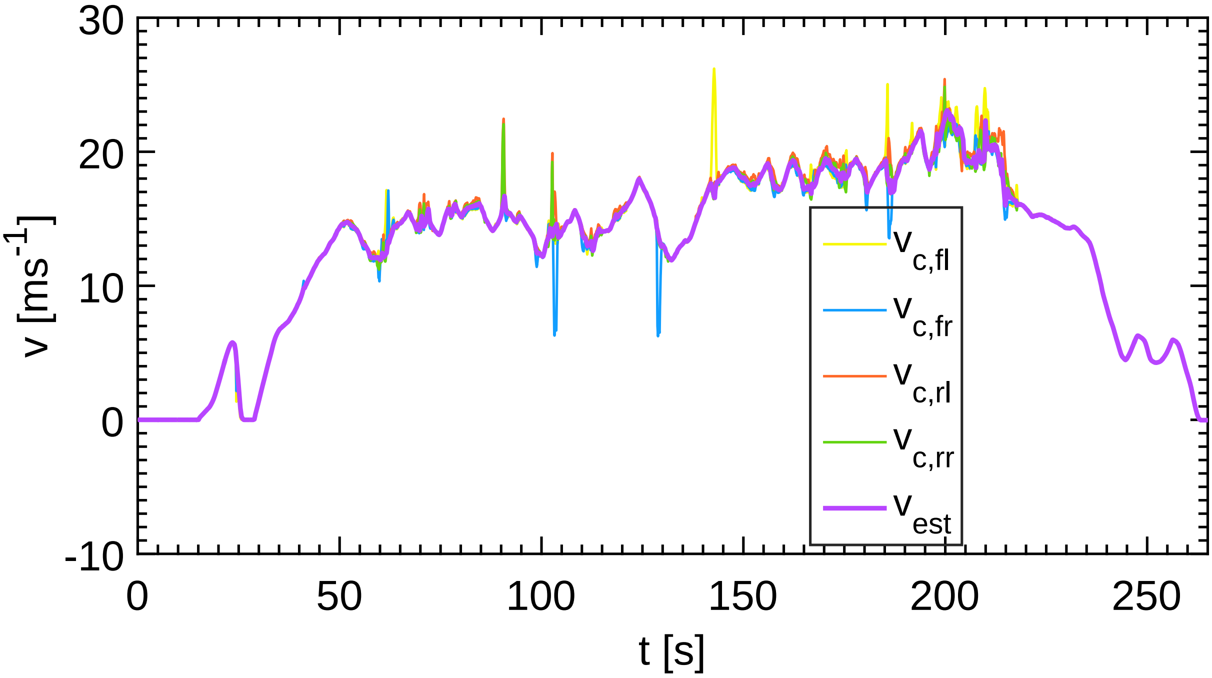


Figure 5.13: Wheel speeds and vehicle speed estimate in whole data sequence

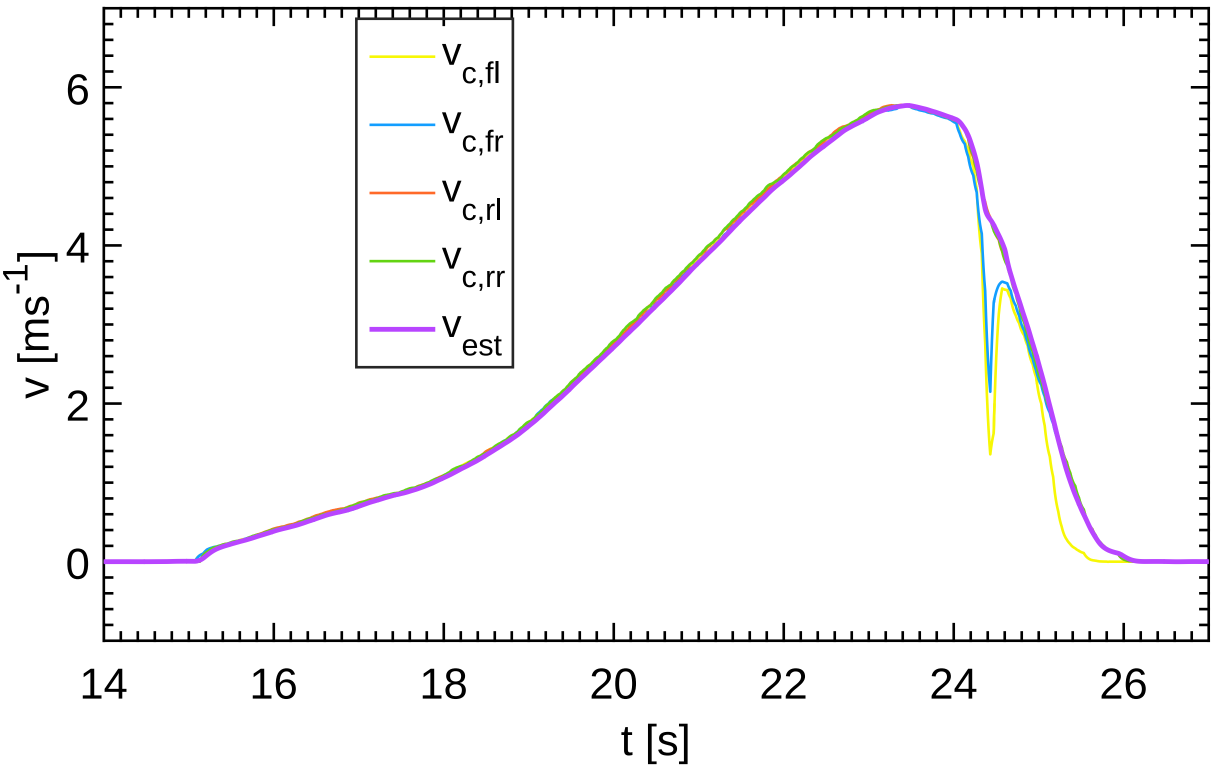


Figure 5.14: Detail of short braking situation with wheel-lock

6. Conclusion

In this thesis, first the research of former use of state observers in order to estimate the vehicle speed made is presented. It brings a summary of methods used in previous work in vehicle speed estimation including a brief insight. Advantages and drawbacks of the methods were considered and commented, partly forming the way of further algorithm development.

Subsequently the theoretical background was build up. Terms such as state observer, system observability are clarified here and useful tools as Luenberger observer or Kalman Filter are introduced. A focus was given to describe the terms and tools clearly and mainly in a range of vehicle state estimation. It was attempted to form the description in comprehensible manner, to be understood also by reader with no previous experience. Therefore, it could serve as a base for practical use in simple private applications.

Following from the theoretical background, the implementation of all parts found in algorithm is introduced. The algorithm which estimates the vehicle speed was successfully developed. This was made in accordance with given requirements.

In the beginning, pre-processing of the wheel rotational speeds, consisting mainly of translation of the wheel speeds in ground contact point to the center-of-gravity, is shown. Its importance is demonstrated by Figures 3.5 and 3.6.

Afterwards, the slope estimation carried out by Luenberger observer is presented. Estimate from Luenberger observer is post-processed by introduced limitations. Their positive influence on the estimation result was proven. The estimated slope χ_{road} serves for correction of the longitudinal acceleration, which is essential element of the vehicle speed estimation. In situations where are all the wheels in over-slip, is the longitudinal acceleration crucial.

The reason can be found in the design of the estimation algorithm under the adaptive Kalman filter with matrix H handled by the wheel slip criteria. Set up of the matrix regulates which measurement can and cannot influence the estimation value in the next step. This gives the algorithm an ability to disqualify the measurements corrupted by a process noise and to prevent the estimation result from the error being introduced.

Therefore, two main criteria are implemented. First based on observing the acceleration, second comparing demanded torque with computed torque considered to be maximally transmittable on the road surface without wheel slipping.

Results from testing the algorithm against the model vehicle are presented. The rsults also show, how the wished result of vehicle speed estimation during the wheel over-slip should look like, by comparison with do model vehicle speed. Algorithm was tested against the model data only in the very beginning of its development. Then it had to be recalibrated to deal with the real conditions, captured on the measurements obtained from test drives with the real vehicle. Therefore, the final version of the algorithm will not necessarily produce the flawless estimation when using the model data.

The estimation results obtained from testing against the data recorded in a real vehicle are more interesting. During the development the estimation algorithm was calibrated on and tested against several data. The driving situations selected for the presentation have been carefully picked due to some specific phenomenon captured. Thus, the value of the developed estimation algorithm can be presented.

The vehicle speed estimate is shown in the situation, when an over-slip of all wheels due to rapid acceleration on surface with low friction takes place. The algorithm did its job properly and eliminated the corrupted measurements via wheel slip criteria. The estimation algorithm also worked well during the emergency braking with ABS activated or by μ-split. The robustness of the algorithm was tested on the noisy data captured during driving in the circle on icy road. Here the results were again satisfactory.

In the end, couple of implementation ideas for further development can be proposed due to experience gained.

It is assumed, that there is room to grow in employment of more signals and creating the rule base according to the state detected. As a brief example can serve the use of the signal from ABS which indicates that the ABS is active. Then, knowing this fact and also knowing how the vehicle speed should look like, the estimation can be switched to follow defined pattern and therefore not to be dependent on the external criteria. Setting such a base of cases will take much effort.

However, the bigger gap is seen in an inaccuracy of the acceleration. There are many critical situations, when the observer can only rely on the integration of longitudinal acceleration to achieve the vehicle speed estimate, as all wheels are in over-slip. Then, especially in over-slip with long duration the integration error will be growing, as there is no reference measurement available. Improvement of estimation in such a situation can be gained by correcting the acceleration via Kalman filter. For this, the model describing the relationship between torques and acceleration can to be built. Torque during the acceleration is known from the motor torque signals. There is however no signal indicating braking torque directly. Part of it can be also captured by motor torque measurement, as the electric vehicles are equipped with recuperation function activating during braking. Here the negative torque made by brakes is problematic. Probably there is an option to obtain it from the measurement of brake fluid pressure with a consequent calculation of braking forces on brake discs. Nevertheless, this will require deep knowledge of the vehicle parameters. Also, the model which is to be set up is complex and therefore it would be time demanding to derive it precisely enough.

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Nomenclature

ABS	anti-lock braking system
ESP	electronic stability program
$4WD$	four-wheel drive
RWD	rear-wheel drive
x	state vector
u	input vector
y	output vector
z	measurement vector
k	time step
T_s	duration of time step
t	time
e	error
$\dot{e}$	error dynamics
λ	vector of eigenvalues
A	state/system matrix
B	input matrix
C	output matrix
H	measurement matrix
L	Luenberger gain
l	element of Luenberger gain
K	Kalman gain
I	identity matrix
Q	process noise covariance
R	measurement noise covariance
w_k	process noise
v_k	measurement noise
ψ	yaw

$\dot{\psi}$	yaw rate
ϕ	roll
χ	pitch
χ_{road}	road slope
ω_{fl}	rotational speed of front left wheel
ω_{fr}	rotational speed of front right wheel
ω_{rl}	rotational speed of rear left wheel
ω_{rr}	rotational speed of rear right wheel
v_{fl}	wheel speed in ground contact point of front left wheel
v_{fr}	wheel speed in ground contact point of front right wheel
v_{rl}	wheel speed in ground contact point of rear left wheel
v_{rr}	wheel speed in ground contact point of rear right wheel
$v_{c,fl}$	front left wheel speed corrected to center-of-gravity
$v_{c,fr}$	front right wheel speed corrected to center-of-gravity
$v_{c,rl}$	rear left wheel speed corrected to center-of-gravity
$v_{c,rr}$	rear right wheel speed corrected to center-of-gravity
v_{CoG}	vehicle longitudinal speed in center-of-gravity
r_D	dynamic radius of wheel
δ	steering wheel angle
δ_l	left wheel angle
δ_r	right wheel ange
b_f	vehicle front track
b_r	vehicle rear track
l_f	longitudinal distance of center-of-gravity from front axle
l_r	longitudinal distance of center-of-gravity from rear axle
h_{CoG}	vertical position of center of gravity above the ground
m	mass
m_m	mass of middle loaded vehicle

g	gravity acceleration
a_x	longitudinal acceleration
$a_{m,x}$	measured longitudinal acceleration
$a_{c,x}$	corrected longitudinal acceleration
$M_{m,f}$	motor torque on wheels on front axle
$M_{m,r}$	motor torque on wheels on rear axle
$F_{t,x}$	longitudinal traction force
F_N	normal force
$F_{N,f}$	normal force of front axle
$F_{N,r}$	normal force of rear axle
F_{stat}	static force
F_{dyn}	dynamic force
F_{roll}	rolling force
f_{rr}	rolling resistance coefficient
μ	road friction coefficient

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List of Attachments

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2. Sample data recorded in real vehicle