## Czech University of Life Sciences

## Prague

## Faculty of Economics and Management Department of Economics



Econometric Analysis of Selected Aviation Market

## Diploma Thesis

## Author: Lukáš Razim

Diploma Thesis Supervisor: Ing. Lenka Šobrová, Ph.D.
© Prague 2011

## Declaration

I hereby declare that this thesis is my own work and effort and that it has not been submitted anywhere for any award. Where other sources of information have been used, they have been acknowledged.

## Acknowledgement

I am deeply indebted to my supervisor Ing. Lenka Šobrová, Ph.D. whose help, stimulating suggestions and encouragement helped me in all the time of research for and writing of this thesis.

# Ekonometrická analýza vybraného trhu v leteckém odvětví 

## Econometric analysis of selected aviation market

## Souhrn

Tato práce se zaměřuje na ekonometrickou analýzu v leteckém podniku a na způsoby jejího využití pro efektivní provoz společnosti. V první části se zabývá současnou situací a vývojem v letecké dopravě, charakteristikou leteckého dopravce se zaměřením na výnosové a nákladové struktury v leteckém podniku a na ukazatele výkonosti používaných v tomto odvětví. Dále se soustředí na teoretické předpoklady pro úspěšné uplatnění metod ekonometrického modelování. Uvedena je především teorie produkční funkce a základní postupy pro tvorbu ekonometrického modelu. Druhá část práce je případová studie, založená na skutečných údajích z podniku České Aerolinie. Zkoumány jsou vztahy mezi položkami nákladů a výnosů a hlavní faktory, které je ovlivňují. Dále je předmětem výzkumu produkce vzhledem k využití produkčního faktoru- práce. Pomocí produkční analýzy je zkoumáno, zda-li společnost efektivně využívá svých zdrojů a maximalizuje tím svůj zisk.

## Summary

This thesis focuses on econometric analysis in an airline company and its application possibilities for efficient running of such organization. First theoretical part describes the current situation on the aviation market, characteristics of an airline provider, particularly revenue-cost structure and performance indexes used in the airline industry. Then it describes theoretical assumptions and methods for satisfactory application of econometric model. The second part of the thesis is the case study, based on real data sourced from the company Czech Airlines. The subject of analysis is the relationship among items of revenue and cost structure and the most affecting factors. Further the production is analyzed with respect to the labor as a production factor. By using production analysis it is surveyed whether the company effectively allocates its resources and maximizes its profit.

## Klićčová slova <br> Letecký dopravce • výnosy • náklady • ukazatele výkonosti • ekonometrický model • produkční funkce - prognóza • Czech Airlines

## Key words

Air carrier • revenues • cost • performance indexes • econometric model $\bullet$ production function $\bullet$ prognosis • Czech Airlines

## Table of Contents

1. Introduction ..... 4
2. Objectives of thesis and methodology ..... 6
2.1 Objectives ..... 6
2.1 Methodology ..... 7
3. Literature overview ..... 8
3.1 Current situation in the airline industry ..... 8
3.1.1 Flight network ..... 8
3.1.2 Airline types and strategies ..... 9
3.1.3 Strategic alliances ..... 9
3.1.4 Technology ..... 11
3.1.5 Airline and economic development ..... 13
3.2 Air Carrier ..... 14
3.2.1 Financial and operating indexes ..... 14
3.2.2 Revenue and cost structure ..... 17
3.3 Applied econometrics ..... 20
3.3.1 Production econometrics ..... 20
3.3.2 Econometric modeling ..... 24
4. Case study ..... 33
4.1 Czech Airlines ..... 33
4.2 Analysis of production ..... 37
4.3 Revenue-cost analysis ..... 47
5. Discussion ..... 57
6. Conclusion ..... 60
7. Bibliography ..... 61
8. Supplements ..... 63

## 1. Introduction

The airline industry is one of the most rapidly growing parts of world economic system, although it is also one of the most sensitive industries in terms of any unexpected deviations. It is an industry which is strongly dependent on oil price development, exchange rate development and many other aspects, which are parts of entire economic environment. The overall economic recession in recent years made many airlines to change the way of making its business or even go bankrupt. One of them was for example Sky Europe or Alitalia. These airlines and many others that took up struggle or even ended had a strong presence at the Czech market. Therefore it is quite obvious that these troubles must have been same for Czech national carrier, Czech Airlines. The financial crisis affected millions of people, who are the customers and if the customers are in trouble, airline is in trouble as well. According to The International Air Transport Association (IATA) airline industry suffered an overall loss of \$9.4 billion. IATA is an international association gathering most of the air transportation providers with the aim to improve the quality in aviation industry. The airline members of IATA cover approximately about $95 \%$ of total air transportation in the world. The most affected regions were North America and Europe, where is concentrated most of the air transportation providers. The domestic market in North America covers approximately $30 \%$ of total world supply, while the European domestic market covers around $10 \%$. Additionally the international market between North America and Europe brings $12.6 \%$ to the total amount of air transportation in the world. However not only Europe and North America suffered but also fast growing economies of Asia. The domestic air transportation within Asia covers similar share of $10 \%$ as Europe and it is growing rapidly. In terms of international market it is around $7 \%$ for Asian-European routes and $6 \%$ for Asia and North America. According to the United Nations forecast it is expected that the volume of transportation will increase four times between 1990 and 2050. As the world is getting more globalized transportation and infrastructure plays more important role.

During the time of the crisis it is complicated for the airlines to perform on desired level. The customers put pressure not only on airlines but on every company by saving money. This money saving lower the prices at the very bottom, often even under
the break even minimum. This case has happened in airline industry as well. The airlines were forced to lower the prices of tickets on the level, which might have seemed even ridiculously low to attract the new customers. And even this approach did not help in many cases. Therefore the key issue to work on during the crisis became cutting the costs. Each company deals with three basic fundamental tasks. The first one is obtaining the production factors such as land, labor and capital. Second one is combining these factors to produce a certain product or service. And finally it needs to be sold on the market. The second point becomes the most crucial one in this matter. It can be assumed that everything can be sold on the market under certain conditions. However these conditions need to be achieved. Companies need to allocate its resources to produce in the most effective way. Airline business is one of the most complicated businesses in general. It is not about producing one or more product like in most of other companies, but it is about providing the complex set of services on international level. Coordinating such business requires very sophisticated skills in economic, political and social areas. At the same time the growing airline industry and world trends set some goals to meet the requirements of the customers. First of all it is the never-ending necessity to improve the safety and security of air transportation. Another important issue is the environment, which has already reached critical figures in many areas. Airlines as producers are strongly required to minimize the impact of their business on the environment. The negative effect of economic recession has shown probably the major weaknesses of current business organizations. Therefore they try redeveloping and running more efficiently. The aircraft producers are strongly required to develop large seat-capacity airplanes, available to fly long distance with very efficient characteristics at the same time. For those purposes airports are forced to be modified to be able to meet such needs. As the airline industry grows, the frequency and volume of aircrafts in the sky rises. So the air navigation control centers are required to provide safe and fluent traffic for the airplanes. Such and many other trends influence all airline related industries and most of all airlines themselves. Perspective airline needs to be flexible and able to follow these trends in real time. This requires the airlines to work continuously on improving internal processes aimed to maximize revenue, minimize cost and therefore have a maximum profit. If such requirements are fulfilled, the company value rises and equity of shareholders as well.

## 2. Objectives of thesis and methodology

### 2.1 Objectives

The objective of the thesis is to analyze the relationships among individual items of cost and revenue structure of Czech Airlines. That consists particularly of how those individual items affect the total cost and revenue respectively. The other particular aim is to analyze whether the company properly allocates its resources, namely labor. The main purpose of the thesis is to show how to improve efficiency and maximize the output in the company by using techniques of econometric modeling.

The first hypothesis is that Czech Airlines improperly allocates one of its production factors- labor, so that it does not maximize the output of the company. The hypothesis will be confirmed or rejected, based on the results of analysis of production of the company.

Following hypothesis regard second part of the case study, which is the econometric model. It is assumed that according to the economic theory, revenue and cost have the simultaneous development. If the amount of production is increased so as the revenue, it is usually followed by increasing cost for material, labor and inputs generally. And this is actually assumed to happen in reverse as well. So the basic hypothesis is that total revenue and total cost of the company is increasing or decreasing at the same time.

Last two hypothesis regard individual items, influencing cost and revenue. First one is the number of passengers as the most affecting factor of total revenue. It is expected that increasing number of passengers brings growth in revenue while decreasing number of passengers brings the decline. On the cost side it is expected that the most affecting factor is the aviation fuel and therefore is the major item of company's expense, which is the last hypothesis.

### 2.1 Methodology

The first thematic chapter literature overview is rather theoretical background and has two main subchapters. The first one describes current situation in the world airline industry. It briefly describes the basic airline's operations, types of airline providers and their strategies. Then it is focused on air carrier rather from internal viewpoint. That consists of characterization of indexes used to measure the performance and description of cost/revenue structures within airline company.

Second part of the literature review briefly introduces those essentials of econometrics that have been later used in the case study. There are described methods and characteristics of production function and econometric model as main tools of the analysis. The parts of the text in the thesis which are italicized are quotations. The source of quotation is noted in the square brackets following the quoted text part. Those text parts or tables, where predominantly one source has been used, are quoted in parentheses in smaller font after the paragraph or below the table.

The second thematic chapter is the case study of Czech Airlines, which analyses the resource allocation, namely production factor- labor. For that purpose was constructed production function based on time series with yearly basis and 13 observations. For analyzing the basic trend of development of airline industry both linear and non-linear functions have been used. Specifically it was the linear function and power function. The more appropriate one has been chosen based on the value of determination coefficient. The second part of the case study analyses the relationships between items of cost/revenue structure of the company. For the purposes of the analysis the econometric model was made. It is based on time series with monthly data starting in December 2007 ending by April 2010, which makes 29 observations. The complete data panel is available in the attachment. The software used for calculations was Microsoft Office Excel. For the parameter estimation of the econometric model two-stage least squares and method of variance ratio minimization have been used. The more corresponding one to the basic economic assumptions has been chosen and the results interpreted.

## 3. Literature overview

### 3.1 Current situation in the airline industry

### 3.1.1 Flight network

First of all it is important to realize what actually the product of any airline is. Whether it is passenger or cargo oriented airline the final output is service. In other words transportation from the point of origin to the point of final destination. Generally the demand for transportation is rising all over the world as it is getting more globalized. To connect all of these points where the demand raises airlines create networks. To build a sophisticated network that would provide opportunity to realize maximum profit is the key issue of every airline. The basic network and simplest network is called Hub and Spoke. Hub and Spoke system is simply the transportation from the point of origin to the point of destination through transit station, which is called Hub. This is traditional and from historical point of view most often used model of network. The largest hub airport in the world is Atlanta, Georgia in the United States. This airport has become the hub airport for Delta Airlines long time ago. Other important hubs are for example in Paris, serving for Air France-KLM, London for British Airways, Frankfurt for Lufthansa, Seoul for Korean Airlines and many others. Regarding cargo oriented hub airport the biggest one is in Memphis, Tennessee which serves for FedEx Express, which is the largest air freight forwarder in the world. However recent development showed that this network model is not the most effective one because it does not follow the trends of globalization. The world is no more concentrated in a few areas in terms of strong economic nations and it is also much more open for international trade and movement these days. So this model of network becomes inefficient and not able to follow this trend in terms of covering the rising demand. Therefore there has been rising the necessity for airlines to create line networks. For such model of network is typical that the spokes are also connected and the transportation does not have to be provided through the transit stations. This model started to appear in case of low cost airlines in recent decades. Low cost airlines provide particularly point to point transportation for short distances with higher frequency. They
tend to connect points where is great demand is generated, but not necessarily concentrate it to one point such as hub. Other airlines started focusing on this as well and even if it is not easy to make all airports in the network equal they have been making progress so far. They try to move some parts of their business to other airports and make at least several hub airports and other as spokes. Most of the world leading airlines added at least one more hub airports in recent decades. Delta Airlines added New York, Lufthansa added Munich, Korean Airlines added Los Angeles etc.

### 3.1.2 Airline types and strategies

Regarding the network model has been mentioned the low cost airline. In the past there was one traditional model of airline which had similar characteristics for all. But recent trend shows that airlines are getting more differentiated to meet the specific needs of customers. The target group of low cost airline is the price oriented group of the market. Typical representative is for example Irish Ryanair. Such airline focuses on minimizing the cost in terms of lowering the quality of transportation in order to minimize the price of the ticket. In the aircrafts there is usually only one class, no refreshment, cabin crew represent more functions, airports used are not the main hubs, cargo transportation is limited etc. The other group of airlines is targeted on timesensitive customers. German Lufthansa is one of such airlines representing this group. Lufthansa tries to cover all the local markets and provide very reliable connections and maximum frequency. British Airways is one of the companies representing an airline targeted on business customers. The aim is to ensure the maximum comfort and quality of the services not only during the flight but for all the actions that need to be taken before the flight. Obviously the target group is the higher class society. Finnair is the airline focusing on transferred passengers on long hauls to Asia. Due to the location of its hub airport in Helsinki, where pass most of the flight paths on the way from Europe to Asia, it is quite a competitive advantage. All the flights are therefore shorter by several hours.

### 3.1.3 Strategic alliances

Even if most of above mentioned airlines are markedly differed, they all have a common goal to achieve. Therefore they have been tending to gather into strategic
alliances in a recent past. Airline alliances are being established for many reasons. The essential of any alliance is the cooperation between its members. Cooperation between airlines started already approximately thirty years ago in 1980's. The most typical one for airline industry is called code sharing. Every airline must have a code according to the IATA. This code is usually shown in the flight number by two first digits. To easily describe how code sharing works it can be considered two airlines operating between two cities. The cities can be for example Atlanta and Prague, representing the hub airports of Delta Airlines and Czech airlines respectively. It is assumed both airlines operate one flight a day which is usually half empty of passengers. In that case it is more effective to send only one aircraft a day and rent the empty seat capacity to the second provider. In that case there is one more flight number for only one real flight, in this case Czech Airlines and Delta Airlines flight number for one flight. This is just a simple example and in reality the airlines can share even ten or more codes for one flight. Therefore it is very usual when a passenger buys a ticket from a certain airline company but the flight is actually operated by another airline. On the same concept are built the alliances. They try together to create a network that would spread an area as large as possible. This fact also refers to structure of the alliances. They are usually composed of few world leading airlines and smaller ones. However the big ones within one alliance are usually from different geographical areas. For instance SkyTeam alliance has Delta Airlines covering the US market, Air France-KLM for European market and Korean Air for Asian market. Star Alliance has United Airlines in the US, Lufthansa for Europe, Asiana Airlines for Asia. Oneworld alliance leading members are American Airlines in the US, British Airways for Europe and Cathay Pacific for Asia. Table 3.1.1 shows the structure of three biggest airline alliances.

|  | Star Alliance | SkyTeam | Oneworld |
| :--- | :---: | :---: | :---: |
| Passengers per year | 627.52 million | 385 million | 335.534 million |
| Destinations | 1172 | 898 | 871 |
| Revenue | \$156.8 billion | $\$ 97.9$ billion | $\$ 89.875$ billion |
| Market share | $29.3 \%$ | $20.6 \%$ | $23.2 \%$ |
| Members | Adria Airways <br> Aegean Airlines <br> Air Canada <br> Air China | Aeroflot <br> Aeroméxico <br> Air Europa <br> Air France | American Airlines <br> British Airways <br> Cathay Pacific <br> Finnair |


|  | Air New Zealand ANA <br> Asiana Airlines <br> Austrian Airlines Blue1 <br> BMI <br> Brussels Airlines <br> Continental Airlines <br> Croatia Airlines EgyptAir <br> LOT Polish Airlines Lufthansa SAS <br> Singapore Airlines <br> South African Airways Spanair <br> Swiss Air Lines <br> TAM Airlines <br> TAP Portugal <br> Thai Airways International Turkish Airlines United Airlines US Airways | Alitalia <br> China Southern <br> Czech Airlines Delta Kenya Airways KLM <br> Korean Air TAROM <br> Vietnam Airlines | Iberia Japan Airlines <br> LAN <br> Malév <br> Mexicana Qantas <br> Royal Jordanian S7 Airlines <br> Canadian Airlines |
| :---: | :---: | :---: | :---: |

Table 3.1.1- Strategic alliances in airline industry (sourced from wikipedia.org)

### 3.1.4 Technology

The basic mean to run an airline business is the sophisticated technological equipment. In terms of an airline the first thing that comes up everyone's mind is the aircraft. Aircraft fleet is the crucial factor that determines the airline's network and structure. The current situation on the market of aircraft manufactures is that only two big aircraft producers are in charge of entire industry. Boeing and Airbus industries cover nearly entire market and they are leading manufacturers of commercial aircrafts in the world. The requirements for aircraft producers are getting more important in recent years and without flexibility manufacturers would not be able to succeed on the market. The current trend requires the aircrafts to offer most economic solutions, providing highest revenue and lowest cost. Therefore emphasis is placed on high seat-capacity aircrafts. Boeing had been in charge for several decades with B747 Jumbo, which was the largest commercial aircraft in the world at that time. Passenger version can carry approximately 500 passengers, depending on the fare class structure. Cargo version is able to freight more than 100t and both versions are able to pass around 12000 km distance fully loaded. However several years ago Airbus A380 started being sold for commercial purposes and became the biggest one. It can carry approximately 800 passengers and able to reach 15000 km distance. The fight between these two aircrafts
is expected to become key issue on the aircraft market in future decades. First of all, the higher seat capacity the more tickets can be sold which brings higher revenue to the prejudice of higher cost. If the aircraft cannot offer large seat capacity it needs to be more effective in terms of economic characteristics. These are the two main strategic ways which became driving forces for the market leaders. Airbus industries focus on development of high seat capacity aircrafts, while Boeing tries to offer most economic solutions. Boeing is currently working on development of new B747 made of specific material that would extremely lower the weight and required much less fuel for transportation. Both of these machines are wide-body aircrafts with long distance range. Long distance flights bring the highest average revenue and therefore are the main source of company's income. However not many airlines are located in destinations that could create such a huge demand from one point. Therefore they use narrow body aircrafts at the regional or local level to concentrate passengers at the hub. According to the analysis of Boeing, Current Market Outlook 2005, there was 16800 aircrafts in total in the world fleet in 2004. Narrow body aircrafts make $53 \%$ of the total number of aircrafts. The prognosis for 2024 says that total number will increase, almost double to 35300 machines. However narrow body aircrafts will participate only by $36 \%$. Figures in the graph 3.1.1 describe the structure of aircraft types in 2004 and expected in 2024 according to Boeing analysis.

2004



$$
\begin{aligned}
& \text { Narrow body aircrafts (<50t) } \\
& \text { Middle body aircrafts (40-65t) } \\
& \text { Wide body aircrafts (>65t) }
\end{aligned}
$$

### 3.1.5 Airline and economic development

Aviation industry is very closely related to the economic development of the country. The economic activity can create the demand for air transport. The impact on aviation is particularly obvious in case of developing or transition economies. Airline industry helps such countries to integrate into global infrastructure which brings further economic development. 10\% rise in connectivity, relative to a country's GDP, will boost labor productivity levels by 0.07\% [Sourced from Žihla Z.: Provozování leteckých podniků a letišt']. In the following table 3.1.2 we can see economic rates of return from aviation investment for several developing economies from different geographical areas.

|  | Kenya | Cambodia | Jordan | El Salvador | Jamaica |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Investment (USD <br> million) | 351 | 538 | 360 | 488 | 168 |
| Increase in <br> national <br> connectivity/GDP | $59 \%$ | $46 \%$ | $55 \%$ | $35 \%$ | $28 \%$ |
| Impact on <br> GDP(\%) | $0.42 \%$ | $0.32 \%$ | $0.39 \%$ | $0.25 \%$ | $0.20 \%$ |
| Impact on <br> GDP(USD million) | 209 | 100 | 100 | 85 | 26 |
| Annual Economic <br> rate of return | $59 \%$ | $19 \%$ | $28 \%$ | $16 \%$ | $16 \%$ |

Table 3.1.2- Economic rates of return from aviation investment (sourced from iata.org)

The provider of airline transportation is generally the key subject in aviation business. All other commercial units are strongly related to the airline business however they often stand and operate as superior unit, such as air navigation services or other supporting business providers. To operate such business company must meet a lot of requirements in several areas. Air traffic operations including proper planning of flight schedule and flight crews. The airline must be able to ensure technical background for its operations, which consists particularly of reliable, safety, secured and environment friendly aircraft fleet. By commercial services as marketing or sales must be able to sell its product or service. Ground handling services include all the supporting activities from the operational point of view which are necessary to run an airline business such as, ramp maintenance, fueling, catering, loading, check-in, cleaning, etc. The
management of the company must be able to make important decisions about investing in the aircraft fleet, flight network or other financial opportunities. And finally quality control is necessary to check the smooth running of the business from the independent viewpoint and indicate any irregularities that could possibly threaten the safety, quality or economic result that the company is trying to reach. One of these areas regarding the revenue and cost structure within the airline company is in details described in the following subchapter.

### 3.2 Air Carrier

### 3.2.1 Financial and operating indexes

To evaluate and compare the economic performance in the airline industry are used many indexes. Although all the airlines operate in the same or similar area on the first sight, they are in fact very different. As it was mentioned they differ by network structure, by internal organization, aircraft fleet, location and many others. Only the fact that they are located in different countries is the reason of different standards, methods and procedures in the companies even if they seem to have similar characteristics. For those reasons the economic indexes used in the airline industry are highly standardized and as much universal as possible so they could offer an objective sight to the independent observer. Essentially the indexes are divided into two subcategoriesfinancial and operating indexes.

## Operating Indexes

## - Break-even load factor

It is similar to the break-even point in production economics of the company. If the airline operates at this level of utilization, it means that the transportation revenues are equal to the operating cost. At this point company operates at zero economic profit. The break-even load factor is usually in the range from 50$80 \%$.

## - Block time

It is the time during the aircraft is actually in operation. That includes from the moment when it leaves the standing point by the terminal at the airport of origin until it arrives to the standing point by the terminal at the airport of its final destination. This indicator is widely used to measure the average utilization of the aircraft. Obviously the airlines that focus on long distance hauls have higher aircraft utilization than those specializing on short distance flights.

## - Distance flown

Is simply the distance between the airports which the aircraft needs to pass. This indicator is often used for some time period for the entire network. Moreover it is important index for calculating following indexes.

## - Available seat kilometer/Available ton kilometer

ASK is one of the basic airline indexes. It is calculated as a flown distance multiplied by seat capacity of the aircraft. For example if there was 100 seat capacity airplane on the 200 km distance, ASK would be equal to 20000 . This measurement takes into consideration the aircraft fleet structure. Generally together with available ton kilometers it is considered to be as a product of airline company. However airlines offer to sell not only seat but also space in lower deck in the cargo compartment. For those purposes every seat is converted into weight and ATK measures the overall available capacity including both passenger and cargo side

## - Revenue seat kilometer/Revenue ton kilometer

RSK is similar to the ASK, however in this case it is the number of seats that have been actually occupied multiplied by the distance. Similarly RTK is the number of tones that have been accommodated multiplied by the distance.

## - Average stage length

Average stage length is the total distance flown divided by the number of flight segments. This measure is used to evaluate the airline productivity according to their network.

- Load factor

Passenger or weight load factor is the share of RSK on ASK or RTK on ATK respectively. It is the basic index measuring the airline performance. The average load factor of world leading airlines is in the range $60-80 \%$.

## Financial indexes

- Traffic revenues/Other revenues

Or also called transportation revenues or operating revenues represent the sum of all income from operating activities. All revenues from other activities including financial revenues are other revenues or non-operating revenues.

## - Operating costs/Other costs

Operating costs are those which include expenses related to transportation. If the overhead costs are not included, they are called direct operating costs which will be described later in this chapter. Overhead costs are sometimes considered to be a part of all other costs which are usually represented by taxes, lease payments or other fees

## - Yield per passenger/ton of cargo

If the total operating revenues are divided by the number of revenue seat kilometers or revenue ton kilometers the result is the yield per one passenger or one ton.

- Cost per ASK/ATK

Is the number of total cost divided by number of available seat kilometers or available ton kilometers respectively.

### 3.2.2 Revenue and cost structure

In the past the airline business was quite specific element of the country's economy. There was a concept of national airline, which stated indirectly that one airline within the country had actually monopoly position. All the financial problems that raised by such airline were solved and covered by government. Airlines therefore were not forced to form some strategies, alliance or generally operate at the maximum efficiency and productivity. However in recent decades the sky has opened and the airline market has been proceeding through huge liberalization. This concept has been broken which brought to the airline business many new things. Nowadays if the company gets into financial troubles it needs to solve on its own. No one can expect any help from government or external parties. Even if it is often in national interest it is strictly prohibited in developed countries like the EU and the supervising authorities watch over these affairs. This approach forces the airline providers to cooperate with each other, form alliances, create different strategies, differentiate the product, etc. That is why the market noticed such great boom of low cost airlines, cargo airlines, full service airlines and others in last decades. These all things are basically intended to maximize the profit, which means to minimize the cost and maximize the revenue. By doing and improving in this matters airlines resist the unwanted external effects like for example competition, aviation fuel price, etc. The profitability in the airline industry is actually very low and strongly cyclical. In the following subsection will be described the revenue and cost structure within the airline company.

## Revenues

The total revenues of the airline consist of three main subcategories, which are operating revenues, non-operating revenues and other revenues.

## - Operating revenues

Source for operating revenues is the income from operating activities, which means regular domestic and international air transport as well as charter and cargo operations.

## - Non-operating revenues

These kinds of revenues are sourced from supporting or supplement activities of the company, which can be ground handling services, catering, crew training etc.

- Other revenues

Other revenues are essentially financial revenues, which include particularly positive interest or differences gained from currency exchange, etc.

Following table 3.2.1 shows the total revenue per one revenue ton kilometer as well as the cost per one available ton kilometer and the weight load factor by selected airlines with different characteristics.

| Airline | Revenue per RTK | Cost per ATK | Load factor |
| :---: | :---: | :---: | :---: |
| Units >> | USD | USD | \% |
| Ryanair | 0.73 | 0.49 | 85.2 |
| easyJet | 1.12 | 0.86 | 81.4 |
| Southwest Airlines | 0.83 | 0.40 | 70.7 |
| Delta Airlines | 0.95 | 0.58 | 77.6 |
| British Airways | 1.00 | 0.60 | 75.6 |
| Korean Air | 0.53 | 0.37 | 71.4 |
| Czech Airlines | 1.19 | 0.80 | 62.0 |

Table 3.2.1- Financial measures of selected airlines 2005 (source: World airline report 2005)

## Costs

In this section subcategories of costs are described. Essentially for the purposes of airline providers they are divided into direct and indirect costs. Cost calculation is also highly standardized which offers to compare the performance of individual companies as in the previous table. Direct costs are those that are directly dependent on aircraft operations while indirect costs are not. The items of direct and indirect costs are as follows:

## Direct costs

- Wages of pilots and cabin crew
- Aviation fuel
- Airport fees
- Air navigation fees
- Aircraft maintenance
- Depreciation
- Lease payments
- Aircraft insurance


## Indirect costs

- Station costs (for operations at different airports)
- Handling (check-in, loading)
- Customer service (onboard/transit services)
- Sales/reservation (distribution, inventory)
- Commissions (overriding commission for agents or airlines)
- Promotion (advertising)
- Overhead costs (administration)
- Other costs

This is the basic categorization of operating costs however in reality the airline cost structures might slightly differ according to their characteristics. Some of the items of costs that are expected to have significant influence on total cost or total revenue are included in the case study. Such as cost structures might differ, share of those items on total costs might differ as well, according to the airline's nature and characteristics. The following table 3.2 .2 shows the different share of such items by low cost and full service airline. The table demonstrates the share of each of cost categories on total cost by low cost airline and full service airline in average.

| Category | Low Cost (\%) | Full Service (\%) |  |
| :---: | :--- | ---: | ---: |
| Cost Wages 10.2 <br>  Fuel 8.3 |  |  |  |
|  | Airport fees | 13.1 | 9.4 |
|  | Navigation fees | 9.3 | 12.1 |
|  | Maintenance | 13.9 | 4.1 |
|  | Depreciation | 0.5 | 7.6 |
|  | Lease payments | 19.1 | 2.6 |
|  | Insurance | 0.5 | 12.4 |


| Share of direct cost on total cost |  | 75 | 54 |
| :--- | :--- | ---: | ---: |
| Indirect | Station cost | 0.2 | 13.8 |
|  | Handling | 7.4 | 4.1 |
|  | Customer service | 1 | 6.4 |
|  | Sale/reservation | 4.3 | 4.7 |
|  | Commission | 0.2 | 7.9 |
|  | Promotion | 6.4 | 3.1 |
|  | Overhead cost | 4 | 4.4 |
|  | Other cost | 1.4 | 1.4 |
| Share of indirect cost on total cost |  | 25 | 46 |
| Cost total |  | 100 | 100 |

Table 3.2.2- Share of cost categories on total cost by low cost and full service airline (source: R.Doganis: The airline business in the $21^{\text {th }}$ century)

### 3.3 Applied econometrics

### 3.3.1 Production econometrics

Econometric analysis of production and costs consists of analyzing relationships between factors of production and product. From this relation four relationships can be derived which is factor- product, factor- factor, product- factor or product- product. This part of the thesis as well as the case study is focused on the first mentioned relation factor-product.

Production function is the technological causal relationship indicating amount of production which can be produced with the given inputs, so it is the relation with production factor on one side and output on the other side[Sourced from Pojkarová K.: Ekonometrie a prognostika v dopravě]. For the purposes of the thesis only one factor production function is described. However in reality more factor production functions are preferred, because they include the influence of more factors and become therefore objective. Production function with two production factors is represented as production surface. One factor production function can be simply demonstrated as relation 3.3.1.

$$
\begin{equation*}
\mathbf{y}=\mathbf{f}(\mathbf{x})+\mathbf{u} \tag{3.3.1}
\end{equation*}
$$

The total production is represented by ' $y$ ' and the amount of production factor by ' $x$ '. There are three possibilities of shape of production function depending on the
additional growth with additional units of input. Linear type of the function refers to the constant additional product. Degressive type refers to the decreasing additional product and progressive type refers to the increasing additional product. However the most used one referring to the economic theory is the neoclassical production function which is combination of progressive and degressive one. From the production can be derived other also other specific production functions necessary for the complex evaluation. First one is average production function. Average production function, average amount of production in physical units is determined by the relationship of gross production and total amount of production factor. It is such amount of production falling in average on one unit of production factor [Sourced from Tvrdoň J.: Ekonometrie]. Average production function can be determined as in the relation 3.3.2.

$$
\begin{equation*}
\mathbf{y}_{\text {ap }}=\frac{y}{x} \tag{3.3.2}
\end{equation*}
$$

As well in this case ' $y$ ' and ' $x$ ' represent the amount of production and production factor respectively. It can be notified as well that average production function is tangens of angle which is between horizontal axis and line connecting the inception and the certain point at the production function. The shape is therefore dependent on the production function. In case of neoclassical development the average production function is increasing until the point, where obviously the line between the inception and the point on the production function becomes tangent to the production function. From this point now on the average production function is decreasing. For the calculation where the function has its maximum can be used first derivation equal to zero or another method which will be explained later with relation to the marginal production function.

Marginal production function represents the relationship between additional amount of production and additional unit of production factor. It is the addition of gross production per a unit of factor addition [Sourced from Tvrdoň J.: Ekonometrie]. Mathematically it can be demonstrated as the relation 3.3.3.

$$
\begin{equation*}
\mathrm{mP}=\frac{\Delta y}{\Delta x} \tag{3.3.3}
\end{equation*}
$$

From the above formula is obvious that the marginal production function can be calculated as a first derivation of production function. The shape of the marginal production is increasing until the so called inflex point. Inflex point is the point where the shape of total production function changes from progressively increasing to degressively increasing. At this point marginal function has its maximum and after that is decreasing. Since the marginal production function is the first derivation of total physical production function it can be easily estimated that if marginal production is equal to zero, total production function is at its maximum. Each additional unit of production factor brings negative additional product which means that the production function is decreasing. As it has been already mentioned the maximum of average production function can be also calculated by using marginal production function. Since the average production function is tangens of the angle between horizontal axis and the line linking the inception and the specific point of the production function, at its maximum it is actually the tangent. And mathematically tangent is actually first derivation of the function. Therefore the maximum of average production function is at the point where the amount of average and marginal production is equal.

The calculation of these specific points is important for determining the rational stages of production. There are three stages of production that are specifically identified and have different characteristics regarding rationality.

- 1. Stage- the range of this stage starts at the very beginning and continues until the point where average production function has its maximum. This stage is characterized as irrational. The stage is irrational because the resources are not properly allocated and the production is not maximized to the end point of efficiency [Sourced from Tvrdoň J.: Ekonometrie].
- 2.Stage- the following second stage continues from the point where is the top of the average production function until the point where is the top of total production function. This stage is characterized as rational one. During the second stage the maximum productivity of labor and maximum amount of additional production is reached, although the
marginal and average production are decreasing [Sourced from Tvrdoň J.: Ekonometrie].
- 3.Stage- the last stage of production continues further starting at the top of production function. Obviously this stage is irrational because with increasing amount of input, the output is decreasing.


## Production elasticity

The stages of production are also highly related to the production elasticity. Production elasticity is the ratio of change in output to the change of input. Specifically it means by how much percent the total product is changed in case that production factor is changed by $1 \%$. The mathematical form of production elasticity (or any elasticity) is as in the relation 3.3.4.

$$
\begin{equation*}
\mathrm{P}=\frac{\frac{y 2-y 1}{\mathrm{y} 1}}{\frac{x 2-x 1}{\mathrm{x} 1}}=\frac{\Delta y}{\Delta x} \frac{x}{y} \tag{3.3.4}
\end{equation*}
$$

(Sourced from Hušek R.: Ekonometrická analýza)
Where variables with index 1 represent the original values of production and production factor respectively. Variables with index 2 represent new values of production and production factor. In case that the elasticity is higher than one, it means that if the production factor is changed by $1 \%$ the total production changes by more than $1 \%$ and the production is elastic. If the coefficient is equal to one, it is unit elasticity. Unit elasticity is performed only during the change from first stage of production to the second stage. The $1 \%$ change of input brings the same percentual change in output. It is the first stage of production function which is has the coefficient of elasticity higher than one. The second- rational stage of production has the coefficient of elasticity between one and zero. The production elasticity is low, it means $1 \%$ change of production factor brings less than $1 \%$ change in product. The elasticity is getting lower as it moves along the curve to the top. At the top of production function the elasticity is equal to zero. Change in production factor brings no change in output and that is the end of second stage. In the third stage the production is inelastic, because it is lower than zero.

## The profit maximizing output

The profit maximizing level of output is reached at the point, where the price of additional amount of production factor for last unit of production is equal to marginal revenue for last unit of production[Sourced from Čechura a kol.: Cvičení z ekonometrie]. Since marginal revenue for last unit of production is equal to the price of the product, it is obvious that profit maximizing level of output in case that marginal production is equal to the ratio of production factor price and price of the product as in the relation 3.3.5.

$$
\begin{equation*}
\mathbf{M P}=\frac{P x}{P y} \tag{3.3.5}
\end{equation*}
$$

### 3.3.2 Econometric modeling

Econometrics is the application of statistical techniques and analyses to the study of problems and issues in economics [Sourced from Barreto H.: Introductory econometrics]. That is probably the most simplified definition of isometric modeling. The main goal of econometrics is to model and simulate real economic situations by using statistical and mathematical techniques. The econometric model is based on the economic model which emerges from the economic theory. The economic model changes to econometric model at the time when the stochastic variable is included in the model, which is also the basic difference between economic and econometric model. There are several types of variables apart from the stochastic one, which are included in the model.

- Endogenous (explained)- endogenous variables are those that are explained by the model. $\beta$ is usually the parameter related to endogenous variable.
- Exogenous (explaining)- those variables that explain the endogenous variables. Parameter related to exogenous variable is usually $\gamma$.
- Predetermined- set of all exogenous variables, exogenous lagged variables and endogenous lagged variables in the model
- Stochastic variable- it is the difference between real and theoretical value which is caused by errors in calculations


## The linear regression model

The multiple linear regression model is used to study the relationship between a dependent variable and one or more independent variables [Sourced from Greene W.H.: Econometric Analysis]. The form of the linear regression model is as in the relation 3.3.1.

$$
\begin{equation*}
\mathbf{y}=\mathbf{f}\left(\mathbf{x}_{1}, \mathbf{x}_{2}, \ldots, \mathbf{x}_{\mathbf{k}}\right)+\mathbf{u}=\boldsymbol{\beta}_{1} \mathbf{x}_{1}+\boldsymbol{\beta}_{2} \mathbf{x}_{2}+\ldots+\boldsymbol{\beta}_{\mathrm{k}} \mathbf{x}_{\mathrm{k}}+\mathbf{u} \tag{3.3.1}
\end{equation*}
$$

Endogenous variable is represented by ' $y$ 'and exogenous variables by ' $x$ '. There are several assumptions that need to be fulfilled if the model generated parameters are supposed to be correct. The assumptions of classical linear regression model are following:

- Linearity- the model specifies a linear relationship between y and $\mathrm{x}_{1}, \ldots$, $\mathrm{x}_{\mathrm{k}}$.
- Full rank- there is no exact linear relationship among any of the independent variables in the model. This assumption is necessary for estimation of the parameters of the model.
- Exogeneity of the independent variables- this states that the expected value of the disturbance at observation in the sample is not a function of the independent variables observed at any observation, including this one. This means that the independent variables will not carry useful information for prediction of $u$.
- Homoscedasticity and nonautocerrelation- each disturbance has the same finite variance and is uncorrelated with every other disturbance. This assumption limits the generality of the model.
- Exogenously generated data- the data of exogenous variables may be any mixture of constants and random variables. The process generating the data outside the assumptions of the model- that is independently of the process that generates $u$.
- Normal distribution- The disturbances are normally distributed

Classical linear regression model assumptions(Sourced from Greene W.H.: Econometric Analysis)

## Multicolinearity

When there is a high correlation between two explanatory variables in the model than there is a presence of multicolinearity. If the multicolinearity is perfect in the sense of the regression coefficients of the $X$, variables are indeterminate and their standard errors are infinite. If multicolinearity is less than perfect, the regression coefficients, although determinate, possess large standard errors (in relation to the coefficients themselves), which means the coefficients cannot be estimated with great precision or accuracy [Sourced from Gujarati D.: Basic Econometrics]. The presence of multicolinearity can be identified by using correlation matrix. Presence of high multicolinearity can be reduced by including dummy variable, vector of differences between real and average value or complete exclusion of the variable.

## Ordinary least squares

In case the model fulfills all the assumptions it can be finally processed the estimation of parameters. The method which is used for the this procedure is called ordinary least square since it is based on minimizing of the sum of squares of differences between theoretical and real value of endogenous variable. For that purpose can be used the formula 3.3.2

$$
\begin{equation*}
\gamma=\left(\mathbf{X}^{T} \mathbf{X}\right)^{-1} \mathbf{X}^{T} \mathbf{y} \tag{3.3.2}
\end{equation*}
$$

where $\gamma$ is the vector of estimators
$X$ is the matrix of exogenous variables
y is the vector of endogenous variable

## Hypothesis testing

After all estimators are calculated they need to be tested whether they refer to the assumed hypotheses. There are three steps of testing- economic testing, statistical testing and econometric testing.

- Economic testing- the hypotheses that are formulated with regards to the parameters of the model are tested whether they respond to the basic economic assumptions. This basically includes intensity and direction of the influence of variables on each other.
- Statistical testing- statistical testing confirms or rejects the hypothesis regarding statistical significance of estimators
- Econometric testing- econometric testing includes all the above mentioned assumptions for classical linear regression model

Economic testing of the estimated model is based on the economic assumptions and criteria and it is necessary for economic interpretation and usage of the resulting estimators. The testing actually consists of verification of plus/minus signs and the size of numerical values estimated parameters. In case the calculated values are in accord with expected values regarding plus/minus signs and the size of numerical values of parameters, the model can be considered to be in accord with theoretical economic assumptions and is a simplified depiction of real economic problem or system [Sourced from Hušek R.: Ekonometrická analýza].

Statistical testing consists of determining the coefficients of correlation and determination and statistical significance of the estimators. The correlation coefficient is
based on relation 3.3.3. $\mathrm{Su}^{2}$ represents the residual variance which is the sum of square differences between theoretical and real values divided by number of observations in the model. Sy ${ }^{2}$ represents the total variance which is the sum of square differences between average and real values divided by number of observations.

$$
\begin{equation*}
\mathrm{R}=\sqrt{1-\frac{S u 2}{S y 2}} \tag{3.3.3}
\end{equation*}
$$

The result of correlation coefficient is in interval <-1; 1> and determines the tightness and dependence between variables. The closer the correlation coefficient is to zero the lower is the tightness and dependence between variables.

The determination coefficient is based on relation 3.3.4. $\overline{\text { S2u }}$ represents adjusted residual variance calculated as sum of square differences between theoretical and real values divided by degrees of freedom which is the number of observations minus number of explanatory variables in the selected equation.

$$
\begin{equation*}
\mathbf{R}^{2}=1-\frac{\overline{s 2 u}}{S y 2} \tag{3.3.4}
\end{equation*}
$$

The determination coefficient results in the range <0; 1> and determines how many percent of dependent variable is explained by exogenous variables.

For the testing of statistical significance of estimators several steps need to be done. First of all adjusted residual variance needs to be calculated which has been already done. Then the adjusted residual variance multiplied by matrix $\left(X^{T} X\right)^{-1}$ results in a matrix which has deviation of parameters on the leading diagonal. The square root of parameter deviation is the standard error. As a ratio of absolute value of parameter and standard error the $t$-value can be obtained. If the $t$-value is higher than the tabled $t$-test value for the selected level of significance the hypothesis is confirmed and the parameter is statistically significant.

The econometric testing consists of verification of specifications of the model, necessary for satisfactory application of specific econometric methods, tests and techniques. The purpose of assumptions is actually statistical testing, because with the help of these tests we determine validity and legitimacy of the statistical criteria,
particularly in case of small range of observation [Sourced from Hušek R.: Ekonometrická analýza]. The most often used methods of econometric testing are residual analyses, Durbin-Watson test indicating the presence of autocorrelation and RESET test.

## Simultaneous equation models

So far there have been mentioned only single-equation models. However in practice more equation models are used. Especially in macroeconomics where the economists try to describe the whole system, in which there are a lot of factors influencing each other. If there are several endogenous variables in the model, which have both endogenous and exogenous character, determined by the econometric model, the system is called simultaneous equation model [Sourced from Hušek R.: Ekonometrická analýza].

## Identification problem

Before the parameter estimation of the simultaneous equation model identification is necessary. By identification problem we mean whether numerical estimates of the parameters of a structural equation can be obtained from the estimated reduced-form coefficients. If this can be done, we say that the particular equation is identified. If this cannot be done, then we say that the equation under consideration is unidentified or under identified [Sourced from Gujarati D.: Basic Econometrics].

A necessary condition for identification of any structural equation is that the number of excluded exogenous variables from this equation are greater than or equal to the number of right hand side included endogenous variables [Sourced from Baltagi B.H.: Econometrics].

## Two-Stage Least Squares

Two-stage least square method is the most commonly used method for estimating parameters in a simultaneous equation system. The approach involves first using ordinary least square (OLS) method to estimate the reduced form equations. The predicted values of endogenous variables are then used in an OLS regression of the identified structural form equation of interest to estimate the parameters [Sourced from Ajmani V.: Applied Econometrics Using the SAS System].

In the first stage the theoretical values of endogenous variables in the equation are estimated according to the relation 3.3.5.

$$
\begin{equation*}
\widehat{Y}_{2}=\mathbf{X}\left(\mathbf{X}^{\mathrm{T}} \mathbf{X}\right)^{-1} \mathbf{X}^{\mathrm{T}} \mathbf{Y}_{2} \tag{3.3.5}
\end{equation*}
$$

Where Y2 ..... matrix of endogenous explanatory variables
$\mathrm{X}_{*} \ldots .$. matrix of predetermined variables included in the equation
$\mathrm{X}_{* *}$..... matrix of all predetermined variables excluding the equation
$\mathrm{X}=\left[\mathrm{X}_{*,} \mathrm{X}_{* *}\right]$ matrix of all predetermined variables in the model
The second stage consists of calculation of vector of structural parameters as in the relation 3.3.6.

$$
\left[\begin{array}{c}
\beta_{2}  \tag{3.3.6}\\
\gamma_{1 *}
\end{array}\right]=\left[\begin{array}{ll}
\hat{Y}_{2}^{T} \hat{Y}_{2} & Y_{2}^{T} X_{*} \\
X_{*}^{T} Y_{2} & X_{*}^{T} X_{*}
\end{array}\right]^{-1}\left[\begin{array}{c}
\hat{Y}_{2}^{T} \\
X_{*}^{T}
\end{array}\right] y_{1}
$$

The resulted estimated parameters are in the order firstly as in the matrix of endogenous variables and further in the same order as in the matrix $X_{*}$. The first of two sub matrices is called covariance matrix. The testing of the results is the same procedure
as in the case of OLS method except of the statistical testing, where the covariance matrix is used instead of $\left(\mathrm{X}^{\mathrm{T}} \mathrm{X}\right)^{-1}$ matrix.
(Sourced from Čechura L. a kol.: Cvičení z ekonometrie)

## Elasticity

The elasticity represents the percentual change, if the affecting variable changes by one percent. There are several types of elasticities, however they differ only according to the variable which affects and which is affected. For example in case of production elasticity we are talking about percentual change of production if the production factor changes by one percent. The basic one, often used in economic theory, is the income elasticity, which is the percentual change of consumption, caused by one percent change in income. Direct price elasticity is the percentual change of consumption if the price changes by one percent. Cross-price elasticity is the percentual change of consumption if the price of another commodity changes by one percent and many other. Nevertheless, all the elasticities are generated on following basis:

$$
\mathrm{E}=\frac{\partial y}{\partial x} \cdot \frac{x}{y}
$$

In this case study elasticity shows how individual items of the model affect each other. To show elasticities only one result of the two method procedure has been chosen, specifically the result of two stage least square method. Elasticity has been calculated for each period separately. To find out the general result there was made an average of all periods and calculated as well.

## Prognosis

One of the objectives of econometric modeling is prognosing or prediction of values of explained endogenous variables out of the range of observation. Econometric prognosis or forecast is the quantitative estimation of probability of future value of the
specific variable by using both past and present information, represented by the estimated econometric model [Sourced from Pojkarová K.: Ekonometrie a prognostika v dopravě].

There are several methods of forecast calculations, however for the purposes of the study only the basic one based OLS procedure is mentioned. The procedure consists of several steps. First one is to check whether the model is appropriate for econometric forecasting, which includes all the econometric assumptions and particularly calculation of standardized deviation as in the following relation 3.3.7.

$$
\begin{equation*}
\mathbf{N}_{\mathrm{it}}=\frac{\hat{y} i t-y i t}{s y i} \tag{3.3.7}
\end{equation*}
$$

Where $\hat{y}_{\mathrm{it}}$..... theoretical value of i - endogenous variable at the time t
$y_{i t}$...... real value of $i$ - endogenous variable value at the time $t$
$S_{y i}$..... standard error of i-endogenous variable based on square root of total variance

Standardized deviation can be calculated for whole model or all equations separately. In case it is equal to zero, it means model is perfect for forecasting and the prognosis is guaranteed. If it is equal to one, the forecasted value is same as the average value. If it is in the interval from zero to one, the model is appropriate for forecasting. In case it is more than one, the model is not appropriate for forecasting. The prognosis then is formulated as in the relation 3.3.8.

$$
\begin{equation*}
\widehat{\boldsymbol{y}}_{\mathrm{n}+\mathrm{j}}=\mathbf{M} \widehat{\mathbf{X}}_{\mathrm{n}+\mathrm{j}} \tag{3.3.8}
\end{equation*}
$$

Where n ..... number of observations
j ..... prognostic horizon
M ..... multiplication matrix

## 4. Case study

### 4.1 Czech Airlines

The company has been established as Czechoslovak state airlines on October $29^{\text {th }}$ 1923. In the beginning they provided passenger and cargo transportation within Czechoslovakia. In 1929 they became member of International Air Transport Association (IATA) and realized first international flight in 1930 to Zagreb. In 1937 was opened the new Prague airport in Ruzyně, which has become an airline's hub and remains until today. In 1939 at the time when the occupation started Czechoslovak airlines stopped operating and were closed during The World War $2^{\text {nd }}$ until 1945 when it was relaunched. In the postwar period the company started being oriented particularly on Soviet Union, regarding not only the destinations but also integrating soviet aircrafts into the airline fleet and others. From 1960s they started spreading its network to long distance destinations. Such destinations were for example Havana, New York, Montreal, Dubai, Ho Chi Minh City, Hanoi, Bangkok, etc. In 1991 Czech airlines purchased first western manufactured aircraft Airbus A310 and since then started to orient on aircrafts from these manufacturers. At the end of 1992 airline became a jointstock company where the majority shareholder was state and Air France, however this share was shortly after that in 1994 this share was bought by Czech Consolidation Bank. In 2001 Czech airlines became a member of SkyTeam, a second world largest airline alliance. In 2008 the company got into serious financial troubles. The world economic situation started escalating and the airline's debt was huge. Czech airlines started massive restructuralisation consisting of dividing the company into subsidiaries. Several of these subsidiaries have been further sold to at least partially cover the increasing debt. In spite of all this effort to erase the debt company's management still was not able to make the airline profitable, particularly due to bad cooperation with labor unions. At the end of 2009 the company was offered for sale however at the last minute labor unions accepted the management's requirements about huge wage cuts and Czech airlines was merged with the company Prague Airport.

Currently Czech airlines is maintaining hub and spoke structure of its network with the main center in Prague. The main focus is targeted on providing sophisticated
network for transit passengers between Europe and post-soviet countries in Eastern Europe and Asia. From June 2011 airline is going to launch a new long range flight to south East Asia to Hanoi to meet the needs of the market where is a demand due to the Vietnamese minority living in the Czech Republic. Czech Airlines is a full service based airline providing most of the traditional services that can be offered in airline business. Such services is for instance various range class fares to buy within business and economy class as well, onboard refreshment, business lounge and VIP services, assistance services, etc. In recent years the trend shows quick development in electronic services in airline industry generally, which Czech airlines successfully follows. There is a possibility not only to buy a flight ticket on the internet but to do a self checkin as well, choose a seat in the aircraft or even book the hotel or rent a car and other which all can be important factor to satisfy the customer needs.

The major shareholder of the company with more than $90 \%$ share is the Department of finance of Czech Republic. The share owners are shown on the graph 4.1.1. The merge of Czech airlines and Prague Airport was actually supposed to be just a temporary solution until the proper buyer would be found or government come with another solution. Second thing happened and in November 2010 was announced that new joint stock


Graph 4.1.1- Czech airlines shareholders(source: Yearly report 2010) company would be established integrating Czech Airlines and Prague Airport called Czech Aeroholding. Czech Airlines has currently focuses mainly on its own core business, which is air transportation only. For other activities that used to be related with the company in the past have been established several subsidiaries. CSA Services, s.r.o. provides supporting ground services necessary for airline operations. Czech Airlines Technics, a.s. ensures technical background for aircraft fleet. Czech Airlines Handling, s.r.o. provides ground handling services for the company as well as for airline
partners, particularly SkyTeam members operating in Prague. HOLIDAYS Czech Airlines, a.s. operates as a charter division cooperating with travel agencies. Czech Airlines CARGO, a.s. is the only operates as an independent commercial unit. Some of the subsidiaries have been already sold in the past due to increasing debt and lack of financial means. One of them was Czech Airlines Catering, providing onboard refreshment and meal services or Czech Airlines Duty Free operating duty free shops at Prague Airport. The following table 4.1 .1 shows the some of the economic figures of the company in 2009 and 2008.

|  | 2008 | 2009 |
| :--- | ---: | ---: |
| Sales(1000 CZK) | 22532908 | 19789620 |
| Operational earnings(1000 czk) | 696048 | -3539760 |
| Earnings before tax(1000 cZk) | 499149 | -3688239 |
| Earnings after tax(1000 CZK) | 470057 | -3765125 |
| Own equity(1000 CZK) | 101686 | -2352045 |
| Capital stock(1000 czk) | 2735510 | 2735510 |

Table 4.1.1- CSA group economic figures according accounting standards(source: Yearly report 2010)

As it is obvious from the table airline noticed almost $14 \%$ decrease in sales. This is most probably related to the sale and termination of lease of some of the aircrafts of the fleet (most of the company's aircrafts are not owned by Czech Airlines, but they are rented as an operational lease). By maintaining these activities company was able to balance the huge debt however it was just postponing the situation that came later. According to financial standards in 2009 company showed huge deficit in earnings and equity.

Regarding the aircraft fleet company has currently 38 aircrafts for its operations. Most of them are the machines manufactured in French Airbus Industries and US Boeing. At this moment Czech Airlines 18 aircrafts of Airbus, 8 aircrafts of Boeing and 12 aircrafts of ATR. ATR is an Italian-French aircraft manufacturer producing less seat capacity machines for short distances. The average age of aircrafts was 9,6 years at the end of 2009. All of the company's aircrafts are narrow body type. Czech Airlines used to own several wide aircrafts Airbus A310 however all of them have been sold or lease terminated. From June 2011 will become the part of fleet one wide body aircraft-

Boeing 747 freighter version. This jumbo machine will be used for cargo purposes to south East Asia, specifically Hanoi and Hong Kong.

In 2009 Czech Airlines operated in total 40989 flights in 115 destinations of which 69 are regular and 46 are charter destinations. The total distance that has been flown with the fleet was 84,5 million kilometers. The total number of passengers in 2009 was 5,46 million and number of revenue ton kilometers was 7,6 billion which means that the seat capacity has been utilized by $68,1 \%$. 813 tons of air freight has been forwarded. After converting into revenue ton kilometers and including cargo operations the utilization of total airline capacity was $56,6 \%$.

Czech Airlines employs 4172 people. However this number is related to the end of year 2009 and the company proceeded crucial changes in employee structure in 2010, but it is the latest information that has been published. On the table 4.1.2 is shown the structure of employees and compared with the number of employees at the end of the year 2008.

| Category | 2008 |  | difference |
| :--- | ---: | ---: | ---: |
| Mechanics/Workers | 1232 | 1105 | -127 |
| Management | 76 | 56 | -20 |
| Administrative | 1831 | 1662 | -169 |
| Cabin crew | 968 | 879 | -89 |
| Pilots | 535 | 470 | -65 |
| Total | $\mathbf{4 6 4 2}$ | $\mathbf{4 1 7 2}$ | $\mathbf{- 4 7 0}$ |

Table 4.1.2- CSA employee structure(source: Yearly report 2010)
If the available ton kilometer is considered as a company's product it is 300 ths ATK for one employee in 2009. That makes the company's productivity deeply below average, which is around 500 ths ATK for one employee. Although it is only 300 ths, Czech Airlines noticed very good progress, since in 2006 it was 224 ths ATK. The most productive airlines are generally Asian companies such as Singapore Airlines, Korean Air or Cathay Pacific. The following table 4.1 .3 shows the productivity of selected airlines.

| Airline | No of employees | ATK(1000 km) | Productivity |
| :--- | ---: | ---: | ---: |
| Air France | 60086 | 22748036 | 378,6 |
| American Airlines | 73495 | 40769688 | 554,7 |


| British Airways | 47936 | 22829354 | 476,2 |
| :--- | ---: | ---: | ---: |
| Cathay Pacific | 15806 | 17741385 | 1122,4 |
| Continental Airlines | 42263 | 17189355 | 406,7 |
| Czech Airlines | 5440 | 1221910 | 224,6 |
| Delta | 48537 | 29776444 | 613,5 |
| KLM | 25027 | 14641087 | 585,0 |
| Korean Air | 16544 | 18782446 | 1135,3 |
| Lufthansa | 92203 | 26608394 | 288,6 |
| Singapore Airlines | 14824 | 2286786 | 1 |

Table 4.1.3- Airline employee productivity 2006(source: IATA, WATS 2006)

### 4.2 Analysis of production

The production function is defined as the technological relationship production factor-product. In case of an airline the product is service. It can be expressed as one seat that is offered for transportation, however transportation of passengers is not the only business of the airline but transportation of cargo as well. In case of Czech Airlines cargo business is highly dependent on passenger business, because the company does not own any aircrafts in cargo version. Destinations where is the high demand for passenger transport are different from those where is the demand for cargo transport. For example in case of China, Shanghai, Hong Kong and Beijing are the top destinations in terms of passenger traffic and probably in cargo traffic as well. However recent trends noticed huge movement of industrial companies from the big cities to countryside or smaller cities. Most of the cargo oriented airlines have started to focus on smaller cities where foreign investors establish their facilities. Czech Airlines cargo division has very limited operational range in this matter, because passenger business has the number one priority. Nevertheless cargo is also source of company's income and must be included as a product. Loading capacity is different by aircraft. As it was mentioned in previous chapters TKM (ton-kilometer) is being used as an index of airline's capacities that can be offered. Tonnekilometer includes both offered seat capacity which is transformed into weight units and available space for cargo transport. ATK is an abbreviation for available ton-kilometer, it is actually the maximum TKM
that airline can offer for sale. Revenue tonnekilometer (RTK) is the number of TKM that have been actually sold. If the total number of revenue tonnekilometers is divided by total number of available tonnekilometers we get load factor, which is very significant index used in airline industry.

As a production factor labor was used. By one labor unit is meant airman- pilot. However the model can be applied for any other group of workers or for the total number of workers in the company. This group does not include cabin crew or flight attendants, only pilots. The reason why pilots have been chosen for the model is that they are most expensive labor unit of any airline company. Of course there might be several positions at top management level where the salaries are even higher, however they are not as of large number as the number of pilots. The salaries of pilots and aviation fuel are two largest items of cost of any airline company. The personnel cost in total in airline company have around $25-35 \%$ share of the total cost. In the table 4.2.1 is described the development of total number of pilots.

The data is sourced from the Czech airlines yearly reports in the period between 1997 and 2009. The number of available tonnekilometers increased during twelve years period by approximately $150 \%$. As the company's fleet structure was changing, additional pilots have been hired. From the table is obvious that the relative ratio of pilots to the available tonnekilometers is

| Year | Available ton <br> kilometer(ATK) | No of pilots |
| :---: | ---: | :---: |
| units >> | 1000 km |  |
| 1997 | 501215,4 | 253 |
| 1998 | 507699,5 | 272 |
| 1999 | 550689,3 | 302 |
| 2000 | 587929,6 | 327 |
| 2001 | 625308,4 | 341 |
| 2002 | 639795,3 | 331 |
| 2003 | 770306,9 | 386 |
| 2004 | 973014,9 | 424 |
| 2005 | 1207400,0 | 525 |
| 2006 | 1221910,7 | 524 |
| 2007 | 1289365,8 | 534 |
| 2008 | 1320328,4 | 535 |
| 2009 | 1251872,3 | 470 |

Table 4.2.1- ATK and pilot development(sourced from CSA internal reports) lower in 2009 than in 1997. This can be caused by the innovating technology, renewing the fleet that does not require so much labor unit. In 1997 there could have been 1981 ATKs offered by one pilot, while in 2009 already 2663,5 ATKs was offered by one
pilot. It means there is a progressive relationship between factor of production, which are pilots in this case and product which is available tonnekilometer. However only pilots cannot bring the additional units of output. The other production factor- capital, which is in this case number of aircrafts, is the limiting factor. Therefore the production function will be neoclassical type. At the beginning there will be increasing growth of production higher than one with each additional unit of production factor. After that in the second part of the production function as the production factor will be cumulated the additional output will start to decrease until the function achieves its top where the marginal product is equal zero. Following graph 4.2.1 shows the graphical form of production function.

## Production



Graph 4.2.1- Production function(source: own calculation)
Above graphics represents production function of Czech Airlines. On the horizontal axis there is production factor, in this case labor- pilots. On vertical axis it is the product, which is number of available tonnekilometers that the company can offer. The production function is represented by trend line. The equation of the function is displayed on the chart, it is polynomial regression type of third order. The coefficient of determination says $98.4 \%$ influence. The function has a classic development. In the first stage it is progressively increasing. The inflex point where the progressively increasing function changes to degresively increasing function is somewhere between 400 and 500
units of production factor as it is obvious from the graph. The calculation of the specific point of change is shown later. The production factor reaches its top between 500 and 600 units of labor. At this point the production factor- labor is highly concentrated and there is a lack of other production factor. The production function is the key function to determine other partial functions which are necessary for complex evaluation of production. One of the ways to calculate the coordinates of the point on the top of the production function is setting the first derivation of production function equal to zero as it is shown in relation 4.2.1. Resulting labor units give the maximum product.

$$
\begin{equation*}
Y^{\prime}=-0.2775 x^{2}+222.2 x-39957 \tag{4.2.1}
\end{equation*}
$$

One of the roots of the equation in the required interval is $\mathrm{x}=528.028$
This says that company is producing maximum output with using 528.028 units of labor. In this case it means, the Czech Airlines can offer maximum number of tonnekilometers by hiring 528 pilots, because 529 is already out of rational stage of production. If this value is simply inputted for the x variable, the maximum product is the result as it is in the relation 4.2.2.
$Y=1272695,87$

The maximum product is equal to 1272 695,87 tonnekilometers assuming no changes in the network and aircraft working load. Then it is shown the average and marginal production functions and their

| Production factor | Total physical product | Average product |
| :---: | :---: | :---: |
| No of pilots | Available tonne <br> kilometre(ATK) | Available tonne <br> kilometre(ATK) |
| 280 | 504620 | 1802,21 |
| 300 | 527300 | 1757,67 |
| 320 | 572260 | 1788,31 |
| 340 | 635060 | 1867,82 |
| 360 | 711260 | 1975,72 |
| 380 | 796420 | 2095,84 |
| 400 | 886100 | 2215,25 |
| 420 | 975860 | 2323,48 |
| 440 | 1061260 | 2411,95 |
| 460 | 1137860 | 2473,61 |
| 480 | 1201220 | 2502,54 |
| 500 | 1246900 | 2493,80 |
| 520 | 1270460 | 2443,19 |
| 540 | 1267460 | 2347,15 |
| 560 | 1233460 | 2202,61 |
| 580 | 1164020 | 2006,93 |
| 600 | 1054700 | 1757,83 |

Table 4.2.2- Average production(source: own calculation) characteristics.

Unit or average production is the amount of production which has been produced spread by equal share for every unit of production factor. So actually it is the ratio of production and units of production factor.

The table 4.2.2 shows the values of average product with using different number of production factor units. From the table is obvious that the average production reaches its maximum with using approximately 480 pilots. At the point where is the top of the average production function begins the rational stage of production which will be shown and described in details later in this chapter. At the same time the top of the average production function is point where it crosses with marginal production function. The ratio of the product and the production factor obviously is equal to tangens of the angle which is between horizontal axis and the line that connects the inception and the specific point. Therefore all points of production function represent the tangens of the specific angle. On the bellow graph 4.2 .2 there is average production function(or average physical product). As well as in the previous case number of pilots on horizontal axis and number of available ton kilometers on the vertical axis.

## Average physical product



Graph 4.2.2- Average production function(source: own calculation)

The other very important tool to evaluate the company's output based on production function is the marginal production. Marginal product is the product, which
has been produced by adding one more unit of production. In case of this study it means how much is the increase of available ton kilometers if one more pilot is hired. The marginal production can be calculated as first derivation of the production function as in the relation 4.2.3. The function equation is as follows:

$$
\begin{equation*}
Y^{\prime}=-0,2775 x^{2}+222,2 x-39957 \tag{4.2.3}
\end{equation*}
$$

The function of marginal production is increasing in case that the total production function is increasing progressively. From the point where the total production function starts to increase degresively the marginal production function is decreasing. The point where is the top of the marginal production function is so called inflex point of total production. It is the point where the function changes from progressively increasing to degresively increasing. The point where is the top of total production the marginal production function crosses the horizontal axis and is lower than zero. The table 4.2.3 shows the calculated values of marginal production function. All figures were calculated by inputting to above mentioned function equation of marginal production. The inflex point- the point where the total production changes from progressively increasing to

| Production factor | Marginal product <br> Available tonne <br> kilometre(ATK) |
| :---: | :---: |
| No of pilots | 503 |
| 280 | 1728 |
| 300 | 2731 |
| 320 | 3512 |
| 340 | 4071 |
| 360 | 4408 |
| 380 | 4523 |
| 400 | 4416 |
| 420 | 4087 |
| 440 | 3536 |
| 460 | 2763 |
| 480 | 1768 |
| 500 | 551 |
| 520 | -888 |
| 540 | -2549 |
| 560 |  |

Table 4.2.3- Marginal product(source: own calculation) dergesively increasing, can be calculated as a first derivation of marginal production function or as a second derivation of total production function respectively. If the resulting equation is equal to zero, we can easily reach the coordinates of inflex point. Inflex point is very important point to determine rational and irrational stages of production, which will be shown later in this chapter. On the graph 4.2.3 there is marginal production function based on the same study. On the horizontal axis there is number of production factor units- pilots and on the vertical axis is the additional or marginal product- number of available tonnekilometers.

## Marginal physical product



Graph 4.2.3- Marginal production function(source: own calculation)

As it has been already mentioned the marginal production function is equal to the average production function at the point where average production function has its maximum. At that point average production is equal to the tangens of angle which clenches the tangent to the production function and horizontal axis. Now the coordinates of the points which are crucial from the production evaluation point of view will be calculated.

There are two ways how to calculate the above mentioned point. The first one is to find the roots of equation which is first derivation of average production function. The second one is to find the point where marginal production is equal to average production. The first derivation of average production function is as according the relation 4.2.4:

$$
\begin{equation*}
Y_{A}^{\prime}=-0,185 x+111,1-5012900 x^{2} \tag{4.2.4}
\end{equation*}
$$

One of the roots in the required interval is equal to 485,7 .

The second option is to set the equation of average production and marginal production equal as described in relation 4.2.5:

$$
-0,0925 x^{2}+111,1 x-39957+5012900 x^{-1}=-0,2775 x^{2}+222,2 x-39957
$$

As well in this case 485,7 is one of the roots of the equation.

This point is the border point until which the company is in the first stage of production. It is the irrational stage of production, since the production cannot be maximized due to inefficient allocation of resources.

After hiring at least 486 pilots the Czech Airlines get into the second stage of production which is rational. This stage continues until the total production reaches its top and marginal production is equal to zero. Therefore it can be easily calculated as in the relation 4.2.6:

$$
\begin{equation*}
0=-0,2775 x^{2}+222,2 x-39957 \tag{4.2.6}
\end{equation*}
$$

One of the roots of this equation is 528,03 , which means that the second stage of production is between 486 and 528 units of labor. The second stage is the rational stage because maximum product and maximum additional product is reached.

The third stage of production is obviously irrational as well as the first stage. Nevertheless in the third stage each additional unit of production factor brings negative product. On the graph 4.2.4 there is clearly defined graphically the rational stage of production and the relationships between individual production functions.


Graph 4.2.4 - Rational stage of production(source: own calculation)
Optimization and proper allocation of production factors becomes a key issue of airline's decision making. The production is optimized at the moment when cost of production factor unit is equal to the price of unit of production. In other words the maximum profit is being performed by using such quantity of production factor, which is attained at the point, where the ratio of production factor and price is equal to marginal production as in the relation 4.2.7.

$$
\begin{equation*}
\mathbf{M P}=\frac{\partial y}{\partial x}=\frac{P x}{P y} \tag{4.2.7}
\end{equation*}
$$

Where $P_{x}$ represents the price of one unit of production factor and $P_{y}$ represents the price for one unit of product. For this calculation following cumulative background data in the table 4.2.4 for the period January - April 2010 was used.

| No of pilots |  | 460 |
| :---: | ---: | ---: |
| Pilot wage(average) | CZK | 148400 |
| Transportation revenues | CZK | 1427500000 |
| Revenue ton kilometers |  | 169159000 |
| Price of 1 ton kilometer(TKM) | CZK | 8,439 |
| Production Price of 1 TKM | CZK | 1,614 |

Table 4.2.4 - Background data(Sourced from CSA internal reports)
At that time 460 pilots were employed by Czech Airlines and the total amount spent for pilot wages was more than CZK 68 million per month. The overall transportation revenue for the whole four month period was CZK 1,4 billion. The number of revenue ton kilometers was approximately 169 billion which makes the price of one tonnekilometers CZK 8,439. Production price regarding pilots as a one of the factors of production is CZK 1,614. To calculate the optimum use of production units we will use the above mentioned formula and input the figures as in the relation 4.2.8:

$$
\begin{equation*}
0,191=-0,2775 x^{2}+222,2 x-39957 \tag{4.2.8}
\end{equation*}
$$

One of the roots of this equation in the required interval is $\mathrm{x}_{1}=528,03$, which is nearly same as the top of the production function. Optimization formula therefore says that hiring 528 pilots brings the maximum profit. The amount of production in such case is equal to 1272696000 available tonnekilometers. Offering such quantity of production would generate profit of CZK 10,7 billion in case that all the tonnekilometers would be sold.

### 4.3 Revenue-cost analysis

The econometric model used in this case study describes the relationships between individual items of revenue-cost structure of Czech Airlines. As an explained endogenous variables are total revenue and total cost. These two are assumed to simultaneously affect each other, therefore the model is simultaneous. Furthermore they are explained by range of exogenous variables which are described later on in details. The difference between revenues and cost is the subject to the last identity equation of the model as the profit. The resulting model looks as follows:

$$
\begin{aligned}
& y_{1}=y_{2}+x_{1}+x_{2}+x_{3}+x_{4}+x_{5}+x_{6}+x_{7}+u_{1} \\
& y_{2}=y_{1}+x_{1}+x_{8}+x_{9}+x_{10}+x_{11}+x_{12}+x_{13}+u_{2} \\
& y_{3}=y_{1}-y_{2}
\end{aligned}
$$

where $y_{k}$ represents endogenous variables, $\mathrm{x}_{\mathrm{k}}$ represent exogenous variables and $\mathrm{u}_{\mathrm{k}}$ the stochastic error. Further on are described the individual variables:
$y_{1} \quad$ Total revenues(10 000000 CZK)
$y_{2} \quad$ Total $\operatorname{cost}(10000000 \mathrm{CZK})$
$\mathrm{x}_{1} \quad$ Unit vector
$\mathbf{x}_{2} \quad$ Transportation revenues( $\mathbf{1 0} 000000$ CZK)
$x_{3} \quad$ Other revenues( 1000000 CZK)
$x_{4} \quad$ Number of passengers on regular flights( $\mathbf{1 0 0 0}$ passengers)
$\mathrm{x}_{5} \quad$ Cargo(10 t)
$x_{6}$ Number of passengers on charter flights(100 passengers-1 ${ }^{\text {st }}$ differences)
$\mathbf{x}_{7} \quad$ Exchange rate $(\boldsymbol{\epsilon} / \mathbf{0}, \mathbf{1 C Z K})$
$\mathrm{x}_{8} \quad$ Aviation fuel(1000 000 CZK)
x9 Transportation $\operatorname{cost}(1000000$ CZK)
$\mathrm{x}_{10} \quad$ Purchases for next sale ( 100000 CZK)
$\mathrm{x}_{11} \quad$ Wages and insurance( $\mathbf{1 0 0 0} 000$ CZK)
$\mathrm{x}_{12} \quad$ Other $\operatorname{cost}(1000000 \mathrm{CZK})$
$\mathrm{x}_{13}$ Depreciation(1000 000 CZK)

## Model Identification

The identification is necessary to realize whether there is sufficient number of exogenous variables that would explain endogenous variables. Number of endogenous variables decreased by one needs to be at least equal or lower than number of exogenous variables in the model apart from analyzed equation. This model is over identified since the number of exogenous variables exceeds the number of endogenous variables several times in all equations of the model.

## Economic and econometric form of the model

The formal economic form of the model is as follows:

$$
\begin{aligned}
& \mathbf{y}_{1}=f\left(\mathbf{y}_{2} ; \mathrm{x}_{1} ; \mathrm{x}_{2} ; \mathrm{x}_{3} ; \mathrm{x}_{4} ; \mathrm{x}_{5} ; \mathrm{x}_{6} ; \mathrm{x}_{7}\right) \\
& \mathrm{y}_{2}=\mathrm{f}\left(\mathrm{y}_{1} ; \mathrm{x}_{1} ; \mathrm{x}_{8} ; \mathrm{x}_{9} ; \mathrm{x}_{10} ; \mathrm{x}_{11} ; \mathrm{x}_{12} ; \mathrm{x}_{13}\right) \\
& \mathrm{y}_{3}=\mathrm{f}\left(\mathrm{y}_{1} ; \mathrm{y}_{2}\right)
\end{aligned}
$$

The econometric form of model including parameters is following:

$$
\begin{aligned}
& \mathbf{y}_{1 \mathrm{t}}=\boldsymbol{\beta}_{12} \mathbf{y}_{2 \mathrm{t}}+\gamma_{11} \mathbf{x}_{1 \mathrm{t}}+\gamma_{12} \mathbf{x}_{2 \mathrm{t}}+\gamma_{13} \mathbf{x}_{3 \mathrm{t}}+\gamma_{14} \mathbf{x}_{4 \mathrm{t}}+\gamma_{15} \mathbf{x}_{5 \mathrm{t}}+\gamma_{16} \mathbf{x}_{6 \mathrm{t}}+\gamma_{17} \mathbf{x}_{7 \mathrm{t}}+\mathbf{u}_{1 \mathrm{t}} \\
& \mathbf{y}_{2 \mathrm{t}}=\boldsymbol{\beta}_{21} \mathbf{y}_{1 \mathbf{t}}+\gamma_{21} \mathbf{x}_{1 \mathrm{t}}+\gamma_{28} \mathbf{x}_{8 \mathrm{t}}+\gamma_{29} \mathbf{x}_{9 t}+\gamma_{210} \mathbf{x}_{10 \mathrm{t}}+\gamma_{211} \mathbf{x}_{11 \mathrm{t}}+\gamma_{212} \mathbf{x}_{12 \mathrm{t}}+\gamma_{213} \mathbf{x}_{13 \mathrm{t}}+ \\
& \mathbf{u}_{2 \mathrm{t}} \\
& \mathbf{y}_{3 \mathrm{t}}=\mathbf{y}_{1 \mathrm{t}}-\mathbf{y}_{2 \mathrm{t}}
\end{aligned}
$$

The time series used in the model are on the monthly basis starting in December 2007 and finishing by April 2010. The complete data panel is available in the attachment.

## Multicolinearity

Originally four more exogenous variables were included in the model, however due to high colinearity between them they were excluded. Such variable was for instance number of flights, which was highly correlated to number of passengers, similarly as the number of flight hours which was the second variable. Then two more
variables- seat load factor and freight load factor which represent the ratio of available seat kilometers/revenue seat kilometers and available ton kilometers and revenue ton kilometers respectively. Nevertheless these indexes are also highly correlated to number of passengers and tons of cargo transported. Last and slightly correlated was number of passengers on charter flights with number of passengers on regular flights. By charter flight is understood the flight which is not included in airline timetable. Mostly it regards the flights which are rented by travel agencies. This variable has been modified into $1^{\text {st }}$ differences series and included in the model. Otherwise there is no high correlation between variables in the model. The only exception is the transportation revenues which is highly correlated to total revenues, however the total revenues is the endogenous variable which is suitable for following calculation. The table 4.3.1 is the correlation matrix showing the correlation values between individual variables.

|  |  |  |  | $\begin{aligned} & \text {-1 } \\ & \stackrel{+}{0} \\ & \stackrel{0}{0} \\ & \end{aligned}$ |  |  |  |  | $$ | $\begin{aligned} & \text { D} \\ & \frac{0}{0} \\ & \frac{0}{0} \\ & \stackrel{1}{0} \\ & \stackrel{\rightharpoonup}{0} \\ & \stackrel{\rightharpoonup}{3} \end{aligned}$ |  | $\begin{aligned} & \stackrel{(1}{2} \\ & \text { OiO } \end{aligned}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Total revenues |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Transportation revenues | 0,99 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Other revenues | 0,28 | 0,13 |  |  |  |  |  |  |  |  |  |  |  |  |
| Total cost | 0,72 | 0,68 | 0,39 |  |  |  |  |  |  |  |  |  |  |  |
| Aviation fuel | 0,70 | 0,72 | 0,05 | 0,75 |  |  |  |  |  |  |  |  |  |  |
| Transportation cost | 0,53 | 0,50 | 0,31 | 0,72 | 0,39 |  |  |  |  |  |  |  |  |  |
| Purchases for next sale | 0,60 | 0,55 | 0,48 | 0,70 | 0,60 | 0,36 |  |  |  |  |  |  |  |  |
| Wages and insurance | 0,54 | 0,48 | 0,50 | 0,63 | 0,41 | 0,56 | 0,67 |  |  |  |  |  |  |  |
| Other cost | 0,30 | 0,23 | 0,48 | 0,62 | 0,10 | 0,50 | 0,34 | 0,40 |  |  |  |  |  |  |
| Depreciation | 0,01 | 0,00 | 0,09 | 0,32 | 0,04 | -0,19 | 0,09 | -0,24 | 0,30 |  |  |  |  |  |
| No of pax (regular) | 0,75 | 0,77 | 0,02 | 0,45 | 0,45 | 0,30 | 0,38 | 0,35 | -0,03 | 0,11 |  |  |  |  |
| Cargo | 0,46 | 0,40 | 0,54 | 0,51 | 0,13 | 0,51 | 0,32 | 0,45 | 0,69 | 0,07 | 0,08 |  |  |  |
| No of pax (charters) 1stdiff | 0,18 | 0,17 | 0,14 | 0,13 | 0,27 | 0,24 | -0,06 | -0,11 | -0,14 | -0,08 | 0,08 | 0,02 |  |  |
| €/CZK Exchange rate | -0,61 | -0,63 | 0,03 | -0,11 | -0,15 | -0,13 | -0,03 | -0,05 | -0,06 | 0,07 | -0,58 | -0,33 | 0,01 |  |

Table 4.3.1- Correlation matrix(source: own calculation)

## Parameter estimation- Two stage least square method

For the estimation of the parameters in the model have been used two stage least square method and the method of variance ratio minimization. The estimation has been
processed separately for each equation except of identity equation, where the parameters are already known.

By using two stage least square method for both equations following values of parameters have been estimated:

$$
\begin{array}{ll}
\beta_{12}=0,00000000010559 & \beta_{21}=-0,00000000000665 \\
\gamma_{11}=0,00000003195754 & \gamma_{21}=-0,00000000128580 \\
\gamma_{12}=0,99999999994863 & \gamma_{28}=0,100000000001014 \\
\gamma_{13}=0,09999999998167 & \gamma_{29}=0,100000000006924 \\
\gamma_{14}=-0,00000000001095 & \gamma_{210}=0,0100000000000557 \\
\gamma_{15}=-0,00000000006834 & \gamma_{211}=0,0999999999947626 \\
\gamma_{16}=0,000000000003651 & \gamma_{212}=0,099999999985634 \\
\gamma_{17}=-0,0000000001051 & \gamma_{213}=0,10000000000599
\end{array}
$$

## Verification

Economic verification evaluates the direction and intensity of exogenous variables on endogenous variables. As expected in case of revenues the most significant item to affect it is the income received from transportation. The second is the income from the other airline activities. The rest of variables which are included in the model have much lower overall affect. However increasing cost usually goes together with increasing revenues. Unexpectingly the number of passengers on regular flights and cargo has negative effect on revenues while transportation of passengers on charter flights increases revenues. As the exchange rate increases total revenue decreases which says that Czech Airlines probably receives most of its income in Czech currency.

In case of second equation it has been estimated in opposite that increasing revenues goes together with decreasing cost. Regarding other variables they are much more balanced than in case of revenues. Transportation cost and aviation fuel are the major items of total cost. Both are increasing while the total cost is increasing. Depreciation is also worthy of notice, it plays the important part while it is increasing.

Purchases for next sale, wages, insurance and other cost are the less significant items in terms of affecting the total cost however all of them influence positively as well.

Regarding statistical verification in the first equation the number of observations was 29 while number of explanatory variables was eight. The number of degrees of freedom is therefore 21 . Since the residual variance is extremely low number while total variance is not, correlation coefficient is

| n | no of observations | 29 |
| :---: | :---: | :---: |
| p | no of explanatories | 8 |
| $\mathrm{n}-\mathrm{p}$ | freedom degrees | 21 |
| $\mathrm{Su}^{2}$ | residual variance | $2,61119 \mathrm{E}-17$ |
| $\mathrm{Sy}^{2}$ | total variance | 1263,3 |
| R | Correlation coef | 1 |
| $\mathrm{aSu}^{2}$ | adjusted $\mathrm{Su}^{2}$ | $9,84242 \mathrm{E}-22$ |
| $\mathrm{R}^{2}$ | Determination coef | 1 |

4.3.2 - Statistical verification figures(source: own calculation) very close to one. The table 4.3 .2 shows the important values to verify the equation of the model statistically. The second table 4.3 .3 shows the statistical significance of individual parameters. Since some of the values in both tables are extremely low they contain E as it is general format of number in Microsoft office excel. The number which follows E represents the change in number of decimal digits in a positive or negative way.

|  | $\mathrm{y}_{1}$ | $\mathrm{x}_{1}$ | $\mathrm{x}_{2}$ | $\mathrm{x}_{3}$ | $\mathrm{x}_{4}$ | $\mathrm{x}_{5}$ | $\mathrm{x}_{6}$ | $\mathrm{x}_{7}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{~S}_{\mathrm{ii}}$ | $1,984 \mathrm{E}-25$ | $9,632 \mathrm{E}-20$ | $1,719 \mathrm{E}-25$ | $2,03 \mathrm{E}-26$ | $1,84 \mathrm{E}-26$ | $2,227 \mathrm{E}-25$ | $6,476 \mathrm{E}-28$ | $1,129 \mathrm{E}-24$ |
| $\mathrm{~S}_{\mathrm{bi}}$ | $4,454 \mathrm{E}-13$ | $3,103 \mathrm{E}-10$ | $4,146 \mathrm{E}-13$ | $1,425 \mathrm{E}-13$ | $1,356 \mathrm{E}-13$ | $4,72 \mathrm{E}-13$ | $2,545 \mathrm{E}-14$ | $1,063 \mathrm{E}-12$ |
| t -value | 237,07291 | 102,97312 | $2,412 \mathrm{E}+12$ | $7,019 \mathrm{E}+11$ | 80,738893 | 144,79785 | 143,47892 | 98,872174 |
| $\mathrm{t}(\alpha=0,005)$ | 3,1352 | 3,1352 | 3,1352 | 3,1352 | 3,1352 | 3,1352 | 3,1352 | 3,1352 |
| S or I | S | S | S | S | S | S | S | S |

Table 4.3.3- First equation t-test(source: own calculation)

To evaluate the statistical significance of the chosen parameters the $t$-test has been used. The $t$-value has been compared with table value of significance level of $\alpha=$ 0,005 . All parameters in the first equation are significant according to the $t$-test.

Regarding econometric verification all the assumptions necessary for application of econometric model have been fulfilled. The multicolinearity test has been described previously. The Durbin-Watson test of autocorrelation of residuals is equal to 2,2416.

The statistical verification for the second equation result in very similar values. Since residual variance and adjusted residual variance are extremely low, the correlation coefficient and determination coefficient are very close to one, however not equal to one. On the table 4.3.4 there are shown values of t -test for second equation. As in the first case, all estimated parameters have resulted to be statistically significant. The significance level was at $99,5 \%$ as well as in the previous case.

|  | $\mathrm{y}_{2}$ | $\mathrm{x}_{1}$ | $\mathrm{x}_{8}$ | $\mathrm{X}_{9}$ | $\mathrm{x}_{10}$ | $\mathrm{X}_{11}$ | $\mathrm{X}_{12}$ | $\mathrm{X}_{13}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{~S}_{\mathrm{ii}}$ | $3,502 \mathrm{E}-27$ | $1,62 \mathrm{E}-22$ | $3,36 \mathrm{E}-28$ | $3,859 \mathrm{E}-28$ | $2,704 \mathrm{E}-28$ | $1,534 \mathrm{E}-27$ | $2,257 \mathrm{E}-27$ | $3,104 \mathrm{E}-28$ |
| $\mathrm{~S}_{\mathrm{bi}}$ | $5,917 \mathrm{E}-14$ | $1,273 \mathrm{E}-11$ | $1,833 \mathrm{E}-14$ | $1,964 \mathrm{E}-14$ | $1,644 \mathrm{E}-14$ | $3,917 \mathrm{E}-14$ | $4,75 \mathrm{E}-14$ | $1,762 \mathrm{E}-14$ |
| $\mathrm{t}-\mathrm{value}$ | 112,42038 | 101,02506 | $5,455 \mathrm{E}+12$ | $5,09 \mathrm{E}+12$ | $6,082 \mathrm{E}+11$ | $2,553 \mathrm{E}+12$ | $2,105 \mathrm{E}+12$ | $5,676 \mathrm{E}+12$ |
| $\mathrm{t}(\alpha=0,005)$ | 3,1352 | 3,1352 | 3,1352 | 3,1352 | 3,1352 | 3,1352 | 3,1352 | 3,1352 |
| S or I | S | S | S | S | S | S | S | S |

Table 4.3.4-Second equation t-test(source: own calculation)

The Durbin-Watson test is equal to 1,7161 .

## Elasticity

The elasticity represents the percentual change, if the affecting variable changes by one percent. All the figures that have been calculated in following tables are based on relation 4.3.1

$$
\begin{equation*}
\mathrm{E}=\frac{\partial y}{\partial x} \cdot \frac{x}{y} \tag{4.3.1}
\end{equation*}
$$

(Sourced from Tvrdoñ J.: Ekonometrie)

In this case study elasticity shows how individual items of the model affect each other. The results of the two stage least square method correspond properly to the economic assumptions rather than the results of method of variance ratio minimization. Therefore only the results of the two stage least square method have been chosen for the elasticity calculation. Elasticity has been calculated for each period separately. To find out the general result there was made an average of all periods and calculated as well. The table 4.3 .8 shows the individual values and the average at the bottom. As well in
this case has been used excel for the calculation and most of the values result in extremely low numbers. Those have been therefore expressed in general form including E.

| Year | Month | ELASTICITY |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| variables >> |  | $\mathrm{y}_{2}$ | $\mathrm{x}_{1}$ | $\mathrm{X}_{2}$ | $\mathrm{X}_{3}$ | $\mathrm{X}_{4}$ | $\mathrm{x}_{5}$ | $\mathrm{x}_{6}$ | $\mathrm{X}_{7}$ |
| 2007 | Dec | 1,278E-10 | 1,755E-10 | 0,8778387 | 0,1221613 | -2,01E-11 | -7,63E-11 | 2,39E-12 | -1,52E-10 |
| 2008 | Jan | 1,249E-10 | 2,159E-10 | 0,9072282 | 0,0927718 | $-2,25 \mathrm{E}-11$ | -7,26E-11 | -2,02E-12 | -1,85E-10 |
|  | Feb | 1,179E-10 | 2,251E-10 | 0,881135 | 0,118865 | $-2,24 \mathrm{E}-11$ | -7,58E-11 | -3,39E-14 | -1,88E-10 |
|  | Mar | 1,103E-10 | 1,716E-10 | 0,9085083 | 0,0914917 | $-2,25 \mathrm{E}-11$ | -7,6E-11 | 4,901E-12 | -1,42E-10 |
|  | Apr | 9,92E-11 | 1,686E-10 | 0,8970184 | 0,1029816 | -2,43E-11 | -6,23E-11 | -3,11E-12 | -1,39E-10 |
|  | May | 1,005E-10 | 1,562E-10 | 0,8916668 | 0,1083332 | -2,49E-11 | -5,38E-11 | 2,456E-12 | -1,29E-10 |
|  | Jun | 8,383E-11 | 1,296E-10 | 0,9272306 | 0,0727694 | $-2,11 \mathrm{E}-11$ | -4,53E-11 | 6,156E-12 | -1,04E-10 |
|  | Jul | 1,004E-10 | 1,485E-10 | 0,9287035 | 0,0712965 | -2,44E-11 | -5,49E-11 | 6,139E-12 | -1,15E-10 |
|  | Aug | 9,463E-11 | 1,478E-10 | 0,9334569 | 0,0665431 | -2,45E-11 | -4,77E-11 | 6,998E-13 | -1,18E-10 |
|  | Sep | 8,619E-11 | 1,318E-10 | 0,9373858 | 0,0626142 | $-2,11 \mathrm{E}-11$ | -4,73E-11 | -4,61E-12 | -1,06E-10 |
|  | Oct | 1,216E-10 | 1,568E-10 | 0,9407506 | 0,0592494 | $-2,32 \mathrm{E}-11$ | -6E-11 | -8,87E-12 | -1,28E-10 |
|  | Nov | 1,236E-10 | 2,01E-10 | 0,8891016 | 0,1108984 | -2,36E-11 | -7,32E-11 | -7,12E-12 | -1,66E-10 |
|  | Dec | 1,271E-10 | 1,876E-10 | 0,878543 | 0,121457 | -1,51E-11 | -6,61E-11 | 2,117E-12 | -1,61E-10 |
| 2009 | Jan | 1,328E-10 | 2,297E-10 | 0,8926886 | 0,1073114 | -2,17E-11 | -6,33E-11 | -1,8E-12 | -2,05E-10 |
|  | Feb | 1,52E-10 | 2,555E-10 | 0,8788918 | 0,1211082 | -2,17E-11 | -8,07E-11 | -1,39E-12 | -2,39E-10 |
|  | Mar | 1,275E-10 | 1,976E-10 | 0,8972559 | 0,1027441 | -2,35E-11 | -6,61E-11 | 2,038E-12 | -1,77E-10 |
|  | Apr | 1,328E-10 | 1,978E-10 | 0,883679 | 0,116321 | -2,76E-11 | -6,18E-11 | 4,935E-12 | -1,74E-10 |
|  | May | 1,277E-10 | 1,877E-10 | 0,9253648 | 0,0746352 | -2,71E-11 | -5,61E-11 | 1,866E-14 | -1,65E-10 |
|  | Jun | 1,049E-10 | 1,636E-10 | 0,9310301 | 0,0689699 | $-2,44 \mathrm{E}-11$ | -4,89E-11 | 6,797E-12 | -1,43E-10 |
|  | Jul | 1,209E-10 | 1,744E-10 | 0,9190007 | 0,0809993 | $-2,93 \mathrm{E}-11$ | -5,39E-11 | 6,271E-12 | -1,48E-10 |
|  | Aug | 1,147E-10 | 1,6E-10 | 0,9358827 | 0,0641173 | $-2,71 \mathrm{E}-11$ | -4,52E-11 | 1,628E-12 | -1,35E-10 |
|  | Sep | 1,111E-10 | 1,732E-10 | 0,9410369 | 0,0589631 | $-2,88 \mathrm{E}-11$ | -5,43E-11 | -8,23E-12 | -1,44E-10 |
|  | Oct | 1,164E-10 | 1,951E-10 | 0,9139739 | 0,0860261 | $-2,98 \mathrm{E}-11$ | -6,51E-11 | -1,08E-11 | -1,66E-10 |
|  | Nov | 1,07E-10 | 2,257E-10 | 0,8407334 | 0,1592666 | -3,02E-11 | -7,19E-11 | -5,27E-12 | -1,92E-10 |
|  | Dec | 1,336E-10 | 2,101E-10 | 0,8029487 | 0,1970513 | -2,75E-11 | -7,45E-11 | 9,725E-13 | -1,8E-10 |
| 2010 | Jan | 1,305E-10 | 2,734E-10 | 0,9238473 | 0,0761527 | -3,03E-11 | -7,52E-11 | -5,58E-13 | -2,35E-10 |
|  | Feb | 1,326E-10 | 3,141E-10 | 0,9527246 | 0,0472754 | $-3,17 \mathrm{E}-11$ | -8,6E-11 | -2,27E-12 | -2,68E-10 |
|  | Mar | 1,18E-10 | 2,401E-10 | 0,9603966 | 0,0396034 | -3,1E-11 | $-7,28 \mathrm{E}-11$ | 4,096E-12 | -2,02E-10 |
|  | Apr | 1,133E-10 | 2,506E-10 | 0,9368865 | 0,0631135 | $-2,87 \mathrm{E}-11$ | -6,73E-11 | -1,53E-12 | -2,09E-10 |
| Average |  | 1,146E-10 | 1,867E-10 | 0,9096038 | 0,0903962 | -2,49E-11 | -6,2E-11 | $-1,53 \mathrm{E}-14$ | $-1,58 \mathrm{E}-10$ |

Table 4.3.8- Elasticity $1^{\text {st }}$ equation based on results of TSLSM(source: own calculation)
The figures in the table show the percentual change of total revenue if the corresponding variable changes by one percent. From these results we can see that total cost, number of passengers on both regular and charter flights, amount of cargo and
exchange rate have very slight influence on total revenue in the observed period. However the transportation and other revenues are the most significant. Resulting values say that one percent change in transportation revenues cause $0,9 \%$ change in total revenue while revenue from other activities cause $0,09 \%$ change in total revenue. The second table 4.3.9 shows the elasticity figures based on calculation of second equation.

| Year | Month | ELASTICITY |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| variables >> |  | $\mathrm{y}_{1}$ | $\mathrm{X}_{1}$ | $\mathrm{x}_{8}$ | X9 | $\mathrm{X}_{10}$ | $\mathrm{X}_{11}$ | $\mathrm{X}_{12}$ | $\mathrm{X}_{13}$ |
| 2007 | Dec | -5,5E-12 | -5,8E-12 | 0,158817 | 0,450319 | 0,02286 | 0,198577 | 0,121245 | 0,048183 |
| 2008 | Jan | -5,6E-12 | -7,3E-12 | 0,220695 | 0,39461 | 0,026983 | 0,217183 | 0,066684 | 0,073845 |
|  | Feb | -6E-12 | -8,1E-12 | 0,197187 | 0,464523 | 0,014566 | 0,234653 | 0,081037 | 0,008034 |
|  | Mar | -6,4E-12 | -6,6E-12 | 0,197702 | 0,44259 | 0,01945 | 0,200262 | 0,079908 | 0,060088 |
|  | Apr | -7,1E-12 | -7,2E-12 | 0,219792 | 0,415219 | 0,029925 | 0,271913 | 0,104577 | -0,04143 |
|  | May | -7E-12 | -6,6E-12 | 0,23854 | 0,391968 | 0,030826 | 0,207096 | 0,077803 | 0,053767 |
|  | Jun | -8,4E-12 | -6,6E-12 | 0,280031 | 0,356721 | 0,026499 | 0,199032 | 0,070783 | 0,066934 |
|  | Jul | -7E-12 | -6,3E-12 | 0,286972 | 0,408103 | 0,022906 | 0,204126 | 0,061551 | 0,016343 |
|  | Aug | -7,4E-12 | -6,6E-12 | 0,299115 | 0,352369 | 0,024048 | 0,200512 | 0,065155 | 0,058801 |
|  | Sep | -8,1E-12 | -6,5E-12 | 0,267919 | 0,427307 | 0,025207 | 0,196513 | 0,06126 | 0,021794 |
|  | Oct | -5,8E-12 | -5,5E-12 | 0,223615 | 0,332673 | 0,021088 | 0,165327 | 0,108759 | 0,148539 |
|  | Nov | -5,7E-12 | -6,9E-12 | 0,250411 | 0,406065 | 0,030084 | 0,211659 | 0,083519 | 0,018262 |
|  | Dec | -5,5E-12 | -6,3E-12 | 0,243377 | 0,461879 | 0,021886 | 0,204763 | 0,097586 | -0,02949 |
| 2009 | Jan | -5,3E-12 | -7,3E-12 | 0,254344 | 0,35566 | 0,027545 | 0,224682 | 0,069593 | 0,068176 |
|  | Feb | -4,6E-12 | $-7,1 \mathrm{E}-12$ | 0,272902 | 0,34792 | 0,021679 | 0,208116 | 0,077369 | 0,072015 |
|  | Mar | -5,5E-12 | -6,6E-12 | 0,250365 | 0,405739 | 0,023965 | 0,199173 | 0,067488 | 0,053271 |
|  | Apr | -5,3E-12 | -6,3E-12 | 0,252727 | 0,385845 | 0,027998 | 0,187544 | 0,065297 | 0,080589 |
|  | May | -5,5E-12 | -6,2E-12 | 0,265761 | 0,377014 | 0,026573 | 0,19994 | 0,063709 | 0,067003 |
|  | Jun | -6,7E-12 | -6,6E-12 | 0,270723 | 0,478111 | 0,025562 | 0,232849 | 0,057865 | -0,06511 |
|  | Jul | -5,8E-12 | -6,1E-12 | 0,263908 | 0,41937 | 0,014805 | 0,193221 | 0,060184 | 0,048512 |
|  | Aug | -6,1E-12 | -5,9E-12 | 0,229124 | 0,393029 | 0,026126 | 0,174632 | 0,071516 | 0,105574 |
|  | Sep | -6,3E-12 | -6,6E-12 | 0,251292 | 0,429028 | 0,024832 | 0,221333 | 0,062823 | 0,010693 |
|  | Oct | -6E-12 | -7,1E-12 | 0,158203 | 0,429369 | 0,025253 | 0,287914 | 0,07557 | 0,023691 |
|  | Nov | -6,6E-12 | -9E-12 | 0,16278 | 0,442677 | 0,029535 | 0,245314 | 0,084972 | 0,034722 |
|  | Dec | -5,3E-12 | -6,7E-12 | 0,13613 | 0,398788 | 0,019249 | 0,202299 | 0,092329 | 0,151205 |
| 2010 | Jan | -5,4E-12 | -8,9E-12 | 0,194395 | 0,440184 | 0,026583 | 0,192213 | 0,067446 | 0,07918 |
|  | Feb | -5,3E-12 | -1E-11 | 0,190839 | 0,460575 | 0,008533 | 0,196398 | 0,074203 | 0,069452 |
|  | Mar | -6E-12 | -8,6E-12 | 0,204295 | 0,464492 | 0,007813 | 0,177343 | 0,078673 | 0,067384 |
|  | Apr | -6,2E-12 | -9,4E-12 | 0,179407 | 0,507778 | 0,007483 | 0,218033 | 0,081741 | 0,005559 |
| Average |  | -6,1E-12 | -6,9E-12 | 0,230697 | 0,412247 | 0,023104 | 0,208259 | 0,077174 | 0,048519 |

Table 4.3.9- Elasticity $2^{\text {nd }}$ equation based on results of TSLSM(source: own calculation)

The figures in the table show that total revenue have the minimum impact on total cost. However other items are more affecting. The largest influence comes obviously from the price of aviation fuel. If the cost for fuel changes by one percent the total cost changes by $0,4 \%$. One percent change in transportation cost cause $0,02 \%$ change. Purchases for next sale cause $0,2 \%$ if they change by $1 \%$. Wages, insurance and all other personal cost cause $0,08 \%$ change if they change by $1 \%$. And finally the depreciation cause $0,05 \%$ if changed by $1 \%$.

## Prognosis

This econometric model fulfills all the assumptions necessary for econometric prognosing. The resulted parameters correspond to the economic assumptions and the theory. There is no multicolinearity among exogenous

| Year | Month | $\mathrm{N}_{1 \mathrm{t}}$ | N2t |
| :---: | :---: | :---: | :---: |
| 2007 | Dec | -5,1E-11 | -2,876E-11 |
| 2008 | Jan | -2,01E-11 | 1,4718E-12 |
|  | Feb | -4,03E-11 | -2,023E-11 |
|  | Mar | -9,11E-11 | 1,0748E-11 |
|  | Apr | 7,06E-10 | 1,306E-10 |
|  | May | -8,12E-11 | -2,022E-11 |
|  | Jun | -7,88E-11 | -2,933E-11 |
|  | Jul | 2,68E-13 | -4,337E-12 |
|  | Aug | -4,37E-11 | -2,139E-11 |
|  | Sep | -1,42E-10 | 1,1646E-13 |
|  | Oct | -3,6E-12 | -1,114E-11 |
|  | Nov | -5,26E-11 | -2,728E-11 |
|  | Dec | 3,45E-11 | -2,986E-11 |
| 2009 | Jan | 1,95E-11 | -2,022E-11 |
|  | Feb | -9,48E-12 | -1,746E-11 |
|  | Mar | -1,94E-11 | 1,323E-11 |
|  | Apr | 2,43E-11 | 2,7277E-11 |
|  | May | 2,88E-11 | 1,2772E-11 |
|  | Jun | -6,26E-12 | -1,312E-11 |
|  | Jul | 4,37E-11 | 3,2763E-11 |
|  | Aug | 5,2E-11 | 3,8372E-11 |
|  | Sep | -5,49E-11 | -4,013E-12 |
|  | Oct | -1,04E-10 | -4,366E-11 |
|  | Nov | -1,54E-10 | -3,377E-11 |
|  | Dec | -6,31E-11 | 1,7745E-11 |
| 2010 | Jan | 5,14E-12 | 1,8598E-11 |
|  | Feb | 2,7E-12 | 8,6247E-12 |
|  | Mar | 5,61E-13 | 1,811E-11 |
|  | Apr | -6,56E-12 | -7,476E-12 | variables as it has been described Table 4.3.10-Standardized deviation(Source: own calculation) earlier in this chapter. The tightness and dependence between variables is very high as the correlation and determination coefficients are nearly one. According to the above calculated t-test all of the parameters are statistically significant. The Durbin-Watson test of autocorrelation of residuals is equal 2,2416 in case of first equation and 1,7161 for second equation. The last condition to fulfill all the assumptions is the standardized deviation. The standardized deviation in this case study has been calculated according to following relation 4.3.2.

$$
\begin{equation*}
\mathbf{N}_{\mathrm{it}}=\frac{\hat{\mathrm{y}} i t-y i t}{S y i} \tag{4.3.2}
\end{equation*}
$$

The table 4.3.10 shows the values of standardized deviation for both first and second equation. The standardized deviation for both endogenous variables for each year of the time series has been calculated as well and the table with resulting values is the part of the attachment at the end of the thesis. Results of all tests necessary to fulfill the assumptions for prognosing have been positive, so the prognosis can be applied for the model.

To calculate the forecast for the first endogenous variable- total revenue it was used ordinary least square method. The vector of endogenous variable keeps as a vector of explained variable in the new model. As explaining variables were used two- the unit vector $\mathrm{x}_{1 t}$ and time vector as $\mathrm{x}_{2 \mathrm{t}}$. After the parameters estimation the values were inputted in the equation and give result as in the relation 4.3.3.

$$
y_{1 t}=203,7684-2,1743 x_{2 t}+u_{1 t}
$$

According to the negative direction it is obvious that according to the prognosis the total revenue will be decreasing in a short or medium term period.

For the second endogenous variabletotal cost similar method has been processed. After estimating the parameters following values have been calculated representing the relation 4.3.4.
$\mathbf{y}_{2 t}=\mathbf{2 0 6}, 7466-1,3968 \mathbf{x}_{2 t}+\mathbf{u}_{2 t}$

As well in this case prognosis shows negative trend. The calculated values are shown in the table 4.3.11. Total revenue represents the variable $\mathrm{y}_{1}$ and total $\operatorname{cost} \mathrm{y}_{2}$.

| Yea |  | Month | Total Revenue | Total Cost |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  | 10 mil CZK | 10 mil CZK |
| 2009 |  | Nov | 141,6 | 143,5 |
|  |  | Dec | 152,1 | 192,5 |
| 2010 |  | Jan | 116,9 | 144,4 |
|  |  | Feb | 101,7 | 127,8 |
|  |  | Mar | 133,1 | 148,7 |
|  |  | Apr | 127,5 | 136,9 |
|  |  | May | 138,5 | 164,8 |
|  |  | Jun | 136,4 | 163,4 |
|  | O | Jul | 134,2 | 162,0 |
|  | O | Aug | 132,0 | 160,7 |
|  |  | Sep | 129,8 | 159,3 |
|  |  | Nov | 127,7 | 157,9 |

Table 4.3.11- Revenue-cost forecast(source: own calculation)

## 5. Discussion

From the above calculated figures of production function it is obvious that Czech Airlines does not allocate resources properly. It has been calculated that the rational stage of production is performed by hiring from 486 up to 528 pilots. However at the end of the period which was subject to this analysis were hired 480 pilots. Technically it means that the company does not fully uses its resources, in this case aircrafts are not fully utilized. Further on has been calculated that Czech Airlines would generate maximum profit by hiring 528 pilots. In such case company could possibly offer 1272696000 available ton kilometers. For describing the development of total production of available capacity by Czech Airlines trend function has been used. As it is obvious the capacity that was available to offer by the company was continuously


Graph 5.1- Czech Airlines available capacity trend development (source: CSA internal report) increasing until 2008 when it noticed decline. The graph 5.1 . 1 shows the capacity development together with linear trend function. Linear trend function has been chosen to describe the increasing tendency. Power function has been also applied however it showed lower determination coefficient and therefore was not preferred. The both function equations are shown in the table 5.1.1 as well with the value of determination coefficient.

| Type of trend function | Trend function | Determination coefficient |
| :--- | :--- | :--- |
| linear | $\mathbf{y}=\mathbf{8 1 9 8 4 x}+\mathbf{3 0 6 6 3 6}$ | $\mathbf{R}^{\mathbf{2}}=\mathbf{0 , 9 1 0 9}$ |
| power | $\mathbf{y = 3 7 2 8 1 2 \mathbf { x } ^ { \mathbf { 0 , 4 5 5 5 } }}$ | $\mathbf{R}^{\mathbf{2}}=\mathbf{0 , 7 9 7 4}$ |

Table 5.1- Trend development of Czech Airline capacity 1997-2009(source: own calculation)
The behavior of the company did not have to necessarily be a result of improper resources allocation. At the end of the year 2009 company got into serious financial troubles and almost became insolvent as it is obvious from the table when the
sharp decline was noticed. However it was saved partially by the help of government. At that time crucial changes processed within the company, a lot of employees have been made redundant and there was sharp cut in wages for rest. These changes made many employees leave on their own, especially pilots. The other reason could have been the planned sale of aircrafts. As it has been mentioned Czech Airlines was in serious financial troubles and there was no other solution than start to sell up the airline's property to avoid bankruptcy. Selling the aircraft is much more easier from the administrative point of view than selling company's buildings or subsidiaries. Therefore lowering the number of pilots might have been the reaction for forthcoming reduction of aircraft fleet.

For the second part of the case study econometric analysis has been used. The aim was to identify the relationships between individual items of revenue and cost structure within Czech Airlines and how they affect each other. The resulting values of first equation show the most significant influence of transportation revenues and revenues from other activities on the total revenue. Specifically in case of the effect of transportation revenues it is $0,91 \%$ change in total revenue if the transportation revenue changes by one percent. Revenues from other activities affect the total revenue by $0,09 \%$ if they change by one percent. According to the results of the model the rest items, namely number of passengers, cargo, charter flights and exchange have minimum effect on the total revenue despite the fact they are highly related. The second equation regarding total cost showed much more balanced values. In terms of most affecting item it was the cost of transportation. If this cost changes by one percent the total cost changes by $0,41 \%$. The aviation fuel as the second most significant factor would influence $0,23 \%$. Purchases for next sale and personal cost would affect the total cost by 0,02 and $0,21 \%$ respectively. Depreciation would cause $0,05 \%$ change. One percent change in all other cost than the above mention would influence the total cost by $0,08 \%$. The transportation and other revenue evince minimum influence.

The results of two stage least square method did not confirm the primar hypothesis. It was assumed that total revenue of the company increase and decrease simultaneously at the same time as the total cost. The resulting values confirm that increasing cost is related to increasing total revenue as it is figured out in the first
equation of the model. However the second equation of the model proves that increasing total revenue affects the total cost negatively. The total revenue can be increasing whether the total cost is either increasing or decreasing.

The second hypothesis also was not confirmed. It assumed that the number of total passengers is the most significant factor that affects the total revenue. The results show that the number of passenger has minimum influence and even in negative direction, which is completely against economic assumptions. This effect might have been caused by the period of observing. The situation during years 2008 and 2009 completely smashed down the financial structures not only in Czech Airlines but in all airline company and in aviation business generally. A lot of airlines went bankrupt and before they passed they were trying hard to attract the customers and increase the number of passengers. However the prices of flight tickets all over the world went down rapidly in many cases much lower bellow the break-even price. So even in case airline were successful to attract the customers the total revenue was decreasing and this probably similar as in case of Czech Airlines. The results would be more demonstrable if the average price of flight ticket would have been included in the model.

The third hypothesis regarding revenue/cost structure assumed that aviation fuel is the most affecting factor influencing the total cost. However according to the results of the model the total cost that cost related to the transportation have far more higher effect on the total cost. One percent change in expenditures for aviation fuel causes $0,23 \%$ change in total cost, while one percent change in cost related to transportation causes $0,41 \%$ change.

By using prognosing methods has been estimated the following development of the total revenue and cost amount. In case of total revenue the trend showed negative growth as well as in case of total revenue. However the ratio of decreasing revenue is almost twice higher than in terms of cost. The estimated data was not compared with real figures because those values have not been published by the company. However the data are expected to be slightly distorted in short term because of unexpected volcanic eruptions in Iceland in April and May 2010, which slowed down the airline industry for several weeks and caused large loss for all airlines operating in Europe.

## 6. Conclusion

From the calculated results it is obvious that the airline industry is continuously growing, although the growth is not stable. To describe the trend development the linear function has been chosen, however only with $91,1 \%$ coefficient of determination. The overall economic recession which has started in 2008 caused huge decline in this industry and showed that aviation industry among others is strongly dependent on the stage of economy.

It has been analyzed that the Czech Airlines improperly allocates its resources. The analysis based on the time period 1997-2009 showed that company should be hiring between 486 and 528 pilots to run most efficiently, which is the rational production stage and 528 pilots was calculated as profit maximizing output. However only 460 pilots were employed by Czech Airlines at the time when the analysis was made.

The results showed major influence of transportation cost on total cost of the company. One percent change in transportation cost causes $0,41 \%$ change in total cost, while aviation fuel causes $0,23 \%$. Other items of cost structure have lower level of influence. On the revenue side as well revenues from transportation and other activities are the most affecting factors. Transportation revenue influence by $0,91 \%$ change in total revenue and revenues from activities by $0,09 \%$. Other factors showed minimum effect. Surprisingly number of passengers resulted as a negative factor with minimum effect on total revenue, which is probably caused by the extremes during the period of observations.

At the end was calculated the forecast for following 6 months after the last observation. The forecast prognosed negative development for both revenue and costs, however with costs increasing by higher ratio.

## 7. Bibliography

(1) DOUGHERTY, Ch. Introduction to Econometrics. USA : Oxford University Press, 2002. 464 p. ISBN 0-19-877643-8.
(2) GUJARATI, D.N. Essentials of econometrics. USA : McGraw-Hill/Irwin, 1992. 554 p. ISBN 0-07-025194-0.
(3) GUJARATI, D.N. Basic Econometrics. USA : McGraw-Hill, 2004. 1002 p. ISBN 0-07-059793-6.
(4) AJMANI, V. Applied Econometrics Using the SAS System. USA : John Wiley \& Sons, 2009. 322 s. ISBN 0-47-012949-2.
(5) BARRETO, H.; HOWLAND, M. Introductory econometrics : Using Monte Carlo Simulation with Microsoft Excel. USA : Cambridge University Press, 2006. 800 s. ISBN 0-52-184319-7.
(6) BIERENS, H.J. Introduction to the Mathematical and Statistical Foundations of Econometrics. USA : Cambridge University Press, 2003. 434 s. ISBN 0-521834317.
(7) DAVIDSON, R.; MACKINNON, J.G. Econometric theory and methods. UK : Oxford University Press, 2004. 692 s. ISBN 0-19-512372-7.
(8) GREEN, W.H. Econometric Analysis. USA : Prentice Hall, 2003. 1056 s. ISBN 0-13-066189-9.
(9) KUAN, Ch.M. Introduction econometric theory. Taiwan : Academia Sinica, 2000. 202 s.
(10) WOOLDRIDGE, J.M. Introductory econometrics : A modern approach. USA : South-Western College Pub., 2003. 820 s. ISBN 0-32-411364-1.
(11) LIND, D.A.; MARCHAL, W.G.; WATHEN, S.A. Statistical techniques in business \& economics. USA : McGraw-Hill/Irwin, 2005. 704 s. ISBN 0-07-286824-4.
(12) LIPSEY, R.G.; CHRYSTAL, K.A. Economics. UK : Oxford University Press, 2004. 665 s. ISBN 0-07-054879-X.
(13) SAMUELSON, P.A.; NORDHAUS, W.D. Economics. USA : McGraw-Hill, 2010. 861 s. ISBN 0-07-070071-0.
(14) HUŠEK, R. Ekonometrická analýza. Praha : Oeconomia, 2007. 368 s. ISBN 978-80-245-1300-3.
(15) TVRDOŇ, J. Ekonometrie. Praha : Česká zemědělská univerzita v Praze, 2007. 228 s. ISBN 978-80-213-0819-0.
(16) ČECHURA, L., et al. Cvičení z ekonometrie. Praha : Česká zemědělská univerzita v Praze, 2008. 102 s. ISBN 978-80-213-1825-0.
(17) POJKAROVÁ, K. Ekonometrie a prognostika v dopravě. Pardubice : Univerzita Pardubice, 2006. 97 s. ISBN 80-7194-868-3.
(18) PRU゚ŠA, J. Svět letecké dopravy. Praha : Galileo CEE, 2007. 315 s. ISBN 978-80-239-9206-9.
(19) ŽIHLA, Z. Provozování leteckých podniků a letišt'. Praha : Vysoká škola obchodní v Praze, 2008. 184 s
(20) Wikipedia [online]. 2011 [cit. 2011-04-05]. Airline alliance. Available at WWW: [http://en.wikipedia.org/wiki/Airline_alliance](http://en.wikipedia.org/wiki/Airline_alliance).
(21) IATA [online]. 2011 [cit. 2011-04-05]. Traffic and capacity analysis. Available at WWW:
[http://www.iata.org/whatwedo/economics/Pages/traffic_analysis.aspx](http://www.iata.org/whatwedo/economics/Pages/traffic_analysis.aspx).
(22) BALTAGI, B.H. Econometrics. USA : Springer, 2008. 403 s. ISBN 978-3-540-76515-8.
(23) Czech Airlines Internal company data.

## 8．Supplements

|  |  | OMOM |  | Nic\|c|in |  | 웅 | $\begin{aligned} & \underset{\sim}{g} \\ & \underset{\sim}{g} \\ & \hline \end{aligned}$ |  | O |  |  |  | ｜oid | $\begin{aligned} & \text { ơ } \\ & \stackrel{\sim}{+} \\ & \underset{\sim}{\infty} \\ & \hline \end{aligned}$ |  |  | （on |  |  |  |  | O | O－ | O | O | O |  | ¢ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{gathered} \infty \\ \underset{\sim}{\mathrm{a}} \\ \hline \end{gathered}$ | $\stackrel{\sim}{\underset{i}{\prime}}$ | $\stackrel{\rightharpoonup}{i}$ | So |  | O |  |  |  | 寸্ণী |  | \％ | $\begin{aligned} & \stackrel{\rightharpoonup}{6} \\ & \stackrel{+}{+} \end{aligned}$ |  | $\begin{aligned} & \infty \\ & \infty \\ & \sim \\ & \sim \end{aligned}$ | － | O |  | $\begin{gathered} o \\ \hline \\ \dot{\infty} \end{gathered}$ | $\begin{array}{\|c} \underset{N}{n} \\ \underset{\sim}{2} \end{array}$ | $\stackrel{N}{\substack { \text { ¢ } \\ \begin{subarray}{c}{\text { ¢ }{ \text { ¢ } \\ \begin{subarray} { c } { \text { ¢ } } } \\{\hline}\end{subarray}}$ | $\stackrel{\rightharpoonup}{\text { d }}$ | N | $\stackrel{\stackrel{\circ}{\infty}}{\substack{1 \\ \hline}}$ | $\stackrel{\text { ¢ }}{\substack{\text { ² }}}$ |  | U |
|  |  | $\underset{\sim}{\sim}$ | $\begin{aligned} & \text { Ñn } \\ & \end{aligned}$ | ${\underset{A}{n}}_{\substack{i \\ i \\ \hline}}$ | $\underset{\substack{\infty \\ \underset{\sim}{2} \\ \hline \\ \hline}}{ }$ | $\underset{y}{6}$ | $\begin{aligned} & \hat{O} \\ & \dot{O} \\ & \end{aligned}$ | $\begin{aligned} & \infty \\ & \underset{\sim}{1} \\ & \underset{y}{1} \end{aligned}$ | $\begin{array}{ll} 0 \\ 0 \\ 0 \\ 0 \\ & 0 \\ \end{array}$ | $\stackrel{\circ}{\text { Non }}$ |  |  |  | $\begin{aligned} & \stackrel{\rightharpoonup}{+} \\ & \stackrel{\rightharpoonup}{\vec{j}} \end{aligned}$ |  | $\begin{aligned} & n \\ & \underset{\sim}{6} \\ & \end{aligned}$ |  | $\begin{aligned} & \stackrel{\circ}{0} \\ & \stackrel{n}{i} \end{aligned}$ |  | $\begin{aligned} & \text { O} \\ & \underset{\sim}{n} \end{aligned}$ |  | O | $\begin{gathered} \underset{\sim}{\infty} \\ \dot{d} \\ \underset{\sim}{6} \end{gathered}$ | $\begin{gathered} o \\ \underset{\sim}{\infty} \\ \end{gathered}$ | $\cdots$ |  |  | N |
|  |  |  |  |  |  | $\stackrel{\circ}{\circ}$ | $\begin{aligned} & \dot{O} \\ & \dot{C} \\ & \dot{f} \end{aligned}$ |  |  |  |  |  | － | $\begin{aligned} & \underset{\sim}{N} \\ & \stackrel{+}{\dot{A}} \end{aligned}$ |  |  | － |  |  |  |  | 尃 | \} | － | $\stackrel{\rightharpoonup}{\underset{\sim}{\sim}}$ | $\stackrel{\infty}{\text { ¢ }}$ | ？ | ¢ |
|  |  |  | $\begin{aligned} & \vec{\lambda} \\ & \underset{\sim}{\mathrm{A}} \end{aligned}$ | $\dot{~ H}$ | $\begin{aligned} & \text { Non } \\ & \underset{c}{c} \end{aligned}$ | $\mathfrak{i}$ | $\begin{gathered} o \\ 0 \\ \underset{\sim}{2} \\ \hline \end{gathered}$ | $\mathfrak{c}$ | $\underset{\sim}{\mathrm{N}}$ |  |  |  | $\begin{aligned} \substack{3 \\ \vdots \\ \vdots} \\ \vdots \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{\mathrm{G}} \\ & \stackrel{\text { Nu}}{ } \end{aligned}$ |  | $\begin{aligned} & \stackrel{0}{6} \\ & \stackrel{\oplus}{6} \end{aligned}$ |  | $\begin{aligned} & \stackrel{O}{9} \\ & \stackrel{\rightharpoonup}{6} \\ & \end{aligned}$ |  | oig |  | $\begin{aligned} & \underset{\sim}{\infty} \\ & \underset{y}{c} \end{aligned}$ | $\begin{gathered} \stackrel{\rightharpoonup}{\infty} \\ \stackrel{子}{8} \end{gathered}$ | $\begin{aligned} & 0 \\ & 0 \\ & \text { din } \end{aligned}$ | N | $\stackrel{4}{4}$ | $\stackrel{\rightharpoonup}{\square}$ | $\stackrel{0}{0}$ |
|  |  |  | $\begin{aligned} & \stackrel{0}{n} \\ & \infty \\ & \underset{\sim}{0} \end{aligned}$ |  |  | $\begin{gathered} 0 \\ \vdots \end{gathered}$ | $\begin{gathered} 0 \\ 0 \\ 0 \\ 0 \\ \end{gathered}$ | $\underset{\infty}{\infty}$ | $\stackrel{\sim}{\infty}$ | ત |  |  | － | $\left\lvert\, \begin{gathered} \underset{\sim}{\infty} \\ \underset{\sim}{\mathbf{N}} \\ \hline \end{gathered}\right.$ |  | $\begin{aligned} & \hat{\omega} \\ & \stackrel{\rightharpoonup}{\mathrm{N}} \end{aligned}$ | $\underset{\sim}{\text { N／}}$ |  |  | $\begin{gathered} \tilde{O} \\ \underset{\sim}{0} \\ \underset{\sim}{2} \end{gathered}$ |  | N | $\stackrel{\text { \％}}{\substack{\text { a }}}$ | $\stackrel{\sim}{N}$ | べ |  | ¢ | 析 |
|  | 2 $\vdots$ $\vdots$ $\vdots$ $\vdots$ $\vdots$ $\vdots$ |  | $\begin{aligned} & 0 \\ & \underset{\sim}{i} \\ & \underset{\sim}{2} \end{aligned}$ | $\underset{n}{2}$ |  |  | $\begin{gathered} \substack{o \\ \\ \dot{\infty} \\ \infty \\ \hline} \end{gathered}$ |  |  |  |  |  | － | $\begin{aligned} & \stackrel{\rightharpoonup}{0} \\ & \stackrel{+}{\dot{N}} \\ & \end{aligned}$ |  |  | 促 |  |  | $\begin{aligned} & n \\ & \underset{N}{n} \\ & \underset{m}{\infty} \end{aligned}$ |  |  |  | $\underset{\mathcal{F}}{ }$ | $\stackrel{\substack{N\\}}{ }$ | H | $\begin{aligned} & \text { ợ } \\ & \stackrel{\oplus}{6} \\ & \hline \end{aligned}$ |  |
|  | $\begin{array}{\|c} \stackrel{y}{*} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \end{array}$ |  | $\begin{aligned} & \stackrel{\rightharpoonup}{\circ} \\ & \stackrel{\sim}{i} \end{aligned}$ |  | $\begin{gathered} \substack{n \\ \vdots \\ \vdots \\ \vdots \\ \\ \\ \hline} \end{gathered}$ |  | $\begin{gathered} a \\ 0 \\ \vdots \\ \vdots \end{gathered}$ | $\mathfrak{c}$ | 0 <br> 0 <br> $\vdots$ |  |  |  |  | $\begin{array}{\|l} \underset{\sim}{0} \\ \underset{\sim}{0} \\ \hline \end{array}$ |  | $\begin{gathered} \underset{\sim}{N} \\ \underset{i}{0} \\ \hline \end{gathered}$ |  | $\begin{aligned} & \overrightarrow{\underset{~}{4}} \\ & \stackrel{\rightharpoonup}{6} \end{aligned}$ |  | $\mathfrak{N}$ |  | $\begin{aligned} & \text { ư } \\ & \substack{0 \\ \text { 号 }} \end{aligned}$ | $\begin{aligned} & \stackrel{\circ}{\infty} \\ & \stackrel{y}{\dot{\prime}} \end{aligned}$ | ¢ | O－ | $\begin{aligned} & \text { on } \\ & \stackrel{\rightharpoonup}{i} \end{aligned}$ | $\begin{aligned} & 0.0 \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ |  |
|  |  |  | Sos |  | $\begin{gathered} \underset{\sim}{n} \\ \underset{\sim}{n} \\ \underset{\sim}{n} \end{gathered}$ |  |  | $\mathfrak{c}$ |  | A | تِ |  |  | $\left\|\begin{array}{c} \underset{\sim}{n} \\ 0 \\ 0 \\ 0 \end{array}\right\|$ | Nin |  | 柋 | $\begin{aligned} & \underset{\sim}{\infty} \\ & \underset{\sigma}{\infty} \\ & \underset{\sigma}{2} \end{aligned}$ |  | $\mathfrak{c}$ |  | ¢ | N్య | $0$ | ה |  |  |  |
|  |  |  |  | $\stackrel{\rightharpoonup}{2}$ |  | Ry |  |  |  |  | Oi |  |  |  |  | in |  |  |  | $\begin{aligned} & \text { a } \\ & \underset{9}{9} \\ & \text { g} \end{aligned}$ |  | $\mathfrak{c}$ | $\begin{array}{\|l\|} \hline \underset{\substack{0}}{ } \\ \underset{\sim}{i} \end{array}$ | O | O | $\begin{aligned} & \overrightarrow{n_{0}} \\ & \underset{\sim}{\dot{2}} \end{aligned}$ | or |  |
|  |  | $\begin{gathered} \text { G} \\ \substack{i} \\ \underset{\sim}{N} \\ \underset{\sim}{2} \end{gathered}$ |  | $\overbrace{0}^{0}$ | $\begin{aligned} & \underset{\sim}{0} \\ & \infty \\ & \end{aligned}$ | Br |  | $\begin{gathered} \underset{O}{O} \\ \dot{d} \\ \hline \end{gathered}$ |  |  |  | $\stackrel{\rightharpoonup}{n}$ |  |  |  | 뭄 | － |  |  | $\begin{aligned} & \infty \\ & \infty \\ & \infty \\ & \underset{\sim}{\infty} \end{aligned}$ | $\overbrace{0}^{\circ}$ | － |  | $\cdots$ | $\begin{gathered} \underset{\sim}{\underset{Z}{2}} \\ \underset{\sim}{\dot{G}} \end{gathered}$ | $\stackrel{\infty}{\text { ® }}$ | $\begin{aligned} & \underset{\substack{0 \\ 0 \\ 0 \\ 0}}{ } \end{aligned}$ |  |
| ¢ |  |  |  |  | $\begin{aligned} & 8 \\ & \vdots \\ & \vdots \\ & \\ & \end{aligned}$ | fo | $\begin{gathered} \underset{C}{C} \\ \underset{\sim}{2} \\ \hline \end{gathered}$ | $\begin{aligned} & \stackrel{\sim}{0} \\ & \underset{\sim}{n} \\ & \end{aligned}$ |  | $\underset{A}{0}$ |  |  |  | $\begin{aligned} & \text { O} \\ & \underset{\sim}{n} \\ & \underset{\sim}{n} \end{aligned}$ |  |  |  | $\begin{aligned} & \stackrel{\rightharpoonup}{N} \\ & \stackrel{\rightharpoonup}{\dot{\sim}} \end{aligned}$ | thiction | $\begin{aligned} & \infty \\ & 0 \\ & \dot{\sim} \\ & \underset{\sim}{\infty} \end{aligned}$ |  | O |  |  | ¢ | ${ }_{\sim}^{\circ}$ |  |  |
|  |  |  |  | $\vec{A}$ |  |  | $\underset{\sim}{\underset{\sim}{\infty}} \underset{\sim}{\underset{\sim}{\infty}}$ |  |  |  |  |  |  | $\begin{aligned} & \tilde{\sim} \\ & \underset{\sim}{\sigma} \\ & \underset{\sim}{\circ} \end{aligned}$ | $\underset{\sim}{N}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{n} \\ & \underset{\sim}{u} \\ & \underset{\sim}{n} \end{aligned}$ |  |  |  | $\mathfrak{c}$ |  | － |  | 宗 | ¢ | ¢ু． | \％ | 析 |
|  |  |  |  | Buccucie |  | ? | $\begin{aligned} & n \\ & \substack{0 \\ \underset{\sim}{0} \\ \hline} \end{aligned}$ | $\stackrel{\rightharpoonup}{n} \stackrel{\rightharpoonup}{n}$ |  |  | Com |  |  | $\underset{\sim}{\text { On}}$ |  |  |  |  |  |  |  | － |  | ก | $\left.\begin{aligned} & \ddot{\infty} \\ & 0 \\ & 0 \\ & 0 \end{aligned} \right\rvert\,$ | O | $\begin{array}{\|l} \hline \underset{\substack{0 \\ \underset{\sim}{2} \\ \hline}}{ } \end{array}$ |  |
|  |  | ¢ 둑 | ® | ${ }^{\text {a }}$ | 눈 | $\sum^{\text {a }}$ | $\bigcirc$ |  | 管 | ～～ | ¢ ${ }_{\text {¢ }}^{2}$ | 岁 | 득 | \％ | $\stackrel{\text { n }}{ }$ | 훈 | $\frac{\square}{4}$ | § |  | 年 | ～ | せ | ${ }_{2}^{3}$ | \％ | $\stackrel{\text { coun }}{0}$ | 윤 | $\stackrel{\text { L }}{ }$ | $\stackrel{1}{0}^{\text {2 }}$ |
| ® | $\hat{\hat{\hat{n}}}$ ¢5 |  |  |  |  |  |  | ర్లి |  |  |  |  |  |  |  |  |  |  | O |  |  |  |  |  |  |  |  |  |

Table 8．1－Czech Airlines revenue－cost data panel（sourced from CSA internal monthly reports）

| Year | Weight load factor | No of flights | No of passengers | No of kilometres flown | No of productive hours | Revenue passenger kms flown(RPK) | Seat load factor | Revenue tonne kms flown(RTK) | Freight tonne kms flown(FTK) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| units >> | \% |  | 1,000 | 1000 km |  | 1000000 km | \% | 1000000 km | 1000000 km |
| 1989 | 68.6 | 10,998.5 | 1,490.7 | 29,418.1 | 40,451.7 | 2,626.0 | 72.9 | 263.7 | 23.6 |
| 1990 | 64.4 | 10,012.5 | 1,285.2 | 27,289.2 | 37,032.8 | 2,346.9 | 68.6 | 236.2 | 21.2 |
| 1991 | 56.8 | 8,927.5 | 974.5 | 25,263.5 | 34,439.9 | 2,103.8 | 61.0 | 227.3 | 31.3 |
| 1992 | 52.4 | 10,211.5 | 1,078.2 | 29,195.6 | 40,393.9 | 2,423.2 | 61.1 | 246.6 | 28.5 |
| 1993 | 49.2 | 11,570.5 | 1,146.1 | 27,011.5 | 43,665.6 | 2,106.4 | 57.4 | 214.7 | 25.1 |
| 1994 | 54.9 | 11,262.0 | 1,239.7 | 26,736.1 | 43,023.3 | 2,245.3 | 63.9 | 224.8 | 22.7 |
| 1995 | 57.3 | 12,929.5 | 1,488.3 | 30,045.8 | 48,944.1 | 2,640.0 | 66.6 | 264.4 | 26.8 |
| 1996 | 56.1 | 13,505.5 | 1,612.1 | 31,675.0 | 51,134.0 | 2,725.4 | 66.0 | 268.7 | 23.5 |
| 1997 | 56.9 | 14,489.5 | 1,733.7 | 33,583.6 | 53,687.5 | 2,897.9 | 66.8 | 285.4 | 24.6 |
| 1998 | 56.8 | 15,205.0 | 1,801.8 | 34,567.0 | 54,949.0 | 2,910.6 | 66.3 | 288.3 | 26.3 |
| 1999 | 56.6 | 17,718.5 | 2,064.1 | 38,627.7 | 62,311.0 | 3,149.5 | 65.8 | 311.6 | 28.2 |
| 2000 | 61.4 | 20,319.0 | 2,461.7 | 41,878.2 | 68,809.0 | 3,622.6 | 70.4 | 360.9 | 34.8 |
| 2001 | 62.1 | 22,760.5 | 2,877.3 | 45,483.7 | 74,687.0 | 3,994.3 | 70.8 | 388.3 | 28.9 |
| 2002 | 63.7 | 24,453.0 | 3,065.0 | 47,045.2 | 78,210.0 | 4,178.3 | 71.3 | 407.6 | 31.5 |
| 2003 | 64.7 | 27,021.5 | 3,591.5 | 54,517.1 | 88,769.0 | 5,084.1 | 72.7 | 498.1 | 40.5 |
| 2004 | 62.3 | 32,702.0 | 4,345.4 | 67,971.7 | 110,124.0 | 6,232.1 | 70.5 | 606.0 | 45.2 |
| 2005 | 61.9 | 38,676.0 | 5,217.6 | 82,960.6 | 132,831.0 | 7,816.8 | 70.1 | 747.9 | 44.4 |
| 2006 | 63.2 | 39,143.5 | 5,469.9 | 82,755.2 | 131,503.0 | 8,074.5 | 71.8 | 771.7 | 45.0 |
| 2007 | 57.7 | 39,850.0 | 5,492.2 | 82,928.8 | 131,614.0 | 7,788.5 | 68.4 | 744.5 | 43.6 |
| 2008 | 56.2 | 41,742.5 | 5,626.0 | 86,705.0 | 138,199.0 | 7,841.2 | 67.1 | 742.5 | 36.8 |
| 2009 | 56.6 | 40,989.0 | 5,464.6 | 84,458.2 | 135,399.7 | 7,557.5 | 68.1 | 708.7 | 28.5 |

Table 8.2- Czech Airlines performance indexes 1989-2009(sourced from CSA yearly reports)

