VYSOKÉ UČENÍ TECHNICKÉ V BRNĚ
Fakulta strojního inženýrství
Ústav mechaniky těles, mechatroniky a biomechaniky

Ing. Oldřich Ševeček

SOLUTION OF GENERAL STRESS CONCENTRATORS IN ANISOTROPIC MEDIA BY COMBINATION OF FEM AND THE COMPLEX POTENTIAL THEORY

ŘEŠENÍ OBECNÝCH KONCENTRÁTORŮ NAPĚTÍ V ANISOTROPNÍCH PROSTŘEDÍCH POMOCÍ KOMBINACE MKP A TEORIE KOMPLEXNÍCH POTENCIÁLŮ

ZKRÁCENÁ VERZE PH.D. THESIS

Obor: Engineering mechanics

Školitel: prof. RNDr. Michal Kotoul, DrSc.

Brno 2009

KEYWORDS:
Anisotropic bi-material, two-state integral, generalized stress intensity factor, T-stress, FEM, bridged crack, fracture criteria.

KLÍČOVÁ SLOVA:
Anisotropní bimateriál, dvoustavový integrál, zobecněný faktor intensity napětí, T-napětí, MKP, přemostěná trhlina, lomová kritéria.

CONTENT

1 INTRODUCTION

The increasing use of fibre-reinforced composites (or other modern materials) in high performance structures has brought a renewed interest in the analysis of cracks in anisotropic and moreover heterogeneous media. Without the tools for the assessment of fracture-mechanics behaviour of these materials it is impossible to apply them into any critical machine parts, where the unexpected failure can have a catastrophic consequence. Therefore, there is a necessity to correctly assess the singular points in constructions (potential stress concentrators) and be able to predict their next behaviour during the operation. Lot of recent works has been focused on the description of general stress concentrators in isotropic media. As a consequence this field is explored quite well. However, in case of the anisotropic materials, there are certain complications which generally disallow applying the same approaches as for isotropic materials. Therefore, it is necessary to find other possible ways how to involve anisotropy into the assessment of general stress concentrators – see Fig. 1.

The existence of material interfaces in composites, especially in laminates, brings other problems in the analysis of cracks – the problem of cracks terminating at the interface of two anisotropic (most often orthotropic) solids and the problem of interfacial cracks [10]. These problems are also encountered in the technology of protective coatings. For the assessment of crack behaviour in the aforementioned situations it is essential to investigate and describe the stress field near the crack tip. Although the FE analysis is capable of capturing the singular stress behaviour near a corner or a crack tip in homogeneous regions with a refined mesh of conventional elements, this traditional FE approach fails to accurately capture the appropriate singular behaviour near a corner or a crack tip at the junction of dissimilar materials. A very promising approach to an accurate calculation of the near crack tip fields consists in the application of so-called two-state (or mutual) conservation integrals - [16], [20], [40]. The two-state conservation integrals, e.g. in conjunction with a displacement-based FEM provide an efficient tool for calculating the stress intensities and elastic T-stresses without need of the very fine mesh in the singular point vicinity. This is a major advantage over the singular finite elements [46], and other various special techniques such as the boundary collocation or the X-FEM.

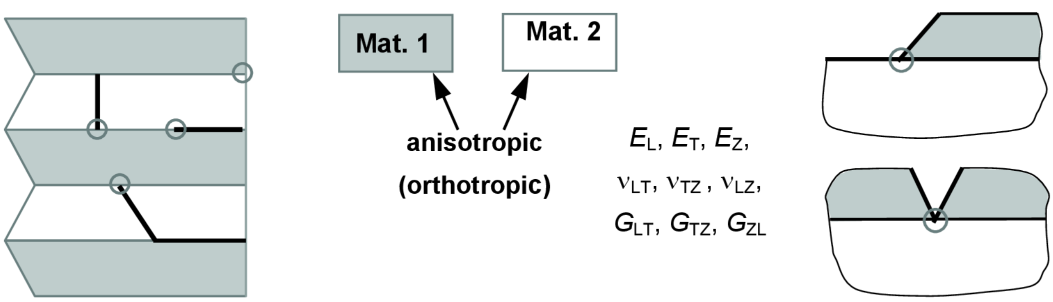


Fig. 1. Different types of the general stress concentrators – crack terminating at the interface of two dissimilar materials, interfacial crack, notch and V-notch and a general multimaterial wedge.

The problem complexity can be further increased by presence of the bridging phase which can significantly influence the resulting stress field in the vicinity of the crack tip (e.g. in laminated structures composed of layers reinforced by long fibres). In spite of the crack existence in some layer, there may be present intact bridging fibres which positively influence the fracture behaviour of the structure. Therefore, this fact should be also involved in the stress field analysis which stands as a basis for the subsequent fracture-mechanics behaviour assessment. The main objective of this assessment and of the whole described problems is to understand the mechanism of competition between the crack deflection along the interface and penetration into the adjoining material and be able to design such a construction which will exhibit the desired behaviour.

2 PRESENT STATE OF THE SOLVED PROBLEMS

2.1 DESCRIPTION OF THE STRESS FIELD IN THE VICINITY OF THE GENERAL STRESS CONCENTRATOR

2.1.1 Singularity analysis

In the first stage of the analysis of the stress field induced by the general stress concentrator, the eigenvalues and eigenvectors pertaining to the given singularity have to be found. These eigenvalues determine the stress singularity exponent δ_i-1 – the order of the stress singularity in the Williams-like stress asymptotic expansion:

$$\sigma_{ij\,(tip)} = H_1 \cdot r^{\delta_1 - 1} \cdot f_{ij1}(\phi,\alpha,\beta,\delta_1,\theta) + H_2 \cdot r^{\delta_2 - 1} \cdot f_{ij2}(\phi,\alpha,\beta,\delta_2,\theta) + \ldots T^{(1)}\delta_{i1}\delta_{j1} + T^{(2)}\delta_{i2}\delta_{j2} + O(r^{\delta}). \quad (1)$$

The eigenvectors determine the shape and distribution of the stress field (see functions $f_{ij}(\ldots)$ in (1)). For the singularity analysis, two main categories of numerical methods are available – explicit and implicit methods - [34]:

a) Explicit methods

An explicit form of the transcendent equation for the eigenvalues of the singular problem – roots of this equation are the eigenvalues of the singular problem operator is derived. Analytical solution was proposed e.g. by Williams or Westergaard and used in works [15] or [33] for a solution of the problem of crack terminating at the interface in isotropic solids. However, from practical point of view, this solution is limited to isotropic materials (or very special cases of anisotropy) and at present, only to the problems of maximum tri-material wedge. However, due to the very difficult manipulation with the long expressions some other approaches based on the semi-analytical solution are preferable to use.

- L.E.S. method

The method is named after Lechnitskii [27], Eshelby [8] and Stroh [43] who introduced the complex potentials for anisotropic bodies. The complex potentials formally satisfy the equilibrium, the compatibility equations and the elastic/strain laws but the specific form of the solution is gained by matching boundary conditions. The stresses σ_{ij}, displacements U_i and a resulting force T_i along the half-line leading from the origin of coordinate system is possible to write as follows:

$$\sigma_{ij} = H \cdot r^{\delta-1} \cdot f_{ij}(\theta,...)\,,\; U_i = H \cdot r^{\delta} \cdot g_i(\theta,...),\; -T_i = H \cdot r^{\delta} \cdot F_i(\theta,...), \tag{2}$$

where $f_{ij}(\theta,...)$, $g_i(\theta,...)$ and $F_i(\theta,...)$ are the functions of the polar coordinate θ, material elastic properties (given by material stiffness matrix), further of the characteristic material eigenvalues p_i and mainly of the searched singularity eigenvalue δ. The means of obtaining the complex numbers p_i have been proposed by Lechnitskii [27], Eshelby et al. [8] and summarized by Suo in [44].

- *Transfer matrix method:*

The procedure originally developed by Ting [48], [49] is an efficient tool for the singular characterization of non-degenerate anisotropic multimaterial corners. Ting's procedure makes use of a transfer matrix, which transfers the displacements and stress function vector components from one edge of the material wedge to the other.

It is worthy of note that Ting's procedure directly yields a linear system whose size is 3×3 or 6×6, irrespective of the number of materials N, contrary to traditional analytical procedures leading to a linear system of ($6N \times 6N$).

- *Continuously distributed dislocation (CDD) technique*

This technique can be used for the modelling of arbitrary cracks (opened or closed ones) [13] and it is based on the so-called Bueckner's principle. The basic idea is to use the superposition of the stress field present in the uncracked body, together with the unknown distribution of edge dislocations, chosen so that the crack faces become traction free. The goal is to compile an integral equation where the appropriate fundamental solution for the isolated dislocation is integrated along the crack line. By solving resulting Fredholm's integral equation the dislocation density is found. When the dislocation density is known, arbitrary stress or displacement component in the vicinity of the crack tip can be calculated.

- *Babuska's method:*

The characteristic eigenvalues and eigenvectors can also be evaluated using the method developed by Papadakis and Babuska in [34]. Their method can be used with multi-material wedges, with anisotropic materials and general boundary conditions under the assumption of plane strain. A special iterative procedure named Shoot was developed to solve the eigenvalue problem. This method has been also used in work [22] for calculation of eigenvalues of the multimaterial wedge.

b) Implicit methods

These methods do not lead to the closed form of the equation for the eigenvalues, they are slower, however they can be used also for the anisotropic materials and multi-material wedges as well. E.g. a method based on the variational formulation of the solved problem is available [25]. The main idea is to replace the classical formulation by the variational one [34]. The classical approximation for the finding of functional minimum using FEM leads to the homogenous system of algebraic equations for eigenvalues and eigenvectors.

2.1.2 Description of the singular stress field

Crack in anisotropic homogenous body

The problem of a crack in general anisotropic material under LEFM conditions is presented in work [41]. Three methods are presented for the calculation of the stress intensity factors for various anisotropic materials. All of the methods employ the displacement field obtained by means of the finite element method. The first one is known as displacement extrapolation and requires the values of the crack face displacements. The other two are conservative integrals based upon the J-integral.

Crack terminating at the interface of two different anisotropic materials

Number of works has been devoted to the problem of singularity analysis of cracks terminating at the interface in anisotropic media – e.g. [28], [49]. Ting in [49] studies the order of stress singularities at the tip of a crack which is normal to and ends at an interface between two anisotropic elastic layers in a composite material. Work [28] extends this study on problem of inclined crack at the bi-material interface. Equations for determining the stress singularity exponent are derived. The works are based on the complex potential theory analyzed in more details in [48].

Multimaterial wedge in anisotropic media

In the paper [3] the singular stress states induced at the tip of linear elastic multimaterial corners are characterized in terms of the order of stress singularities and angular variation of stresses and displacements. Linear elastic materials of an arbitrary nature are considered, namely anisotropic, orthotropic, transversely isotropic, isotropic, etc. This work is based on an original idea of Ting [48] in which an efficient procedure for a singularity analysis of anisotropic non-degenerate multimaterial corners is introduced by means of the use of transfer matrices.

2.1.3 Overview of references focusing on the GSIFs and T-stress calculation

There are several approaches for the calculation of the generalized stress intensity factor. One of the simplest is based on the comparison of numerical calculations of the stress (or displacement) field in front of the crack tip (e.g. by FEM) with the appropriate analytical expressions for stresses or displacements. GSIF is then extracted for $r \to 0$ – see e.g. work [33]. This approach is called a “direct method” and it can be used for cases where only one singularity is present. The accuracy of this method is strongly dependent on the element size at the crack tip.

Another, much more effective method, which can be used, for the GSIF (eventually T-stress) calculation is based on the method of two state (interactive) integrals in combination with FEM – e.g. [7], [19]. This method enables to determine the local stress field parameters in the vicinity of the crack tip using the deformation and stress field in the remote points, where the numerical results obtained e.g. using FE analysis are more accurate. The two-state integrals, which are path independent, are based on the J-integral [10], [14] or M-integral [11]. The application of the two-state integrals requires knowledge of the so-called auxiliary solution in the form of eigenfunctions of the appropriate singular problem [19].

The auxiliary solution has been found for the semi-infinite or finite crack, generally terminating at the interface of two anisotropic materials. In the connection with a description of V-notches or other general stress concentrators it is necessary to point out that J-integral is not path independent. On contrary the two-state M-integral is path independent for the case of V-notch configurations [11].

GSIF can also be determined using the so-called Ψ-integral [7]. This method which turned out to be very efficient is an implication of the Betti's reciprocity theorem. Major advantage of this integral consists in its path independency also for cases of multimaterial wedges in anisotropic media [42]. The reciprocal theorem of elastostatics states that in the absence of body forces and residual stresses the reciprocal theorem states that the following integral is path independent

$$\Psi(\mathbf{U},\mathbf{V}) = \int_{\Gamma}\left[\sigma_{ij}(\mathbf{U})n_i V_j - \sigma_{ij}(\mathbf{V})n_i U_j\right]\mathrm{d}s, \tag{3}$$

where Γ is any contour surrounding the crack tip and **U**, **V** are two admissible displacement fields. The asymptotic expansion of the displacements $\mathbf{U}(x)$ is possible to write in the following form

$$\mathbf{U}(x) = \mathbf{U}(0) + H_1 r^{\delta_1}\mathbf{u}_1(\theta) + H_2 r^{\delta_2}\mathbf{u}_2(\theta) + Tr^{\delta_3}\mathbf{u}_3(\theta) + \ldots = \sum_{i=0}^{\infty} k_i r^{\delta_i}\mathbf{u}_i(\theta),\ \ \delta_3 = 1, \tag{4}$$

where H_1 ,H_2 are the generalized stress intensity factors $\mathbf{u}_i(\theta),\ i=1,2$ are the angular distribution of the displacements corresponding to the singular terms in the stress asymptotic expansion and $\mathbf{u}_3(\theta)$ is the angular distribution of displacements for the T-stress. In the following we will consider $\mathbf{U}(0)=0$. T-stress is a non-singular stress component $\sigma_{22}(0,x_2)$ acting at the crack tip, $T = \sigma_{22}(0,x_2)\big|_{x_2\to 0^-}$. Due to the elastic mismatch, there exists also the non-singular stress component σ_{11} ahead of the crack tip (in the material M1), contrary to homogeneous materials, where T-stress is the only non-singular in-plane stress component. If the following displacement fields are considered $\mathbf{U} = \mathcal{U}_i(x) = r^{\delta_i}\mathbf{u}_i(\theta)$, $\mathbf{V} = \mathcal{U}_j(x) = r^{\delta_j}\mathbf{u}_j(\theta)$, (where δ_i, δ_j are obtained by solving the eigenvalue problem, see 2.1.1), it can be proved that the contour integral Ψ is equal to zero for $-\delta_i \neq \delta_j$ and non-zero if $-\delta_i = \delta_j$ - [25], [50],. Since the basis function corresponding to coefficients $k_1 = H_1$, $k_2 = H_2$, $k_3 = T$ in the asymptotic expansion for **U** are $r^{\delta_1}\mathbf{u}_1(\theta), r^{\delta_2}\mathbf{u}_2(\theta), r^{\delta_3}\mathbf{u}_3(\theta)$, it holds

$$\Psi\left(\mathbf{U}, r^{-\delta_1}\mathbf{u}_{-1}\right) = \sum_{i=1}^{\infty} k_i \Psi\left(r^{\delta_i}\mathbf{u}_i, r^{-\delta_1}\mathbf{u}_{-1}\right) = k_1\Psi\left(r^{\delta_1}\mathbf{u}_1, r^{-\delta_1}\mathbf{u}_{-1}\right), \tag{5}$$

where $\Psi\left(r^{\delta_1}\mathbf{u}_1, r^{-\delta_1}\mathbf{u}_{-1}\right)$ is computed analytically along the path Γ_1 surrounding the crack tip with diameter approaching zero, while $\Psi\left(\mathbf{U}, r^{-\delta_1}\mathbf{u}_{-1}\right)$ is computed along Γ_2 which is any remote integration path with finite diameter (see Fig. 2). Thus, the GSIF $H_1 = k_1$ can be computed as follows:

$$H_1 = \frac{\Psi\left(\mathbf{U}, r^{-\delta_1}\mathbf{u}_{-1}\right)}{\Psi\left(r^{\delta_1}\mathbf{u}_1, r^{-\delta_1}\mathbf{u}_{-1}\right)} \doteq \frac{\Psi\left(\mathbf{U}^h, r^{-\delta_1}\mathbf{u}_{-1}\right)}{\Psi\left(r^{\delta_1}\mathbf{u}_1, r^{-\delta_1}\mathbf{u}_{-1}\right)} = \frac{\int_{\Gamma_2}\left[\boldsymbol{\sigma}\left(\mathbf{U}^h\right)\cdot\mathbf{n}\cdot r^{-\delta_1}\mathbf{u}_{-1} - \boldsymbol{\sigma}\left(r^{-\delta_1}\mathbf{u}_{-1}\right)\cdot\mathbf{n}\cdot\mathbf{U}^h\right]\mathrm{d}s}{\int_{\Gamma_\varepsilon}\left[\boldsymbol{\sigma}\left(r^{\delta_1}\mathbf{u}_1\right)\cdot\mathbf{n}\cdot r^{-\delta_1}\mathbf{u}_{-1} - \boldsymbol{\sigma}\left(r^{-\delta_1}\mathbf{u}_{-1}\right)\cdot\mathbf{n}\cdot r^{\delta_1}\mathbf{u}_1\right]\mathrm{d}s} \tag{6}$$

Similarly the GSIF $H_2 = k_2$ is calculated. Observe, that the dual displacement fields (so called extraction solutions) $r^{-\delta_i}\mathbf{u}_{-i}(\theta)$ are singular at the crack tip, hence they have unbounded energy near the crack tip and thus corresponds to some concentrated sources at the crack tip. Since the exact solution **U** is not known, a finite element solution $\mathbf{U}^h$ can be used as an approximation for **U** so to obtain an approximation for GSIFs see e.g. [37], [38].

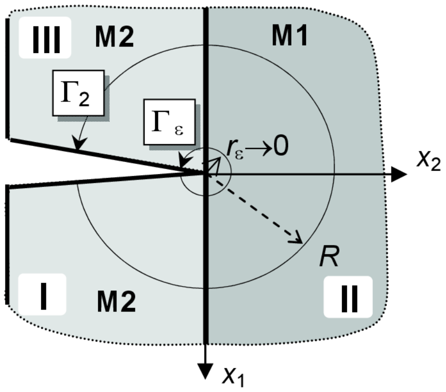


Fig. 2 Integration paths surrounding the singular point.

2.1.4 Description of the non-singular stress field

The non-singular term in the Williams asymptotic stress expansion – the T-stress - is often neglected. However, when aiming to describe the stress field in the vicinity of the crack tip more exactly, it should be also paid the same attention to the T-stress as to GSIF. The T-stress term is related to the characteristic eigenvalue $\delta = 1$, so it is no more singular. In these cases there are several other possibilities and approaches how the T-stress can be obtained:

Calculation of the T-stress using FEM

Estimation of the T-stress using the FE analysis is possible with a quite good accuracy for cracks in the homogenous materials. However in case of the general stress concentrators this analysis becomes controversial due to the presence of media discontinuity at the interface. This approach can be used only as a first approximation, but cannot be taken as an accurate solution. The T-stress is estimated by this method as a stress in direction of the crack face at distance $r \rightarrow 0$ from the crack tip. The estimation is strongly dependent on the mesh refinement in the vicinity of the crack tip.

Calculation of the T-stress using contour integrals

Calculation of the T-stress in the anisotropic linear elastic homogenous solid is presented in papers [53] and [45]. The T-stress is calculated using the path independent line integral and Betti's reciprocal work theorem, together with selected auxiliary fields. However, all the presented theory is applicable only to the case of crack in the homogenous body. In case of the general stress concentrators the application of J-integral is not possible, because it become path dependent. Therefore it is necessary to use other path-independent integral such as for example the Ψ-integral as it was proposed in the previous chapter for the calculation of GSIF.

Similar arguments which lead to Eq. (6), apply also for T-stress calculation, i.e. $T = k_3$ can be computed as follows

$$T = \frac{\Psi\left(\mathbf{U}, r^{-1}\mathbf{u}_{-3}\right)}{\Psi\left(r\mathbf{u}_3, r^{-1}\mathbf{u}_{-3}\right)} \doteq \frac{\Psi\left(\mathbf{U}^h, r^{-1}\mathbf{u}_{-3}\right)}{\Psi\left(r^1\mathbf{u}_3, r^{-1}\mathbf{u}_{-3}\right)} = \frac{\int_{\Gamma_2}\left[\boldsymbol{\sigma}\left(\mathbf{U}^h\right)\cdot\mathbf{n}\cdot r^{-1}\mathbf{u}_{-3} - \boldsymbol{\sigma}\left(r^{-1}\mathbf{u}_{-3}\right)\cdot\mathbf{n}\cdot\mathbf{U}^h\right]\mathrm{d}s}{\int_{\Gamma_\varepsilon}\left[\boldsymbol{\sigma}\left(r^1\mathbf{u}_3\right)\cdot\mathbf{n}\cdot r^{-1}\mathbf{u}_{-3} - \boldsymbol{\sigma}\left(r^{-1}\mathbf{u}_{-3}\right)\cdot\mathbf{n}\cdot r^1\mathbf{u}_3\right]\mathrm{d}s}. \quad (7)$$

Similarly like with GSIF a finite element solution $\mathbf{U}^h$ can be used as an approximation for $\mathbf{U}$ so to obtain an approximation for T.

In Eq. (7), $\mathbf{u}_{-3}(\theta)$ denotes the auxiliary solution for the T-stress. Physically, this solution corresponds to the concentrated moment about x_3 acting at the crack tip.

Calculation of the T-stress using CDD technique

As suggested by Broberg [4], the T-stress can also be determined using dislocation arrays. This approach has been widely discussed in [37] .

Modelling of a finite crack perpendicular to the bi-material interface (of isotropic materials), and terminating in front of the interface at distance b, is presented in [17] and [52]. The complete solutions of the problem, including the T-stress and the stress intensity factors are obtained.

2.2 CRACK BRIDGING PROBLEM

Fibre reinforced ceramic materials have promising potential e.g. for high-temperature applications. Under the tensile loading of the composite in the fibre direction, the brittle matrix can undergo extensive cracking normal to the fibres, but the associated matrix cracking stress may be substantially greater than the critical fracture stress of the unreinforced ceramic - see [6]. Furthermore, with the intact fibres, the composite material can continue to sustain additional load up to the fibre bundle fracture stress.

2.2.1 Bridging models

These models describe a relation between the bridging stress and the crack face opening. For a simple sliding with a constant τ_s, Aveston et al.[1]; Budiansky et al.[6] and [5]; Marshall et al. [29] suggested a model of bridging fibres represented by a continuous distribution of bridging springs obeying the quadratic bridging law

$$v(x_2) = \left(\frac{\sigma_{br}(x_2)}{\beta}\right)^2 \quad \text{where} \quad \beta = \sqrt{\frac{4c_\mathrm{f}^2 E_\mathrm{f} E^2 \tau_s}{\mathrm{R_f}\left(1-c_f\right)^2 E_\mathrm{m}^2}}, \quad (8)$$

where $v(x_2)$ is the displacement of the upper crack face, $\mathrm{R_f}$ is the fibre radius, E_f, E_m are material characteristics of the fibre and the matrix respectively, c_f fibre fracture volume and τ_s is a interface slipping shear resistance stress.

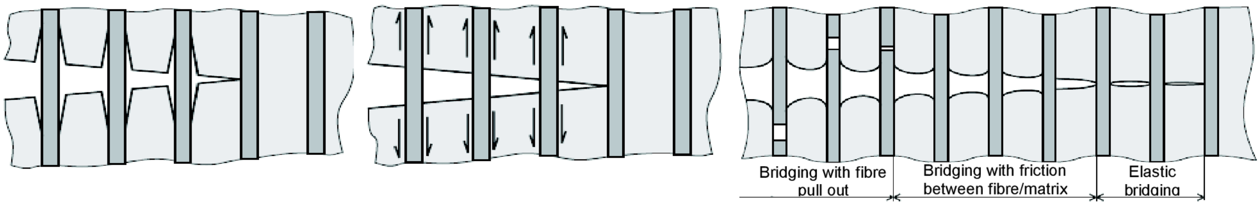


Fig. 3 Interface fibre/matrix: a) decohesion of fibre in matrix; b) frictional constraint fibre/matrix; c) characteristic bridging areas and corresponding types of constraints fibre/ matrix.

Under the assumption that the strength of the fibres, σ_{0f}, has a single, deterministic value, failure occurs when the bridging spring stress at the original crack tips reaches $\sigma = c_f\ \sigma_{0f}$. Since the stress on the fibres has a maximum value in the plane of the matrix crack so the fibres are always broken in the plane of the crack. The prediction of the composite toughness and strength may be so unduly conservative. The reason is that with dispersion in the fibre tensile strength, fibres may fracture within the matrix rather than at bridged faces of the crack, thereby leading to frictionally constrained fibre pullout before final failure occurs, and so leading to enhanced composite strength. Apparently, fractured fibres still contribute to the bridging stresses as they have to be pulled out from the matrix – see Fig. 3 c). The relative contribution of intact fibres and broken ones within the matrix, is analyzed assuming that the fibre strength follows the Weibull statistics [47]. This gave an expression for the average stress transferred by the fibres across crack:

$$\hat{\sigma}_{br} = \underbrace{\sigma_{br} \exp\left[-\left(\frac{\sigma_{br}}{c_f \Sigma}\right)^{m_w+1}\right]}_{\text{fraction of intact fibres}} + \underbrace{\sigma_P \left\{1 - \exp\left[-\left(\frac{\sigma_{br}}{c_f \Sigma}\right)^{m_w+1}\right]\right\}}_{\text{fraction of broken fibres}}, \tag{9}$$

where σ_P is the average stress exerted by the broken fibres pulled out from the matrix, and $\exp[-\left(\sigma_{br}/c_f\Sigma\right)^{m_w+1}]$ stands for the fraction of intact fibres in the crack wake. Σ is the fibre strength distribution and m_w the Weibull modulus [47]

2.2.2 Generalized bridging stress intensity factor

To quantitatively express the influence of the bridging fibres on the resulting stress field the value of the generalized bridging stress intensity factor H_{br} caused by the bridging stress have to be calculated. As a result, the local generalized stress intensity factor H_{tip}=H_{appl} -H_{br} acting in the very crack tip is lower than the remote applied stress intensity H_{appl}. One of the possible ways how to calculate the influence of the bridging effect can be found e.g. in [21] or [32]. The generalized bridging stress intensity factor is calculated using the following formula:

$$H_{br} = \int_{-h}^{0} W\left(x_2, h\right) \sigma_{br}\left(x_2\right) \mathrm{d}\, x_2\ , \tag{10}$$

where $W(x_2,h)$ is the weight function which can be obtained numerically using the FE analysis as was proposed for example in [39]. The weight function depends on the component geometry, but it is independent of the applied loading. This technique can be modified also for the solution of the plane crack problems.

The bridging stress σ_{br} can be calculated using the recurrent formulas and suitable bridging models as is presented in paper [21]. After the weight function and bridging stress is calculated, the generalized bridging SIF H_{br} can be determined and the local GSIF H_{tip} as well. The SIF H_{appl} can be calculated on the unbridged configuration e.g. using some of the two state integral method.

The bridging crack problems can also be solved efficiently using the already mentioned CDD technique. The solution can be worked out due to recent findings of authors in [18]. An integral equation is obtained by choosing the dislocation distribution to meet the traction conditions along the line of the crack and within crack bridging zone:

$$\frac{\left(\mathrm{Im}\left(\mathbf{A}^{II}\mathbf{M}^{II}\right)\right)^{-1}_{ik}}{2\pi}\int_{-h}^{0}\frac{f_k\left(x_{2o}\right)\mathrm{d}x_{2o}}{x_2-x_{2o}}+\int_{-h}^{0}N_{1ik}\left(x_2,x_{2o}\right)f_k\left(x_{2o}\right)\mathrm{d}x_{2o}=\sigma_{1i}^{appl}\left(x_2\right)+\delta_{1i}\sigma_{br}\left(v\left(x_2\right)\right). \quad (11)$$

Here, N_{1ik} are regular kernels in the closed interval [-h,0] (along the crack), $\sigma_{1i}^{appl}\left(x_2\right)$ denotes the negated stresses in x_1=0 produced by the given boundary loads, acting on a specimen with boundary $\partial\Omega$, but without cracks and dislocations. σ_{br} is the bridging stress as a function of the upper crack face displacement. $f_k(x_{2o})$ is the unknown dislocation density. Once the dislocation density $f_k(x_{2o})$ is found, the displacement of the upper crack face $v(x_2)$ is also known and from $\sigma_{br}\left[v\left(x_2\right)\right]$, the bridging stress as a function of position follows. After the bridging stress and dislocation density is known, arbitrary stress component in front of the crack tip can be calculated. Afterwards the resulting local GSIF is obtained as the following limit:

$$H_{tip}=\lim_{r\to 0} r^{1-\delta}\sigma_{11}\left(r,\theta=\pi/2\right). \quad (12)$$

2.3 PROBLEMS OF FRACTURE CRITERIA

It is now well established that the increase of the toughness of ceramics laminates or ceramic-matrix composites can be achieved by introducing weak interfaces between layers or between the fibre and the matrix [30]. Deflection along the interface then results in a crack blunting and this effect increases the required energy for the next crack propagation. Understanding the mechanism of the crack deflection along the interface is thus essential to determine, for example, the suitable interlayer and the optimum interface toughness which are necessary to favour this phenomenon [23]. Various attempts have been made to attain this objective.

The discontinuity in the elastic properties at the interface strongly influences the behaviour of the energy release rate of the crack in the vicinity of the interface. In the case of a strong singularity (crack lies in a stiffer material and a characteristic eigenvalue δ<1/2), the energy release rates $G_p(a_p=0)$, $G_d(a_d=0)$ for a crack terminating at the interface are infinite and interface penetration or deflection is thus possible at any finite load level. In contrast, the presence of a weak singularity (crack lies in a softer material, δ>1/2) implies that the energy release rates $G_p(a_p=0)$,

$G_d(a_d=0)$ for a crack terminating at the interface are zero and interface penetration or deflection is not predicted for any applied load. Due to this fact, the classical differential theory cannot be used for the case of cracks propagating near the interface. This problem may be overcome with the help of the so-called *finite fracture mechanics* [31], where the crack increment of a finite length is used instead of the infinitesimal one – for details see section 5.2.

3 APPLICATION OF METHODS FOR THE STRESS FIELD DESCRIPTION IN THE VICINITY OF THE GSC

3.1 CDD TECHNIQUE

The semi-infinite crack can be modelled as an array of continuously distributed edge dislocations along the negative x_2-axis, see Fig. 4.

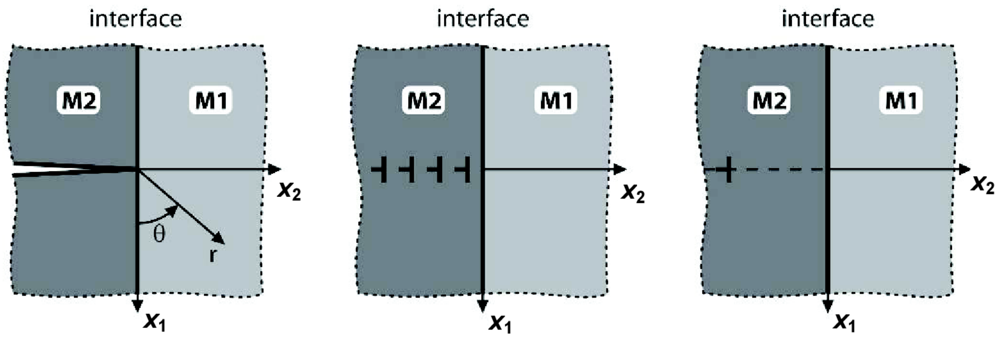


Fig. 4 Semi-infinite crack terminating perpendicular to the interface of two anisotropic materials.

The potential functions for an isolated dislocation located at the point (x_1,x_2) in an infinite homogeneous anisotropic medium is

$$\Phi_{\alpha o}(z)=q_\alpha \ln(z-\varsigma_\alpha),\ \text{where}\ \ \varsigma_\alpha = x_1 + p_\alpha x_{2o},\ \ \alpha=1,..3\,,\ \ q_\alpha = \frac{1}{4\pi} M_{\alpha k} d_k, \tag{13}$$

where the vector d_k is related to the Burgers vector b_i through the equation

$$b_i = B_{ik} d_k,\ \ \text{with}\ B_{ik} = \frac{i}{2}\sum_\alpha \left(A_{i\alpha}M_{\alpha k} - \bar{A}_{i\alpha}\bar{M}_{\alpha k}\right) = -\,\text{Im}\left(\sum_\alpha A_{i\alpha}M_{\alpha k}\right), \tag{14}$$

where the matrix $M_{\alpha k}$ is defined as the inverse of $L_{i\alpha}$, $M_{\alpha k} L_{k\beta} = \delta_{\alpha\beta}$. The quantities p_α, $A_{i\alpha}$, $L_{i\alpha}$ are given by Lekhniskii [27]. For the plane deformation, the elastic field can be represented in terms of the complex potential functions $\Phi_1(z_1)$, $\Phi_2(z_2)$, $\Phi_3(z_3)$, each of which is holomorphic in its arguments $z_\alpha = x_1 + p_\alpha x_{2o}$. Here, p_α are three distinct complex numbers with positive imaginary parts, which are obtained as the roots of the characteristic equation

$$\det\left[c_{i1k1} + p\left(c_{i1k2} + c_{i2k1}\right) + p^2 c_{i2k2}\right] = 0, \tag{15}$$

where c_{ijkl} is the tensor of elastic constants, i.e. $\sigma_{ij} = c_{ijkl}u_{k,l}$, which satisfies the symmetry conditions $c_{ijkl} = c_{ijlk} = c_{jikl} = c_{klij}$.

With these holomorphic functions, the representation for the displacements U_i and stresses σ_{ij} is

$$U_i = 2\mathrm{Re}\left[\sum_{\alpha=1}^{3} A_{i\alpha}\Phi_\alpha\left(z_\alpha\right)\right],\ \sigma_{2i} = 2\mathrm{Re}\left[\sum_{\alpha=1}^{3} L_{i\alpha}\Phi'_\alpha\left(z_\alpha\right)\right],\ \sigma_{1i} = -2\mathrm{Re}\left[\sum_{\alpha=1}^{3} L_{i\alpha}p_\alpha\Phi'_\alpha\left(z_\alpha\right)\right]. \quad (16)$$

The solution for the stress field produced by an isolated dislocation located at point (x_{1o}, x_{2o}) with the Burgers vector b_i in an infinite anisotropic bi-material can be written e.g. for stress component σ_{1i} as follows

$$\sigma_{1i}\left(x_1, x_2\right) = -\frac{1}{4\pi}\sum_\alpha L_{i\alpha}^{II} p_\alpha^{II}\left[\sum_\beta\left(G_{\alpha\beta}\bar{M}_{\beta k}^{II}\frac{d_k}{z_\alpha - \bar{\varsigma}_\beta}\right) + M_{\alpha k}^{II}\frac{d_k}{z_\alpha - \varsigma_\alpha}\right] + C.C.,\ z \in 2, \quad (17)$$

where superscript I and II refers to the material 1 and 2 respectively - see Fig. 4. Similar relations are derived also for other stress and displacement components.

The asymptotic stress field near the crack tip is modelled as a continuous distribution of dislocations with density function

$$f_k\left(x_{2o}\right) = Hv_k\left(-x_{2o}\right)^{\delta-1},\quad x_{2o} < 0, \quad (18)$$

By integration of (17) where δ is the unknown stress singularity exponent, v_k are the components of corresponding eigenvector, and H is the generalized stress intensity factor (GSIF). Substitute Eq. (18) into the integral equation, integrate and apply the traction-free condition on the plane of the crack to obtain

$$\mathrm{Re}\left\{\left[\sum_\alpha\sum_\beta L_{i\alpha}^{II} G_{\alpha\beta}\bar{M}_{\beta k}^{II}\left(-\frac{\bar{p}_\beta^{II}}{p_\alpha^{II}}\right)^{-\delta}\csc\left(\pi\delta\right) - \delta_{ik}\cot\left(\pi\delta\right)\right]\right\} v_k = 0. \quad (19)$$

Eq. (19) can be briefly written as

$$\mathbf{D}\left(\delta\right)\mathbf{v} = 0,\ \text{where}\ D_{ik}\left(\delta\right) = \mathrm{Re}\left\{\left[\sum_\alpha\sum_\beta L_{i\alpha}^{II} G_{\alpha\beta}\bar{M}_{\beta k}^{II}\left(-\frac{\bar{p}_\beta^{II}}{p_\alpha^{II}}\right)^{-\delta}\csc\left(\pi\delta\right) - \delta_{ik}\cot\left(\pi\delta\right)\right]\right\}. \quad (20)$$

The parameter δ is calculated from the characteristic equation Det[**D**(δ)]=0. Substituing δ back into relations like (17), the arbitrary stress or displacement component can be calculated.

3.2 L.E.S. METHOD

Choose the coordination system so that the material containing crack is in the area $x_2<0$. Both of these materials are homogenous and linear elastic and the Hooke´s law is valid for the deformations:

$$\varepsilon_i = \sum_{j=1,2,6} s_{ij}\sigma_j,\quad (i = 1,2,6). \quad (21)$$

Where s_{ij} is a compliance matrix and the Eq. (21) holds for the case of the plane stress. In the case of plane strain it is necessary to perform a conversion of the compliance matrix components.

The subscripts 1,2,3 denotes the appropriate material direction, where the direction 1 is called the Longitudinal (L), 2 – Transversal (T) and 3 as Z. Both orthotropic materials are characterized by the complex numbers p_i, Im(p_i)>0, (see also the preceding section) where i=1,2 and Im(.) denotes the complex number imaginary part. Numbers p_i are depending on material characteristics and can be obtained as the roots of the 4th order equation

$$\lambda p^4 + 2\rho\lambda^{1/2}p^2 + 1 = 0, \text{ where } \lambda = \frac{s_{11}}{s_{22}},\ \rho = \frac{2s_{12}+s_{66}}{2\sqrt{s_{11}s_{22}}}. \tag{22}$$

The eq. (22) is a special case of the characteristic equation of the 6th order presented by Lechnitskii [26]. The case ρ=1 corresponds to material with cubic symmetry and λ=ρ=1 corresponds to isotropic material. These are the so-called degenerate cases of anisotropy, where the LES formalism cannot be applied directly. One of the ways to overcome the problems with material symmetry was introduced by Suo [44]. This is an analogy to the Muschelishvilli´s complex potential method.

For the two aligned orthotropic media, it is possible to define two generalized Dundurs' parameters α and β - see [12], which are the only bi-material constants that enter the solution for the problem involving dissimilar materials with prescribed tractions at the boundary. Thus, the solution for the problem under consideration should depend upon λ and ρ for each material and the two bi-material parameters α and β. Both parameters α and β can take the value from interval (-1, 1). For the case of anisotropic material, i.e. for $\rho\neq1$, it is possible to write the relations for displacements U_i, stresses σ_{ij}, and the resulting force T_i along the half-line leading from the CS origin:

$$\begin{aligned} U_i &= 2\,\mathrm{Re}\left\{\sum_{j=1}^{2} A_{ij} f_j(z_j)\right\},\ T_i = -2\,\mathrm{Re}\left\{\sum_{j=1}^{2} L_{ij} f_j(z_j)\right\} \\ \sigma_{2i} &= 2\,\mathrm{Re}\left\{\sum_{j=1}^{2} L_{ij} f_j'(z_j)\right\},\ \sigma_{1i} = -2\,\mathrm{Re}\left\{\sum_{j=1}^{2} L_{ij} p_j f_j'(z_j)\right\} \end{aligned} \tag{23}$$

where $z_j = x_1 + \mathrm{i}p_j x_2$, (.)′ denotes differentiation with respect to z_j and the matrices A_{ij}, L_{ij} holds contains characteristics p_i and components s_{ij} – see [36] .

The exponent δ depends on the local boundary conditions and the material characteristics of both materials. The unknown potentials $f_j(z_j)$, $\varphi(z)$ and $\psi(z)$ are sought in the following form $f_j^J(z_j^J) = \phi_j^J z_j^{J\delta}$, $\varphi^J(z) = \phi_1^{*J} z^\delta$, $\psi^J(z) = \phi_2^{*J} z^\delta$, where j=1, 2, J=I, II, III and ϕ_j^J and ϕ_j^{*J} are vectors of complex coefficients. The superscripts denotes the appropriate bi-material region (see Fig. 2), the subscript denotes either the pertinence to characteristic number p_i^J of the orthotropic material or pertinence to the potential of the isotropic material. The coordinates z_j and z are considered as polar (see Fig. 2) $z_j^J = r\left(\cos\theta + p_j^J \sin\theta\right)$, $z = r\left(\cos\theta + \mathrm{i}\sin\theta\right)$.

In the crack tip region, the following boundary conditions have to be satisfied

$$T_i = 0 \text{ for } \theta = -\phi, 2\pi-\phi\ ;\ U_i^I = U_i^{II},\ T_i^I = T_i^{II} \text{ for } \theta = 0\ ;\ U_i^{II} = U_i^{III},\ T_i^{II} = T_i^{III} \text{ for } \theta = \pi, \tag{24}$$

where ϕ is an angle formed by the crack and the interface. The goal is to find the unknown singularity exponent δ and the corresponding unknown eigenvectors ϕ_j^J or ϕ_j^{*J} so that the boundary conditions (24) are satisfied. Substituting the assumed form of the potential solution $f_j(z_j)$, $\varphi(z)$ and $\psi(z)$ into (23) one obtain for the case of the anisotropic media the following relations

$$\mathbf{U}^J = \mathbf{A}^J \mathbf{Z}^{J\delta} \Phi^J + \overline{\mathbf{A}}^J \overline{\mathbf{Z}}^{J\delta} \overline{\Phi}^J, \qquad -\mathbf{T}^J = \mathbf{L}^J \mathbf{Z}^{J\delta} \Phi^J + \overline{\mathbf{L}}^J \overline{\mathbf{Z}}^{J\delta} \overline{\Phi}^J, \tag{25}$$

For the bi-material, composed of two anisotropic media, combining the boundary conditions (24) and relations for the displacements and resulting force (25) one gets the homogenous algebraic equation system of size 12x12 which is is possible to reduce on the system of two equations

$$\mathbf{K}(\delta)\mathbf{v}^I = \mathbf{0}, \tag{26}$$

where $\mathbf{0}$=0_i is a vector 2×1 and for the vector v_i^I holds $\mathbf{v}^I = (1/H)\cdot \mathbf{L}^I \Phi^I$, where H is a generalized stress intensity factor (GSIF). The following relation has to hold, in order to the solution of the equation system (26) exist

$$\det\left[\mathbf{K}(\delta)\right] = 0. \tag{27}$$

The relation (27) leads to nonlinear equation with parameter δ, which has at least two roots on interval (0, 1). Substituing δ back into the relations for $f_j(z_j)$, $\varphi(z)$ and $\psi(z)$ and Eq. (23) the complete stress and displacement field is obtained.

Analogical relations as in (23)-(25) can be used also for the stress and displacement field description in the case of isotropic material. The matrices A_{ij} and L_{ij} in (23) are then simplified - see [36].

4 SOLUTION OF THE CRACK BRIDGING PROBLEM

4.1 BRIDGED CRACK MODELLING USING THE WEIGHT FUNCTION METHOD

The weight function method allows setting up a bridging stress-crack opening displacement relationship (see section 2.2) by analysing the experimental crack opening displacement data and solving an integral equation. Once the weight function(s) are known the bridging intensity factor can be easily calculated for any bridging stress distribution by evaluating the integral of the form of Eq.(10). The weight function method has been extensively used to the modelling of bridged crack problems [9]. For a complicated domain, the weight function has to be obtained numerically, e.g. by FEM - [39] (performing a number of calculations of the GSIF due to unit line load applied to the crack face at arbitrary points - Fig. 5). To this end, an application of the reciprocal theorem seems to be very efficient - see 2.1.3. To calculate the COD and the bridging stress, the special recurrent calculations are applied – see [21] .

Now assume that a pair of line forces acts on the crack faces at a point x_{2b} - Fig. 5. Other loading is absent. Eq. (3) modifies with help of Eq. (5) as

$$\int_{\Gamma_3} \left[\sigma_{ij}(\mathbf{U}) n_i\, r^{-\delta_1} u_{-1_j} - \sigma_{ij}\left(r^{-\delta_1}\mathbf{u}_{-1}\right) n_i U_j \right] \mathrm{d}s + 2\mathbf{F} r^{-\delta_1}\mathbf{u}_{-1} = H\Psi\left(r^{\delta_1}\mathbf{u}_1, r^{-\delta_1}\mathbf{u}_{-1}\right). \tag{28}$$

Γ_3 is an arbitrary contour enclosing a domain containing both the crack tip and the pair of line forces. By definition, the weight function $W(x_{2b},h)$ follows as

$$W \equiv \frac{H}{|\mathbf{F}|} = \frac{1}{|\mathbf{F}|} \frac{\int_{\Gamma_3} \left[\sigma_{ij}(\mathbf{U}) n_i\, r^{-\delta_1} u_{-1_j} - \sigma_{ij}\left(r^{-\delta_1}\mathbf{u}_{-1}\right) n_i U_j \right] \mathrm{d}s + 2\mathbf{F} r^{-\delta_1}\mathbf{u}_{-1}}{\Psi\left(r^{\delta_1}\mathbf{u}_1, r^{-\delta_1}\mathbf{u}_{-1}\right)}. \tag{29}$$

FE solution $\mathbf{U}^h$ was used as an approximation for $\mathbf{U}$ in Eq. (29). Having calculated a value of the weight function W for sufficiently large number of line force positions, the generalized bridging SIF, H_{br}, can be using relation (10) obtained for an arbitrary bridging stress distribution $\sigma_{br}(x_2)$.

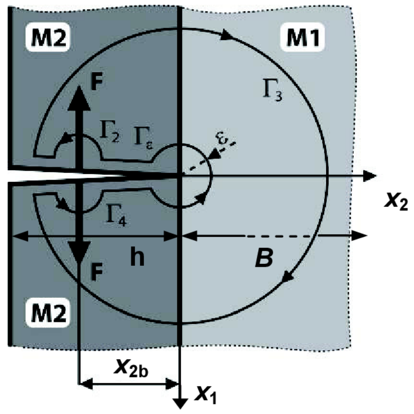
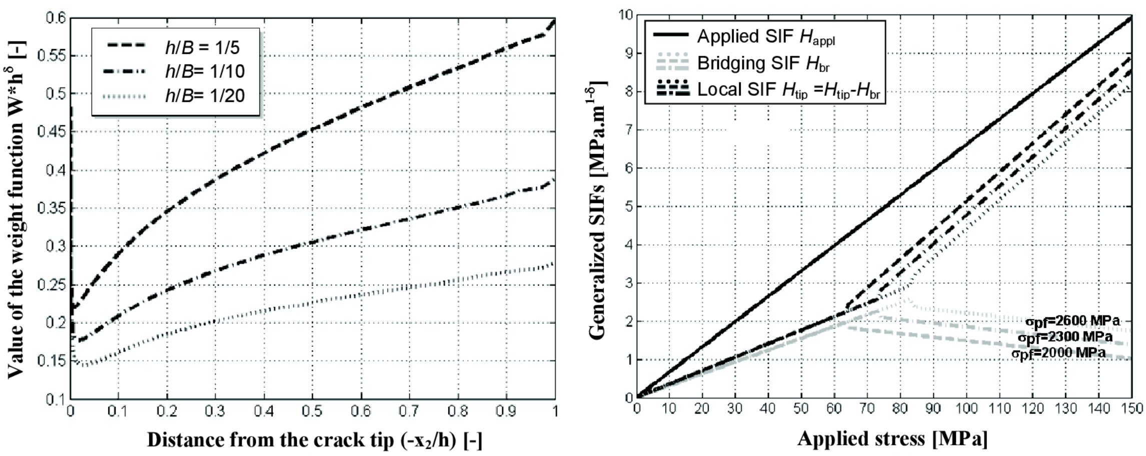


Fig. 5 A pair of line forces acting on the crack faces and the integration path.

With elastic constants of two layers M1 and M2 E_L=137GPa, E_T= E_Z=10.8GPa G_{ZT} = 3.36 GPa, G_{ZL} = G_{TL} = 5.65 GPa, ν_{TZ} = 0.49 and ν_{ZL} = ν_{TL} = 0.238, the weight functions were calculated for several ratios of the layer thicknesses h/B. Note that for M1 the L-direction is parallel with x_2-axis and for M2 parallel with x_1-axis. The bridging model (9) was applied with the fibre volume fraction c_f = 0.4, the fibre radius R_f = 7 µm, the sliding resistance τ_s = 6 MPa, the fibre Young modulus E_f = 228 GPa, and the matrix Young modulus E_m = 76 GPa. An example parametric study was performed in order to examine an influence of the fibre characteristic strength σ_{0f} - see Fig. 6. The product $W.h^\delta$ is plotted against the dimensionless distance from the crack tip $-x_2/h$. The same figure also shows examples of the resulting SIFs - H_{appl}, H_{br} and H_{tip}, calculated as a function of the tensile loading σ_0 for several fibre strengths of the bridging model (9).



a) b)

Fig. 6 a) Bimaterial normalized weight function against the dimensionless distance from the crack tip for several values of the ratio h/B. b) Remote, bridging, and local GSIFs plotted as functions of the applied tensile loading σ_0 for several values of the fibre characteristic strength σ_{0f}.

5 FRACTURE CRITERION FOR THE ASSESSMENT OF THE GENERAL STRESS CONCENTRATOR STABILITY

5.1 CRACK ONSET CRITERION

As it was presented in the work [24], both energy and stress criteria are necessary conditions for fracture but neither one nor the other are sufficient. Experiments by Parvizi et al. - [35] on transverse cracking in cross-ply laminates corroborate this assumption. When fracture occurs, the two criteria (energy and strength) are fulfilled simultaneously, even if one often hides the other. Both are necessary conditions and together they seem to form a sufficient one. Giving both the toughness G_c and the strength σ_c it is possible to define a characteristic length for the crack onset. The failure is assumed to be a sudden and quasi-spontaneous mechanism as proposed e.g. by [2], [35] and [51].

5.2 PERTURBATION ANALYSIS

In the case of a matrix crack impinging on the interface, a differential energy analysis is unsuitable due to the discontinuity in the elastic properties: finite crack extensions a_d, a_p are to be considered (instead of infinitesimal one) and the competition between deflection and penetration at the interface is evaluated using the condition that the crack will follow the path which maximizes the additional energy ΔW released by the fracture. If crack deflection occurs preferentially to penetration at the interface, the following condition must be satisfied:

$$\Delta W_d = \delta W_d - G_c^{\mathrm{i}} a_d > \Delta W_p = \delta W_p - G_c^1 a_p, \tag{30}$$

where G_c^{i} is the interface toughness, G_c^1 is the toughness of the material M1 and δW is a change of the potential energy between the original and new crack position. Matched asymptotic procedure is used to derive δW - [27]. A perturbation of the domain Ω^{in} is introduced as shown in Fig. 7. The perturbation is a deflected (singly, doubly) crack extension of length a_d or penetrating crack extension of length a_p with the small perturbation parameter ε defined as $\varepsilon = a / L_c \ll 1,\ a = a_p, a_d,$

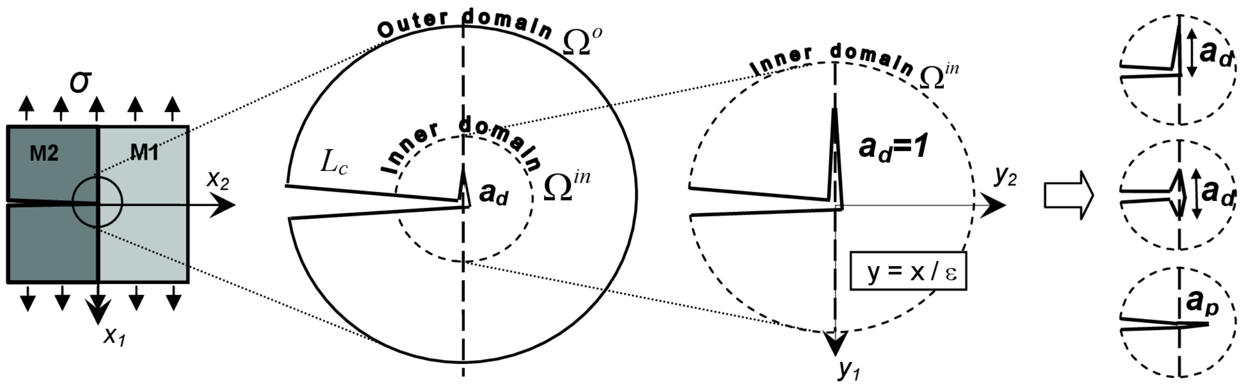


Fig. 7 Outer and Inner domain used in the matched asymptotic analysis (in case of the singly deflected crack) - zoomed-in view of crack neighbourhood perturbed by a small crack extension.

where L_c is the characteristic length of Ω^0. A second scale to the problem can be introduced, represented by the scaled-up coordinates

$$y_i = \frac{x_i}{\varepsilon}, \text{ or } (y_1, y_2) = \left(\frac{x_1}{\varepsilon}, \frac{x_2}{\varepsilon}\right), \tag{31}$$

which provides a zoomed-in view into the region surrounding the crack.

5.2.1 Matched asymptotic procedure

To derive the change of potential energy, consider a perturbation of the domain Ω with crack impinging the interface; the perturbation is a deflected crack extension of length a_d or penetrating crack extension of length a_p with the small perturbation parameter ε. The displacement $\mathbf{U}^\varepsilon$ of the perturbed elasticity problem due to the crack extension can be expressed in terms of the regular coordinate x and the scaled-up coordinate y - (31) as $\mathbf{U}^\varepsilon(x) = \mathbf{U}^\varepsilon(\varepsilon y) = \mathbf{V}^\varepsilon(y)$. Consider now the asymptotic expansion for $\mathbf{U}^\varepsilon$ (which is also known as the ‘‘outer expansion’’) and for $\mathbf{V}^\varepsilon$ (which is also known as the ‘‘inner expansion’’)

$$\mathbf{U}^\varepsilon(x) = f_0(\varepsilon)\mathcal{U}_0(x) + f_1(\varepsilon)\mathcal{U}_1(x) + ... = \sum_{i=0}^{\infty} f_i(\varepsilon)\mathcal{U}_i(x), \text{ outer expansion}, \tag{32}$$

where $\lim_{\varepsilon\to 0} f_{i+1}(\varepsilon)/f_i(\varepsilon) = 0, \ \forall i = 1,2,...$ and $\{\mathcal{U}_1, \mathcal{U}_2, ...\}$ form a set of linearly independent basis functions, and the inner asymptotic expansion is possible to write in the following form – for more details see [50]:

$$\mathbf{V}^\varepsilon(y) = F_0(\varepsilon)\mathcal{V}_0(y) + F_1(\varepsilon)\mathcal{V}_1(y) + ... = \sum_{i=0}^{\infty} F_i(\varepsilon)\mathcal{V}_i(y), \text{ inner expansion}, \tag{33}$$

where $\lim_{\varepsilon\to 0} F_{i+1}(\varepsilon)/F_i(\varepsilon) = 0, \ \forall i = 1,2,...,\ F_0(\varepsilon) = 1,\ \mathcal{V}_0(y) = \mathbf{U}^0(0) = 0$ and $\{\mathcal{V}_0, \mathcal{V}_1, \mathcal{V}_2, ...\}$ form a set of linearly independent basis functions. The basis functions $\{\mathcal{U}_i\}$ satisfy the elasticity problem on the same domain $\Omega \approx \Omega^\varepsilon$ but with zero body force and with homogeneous boundary conditions. From the matching conditions of the outer and inner asymptotic expansion, the asymptotic expansion coefficients $f_0(\varepsilon)$, $f_1(\varepsilon)$,.. and $F_0(\varepsilon)$, $F_1(\varepsilon)$,…can be found:

$$\begin{aligned}
&\mathbf{U}^\varepsilon(x = \varepsilon y) = H\varepsilon^{\delta_1}\rho^{\delta_1}\mathbf{u}_1(\theta) + T\varepsilon\rho\mathbf{u}_3(\theta) + f_1(\varepsilon)\left[K_{1d(p)}\varepsilon^{-\delta_1}\rho^{-\delta_1}\mathbf{u}_{-1}(\theta) + ...\right] + f_2(\varepsilon)\varepsilon\rho\mathbf{u}_3(\theta) + ... = \\
&= \mathbf{V}^\varepsilon(y) = F_1(\varepsilon)\left[\rho^{\delta_1}\mathbf{u}_1(\theta) + K_{1d(p)}\rho^{-\delta_1}\mathbf{u}_{-1}(\theta) + K_{2d(p)}\rho^{-1}\mathbf{u}_{-3}(\theta) + ...\right] + \\
&+ F_2(\varepsilon)\left[\rho\,\mathbf{u}_3(\theta) + K'_{1d(p)}\rho^{-\delta_1}\mathbf{u}_{-1}(\theta) + K'_{2d(p)}\rho^{-1}\mathbf{u}_{-3}(\theta) + ...\right] + \\
&+ F_3(\varepsilon)\left[\rho^{\delta_1}\mathbf{u}_1(\theta) + K_{1d(p)}\rho^{-\delta_1}\mathbf{u}_{-1}(\theta) + K_{2d(p)}\rho^{-1}\mathbf{u}_{-3}(\theta) + ...\right] + ...
\end{aligned} \tag{34}$$

The corresponding terms (with the same power exponent δ) are to be compared and from this comparision follows e.g. that $F_1(\varepsilon) = H\varepsilon^{\delta_1}$, $F_3(\varepsilon) = T\varepsilon$ or $f_1(\varepsilon) = H\varepsilon^{2\delta_1}$.

Finally, the following asymptotic expansion $\mathbf{V}^\varepsilon(y)$ is obtained (by substitution of the obtained coefficients $F_i(\varepsilon)$ and $f_i(\varepsilon)$ into (34)):

$$\mathbf{U}^{\varepsilon}(x=\varepsilon y)=\mathbf{V}^{\varepsilon}(y)=H\varepsilon^{\delta_1}\left[\rho^{\delta_1}\mathbf{u}_1(\theta)+K_{1d(p)}\rho^{-\delta_1}\mathbf{u}_{-1}(\theta)+K_{2d(p)}\rho^{-1}\mathbf{u}_{-3}(\theta)+...\right]+ \\ +T\varepsilon\left[\rho\,\mathbf{u}_3(\theta)+K'_{1d(p)}\rho^{-\delta_1}\mathbf{u}_{-1}(\theta)+K'_{2d(p)}\rho^{-1}\mathbf{u}_{-3}(\theta)+....\right]+... \tag{35}$$

The terms in (35) are ordered with respect to the increasing power of parameter ε.

With an eye on applications we will distinguish between two cases: a) crack perpendicularly impinging an interface, b) inclined crack impinging a interface.

5.2.2 Crack perpendicularly impinging an interface

The asymptotic expansion of the displacements for the initial state $\mathbf{U}^0(x)$ (main crack terminating on the interface and no crack extension of length $a_{d(p)}$ is present) is possible to write in the following form

$$\mathbf{U}^0(x)=\mathbf{U}^0(0)+H_1r^{\delta_1}\mathbf{u}_1(\theta)+H_2r^{\delta_2}\mathbf{u}_2(\theta)+Tr^{\delta_3}\mathbf{u}_3(\theta)+...=\sum_{i=1}^{\infty}k_ir^{\delta_i}\mathbf{u}_i(\theta),\ \ \delta_3=1. \tag{36}$$

The meaning of the individual terms have been already discussed in the sections 2.1.3 and 2.1.4. In the following we will consider $\mathbf{U}^0(0)=0$.

The outer asymptotic expansion of $\mathbf{U}^{\varepsilon}$ (when the small crack extension has originated) is possible to write as

$$\mathbf{U}^{\varepsilon}(x)=\mathbf{U}^0(x)+f_1(\varepsilon)\left[K_{1d(p)}r^{-\delta_1}\mathbf{u}_{-1}(\theta)+...\right]+... \tag{37}$$

Where $\{\mathcal{V}_0,\mathcal{V}_1,\mathcal{V}_2,...\}$ are linearly independent basis functions of the inner expansion (33) as follows

$$\mathcal{V}_0(y)=\mathbf{U}^0(0)=0,\quad \mathcal{V}_1(y)=\rho^{\delta_1}\mathbf{u}_1(\theta)+K_{1d(p)}\rho^{-\delta_1}\mathbf{u}_{-1}(\theta)+K_{2d(p)}\rho^{-1}\mathbf{u}_{-3}(\theta)+....,\ \rho=\frac{r}{\varepsilon}, \\ \mathcal{V}_2(y)=\rho\,\mathbf{u}_3(\theta)+K'_{1d(p)}\rho^{-\delta_1}\mathbf{u}_{-1}(\theta)+K'_{2d(p)}\rho^{-1}\mathbf{u}_{-3}(\theta)+....,\ \rho=\frac{r}{\varepsilon} \tag{38}$$

The first terms on the right hand side of (38) express the asymptotic behaviour of the functions $\mathcal{V}_i$ for $\rho\to\infty$. The coefficients $K_{1d(p)}$ and $K_{2d(p)}$ are computed on the inner domain Ω^{in}, which is unbounded for $\varepsilon\to 0$ but in the model employed in the finite element calculation, Ω^{in} is approximated by a circular region with radius R much larger than the crack extension length $a_{d(p)}$. On the circle boundary, the condition of the type $\mathbf{U}|_{\partial\Omega^{in}}=\rho^{\delta_1}\mathbf{u}_1(\theta)$ is prescribed. $K_{1d(p)}$ and $K_{2d(p)}$ are calculated similarly as H or T-stress as follows:

$$K_{1d(p)}=\frac{\Psi\left(\mathcal{V}_1^h(\rho,\theta),\rho^{\delta_1}\mathbf{u}_1\right)}{\Psi\left(\rho^{-\delta_1}\mathbf{u}_{-1},\rho^{\delta_1}\mathbf{u}_1\right)},\ K_{2d(p)}=\frac{\Psi\left(\mathcal{V}_1^h(\rho,\theta),\rho\mathbf{u}_3\right)}{\Psi\left(\rho^{-1}\mathbf{u}_{-3},\rho\mathbf{u}_3\right)},\ \mathcal{V}_1^h\text{ - FE approx. to }\mathcal{V}_1. \tag{39}$$

The coefficients $K'_{1d(p)}$ and $K'_{2d(p)}$ in (38) are calculated in a similar way with the boundary condition $\mathbf{U}|_{\partial\Omega^{in}}=\rho\mathbf{u}_3(\theta)$ prescribed on the circular region boundary.

A necessary condition for a crack to deflect along the interface is $G_c^i/G_c^1<G_d/G_p$ and vice versa, if the inequality is of the opposite sign, the penetration is preferred before the deflection. Note that the criterion $G_c^i/G_c^1<G_d/G_p$ implicitly assumes the considered finite crack extensions of both, deflected and penetrating crack, of the

same lengths (a_d=a_p). G_c^{i} is the interface toughness, G_c^{1} is the toughness of the next layer, G_d is the energy release rate (ERR) for a crack deflected at the interface and G_p is the ERR for a penetrating crack. The values of toughnesses G_c^{i} and G_c^{1} have to be specified by virtue of experiments.

The incremental ERR $G_{d(p)}$, related to the unperturbed state $\mathbf{U}^0$ (without the crack extension) and perturbed state $\mathbf{U}^\varepsilon$ (with the finite crack extension), is defined as

$$G_{d(p)} = -\frac{\delta W}{a_{d(p)}} = -\frac{W^{\varepsilon} - W^{0}}{\varepsilon_{d(p)} L} = -\frac{1}{2\varepsilon_{d(p)} L}\int_{\Gamma}\left(\sigma_{kl}\left(\mathbf{U}^{\varepsilon}\right) n_k U_l^0 - \sigma_{kl}\left(\mathbf{U}^{0}\right) n_k U_l^{\varepsilon}\right) ds = -\frac{1}{2\varepsilon_{d(p)} L}\Psi\left(\mathbf{U}^{\varepsilon},\mathbf{U}^{0}\right), \quad (40)$$

where δW is the potential energy change, $\varepsilon_{d(p)} = a_{d(p)}/L_c$, H – GSIF (6) and T is a T-stress (7).

Observe, that line Γ is any contour surrounding the crack tip and the crack increment and starting and finishing on the stress-free faces of the primary crack.

The ratio of the debonding to the penetrating ERR follows from (40) as

$$\frac{G_d}{G_p} = \frac{K_{1d}\Psi_1 + \left(K'_{1d}\Psi_1 + K_{2d}\Psi_2\right)\eta_{d(p)}}{K_{1p}\Psi_1 + \left(K'_{1p}\Psi_1 + K_{2p}\Psi_2\right)\eta_{d(p)}}\left(\frac{a_d}{a_p}\right)^{2\delta_1 - 1}, \text{ where } \eta_{d(p)} = \frac{T}{H}\varepsilon_{d(p)}^{1-\delta_1}, \quad (41)$$

$$\Psi_1 = \Psi\left(\rho^{-\delta_1}\mathbf{u}_{-1}(\theta), \rho^{\delta_1}\mathbf{u}_1(\theta)\right), \Psi_2 = \Psi\left(\rho^{-1}\mathbf{u}_{-3}(\theta), \rho\mathbf{u}_3(\theta)\right).$$

To demonstrate the calculations of the ERRs ratio (41) a parametric study displayed in the Fig. 8 was made:

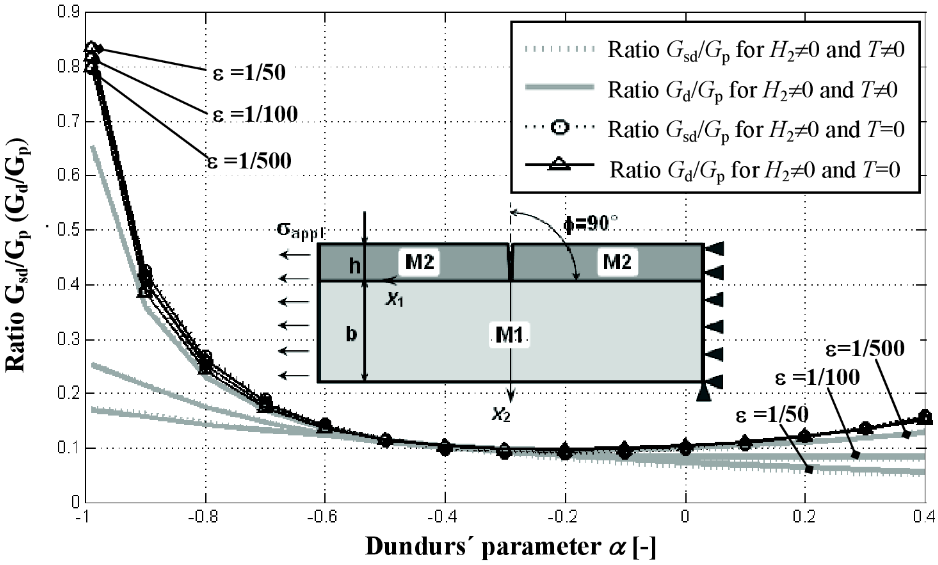


Fig. 8 The ratio of G_{sd}/G_p (G_d/G_p) as a function of Dundurs' parameter α for several values of the characteristic crack extension size ε. (sd-single deflection, d-double deflection, p-penetration).

The ratios of G_{sd}/G_p and G_d/G_p were calculated as a function of the Dundurs′ parameter α and by setting Dundurs' parameter β=0. Material M1 (Fig. 7) of the surface layer was considered as an isotropic material with constant elastic properties E=60GPa, ν=0.238. The elastic properties of the orthotropic material were computed for each value of α using the equations in detail described in the thesis. They relate

Dundurs' parameters α, β, components of the material compliance matrix s_{ij} – see Eq. (21) and parameters λ, ρ.- see Eq. (22) (the parameters λ_1=0.1, ρ_1=2, λ_2=1 and ρ_2=1 were considered). The singularity exponent δ, GSIF H and T-stress T, were calculated using the theory described in sections 2.1, 3.1 and 3.2.

When the T-stress term is considered, one can observe, that with decreasing ε the ratios G_{sd}/G_p (G_d/G_p) approach the limiting case when the T-stress is not considered. In other words, when the crack extension is too small, influence of the T-stress is not measurable.

Observe also (in Fig. 7) the difference between the single and double deflection ERR ratios is very slight. This implies that for the perpendicular crack it is not possible to decide for a certainty whether the single or double deflection will occur. The resulting behaviour will depend also on some other factors like the loading, geometry or bonding imperfections which will initiate one of these modes of deflection.

5.2.3 Inclined crack impinging a interface

Asymptotic expansion for the primary inclined crack before the perturbation inception takes place reads

$$\mathbf{U}^0(x) = \mathbf{U}^0(0) + H_1 r^{\delta_1}\mathbf{u}_1(\theta) + H_2 r^{\delta_2}\mathbf{u}_2(\theta) + \dots, \tag{42}$$

The determination of the coefficients $K_{1d(p)}, K_{2d(p)}, K'_{1d(p)}, K'_{2d(p)}$, proceeds in a similar fashion as the coefficients K in the section 5.2.2, $K_{1d(p)}, K_{2d(p)}$ are calculated in the inner domain whose remote boundary $\partial\Omega^{in}$ is subjected to the boundary condition $\mathbf{U}|_{\partial\Omega^{in}} = \rho^{\delta_1}\mathbf{u}_1(\theta)$ and the coefficients $K'_{1d(p)}, K'_{2d(p)}$ are calculated in the inner domain whose remote boundary $\partial\Omega^{in}$ is subjected to the boundary condition $\mathbf{U}|_{\partial\Omega^{in}} = \rho^{\delta_2}\mathbf{u}_2(\theta)$.

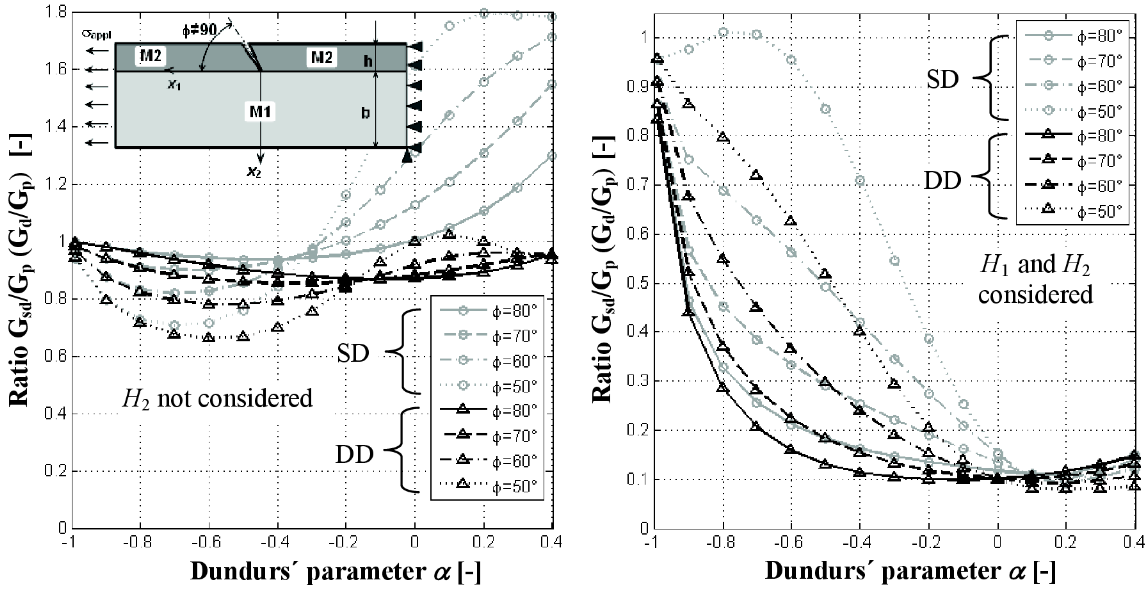
The ratio of the debonding to the penetrating ERR exhibits a similar form as for perpendicular crack:

$$\frac{G_d}{G_p} = \frac{K_{1d}\Psi_1 + (K'_{1d}\Psi_1 + K_{2d}\Psi_2)\eta_d + K'_{2d}\Psi_2\eta_d^2}{K_{1p}\Psi_1 + (K'_{1p}\Psi_1 + K_{2p}\Psi_2)\eta_p + K'_{2p}\Psi_2\eta_p^2}\left(\frac{a_d}{a_p}\right)^{2\delta_1 - 1}, \ \eta_d = \frac{H_2}{H_1}\left(\frac{a_d}{L}\right)^{\delta_2-\delta_1}, \ \eta_p = \frac{H_2}{H_1}\left(\frac{a_p}{L}\right)^{\delta_2-\delta_1} \tag{43}$$

$$\Psi_1 \equiv \Psi\left(\rho^{\delta_1}\mathbf{u}_1(\theta), \rho^{-\delta_1}\mathbf{u}_{-1}(\theta)\right), \quad \Psi_2 \equiv \Psi\left(\rho^{\delta_2}\mathbf{u}_2(\theta), \rho^{-\delta_2}\mathbf{u}_{-2}(\theta)\right),$$

In the same manner (and the same materials) as in Section 5.2.2 and Fig. 8, the appropriate ratios G_{sd}/G_p and G_d/G_p for the inclined crack were calculated - see Fig. 9. In this case only the singular terms (characterized by H_1 and H_2) of the asymptotic expansion were taken into account for calculation of the ratio (43).

By comparison of Fig. 9 a) and b) one can conclude that the influence of the second singular term on the ratios $G_{(s)d}/G_p$ is very significant and it can strongly affect the resulting verdict about the further propagation direction. The second singular term seems to be here more dominant for the fracture criterion than the first singular term. The other general conclusion which can be drawn for the inclined cracks is that the single deflection is preferred before the double deflection.



a) b)

Fig. 9 Ratios G_{sd}/G_p and G_d/G_p as a function of Dundurs' parameters α for several values of the crack inclination angle ϕ (the characteristic crack extension length ε=1/100) a) the case when the second singular term is not considered (H_2=0); b) both H_1 and H_2 singular terms are considered.

6 CONCLUSION

It is possible to conclude, that the main aims of the thesis were achieved. Briefly speaking a complex computational tool for the assessment of the general stress concentrators in anisotropic media (especially cracks terminating at the interface of two dissimilar materials) was created.

The main particular outputs of this work are possible to summarize as follows:

☞ Using the complex potential theory and the Lechnitskii-Stroh formalism a technique for the calculation of the stress singularity exponents and description of the singular (regular) stress field in the vicinity of general stress concentrator in anisotropic media have been developed.

☞ Next to this approach, the Continuously Distributed Dislocation technique has been employed to attain the same objective (singularity analysis, description of the stress and displacement field and the GSIFs calculations).

☞ Using the Betti´s reciprocal theorem and the two state (integration path independent) Ψ-integral, a powerful tool for the calculation of the GSIFs and T-stresses has been developed. It takes advantage of the FE analysis, which was performed within the code ANSYS 10.0, and the post-processing, which has been programmed in the mathematical software MATLAB 7.1, including the integration process for GSIF (T-stress) calculation.

☞ A technique for the involvement of the possible crack bridging effect into the resulting stress field in the vicinity of the bridged crack was developed. The two different bridging models were introduced and compared – a simple Budiansky´s model and advanced statistical bridging model.

- ☞ A suggestion of the suitable fracture criterion for the general stress concentrators has been made. The theory of the finite fracture mechanics in combination with the matched asymptotic expansions technique has been employed here. A relation for the energy release rate of the crack terminating at the interface of two different (anisotropic) materials has been derived. This relation can involve two parameters. Either the leading singular term of the Williams-like asymptotic expansion together with the T-stress (for perpendicular cracks) or the two singular terms (for inclined cracks). The FE analysis and Ψ-integral are used for the mentioned calculations.
- ☞ A direction of the prospective crack extension (crack penetration across the interface or the crack deflection) has been studied (for the crack terminating at the interface of the orthotropic substrate and the isotropic surface layer). If the crack penetrates to the next material, then it follows that penetration direction which maximizes the ERR for the chosen small crack extension.
- ☞ It was shown, for the case of the crack perpendicular to the interface, that in some cases, also a consideration of the T-stress can influences the resulting direction of the propagation – see Fig. 8. Due to this reason, it is recommended to take also the T-stress into the account. On the contrary, for the case of the inclined cracks, it is always recommended to consider both singular terms from the Williams's like asymptotic expansion for the definition of the fracture criteria (especially for cracks inclined more than 10° from the perpendicular state) – see Fig. 9. The T-stress term was not proved to exist for the inclined cracks, at least for investigated configurations. The existence of the T-stress is closely related to the existence of the root $\delta=1$ of the eigenvalue-equation pertaining to a particular singularity problem. This is a necessary condition but it is still not clear whether this condition is also sufficient one. Further investigations are needed.

REFERENCES

[1] Aveston, J., Cooper, G.A. & Kelly, A. (1971) The properties of fibre composites , *In proc. of National Physical Laboratory*, Guilford, IPC Science and Technology Press, Teddington, U.K., pp.15-26.

[2] Aveston, J. & Kelly, A. (1973) Theory of multiple fracture of fibrous composites, *J. Mater. Science*, Vol.8, pp. 352-362.

[3] Barroso, A., Mantič, V. & Paris, F. (2003) Singularity analysis of anisotropic multimaterial corners, *Int. J. of Fracture*, Vol. 119, pp. 1-23.

[4] Broberg, K. B. (2005) A note on T-stress determination using dislocation arrays, *Int. Journal of Fracture*, Vol.131, 2005, pp. 1-14.

[5] Budiansky, B. & Amazigo, J. C. (1989) Toughening by aligned, frictionally constrained fibres, *J.Mech. Phys. Solids*, Vol. 37, pp.93-109.

[6] Budiansky, B., Hutchinson, J. W. & Evans A.G. (1986) Matrix fracture in fiber-reinforced ceramics, *J. Mech. Phys. Solids,* Vol. 34, pp. 167-189.

[7] Desmorat, R. & Leckie, F. A. (1998) Singularities in bi-materials: parametric study of an isotropic/anisotropic joint. *Eur. J. Mech. A/Sol.,*Vol. 17, pp. 33-52.

[8] Eshelby, J. D., Read, W. T. & Shockley, W. (1953) Anisotropic Elasticity with Application to Dislocation Theory, *Acta Metallurg.*, Vol. 1, pp. 251-259.

[9] Fett, T., Munz, D. T. & White, K.W. (2000) Bridging stress determination by evaluation of the R-curve, *J. of Eur. ceram. soc.*, Vol. 20, pp. 2143-2148.

[10] Gross, D. & Seelig, T. (2006) Fracture Mechanics – With Introduction to Micromechanics, *Springer*, Berlin.

[11] Gröger, R. (2003) Characterization of fracture-mechanical behavior of bi-material V-notches using BEM, *Dissertation*, FME BUT Brno, Brno.

[12] Gupta, V., Argon, A. S. & Suo, Z. (1992) Crack deflection at an interface between two orthotropic media, *J. Applied Mechanics*, Vol.59, pp. 79-87.

[13] Hills, D. A. & Kelly, D. N. (1996) Solution of Crack Problems – The distributed dislocation technique, *Kluwer Academic publisher*, Dordrecht.

[14] Chang, J. H. & Wu, D. J. (2003) Calculation of mixed-mode stress intensity factors for a crack normal to a bi-material interface using contour integrals, *Engineering Fracture Mechanics*, Vol.70, pp.1675-1695.

[15] Chen, D. H. (1994) A crack normal to and terminating at a bi-material interface, *Eng. Fr. Mech.*, Vol.49, pp.517-532.

[16] Chen, F. H. K. & Shield, R. T. (1977) Conservation laws in elasticity of the J-integral type, *Z. Angw. Math. Phys. (ZAMP)*, Vol.28, pp.1-22.

[17] Chen, S. H., Wang, T. C. & Sharon, K. (2003) A crack perpendicular to the bimat. interface in finite solid, *Int. J. of Sol. Struct.*, Vol. 40, pp.2731-2755.

[18] Choi, S. T. & Earmme, Y. Y. (2002) Elastic study on singularities interacting with interfaces using alternating technique Part I.: Anisotropic trimaterial, *Int. J. Solid Structures*, Vol.39, pp.943-957.

[19] Im, S. & Kim, K. S. (2000) An application of two-state M-integral for computing the intensity of the singular near-tip field for a generic wedge, *Journal of the Mechanics and Physics of Solids*, Vol.48, pp.129-151.

[20] Kfouri, A. P. (1986) Some evaluations of elastic T-term using Eshelby´s method, *Int. J. of Fracture*, Vol.30, pp.301-315.

[21] Kotoul, M., Vysloužil, T. & Dlouhý, I. (2005) Modelling the R-curve behaviour in chevron-notched specimen using weight functions , *In Advances in Fract. and Damage Mech. IV*, Eastleigh, U.K., pp.217-225, No.1, pp.17-22.

[22] Krejčíř, M. (2002) Výpočet vlastních hodnot asymptotického rozvoje napětí v okolí multimateriálového vrubu, *Diploma work*, FME BUT Brno.

[23] Lacroix, C., Leguillon, D. & Martin, E. (2002) The Influence of an interphase on the deflection of a matrix crack in a ceramic-matrix composite, *Composite science and technology*, Vol. 62, pp. 519-523.

[24] Leguillon, D. (2002)Strength or toughness? A criterion for crack onset at a notch, *Europ. Journal of Mechanics A/Solids*, Vol. 21, pp. 61-72.

[25] Leguillon, D. & Sanchez-Palencia, E. (1987) Computation of Singular Solutions in Elliptic Problems and Elasticity, *Masson*, Paris.

[26] Lekhnitskii, S. G. (1950) Těoria uprugosti anisotrop. těla, *Gostěchizdat*, Mos.

[27] Lekhnitskii, S. G. (1963) Theory of Elasticity of an Anisotropic Body, *Holden-Day*, San Francisco.

[28] Lin, Y. Y. & Sung, J. C. (1997) Singularities of an inclined crack terminating at an anisotropic bimat. interface, *Int. J. Sol. Struct.*, Vol.34, pp.3727-3754.

[29] Marshall, D.B. & Cox, B. N. (1988) A J-integral method for calculating steady-state matrix cracking stresses in composites, *Mech. of mat.*, Vol.7, pp.127-133.

[30] Martin, E., Peters, P.W.M., Leguillon, D. & Quenisset, J.M. (1998) Conditions for matrix crack deflection at an interface in ceramic matrix composites, *Material science and engineering*, Vol. A250, pp.291-302.

[31] Martin, E., Leguillon, D. & Lacroix, C. (2001) A revisited criterion for crack deflection at an interface in a brittle bi-material, *Composites Science and Technology*, Vol. 61, pp.1671-1679.

[32] Munz, D. & Fett, T. (1999) Ceramics – Mechanical Properties, Failure Behaviour, Materials Selection, *Springer*, Berlin.

[33] Náhlík, L. (2002) Šíření únavových trhlin v okolí rozhraní dvou elastických materiálů, Dissertation (in Czech), FME BUT a IPM ASCR Brno.

[34] Papadakis, P. J. & Babuška, I. (1995) A numerical procedure for the determination of certain quantities related to the SIFs in two-dimensional elasticity, *Computer meth. in appl. mech. & eng.*, Vol. 122, pp. 69-92.

[35] Parvizi, A., Garret, K. W. & Bailey, J. E. (1978) Constrained cracking in glass fibre-reinforced epoxy cross-ply laminates, *J. Mat. Sci.*, Vol.23, pp. 195-201.

[36] Profant, T., Ševeček, O. & Kotoul, M. (2006) Singularita napětí polonekonečné trhliny kolmé k rozhraní dvou ortotropních materiálů, *In proc. of Dynamika tuhých a deformovatelných těles*, Ústí n. Labem, pp. 175-182.

[37] Profant, T., Ševeček, O. & Kotoul, M. (2008) Calculation of K-factor and T-stress for cracks in anisotropic bimat., *Eng. Fr. Mech.*, Vol.75, pp. 3707-3726.

[38] Profant, T., Ševeček, O. & Kotoul, M. (2007) The analysis of the stress and displacement field near the surface crack tip terminating perpendicular to the interface between two orthotropic materials, *Materials Science Forum*, Trans Tech Publications, Vol.567, No.1, pp.137-140.

[39] Sarrafi-Nour, G. R., Coyle., T. W. & Fett, T. (1998) A weight function for the crack surface tractions in chevron-notched specimens, *Engineering fracture mechanics*, Vol.59, pp. 439-445.

[40] Shih, C. F. & Asaro, R. J. (1988) Elastic-plastic analysis of cracks on bi-material interfaces: Part I, *ASME J.Appl. Mech.*, Vol. 55, pp. 299-316.

[41] Sills, L.B., Hershkovitz I. & Wavrzynek, P.A. (2005) Methods for calculating SIFs in anisotropic materials: Part I, *Eng. Fr.Mech.*, Vol. 72, pp. 2328 – 2358.

[42] Stern, M. & Soni, M. L. (1976) On the computation of stress intensities at fixed-free corners, *Int. J. Solid Structures*, Vol. 12, pp. 331-337.

[43] Stroh, A. N. (1958) Dislocation and Cracks in Anisotropic Elasticity, *Phil. Mag.*, Vol. 7, pp. 625-646.

[44] Suo, Z. (1990) Singularities, interfaces and cracks in dissimilar anisotropic media, *Proc. of Royal Society of London,* Vol. A 427, pp. 331-358.

[45] Sutradhar, A. & Paulino, G. H. (2004) Symmetric Galerkin boundary element computation of T-stress and SIFs for mixed mode cracks by the interaction integral method, *Engng. Anal. with Boundary Elems.*, Vol. 28, pp. 1335-1350.

[46] Tan, M. A.& Meguid, S. A. (1997) Analysis of bi-material wedges using a new singular finite element, *Int. Journal of Fracture*, Vol.88, pp. 373-391.

[47] Thouless, M.D. & Evans, A.G. (1988) Effects of pull-out on the mechanical properties of ceramic matrix composites, *In Acta Metall,* pp.517-522.

[48] Ting, T. C. T. (1996) Anisotropic Elasticity, *Oxford University Press.*

[49] Ting, T. C. T. & Hoang, P. H. (1984) Singularities at the tip of a crack normal to the interface of an anisotropic layered composite, *International Journal of Solids and Structures*, Vol. 20, pp. 439-454.

[50] Vu-Quoc, L. & Tran, V. X. (2006) Singularity analysis and fracture ERR for composites, *Comp. Meth. Appl. Mech. Eng*, Vol.195, pp.5162-5197.

[51] Wang, A. S. D. & Crossman, F. W. (1980) Initiation and growth of transverse cracks and edge delamination in composite laminates, Part I. An energy method, *J. Comp. Mater. Suppl.*, Vol.14, pp. 71-108.

[52] Wang, T. C. & Stahle, P. (1998) Stress state in front of a crack perpendicular to bi-material interface, *Eng. Fr. Mech.*, Vol.59, pp. 471-485.

[53] Yang, S., & Yuan, F. G. (2000) Determination and representation of the stress coefficient terms by path independent integrals in anisotropic cracked solids, *Int. Journal of Fracture*, Vol.101, pp. 291-319.

AUTHOR´S PUBLICATIONS

[I] Ševeček, O. & Kotoul, M. (2005) Řešení obecných koncentrátorů napětí v anisotropních prostředích, *In proc. of Multilevel design of advanced materials 2005*, FME BUT Brno, pp.215-222.

[II] Ševeček, O., Profant, T. & Kotoul, M. (2006) Výpočet zobecněného součinitele intensity napětí pro trhlinu kolmou na rozhraní dvou anisotropních materiálů, *In proc. of Applied Mechanics 2006*, Srní, pp.93-94.

[III] Ševeček, O., Kotoul, M., Profant, T. (2006) Fracture mechanics problems in anisotropic bimat. media, *In proc. of Únava a Lomová mechanika*, Žinkovy.

[IV] Ševeček, O., Kotoul, M. & Profant, T. (2006) A construction of the fundamental solution for the dislocation in the anisotropic bi-material body by force of the FEM, *In proc. of Eng. Mechanics 2006*, Svratka, pp. 350-351.

[V] Profant, T., Kotoul, M. & Ševeček, O. (2006) Stress field analysis near the semi-infinite crack tip terminating perpendicular to the interface between two orthotropic materials, *In proc. of Eng. Mechanics 2006*, Svratka, pp. 304-305.

[VI] Profant, T., Ševeček, O. & Kotoul, M. (2006) Singularita napětí polonekonečné trhliny kolmé k rozhraní dvou ortotropních materiálů, *In proc. of Dynamika tuhých a deformovatelných těles*, Ústí n.Labem, pp. 175-182.

[VII] Kotoul, M., Ševeček, O. & Profant, T. (2006) Calculation of K-Factor and T-Stress for Crack in Anisotropic bi-materials, *In proc. of 16th Europ. Conf. of Fracture Failure Analysis of Nano and Eng. Mat. & Structures*, ISBN 1-4020-4971-4, Alexandroupolis, Greece.

[VIII] Profant, T., Kotoul, M. & Ševeček, O. (2006) Analysis of edge bridged crack in anisotropic bi-material half-space, *Key Engineering Materials,* ISSN 1013-9826, Vol. 324-325, pp. 1143-1148.

[IX] Ševeček, O., Profant, T. & Kotoul, M. (2006) Problematika přechodu trhlin přes rozhraní dvou ortotropních materiálů, *In Proc. of Computational Mechanics 2006*, Nečtiny, pp. 601-608.

[X] Profant, T., Ševeček, O. & Kotoul, M. (2006) Analýza singularity napětí u trhlin na rozhraní dvou ortotropních materiálů, *In Proc. of Computational Mechanics 2006*, Nečtiny, pp. 483-490.

[XI] Ševeček, O. & Profant, T. (2006) Problematika obecných koncentrátorů na rozhraní dvou anisotropních materiálů, *In proc. of Multilevel Design of Advanced Materials 2006*, IPM ASCR Brno, pp. 155-162.

[XII] Ševeček, O., Vysloužil, T., Profant, T. & Kotoul, M. (2007) A solution of the bridging problem of a crack in the thin orthotropic layer with tip on the interface of an orthotropic substrate, *In proc. of Applied Mechanics 2007*, Malenovice, pp.207-208.

[XIII] Ševeček, O., Profant, T. & Kotoul, M. (2007) Crack propagation criteria for the crack terminating on the interface of a thin orthotropic layer and an orthotropic substrate, *In proc. of Engineering Mechanics 2007*, ISBN 978-80-87012-06-2, Svratka, pp. 350-351.

[XIV] Kotoul, M., Profant, T., Ševeček, O. & Krejčíř, M. (2007) Modelling of cracks crossing an interface between dissimilar elastic anisotropic materials, *Mat. Science Forum*, Trans Tech Publications, ISSN 0255-5476, Vol.567, No.1, pp.17-22.

[XV] Profant, T., Ševeček, O. & Kotoul, M. (2007) The analysis of the stress and displacement field near the surface crack tip terminating perpendicular to the interface between two orthotropic materials, *Materials Science Forum*, Trans Tech Publications, Vol.567, No.1, pp.137-140.

[XVI] Kotoul, M., Profant, T., Ševeček, O. & Krejčíř, M. (2007) Solution methods for general stress concentrators in anisotropic heterogeneous media, *Key Engineering Materials*, Vol. 348-349, pp. 677-680.

[XVII] Profant, T., Ševeček, O. & Kotoul, M. (2008) Calculation of K-factor and T-stress for cracks in anisotropic bi-materials, *Engineering Fracture Mechanics*, Vol. 75, pp. 3707-3726.

[XVIII] Kotoul, M., Profant, T. & Ševeček, O. (2007) Dislocation tri-material solution in the analysis of bridged crack in anisotropic bi-material half-space, *International Journal of Fracture*, Vol. 147, pp. 199-217.

[XIX] Ševeček, O. & Profant, T. (2007) Problems of the crack propagation across the interface of two orthotropic materials, *In proc. of Multilevel Design of Advanced Materials 2007*, IPM ASCR, Ostrava, pp. 53-60.

[XX] Kotoul, M., Profant, T. & Ševeček, O. (2008) Analysis of multiple cracks in thin coating on orthotropic substrate, *In proc of: European Conference on Fracture 17*, Brno, pp. 2159-2166.

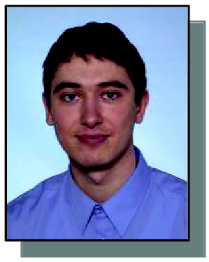
[XXI] Profant, T., Kotoul, M. & Ševeček, O. (2008) Stronger and weaker singularity of an inclined crack terminating at the orthotropic bi-material interface, *In proc. of European Conference on Fracture 17*, Brno, pp. 1150-1157.

[XXII] Ševeček, O., Kotoul, M. & Profant, T. (2008) Crack propagation problems in orthotropic bi-materials, *In proc. of Multilevel Design of Advanced Materials 2008*, IPM ASCR, Velké Bílovice, pp. 7-14.

[XXIII] Kotoul, M., Ševeček, O. & Profant, T. (2009) Analysis of multiple cracks in thin coating on orthotropic substrate under mechanical and residual stresses, *Under review in: Engineering Fracture Mechanics*.

AUTHOR´S CURRICULUM VITAE

Oldřich Ševeček



Date of birth: 30. 01. 1980
Address: Jesenická 67, Šumperk 787 01
E-mail: sevecek@seznam.cz
Nationality: Czech
Marital status: Single

Education:

2004 - 2008 *Ph.D. Study* – Brno University of Technology, Faculty of Mechanical Engineering, Technická 2, 616 69 Brno
Specialization: D2312 – Engineering mechanics
Ph.D. Thesis: Solution of general stress concentrators in anisotropic media by combination of FEM and the complex potential theory

1998 - 2004 *M.Sc. study program* – FME BUT, Technická 2, 616 69 Brno
Specialization: Applied mechanics
Diploma work: Computational and experimental analysis of the first vibration shape at simultaneous and contra-rotating precession of model rotor kit RK4

1994 - 1998 Vyšší odborná škola a Střední průmyslová škola, Gen.Krátkého 1 787 01 Šumperk
Specialization: Management of engineering

Work experience:

2008 – onwards FEM Specialist – Crash simulations, ŠKODA AUTO, a.s., Václava Klementa 869, 293 60 Mladá Boleslav.

2004 – 2007 Teacher of subject Technical Mechanics II on FME BUT
Cooperation on solution of grant projects GAČR: 106/05/H008, 101/05/0320, 106/06/0724, 101/05/P290

2007 Solution of the grant project FRVŠ 2421/2007/G1

Personal skills:

Languages: English: communication level, technical English
Russian, German: basics

Computer skills: Windows, Linux, MS Office, FE-codes ANSYS, ABAQUS, PAMCRASH, mathematical software MATLAB, MAPLE, CAD softwares AutoCAD, Catia, Inventor.

Driving license: Category B

Hobbies: Travelling, photography, sports, IT

Awards:

06 / 2004 Dean´s award for excellent study results and the participation in the science-research activities on FME BUT

ABSTRACT

The presented Ph.D. thesis has originated in the framework of the postgraduate study under tuition of my supervisor Prof. RNDr. Michal Kotoul, DrSc. The thesis focuses to the solution of the problems of general stress concentrators in anisotropic media. Particularly, it is a problem of cracks terminating on the interface of two dissimilar materials or problems of general multi-material wedge. The work is possible to sectionalize into three parts. The first part is dedicated to the search study in the area of interest, the second part to the methods chosen for the achievement of the thesis aims. These aims are as follows: the description of the stress field in the vicinity of the general stress concentrator, the inclusion of the effect of crack bridging into the resulting stress field, and definition of the fracture criteria for the crack impinging at the interface in dissimilar anisotropic media. The last, third, part contains several demonstrative examples – applications of methods on specific bi-material models. For the description of the stress field the so-called Lechnitskii-Stroh formalism and continuously distributed dislocation technique, exploiting the complex potential theory. The first step is the singularity analysis of the general stress concentrator, next the calculation of the generalized stress intensity factor and of the T-stress. The obtained asymptotic expansion for stresses and displacements is subsequently used for the fracture criterion definition, where the theory of Finite Fracture Mechanics and matched asymptotic expansions is used for its derivation. All the needed calculations are performed in the mathematical softwares MAPLE 10.0 and MATLAB 7.1 and in the finite element system ANSYS 10.0. The two-state Ψ-integral is widely applied in this work – especially for the calculation of the generalized stress intensity factor or T-stress, calculation of the bridging effect and for the application of the fracture criteria.