Czech University of Life Sciences Prague

Faculty of Economics and Management

Department of Management (FEM)

Bachelor Thesis

Using the AHP for choosing an apartment for temporary residence in Prague.

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CZECH UNIVERSITY OF LIFE SCIENCES PRAGUE

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Economics Policy and Administration
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Thesis title

Using the AHP for choosing an apartment for temporary residence in Prague.

Objectives of thesis

The main objective is an application of the AHP method for the help to students in the choice of the rental apartment.
Partial goals:
-Conduct theoretical researches based on the multi-criteria analysis,
-Investigate advantages and shortcomings of the AHP method,
-Construct hierarchical structure of the selected task,
-Collect real data via online poll of students,
-Carry out calculations for finding a compromise solution,
-Interpret the results and make practical conclusions.

Methodology

The paper consists of two parts: theoretical and practical. The theoretical part is based on the study and analysis of data taken from literary sources. This part focuses on describing the AHP method as one of the main methods widely used in multi-criteria analysis.

At the beginning of the practical part, a small analysis of the rental housing market in Prague is carried out. The main part of the practical work is devoted to the collection and analysis of the necessary information from real sources, the selection of relevant criteria with the calculation of their weights, their consistency and further multi-criteria analysis using the AHP method.

The proposed extent of the thesis

35-40 pages

Keywords

Analytical Hierarchy Process (AHP), rental market, multiple criteria,, decision making.

Recommended information sources

Bhushan, Navneet; Kanwal Rai (January 2004). Strategic Decision Making: Applying the Analytic Hierarchy Process. London: Springer-Verlag. ISBN 978-1-85233-756-8.

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Declaration

I declare that I have worked on my bachelor thesis titled "Using the AHP for choosing an apartment for temporary residence in Prague." by myself and I have used only the sources mentioned at the end of the thesis. As the author of the bachelor thesis, I declare that the thesis does not break any copyrights.

In Prague on 15.03.2021 ______________________________

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Using the AHP for choosing an apartment for temporary residence in Prague.

Abstract

This work is devoted to the study of the possibility and feasibility of using one of the methods of multi-criteria analysis in choosing the best option for rented housing in Prague.
The theoretical part describes the basic concepts of multi-criterial decision-making analysis and the AHP method, as one of the widely used methods in practice when conducting multi-criteria analysis in various fields.
In practical terms, an analysis of the rental housing market in Prague is carried out. Based on the collected data conducted online by a survey of foreign and non-resident students in need of rented housing, relevant criteria were determined, and the necessary calculations were made. Based on studies and the results obtained, it is concluded that it is possible to use the AHP method when choosing rented housing.

Keywords: Analytical Hierarchy Process (AHP), criteria, rental market.

Využití AHP pro výběr bytu k přechodnému pobytu v Praze.

Abstrakt

Tato práce je věnována studiu možnosti a proveditelnosti využití jedné z metod multikriteriální analýzy při výběru nejlepší možnosti nájemního bydlení v Praze.

Teoretická část popisuje základní pojmy z multi-kriteriální rozhodování analýza a metoda AHP, jako jeden z široce používaných metod v praxi při provádění multikriteriální analýzy v různých oblastech.

Z praktického hlediska se provádí analýza trhu nájemního bydlení v Praze. Na základě shromážděných údajů provedených online průzkumem zahraničních a nerezidentních studentů, kteří potřebují pronajaté bydlení, byla stanovena příslušná kritéria a byly provedeny nezbytné výpočty. Na základě studií a získaných výsledků se dospělo k závěru, že při výběru pronajatého bydlení je možné použít metodu AHP.

Klíčová slova: Analytický hierarchický proces (AHP), kritéria, nájemní trh.

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1 Introduction

Thomas L. Saaty has said that all people make decisions and everything we do is the result of a decision.[1] Not all of the information we have at our disposal is useful for our judgments. If decisions are made only intuitively, a decision maker may think that all information is useful, and the more information is available, the better. However, it is not so. Currently, the decision-making theory is developing steadily, new problems are being investigated and new approaches are emerging.

When choosing solutions, the analysis of their consequences plays a major role. Unfortunately, for the vast majority of human decisions, the consequences cannot be accurately calculated and evaluated. A person can only assume that a certain solution will lead to a certain result. Such an assumption, of course, may turn out to be erroneous, because it is far from always possible to consider all the factors influencing the result of the decision being made. However, being inferior to a computer in terms of speed and accuracy of calculations, a person has a unique ability to quickly assess the situation, highlight the main thing and discard the secondary ones, measure the conflicting assessments, and make up for uncertainty by guessing. At the same time, the number of erroneous decisions is large, and the strength and depth of their negative impact grows along with the power of human society. In this regard, the question arises about the meansthat can help a person in making decisions. One important group of such tools are multi- criteria decision support methods.

The methods of supporting a person in the decision-making process must have a theoretical basis. Decision theory is engaged in the study of various aspects of the decision-making process, both by individuals and by groups of individuals. The development of this theory contributes to the development of methods that can provide real help to people in the decision-making process. At present, such methods are usually implemented in the form of complexes of computer programs, often called decision support systems (DSS). Since the application of decision support systems is quite laborious, these methods and systems are most often used in those tasks where the costs of developing and mastering the DSS pay off.

First of all, it should be emphasized that the theory of decision making, and especially the methods developed on its basis, is an applied discipline, which is not a rigorous mathematical science such as optimization theory but is based largely on knowledge about the people involved in the decision-making process. Therefore, understanding the essence of the decision-making process and human capabilities in this process is the basis for the creation and analysis of methods designed to support decision-makers.

This thesis examines the use of the AHP method developed by T. Saaty to solve the problem of multi-criteria choice.

2 Objectives and Methodology

2.1 Objectives

Primary goal.

To make a choice of an apartment in Prague 6 using AHP method and in the same time determine the possibility and feasibility of using the AHP method to help students tochoose a rent an apartment.

Partial goals

- To study the AHP method.
- To examine the advantages and disadvantages of the AHP method.
- To build a hierarchical structure of the selected task.
- To collect and process initial data including an online survey of University students to determine the relevant criteria and their weights.
- To carry out calculations to find a compromise solution.
- To make a conclusion about the possibility of using the AHP method when choosing an apartment.

2.2 Methodology

The paper consists of two parts: theoretical and practical. The theoretical part is based on the study and analysis of data taken from literary sources. This part focuses on describing the AHP method as one of the main methods widely used in multi-criteria analysis.

At the beginning of the practical part, a small analysis of the rental housing market in Prague is carried out. The main part of the practical work is devoted to the collection and analysis of the necessary information from real sources, the selection of relevant criteria with the calculation of their weights, their consistency and further multi-criteria analysis using the AHP method.

3 Literature Review

3.1 Problems of Multi-criteria Choice

3.1.1 General Problem Statement

Currently, in many branches of science, one must deal with the problems of multi-criteria choice. In general, these problems are posed as follows: let us assume that within the framework of solving a certain problem, a set of its solutions or alternatives U = {u1, u2, …, un} has been revealed. The choice of this or that decision by a decision maker (hereinafter DM) depends on the set of criteria V= {v1, v2, ..., vn} that determines the DM's preference (ratio R) of a particular decision. Usually, it is required either to find the most preferable solution (an object from the set U) according to the set of all criteria V, orto order the solutions (objects) on the set U.[2]

3.1.2 Approaches to Solving

There are many methods for solving multicriteria decision-making problems:

- Method for calculating compromise curves - analytical method.
- Group of methods ELECTRA.
- Methods of random search.
- STEM method (STEpMethod).
- Podinovsky's method.
- Geoffrion/Dyer/Feinberg method.
- Evolutionary methods.
- Methods using visualization of points and curves.
- Zionts/ Wallenius procedure.
- Analytic Hierarchy Process (AHP).

Each of these methods is used for a certain class of problems associated with one or another data dimension, data constraints, problem statement, and other conditions. Among these methods, the best-known is the Analytic Hierarchy Process (AHP) method, which was developed by an American mathematician Thomas Saaty in the early 1970s.

The AHP method is now widely used all over the world for decision-making in a variety of situations - from management at the interstate level to solving sectoral and private problems in business, industry, healthcare, and education. This method belongs to the class of criteria and occupies a special place due to thefact that it does not prescribe any "correct" solution to DM but allows them to interactively find such an option (alternative) that best matches their understanding of the essence of the problem and requirements to its solution. The approach is endowed with such opportunities due to the fact that, along with mathematics, it is based on psychological aspects. The method makes it possible to structure a complex decision- making problem in the form of a hierarchy in an understandable and rational way, to compare and quantify alternative solutions.[3]

3.2 The AHP Method

The analysis of the decision-making problem in the AHP begins with the construction of a hierarchical structure that includes the goal, criteria, alternatives, and other factors being considered which influence the choice. This structure reflects DM's understanding of the problem. Each element of the hierarchy can represent various aspects of the problem being solved, and both material and non-material factors, measurable quantitative parameters and qualitative characteristics, objective data and subjective expert assessments can be considered. In other words, the analysis of a decision-making situation in the AHP resembles the procedures and methods of argumentation that are used on the intuitive level. The next stage of the analysis is to determine the priorities representing the relative importance or preference of the elements of the constructed hierarchical structure using the procedure of pairwise comparisons. Dimensionless priorities allow for meaningful comparison of disparate factors, which is the hallmark of the AHP. At the final stage of the analysis, the synthesis (linear convolution) of priorities on the hierarchy is performed, because of which the priorities ofalternative solutions are calculated relative to the main goal. The best alternative is the one with the highest priority value.

3.2.1 Hierarchical Structure

Very often, when analyzing the structure of interest to us, the number of elements and their interrelationships is so great that it exceeds the ability of the researcher to perceive information in full. In such cases, the system is divided into subsystems, much like a computer circuit, consisting of blocks and their interconnections, and each block has its own circuit.[1]

The first step of the AHP is to build a hierarchical structure that combines the goalof the choice, criteria, alternatives, and other factors that influence the choice of a solution. Building such a structure helps to analyze all aspects of the problem and delve deeper into the essence of the problem.

Hierarchical structure is a graphical representation of a problem in the form of an inverted tree, where each element, except for the topmost one, depends on one or more higher-located elements. Often in different organizations, the distribution of authority, leadership and effective communication between employees is organized in a hierarchical manner.

Hierarchical structures are used to better understand complex reality: we decompose the problem under study into its component parts; then we split the resulting elements into their component parts, etc. At each step, it is important to focus on understanding the current element, temporarily abstracting from all other components. When conducting such an analysis, an understanding of the complexity and versatility of the subject under study comes.

The hierarchical structures used in the AHP are a tool for qualitatively modeling complex problems. The top of the hierarchy is the main goal; elements of the lower level represent a variety of options for achieving the goal (alternatives); elements of intermediate levels correspond to criteria or factors that link the goal to the alternatives.

In general, the hierarchical model can be represented as follows: (Fig. 1) at the topmost level is the global goal (focus of the hierarchy), then the criteria, after that the sub-criteria, and so on to the lowest level - alternatives.

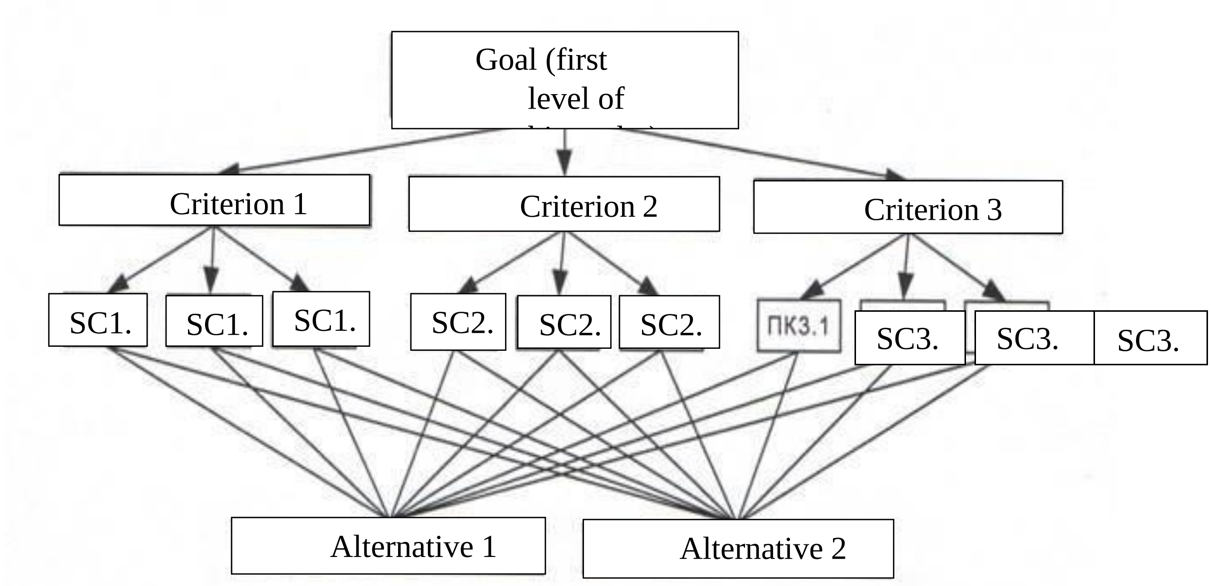

Figure 1. The Saaty's AHP Hierarchical Model

A number of the AHP modifications can be distinguished which are determined bythe nature of the relationship between criteria and alternatives located at the lowest level of the hierarchy, as well as by the method of comparing alternatives [7]. By the nature of the relationship between criteria and alternatives, two types of hierarchies are determined.The first type includes those where each criterion related to alternatives is associated with all alternatives being considered (a type of hierarchy with the same number and functional composition of alternatives under the criteria). The second type of hierarchy includes those in which each criterion that is related to alternatives is not associated with all considered alternatives (a type of hierarchy with a different number and functional composition of alternatives under the criteria).

Advantages of Hierarchies [1]

1. System breakdown can be used to describe how changes in priorities at higher levels affect the priorities of items at lower levels.

2. Hierarchies provide more detailed information about the structure and function of the system at the lower levels and provide consideration of factors and their goals atthe higher levels. To satisfy the constraints on level elements, it is best to reproduce them at the next higher level. For example, nature can be seen as a factor whose purpose is to use a certain material and which obeys certain laws as restrictions.

3. Natural systems constructed hierarchically, that is, by modular construction and then assembly of modules, are built much more efficiently than systems assembled as a whole.

4. Hierarchies are stable and flexible; they are stable in the sense that small changes have little effect, and flexible in the sense that additions to a well-structured hierarchy do not destroy its characteristics.

3.2.2 Prioritization

Once the hierarchy is built, participants in the process use the AHP to prioritize allnodes in the structure. Prioritization information is collected from all participants and mathematically processed.

Priorities are numbers that are associated with the nodes of the hierarchy. They represent relative weights of the elements in each group. Like probabilities, priorities are dimensionless quantities that can range from zero to one. The higher the priority value, the more significant the corresponding element is. The sum of the priorities of elements subordinate to one element above the underlying level of the hierarchy is equal to one. The goal priority is 1.0 by definition.

Let us look at an example that explains the priority calculation technique. A set of matrices of pairwise comparisons is constructed. [4]

In order to do this, elements of two types are distinguished in the hierarchy: “parent” elements and “descendant” elements.

“Descendant” elements have an effect on corresponding elements of the higher level of the hierarchy which are their “parent” elements.

Pairwise comparison matrices are built for all "descendant" elements related to the corresponding "parent" element.

For each "parent" element, a square matrix is constructed with a dimension equal to the number of n elements of the lower level ($A_1, A_2, \dots, A_n$), which is its "descendant" element.

$$A = (a_{ij})\ (i, j = 1, 2, \dots, n)$$

If the elements ($A_1, A_2, \dots, A_n$) can be estimated quantitatively by a parameter (weight, cost, time, etc.), then their paired comparison can be carried out by comparing the quantitative values of this parameter for each element ($\beta_1, \beta_2, \dots, \beta_n$). Then the ratiosof these quantitative values are entered into the corresponding cells of the matrices.

If the values ($\beta_1, \beta_2, \dots, \beta_n$) are not known in advance, then pairwise comparison of elements ($A_1, A_2, \dots, A_n$) is made using subjective judgments, numerically assessed on the scale of relative importance.

Table 1. Scale of Relative Importance

Intensity of relative importance	Definition
1	Equal importance
3	Moderate superiority
5	Substantial superiority
7	Significant superiority
9	Very strong superiority
2,4,6,8	Intermediate judgements

Rules for Comparing Elements and Filling in Matrices

1. If A_i element dominates over A_j element, then the cell at the intersection of A_i row and A_j column is filled in with a numerical value in accordance with the scale of relative importance, and the cell at the intersection of A_j row and A_i column is filled in with the fraction inverse to this value.

If $a_{ij} = \alpha$, then $a_{ji} = 1/\alpha,\ \alpha \neq 0$.

2. If A_j element dominates over A_i element, then the opposite happens - the numeric value of relative importance is written down into the cell at the intersection of A_j row and A_i column, and its reciprocal value (reciprocal fraction) is written into the cell at the

intersection of A_i row and A_j column.

3. If A_i and A_j elements are considered to be the same, then 1 is written in both cells, i.e., A_i has the same relative importance as A_j, then $a_{ij} = 1$, $a_{ji} = 1$; in particular, $a_{ii} = 1$ for all i.

For example:

Table 2. Relative importance a Aj

	A1	A2	A3	A4
A1	1	3	5	5
A2	1/3	1	2	3
A3	1/5	1/2	1	1
A4	1/5	1/3	1	1

When conducting pairwise comparisons of qualitative characteristics, an expert or a decision maker must be able to calculate the absolute significance of each of the compared elements and make a comparison taking into account this significance. Taking this feature into account, it is advisable to simplify the comparison procedure by assigning a significance value for each comparison element at the initial stage. The simplest way is to use the familiar assessment system: excellent, very good, good, bad, very bad. Further, when constructing matrices of pairwise comparison, the decision maker compares the objects using these assessments, based on his subjective judgments. Moreover, if by some criterion two objects have “good”, but one of them is slightly better, and the other is slightly worse, then the first (the best) can be given a slight preference over the other.

Priority Vector Calculation

The next step is to calculate the priority vector for the given matrix. In mathematical terms, this is the calculation of the main eigenvector, which, after normalization, becomes the priority vector. In the absence of a computer that can accurately solve this problem, rough estimates of this vector can be obtained in the following four ways: [1]

Method 1

Sum up the elements of each row and normalize by dividing each sum by the sum of all the elements; the sum of the results obtained will be equal to one. The first element of the resulting vector will be the priority of the first object, the second – the second object, and so on.

Method 2

Sum up the items in each column and get the reciprocal of these sums. Normalize them so that their sum equals one, dividing each reciprocal by the sum of all reciprocal values.

Method 3

Divide the elements of each column by the sum of the elements of that column (that is, normalize the column), then add the elements of each resulting row and divide that sum by the number of elements in the row. It is a normalized column averaging process.

Method 4

Multiply *n* elements of each row and extract the *n*th root.

$$R_i = \sqrt[n]{\prod_{j=1}^{n} C_{ij}} \qquad (3.1)$$

Normalize the resulting numbers.

$$w_i = \frac{R_i}{\sum_i^n R_i} \qquad (3.2)$$

T. Saaty has noted that by comparing the results obtained using all four methods, the accuracy increases from 1 to 2 and further to 3, but at the same time the calculations become more complicated. If the matrix is consistent, then in all four cases the priority vectors will be the same. In case of inconsistency, a very good approximation can only be obtained using method 4.

The computation of priority vectors is carried out in the direction from the lower levels to the upper ones, taking into account the specific connections between elements belonging to different levels.

The calculation is carried out by multiplying the corresponding vectors and matrices.

This is an example of alternatives priority vectors calculation for elements of the secondlevel in a multilevel hierarchy (Fig. 2):

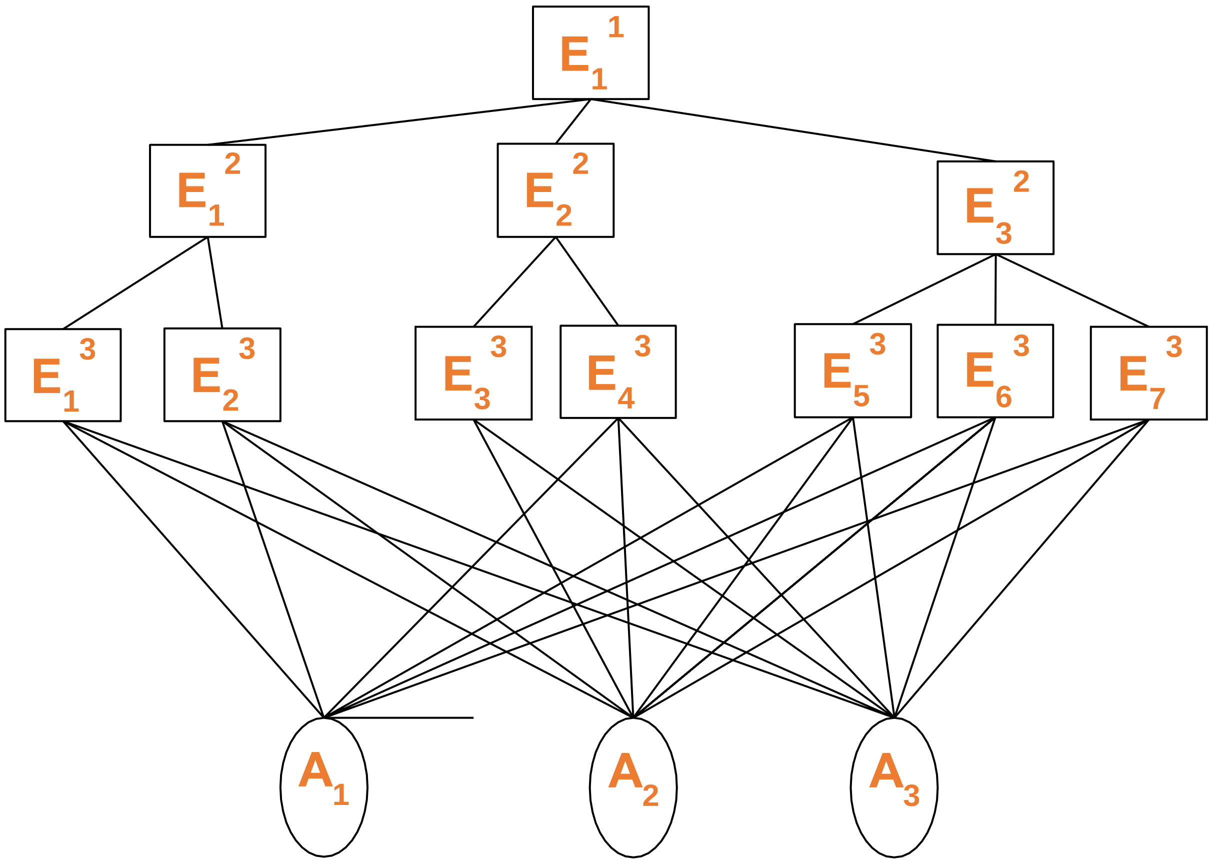

Figure 2. Multilevel hierarchy

It is calculated as follows:

$$W_{E_1^2}^{A} = [W_{E_1^3}^{A}, W_{E_2^3}^{A}]W_{E_1^2}^{E}$$

$$W_{E_2^2}^{A} = [W_{E_3^3}^{A}, W_{E_4^3}^{A}]W_{E_2^2}^{E}$$

$$W_{E_3^2}^{A} = [W_{E_5^3}^{A}, W_{E_6^3}^{A}, W_{E_7^3}^{A}]W_{E_3^2}^{E}$$

The resulting vector of priorities of alternatives relative to the root vertex of E_1^1 hierarchy is calculated as follows:

$$W_{E_1^1}^{A} = [W_{E_1^2}^{A}, W_{E_2^2}^{A}, W_{E_3^2}^{A}]W_{E_1^1}^{E}$$

3.3 Consistency of Judgment

3.3.1 Assessing Matrix Consistency

The AHP procedure has a built-in criterion for the quality of an expert's work - the consistency index (CI), which provides information on the degree of violation of the numerical (cardinal) and transitive (ordinal) consistency of expert judgments. Cardinality check consists in monitoring certain numerical characteristics, deviation from which indicates the presence of errors in the formalization of expert judgments. In other words, if some rules for encoding expert judgments are adopted, for example, from zero to one, then expert judgments should not go beyond the set of values established by these rules, that is, be negative or greater than one. Transitivity allows you to check the logic of the expert's thinking. If the expert believes that factor A exceeds factor B, and factor B, in turn, exceeds factor C, then in a pairwise comparison factor C should not exceed factor A,that is, the inequality A> B> C must be satisfied. Lack of consistency can be a serious limiting factor for the study of some problems. [5]

If a matrix is analyzed with the results obtained using precise physical measurements (for example, height, mass, etc.), then the values of the matrix elements are transitive: if a certain object A_1 is preferable to object A_2 by a factor of *k*, and object A_2 is preferable to object A_3 by a factor of *m*, then object A_1 is preferable to object A_3 by a factor of $k * m$.

In practical tasks, quantitative (cardinal) and transitive (ordinal) consistency is violated, since human sensations cannot be expressed by an exact formula. In real life, it is difficult to achieve such an examination accuracy, therefore, it is necessary to introduce a parameter that determines how much the consistency indices differ for an arbitrary matrix filled in by an expert.

It is known that the consistency of a positive inverse symmetric matrix is equivalent to the requirement that its maximum eigenvalue λmax be equal to n. Note that the inequality λmax ≥ n is always true. The closer λmax is to n (the number of objects or types of actions in the matrix), the more consistent is the result. λmax is calculated by the formula:

$$\lambda_{max} = \sum_{i=1}^{n} \lambda_i, \qquad \lambda_i = \sum_{i=1}^{n} C_{ij} W_{ij} \qquad (3.3)$$

One can also estimate the deviation from consistency by the difference λmax -n, divided by (n −1), this value will be called the consistency index (CI).

$$I_s = \frac{\lambda_{max} - n}{n-1} \qquad (3.4)$$

The consistency index of the inversely symmetric matrix generated randomly on ascale from 1 to 9 with the corresponding inverse values of the elements is called the random index (RI).

Below are the matrix order (the first line) and the average RI (the second line):

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0,00	0,00	0,58	0,90	1,12	1,24	1,32	1,41	1,45	1,49	1,51	1,48	1,56	1,57	1,59

How bad the consistency for a particular problem is can be estimated by comparing the value of the quantity (CI) we obtained with its value from randomly selected judgments and the corresponding inverse values of a matrix of the same size (SI).

The ratio of the CI to the average SI for a matrix of the same order is called the consistency relation (CR). The CR value, which is less than or equal to 0.10 is considered acceptable.

3.3.2 Assessing Hierarchy Consistency

After solving the problem of hierarchical synthesis, the homogeneity of the entire hierarchy is estimated by summing up consistency indices of all levels reduced by "weighting" to the first hierarchical level where the root vertex is located. The number of steps in the algorithm for calculating consistency is determined by a specific hierarchy.

At two or more levels of partitioning into clusters, in addition to the consistency of each matrix of pairwise comparisons, it is advisable to check the hierarchy consistency relation (HCR) by the following formula:

$$M = CI_{E_1^1} + W_{E_1^2}^{E} \cdot CI_{E_1^2} + W_{E_2^2}^{E} \cdot CI_{E_2^2} + W_{E_3^2}^{E} \cdot CI_{E_3^2} \quad (3.5)$$

For random inversely symmetric matrices, the same calculated value is:

$$\tilde{M} = RC_{E_1^1} + W_{E_1^2}^{E} \cdot RC_{E_1^2} + W_{E_2^2}^{E} \cdot RC_{E_2^2} + W_{E_3^2}^{E} \cdot RC_{E_3^2} \quad (3.6)$$

where RC is a vector whose elements are equal to the random indices of matrices of the corresponding dimension.

The consistency ratio of the hierarchy is

$$M / \sim M. \quad (3.7)$$

If the resulting value does not exceed 0.10, the hierarchy is considered consistent.

3.4 Hierarchical Synthesis

Create a matrix where the columns are the priority vectors of each of the alternatives for aspecific criterion. If there are N alternatives and M criteria in the hierarchy, the matrix has N rows and M columns.

In order to obtain the alternatives assessment, it is necessary to multiply the resulting matrix by the criteria priorities vector, i.e., N*M dimension matrix is multiplied by M dimension vector. As a result, N dimension vector is obtained whose elements values correspond to the alternatives preference from the point of view of the set goal attainability.

The highest value alternative should be selected from the vector obtained.

3.5 Method Application Procedure

Summing up the results of the AHP method examination, we can determine the followingprocedure for its application:

- State a purpose, select criteria and alternatives.
- Build a qualitative model of a problem in the form of a hierarchy, including a goal, alternative options for achieving the goal and criteria for assessing the quality of alternatives.
- Determine priorities of all elements of the hierarchy using the method of pairwise comparisons.
- Check judgments for consistency.
- Correct the judgments.
- Synthesize global alternatives priorities by linear convolution of the hierarchy elementspriorities.
- Make a decision based on the results obtained.

3.6 Advantages and Disadvantages of the AHP Method

3.6.1 Advantages

At first glance, it seems that AHP is an ideal tool for solving a wide range of multifactorial problems where expert methods are used as key methods. This is largely true, and we shall point out the main reasons for this:

Pairwise Comparisons.

The main advantage of the method is the fact that criteria weights and assessments by subjective criteria are not assigned by a direct volitional method but are established on the basis of pairwise comparisons.

Comparison of objects in pairs is inherent in human nature [6]. The absence of the need toconstantly keep in sight all the factors, or at least a group of homogeneous factors, allows an expert to focus on a specific problem: by how much factor A exceeds factor B or is inferior to it. As a consequence, more accurate results should be expected.

Hierarchical Structure

Another advantage is the presentation of a problem in the form of a hierarchy (tree). A hierarchical structure simplifies the situation allowing you to decompose a complex decision-making problem.

Complementarity of the Original Matrix.

In the practice of systems research, situations often arise when the number of influencing factors changes. This occurs both due to the cyclical nature of natural processes and due to the changes in socio-economic conditions. Then you must add, reduce or replace some factors with others. When using the AHP, this only leads to the need to compare the newly formed pairs or to delete the rows and columns of the matrix of paired comparisons corresponding to the factors removed from consideration, i.e., to the formation of a minor matrix. The results obtained from previous surveys are saved, and a complete update of the questionnaire, as it happens in other cases, is not required. Taking into account thatthe AHP procedure, in essence, is reduced to finding the eigenvector of the corresponding matrix belonging to the maximum eigenvalue, from the “technical” point of view, the inclusion of additional factors is an increase in the dimension of the corresponding linear space by adding direct summands.

Verbal-numerical Scale.

Conventional numerical scales are not always convenient for comparing factors expressedin different dimensions and concepts. It is especially difficult to compare factors whose indicators, on the one hand, are quantitative values, and on the other, qualitative ones. Verbal-numerical scales, one of whose variants is the Saaty scale, are designed to assess such inconsistencies of influencing factors indices.

Built-in Criterion for the Expert's Work Quality.

According to survey results, experts are usually subject to verification. Various numericalindices developed for both group and individual surveys are usually used for such verifications. In this case, the question of the optimal criterion is open, and its choice is arbitrary. In this sense, the presence in the AHP of such a characteristic (parameter) as the consistency relation is very convenient, especially when creating an automated software and hardware complex.

3.6.2 Disadvantages of the Method

However, not all advantages of the AHP are so obvious. A number of questions arise in the interpretation of the results and they are connected, first of all, with the criterion of the expert's work quality, i.e., with the consistency relation.

Using Transitivity for Qualitative Indices.

The transitivity relation works well when all characteristics of the system under study can be represented by numerical values. But as soon as this becomes impossible, the requirement of the presence of transitivity often conflicts with the logic of the researcher.

"Reverse" Logic.

The criteria for the expert's work quality, for the most part, and the consistency relation aswell, are based on deviations from a certain statistical characteristic, for example, the mathematical expectation. Like all criteria of statistical nature, the consistency relation is formal and, in some cases, leads to results that are difficult to interpret.

When analyzing the results of experts' surveys that fall within the limits of a given quality level (OS <10-20%), there have almost always been several cases where the weight coefficients sharply differ from the majority, or even have directly opposite nature: those factors which the majority of experts considered the most significant, these experts assessed as less significant and - vice versa. When averaging the results of all experts that meet a given criterion, which is usually done to obtain generalized estimates, this leads to a shift in the average values of the weight coefficients.

3.7 Practical Application of the Method

The AHP can be used successfully to solve simple problems, but its effectiveness is manifested in the search for solutions to complex problems that require a systematic approach and the involvement of a large number of experts. The following are problems where the AHP can be applied to find a solution:

Multi-criteria choice problems. Selection of one alternative from the available set of alternatives based on some criteria.

Ranging. Multicriteria ordering of a given set of alternatives.

Determination of alternatives and criteria priorities in multi-criteria choice problems.

Resource allocation. Distribution of resources between alternatives from a given set.

Comparative analysis. Development of recommendations for optimizing internal processes of an organization based on the successful experience of competitors.

Quality control. Analysis of various aspects of quality and ways to improvequality.

There are many examples of successful application of the AHP to solve complex problems, in particular the following:

Development of a strategy aimed at reducing the negative impact of global climate change (Fondazione Eni Enrico Mattei).

Calculation of the aggregate quality index of software systems (Microsoft Corporation).

Choice of specialization when studying at the University (Bloomsburg University of Pennsylvania).

Deciding on the location of offshore enterprises (University of Cambridge).

Assessment of the risks associated with the functioning of oil pipelines running in the country (American Society of Civil Engineers).

Development of a strategy for the most effective management of the US watersheds (U.S. Department of Agriculture).

Sphere of education and research.

Although there is no need for special training for practical application of the AHP,the fundamentals of the method are taught in many educational institutions. In addition, this method is widely used in quality management and is read in many specialized programs such as Six Sigma, Lean Six Sigma, and QFD. About one hundred Chinese universities offer courses on the fundamentals of the AHP, and many applicants for scientific degrees choose the AHP as an object of scientific and thesis research. Morethan 900 scientific articles have been published on this topic. There is a Chinese scientificjournal specializing in the AHP. The International Symposium on Analytic Hierarchy Process (ISAHP), which brings together both scientists and practitioners working with theAHP, is held every two years.

4 Practical Part

This part of the thesis is devoted to the practical application of theoretical materials in the field of multiple criteria analysis, in particular the Analytic Hierarchy Process (AHP) method.

To write the practical part of the paper, the materials of the site for searching rental housing in Prague (www.bezrealitky.cz), as well as data from the results of an online survey of university students living in Prague, were used. [7]

All calculations were performed using the AHP method. The procedure for applying the method is described in the theoretical part.

4.1 Introduction

Prague is one of the most beautiful and romantic cities on the planet. It is the capital and main economic, political and cultural center of the Czech Republic. Its population is over 1.3 million people. Prague is one of the largest tourist centers in Europe. The historic city center is in the UNESCO World Heritage List.

In addition, the city is a recognized center of higher education in the Czech Republic and Eastern Europe. The Czech Republic attracts applicants from all over the world, who want to get a quality education in European universities. Czech universities are known all over the world for their prestige, quality of education and free education in Czech.

In Prague, there are 8 state-funded higher education institutions and 21 higher education institutions owned by private companies or foundations.

The total number of students in universities and higher schools is more than 130 thousand people.

Against the background of a large influx of tourists and students from all over the world, one of the urgent problems for many students is the search and selection of rental housing.

Renting a home in Prague depends on many factors: the number of rooms, the presence of a separate kitchen, the size of the area, the height of the floor, the availability of furniture and equipment, the neighborhood, utility bills. It also plays a role how many people are going to live in an apartment, because in Prague the cost of utilities is quite tangible, and the amount of consumed resources and generated waste is calculated for each.

It can be quite difficult to find housing that would meet the stated requirements (the location of housing in a certain neighborhood, the number of rooms, the availability of furniture and household appliances, etc.), while counting on a certain cost.

The variety of possible options and criteria makes the selection task even more difficult. The desire to find the best option is quite natural for any person. Multi-criteria analysis methods can help students choose the best rental housing option. We will consider using the Analytic Hierarchy Process (AHP) method for this.

4.2 Practical application of the AHP method

In the theoretical part (section 3.2.5), the procedure for applying the method was determined. We will adhere to this sequence.

4.2.1 Statement of goal, selection of criteria and alternatives

The goal is to choose the best option for a rented apartment for a long-term stay in Prague.

Criteria

When choosing rental housing, different requirements are taken into account. The nature and variety of these requirements depends on the preferences of the decision maker. Many factors influence the choice of an apartment, here are the main ones:

1. Total costs of renting an apartment

The cost of renting an apartment is an important factor in choosing the best option.The total cost, besides the monthly rent set by the owner, includes utility bills (poplatky). This amount depends on the size of the apartment, the type of heating, thermal insulation,the number of people living, the presence or absence of an elevator, and many other factors. It is also necessary to pay a deposit in case of damage or severe wear and tear of property (kauce), set by the owner of the apartment.

2. Location of the apartment

Tenants are guided by different criteria when choosing a home. But, as a rule, location and comfortable living in the chosen neighborhood are of great importance for them. Of particular importance here is: the proximity of public transport stops, service sector objects (trade, food, sports, beauty, etc.), and the location of the apartment on a quiet street not loaded with transport.

3. Characteristics of the apartment

When choosing a home, special attention is paid to its characteristics. The area of the apartment, the floor on which the apartment is located, the presence of household appliances and furniture must be taken into account.

In our case, to assess alternatives, the following criteria are determined, with the help of which the choice of the best option will be made:

1. Rental cost (CZK/ month)
2. The cost of utility bills (CZK/ month)
3. The amount of the deposit (CZK)
4. The proximity of public transport stops
5. The proximity of service sector objects
6. Location of the apartment in a quiet neighborhood
7. Apartment area (m^2)
8. The floor on which the apartment is located
9. The presence of household appliances
10. The presence of furniture

Criteria 1, 2, 3, 7 have quantitative characteristics, criteria 4, 5, 6, 8, 9, 10 have qualitative characteristics. Criteria 1, 2, 3 are minimized, criterion 7 is maximized.

In order to assess the quality criteria, let us establish the following scale:excellent, very good, good, bad, very bad.

Alternatives

When choosing apartment options, we used the website www.bezrealitky.cz for their assessment, 5 options were selected with the help of this site. Apartments were chosen in Prague 6 area, because the University is located there. The layout of the apartments was set as 1 + 1 (one-room apartment with a separate kitchen); it is the best option for students.

Apartment options with their characteristics according to the given criteria are summarized in Table 3 The qualitative characteristics were assessed on a scale: excellent, very good, good, bad, very bad.

We have divided all the criteria into three subgroups according to their common characteristics. The first group C1 - according to the apartment’s price characteristics, the second group C2 - according to the characteristics of the apartment’s location, the third group C3 - according to the characteristics of the apartment’s conditions. For the convenience in calculations, all the criteria have been assigned the following designations:

C_1^1 - Rental cost

C_1^2 - The cost of utility bills

C_1^3 - The amount of the deposit

C_2^1 - The proximity of public transport stops

C_2^2 - The proximity of service sector objects

C_2^3 - Location of the apartment in a quiet neighborhood

C_3^1 - Apartment area

C_3^2 - The floor on which the apartment is located

C_3^3 - The presence of household appliances

C_3^4 - The presence of furniture

Table 3. Summary table of alternatives. (Source: own calculation/source)

		A_1	**A_2**	**A_3**	**A_4**	**A_5**
C_1	**C_1^1**	15000	13000	9000	13875	8300
	C_1^2	2500	3000	600	4625	2500
	C_1^3	15000	16000	9000	15000	11000
C_2	**C_2^1**	good	bad	very bad	very good	bad
	C_2^2	very bad	good	very good	excellent	bad
	C_2^3	bad	very good	excellent	good	bad
C_3	**C_3^1**	47	56	30	28	31
	C_3^2	good	good	bad	very good	good
	C_3^3	very good	very good	bad	excellent	very bad
	C_3^4	very good	very good	bad	excellent	very bad

4.2.2 Building a hierarchical structure of the selected task

When building a hierarchical structure, we divided the criteria into subgroups and designated all elements of the structure with alphabetic symbols to simplify the further execution of calculations. (Fig. 3)

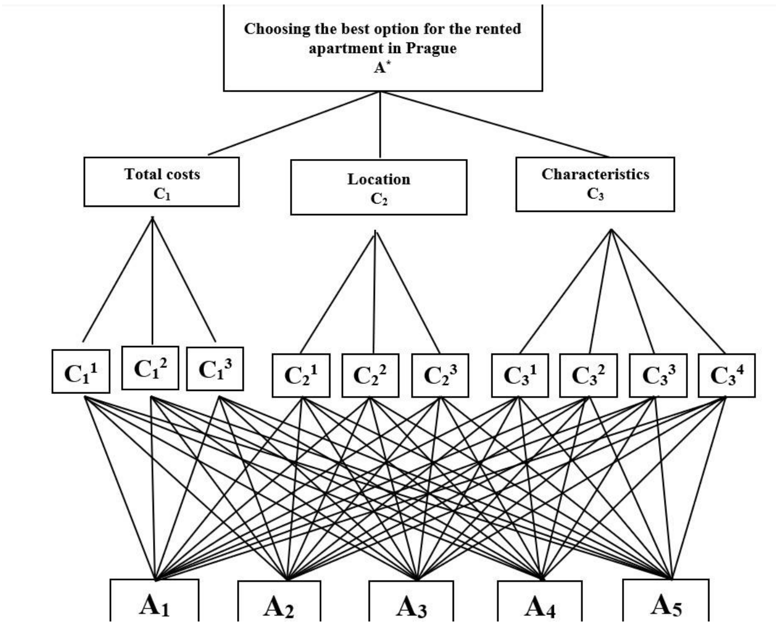

Figure 3. Hierarchical structure of the selected task

4.2.3 Analysis of the Results Obtained

We can see that A3 alternative is better than A4 alternative with a slight advantage (0.260 and 0.246 points respectively). Although it yields significantly according to C3 criterion - the apartment's characteristics and has the same results according to C2 criterion - the apartment's location, it is superior according to C1 criterion - the total costs, which plays the decisive role.

A5 alternative has the best result according to C1 criterion - the total costs. Nevertheless, it is on the third place due to its low results according to the other criteria.

A1 and A2 alternatives have good results according to C2 criterion - the apartment's location and C3 criterion - the apartment's characteristics. However, they could not compete with the others due to their poor results according to C1 criterion, i.e., very high total rental costs for an apartment.

Conclusion: When applying the AHP method in choosing the best alternative, the final result is influenced by:

a) High values of the weighting factors.

c) Qualitative or quantitative characteristics according to the selected criteria, especially those with the highest weight coefficients.

In our calculations, we have taken the averaged values of 25 respondents, where the total rental cost of an apartment has been determined as a priority criterion. However, each person has their own individual preferences. Someone might think that the price is not as important as the location of the apartment, and someone may believe that the location is less important than the availability of furniture andhousehold appliances in the apartment. Therefore, when making their choice, eachperson must first of all decide on the selection criteria and their importance.

When choosing a large number of criteria and possible alternatives, there is a complexity in the calculations related primarily to their length. So, in our example of applying the method when choosing the best option for a rental apartment with 10 criteria and 5 alternatives distributed in a multi-level hierarchy, 340 different calculations have been made and summarized in 18 tables. This process has taken a total of about 5 hours. If we use computer decision support systems, then the time will certainly decrease by several times, but the question whether it is possible to reduce the time spent on the selection process to a minimum and do it without lengthy calculations remains. Let us try to figure it out using our example from one person's (a university student choosing rental housing) perspective. He acts as both an expert and a decision-maker. First of all, we have to define the criteria and alternatives and build a hierarchical structure (see Fig. 3). The structure shows that there are a lot of criteria, which inevitably leads to lengthy calculations. Let us try to reduce their number.

a) Let's define the unimportant criteria.

Thus, C13 criterion (collateral amount) can be excluded, since the funds invested in the form of collateral will ultimately be returned. This criterion has a low weighting factor.

C23 criterion (location of the apartment in a quiet place) is also not particularly important for consideration and can be excluded.

C31 criterion (apartment area) is usually interconnected with the price indicator, the larger the area, the higher the price, therefore it is already indirectly involved in the selection process and, therefore, it can be excluded.

C32 criterion (which floor is the apartment on) is not particularly important, since in most cases either the floor is low or the building has an elevator. We can exclude it.

c) Combine the remaining criteria of the same nature.

C11 criterion (rental cost) and C12 criterion (cost of utility bills) can be combinedinto C1 criterion (total costs).

C21 criterion (proximity of public transport stops) and C22 criterion (proximity of service facilities) can be combined into C2 criterion (location).

C33 criterion (availability of household appliances) and C34 criterion (availabilityof furniture) can be combined into C3 criterion (characteristics of an apartment).

By performing these steps, we have simplified the hierarchical structure.

Table 4. Summary table of alternatives. (Source: own calculation/source)

	A_1	**A_2**	**A_3**	**A_4**	**A_5**
C_1	17500	16000	9600	18500	10700
C_2	bad	very good	excelle nt	very good	bad
C_3	very good	very good	bad	excellent	very bad

Let us arrange the weights and reduce the considered alternatives.

Option 1.

Let us assume that the decision maker has defined his priorities as follows:C1 (Total costs) - 0.45

C2 (Location) - 0.25

C3 (Apartment characteristics) - 0.30

In this case, it can be seen that A1, A2, A4 alternatives have higher cost results, so they are unable to compete with the other alternatives due to a high weight coefficient of this criterion. A5 alternative looks very weak in terms of C2 and C3 criteria, therefore, it should also be excluded from consideration. Thus, the remaining A3 alternative is the best one. No calculations required.

Option 2.

Let us say that the decision maker has defined his priorities as follows:C1 (Total Costs) - 0.30

C2 (Location) - 0.25

C3 (Apartment characteristics) - 0.45

A3 and A5 alternatives have the worst results according to C3 criterion, which has the highest weight coefficient, and therefore they are excluded from consideration. A1 alternative has approximately the same results as A2 and A4 alternatives according to C1 criterion, but much worse results according to C2 criterion. We should also exclude it from consideration. Of the remaining A2 and A4 alternatives, the best can be calculated using the AHP method.

Option 3.

Let us assume that the decision maker has defined his priorities as follows:C1 (Total Costs) - 0.30

C2 (Location) - 0.45

C3 (Apartment characteristics) - 0.25

A1 and A5 alternatives have the worst results according to C2 criterion, which has the highest weight coefficient, and therefore are excluded from consideration. The remaining A1, A2, A3 alternatives have relatively equal results according to the criteria, therefore, in order to make a choice, it is necessary to carry out calculations using the AHP method.

This analysis has shown that in all of the cases the number of required mathematical actions has significantly decreased as well as the time spent making the calculations.

Conclusion: The application of the AHP method becomes faster and simpler if the number of criteria and alternatives has previously been minimized by the decision maker by excluding the unimportant criteria and the worst alternatives from the hierarchical structure.

4.3 Prioritizing all the hierarchy elements. Checking judgments for consistency

At this stage, the decision maker performs pairwise comparisons of the elements of each level. Comparison results are converted to numbers. The coefficients of significance are calculated for the elements of each level. At the same time, the consistency of the decision maker's judgments is checked.

4.3.1 Prioritizing criteria levels and checking matrix and hierarchical structure.

To construct a criteria matrix in this paper, an online survey was conducted among students of the University.

Judgments obtained through questionnaires.

We offer a simple illustration of how judgments can be obtained using a questionnaire. In the left column, we list all the criteria that need to be comparedin terms of superiority with other criteria from the right column. In total, each column contains [n (n - 1) / 2] alternatives. Then experts should note the judgmentthat expresses the superiority of the element from the left column over the corresponding element from the right column located in the same row. If such superiority actually takes place, then one of the positions to the left of the equality will be marked. Otherwise, equality or some position to the right will be marked. We do the same for other criteria [1]. This whole procedure is needed in order to determine the superiority of the selected criteria and, ultimately, to determine the criteria weights using the Saaty matrix (Section 3.2.2 of the Theoretical Part)

The following form was used for the survey:

Questionnaire of preferences by criteria when choosing a rental apartment

You need to compare the specified criteria with each other and prioritize the significance of these criteria for you in comparison with each other. Only one box with the value of the degree of significance for you of one or another criterion is selected and marked in each row.

Table 5. Nine-point scale used in comparison. (Source: own calculation/source)

Significance degree	Characteristic	Explanation
1	Equal significance	Both compared criteria are of equal importance to you when choosing a rental apartment.
3	Some predominance of the significance of one criterion over another one (weak significance)	You give a slight preference to one criterion over another one.
5	Substantial or strong significance	You give a strong preference to one criterion over another one.
7	Very strong or obvious	The preference for one criterion

	significance	over another one is very strong. Its superiority is clear.
9	Absolute significance	Evidence in favor of the preference for one criterion over another one in the highest degree of preference.
2, 4, 6, 8	Intermediate values between adjacent scale values	The situation when a compromise solution is needed.

Table 6. Main criteria comparison. (Source: own calculation/source)

Subcriterion	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	Subcriterion
Total costs of renting an apartment																		Location of the apartment (proximity to infrastructure facilities)
Total costs of renting an apartment																		Characteristics of the apartment (area, furniture and household appliances, floor, wall material)
Location of the apartment (proximity to infrastructure facilities)																		Characteristics of the apartment (area, furniture and household appliances, floor, wall material)

Table 7. Criterion sub-criteria comparison table - total cost of renting apartment. (Source: own calculation/source)

Subcriterion	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	Subcriterion
Rental cost																		Utility bills cost
Rental cost																		Deposit amount

Utility bills cost																		Deposit amount

Table 8. Criterion sub-criteria comparison table - location of the apartment. (Source: own calculation/source)

Subcriterion	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	Subcriterion
The proximity of public transport stops																		The proximity of service sector objects (trade, food, sports, health, beauty, etc.)
The proximity of public transport stops																		The location of the apartment on a quiet street not loaded with transport
The proximity of service sector objects (trade, food, sports, health, beauty, etc.)																		The location of the apartment on a quiet street not loaded with transport

Table 9. criterion sub-criteria comparison table - characteristics of the apartment (Source: own calculation/source)

Subcriterion	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	Subcriterion
Total area of the apartment																		The floor on which the apartment is located
Total area of the apartment																		The presence of furniture in the apartment
Total area of the apartment																		The presence of household appliances in the apartment
The floor on which the																		The presence of furniture in the apartment

apartment is located																		
The floor on which the apartment is located																		The presence of household appliances in the apartment
The presence of furniture in the apartment																		The presence of household appliances in the apartment

The survey results are reflected in each table as dots. Each dot denotes a preference given by one of the students in his judgments.

As a result, according to the survey data, the significance of the established criteria was determined and matrices of the corresponding levels of the hierarchy were compiled.

Calculation of the weight coefficients of the criteria levels

Let's calculate the weight coefficients using the formulae (3.1, 3.2).

Multiply *n* elements of each row and extract the *n*th root.

$$R_i = \sqrt[n]{\prod_{j=1}^{n} C_{ij}} \quad (3.1)$$

Normalize the resulting numbers.

$$w_i = \frac{R_i}{\sum_i^n R_i} \quad (3.2)$$

All data are summarized in Tables 4.2, 4.3, 4.4, 4.5.

Table 10. Calculations of criteria weights. (Source: own calculation/source)

	C_1	C_2	C_3	W_i
C_1	1	2	2	0,493
C_2	1/2	1	1/2	0,196
C_3	1/2	2	1	0,311
∑				1,0

Let's determine the matrix consistency.

We find λmax by the formula (3.3) λmax = 3,004, then we find the consistency index by the formula (3.4)

Is = (3,054-3)/(3-1)=0,027

The random index for the third-order matrix is 0,58, we find the consistency relation:

CR = 0,027/0,58 = 0,046 ≤ 0,10, therefore, the matrix is consistent.Let's calculate the subcriteria weights

Table 11. Calculation of C1 criterion sub-criteria weights. (Source: own calculation/source)

C_1	**C_1^1**	**C_1^2**	**C_1^3**	**W_i**
C_1^1	1	3	4	0,625
C_1^2	1/3	1	2	0,239
C_1^3	1/4	1/2	1	0,136
∑				1,0

Let's determine the matrix consistency.

λmax = 3,018, Is = (3,018-3)/(3-1)=0,009 CR = 0,009/0,58 = 0,016 ≤ 0,10, therefore, the matrix is consistent.

Table 12. Calculation of C2 criterion sub-criteria weights. (Source: own calculation/source)

C_2	**C_2^1**	**C_2^2**	**C_2^3**	**W_i**
C_2^1	1	2	7	0,592
C_2^2	1/2	1	5	0,333
C_2^3	1/7	1/5	1	0,075
∑				1,0

Let's determine the matrix consistency.

λmax = 3,014, Is = (3,014-3)/(3-1)=0,007 CR = 0,007/0,58 = 0,012 ≤ 0,10, therefore, the matrix is consistent.

Table 13. Calculation of C3 criterion sub-criteria weights. (Source: own calculation/source)

C_3	C_3^1	C_3^2	C_3^3	C_3^4	W_i
C_3^1	1	1/3	1/4	1/5	0,071
C_3^2	3	1	1/2	1/3	0,164
C_3^3	4	2	1	1/3	0,251
C_3^4	5	3	3	1	0,514
$\sum$					1,0

Let's determine the fourth-order matrix consistency.

λmax = 4,111, Is = (4,111-4)/(4-1)=0,037 CR = 0,037/0,9 = 0,041 $\leq$ 0,10, therefore, the matrix is consistent.

4.3.2 Prioritizing alternatives level and checking matrix consistency

Based on the data (table), we determine the priorities of alternatives in relation tothe criteria, compile matrices and calculate the weight coefficients.

Table 14. Matrix of pairwise comparisons of suppliers according to C11 criterion "Rental cost". (Source: own calculation/source)

C_1^1	A_1	A_2	A_3	A_4	A_5	W_i
A_1	1	1/4	1/8	1/3	1/9	0,033
A_2	4	1	1/5	2	1/6	0,097
A_3	8	5	1	6	1/2	0,329
A_4	3	1/2	1/6	1	1/7	0,065
A_5	9	6	2	7	1	0,476
$\sum$						1,0

Let's determine the fifth-order matrix consistency.

λmax = 5,194, Is = (5,194-5)/(5-1)=0,048 CR = 0,048/1,12 = 0,043 $\leq$ 0,10, therefore, the matrix is consistent.

Table 15. Matrix of pairwise comparisons of suppliers according to C12 criterion "Utility bills cost". (Source: own calculation/source)

C_1^2	A_1	A_2	A_3	A_4	A_5	W_i
A_1	1	2	1/7	5	1	0,128
A_2	1/2	1	1/8	3	1/2	0,074
A_3	7	8	1	9	7	0,635
A_4	1/5	1/3	1/9	1	1/5	0,035
A_5	1	2	1/7	5	1	0,128
$\sum$						1,0

Let's determine the fifth-order matrix consistency.

$\lambda max = 5{,}224$, Is = (5,224-5)/(5-1)=0,056 CR = 0,056/1,12 = 0,05 $\leq$ 0,10, therefore, the matrix is consistent.

Table 16. Matrix of pairwise comparisons of suppliers according to C13 criterion "Deposit amount". (Source: own calculation/source)

C_1^3	A_1	A_2	A_3	A_4	A_5	W_i
A_1	1	2	1/6	1	1/4	0,082
A_2	1/2	1	1/7	1/2	1/5	0,051
A_3	6	7	1	6	3	0,520
A_4	1	2	1/6	1	1/4	0,082
A_5	4	5	1/3	4	1	0,265
$\sum$						1,0

Let's determine the fifth-order matrix consistency.

$\lambda max = 5{,}104$, Is = (5,104-5)/(5-1)=0,026 CR = 0,026/1,12 = 0,023 $\leq$ 0,10, therefore, the matrix is consistent.

Table 17. Matrix of pairwise comparisons of suppliers according to C21 criterion "The proximity of public transport stops". (Source: own calculation/source)

C_2^1	A_1	A_2	A_3	A_4	A_5	W_i
A_1	1	4	7	1/2	3	0,284
A_2	1/4	1	4	1/6	1/2	0,084
A_3	1/7	1/4	1	1/8	1/5	0,035
A_4	2	6	8	1	5	0,471
A_5	1/3	2	5	1/5	1	0,126
$\sum$						1,0

Let's determine the fifth-order matrix consistency.

λmax = 5,198, Is = (5,198-5)/(5-1)=0,05 CR = 0,05/1,12 = 0,044 $\leq$ 0,10, therefore, the matrix is consistent.

Table 18. Matrix of pairwise comparisons of suppliers according to C22 criterion "The proximity of service sector objects". (Source: own calculation source)

C_2^2	A_1	A_2	A_3	A_4	A_5	W_i
A_1	1	1/4	1/8	1/9	1/2	0,035
A_2	4	1	1/3	1/5	3	0,119
A_3	8	3	1	1/3	7	0,279
A_4	9	5	3	1	8	0,516
A_5	2	1/3	1/7	1/8	1	0,051
$\sum$						1,0

Let's determine the fifth-order matrix consistency.

λmax = 5,175, Is = (5,175-5)/(5-1)=0,044 CR = 0,044/1,12 = 0,039 $\leq$ 0,10, therefore, the matrix is consistent.

Table 19. Matrix of pairwise comparisons of suppliers according to C23 criterion "The location of the apartment in a quiet neighborhood". (Source: own calculation/source)

C_2^3	A_1	A_2	A_3	A_4	A_5	W_i
A_1	1	1/5	1/7	1/2	1	0,057
A_2	5	1	1/3	4	5	0,275
A_3	7	3	1	5	7	0,514
A_4	2	1/4	1/5	1	2	0,097
A_5	1	1/5	1/7	1/2	1	0,057
$\sum$						1,0

Let's determine the fifth-order matrix consistency.

λmax = 5,107, Is = (5,107-5)/(5-1)=0,027 CR = 0,062/1,12 = 0,024 $\leq$ 0,10, therefore, the matrix is consistent.

Table 20. Matrix of pairwise comparisons of suppliers according to C31 criterion "Apartment area". (Source: own calculation/source)

C_3^1	A_1	A_2	A_3	A_4	A_5	W_i
A_1	1	1/3	4	5	6	0,279
A_2	3	1	5	8	7	0,519
A_3	1/4	1/6	1	2	1	0,082
A_4	1/6	1/8	1/2	1	1/2	0,046
A_5	1/5	1/7	1	2	1	0,074
$\sum$						1,0

Let's determine the fifth-order matrix consistency.

λmax =5,117, Is = (5,117-5)/(5-1)=0,029 CR = 0,029/1,12 = 0,026 $\leq$ 0,10, therefore, the matrix is consistent.

Table 21. Matrix of pairwise comparisons of suppliers according to C32 criterion "The floor on which the apartment is located". (Source: own calculation/source)

C_3^2	A_1	A_2	A_3	A_4	A_5	W_i
A_1	1	1/3	5	1/3	1	0,134
A_2	3	1	6	1/2	3	0,296
A_3	1/5	1/6	1	1/7	1/5	0,038
A_4	3	2	7	1	3	0,398
A_5	1	1/3	5	1/3	1	0,134
$\sum$						1,0

Let's determine the fifth-order matrix consistency.

$\lambda max = 5,166$, Is = (5,166-5)/(5-1)=0,041 CR = 0,041/1,12 = 0,037 $\leq$ 0,10, therefore, the matrix is consistent.

Table 22. Matrix of comparisons of suppliers according to C33 criterion "The presence of household appliances". (Source: own calculation/source)

C_3^3	A_1	A_2	A_3	A_4	A_5	W_i
A_1	1	1	5	1/2	7	0,243
A_2	1	1	5	1/2	7	0,243
A_3	1/5	1/5	1	1/7	3	0,064
A_4	2	2	7	1	9	0,417
A_5	1/7	1/7	1/3	1/9	1	0,033
$\sum$						1,0

Let's determine the fifth-order matrix consistency.

$\lambda max = 5,103$, Is = (5,103-5)/(5-1)=0,026 CR = 0,026/1,12 = 0,023 $\leq$ 0,10, therefore, the matrix is consistent.

Table 23. Matrix of pairwise comparisons of suppliers according to C34 criterion "The presence of furniture". (Source: own calculation/source)

C_3^4	A_1	A_2	A_3	A_4	A_5	W_i
A_1	1	2	4	1/2	5	0,265
A_2	1/2	1	3	1/3	4	0,168
A_3	1/4	1/3	1	1/6	2	0,071
A_4	2	3	6	1	8	0,450
A_5	1/5	1/4	1/2	1/8	1	0,046
$\sum$						1,0

Let's determine the fifth-order matrix consistency.

$\lambda max = 5{,}059$, Is = (5,059-5)/(5-1)=0,015 CR = 0,015/1,12 = 0,013 $\leq$ 0,10, therefore, the matrix is consistent.

4.4 Hierarchy consistency

Let's determine the hierarchy consistency by the formula (3.5, 3.6, 3.7)

Table 24. Hierarchy consistency calculation. (Source: own calculation/source)

	$W(C_i)$	$W(C_i^j)$	$W(C_i)$* $W(C_i^j)$	Is	W*Is	RI	W*RI
C_1	**0,493**			0,009	**0,0044**	0,58	**0,2859**
C_2	**0,196**			0,007	**0,0014**	0,58	**0,1137**
C_3	**0,311**			0,037	**0,0115**	0,9	**0,2799**
C_1^1		0,625	**0,308**	0,048	**0,0148**	1,12	**0,3450**
C_1^2		0,239	**0,118**	0,056	**0,0066**	1,12	**0,1322**
C_1^3		0,136	**0,067**	0,026	**0,0017**	1,12	**0,0750**
C_2^1		0,592	**0,116**	0,050	**0,0058**	1,12	**0,1299**
C_2^2		0,333	**0,065**	0,044	**0,0029**	1,12	**0,0728**
C_2^3		0,075	**0,015**	0,027	**0,0004**	1,12	**0,0168**
C_3^1		0,071	**0,022**	0,029	**0,0006**	1,12	**0,0246**
C_3^2		0,164	**0,051**	0,041	**0,0021**	1,12	**0,0571**
C_3^3		0,251	**0,078**	0,026	**0,0020**	1,12	**0,0874**
C_3^4		0,514	**0,160**	0,015	**0,0024**	1,12	**0,1792**

M=0,027+0,0044+0,0014+0,0115+0,0148+0,0066+0,0017+0,0058+0,0029+0,0004+ +0,0006+0,0021+0,002+0,0024= 0,0836

~M=0,58+0,2859+0,1137+0,2799+0,345+0,1322+0,0750+0,1299+0,0728+0,0168+ +0,0246+0,0571+0,0874+0,1792= 2,3795

M/~M = 0,0836/2,3795= 0,035 < 0,10 ⇒ hierarchy is consistent.

4.4.1 Correcting judgements

Revision of judgment is not required, since all pairwise comparison matrices and the hierarchical structure are consistent, CR <0.10

4.5 Synthesis of global priorities of alternatives by linear convolution of priorities of hierarchy elements

4.5.1 Calculation of weights of all the criteria in relation to the goal

It is necessary to reduce the weight of the criterion of each level to the first level of the hierarchy, i.e. the weight of the criterion in the overall structure (hierarchy).

For this, it is necessary to multiply the weight of the criterion in the group by the weight of the group itself. The total sum of the weights for all the criteria must be equal toone.

Table 25. Table of the weight coefficients of criteria in relation to the goal. (Source: own calculation/source)

Group	W group	Criterion	W criterion	Final W
C_1	0,493	C_1^1	0,625	**0,308**
		C_1^2	0,239	**0,118**
		C_1^3	0,136	**0,067**
C_2	0,196	C_2^1	0,592	**0,116**
		C_2^2	0,333	**0,065**
		C_2^3	0,075	**0,015**
C_3	0,311	C_3^1	0,071	**0,022**
		C_3^2	0,164	**0,051**
		C_3^3	0,251	**0,078**
		C_3^4	0,514	**0,160**
∑				**1,0**

4.6 Synthesis as an assessment of alternatives in relation to the goal

The local priorities of the alternatives are multiplied by the priorities of the corresponding level criteria and are summed up for each element according to the criteria. As a result, the global priorities of the alternatives are determined, taking into account the priorities of the criteria. The highest rating will correspond to the alternative with the highest global priority.

Table 26. Calculation of global priorities of alternatives. (Source: own calculation/source)

	Priority vectors										**A_g**
	C_1^1	**C_1^2**	**C_1^3**	**C_2^1**	**C_2^2**	**C_2^3**	**C_3^1**	**C_3^2**	**C_3^3**	**C_3^4**	
	0,308	**0,118**	**0,067**	**0,116**	**0,065**	**0,015**	**0,022**	**0,051**	**0,078**	**0,160**	
A_1	0,033	0,128	0,082	0,284	0,035	0,057	0,279	0,134	0,243	0,265	**0,140**
A_2	0,097	0,074	0,051	0,084	0,119	0,275	0,519	0,296	0,243	0,168	**0,136**
A_3	0,329	0,635	0,520	0,035	0,279	0,514	0,082	0,038	0,064	0,071	**0,260**
A_4	0,065	0,035	0,082	0,471	0,516	0,097	0,046	0,398	0,417	0,450	**0,246**
A_5	0,476	0,128	0,265	0,126	0,051	0,057	0,074	0,134	0,033	0,046	**0,218**
Σ											1,0

4.7 Making a decision based on the results obtained

Table 27. Order of alternatives. (Source: own calculation/source)

	Tradeoff	**rank**
A_1	**0,140**	**4**
A_2	**0,136**	**5**
A_3	**0,260**	**1**
A_4	**0,246**	**2**
A_5	**0,218**	**3**

According to the analysis, the most preferred Alternative is Alternative A_3.

5. Conclusion

Based on the above example, we can conclude that the use of the Analytic Hierarchy Process (AHP) method allows us to include in the hierarchy all the main detailson the problem being analyzed, to use a fairly simple mathematical apparatus to analyze the problem, to carry out this analysis taking into account complex, diverse structural and functional connections.

The considered method allows both one student and a group of students different in their judgments to interact on a problem of interest to them, in particular, on the problem of choosing a rental apartment for living in Prague, to modify their judgments, tointegrate individual judgments regarding the presented alternatives and, ultimately, to seek optimal solutions to the problem under consideration. To speed up mathematical calculations in the selection process, various computer decision support systems (DSS) developed to apply the AHP method can be used. In addition, prior to making the calculations, you can use the option to simplify the hierarchical structure by visually identifying unimportant criteria and worst-case alternatives.

6 References

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