Czech University of Life Sciences Prague

Faculty of Economics and Management

Department of Economics and Management

Bachelor Thesis

Travelling salesman problem in the selected business case

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CZECH UNIVERSITY OF LIFE SCIENCES PRAGUE

Faculty of Economics and Management

BACHELOR THESIS ASSIGNMENT

Zhan Zhunussov

Informatics

Thesis title

Travelling salesman problem in the selected business case

Objectives of thesis

The main objective of the bachelor thesis is to improve a routing process in the chosen company by using selected methods for travelling salesman problem. The route scheduling of selected company will be analyzed and more profitable route scheduling will be proposed.

Methodology

Bachelor thesis consists of theoretical and practical parts. The theoretical part is about reviewing literature related to the travelling salesman problem and finding out proper methods of the solution. The practical part is about collecting the real data and applying selected methods. After the analysis of obtained results, an appropriate conclusion will be made.

The proposed extent of the thesis

30-40 pages

Keywords

Travelling Salesman Problem, Operations research, Approximation methods

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Recommended information sources

APPLEGATE, D L. *The traveling salesman problem : a computational study.* Princeton: Princeton University Press, 2006. ISBN 9780691129938.

GUTIN, G. – PUNNEN, A P. *The traveling salesman problem and its variations.* New York: Springer, 2007. ISBN 0387444599.

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Declaration

I declare that I have worked on my bachelor thesis titled "Travelling salesman problem in the selected business case" by myself and I have used only the sources mentioned at the end of the thesis. As the author of the bachelor thesis, I declare that the thesis does not break any copyrights.

In Prague on 15.03.2022 ____________________________

Acknowledgement

I would like to express my gratitude to Ing. Igor Krejčí, Ph.D. for his guidance, wise and valuable advices, professionalism, desire to help and dedicated time. Furthermore, I want to thank my family for giving me an opportunity to study. Besides, I also appreciate the company's cooperation in providing me with the necessary data.

Travelling salesman problem in the selected business case.

Abstract

The topic of this Bachelor Thesis is closely related to the branch of operations research, an analytical method of decision-making used in applied mathematics. More accurately, this Bachelor Thesis is about approximation and optimization methods of Travelling Salesman Problem. I selected a distribution company for analysing delivery routes in order to decrease the length of these routes and, consequently lowering the enterprise's expenses. The Bachelor Thesis has both theoretical and practical parts.

The theoretical part has a detailed description of logistics, its: meaning in business, history, aims and goals. Besides it, I represent operations research in the theoretical part. Moreover, I reveal the usage of operations research in logistics in this part of the Bachelor Thesis. For instance, I show different approaches for solving the Travelling Salesman Problem in logistics.

The practical Part of this Bachelor Thesis is represented by the direct application of concepts that I gather in the Theoretical Part of the Bachelor Thesis. More precisely, I applied different approaches of route construction in the selected company. Thanks to the participation of the company that provided real data. Using actual data, I constructed the square matrices. These matrices are a basis for alternative routes construction. The arrangement of the alternative solution was reached by the TSPKOSA software that has four methods: Nearest Neighbour Algorithm (sequential), Vogel Approximation Method/Loss Method, Savings Method (parallel), Branch and Bound. I analysed and compared numerical and graphical outcomes with initial routes. Based on the mathematical and statistical evidence, I selected the most appropriate approach and made the following conclusion. Eventually, based on the findings, I made relevant recommendations to the selected firm.

Keywords: Travelling Salesman Problem, Operations research, Approximation methods, Optimization methods, Logistics, Supply chain.

Problém obchodního cestujícího ve vybrané společnosti.

Abstrakt

Téma této bakalářské práce úzce souvisí s oborem operačního výzkumu, což je analytická metoda rozhodování, která je využívána v aplikované matematice. Přesněji řečeno, tato bakalářská práce se zabývá aproximačními a optimalizačními metodami řešení problému obchodního cestujícího. Cílem práce je analýza přepravních tras dopravní společnosti za účelem jejich zkrácení a s tím spojeného snížení nákladů podniku. Práce se dělí na teoretickou a praktickou část.

Teoretická část se podrobně zabývá logistikou, jejím významem v podnikání, historií a cíli. Dalším tématem teoretické části je operační výzkum. Je zde zejména popsáno využití operačního výzkumu v logistice. Na jednotlivých příkladech jsou ukázány různé přístupy k řešení problému obchodního cestujícího v logistice.

Praktickou část této bakalářské práce představuje přímá aplikace poznatků, které byly získány v části teoretické. Přesněji řečeno, jsou zde implementovány odlišné přístupy k plánování tras ve zvolené společností. Na základě reálných dat byly zkonstruovány čtvercové matice sazeb, které jsou základem pro identifikaci alternativních tras. Uspořádání alternativního řešení bylo dosaženo softwarem TSPKOSA, který disponuje čtyřmi metodami: Algoritmus nejbližšího souseda (sekvenční), Vogelova aproximační metoda, metoda výhodnostních čísel (paralelní) a metoda větví a mezí (Branch and bound). Byly analyzovány a porovnávány číselné a grafické výstupy s počátečními cestami. Následně byl zvolen nejvhodnější přístup a definována možná úspora. Na závěr bylo na základě zjištěných skutečností učiněno relevantní doporučení vybrané firmě.

Klíčová slova: Problém obchodního cestujícího, Operační výzkum, Aproximační metody, Optimalizační metody, Logistika, Dodavatelský řetězec.

Table of content

List of figures

List of tables

List of graphs

List of abbreviations

- SCOR - Supply Chain Operations Reference
- OR - Operations Research
- NP - nondeterministic polynomial-time
- NNA – Nearest Neighbour algorithm
- TSP – Travelling Salesman Problem

1 Introduction

From ancient times, when only simple creatures were surrounding huge planet Earth, the main goal and purpose of each living creature were to survive. Since that time, the main goal of living entities has not changed much. They are still focusing on prolonging the period of survival and making existence in this world more comfortable (Dawkins, 2017, p. 30). Applying it to competition between companies on the market nowadays: each company aims to survive, and in order to do that firms are trying to increase profit as much as possible. As a matter of fact, profit is calculated by the difference between expenses and revenues. If the enterprise's revenues are higher than expenses so it has a profit. On the other hand, if the enterprise's revenues are lower than expenses so it has loss accordingly (Zimmerman, 2011, p. 113).

In this Bachelor Thesis, I chose a company that sells products. According to the selected firm's activity, competitive companies tend to decrease the price of their distribution with the intention to sell more products. Price reduction causes an increase in the quantity of demand regarding the demand curve. As soon as equilibrium is reached, decreasing price does not lead to an increase in profit (Mankiw, 2017, p. 77). On the other hand, companies have an opportunity to boost their profitability by means of decreasing their expenses. Unfortunately, a lot of enterprises do not focus on improving in areas where positive changes may occur. Expenses are a giant branch of the firm's outcome (Zimmerman, 2011, p. 113). However, I review and analyse expenses only in the logistic area as this Bachelor Thesis focuses on logistics. For instance, increasing of profit may be caused by efficient construction of distribution routes. The main goal of this Bachelor Thesis is the reduction of delivery routes' length and consequently, augmenting the company's income. I approach key goals by means of operations research, to be more accurate, by approximation and optimization methods of the Travelling Salesman Problem (TSP).

The main reason for choosing this theme is an inspiration if applying rational decision-making in business today by searching for improvements in the area of logistics.

2 Objectives and Methodology

2.1 Objectives

The main objective of this Bachelor Thesis is an improvement of delivery route establishment in the selected company. This primary goal of increasing the efficiency of path scheduling is achieved by using means of approximation and optimization methods of transportation problem. Breaking down the main task into three smaller ones: I analyse each day of route scheduling in the selected business case, and I find the most efficient or in other words the shortest alternative. Furthermore, I make appropriate recommendations according to the research's results if they are needed.

2.2 Methodology

As was mentioned above, this particular Bachelor Thesis has **theoretical** and **practical parts**:

1. **The theoretical part** consists of a review of specific books related to the logistics itself: its history, meaning in business, objectives and approaches. Additionally, there is an overview of literature which is directly regarding operations research and particularly to TSP and approaches for decision making. In fact, the section describing methods used for decision making is closely related to the practical part. Approximation and optimization methods of solving the TSP are characterized and described in this segment of the theoretical part. These approaches are used in the practical part of this Bachelor Thesis.
2. Meanwhile **practical part** of this Bachelor Thesis has a precise and at the same time anonymous description of the selected company, the company's features, and ways of delivery route construction. Moreover, chosen enterprise provided real data which I numerically and graphically represent in this part of the Bachelor Thesis. More accurately, a graphical representation is shown by pictures of maps with routes which was created by means of Mapy.cz. While a numerical representation of the length of paths between addresses is expressed by square matrices that were obtained using Yandex maps. Moreover, alternative route construction was approached by TSPKOSA software.

3 Literature Review

3.1 What is logistics?

Logistics itself has several essential components: transport, inventory, warehousing. These components are fundamental elements of economics. However, logistics has become a big branch of economics comparably not long time ago. To be more precise, it has become very meaningful only in the last 20-25 years (Rushton et al., 2017, p.3). But what is logistics? There are many various definitions. One example of definition was provided by the American Council for Supply Chain Management Professionals (CSCMP):
“Logistics refers to the transportation and storage of materials, parts and products in a supply chain” (Zijm et al., 2019, p. 33).
Another instance of characterizing logistics that is widely respected:
“Logistics = Materials Management + Distribution” (Rushton et al., 2017, p. 3).
From this statement, we can understand that logistics deals with material regulations and its delivery. In other words, constructing the final product on the factory and shipping it to the final customer can be viewed as an instance of logistics. However, logistics can be extended in an even more precisive definition of Supply Chain:
“Supply Chain = Suppliers + Logistics + Customers” (Rushton et al., 2017, p. 3).

3.1.1 Supply chain

As a matter of fact, Supply Chain has seven key aspects: availability of materials, products and information; cost-efficiency; customer orientation; speed; effectiveness; environmental sustainability; social aspects (Zijm et al., 2019, p. 33). Additionally, logistics and supply chain can be interpreted not only in the case of managing and delivering tangible goods but also with respect to information storage and its flow. The focus of this Bachelor Thesis is taken on a cost-efficiency, which means that the given task of distribution has to be accomplished with minimal costs.

3.1.2 History of term "logistic" and logistics itself

The term „logistics" has its root in the Latin language. Logistics derives from the compound of the words „logic" and „statistics". Consequently, logistics is „statistical logic" (Saridogan, 2017, p. 182).

Most of the innovations are used and implemented in the weapons segment of human development throughout the history of mankind. Thus, logistics was firstly used in military slang from a historical perspective. Later, people were gradually implementing logistics in ordinary life.

The first mention of the importance of logistics and transportation dates back to 1776, to the economics book written by the philosopher Adam Smith. In his book called "Wealth of Nations" the author describes how long-distance transport enables market extensions (Zijm et al., 2019, p. 28).

Nowadays, logistics is an important branch of economics. Specialists do a lot of research in this field. Plenty of universities has a specialization in logistics. Many countries have their institutions of logistics. For example, the National Institute for Transport & Logistics in Dublin, the Institute of Supply Chain Management in Newcastle, College of Logistics (VŠLG) Prerov, etc.

Globalization is caused by rapid development in technologies, especially in information technologies: a great example is an internet. Due to that, globalization can be seen in many aspects of people today, and logistics is not an exception in this trend. Logistics and supply chain networks have become much more complicated in comparison to the nearest past:

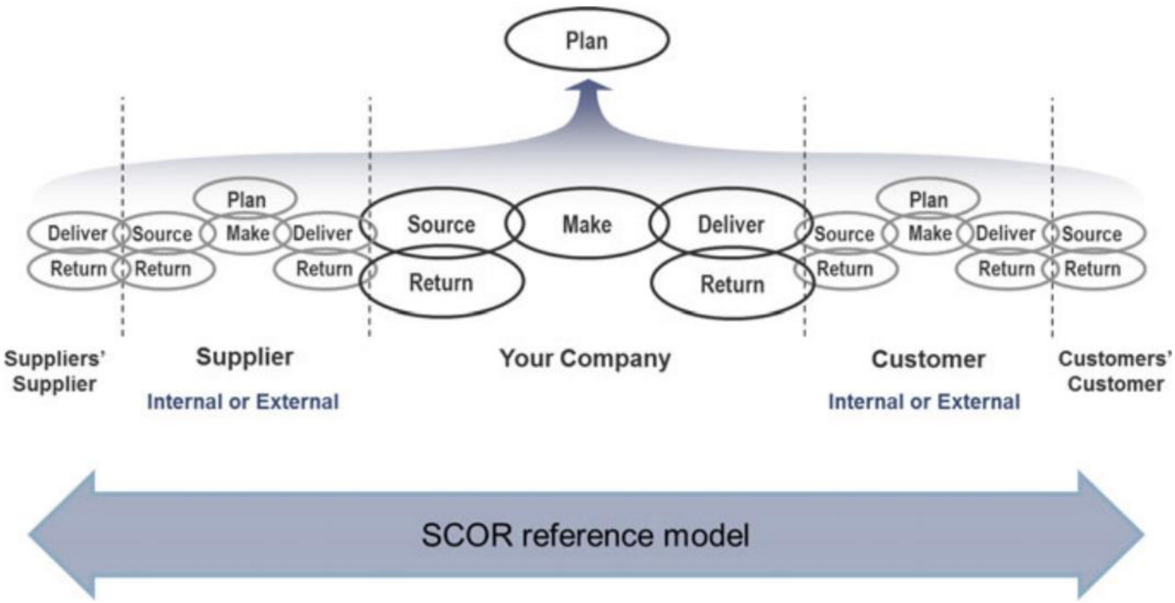
„To service global markets, logistics networks become, necessarily, far more expansive and far more complex" (Rushton et al., 2017, p. 24).

Due to furious development in logistics, each company should implement modern approaches of decision making in logistics.

3.1.3 Logistics objectives

To observe logistics objectives, a supply chain can be viewed as since it is an extension of logistics. The supply chain's main goal is to be sure that the right amount of raw materials, parts and products are available for further usage at the right locations. Many instruments are used in the supply chain in order to achieve the fundamental objective. These instruments are sourcing, acquisition, transport, logistics, manufacturing, stock keeping between subsequent phases, and sales. (Zijm et al., 2019, p. 34)

Figure 1: Supply Chain Operations Reference (SCOR) model



Source: Poluha, 2007, p. 52

From figure 1, we can see that the Supply Chain covers many attributes. And all these attributes have their impact on the logistics branch of the company. For a good firm, it is necessary to pay attention to each small part of SCOR as it may be interpreted as a part of the logistics objective.

In fact, there is a possibility to describe all objectives of logistics with 7 "R":

1. The *Right* goods.
2. In the *Right* quantity.
3. At the *Right* time.
4. At the *Right* place.
5. With the *Right* quality.
6. At the *Right* cost.
7. At the *Right* sustainable impact/footprint.

Logistics should provide all mentioned "R"'s (Zijm et al., 2019, p. 38). This Bachelor Thesis is directly related to "*Right* cost", seeing that the primary goal of the Bachelor Thesis is cost reduction of transportation.

3.1.4 Logistics approaches

Since there are logistics objectives, there should be logistics approaches to achieve these objectives. Traditionally, the logistics decision making process classifies into three decisions: **strategic, tactical** and **operational**, according to the planning horizon (Ghiani et al., 2004, p. 18).

- **Strategic decisions**. This type of decisions has long-perspective effects. It consists of the design of logistics systems and the purchasing of expensive resources such as the location of the warehouse, determination of size or capacity of the plant or warehouse, etc. Strategic decisions tend to use forecasts based on aggregated data. For instance: combining groups of individual products into product families (Ghiani et al., 2004, p. 18).
- **Tactical decisions**. This particular part of decisions includes production and distribution scheduling resource assignment. An example of resource assignment is storage allocation. These decisions are usually made monthly or quarterly. Tactical decisions often use forecasts based on disaggregated data (Ghiani et al., 2004, p. 18).
- **Operational decisions**. The last type of decisions is made every day. It subsists of warehouse order picking, shipment and vehicle dispatching. Determinations are constructed using detailed data (Ghiani et al., 2004, p. 18).

Since the selected company distributes its product to the customer on daily basis, I describe and use methods for operational decision in this Bachelor Thesis.

3.2 Operations research

3.2.1 What is operations research?

The term Operations Research (OR) originates in 1937. In this year, EC Williams was recruited to create research about the operational aspects of the newly developed radar systems. He received the title „Operations Researcher" (Manson, 2006, p. 1). A great definition of OR is given by Murthy (2007, p. 1):

„The subject OPERATIONS RESEARCH is a branch of mathematics - specially applied mathematics, used to provide a scientific base for management to take timely and effective decisions to their problems."

Basically, OR is created to optimize certain decision makings, and the solution depends on the completeness of constructed model (Taha, 2017, p. 34). In other words, OR is used today to increase the efficiency of various types of systems. For instance, in this Bachelor Thesis, I applied OR to improve route scheduling of delivery in the selected business case. Furthermore, the main objective of the OR is to supply a decision-maker with a scientific approach for finding the best solution in problems where various components affect the outcome. The best solution is also known as the optimal solution (Murphy, 2007, p. 7).

The most appropriate OR technique is linear programming. It is designed for models with linear objective and constraint functions (Taha, 2017, p. 34).

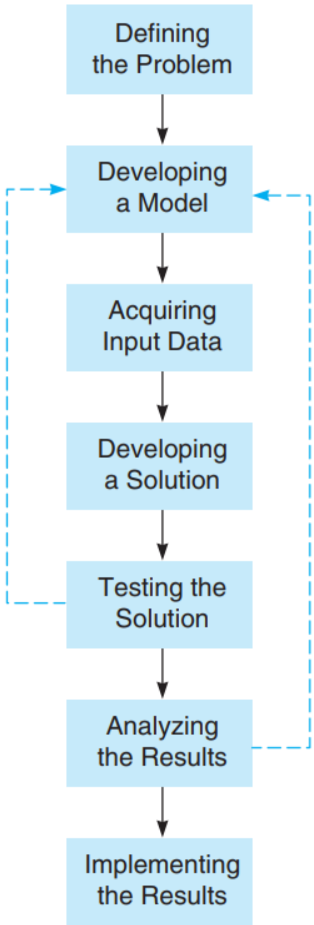
There are three approaches in OR: **qualitative**, **quantitative** and **mixed methods** (Creswell, 2018, p. 41):

- **Qualitative research** is an approach used for understanding social or human problems. In this type of research, questioners are implemented. What's more, data is tended to be collected from the participants. The final report of this has a flexible structure (Creswell, 2018, p. 41).
- On the other hand, **quantitative research** is an approach used for testing objective theories. The main instrument of this research type is analysing the relationship among variables. These variables are measured in a numeric way to analyse it using statistical approaches. The final report has a set structure consisting of introduction, literature and theory, methods, results, and discussion (Creswell, 2018, p. 41). I use and implement this type of research in this Bachelor Thesis. I discuss quantitative research in details in paragraph 3.3.
- In addition, **mixed methods research** is an approach that involves both quantitative and qualitative data, integrating the two forms of data (Creswell, 2018, p. 41).

3.3 Quantitative approach

As was mentioned before, a quantitative approach is used in OR when numerical data needs to be analysed. The quantitative analysis method consists of defining a problem, developing a model, acquiring input data, developing a solution, testing the solution, analysing the results, and implementing the results (Render et al., 2018, p. 22). It is represented in the following figure.

Figure 2: Quantitative approach



Source: Render et al., 2018, p. 22

I have done all steps illustrated in figure 2 in the practical part of my Bachelor Thesis. However, I used different methods in the stage of developing the solution. To be more precise, I applied optimization and approximation approaches in the selected business case.

3.3.1 Optimization

Optimization approach searches for the most appropriate solution in certain decision-making cases. For instance, an organization targets to maximize profit from sales but it may face constraints in terms of its production capacity or a finite demand for its products (Anderson et al., 2014, p. 7).

There are two types of optimizations: *easy* and *NP-hard* (Non-deterministic Polynomial-time). If instance size is big, then *easy* or simple optimization solves problems within an acceptable amount of time. Moreover, it uses *linear programming* technique. On the other hand, *NP-hard* optimization problems cannot be solved within a sensible amount of time if the instance size is large (Ghiani et al., 2004, p. 19). There are many techniques for solving optimization problems, such as *integer programming* – in this technique variables assume integer values; *dynamic programming* – here original model decomposes into smaller and more manageable subproblems; *network programming* - there the problem can be modelled as a network, and *nonlinear programming* – in this approach which functions of the model are nonlinear (Taha, 2017, p. 34)

3.3.2 Approximation

Approximation method is applicable when there are too many customers, so that demand can be seen as a continuous spatial function. Approximation usually proposes closed-form solutions. Moreover, this type of methods may be used as a simple heuristic (Ghiani et al., 2004, p. 20). I provide a detailed explanation of the heuristic in paragraph 3.4.5.1.

3.4 Travelling Salesman problem (TSP)

TSP is the fundamental part of this Bachelor Thesis. Basically, TSP is a NP-hard optimization problem which means that solving that kind of problem is very complicated and time-consuming. The purpose of the TSP is to find a route of a salesman who starts from a home location, visits a defined set of cities and returns to the original location in such a way that the total distance travelled is minimum, and each city is visited exactly once (Gutin & Punnen, 2007, p. 1). The origin of the term "Travelling Salesman Problem" is quite mysterious. There is no documented evidence of the creator of this term (Applegate, 2006, p. 2). However, there is a belief that the expression TSP has roots in the United States. To be more precise, the first reported publication with this term dates to 1949. Nevertheless, the first systematic study of TSP had started by the work of Dantzig, Fulkerson, and Johnson called „Solution of a large scale Travelling Salesman Problem" and finished in 1954 (Gutin & Punnen, 2007, p. 2).

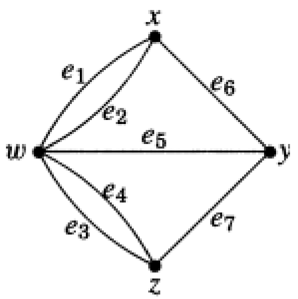
3.4.1 Graph theory

Graph theory is necessary for understanding TSP. This specific theory describes different types of graphs and how they can be applied. I discuss the basics of graph theory in this chapter.

3.4.1.1 Graph

First of all, what is a graph? West (2020, p. 2) gives a great definition:
"A graph G is a triple consisting of a vertex set V(G), an edge set E(G), and a relation that associates with each edge two vertices (not necessarily distinct) called its endpoints." An example is represented in figure 3. In this illustration, the graph has a vertex set is {*x*, *y*, *z*, *w*}, the edge set is {e_1, e_2, e_3, e_4, e_5, e_6, e_7}, and the assignment of endpoints to edges can be read from the picture.

Figure 3: Example of graph

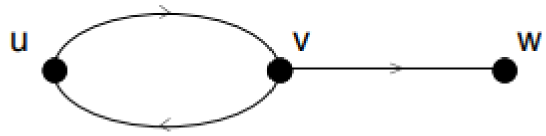


Source: West, 2020, p. 2

3.4.1.2 Types of graphs

Furthermore, graphs divide into two types: directed or nondirected. Definition:
„A directed graph (or digraph) is a graph each of whose edges is directed" (Gross et al., 2018, p. 3).
An example of a digraph is figure 4.

Figure 4: Example of digraph



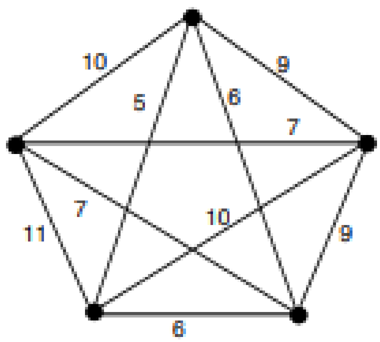
Source: Gross et al., 2018, p. 4

Besides that, a graph can also be weighted or non-weighted. Definition:

„A weighted graph is a graph in which each edge is assigned a number, called its edge-weight" (Gross et al., 2018, p. 39).

As a matter of fact, transition matrices can be constructed from weighted graphs in order to represent numeric values of distances between edges.

Figure 5: Representation of weighted graph



Source: Gross et al., 2018, p. 50

From figure 5 distances between edges can be observed. I use transition matrices in the practical part of this Bachelor Thesis, in order to represent the length of the path between addresses. I provide a detailed description of matrices in paragraph 3.4.4.

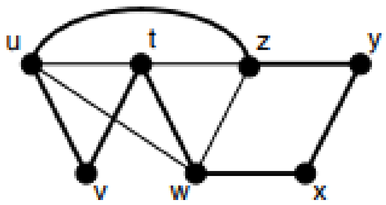
3.4.1.3 Hamiltonian path

Moreover, some paths in a graph can be Hamiltonian paths. Gross (2018, p. 42) provides an explicit definition of the Hamiltonian cycle:

„A Hamiltonian graph is a graph that has a Hamiltonian cycle. And a cycle that includes every vertex of a graph is called a Hamiltonian cycle".

I illustrate the Hamiltonian path in figure 6. Gross (2018, p. 42) highlights the Hamiltonian cycle by bold lines. Furthermore, the edges of the Hamiltonian circle are: {*u, z, y, x, w, t, v,* *u*}. In other words, the path starts at the edge '*u*', goes through all other points, and returns at the edge '*u*'.

Figure 6: Example of Hamiltonian graph



Source: Gross et al., 2018, p. 42

3.4.2 Mathematical definition of TSP

TSP can be described from a mathematical point of view. Mathematical definition of TSP: "Let G = (V, E) be a graph. For each edge e ∈ E, a cost (weight) is prescribed. Then the TSP is to find a tour (Hamiltonian cycle) in G such that the sum of the costs of edges of the tour is as small as possible" (West, 2020, p. 3).
From this statement, I can summarise that TSP searches for a Hamiltonian path with the least cost in a graph.

3.4.3 Types of TSP

The TSP has different types. The first type is *the symmetric* TSP, and the second variation is *the asymmetric* TSP. In fact, the type of TSP is completely dependent on the cost matrix. If the cost matrix is symmetric then TSP is also symmetric, and vice versa TSP is asymmetric if the cost matrix is asymmetric (Gutin & Punnen, 2018, p. 4).

3.4.4 Matrix construction

Squared matrices are essential due to their usage in all approaches for finding the solution for TSP. In fact, the size of the matrix depends on the number of nodes (cities or points) that need to be visited. Moreover, the matrix is "squared" due to the equality of rows and columns (Croall & Mason, p.176). *Xij* in the matrix means the length of the path from node *i* to node *j*. The main diagonal of the matrix is equal to 0 seeing that values on the main diagonal represent a route from and to the same city. Below there is an example of the square matrix used in solving TSP:

Table 1: Example matrix for asymmetric TSP

	City A	City B	City C
City A	0	3	2
City B	2	0	1.7
City C	2.5	1.5	0

Source: construction by own means.

3.4.4.1 Triangle inequality

The crucial structural property of a matrix used in TSP is triangle inequality. The triangle inequality means that for every type of triangle, the sum of the lengths of any two sides must be greater than or equal to the yardage of the remaining side. This fundamental mathematical theorem relates to Secondary schools' programs all over the world. Definition of triangle inequity:

„In a weighted simple graph whose vertices are labelled 1, 2, . . . , n: a condition on the edge-weights c_{ij}, given *by* $c_{ij} \leq c_{ik} + c_{kj}$ for all *i, j*, and *k"* (Gross et al., 2018, p. 296).

In TSP violations of this inequality are met from time to time. Different actions can cause these non-observances. Firstly, a violation can occur because researchers used randomly created networks with intention of creating a generalization of the feasibility of a solution. Whereas randomly generated graphs may create distances that cannot exist in flat space (Euclidian space). That is why random creation of graphs cannot be used in TSP. In the second place, rounding of numeric representations of distances to integers or decimals can cause violations. Due to that rounding should be used carefully. Finally, traffic delays may lead to violations of the triangle inequality. The solutions with violation of triangle inequality may not appropriately show the reality of the routing network. These results may be interpreted as an optimistic or inaccurate solution (Fleming et al., 2013, p. 2).

3.4.5 Approaches for solving TSP

Problems such as TSP can be solved just by describing each alternative and choosing the best variant. Karl Menger in his book “Bericht über ein mathematisches Kolloquium” describes complexity of solution for this kind of method. Applegate (2007, p. 45) translated

his description: “This problem can naturally be solved using a finite number of trials. Rules which reduce the number of trials below the number of permutations of the given point set are not known. The rule that one should go from the starting point to the next nearest point, then to the next nearest point, and so on, does not always produce the shortest path.”
From this statement, I can claim that the complexity of solving TSP lies in a big bunch of possible outcomes. Moreover, to find all possible routes in the case with *n* destinations, it is necessary to calculate *(n-1)!* alternatives (Applegate, 2007, p45). For instance, if TSP is asymmetric and contains 16 different nodes, the number of possible routes equals 15!, which is approximately 1,3 trillion alternatives. Because calculating such amount of solutions will consume an enormous amount of time and considerable resources scientists suggested different methods for solving problems such as TSP. I put into practice four approaches in this Bachelor Thesis. They are Branch and Bound, Vogel’s Approximation Method, Nearest Neighbour Algorithm, Savings Method. Mentioned methods have their pros and cons, which I show and discuss in further chapters.

3.4.5.1 Nearest Neighbour Algorithm (NNA)

This method is the simplest among others that I describe in this Bachelor Thesis. The Nearest Neighbour Algorithm (NNA) is a heuristic algorithm which means that there is a rule helping to choose between alternatives. An example of heuristics is chess. In a chess game, two players are not analysing all possible moves. But they analyse only a few moves which have a crucial effect on the game's result. These few moves are chosen based on the player’s knowledge of the theory of chess and his/her own experience. Of course, the speed of finding the solution affects the optimality of this solution. Regarding NNA, the rule which helps to find the solution is: “from where are you choose the cheapest or in other words the shortest way to go somewhere else” (Gross et al., 2019, p. 287). Advantages of this method are that it is easy to use, and the required time is comparably low. A disadvantage is that the optimality of the founded solution is relatively low. A better comparison of NNA with other approaches I discuss in further chapters of Bachelor Thesis.
Algorithm for NNA: the first step is to create a cost matrix (see paragraph 3.4.4). The main diagonal is filled with zeros. The second step is to set starting point, an initial city. Besides, in this step, it is necessary to cross the column of starting node as the traveller returns to the starting destination only in the end. Thirdly, a cell with the lowest value must be found in

the row of the current city. Then this cell must be highlighted since the traveller goes to this city. The fourth step is to cross the column related to the pointed cell. After that, the third and the fourth steps have to be repeated until the last column is crossed. Eventually, the traveller must return to the initial city.
This algorithm must be applied to all cities. In other words, all cities have to be the starting city once. If TSP is asymmetric, we have to use *backwards* research. In this research rows have to be crossed and the lowest number has to be found in columns for the same matrix. Or it is necessary to apply the same procedure(*forward* research), but for transposed matrix (rows and columns are swapped). Finally, the shortest path between all has to be selected (Šubrt, 2019, p. 104).

3.4.5.2 Vogel Approximation Method (VAM)

The Vogel Approximation Method (VAM) is another heuristic method used for finding the best solution in TSP. VAM uses a comparison between the two lowest numbers in both rows and columns of transition matrix. Comparison finds the option with the least impact. (Hlatká et al., 2017, p. 2). The benefit of this method is almost the same as NNA: VAM is easy to apply and understand. However, the solution may be far from optimal.

Algorithm for VAM: first step is the construction of the cost matrix (see paragraph 3.4.4). Besides, the main diagonal is filled with "–" or X but not zeros as the algorithm is sensitive to numbers. The second step is to create a new row and column that will illustrate the difference between the two lowest numbers in the row and columns accordingly. Thirdly, it is needed to choose the highest number (the largest difference) between the two shortest distances. Then the cell with the lowest number is highlighted, its column and row are crossed, seeing that we do not need these routes anymore. Moreover, the mirror cell, located across the main diagonal, must be crossed. After that, it is necessary to repeat the second and third steps until the solution is found (Hlatká et al., 2017, p. 3).

3.4.5.3 Savings Method

The Savings Method (SM) is another approximation method used for solving TSP. The base of SM is the comparison of lengths of a straight route between any two cities and a route via another selected city. This particular method is much more time-consuming in comparison to NNA and VAM. Nevertheless, the result of SM is more satisfying in the point of finding the best alternative (Kučera, 2013).

Algorithm for SM: the first step is the creation of a cost matrix (see paragraph 3.4.4). Secondly, it is required to choose randomly one city. This city is denoted as city 0. The third step is to compute savings for all pairs of other cities (i,j):

$$s_{ij} = c_{i0} + c_{0j} - c_{ij}$$

The following step is the construction of descending order of straight routes between pairs of cities, according to received values of savings (s_{ij}). Then it is necessary to process the edges using this approach: when by adding an edge we obtain a set of vertex disjoint paths, we add it to the solution. Repeating this step is required until the creation of the Hamiltonian path (see paragraph 3.4.1.3). The final step is to add so-called city 0. A recommendation is to use all the cities as the city 0 (Kučera, 2013, p. 2).

3.4.5.4 Branch and Bound

The Branch and Bound method is an optimization method, unlike previous methods. It means that the computation of large-size problems consumes a huge amount of time and computational power as this method is not heuristic. On the contrary, this method proposes the best outcomes compared to approximation methods. I discover it in the practical part. The Branch and Bound method uses a state-space tree to solve the TSP. Consequently, it is complicated to compute (Violina, 2021).

Algorithm for Branch and Bound: firstly, a cost matrix has to be made (see paragraph 3.4.4). Moreover, the values of the main diagonal have to be infinity. The second step is a reduction of each row and column in a way that there must be at least one zero in each row and column. To accomplish it, subtract the minimum value from each row and column. Then it is necessary to calculate the lower bound of root node. The lower bound calculates by summarizing all reductions. The third step is to choose any other node for expansion. To do it, we have to find out the node with minimum expanding cost.

The formula for finding cost:

L(node) = L (parent node) + Parent (i,j) + total cost of reduction

When an appropriate node is found, repeat step 3 for the next set of nodes. It is mandatory to continue calculating until the route is fully completed (Rastogi et al., 2013)

4 Practical Part

4.1 Practical usage in a selected company

The practical part is completely related to the theoretical part described above. Moreover, this part clearly shows the efficiency of using scientific research in optimizing the logistic part of a company.

I chose a delivery company for the practical part. The delivery company is one of the most suitable examples by the virtue of the fact that the delivery enterprise deals with route construction every day. Usage of TSP approaches is natural in the selected business case. The company daily faces delivering goods to customers from the warehouse from where the driver visits every delivery point and returns to the warehouse.

4.1.1 Company features

A selected company locates in Kazakhstan. I anonymously describe the company due to professional confidentiality.

First of all, the selected company is an online supermarket established in 2011. From that time the firm has enlarged its customer base and the assortment of products. The enterprise works simultaneously with Kazakh, Russian and Uzbek suppliers of goods such as fruits, vegetables, household chemicals etc.

The selected company has different vendors for different types of products. Moreover, all merchandise is stored in a warehouse. Where it is appropriately kept until it arrives at the customers. As a matter of fact, customers accomplish orders through the company's website. After customers complete their orders, drivers deliver products to their addresses from the warehouse.

The company distributes products by rented car type – Lada Largus van. This type of car uses petrol. Petrol usage is 8,1 litres per 100 km.

4.1.2 Company’s delivery routes construction

Route construction starts from receiving orders from customers. Customers can complete their orders through the website at different times. Once the customer completes his/her order, it sends to the operator's PC at the warehouse. At the beginning of the workday, the operator checks received orders and creates a list of purchases and a shipping list. The list of purchases consists of orders themselves. Meanwhile, the shipping list consists of addresses. Then operator sends the order list to the workers for collection in the warehouse. If operators receive a new order, they add it to the lists until the start of the shipment. When orders are packed and ready for delivery, the delivery man receives the shipping list. Then he constructs the route based on the shipping list, his knowledge and experience.

The selected company provided data about five working days. According to the provided data, I constructed the table for the Monday case. Addresses are denoted concerning confidential customer information. Furthermore, destinations are written in a sequence of delivery routes. A destination A1 is a warehouse, others are delivery points. Moreover, the warehouse is always the first and last destination.

Table 2: Sequence and distance of the initial route on Monday

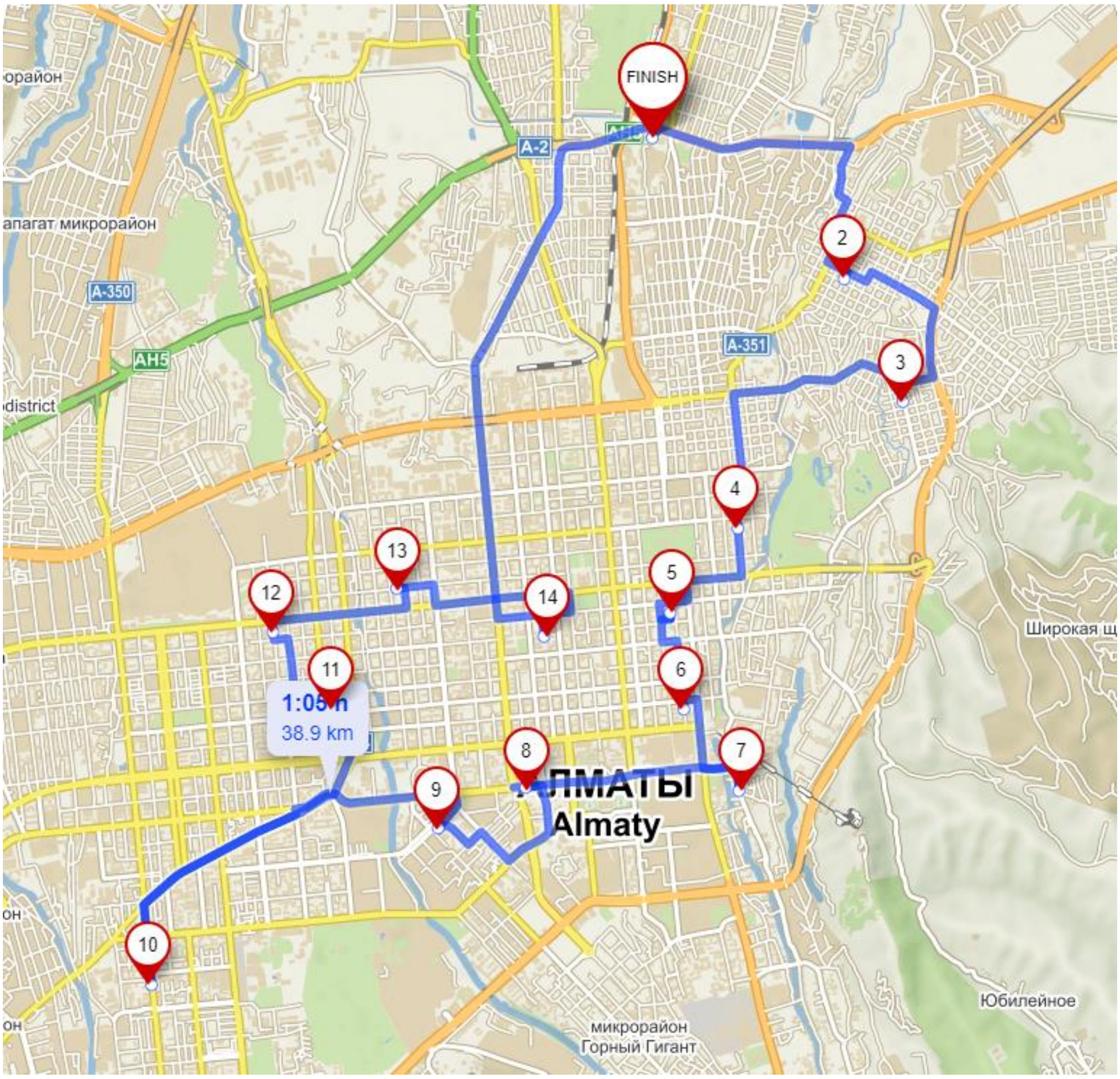
Destination	Distance (km)
A_1	0
A_2	3.6
A_3	2.4
A_4	3.3
A_5	1.8
A_6	1.2
A_7	1.4
A_8	2.5
A_9	2.1
A_{10}	4.6
A_{11}	4
A_{12}	1.5
A_{13}	1.7
A_{14}	2.5
A_1	6.4
Total:	39

Source: own processing, based on provided data by the company

The real distance for Monday is 39 km. The aim is to decrease this value as much as possible. Monday data is used for a comparison between real and alternative routes.

The second way is a graphical representation, which is widely used in operations research, due to that everything is more visible and understandable for everyone once it is illustrated. The following map represents the current situation of Monday.

Figure 7: Graphical representation of real route on Monday



Source: own processing, based on provided data by the company

4.2 Alternative way of routes construction

To show the alternative way of routes construction, I structured the model, which is as close as possible to the actual situation. A carefully constructed model is the most significant condition of quantitative research on the one hand (see paragraph 3.2). And on the other hand, the represented model, as each mathematical model, is a simplification of the authentic world which is why the condition of time windows and delivery time by agreement are not counted.

The matrix of Monday is developed to apply it in TSP methods (see paragraph 3.4.4). Moreover, I should note that there are slight non-observances of triangle inequality in matrices (see paragraph 3.4.4.1). Two reasons cause these violations. The first one is rounding the length of the path to decimals. The second reason is that there are many one-way roads, which leads to an increment of routes that could not occur if all tracks were two-ways.

The matrix features are:

1. Headings of the table in rows and columns are named by the denotations A_n, A_{n+1}, …, which are addresses of the warehouse(A_1) and addresses of delivery points;

2. The size of the matrix is 14x14;

4. Filled cells demonstrate distances in km. The values are obtained owing to the web – yandex.com/maps;

5. The matrix is asymmetric because routes are different. A big amount of one-way roads causes these differences.

6. Values of distances are rounded to decimal numbers to avoid inaccuracy.

Table 3: Monday coefficient matrix

Monday	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}
A_1	0	3.6	4.8	4.1	5.4	6.1	6.5	8.4	9.3	12.4	8.8	8.5	7.1	6.3
A_2	3.5	0	2.4	3.2	4.9	5.6	6	8.1	9	12.1	8.5	8	7.1	5.9
A_3	5.4	2	0	3.3	4	4.7	5.2	7.3	8.2	11.4	7.9	7.5	6.3	5.1
A_4	5	3.1	3.3	0	1.8	2.4	2.8	4.9	5.8	9	5.5	5.1	3.9	2.7
A_5	6	4.8	4.5	1.8	0	1.2	2.4	3.5	4.5	8.1	4.2	4.2	3.1	1.5
A_6	7	5.5	5	2.4	1.1	0	1.4	3	4.5	7.4	4	5.2	4.1	2.5
A_7	7.2	5.9	6.1	2.8	2.6	2.6	0	2.5	3.8	7.1	4.9	6.5	5.3	3.7
A_8	8	7.3	7.5	4.5	3	2.3	2.2	0	2.1	5.3	4.2	4.7	3.6	2.3
A_9	9.2	8.7	8.3	5.7	4.3	3.5	3.4	1.2	0	4.6	3.5	3.9	2.8	3.5
A_{10}	12.2	12.1	11.6	9	7.5	6.8	7	4.9	4	0	4	4.5	5.4	6.9
A_{11}	9.7	9.2	9.4	6.4	5.4	4.5	5.3	3.1	2.9	3.9	0	1.5	2.8	4.5
A_{12}	8.6	8.1	8.3	5.3	4.4	5	6.2	4.1	3.8	4.3	1.3	0	1.7	3.8
A_{13}	7.1	6.7	6.8	3.8	3.1	4.1	5.4	3.2	3	5.4	1.9	1.5	0	2.5
A_{14}	6.4	6.7	6.3	3.7	2.4	2.8	4.1	2.5	3.4	6.9	3.1	3.1	2	0

Source: own processing based on provided data by the company

4.2.1 Software for finding the solution (TSPKOSA)

I used a program called TSPKOSA for finding the optimal solution. Thanks to Ing. Igor Krejčí Ph.D., who supplied me with this tremendous software. TSPKOSA was created in Microsoft Visual Basic 6.5. There are three approximation approaches:
Nearest Neighbour Algorithm (sequential), Vogel Approximation Method/Loss Method, Savings Method (parallel).
Moreover, TSPKOSA has one optimization method: *Branch and Bound*.
I applied all four approaches for each day of the selected business case. Furthermore, it is possible to enter problems with up to 250 nodes. In other words, this software could be used to solve TSP with a big amount of data (Krejčí *et al.*, 2010). The data in the selected business case is small, therefore I can use TSPKOSA with confidence.
By virtue of the fact that all four methods have different approaches, each of them suggests various solutions. Consequently, I observed all methods and chose the most satisfying (the lowest) result between all outcomes. Furthermore, I compared the most reliable outcome with the initial route in order to find possible savings.

4.2.2 Suggested solution for Monday

In this part of the Bachelor Thesis, I describe in detail approachs used in the Monday case. As a matter of fact, I applied the same technique for the other four working days in the selected business case.
I used the Monday matrix (table 3) in TSPKOSA software for finding alternative routes in the Monday case. I applied four methods for solving this TSP and determined the best approach among these four. These methods are NNA, VAM, Savings Method, Branch and Bound. Below I show a numerical and graphical representation of approached outcomes from each method. Comparison between the four methods is necessary since each method suggested a different solution in the Monday case. I selected the best cycle between 4 alternative paths. This route I used to show improvement in comparison with the initial circle.

4.2.2.1 Nearest Neighbour Algorithm (NNA)

I applied NNA for finding alternative solutions (see paragraph 3.4.5.1). The main feature of the NNA is repeating the procedure until the number of routes equals the number of nodes. After that, the shortest route is selected between obtained paths. The program TSPKOSA proposed one minimal cycle. The length of the offered circle is 41.1 km. The result of the NNA for Monday is described in the following table:

Table 4: Sequence and distance of the route constructed by NNA on Monday

Destination	Distance (km)
A_1	0
A_2	3.6
A_3	2.4
A_4	3.3
A_5	1.8
A_6	1.2
A_7	1.4
A_8	2.5
A_9	2.1
A_{13}	2.8
A_{12}	1.5
A_{11}	1.3
A_{10}	3.9
A_{14}	6.9
A_1	6.4
Total:	41.1

Source: own processing based on the TSPKOSA results

The outcome of the NNA shows the route with a length of 41.1 km. In other words, using NNA leads to an increase of the path by 2.1 km compared to the initial route. This result does not satisfy the goal of decreasing the circle. Due to that, this solution cannot be implemented in the selected business case.

4.2.2.2 Vogel Approximation Method (VAM)

I also used VAM (see paragraph 3.4.5.2) or Loss Method for finding alternative routes in the Monday matrix. The fundamental idea of this method is to calculate differences between the two lowest numbers in each row and column. Therefore, the most appropriate difference is chosen. TSPKOSA suggested one minimum cycle with a length of 41.7 km. The outcome is shown in the following table:

Table 5: Sequence and distance of the route constructed by VAM on Monday

Destination	Distance (km)
A_1	0
A_4	4.1
A_5	1.8
A_6	1.2
A_7	1.4
A_{10}	7.1
A_9	4
A_8	1.2
A_{14}	2.3
A_{11}	3.1
A_{12}	1.5
A_{13}	1.7
A_3	6.8
A_2	2
A_1	3.5
Total:	41.7

Source: own processing based on the TSPKOSA results

The result of VAM shows the route with a length of 41.7 that is longer by 2.7 km compared to the initial path. Therefore, this output does not satisfy the aim of decreasing the route. Consequently, I cannot implement this solution in the selected business case.

4.2.2.3 Savings Method

The third method which I used for finding alternative routes is Savings Method. This method is based on savings procedure. This procedure is a calculation of matrix S by the quotation $s_{ij} = c_{i0} + c_{0j} - c_{ij}$. Then the operation is implemented according to the Savings Method algorithm (see paragraph 3.4.5.3). The program TSPKOSA proposed one minimal cycle. I reported the result in the following table:

Table 6: Sequence and distance of the route constructed by Savings Method on Monday

Destination	Distance (km)
A_1	0
A_5	5.4
A_{14}	1.5
A_{13}	2
A_{12}	1.5
A_{11}	1.3
A_{10}	3.9
A_9	4
A_8	1.2
A_6	2.3
A_7	1.4
A_4	2.8
A_3	3.3
A_2	2
A_1	3.5
Total:	36.1

Source: own processing based on the TSPKOSA results

The output of the Savings Method shows the route with a length of 36.1 km. It means that the initial cycle was decreased by 2.9 km, which is approximately 7.4%. Basically, the path improved by 7.4%. I can apply this approach to decrease fuel expenses in the selected business case.

4.2.2.4 Branch and Bound

Lastly, I used Branch and Bound method (see paragraph 3.4.5.4) for searching alternative routes in the Monday matrix. The main concept of this method is finding a reduced matrix of the parent node by reducing the matrix by minimum value in each row and column. The program TSPKOSA offered one cycle with the least length. The output is described in the following table:

Table 7: Sequence and distance of the route constructed by Branch and Bound on Monday

Destination	Distance (km)
A_1	0
A_5	5.4
A_{14}	1.5
A_{13}	2
A_{12}	1.5
A_{11}	1.3
A_{10}	3.9
A_9	4
A_8	1.2
A_6	2.3
A_7	1.4
A_4	2.8
A_3	3.3
A_2	2
A_1	3.5
Total:	36.1

Source: own processing based on the TSPKOSA results

The output of Branch and Bound shows the route with a length of 36.1. The actual cycle is shorter by 2.9 km. Moreover, Branch and Bound suggests the same path as Savings Method. Seeing that, I can implement the Branch and Bound approach as well as the Savings Method in the selected business case.

4.2.3 Comparison of proposed solutions for Monday

All four methods have their proposed solution that differs from each other. As a consequence, a comparison is required to find the best alternative. I compared all numeric results from each method between each other and the initial route for the Monday case in the following table:

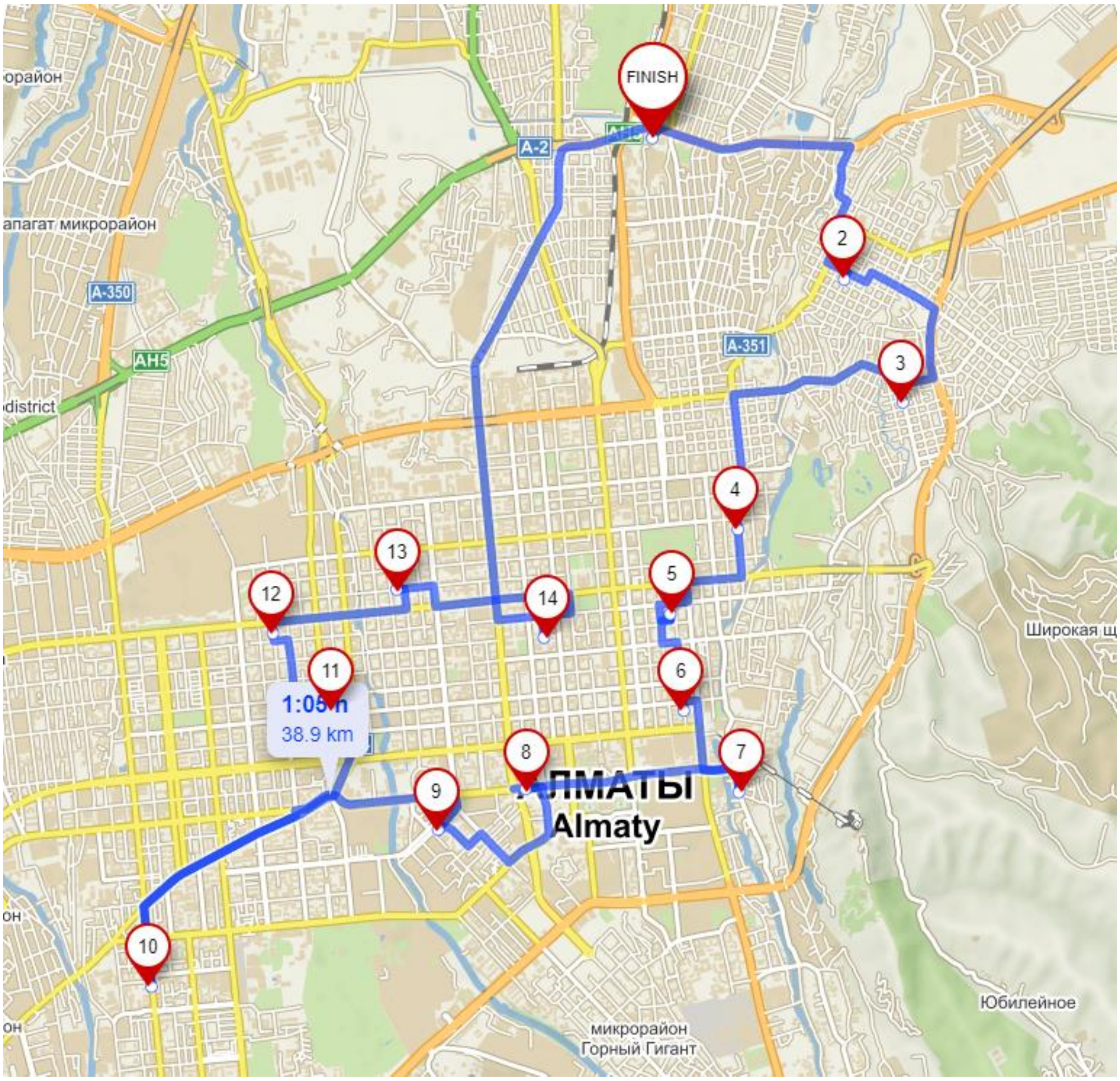
Table 8: Comparison between initial and alternative routes

	Initial	Nearest Neighbour Algorithm	Vogel Approximation Method	Savings Method	Branch and Bound
Distance (km)	39	41.1	41.7	36.1	36.1
Difference from initial (km)	-	+2.1	+2.7	-2.9	-2.9
Difference from initial in %	-	+5.4%	+6.9%	-7.4%	-7.4%

Source: own processing based on the TSPKOSA results

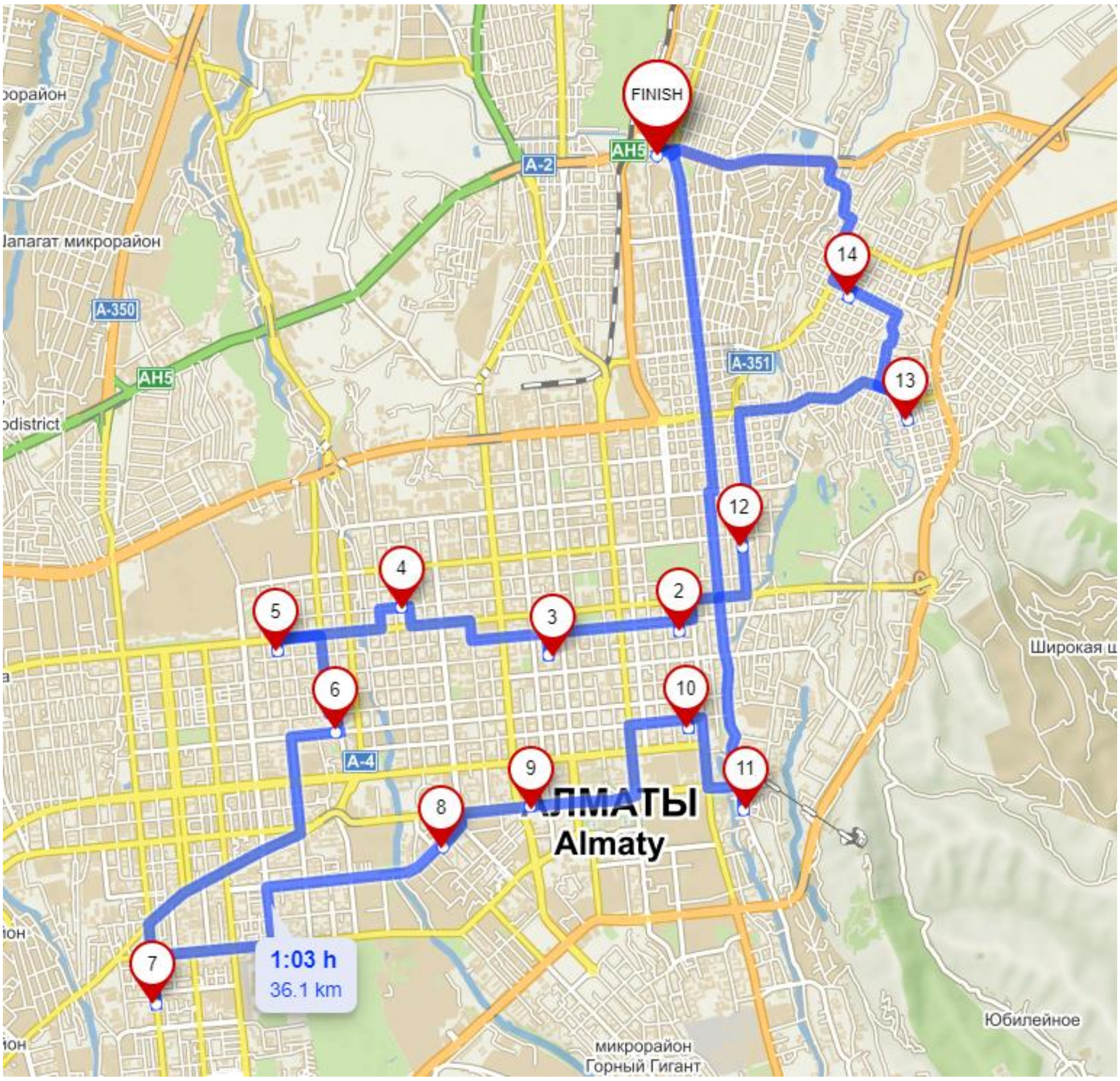
Table 8 represents the length of the initial route and the length of paths proposed by different methods. As it may be visible from table 8, the two best approaches (highlighted by green colour) are Savings Method and Branch and Bound. These methods lead to a decrease of the initial route by around 7%. It proves that I can use TSPKOSA in order to improve route construction. As a consequence of the route improvement, fuel expenses decrease. On the other hand, the company's profit increase, which is the primary goal in the selected business case. I designed a graphical representation of the achieved improvement in figures 8 and 9.

Figure 8: Graphical representation of the initial route



Source: own processing via the Mapy.cz website

Figure 9: Graphical representation of the alternative route



Source: own processing via the Mapy.cz website

Figures 8 and 9 represent the initial route and alternative route proposed by the Savings Method and Branch and Bound approach. Comparing the two paths, we can find out the differences between them. For instance, on the alternative path connections between nodes seem slightly shorter than on the initial path. Moreover, the overall picture looks better, especially when paying attention to the almost perfect construction of the alternative solution.

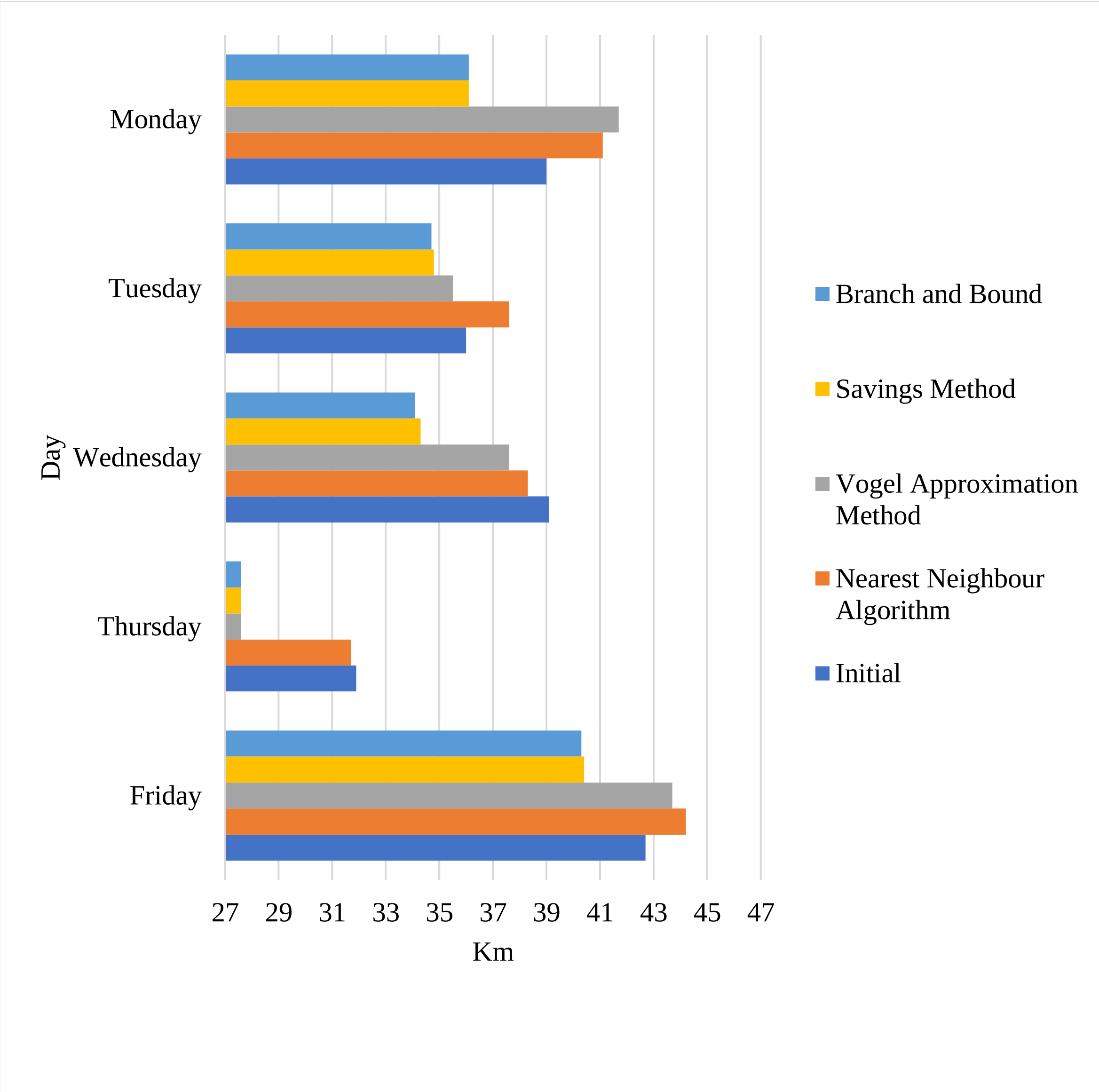
To summarize the comparison between four methods, two methods which showed the best results are: Savings Method and Branch and Bound.

5 Results and Discussion

5.1 Results comparison

After continuous data processing, I have found outcomes of NNA, VAM, Savings Method, Branch and Bound by means of the TSPKOSA software for all five working days. The results from each day are represented in the Appendix. I designed the following table and graph to illustrate achieved results:

Graph 1: Length of the initial and alternative routes for each day of the week



Source: constructed by own means based on TSPKOSA results

Table 9: Comparison between the initial and alternative routes for each day of the week

	Initial	Nearest Neighbour Algorithm	Vogel Approximation Method	Savings Method	Branch and Bound
Monday (km)	39	41.1	41.7	36.1	36.1
Monday change from initial	-	+5.4%	+6.5%	-7.4%	-7.4%
Tuesday (km)	36	37.6	35.5	34.8	34.7
Tuesday change from initial	-	+4.4%	-1.4%	-3.3%	-3.6%
Wednesday (km)	39.1	38.3	37.6	34.3	34.1
Wednesday change from initial	-	-2%	-3.8%	-12.3%	-12.8%
Thursday (km)	31.9	31.7	27.6	27.6	27.6
Thursday change from initial	-	-0.6%	-13.5%	-13.5%	-13.5%
Friday (km)	42.7	44.2	43.7	40.4	40.3
Friday change from initial	-	+3.5%	+2.3%	-5.4%	-5.6%
Week's total (km)	188.7	192.9	186.1	173.2	172.8
Week's total change from initial	-	+2.2%	-1.4%	-8.2%	-8.4%

Source: own processing based on the TSPKOSA results

Table 9 represents the comparison between initial and alternative routes proposed by four different methods. Additionally, there are four different colours: blue represents the initial route, *orange* shows an increase of the initial path, *yellow* illustrates a decrease compared to the actual circle, and *green* highlights the best alternative.

NNA proposed the worst alternative. There is a slight increase in the route's length for Monday, extended by 2.1 km. Furthermore, Tuesday's path is also longer by 1.6 km. Moreover, there is a 1.5 km growth in Friday's cycle. However, the route on Wednesday is lower by 0.8 km and Thursday’s cycle is shorter by 0.2 km. Nevertheless, the total week distance proposed by NNA is longer than the initial by 4.2 km.

Regarding VAM, even though it suggested the total outcome, that is shorter than real by 2.5 km. The length of the path on Monday is longer by 2.7 km. Moreover, Friday’s circle is greater by 2.7 km than the initial solution. The other three days have better results. Tuesday's route differs from the initial one by 0.5 km. Moreover, Wednesday's path is shorter by 1.5 km. Additionally, Thursday's route is smaller by 3.3 km.

Whereas Savings Method shows a considerable improvement for each day: Monday's cycle is lower by 2.9 km; Tuesday's path is lower by 1.2 km; Wednesday's route is lower by 4.8 km; Thursday's circle is lower by 4.3 km; Friday's cycle is lower by 2.3 km. As a result, the week’s total is shortened by 15.5 km. However, this is not the superior solution.

As a matter of fact, Branch and Bound has a similar result as Savings Method. However, Branch and Bound is the best alternative for the week’s total. This approach has an abridgement of the actual path for each day for approximately 3.18 km. The total routes distance for the whole week suggested by the Branch and Bound approach is 172.8 km. It is better than the initial route by 8.42%.

The objective of this Bachelor Thesis is achieved!

5.2 Company’s saving according to accomplished results

The following step is the calculation of potential expenses cut caused by shortens of routes. For calculation and representation of savings results, I used Branch and Bound approach, seeing that it is the most appropriate alternative. As I mentioned above, the company rents car type – Lada Largus van that uses petrol as fuel. Petrol usage is 8.1 litres per 100 km and 0.081 litres per 1 km accordingly. The price of 1 litre of petrol (Petrol Asia) is 169KZT. On account of the exchange rate of 19.02.2022 1 KZT equals 0.0021 EUR or 1 EUR is equal to 484.88 KZT. Therefore, the cost of 1 litre of petrol is 0.3485 EUR.

Table 10: Expenses and savings calculation of the initial and the best alternative route per one car

	Initial	Branch and Bound
Monday (km)	39	36.1
Tuesday (km)	36	34.7
Wednesday (km)	39.1	34.1
Thursday (km)	31.9	27.6
Friday (km)	42.7	40.3
Week's total (km)	188.7	172.8
Week's total change (km)	-	15.9
Week's total change (%)	-	-8.4%
Consumption (litre/km)	0.081	0.081
Petrol (litre)	15.2847	13.9968
Cost of petrol (EUR)	0.3485	0.3485
Total expenses per week (EUR)	5.3273	4.8784
Saving (EUR)	-	0.4489

Source: own processing based on the TSPKOSA results.

The table above represents expenses paid for petrol. The amount of gasoline used for the path, proposed by Branch and Bound, is lower by 1.2879 litres. It is obvious since the length of the circle, suggested by the Branch and Bound approach, is shorter than the actual path. The saved amount of money is 0.4489 EUR for this weak. From the long-term perspective, it can lead to a tremendous loss of profit. I can approximate savings from a 1-year perspective to imagine the losses from a long-term expectation. After 365 days or 52 weeks, the company will amplify its profit by 23.34 euros. If we would assume that the average speed of the delivery car is ~35 km/h on the whole distance, the company could save approximately 30 minutes. In 1 year, the selected company will save 26 hours of driver's labour. It is worth to consider that I analysed routes performed by only one car in this business case. Since the company has several vehicles, the lack of income may be even more significant. It can lead to irreparable harm to the enterprise's economy. Whereas, saving money can cause considerable development and growth of the company.

6 Conclusion and recommendation

Eventually, I have achieved the fundamental goal of this Bachelor Thesis: I discovered that the construction of delivery routes in the selected company is not ideal. Moreover, applying the approaches of TSP can obtain the enhancement in constriction of delivery circles. Branch and Bound approach showed the best results in the practical part. However, this method uses an optimization approach which is more time-consuming in comparison with approximation types. Besides the Branch and Bound approach, the Savings Method also proposed improved routes in the selected business case. Even though the suggested solution is a bit worse (0.2 %), Savings Method requires less time to compute. Due to that, Savings Method is a favourable alternative to the Branch and Bound approach.

Based on results approached in practical part I would strongly recommend to the selected company to use approaches of TSP. This implementation leads to a decrease of enterprise's expenses. To be more precise, I advise implementing Branch and Bound or Savings Method in the chosen enterprise. The selected company can achieve applying the mentioned approaches by TSPKOSA software. TSPKOSA is one of the various applications used for solving TSP. Managers of the picked enterprise can choose any of the programs based on their preferences.

Due to the unstable situation in Eastern Europe, a new crisis appeared. Under this crisis, all companies must operate as perfectly as possible. The current situation affects all aspects of lives such as economic, social etc. Enterprises' distribution expenses grow all over the world. For instance, petrol prices are rising every day and not rational usage can cause financial issues. It is directly related to the topic of the Bachelor Thesis. The slightest loss of revenue can lead to dramatic consequences, especially in such a difficult time.

In this Bachelor Thesis, I analysed and improved the delivery routes of only one driver. I have reduced the fuel costs of this driver by 0.4489 EUR. Seeing that the selected firm has several vehicles, I strongly suggest applying TSP approaches in the logistics department of the chosen company as this cause long-term/continuous improvement of the enterprise's performance. Moreover, the size of the enterprise matters – the bigger the company is, the higher are savings in the absolute value.

Summing up this Bachelor Thesis, I want to add that it is significant to apply rational decision making to all aspects of life, not only logistics. We all should develop ourselves and our environment to create a better world!

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8 Appendix

List of tables

Table 1: The Monday matrix

Monday	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}
A_1	0	3.6	4.8	4.1	5.4	6.1	6.5	8.4	9.3	12.7	8.8	8.5	7.1	6.3
A_2	3.5	0	2.4	3.2	4.9	7.2	6	8.1	9	12.5	8.5	8	7.1	5.9
A_3	5.4	2	0	3.3	4	4.7	5.2	7.3	8.2	11.4	7.9	7.5	6.3	5.1
A_4	5	3.1	3.3	0	1.8	2.4	2.8	4.9	5.8	9	5.5	5.1	3.9	2.7
A_5	6	4.8	4.5	1.8	0	1.2	2.4	3.5	4.5	8.1	4.2	4.2	3.1	1.5
A_6	7	5.5	5	2.4	1.1	0	1.4	3	4.5	7.4	4	5.2	4.1	2.5
A_7	7.2	5.9	6.1	2.8	2.6	2.6	0	2.5	3.8	7.1	4.9	6.5	5.3	3.7
A_8	8	7.3	7.5	4.5	3	2.3	2.2	0	2.1	5.3	4.2	4.7	3.6	2.3
A_9	9.2	8.7	8.3	5.7	4.3	3.5	3.4	1.2	0	4.6	3.5	3.9	2.8	3.5
A_{10}	12.2	12.1	11.6	9	7.5	6.8	7	4.9	4	0	4	4.5	5.4	6.9
A_{11}	9.7	9.2	9.4	6.4	5.4	4.5	5.3	3.1	2.9	3.9	0	1.5	2.8	4.5
A_{12}	8.6	8.1	8.3	5.3	4.4	5	6.2	4.1	3.8	4.3	1.3	0	1.7	3.8
A_{13}	7.1	6.7	6.8	3.8	3.1	4.1	5.4	3.2	3	5.4	1.9	1.5	0	2.5
A_{14}	6.4	6.7	6.3	3.7	2.4	2.8	4.1	2.5	3.4	6.9	3.1	3.1	2	0

Table 2: The Tuesday matrix

Tuesday	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}	A_{15}
A_1	0	3.5	4.7	6.5	6.2	5.7	6.8	8.2	11.2	11.8	11.6	10.1	7.9	6	5.8
A_2	6.5	0	1.8	3.8	3.4	3	4.2	5.6	8.6	9.4	10.1	7.9	5.9	3.4	3.2
A_3	5.4	1.8	0	2	1.7	1.2	2.5	4	6.8	8	8.4	6.3	4.3	2.5	3.6
A_4	7.1	3.8	2	0	1.7	2.7	3	5.2	5.3	6.7	6.9	7	6.1	4	5.4
A_5	6.6	3.3	1.6	1.2	0	1.7	1.8	4	5.4	6.6	7	5.8	5.3	3	4.4
A_6	5.3	2.8	1.1	2.9	1.7	0	2	3	6	6.8	7.5	5.4	3.4	1.3	2.7
A_7	6.6	4.2	2.5	3	1.6	1.7	0	2.6	4.5	5.2	5.6	4.4	3.4	1.3	2.7
A_8	7.2	5.6	3.9	5.7	4.5	3.5	2.9	0	3.1	3.8	4.2	2.3	1	2.4	3
A_9	9.7	8.1	6.4	4.6	4.7	5.6	4	2.5	0	2.1	3.5	4	3.1	4.9	5.4
A_{10}	11	9.5	7.6	5.9	6.1	7.3	5.3	3.9	2.1	0	2.7	3	4.5	6.3	6.8
A_{11}	10.7	9.8	8.2	6.7	6.3	7.5	5.6	4.3	3	2.7	0	2.2	4.4	6.5	7
A_{12}	9.4	7.7	6.1	7.4	6.5	5.6	5	2.8	3.9	4.2	2.7	0	2.2	4.5	5.1
A_{13}	7.3	5.8	4.2	5.9	4.7	3.7	3.1	1	3.6	4.4	4.5	2.3	0	2.7	3.2
A_{14}	5.8	3.6	2.3	4.1	2.9	1.9	1.3	2.2	5.3	6	6.4	4.6	2.2	0	1.4
A_{15}	4.3	2.7	3.8	5.8	4.4	3.4	2.8	3.1	6.4	6.9	7.3	5.5	3.6	1.5	0

Table 3: The Wednesday matrix

Wednesday	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}	A_{13}
A_1	0	4	6.3	6.1	4.5	6	6.8	9	12.4	10.7	8.6	9.2	10.1
A_2	4.8	0	2.6	3.3	1.7	3.5	4.2	6.5	9.8	8	5.9	6.9	8.6
A_3	6.4	2.6	0	2.4	3.1	3.6	3.2	4.8	8.1	6.3	5.2	6.6	7.6
A_4	5.8	3.4	1.6	0	1.9	2.5	1.3	3.2	6.6	5	3.6	4.3	6.4
A_5	4.4	2.1	3.1	1.9	0	1.8	2.6	5.5	8.2	6.6	4.3	5.2	7
A_6	5.4	3.7	3.6	2.5	2.3	0	1.4	3.4	6.7	4.9	2.5	3.4	5.1
A_7	6.5	4.1	2.4	1.2	2.6	1.5	0	3	6.3	4.5	2.8	3.8	5.5
A_8	8.9	6.5	4.2	3.8	5.1	3.6	2.4	0	4.3	2.8	3.1	3.9	6
A_9	11.4	9.9	8.1	6.6	8.4	6.4	5.8	4.2	0	2.1	3.9	3.4	4
A_{10}	9.8	8.1	6.4	5	6.9	4.7	4	2.8	1.9	0	2.2	1.6	3.3
A_{11}	7.6	5.9	4.9	3.6	4.4	2.5	3.1	3	3.9	2.2	0	1	2.9
A_{12}	9	6.9	5.5	4.3	5.4	3.4	3.9	3.7	3.4	1.5	1	0	2.9
A_{13}	9.6	8.6	8	6.8	7.1	5.1	6	5.5	4	3.3	3.2	2.2	0

Table 4: The Thursday matrix.

Thursday	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}
A_1	0	3.4	4.2	4.8	5.4	5.9	6.7	8.2	10.9	9.1	7.6	6.4
A_2	4.3	0	2.1	2.3	2.6	3.6	4.6	6	8.8	7	5.3	3.9
A_3	3.9	2.1	0	2.8	3.4	3.4	2.6	4.1	6.9	5.1	3.5	2.3
A_4	5.3	2.3	2.8	0	0.7	1.5	2.5	3.9	6.8	4.9	4	2
A_5	6	2.6	3.4	0.7	0	1.5	2.7	4.2	7.1	5.2	4.3	2.4
A_6	6.6	3.8	3.5	1.6	1.6	0	1.5	2.8	5.5	4.6	4.4	2.5
A_7	6.5	4.5	2.6	2.5	2.3	1.9	0	2	4.5	3.6	3	1
A_8	7.6	6.3	4.4	4.3	4.1	2.7	2	0	3.2	3	2.3	2.5
A_9	9.9	8.8	6.7	6.6	6.5	5	4.3	3.2	0	3.2	4.4	5
A_{10}	8.3	7	5.1	5	5.4	4.8	3.6	2.8	2.3	0	2.6	3.9
A_{11}	5.9	4.7	2.7	3.7	4.3	4.3	3	2.5	4.7	2.7	0	2.7
A_{12}	6.2	4.3	3	3.1	3	2.5	1.2	1.8	4.6	2.8	2	0

Table 5: The Friday matrix

Friday	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}	A_{15}	A_{16}
A_1	0	6.8	5.7	5.8	7.2	9.5	14.2	11.6	9.6	8.9	9.1	6.6	7.3	6.3	5.1	4.4
A_2	6.3	0	1.1	1.9	4.6	7	11.8	9.1	7.1	6.5	6.6	4.3	4.7	3.8	4	3.6
A_3	6.4	1.8	0	0.9	3.4	5.8	10.5	8	6	5.3	5.6	3.2	3.6	3	3.2	4.5
A_4	6.2	2	1	0	2.5	4.9	9.7	7.1	5	4.6	5.7	3.4	3.1	2.9	3.1	2.7
A_5	8.3	5.5	4.2	3.8	0	3.1	6.9	5.3	3.8	5.5	6.1	4.1	3	3.9	4.3	4.3
A_6	9.2	6.9	5.7	5.2	3.4	0	6.8	3	2.4	3.5	5.4	3.9	2.8	3.7	5.2	5.3
A_7	13.6	11.7	10.4	10	8.1	6	0	2.4	5.3	5.6	5.5	7.5	7.2	8.4	9.9	10
A_8	11.3	9	8.2	7.1	5.5	3.6	2.8	0	2.5	2.9	2.8	4.7	4.4	5.6	7.2	7.2
A_9	9.5	7.4	6	5.1	3.7	2.3	4.8	2.5	0	1.8	3	3.1	2.9	4	5.5	5.6
A_{10}	8.3	6	4.7	4.6	3.8	2.5	6.1	3.8	1.5	0	2.5	1.8	1.6	2.8	4.3	4.3
A_{11}	8.7	6.8	6	5.9	6.1	4.8	5.3	3.2	3.1	2.2	0	2.8	3.7	3.4	4.9	5
A_{12}	7.3	4.9	3.7	3.3	3.9	2.7	7.9	5	2.8	2.2	3.4	0	1.6	1.7	2.6	3.3
A_{13}	6.8	4.7	3.9	2.8	3.1	2.7	7.2	4.9	2.8	2.2	3.4	1.4	0	1.2	2.7	2.8
A_{14}	5.6	4.6	3.8	3.7	4.5	4.1	8.4	6.3	4.1	3.5	3.7	1.4	2.5	0	1.6	1.6
A_{15}	5.4	3.1	2.3	2.2	3	4.6	9.3	6.7	4.7	4.1	4.2	2	2.4	1.4	0	0.8
A_{16}	4.3	3.7	2.9	2.8	3.8	5.3	10	7.7	5.4	4.8	5	2.7	3.1	2.1	0.7	0

Table 6: The sequence of real route and the best alternative route proposed by Branch and Bound on Monday

Real		Branch and Bound	
Destination	Distance (km)	Destination	Distance (km)
A_1	0	A_1	0
A_2	3.6	A_5	5.4
A_3	2.4	A_{14}	1.5
A_4	3.3	A_{13}	2
A_5	1.8	A_{12}	1.5
A_6	1.2	A_{11}	1.3
A_7	1.4	A_{10}	3.9
A_8	2.5	A_9	4
A_9	2.1	A_8	1.2
A_{10}	4.6	A_6	2.3
A_{11}	4	A_7	1.4
A_{12}	1.5	A_4	2.8
A_{13}	1.6	A_3	3.3
A_{14}	2.5	A_2	2
A_1	6.4	A_1	3.5
Total:	39	Total:	36.1

Table 7: The sequence of real route and the best alternative route proposed by Branch and Bound on Tuesday

Real		Branch and Bound	
Destination	Distance (km)	Destination	Distance (km)
A_1	0	A_1	0
A_2	3.5	A_2	3.5
A_3	1.8	A_3	1.8
A_4	2	A_6	1.2
A_5	1.7	A_{14}	1.3
A_6	1.7	A_7	1.3
A_7	2	A_5	1.6
A_8	2.6	A_4	1.2
A_9	3.1	A_9	5.3
A_{10}	2.1	A_{10}	2.1
A_{11}	2.7	A_{11}	2.7
A_{12}	2.2	A_{12}	2.2
A_{13}	2.2	A_{13}	2.2
A_{14}	2.7	A_8	1
A_{15}	1.4	A_{15}	3
A_1	4.3	A_1	4.3
Total:	36	Total:	34.7

Table 8: The sequence of real route and the best alternative route proposed by Branch and Bound on Wednesday

Real		Branch and Bound	
Destination	Distance (km)	Destination	Distance (km)
A_1	0	A_1	0
A_2	4	A_5	4.5
A_3	2.6	A_6	1.8
A_4	2.4	A_{11}	2.5
A_5	1.9	A_{13}	2.9
A_6	1.7	A_{12}	2.2
A_7	1.4	A_{10}	1.5
A_8	3	A_9	1.9
A_9	4.3	A_8	4.2
A_{10}	2.1	A_7	2.4
A_{11}	2.2	A_4	1.2
A_{12}	1	A_3	1.6
A_{13}	2.9	A_2	2.6
A_1	9.6	A_1	4.8
Total:	39.1	Total:	34.1

Table 9: The sequence of real route and the best alternative route proposed by Branch and Bound on Thursday

Real		Branch and Bound	
Destination	Distance (km)	Destination	Distance (km)
A_1	0	A_1	0
A_2	3.4	A_2	3.4
A_3	2.1	A_4	2.3
A_4	2.8	A_5	0.7
A_5	0.7	A_6	1.5
A_6	1.5	A_7	1.5
A_7	1.5	A_{12}	1
A_8	2	A_{10}	2.8
A_9	3.2	A_9	2.3
A_{10}	3.2	A_8	3.2
A_{11}	2.6	A_{11}	2.3
A_{12}	2.7	A_3	2.7
A_1	6.2	A_1	3.9
Total:	31.9	Total:	27.6

Table 10: The sequence of real route and the best alternative route proposed by Branch and Bound on Friday

Real		Branch and Bound	
Destination	Distance (km)	Destination	Distance (km)
A_1	0	A_1	0
A_2	6.8	A_2	6.8
A_3	1.1	A_3	1.1
A_4	0.9	A_4	0.9
A_5	2.5	A_5	2.5
A_6	3.1	A_6	3.1
A_7	6.8	A_9	2.4
A_8	2.4	A_7	4.8
A_9	2.5	A_8	2.4
A_{10}	1.8	A_{11}	2.8
A_{11}	2.5	A_{10}	2.2
A_{12}	2.8	A_{12}	1.8
A_{13}	1.6	A_{13}	1.6
A_{14}	1.2	A_{14}	1.2
A_{15}	1.6	A_{15}	1.6
A_{16}	0.8	A_{16}	0.8
A_1	4.3	A_1	4.3
Total:	42.7	Total:	40.3