Czech University of Life Sciences Prague

Faculty of Economics and Management

Department of Systems Engineering

Bachelor Thesis

Solving Travelling salesman problem for deliveries

Lucas Zuza Araújo

CZECH UNIVERSITY OF LIFE SCIENCES PRAGUE

Faculty of Economics and Management

BACHELOR THESIS ASSIGNMENT

Lucas Zuza Araujo

Informatics

Thesis title

Solving Travelling salesman problem for deliveries

Objectives of thesis

The main objective of this work is to formulate based on the data collected, the delivery route plan for a selected company in order to keep travel costs and distance travelled as low as possible.

Methodology

In the theoretical part of this work, secondary research in the form of a literature review will be done. A selected company will provide a List of delivery locations, current delivery routes and further specifications relevant for comparison and analysis, which will be done in the practical part. The practical part consists of utilizing methods for solving the Travelling salesman problem and formulation of the solution to the problem.

The proposed extent of the thesis
30-40 pages

Keywords
Operations research, route optimization, efficient deliveries, travelling salesman problem.

Recommended information sources

APPLEGATE, David L. *The traveling salesman problem : a computational study.* Princeton: Princeton University Press, 2006. ISBN 9780691129938.
GUTIN, Gregory; PUNNEN, Abraham P. *The traveling salesman problem and its variations.* New York: Springer, 2007. ISBN 0387444599.
HILLIER, Frederick S.; LIEBERMAN, Gerald. *Introduction to operations research.* AIPI: AIPI, 2007. ISBN 1428808523.
TAHA, Hamdy A. *Operations research : an introduction.* Upper Saddle River, N.J.: Pearson/Prentice Hall, 2007. ISBN 0-13-188923-0.

Expected date of thesis defence
2022/23 SS – FEM

The Bachelor Thesis Supervisor
doc. Ing. Igor Krejčí, Ph.D.

Supervising department
Department of Systems Engineering

Electronic approval: 9. 11. 2023
doc. Ing. Tomáš Šubrt, Ph.D.
Head of department

Electronic approval: 23. 11. 2023
doc. Ing. Tomáš Šubrt, Ph.D.
Dean

Prague on 26. 11. 2023

Declaration

I declare that I have worked on my bachelor thesis titled "Solving Travelling salesman problem for deliveries" by myself and I have used only the sources mentioned at the end of the thesis. As the author of the bachelor thesis, I declare that the thesis does not break copyrights of any their person.

In Prague on 30.11.2023 ___________________________

Acknowledgement

I would like to thank doc. Ing. Igor Krejčí, Ph.D. for giving me the opportunity to have him as supervisor and for his great recommendations and support during my work on this thesis.

Solving Travelling salesman problem for deliveries

Abstract

This work addresses Traveling Salesman Problem in a real business case and formulates a delivery plan which minimizes delivery costs. A selected company which deals with deliveries daily, provided data regarding current route plans for five working days and further relevant information which allowed deeper analysis and comparisons.

Literature review on Operations Research, Traveling Salesman Problem and other connected topics was done. Furthermore, the approximation method Nearest Neighbour Algorithm and the optimization method Branch and Bound were used for solving Traveling Salesman Problem for the given problem with the help of the TSPKOSA solver.

The results obtained showed that the current route used by the selected company are not the best when it comes to costs minimization. Additionally, Branch and Bound method achieved better results compared to the other method utilized. Based on results achieved it is safe to conclude that the route planning for the selected company may be improved to reduce delivery costs and its impact on the company's profit. A suggestion for achieving such improvement is to adopt Traveling Salesman Problem methods such as Branch and Bound in the route planning activities of the company.

Keywords: Traveling salesman problem, operations research, deliveries, optimization approach, approximation methods.

Řešení problému cestovního prodejce pro dodávky

Abstrakt

Tato práce řeší problém Traveling Salesman v reálném obchodním případě a formuluje plán dodávek, který minimalizuje náklady na doručení. Vybraná společnost, která se denně zabývá dodávkami, poskytla údaje o aktuálních plánech tras na pět pracovních dnů a další relevantní informace, které umožnily hlubší analýzu a srovnání.

Byla provedena literární rešerše na téma Operační výzkum, Problém cestujícího obchodníka a další související témata. Dále byla použita aproximační metoda Nearest Neighbor Algorithm a optimalizační metoda Branch and Bound pro řešení Traveling Salesman pro daný problém pomocí řešitele TSPKOSA.

Získané výsledky ukázaly, že současná trasa používaná vybranou společností není z hlediska minimalizace nákladů nejlepší. Navíc metoda Branch and Bound dosáhla lepších výsledků ve srovnání s jinou používanou metodou. Na základě dosažených výsledků lze s jistotou učinit závěr, že plánování trasy pro vybranou společnost lze zlepšit, aby se snížily náklady na doručení a jeho dopad na zisk společnosti. Návrhem, jak dosáhnout takového zlepšení, je zavést metody Traveling Salesman, jako je Branch and Bound, do plánovacích aktivit společnosti.

Klíčová slova: Problém cestujícího obchodníka, operační výzkum, dodávky, optimalizační přístup, aproximační metody.

Table of content

List of figures

List of tables

List of abbreviations

- OR – Operations Research
- TSP – Traveling Salesman Problem
- NNA – Nearest Neighbour Method
- VAM – Vogel's Approximation Method
- SM – Savings Method
- B&B – Branch and Bound Method

1 Introduction

The final and most challenging stage of the supply chain is the last mile delivery. Routes planning, traffic, and failed deliveries are only few of the challenging factors behind this important phase of the supply chain. Furthermore, last mile logistics is the costliest leg in supply chain, with costs reaching up to 41% of the total supply chain costs and unsustainably eroding organization's profit. (Jacobs et al., 2018). Among the several actions to be taken to make last mile delivery more efficient and less costly, this thesis will focus attention to route planning.

Although directly related, the focus of this work is not to deep dive in the Logistics subject. The main objective is connected to Traveling Salesman Problem, a widely studied problem in Operations Research. Therefore, the theory part of this thesis will deal with reviewing current knowledge in Operations Research and TSP. With respect to the practical part and main objective of this thesis, a route planning problem will be addressed by means of different algorithms in cooperation with a selected business that deals with products delivery. Data provided by the selected company will be analysed and processed with the goal of solving Traveling Salesman Problem using different approaches and formulating a route plan that minimizes delivery costs.

There are different methods for building efficient delivery routes, and the actual method used for building routes varies according to complexity of the problem and whether manual or computer-based approach is used (Rushton, Croucher, Baker, 2014). For the problem in hand, the methods which will be used to reach the goal of this thesis consist of one approximation method (Nearest Neighbour Algorithm) and one optimization method (Branch and Bound). Solutions obtained through each method will be then analysed and compared. Additionally, savings estimates will also be computed for illustrating the significance of the results achieved.

2 Objectives and Methodology

2.1 Objectives

The main objective of this Bachelor Thesis is to solve Traveling Salesman Problem in a real business case by formulating an alternative delivery route plan that minimizes delivery costs.

2.2 Methodology

In the theoretical part of this thesis, the methodology which will be used is secondary research. Topics which are relevant to the thesis objectives will be reviewed in the form of a Literature review.

The practical part methodology involves working with approximation and optimization algorithms. Two different prominent methods for solving Traveling Salesman Problem will be chosen, analysed, and executed aiming to achieve a solution for the business case. Outcomes will be analysed, and compared aiming to detect the approach which best solves the business problem in hand.

3 Literature Review

3.1 Operations Research

Over the years, small organizations tend to grow and specialize more and more in their activity. The constant growth and development of organizations has also generated a significant increase in their management complexity, which ends up introducing new problems. Among the various problems related to the increase in complexity and specialization of companies, the challenge of effective allocation of resources stands out, a problem that led to the emergence of Operational Research (OR), (S. Hillier, J. Lieberman, 2010).

3.1.1 The Emergence of Operational Research

Attempts to use scientific methods for decision making may be traced back at least a century ago. The term Operations Research however, had its first activities initiated during World War II by British scientists aiming for scientifically based decisions regarding best utilization of war supplies (Taha, 2007).

After the war, the success of OR in the war related management decisions followed by the industrial boom and the increase in complexity of organization, made apparent to business consultants and several people who had worked in OR teams during the war that OR could be applied in the civilian sector. "By the early 1950s, these individuals had introduced the use of OR to a variety of organizations in business, industry, and government" (S. Hillier, J. Lieberman, 2010). Since then OR has been applied in several areas for numerous decision-making problems, such as transportation, health care, manufacturing, construction, telecommunications, financial planning, the military amongst others.

3.1.2 Aspects of Operations Research

Operations Research makes use of scientific methods to seek solutions to decision-making problems. With that been said, there are important aspects of OR, which are worth mentioning. First, there are two elements to the discipline of OR: the practice of OR and the research into methods of the discipline. The pure practice of OR does not add to the knowledge in the OR field, it consists of solving a targeted problem. In the other hand the research into Operations Research seeks to increase understanding and develop better methods, thus contributing to the body of OR knowledge (Manson, 2006). A second aspect of the discipline is that there is no single technique for formulating and solving mathematical models, the nature of the solution method relies on the type and complexity of the mathematical model (Taha, 2007). “An additional characteristic is that OR frequently attempts to search for a best solution (referred to as an optimal solution).” (S. Hillier, J. Lieberman, 2010, p. 3)

3.1.3 Operations Research approaches

There are a vast number of methods to be used when performing OR, and those are classified as Qualitative and Quantitative approaches. Both approaches use a variety of techniques for data gathering and analysis, with the quantitative approach working with numeric data focused on variables and objective facts measuring, while the qualitative approach focuses on working with non-numeric data, exploring, and understanding a social problem (Lawrence Neuman, 2014).

3.1.4 Quantitative approach

In quantitative approach a researcher aims to test objective theories by gathering numerical data, examining, and quantifying the relationship between variables using statistics methods (Cresswell, 2009). A particular characteristic of this approach is that it is exceptionally strong at achieving detailed understanding about large samples and generalizing results beyond the sample under study. Furthermore, quantitative approach can be visualized as a five steps process: defining the problem or question to be answered, collecting data, selecting methods for solving the problem, developing a solution, results interpretation (Swanson, F. Holton III, 2005).

3.1.4.1 Approximation methods

Although OR frequently attempts to search for an optimal solution, depending on the complexity of the problem this requirement needs to be relaxed and a solution considered “good enough” is adopted by means of an Approximation method. One example of this is when dealing with NP-hard optimization problem. Approximation methods are used for trying to find a solution that closely approximates the optimal solution (P. Williamson, B. Shmoys, 2011).

3.1.4.2 Optimization methods

Information technology has made it possible to find solutions to previously unsolvable optimization problems. Human power alone is often incapable of dealing with systems with more than three variables, nevertheless, thanks to the rise of high-speed computers, nowadays several algorithms are available for dealing with optimization problems for systems with hundreds of variables (Adby, Dempster, 1974). Among the numerous prominent optimization techniques, Linear programming, Integer programming, Dynamic programming, Network programming and Nonlinear programming stand out (Taha, 2007).

3.2 The Traveling Salesman Problem

The traveling salesman problem (TSP) is one of the best known and studied problems in Operations Research. TSP aims to discover the least expensive possible route to visit a cluster of different locations and return to a starting point, in other words, the order in which each location should be visited aiming to minimize resources expenditure. It may sound an easy task, especially for a low count of locations to visit, however, as the number of locations increases so does the complexity of the problem. The TSP is of immense importance, as it has many applications in different activities in the real world. It is among the most investigated problems in computational mathematics and has implementations in the field of Logistics, Genetics, Telecommunications amongst others (Applegate, Bixby, Chvátal, Cook, 2006).

3.2.1 Origins

The origin of this problem is not completely certain; however, it is believed that Kalr Menger was one of the first to study the general form of TSP (Davendra, 2010). Besides that, Hassler Whitney and Merrill Flood are credited with the first reference to the problem, in a work done at Princeton University, with Merrill considered as one of the most influential TSP researchers (Applegate, Bixby, Chvátal, Cook, 2006).

3.2.2 Methods

In the 1960s Edmonds raised the question whether there is a good method for solving TSP and to date, this question has not been answered (Applegate, Bixby, Chvátal, Cook, 2006). Even though “The good" method for solving TSP real life situations is yet to be found by mathematicians, it is possible to name commonly used approaches such as, The Nearest Neighbour Algorithm, Vogel’s approximation, Savings Method, and The Branch and Bound Method.

3.2.2.1 Nearest Neighbour Algorithm

Considered the simplest heuristic approach to solve TSP, the Nearest Neighbour Algorithm (NNA) was initially introduced by J. G. Skellam. This approximation algorithm compares the distance between a set of points and its nearest neighbour in a dataset (A. AlSalibi et al., 2013). NNA algorithm for determining TSP minimal cycle is as follows. Select Starting point, visit the point closest to the starting point, proceed to the nearest unvisited point, return to the starting point whenever all other points have been visited once. “We can obtain the best result by running the algorithm over again for each vertex and repeat it for n times.” (Kizilateş, Nuriyeva, 2013, p.112). The output is a short cycle not an optimal one as NNA may miss possible shorter cycles and thus not find a feasible sequence (A. AlSalibi et al., 2013).

3.2.2.2 Vogel’s approximation

One other method that falls in the category of approximation methods is Vogel’s approximation method (VAM). Like NNA, VAM is a heuristic approach that usually achieves better results than other approximation methods used for solving transportation problems (Uddin et al., 2014). The algorithm for Vogel’s approximation method consists of:

1. Balancing the problem: if Σ Supply $\neq \Sigma$ Demand, add dummies aiming for equality,
2. Determining penalty cost: select the cell that have the minimal cost in each row and subtract from another cell having minimal cost in the same row for,
3. Select highest penalty cost: identify the row or column with highest difference and then allocate as much as possible to the cell with lowest transportation cost,
4. Repeat the process: perform previous steps until all allocations are done,
5. Compute feasible allocations and find transportation cost (Korukoğlu, Balli 2011).

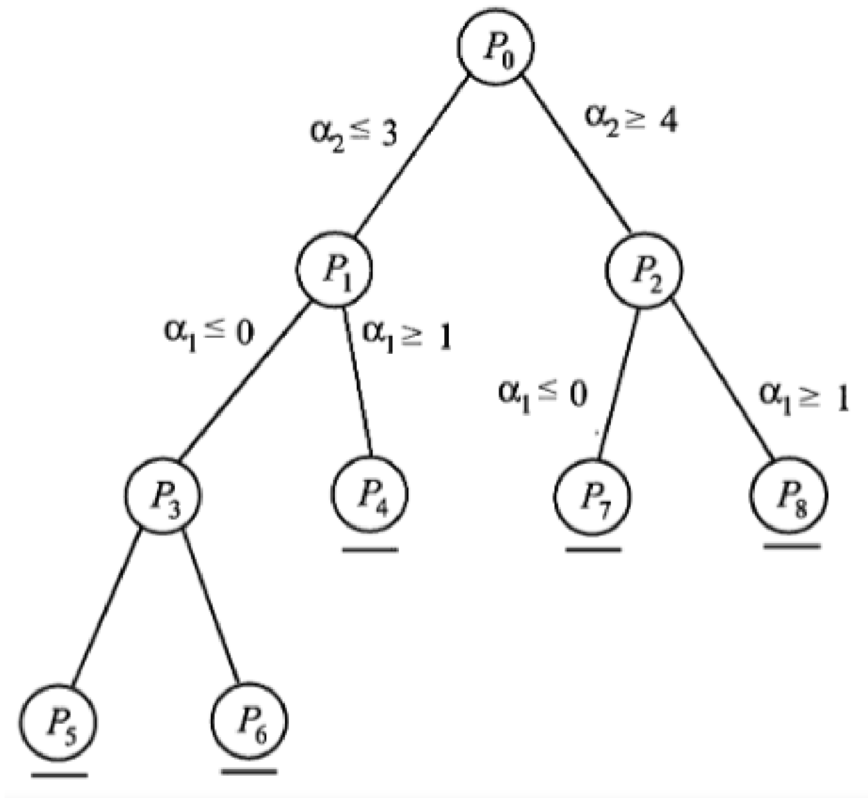
3.2.2.3 Savings Method

An additional method of the same kind as the ones discussed in previous subchapters is Savings Method (SM). The calculation process in this method involves comparing the distance of a route between two points and a route through one other chosen point (Kučera, 2012). A fundamental part and the starting point of this method is calculating savings: $S_{ij} = C_{oi} + C_{oj} - C_{ij}$ where C_{oi} is the distance between a denoted initial point and point i, C_{ij} is the distance between point i to j and S_{ij} is distance savings from point i to j (Tunnisaki, 2023). The process continues with processing of the routes between points starting from those with highest savings S_{ij} using the approach of; adding to the solution every set of disjoint vertex paths obtained by adding an edge. After repeating the previous procedure until a Hamiltonian path has been created including all but the initial point, the initial point is added to close the route cycle (Kučera, 2012).

3.2.2.4 Branch and Bound

Differently from previously discussed methods, Branch and Bound (B&B) method is an optimization method widely used for solving large scale NP-hard combinatorial optimization problems such as TSP, Minimum Spanning Tree, Crew scheduling, and Vehicle routing problem. B&B method systematically aims for the best solution among all the solutions of a given problem (Clausen, 1999). The name Branch and Bound was introduced in 1963 by Little, Murty, Sweeney and Karel, while the concept was only introduced in TSP papers from 1950s by Bock, Croes, Eastman and Rossman and Twery (Applegate, Bixby, Chvátal, Cook, 2006). This approach uses a tree-based data structure (see Figure 1) and consists of breaking the root problem into several sub-problems, calculating bounds on the objective function value over each subset, providing numerous potential solutions for each of them and eliminating certain subsets from further consideration (Balas, Toth 1983). "The idea is that if a bound is greater than or equal to the cost of a tour we have already found, then we can discard the subproblem without any danger of missing a better tour" (Applegate, Bixby, Chvátal, Cook, 2006, p.41).

Figure 1 Example of a branch and bound tree.



Source: (Yong Ching, Ya Jun, 2003, p.260)

3.2.3 Mathematical definition and classification

Prior to speaking about the classifications of TSP, let us familiarize ourselves with a mathematical definition of the problem. For that purpose, "Let G=(V, E) be a graph. For each edge $e \in E$ a cost c_e is prescribed", then, TSP is the problem of finding a Hamiltonian cycle with minimal cost… "let the node set V = {1, 2, …, n}. The matrix $C = (c_{ij})_{n \times n}$, is called the cost matrix, where $(i, j)^{th}$ entry c_{ij} corresponds to the cost of the edge joining node i to node j in G" (Gutin, P. Punnen, 2007, p.3).

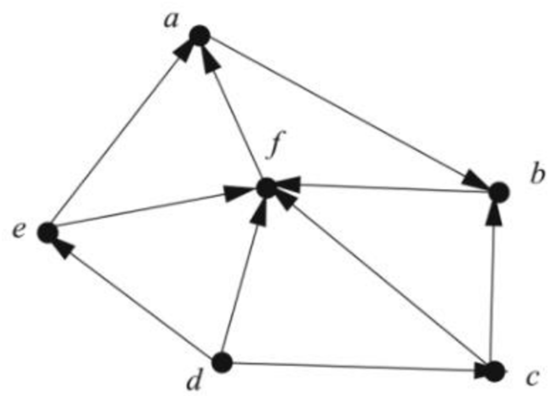
In general, TSP is classified as symmetric travelling salesman problem (STSP), and asymmetric travelling salesman problem (ATSP), where the difference between the two types is given by the nature of the cost matrix (additionally known as distance matrix). If the cost matrix is symmetric $c_{ij} = c_{ji}$ then TSP is classified as STSP. Presuming that the cost matrix is asymmetric $c_{ij} \neq c_{ji}$ in other words, the cost matrix is different than its transpose matrix, then TSP is classified as ATSP (Gutin, P. Punnen, 2007).

3.2.4 Graph theory

Graph theory history traces back to when Leonhard Euler, a Swiss mathematician solved the Königsberg bridge problem in 1736. (Rahman, 2017). A graph consists of a set of vertices (also called, points or nodes) and a set of edges (also known as, lines) joining two vertices. Graph theory is a branch of mathematics which deals with graphs, playing an important role when it comes to understanding TSP.

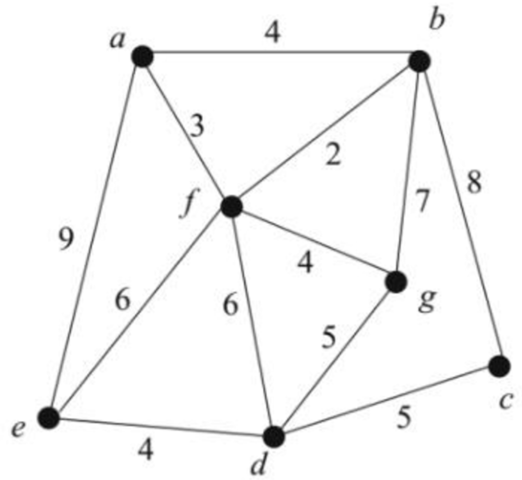
Points and lines are only basic attributes of a graph. Additional attributes such as edge direction and edge weight are two of the most common and essential attributes, enabling graphs to serve as mathematical models for numerous applications (Gross, Yellen, 2004). A graph containing the edge direction attribute is named, Directed graph (also known as, digraph) as illustrated in Figure 2. In the other hand, the graph in Figure X is an example of Weighted graph, name given to graphs that have a weight value assigned to its edges.

Figure 2 Illustration of a directed graph.



Source: (Rahman, 2017, p.12).

Figure 3 Exemplification of a weighted graph.



Source: (Rahman, 2017, p.12).

Named after Willian Rowan Hamilton and his Icosian Game, the Hamiltonian graph is one more important topic in graph theory. Hamilton's game had the challenge of defining a sequence including all vertices with a minimum cost and passing through vertices only once. A graph is classified as Hamiltonian graph "if it contains a spanning cycle (Hamiltonian cycle)" (Gross, Yellen, 2004).

3.2.5 Matrices

Square matrices "(matrices having the same number of rows as columns", (Andrilli, Hecker, 2023)), are built out of the distance between the points and some attributes such as matrix size and classification may vary depending on number of points and symmetricity. As mentioned in chapter 3.2.3, matrices play an important role for solving TSP and I particularly deal with asymmetric matrices of different sizes in the practical part of this work.

3.3 TSP applications

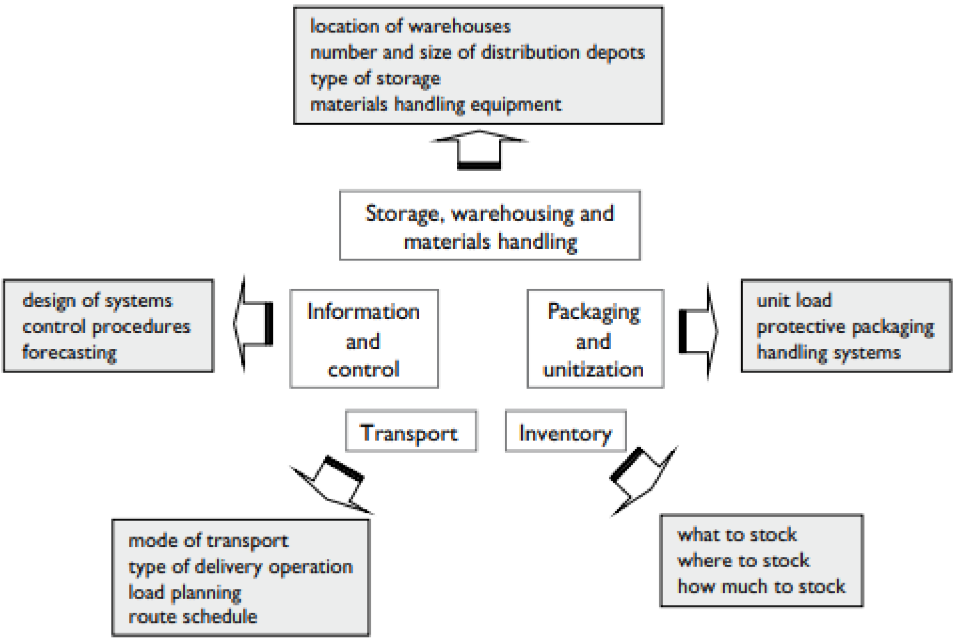
TSP was and continues to be inspired by real and direct applications. Its variations range over a variety of fields. An example of the range of interest in TSP is the adoption of this method in psychological experiments aimed at understanding problem-solving ability of humans (Applegate, Bixby, Chvátal, Cook, 2006). In this chapter some more fields and applications of TSP will be addressed.

3.3.1 Logistics

In simple words logistics may be defined as the systematic and organized movement of products, people, and services. Logistics was originally used for military applications, and quickly became an indispensable part of handling resources. In fact, "the role of logistics has developed such that it now plays a major part in the success of many different operations and organizations." (Rushton, Croucher, Baker, 2014, p.7).

Although logistics is a complex and very well-structured field with different phases and processes, there are major components of Logistics which we can list. "These will include transport, warehousing, inventory, packaging, and information. Some typical examples are given in the figure below." (Rushton, Croucher, Baker, 2014, p.6).

Figure 4 Major components of logistics and associated elements.



Source: (Rushton, Croucher, Baker, 2014, p.6).

All the components and associated elements listed in the figure above require efficient planning for the proper methods to be adopted. When talking about methods, the key component to be highlighted in this chapter is Transport with one of its key elements, the route schedule. The reason behind this is that "a common application of the TSP is of course the movement of people, equipment, and vehicles around tours of duty." (Applegate, Bixby, Chvátal, Cook, 2006, p.59). Furthermore, route schedule is directly related to the practical part of this bachelor's thesis, as the business case and its problem will be addressed using TSP methods.

3.3.2 X-RAY Crystallography

X-Ray Crystallography is a method for determining the structure of molecules. This widely used technique aims to achieve a three-dimensional molecular structure from a crystal of the substance under study (MS, JH, 2000).

The analysis of crystal's structure became an important TSP application once in the mid-1980s Robert Bland and David Shallcross adopted this method for aiming diffractometer in X-ray crystallography (Applegate, Bixby, Chvátal, Cook, 2006). To achieve this "a detector measures the intensity of X-ray reflections of the crystal from various positions. Whereas the measurement itself can be accomplished quite fast, there is a considerable overhead in positioning time since up to hundreds of thousands of positions have to be realized for some experiments." (Matai, Singh, Lal Mittal, 2010, p.3). In this case, positioning involves moving computer-driven motors in between observations and the travel cost is given by estimates of time for this repositioning to happen (Applegate, Bixby, Chvátal, Cook, 2006).

4 Practical Part

4.1 TSP for deliveries

In the practical part of this work, TSP methods were applied in cooperation with a chosen company. The company provided delivery routes, additional basic business information and data relevant for processing and analysis of results. Gathered data was organized and processed aiming to achieve the goal of this bachelor thesis which is solving TSP for deliveries.

With respect to the company's request not to be identified, the name of the company and the names of customers will not be mentioned. Therefore, for reference purposes in this work the company will be denominated as Company XX.

4.1.1 The company

Company XX is a relatively small company headquartered in Brazil, operating in the automotive industry. The company provides a wide variety of products, parts and accessories for cars, motorcycles, and small trucks and possesses products available for pickup and delivery. It has a very stable set of customers and delivers several products on a weekly basis for a list of clients.

The company prepares deliveries to be made in the morning and aims to divide the groups into similar amounts of locations. Routes are prepared on a weekly basis and amended as necessary before each working day. No further information on the planning of routes, such as approaches taken for minimizing delivery costs, was provided by the company in this business case problem.

One other fact about Company XX is that like other companies in the Brazilian market it is partially absorbing delivery costs due to customers not willing to pay in fully for this service. An approach that leads to profit erosion.

4.1.2 Company XX's current route

No information on route construction method used for calculating existing routes was provided by the chosen company. Nevertheless, routes for five working days were provided by Company XX.

As part of the practical part of this bachelor thesis, all the provided routes were processed to obtain distances between the locations, named for referencing purposes and organised in similar tables as the following:

Table 1 Day 1 current route.

Point-to-point	Distance (km)
L1 → L2	1.6
L2 → L3	1.5
L3 → L4	1.3
L4 → L5	0.4
L5 → L6	2.4
L6 → L7	3.3
L7 → L8	7.2
L8 → L9	7.4
L9 → L10	5.8
L10 → L1	3.5
Total distance:	34.4

Source: Own work based on data provided by Company XX.

Tables built for all other five days can be found under the appendix chapter.

4.1.3 Matrix construction

A key step for solving TSP is to build a well-structured cost matrix. To apply the TSP methods for the business case, the cost matrix for each working day was built. Every cost used in the matrix for the five processed days are real world distances in km. Company XX did not provide distances between the delivery locations, a list of locations was provided instead and processed through Google Maps to obtaining real distance between each location. As an example, the following table can be observed:

Table 2 Day 1 matrix.

Day 1	**L1**	**L2**	**L3**	**L4**	**L5**	**L6**	**L7**	**L8**	**L9**	**L10**
L1	0	1.6	0.4	1.1	1.5	3.8	4.9	7.6	4.5	2.9
L2	1.2	0	1.5	2.2	2.6	5	3.2	6.8	5.7	4.1
L3	0.5	1.6	0	1.3	1.7	4	5.1	7.7	4.7	2.8
L4	1.5	2.2	1.8	0	0.4	2.7	3.8	9.1	6	4.4
L5	1.7	2.4	2	1	0	2.4	3.5	9.3	6.2	4.6
L6	4	4.6	4.3	3.3	3	0	3.3	10.2	8.5	6.9
L7	4.7	3.2	5	4	3.7	3.1	0	7.2	9.2	7.6
L8	7.6	6.9	7.9	8.7	9.1	10.7	7.3	0	7.4	8.4
L9	5.3	6.5	5.2	6.2	6.5	8.9	9.9	7.3	0	5.8
L10	3.5	4.7	2.9	4.4	4.8	7.1	8.2	8.2	5.9	0

Source: Own calculation.

Day 1 matrix is composed of ten locations including Company XX's depot which is included as L1 in the matrix. We can observe that the matrix is asymmetric and the reason for this is that the business case problem deals with in-city routes which in most cases are one-way roads. As discussed in section 3.2.3, the nature of the cost matrix defines the classification of TSP and in this case, we can conclude that the problem in hand is an Asymmetric Traveling Salesman Problem. The same matrix construction approach described was used for building all cost matrix in this problem and those can be found in the Appendix chapter.

4.2 Solving the problem

Based on the current route (provided by Company XX) and the matrix with distances between the locations, alternative routes will be calculated for five working days and their respective delivery locations using two different approaches. The approaches chosen to calculate alternative routes fall in the class of approximation methods (NNA) and optimization methods (B&B) as seem in subsections of section 3.2.2. The outcome solution provided by each approach and the current routes will then be compared for deeper interpretation of results and to determine the routes which minimize delivery costs.

4.2.1 TSPKOSA

Developed in Microsoft Visual Basic 6.5 by Ing. Igor Krejčí, RNDr. Petr Kučera, Ph.D., and Ing. Hana Vydrová, TSPKOSA is a program that allows entering problems of up to 250 nodes, which works with four basic TSP methods; Nearest Neighbour Algorithm, Vogel Approximation Method, Savings Method, and Branch and Bound. This powerful program will be used for processing Company XX's problem using two of its four supported methods.

4.2.2 Nearest Neighbour Algorithm

The first method to be used for solving the current issue is the approximation method, Nearest neighbour algorithm (NNA).

Based on Day 1 matrix, NNA approach was applied using TSPKOSA solver and a minimal cycle was obtained with the total length of 34.1km. The alternative route obtained is presented in the following table:

Table 3 Alternative route for Day 1 obtained through NNA.

Point-to-point	Distance (km)
L1 → L3	0.4
L3 → L4	1.3
L4 → L5	0.4
L5 → L6	2.4
L6 → L7	3.3
L7 → L8	7.2
L8 → L9	7.4
L9 → L10	5.8
L10 → L2	4.7
L2 → L1	1.2
Total distance:	34.1

Source: Own work based on obtained results.

When compared with the initial route, a decrease of 0.4km in the total route length is identified in the above provided solution. As the goal of this work is to provide an alternative route which is shorter than the initial route, the solution found using NNA can be considered acceptable for the given problem. Alternative routes obtained for the other four working days are illustrated in similar tables in the Appendix chapter. Furthermore, a detailed comparison and analysis of NNA alternative solutions obtained are done in chapter 5.

4.2.3 Branch and Bound

Now opting for an optimization approach, the alternative method to be used for solving the current issue is the Branch and Bound (B&B).

Once again utilizing TSPKOSA solver and working with the Day 1 matrix, a minimal cycle with the total length of 33.3km was obtained. The result illustrated in the table below:

Table 4 Alternative route for Day 1 obtained through B&B.

Point-to-point	Distance (km)
L1 → L4	1.1
L4 → L5	0.4
L5 → L6	2.4
L6 → L7	3.3
L7 → L8	7.2
L8 → L9	7.4
L9 → L10	5.8
L10 → L3	2.9
L3 → L2	1.6
L2 → L1	1.2
Total distance:	33.3

Source: Own work based on obtained results.

Compared with the initial route, a decrease of 1.1km is observed in the total route length obtained through Branch and Bound method. As the previous solution obtained, the B&B solution for Day 1 can be considered as an acceptable solution. In addition, since the aim of this work is to formulate the least expensive solution for the TSP, the B&B route obtained for Day 1 is not only acceptable but preferable over NNA's solution since it is 0.8km shorter. Just like done with NNA method, further working days were calculated using B&B method and results described in tables presented in the Appendix chapter.

5 Results and Discussion

5.1 Comparison

In addition to calculating and obtaining routes using different TSP methods for Day 1, routes were also obtained for four other days of the week following the same criteria and process as Day 1. Results for the five days are included in the appendix of this work. To illustrate and analyze the difference between the obtained results, a comparison of all five initial routes, NNA routes and B&B routes will be made in the following sections.

5.1.1 Day by day comparison

In this section the results obtained with each route for the five processed days are discussed. For better visualization and interpretation of the results the following table was built illustrating all results for comparison:

Table 5 Day by Day comparison.

	Route	Total distance (km)	Distance difference (km)	Fuel consumption (L)	Fuel expense (BRL)	Fuel savings (BRL)
Day 1	*Initial route*	34.4	-	3.215	BRL 17.36	-
	NNA route	34.1	-0.3	3.187	BRL 17.21	BRL 0.15
	B&B route	33.3	-1.1	3.112	BRL 16.81	BRL 0.56
Day 2	*Initial route*	37.5	-	3.505	BRL 18.93	-
	NNA route	37.1	-0.4	3.467	BRL 18.72	BRL 0.20
	B&B route	35.3	-2.2	3.299	BRL 17.81	BRL 1.11
Day 3	*Initial route*	35.3	-	3.299	BRL 17.81	-
	NNA route	37.1	1.8	3.467	BRL 18.72	-BRL 0.91
	B&B route	35.3	0	3.299	BRL 17.81	BRL 0.00
Day 4	*Initial route*	35.5	-	3.318	BRL 17.92	-
	NNA route	36	0.5	3.364	BRL 18.17	-BRL 0.25
	B&B route	32.8	-2.7	3.065	BRL 16.55	BRL 1.36
Day 5	*Initial route*	57	-	5.327	BRL 28.77	-
	NNA route	53	-4	4.953	BRL 26.75	BRL 2.02
	B&B route	51.1	-5.9	4.776	BRL 25.79	BRL 2.98
Week total	*Initial route*	199.7	-	18.664	BRL 100.78	-
	NNA route	197.3	-2.4	18.439	BRL 99.57	BRL 1.21
	B&B route	187.8	-11.9	17.551	BRL 94.78	BRL 6.01

Source: Own calculation.

A total of six rows and columns represent the results for five different days including the week total and relevant data for the three routes: initial, NNA and B&B. The Distance difference column shows in kilometres (km) the distance difference between the NNA route and Initial route as well as the distance difference between B&B route and Initial route. Solutions obtained through B&B method were more efficient in all weekdays compared to NNA method solutions. In Day 1, B&B route is 1.1km and 0.8km shorter than initial route and NNA route respectively. In Day 2 the distance difference is even greater with B&B route been 2.2km shorter than initial route and 1.8km shorter than NNA route. There is no difference between B&B and initial route for Day 3, in the other hand the NNA route found is 1.8km longer than both other routes. For Day 4 the B&B route proposed is 2.7km shorter than the initial route and 3.2km shorter than NNA route. Lastly, B&B route for Day 5 is 5.9km and 4km shorter than initial route and NNA route respectively.

5.1.2 Fuel expenses and savings

Given that the car used by Company XX for the delivery is a Fiorino, with average fuel consumption of 10.7 km/L, and considering that the fuel costs BRL 5.40 per Liter on average, fuel consumption and expenses were estimated. Since B&B routes were the shortest for each day, it is natural that fuel consumption found was smaller compared to other two routes. The column Fuel savings in Table 5 perfectly illustrates this difference by showing that B&B route proposes a total of BRL 6.01 in fuel savings for the week.

5.1.3 Annual fuel expenses and savings

For analytical purpose and illustration of results significance, the annual recalculation was made considering the week total distance results previously analysed in the preceding chapter. The results obtained are described in the next table:

Table 6 Annual recalculation.

	Route	**Total distance (km)**	**Distance difference (km)**	**Fuel consumption (L)**	**Fuel expense (BRL)**	**Fuel savings (BRL)**
Week total	*Initial route*	199.7	-	18.664	BRL 100.8	-
	NNA route	197.3	-2.4	18.439	BRL 99.6	BRL 1.2
	B&B route	187.8	-11.9	17.551	BRL 94.8	BRL 6.0
Year total	*Initial route*	10,384.4	-	970.505	BRL 5,240.7	-
	NNA route	10,259.6	-124.8	958.841	BRL 5,177.7	BRL 63.0
	B&B route	9,765.6	-618.8	912.673	BRL 4,928.4	BRL 312.3

Source: Own calculation.

Significant reduction in fuel consumption and expenses is observable in the results from NNA and B&B routes, with total savings amounting to BRL 63.00 and BRL 312.30 respectively. Although multiple delivery vehicles are not yet part of Company XX's business reality, for analysis and comparison purposes we can consider that if the company has more delivery vehicles, the estimated fuel savings increase, making the results increasingly sizable over the course of a year.

5.1.4 Time-savings

Considering the total travel time and distance provided by Company XX for each of the initial routes, average speed was calculated and used to estimate time saved by using each of the alternative methods routes. Results are presented in the following table.

Table 7 Day by Day comparison of total travel time.

Day	Route	Total travel time (min.)	Total distance (km)	Average speed (km/h)
Day 1	*Initial route*	60	34.4	**34**
	NNA route	59	34.1	
	B&B route	58	33.3	
Day 2	*Initial route*	76	37.5	**30**
	NNA route	75	37.1	
	B&B route	72	35.3	
Day 3	*Initial route*	63	35.3	**34**
	NNA route	66	37.1	
	B&B route	63	35.3	
Day 4	*Initial route*	57	35.5	**37**
	NNA route	58	36	
	B&B route	53	32.8	
Day 5	*Initial route*	109	57	**31**
	NNA route	101	53	
	B&B route	98	51.1	

Source: Own calculation.

Table 7 is composed of five columns and shows each of the five working days, the three routes for each of the five days, total travel time and total distance for each of the routes together with the average speed. The average speed was calculated using the total distance of each initial route divided by the total travel time of each initial route. The total travel time for NNA and B&B routes was then calculated by dividing the total distance of each NNA and B&B routes by the average speed obtained for each day.

It is possible to observe that B&B routes achieved better results compared to NNA routes for each of the five calculated days just like results discussed in Fuel expenses and savings estimates chapter. In Day 1 and Day 2, NNA route is one minute shorter than initial route. In the other hand, B&B route for Day 1 is two minute shorter than initial route, while in Day 2 it is four minute shorter compared to the 72 minutes of initial route total travel time. In Day 3 the B&B obtained same 63 minutes of duration as initial route meanwhile NNA route turned out to be 3 minutes longer than the other two routes. In Day 4, NNA route total travel time exceeds by 1 minute the total travel time of the initial route, while B&B route is 4 minutes shorter than initial route. In Day 5 both alternative routes achieved better results over the initial route with B&B route obtaining a total of 11 minutes saved against 8 minutes saved using NNA.

5.1.5 Annual time-savings

A considerable amount of time is saved by using B&B method with the week total saving time amounting to 21 minutes. If we consider that this amount will be saved every week for one year, the total saving time amounts to 18 hours and 20 minutes, which is more than two working days.

6 Conclusion

The main goal of this bachelor thesis was to propose a route plan that would keep traveling costs as low as possible for the business case in hand by solving the Travelling Salesman Problem.

In the theoretical part of this work, the literature review provided a good overview and deeper understanding of Operations Research with a special focus on the Traveling Salesman Problem by covering its applications, methods, and history. Furthermore, with the literature review it was possible to acknowledge how prominent is the TSP and its range in numerous fields of study.

In the practical part, the theory reviewed in the theoretical part of this work was applied. The business case problem was solved using different TSP methods and two different alternative solutions were presented for each working day processed. As a result, it was discovered that the current delivery routes used by Company XX are not the cheapest routes available. In the Results and Discussion chapter, the current route was compared to the alternative routes obtained through the TSP methods solution. Based on the comparison results, it is safe to conclude that Branch and Bound approach provided a more feasible solution compared to the solution provided by Nearest Neighbour Algorithm. Additionally, Branch and Bound solution proved to be best than the current route solution by achieving considerable amount of fuel savings with the less expensive routes provided.

Maximizing profit is a natural goal of every business, and costs reductions is a way to achieve this goal. Although the money saved on fuel costs is a considerable amount, the potential benefits gained through using the routes proposed by the B&B method are not limited to fuel savings. In the long term, the wear of tires and other important parts of the vehicle must also be considered. Moreover, time saving was also observed in the results, meaning that this time could potentially be invested in other tasks or that wage costs with delivery drivers could be reduced significantly.

With respect to the main goal of this thesis the conclusion is that the objective of formulating a route plan that minimizes delivery costs was achieved. This result can potentially be useful for Company XX in terms of understanding that there is room for improvement in their current routes planning by for instance adopting TSP approaches such as B&B.

7 References

A. ALSALIBI, Besan, Marzieh BABAEIAN JELODAR and Ibrahim VENKAT, 2013. A Comparative Study between the Nearest Neighbor and Genetic Algorithms: A revisit to the Traveling Salesman Problem [online]. [cit. 2023-11-19]. ISSN 2320–4028.

ADBY, P.R. and M.A.H. DEMPSTER, 1974. Introduction to Optimization Methods [online]. Chapman and Hall [cit. 2023-11-26]. ISBN 978-94-009-5705-3. Available from: doi: 10.1007/978-94-009-5705-3

ANDRILLI, Stephen and David HECKER, 2023. Elementary Linear Algebra. Sixth Edition. US: Elsevier. ISBN 978-0-12-822978-1.

APPLEGATE, David L., Robert E. BIXBY, Vašek CHVÁTAL and William J. COOK, 2006. The Traveling Salesman Problem: A Computational Study. Princeton: Princeton University Press. ISBN 978-0-691-12993-8.

BALAS, Egon and Paolo Toth, 1983. Branch and Bound Methods for The Traveling Salesman Problem [PDF]. Management Science Research Report No. MSRR 488.

CARLSON, Stephan C., 2023. Graph Theory [online]. Indiana: Encyclopaedia Britannica [cit. 2023-08-20]. Available from: https://www.britannica.com/topic/graph-theory

CLAUSEN, Jens, 1999. Branch and Bound Algorithms - Principles and Examples. [PDF]. Department of Computer Science, University of Copenhagen.

CRESWELL, John W., 2009. Research Design: Qualitative, Quantitative, and Mixed Approaches. Third edition. Sage. ISBN 978-1-4129-6556-9.

DAVENDRA, Donald, 2010. Traveling Salesman Problem, Theory and Applications. Rijeka: InTech. ISBN 978-953-307-426-9.

GROSS, Jonathan L. and Jay YELLEN, 2004. Handbook of Graph Theory. Boca Raton: CRC Press. ISBN 1-58488-090-2.

GUTIN, Gregory and P. PUNNEN, Abraham, 2007. The Traveling Salesman Problem and Its Variations. New York: Springer. ISBN 0-306-48213-4.

JACOBS, Kees, Marc RIETRA, Jerome BUVAT, Sumit CHERIAN, Shannon WARNER, Lindsey MAZZA, Amol KHADIKAR and Yashwardhan KHEMKA, 2018. The last-mile delivery challenge: Giving retail and consumer product customers a superior delivery experience without impacting profitability [PDF]. Capgemini Research Institute.

KIZILATEŞ, Gözde a Fidan NURIYEVA, 2013. On the Nearest Neighbor Algorithms for the Traveling Salesman Problem. In: Advances in Computational Science, Engineering and Information Technology. Heidelberg: Springer International Publishing, p. 111-118. Advances in Intelligent Systems and Computing. ISBN 978-3-319-00951-3. Available from: doi:10.1007/978-3-319-00951-3_11.

KORUKOĞLU, Serdar and Serkan BALLI, 2011. An Improved Vogel's Approximation Method For The Transportation Problem [PDF]. Mathematical and Computational Applications Vol. 16, No. 2, pp. 370-381. Association for Scientific Research.

KUČERA, P., 2012. Different Versions of The Savings Method for The Time Limited Vehicle Routing Problem. Acta Universitatis Agriculturae Et Silviculturae Mendelianae Brunensis. (7), p.171-178.

LAWRENCE NEUMAN, W., 2014. Social Research Methods: Qualitative and Quantitative Approaches. Seventh Edition. Pearson Education Limited. ISBN 978-1-292-02023-5.

MANSON, Nj., 2006. Is operations research really research? ORiON. vol. 22, no. 2. DOI: 10.5784/22-2-40

MATAI, Rajesh, Surya SINGH and Murari LAL MITTAL, 2010. Traveling Salesman Problem: An Overview of Applications, Formulations, and Solution Approaches [online]. InTech [cit. 2023-10-15]. Available from: doi:10.5772/12909

MS, Smyth and Martin JH, 2000. X Ray Crystallography. Mol Pathol [online]. Journal of Clinical Pathology (v.53(1), 8-14 [cited 2023-10-15]. Available from: doi:10.1136/mp.53.1.8.

P. WILLIAMSON, David and David B. SHMOYS, 2011. The Design of Approximation Algorithms. Cambridge University Press. ISBN 978-0-521-19527-0.

RAHMAN, Md. Saidur, 2017. Basic Graph Theory. Germany: Springer International Publishing. ISBN 978-3-319-49474-6.

RUSHTON, Alan, Phil CROUCHER and Peter BAKER, 2014. The handbook of logistics and distribution management. 5th ed. London: Chartered Institute of Logistics and Transport. ISBN 978-074-9466-275.

S. HILLIER, Frederick, and Gerald J. LIEBERMAN, 2010. Introduction to Operations Research. Ninth edition. New York: McGraw-Hill. ISBN 978-0-07-337629-5.

SWANSON, Richard A., Elwood F. HOLTON III, 2005. Research in Organizations: Foundations and Methods of Inquiry. Berrett-Koehler. ISBN 978-1-60509-333-8.

TAHA, Hamdy A., 2007. Operations Research: An Introduction. Eighth Edition. Upper Saddle River, NJ: Pearson Prentice Hall. ISBN 0-13-188923-0.

UDDIN, Sharif, Utpal KANTI DAS, Aminur RAHMAN KHAN, Abu HELAL and Ashraful BABU, 2014. Logical Development Of Vogel's Approximation Method (LD-VAM): An Approach To Find Basic Feasible Solution Of Transportation Problem. International Journal of Scientific & Technology Research. 2014 (Vol. 3). ISSN 2277-8616.

YONG CHING, Lim a Yu YA JUN, 2003. A width-recursive depth-first tree search approach for the design of discrete coefficient perfect reconstruction lattice filter bank. IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing [online]. 50(6), 257-266 [cit. 2023-11-28]. ISSN 1057-7130. Available from: doi:10.1109/TCSII.2003.812912.

8 Appendix

Table 1: Day 1 matrix.

Day 1	**L1**	**L2**	**L3**	**L4**	**L5**	**L6**	**L7**	**L8**	**L9**	**L10**
L1	0	1.6	0.4	1.1	1.5	3.8	4.9	7.6	4.5	2.9
L2	1.2	0	1.5	2.2	2.6	5	3.2	6.8	5.7	4.1
L3	0.5	1.6	0	1.3	1.7	4	5.1	7.7	4.7	2.8
L4	1.5	2.2	1.8	0	0.4	2.7	3.8	9.1	6	4.4
L5	1.7	2.4	2	1	0	2.4	3.5	9.3	6.2	4.6
L6	4	4.6	4.3	3.3	3	0	3.3	10.2	8.5	6.9
L7	4.7	3.2	5	4	3.7	3.1	0	7.2	9.2	7.6
L8	7.6	6.9	7.9	8.7	9.1	10.7	7.3	0	7.4	8.4
L9	5.3	6.5	5.2	6.2	6.5	8.9	9.9	7.3	0	5.8
L10	3.5	4.7	2.9	4.4	4.8	7.1	8.2	8.2	5.9	0

Table 2 Initial route for Day 1.

Point-to-point	**Distance (km)**
L1 → L2	1.6
L2 → L3	1.5
L3 → L4	1.3
L4 → L5	0.4
L5 → L6	2.4
L6 → L7	3.3
L7 → L8	7.2
L8 → L9	7.4
L9 → L10	5.8
L10 → L1	3.5
Total distance:	34.4

Table 3 Alternative route for Day 1 obtained through NNA.

Point-to-point	**Distance (km)**
L1 → L3	0.4
L3 → L4	1.3
L4 → L5	0.4
L5 → L6	2.4
L6 → L7	3.3
L7 → L8	7.2
L8 → L9	7.4
L9 → L10	5.8
L10 → L2	4.7
L2 → L1	1.2
Total distance:	34.1

Table 4 Alternative route for Day 1 obtained through B&B.

Point-to-point	Distance (km)
L1 → L4	1.1
L4 → L5	0.4
L5 → L6	2.4
L6 → L7	3.3
L7 → L8	7.2
L8 → L9	7.4
L9 → L10	5.8
L10 → L3	2.9
L3 → L2	1.6
L2 → L1	1.2
Total distance:	33.3

Table 5 Day 2 matrix.

Day 2	**L1**	**L2**	**L3**	**L4**	**L5**	**L6**	**L7**	**L8**	**L9**	**L10**	**L11**	**L12**
L1	0	3.3	5.4	5.1	6.9	6.2	7.3	5.8	7.7	3.5	4.6	1.5
L2	3.6	0	2.5	2.2	4.4	3.6	5.2	4.2	7.2	2.4	6.8	4.5
L3	6.7	2.3	0	2.8	4.2	4.5	6.4	5.9	8.5	4	9	7.9
L4	4.8	1.8	3.4	0	2.4	1.4	3.2	3.2	5.8	1.7	7.1	5.2
L5	7	3.9	4	2.2	0	1.9	3.1	4.2	6.8	3.8	10.1	7.2
L6	5.9	3.1	4.2	2.1	2.6	0	2.2	2.4	4.9	2.7	8.3	6.1
L7	7.8	5.1	5.8	3.4	2.7	1.9	0	3.3	4.7	4.6	8.6	8
L8	6.4	4.3	6.1	3.8	4.4	2.7	3.8	0	3.4	2.4	6.1	5.5
L9	8	6.9	8.7	6.5	6.8	5.2	5.3	3.9	0	5	6.6	7.8
L10	3.6	1.9	3.7	2	3.6	2.9	4	2.3	5	0	5.9	4
L11	4	6.4	8.2	7.5	9	8.4	8.7	6.5	6.4	5.4	0	3.1
L12	1.7	4.5	6.3	5.6	7.2	6.4	7.5	5.8	7.4	3.6	3.2	0

Table 6 Initial route for Day 2.

Point-to-point	Distance (km)
L1 → L2	3.3
L2 → L3	2.5
L3 → L4	2.8
L4 → L5	2.4
L5 → L6	1.9
L6 → L7	2.2
L7 → L8	3.3
L8 → L9	3.4
L9 → L10	5
L10 → L11	5.9
L11 → L12	3.1
L12 → L1	1.7
Total distance:	37.5

Table 7 Alternative route for Day 2 obtained through NNA.

Point-to-point	Distance (km)
L1 → L3	5.4
L3 → L2	2.3
L2 → L4	2.2
L4 → L6	1.4
L6 → L7	2.2
L7 → L5	2.7
L5 → L10	3.8
L10 → L8	2.3
L8 → L9	3.4
L9 → L11	6.6
L11 → L12	3.1
L12 → L1	1.7
Total distance:	37.1

Table 8 Alternative route for Day 2 obtained through B&B.

Point-to-point	Distance (km)
L1 → L10	3.5
L10 → L2	1.9
L2 → L3	2.5
L3 → L4	2.8
L4 → L5	2.4
L5 → L7	3.1
L7 → L6	1.9
L6 → L8	2.4
L8 → L9	3.4
L9 → L11	6.6
L11 → L12	3.1
L12 → L1	1.7
Total distance:	35.3

Table 9 Day 3 matrix

Day 3	**L1**	**L2**	**L3**	**L4**	**L5**	**L6**	**L7**	**L8**	**L9**	**L10**	**L11**
L1	0	2.7	3.9	7	10.2	9	7.8	7.6	7.8	5.3	6.1
L2	3.3	0	1.2	7.1	8.9	6.3	5.1	4.9	5.1	3.7	7.7
L3	4.5	3.3	0	7.5	9.7	5.1	3.9	3.7	3.9	2.5	6.5
L4	8.9	8.6	7.6	0	4.7	4.9	5.2	6.2	6.3	8.3	12.3
L5	10.1	9.8	9.6	3.7	0	3.2	4.4	4.8	4.9	10.3	11
L6	9.6	6.5	5.4	4.9	3.1	0	1.2	1.6	1.7	4.9	8.5
L7	7.7	6.5	4.1	5.3	5.5	1.1	0	1	1.4	4.3	7.8
L8	7.4	6.2	3.8	6.2	4.6	1.6	1	0	0.2	3.5	7
L9	7.7	6.4	4.1	6.3	4.7	1.7	1.2	0.2	0	3.5	6.5
L10	5.1	5.2	2.4	8.3	10.5	5.4	4.2	4	3.5	0	4.2
L11	5.9	8.2	7	13	11.4	8.3	8	6.8	6.7	4.8	0

Table 10 Initial route for Day 3

Point-to-point	Distance (km)
L1 → L2	2.7
L2 → L3	1.2
L3 → L4	7.5
L4 → L5	4.7
L5 → L6	3.2
L6 → L7	1.2
L7 → L8	1
L8 → L9	0.2
L9 → L10	3.5
L10 → L11	4.2
L11 → L1	5.9
Total distance:	35.3

Table 11 Alternative route for Day 3 obtained through NNA.

Point-to-point	Distance (km)
L1 → L2	2.7
L2 → L8	4.9
L8 → L9	0.2
L9 → L7	1.2
L7 → L6	1.1
L6 → L5	3.1
L5 → L4	3.7
L4 → L3	7.6
L3 → L10	2.5
L10 → L11	4.2
L11 → L1	5.9
Total distance:	37.1

Table 12 Alternative route for Day 3 obtained through B&B.

Point-to-point	Distance (km)
L1 → L2	2.7
L2 → L3	1.2
L3 → L4	7.5
L4 → L5	4.7
L5 → L6	3.2
L6 → L7	1.2
L7 → L8	1
L8 → L9	0.2
L9 → L10	3.5
L10 → L11	4.2
L11 → L1	5.9
Total distance:	35.3

Table 13 Day 4 matrix

Day 4	**L1**	**L2**	**L3**	**L4**	**L5**	**L6**	**L7**	**L8**	**L9**	**L10**	**L11**
L1	0	0.9	1.4	2	6.3	1.6	7.9	6.6	6	4.1	0.9
L2	0.8	0	0.4	1.6	5.6	2.1	8.7	7.4	6.8	5	1.8
L3	1.3	0.5	0	1.1	5.2	1.6	9.2	7.9	7.3	5.4	2.2
L4	2.2	1.4	0.9	0	5.5	1.4	10.3	9	8.4	6.6	3.4
L5	5.9	5.5	5.2	4	0	4.6	13.5	12.2	8.1	6.4	6.6
L6	1.3	0.8	0.5	1.2	5.4	0	8.9	7.6	7	5.1	1.9
L7	7.2	8	8.6	9.2	13.5	8.8	0	2.2	6.6	6.9	8.1
L8	7.5	8.3	8.9	9.5	13.8	9.1	3.5	0	5.2	5.7	8.5
L9	6.3	7.2	7.7	8.3	8.3	7.8	7.3	5.6	0	2.4	6
L10	4.5	5.4	5.9	6.5	6.3	5.2	7.5	5.8	2.5	0	3.3
L11	1	1.9	2.4	3	5.1	2.1	8.6	7.3	5.4	3.2	0

Table 14 Initial route for Day 4.

Point-to-point	**Distance (km)**
L1 → L2	0.9
L2 → L3	0.4
L3 → L4	1.1
L4 → L5	5.5
L5 → L6	4.6
L6 → L7	8.9
L7 → L8	2.2
L8 → L9	5.2
L9 → L10	2.4
L10 → L11	3.3
L11 → L1	1
Total distance:	35.5

Table 15 Alternative route for Day 4 obtained through NNA.

Point-to-point	**Distance (km)**
L1 → L2	0.9
L2 → L3	0.4
L3 → L4	1.1
L4 → L6	1.4
L6 → L11	1.9
L11 → L5	5.1
L5 → L10	6.4
L10 → L9	2.5
L9 → L8	5.6
L8 → L7	3.5
L7 → L1	7.2
Total distance:	36

Table 16 Alternative route for Day 4 obtained through B&B.

Point-to-point	Distance (km)
L1 → L11	0.9
L11 → L7	8.6
L7→ L8	2.2
L8 → L9	5.2
L9 → L10	2.4
L10 → L5	6.3
L5 → L4	4
L4 → L6	1.4
L6 → L3	0.5
L3 → L2	0.5
L2 → L1	0.8
Total distance:	32.8

Table 17 Day 5 matrix

Day 5	**L1**	**L2**	**L3**	**L4**	**L5**	**L6**	**L7**	**L8**	**L9**
L1	0	6.3	17.8	13.7	12.7	12.2	11.3	10.8	9.8
L2	6.3	0	13.1	9.2	10.2	7.7	6.8	8.6	4.7
L3	17.7	13	0	9.5	12.3	9.1	10.6	14.6	10.2
L4	14.2	9.7	10.2	0	3.5	2.9	2.8	6.3	5.1
L5	12.6	9.7	13.1	4	0	4.6	4	3.3	6.2
L6	13.6	7	9.5	3	5.2	0	2	6.1	3
L7	12	6.8	10.4	2.5	5.2	1.6	0	4.7	2.5
L8	10.1	7.4	14.2	7.2	3.9	6.8	5.2	0	3.8
L9	9.5	4.7	11.4	4.9	6.4	3.4	2.6	5.1	0

Table 18 Initial route for Day 5.

Point-to-point	Distance (km)
L1 → L2	6.3
L2 → L3	13.1
L3 → L4	9.5
L4 → L5	3.5
L5 → L6	4.6
L6 → L7	2
L7 → L8	4.7
L8 → L9	3.8
L9 → L1	9.5
Total distance:	57

Table 19 Alternative route for Day 5 obtained through NNA.

Point-to-point	**Distance (km)**
L1 → L3	17.8
L3 → L6	9.1
L6 → L7	2
L7 → L4	2.5
L4 → L5	3.5
L5 → L8	3.3
L8 → L9	3.8
L9 → L2	4.7
L2 → L1	6.3
Total distance:	53

Table 20 Alternative route for Day 5 obtained through B&B.

Point-to-point	**Distance (km)**
L1 → L2	6.3
L2 → L9	4.7
L9 → L7	2.6
L7 → L6	1.6
L6 → L3	9.5
L3 → L4	9.5
L4 → L5	3.5
L5 → L8	3.3
L8 → L1	10.1
Total distance:	51.1