Czech University of Life Sciences Prague

Faculty of Economics and Management

Department of Economics

Bachelor Thesis

Travelling around Czech Republic - designing the route using quantitative methods

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CZECH UNIVERSITY OF LIFE SCIENCES PRAGUE

Faculty of Economics and Management

BACHELOR THESIS ASSIGNMENT

Thu Giang Dang

Economics and Management
Economics and Management

Thesis title

Travelling around Czech Republic – designing the route using quantitative methods

Objectives of thesis

The main goal of this bachelor thesis is to optimize the routes for a generic traveler, student for instance, who wants to travel around the Czech Republic during holiday time by car. The starting point of the routes will start at the Prague main train station and after visit all the well-known places in Czech Republic, the student will return to the starting point.

Methodology

The theoretical part of the bachelor thesis will analyse and discuss the Nearest neighbor algorithm, Vogel's approximation algorthim, Savings algorithm methods. The results of these methods will be calculated and discussed in the practical part in order to find out the best result for the problem. The conclusions will be carried out at the end of the bachelor thesis.

The proposed extent of the thesis

30-40

Keywords

travelling salesman, tourism, travel

Recommended information sources

APPLEGATE, D L. *The traveling salesman problem : a computational study.* Princeton: Princeton University Press, 2006. ISBN 9780691129938.

DANTZIG, G B. *Linear programming : 2: theory and extensions.* Madison: Springer, 2003. ISBN 978-0387986135.

GUTIN, G. – PUNNEN, A P. *The traveling salesman problem and its variations.* New York: Springer, 2007. ISBN 0387444599.

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Declaration

I declare that I have worked on my bachelor thesis titled “Travelling around Czech Republic - designing the route using quantitative methods” by myself and I have used only the sources mentioned at the end of the thesis. As the author of the bachelor thesis, I declare that the thesis does not break copyrights of any their person.

In Prague on 23.03.2020 ____________________________

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This one is my gift for you!

Travelling aroud Czech Republic - designing the route using quantitative method

Abstract

The main objective of this Bachelor Thesis is to find an optimal combination of routes through selected destinations around the Czech Republic, of one student for instance, who would like to travel by car. Thus, the student wishing to see the shortest routes for travelling to save more time for the excursion. The bachelor thesis contained two parts: theoretical and practical. The theoretical part underlines an overview of travel and tourism and its categories. Besides that, the general information on logistics such as its objectives, history, development and its features will be given out. Finally, the particular information about operations research along with its characteristics will be brought out in this work.

The practical part describes more into detail about the problems with the obtained results by the TSPKOSA program with three main algorithms which are: Nearest Neighbour algorithm, Vogel's Approximation algorithm and the Savings algorithm. All of the calculations will be analysed and represented by tables and figures. The optimal result will be carried out at the end of the bachelor thesis.

Keywords: Travelling salesman, tourism, travel

Cestování po České republice - návrh trasy pomocí kvantitativní metody

Abstrakt

Hlavním cílem této bakalářské práce je najít optimální kombinaci tras napříč vybranými destinacemi. Praktická část je založena na příkladu studenta, který by chtěl cestovat autem po celé České republice. Z pohledu studenta je podstatné nalézt optimální trasy a minimalizovat tak čas strávený cestováním. Bakalářská práce obsahuje dvě části: teoretickou a praktickou. Teoretická část zahrnuje přehled cestovního ruchu a jeho kategorie. Kromě toho jsou zde také uvedeny obecné informace týkající se logistiky jako jsou například cíle, historie, vývoje a jejich vlastnosti. Nakonec, budou v této části uvedeny informace o kvantitativních metodách vhodných pro optimalizaci tras.

Praktická část podrobněji popisuje problémy a získané výsledky prostřednictvím programu TSPKOSA se třemi hlavními algoritmy: Metoda nejbližšího souseda, Vogelova aproximační metoda a Metoda výhodnostních čísel. Všechny výpočty jsou analyzovány a reprezentovány pomocí tabulek a obrázků. Na konci bakalářské práce je provedeno zhodnocení a prezentace optimálního řešení problému.

Klíčová slova: Obchobní cestujicí, turismus, cestovaní

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1 Introduction

In recent years, the world has witnessed an explosion of traveling activities around the globe. During the period of ancient Egypt and Greece, people have a tendency to travel to temples, holy places, Christian churches and so on… But traveling was only really becoming popular in the late nineteenth century and exploded in the 1960s when the second scientific and technological revolution has brought tremendous achievements in the social economy. When people are getting tired of industrial life manner, they have a tendency to return to nature, to the roots of agricultural civilization in order to relax after a long period of working time.

Czech Republic is an amazing destination for those who want to travel in Europe. With a long-standing splendid beauty along with ancient Baroque architecture, temperate climate and friendly local people, Czech Republic welcomes thousands of tourists from all over the world visiting the country every year. This has greatly contributed to promote the development of the economy in not only the process of building but also preserving the country nowadays as well as in the future. Besides, we cannot deny the benefits of traveling in our busy lifetime. It is not only helps us to gain new experiences in a new country but also contributes to making our spiritual life becomes more knowledgeable and interesting.

The goal of this thesis is to find the shortest routes for student who would like to travel around Czech Republic in the holidays. Assume all of the destinations are chosen based on the interest of the student with the starting point of the trip is started from Prague main train station and after visit all the famous chosen destinations, the student will return to the initial point. The length of the trip will be divided into 10 days with the time set for one day travel is approximately from 09:00 am to 17:00 pm.

2 Objectives and Methodology

2.1 Objectives

The aim of the bachelor thesis is to find the most convenient routes through selected locations in order to traveling around Czech Republic. Assume all the destinations are chosen based on the interest of the student. Every day, the student will have approximately 5 locations to visit. The duration time for each day started from 9:00 am to 17:00 pm with the begin location is at Prague main train stations and after 10 days traveling, the student will come back to the initial point and finish the excursion.

2.2 Methodology

The Bachelor Thesis is divided into two parts:

- The first theoretical part includes:
  - The research of the related literature. To be more precise, it is focused on the study of travel and tourism, tourism flow models, logistics history and its development.
  - Moreover, it is also concentrated on the quantitative approach, operational research as well as the methods by which problems are solved in the practical part which is namely: the nearest neighbor algorithm (sequential), Vogel's approximation algorithm and lastly the savings algorithm (parallel).
- The second practical part includes:
  - Introduction about the traveling scenarios and selected destinations for the excursion. There are 47 destinations that needed to visit during the trip, which are divided into days with up to 5 destinations needed to travel each day.
  - All of the data was collected from TSPKOSA program. The comparison of the overall traveling scenarios with 3 different algorithms. The best option will be carried out and split into 10 days in order to calculate the total time fit for one day travel. The results will be represented in tables by the introduced program and in figures obtained by the chosen website Mapy.cz.

3 Literature Review

3.1 Travel and tourism:

It was first started for a long time ago when primitive man set out, they often traveling for great distances searching for food, clothing, and shelter for survival. During the long history, people have traveled all around the world for the purpose of war, trade, religious, economic gain and so on. Since that time, travel continues to grow and become an important factor in civilization development. (17)

It is said that, tourism began during the industrial revolution along with the rise of inexpensive transportation as well as the rise of the middle class in England. Due to the industry of airline commercial creation after World War II along with jet aircraft development started the increase of international travel which was called today: *tourism*. (17)

Nowadays, tourism has developed not only in economic sector but also in social importance factors. It is worth noticing that over the past few years, the area of services has been growing pretty fast in the economic sector of almost industrialized countries. From time to time, travel and tourism have become one of the biggest segments in the service industry.

The word "*tour*" originally from the Greek "*tornos*" and from the Latin "*tornare*" which means "a lathe or circle, the movement around a central point or axis". The meaning in English is "one's turn", and it is suggested that the word "*tour"* stands for the action of movement around a circle. For this reason, a circle stands for a starting point, which in the end returns back to its beginning. (17)

3.1.1 Tourism Flow Models:

Given the fact that the methodology for predicting the flow of tourist between the various touristic attractions in the geographical area can be different depend on predicting tourist arrivals in the number of tourists at one given attraction can be used in some circumstances as information in deciding how many will be traveling another destination within that area at a subsequent time. It is said that, there are two main approaches and it can be divided into stochastic (probabilistic) and deterministic. (17)

3.1.1.1 Probabilistic Approach

An example is given to explain the probabilities that United States visitors to the Ontario province would make a stop by overnight among ten other economic regions. The model is Markov a chain that transition matrix probability demonstrates the relationships of economic regions as stopping at a location of overnight points. The outcome of the model brings in the average number of overnights stops as well as their distribution throughout the province for visitors first stopping in each part of Ontario. This model however has shown good results and sets the stage for further applications of this approach. (17)

3.1.1.2 Deterministic Approach

A tourism flow model – a part of a multiperiod planning model was developed for tourism development. The difference of this model is that it is firstly focused on predicting the changes in tourist flows which is a result of the implementation of a specific tourism development projects in different locations. It is mainly concentrated on the development of the new project which may cause the change in "touristic attractiveness" of a location and as a result, the existing tourist flows will be redistributed in order to reflect this change. Moreover, the term "saturation index", which is refered to the ratio of projected visitors to the visitor capacity indicated that the closer the index is to one, the less effect any further development that one visitor can have on the attractiveness area, therefore it attracts more tourists flow from that area. (26)

3.1.2 Dimension of travel

The three main elements of dimension travel in tourism are: the purpose of the trip, distance traveled and the last one is the duration of the trip. However, travelers are also being defined by two additional dimensions that are occasionally used are the residence of traveler and the mode of transportation.

1. **Purpose of trip**: the concept behind the tourism dimension was also contains other elements of most travel in these times. Travelers are often being viewed only as tourists. However, business travel is also should be considered as one part of leisure travel for the reason that it is not a daily business routine as usual. (28)

2. **Distance traveled:** Measuring travel away from home is a part of statistical purposes and the total of the round-trip distance between the place of residence and the

destination is often viewed as a statistical measurement factor. The distance which was the view as traveling can vary from 0 to 160 kilometers, therefore, the destinations that are less than the given kilometers are not rated as official estimates of tourism. (28)

3. **Duration of trip:** For the purpose of defining travelers, almost interpretations of tourists involve at least one overnight stay in the area of destination. (28)

4. **Residence of traveler:** When it comes to business purpose needed to define markets or marketing strategy. It is more vital to define the location rather than focus on other demographic elements for instance like their citizenship or nationality. (28)

5. **Mode of transport:** The means of transportation for example airplane, train, auto, train and so forth are used for planning and travelling purposes. (28)

3.1.3 The importance of travel satisfaction in life

Nowadays, a lot of scientific studies have shown that travel satisfaction has a positive impact on the well-being of people's health conditions. It is said that travel satisfaction is an indicator of the trip's contribution to the subjective well-being of the traveler. (23)

In other circumstances, the term subjective well-being is used as a measure of psychological well-being which is related to affect, happiness and satisfaction of one person. Moreover, it is also being used as a tool to make an assessment of one's state. (23)

It is said that the overall mood and happiness of an individual is related to daily travel. In one research, they have pointed out that travel satisfaction had a direct effect on not only cognitive but also affective subjective well-being as a result of daily travel. (2)

Besides, satisfaction with travel can be considered as an indicator of the trip's contribution to the subjective well-being of the traveler, which is easier to measure than well-being in general. (3)

Well-being is often explained as an assessment of each individual based on one's life which is related to their needs on a daily basis. Another way to approach to subjective well-being consists of the cognitive component can be used to indicate an individual's satisfaction based on their positive as well as a negative emotion. (8)

There are several scientific studies shown that higher travel satisfaction lead to the higher overall well-being of one person. Meanwhile, some studies of social exclusion provide evidence that a person with a lower level of well-being is a result of the lack of access to travel. (7)

3.2 Logistics

Basically, logistics is a framework and planning orientation in which through a business, strive to create a plan for the flow of products as well as information. (5)

3.2.1 Objectives of logistics

Every logistics has certain goals, concerns depend on different aspects. Logistics can be divided into *macrologistics and micrologistics.* (12)

The main goal of *macrologistics* is to make sure about the efficient supply of the customers, business in order to establish the traffic flows between sources and destinations within a region, country or around the world. An efficient logistics is necessary so as to one company can achieve the optimal economic development of one country besides suitable laws and capable institutions. (12)

Besides, the main goal of *micrologistics* is to supply depend on private orders, contracts as well as agreements. Therefore, companies and consumers with the most requirement of goods and to cover the mobility demand of individuals. For this purpose, companies and logistics providers plan, set up and operate logistic systems and networks. The main task of *micrologistics* is to operate and realize also including operate logistics system in order to ensure supply networks and transport chains to satisfy the needs of the consumers along with the optimal development of a company. (12)

3.2.2 History of logistics:

The original of the word "*Logistics*" was from Greek, the term "skilled in calculating" was from "*logistikos"* meaning. However, the word "*Logistics*" did not appear until the end of eighteenth century when applied to military administration. (22)

Logistics is a type of activity, has happened many years ago and can be associated with the oldest forms of organized trade. Even some authors consider the organization of pyramid building as the beginning of practical logistics. (22)

Since the 9th century, this term can be noticed in military. Logistics organized the provision of all military needs for example supply of food, weapons, ammunition or controlled movements of military units. (13)

Logistics, as the subject of research, appears in the early 20th century. Logistics did not receive much attention until after World War II. Efficient distribution and supply have fundamentally contributed to the success of the Allied troops on the Western Front. As a result, the post-war tendency was to use logistics in the civilian sphere. Supply problems led to the use of mathematical methods, which were subsequently applied in business logistics. (13)

3.2.3 System of logistics

"The logistics system represents a purposefully arranged set of all technical means, equipment, buildings, roads and workers involved in the realization of the logistics chain. The logistics system can be considered as a special kind of multi-system, which we define as technical-technological, information-communication system and management system. The aim of the company's logistics system is to strengthen and strengthen the company's position as an economic entity in the market". (29)

The logistics system includes three subsystems:

1. **Information system:**

Ensures all data manipulation related to a logistics system. The information system can be divided into three groups:

- Planning system: prepares, creates and optimizes the logistics chain links

- Disposition system: guarantees smooth operation of logistics systems.
- Delivery system: provides support for information management of material flow. (19)

2. **Control system:**

Process information at the point of origin in real-time. Control systems can be divided into two groups:

- Computerized: data processing using technology, which causes a lower error rate and fewer administration demands.
- Uninformatized: data is processed by people, which results in its extensive administration and slow and inefficient management. (19)

3. **Material system:**

Records material and manages it by material security. (19)

3.2.4 Development of logistics:

The origin of logistics was first started in the military. The Sumerians use simple considerations and calculations in order to improve the supply as well as the transfer of troops in the organization. Moreover, in the military, logistics has proved its worth complex operations of allied troops at the end of World War II. (18)

In the civilian sector, the logistics method began to be used after the war. They started to develop inventory management systems along with IT planning. However, at the end of the twentieth century, with the use of sciences established partial scientific approaches to the problems. (18)

In contrast, logistics today not only optimize material but also information and financial flows. Fortunately, with the help of new tracking technologies, it allows to manage and monitor material flow, handing equipment and means of transport. (18)

3.2.5 Supply Chain Management and Logistics in tourism

Regarding logistic in tourism, the supply-delivery chain management concerns not only the internal integration of logistics activity but also the external integration.
The internal logistics of one organization include the following activities:

- The scheduling of strategic (the structure of the distribution)

- The scheduling of demand (the demand o touristic on the destination)
- The scheduling of provision (the production planning and dimensioning)

The main core of the supply-delivery chain management in tourism includes the following activities: logistics management, external integration as well as the elaboration of strategy concerning the specific activity development in different categories of participants. (18)

3.2.6 Logistics decision

It has been demonstrated that when performing and creating a logistics system, it is necessary to take into account some of the basic problems. According to the planning horizon, logistics decisions conventionally divided into 3 parts: strategic, tactical and finally is operational. (9)

Strategic decisions: It contains logistics systems design and the acquisition of capacity sizing, warehouse structure… for the reason that data is usually inadequate and incorrect, strategic decisions employ forecast depends on the aggregate data which makes it has more long-lasting effects. (9)

Tactical decisions: Made on a medium-term basis. It contains the activity of production and distribution planning. Tactical decisions usually employ forecast depends on the disaggregated data. (9)

Operation decisions: Made on real-time or daily basis which include warehouse order picking, shipment as well as vehicle dispatching. Operation decisions depend on very detailed data. (9)

3.3 Quantitative methods for decision support:

The use of quantitative methods is mainly used to reduce intuitive decision-making and eliminate the negative consequences of subjective management problems. (11)
The most common decision-making problem in the proceedings is the decision-making problem, which contributes to solves the goals and needed for its realization. (11)
Some publications talk about quantitative methods also through the concept of operation analysis, for example: "Operational analysis is the application of scientific methods to complex, trade and military. A peculiarity of the approach is the development of a scientific methods model of a system, including the measurement of factors such as chances and risk

from which the outcome of alternative decisions, strategies or management their decision scientifically". (11)

Thus, it is typically focused on decision support in situations where complex solutions to complex or large systems need to be designed. The use of quantitative methods can also be observed in situations where a large number of external and internal factors, with complex interrelationships, affect the problem solution, and the resulting solution has an impact on the entire managed system. These methods are also a useful tool in finding solutions in new situations where the impact of decisions has not yet been mapped, or effective solutions. It is also recommended to apply these methods when the decision has an impact on the economic indicators of the business or to apply routine solutions to standardized problems where the solution algorithm can easily be implemented in an automated company management system. (11)

3.3.1 Operational research:

Operational research is a term which represents for an approach with the aim of problem-solving characterized by system orientation, an interdisciplinary philosophy which is concentrate on quantification of the important aspects of the circumstances into a model as well as the manipulation of this model in term of mathematical, statistical along with computer methodologies so as to improve plans, decisions, and policies in life. (25)

This term was created during World War II when scientists in many different fields were asked to contribute in the military in order to find out the best solutions in various operational problems. These problems vary from find out the best location of radar units in order to release the early warning of air raids to clarifying search strategies for the protection of submarine. (25)

Since then, Operational Research has come to be a standard activity in almost organizations and is educated as a part of the business for instance: system analysis, quantitative analysis and so on. (25)

Under the field of tourism, the term Operational Research used by Cesario happens to be the first applied in the literature (27). Nowadays, with the practice of Operational Research, a lot of planers together with tourism researchers use the concepts as well as methodologies in the world of business. Associated with the help of concepts and methodologies, it is typically deal with some problems in the field of tourist flows, tourist forecasting, planning and policy problems, the measurement and evaluation of the impact of oriented tourism facilities and activities, and the determination of tourist flows. (25)

3.3.2 Traveling salesman problem

The name "traveling salesman problem" for the optimization is believed to have originated in the United States. Possibly because the first report using this term was made public in 1949 (21). The traveling salesman problem is defined as follows: Given a set of nodes and length of travel between pairs and nodes, find the tour of the minimal length that visits each node exactly once and comes back to the initial node. (1)

In the field of operations research literature, the traveling salesman problem is one of the most popular combinational optimization problems. It is divided into two main types: symmetric or asymmetric which is based on accordingly whether the cost and the distance to travel is either symmetric or asymmetric. (4)

The problem is modeled using a complete graph $G = \{V, E\}$ which are including n vertices

which are denoting the cities and m connecting edges. The traveling cost (distance or time) between the vertices it connects is assigned with a weight on each edge. The main objective is to find the shortest routes to pass through n vertices that minimizes the total cost. (14)

In real life, there are many issues that include finding a series of tasks, actions or operations can be known as Travelling Salesman Problem. Moreover, many real-world problems for example like scheduling, routing, networking and so on… is a part of it. (20)

Given a formal mathematical definition of the Traveling Salesman Problem, let $G = \{V, E\}$ (Directed or undirected graph), F be the family of all Hamiltonian cycle (tours) in G . (10)

For each edge $e \in E$ a cost (weight) C_e is prescribed. The problem of the travel salesman problem is to find a tour (Hamiltonian cycle) in G in order to find the smallest sum of the cost on each edge of the tour. (10)

We suppose that G is a completed graph, we could replace the missing edges with the one that have a very large cost. Let set the node $V = \{1,2, \ldots n\}$. The matrix $C = (C_{ij})n \times n$, is called the cost matrix (distance or weight matrix), where the $(ij)^{th}$ entry C_{ij} corresponds to the cost of the edge joining node i to node G. (10)

Besides, depending on the cost of the matrix (the nature of G), Traveling Salesman Problem is divided into two classes:

- If C is asymmetric (given the graph G is undirected) then the problem is called the Symmetric Traveling Salesman Problem (STSP). (10)

- If C is not asymmetric (given the graph G is directed) then the problem is called the Asymmetric Traveling Salesman Problem (ATSP). (10)

While every undirected graph can be seen as a directed graph by duplicating edges in the forward direction and the other from the backward direction. Therefore, STSP can be taken as a special case of ATSP. (10)

3.3.2.1 Nearest neighbour algorithm (sequential)

The nearest neighbor method is the simplest approximation method. Its principle is that the chosen starting point is connected with the place with the most favorable rate.
From there it connects to another location that has not been yet included in the route, which has the most convenient connection to a location that has already been selected before.
After passing all the places, the circuit is closed by returning to the starting point. By choosing each location as the starting point will determine all around trips.
When calculating by using this method, the column in the rate matrix will first be deleted. The cell that has the most favorable rate is found in the row which corresponding to the starting point. This connection creates the first section of the circular route. (24)

3.3.2.2 Vogel's approximation algorithm

Vogel's approximation method is to finds solutions near the optimum, and it is very often used in solving circular traffic problems, but also in single-stage traffic tasks. (15)

1. Calculate the difference between the two lowest rates in each row and column.
2. Select the cell with the lowest rate in the row or column with the highest difference.

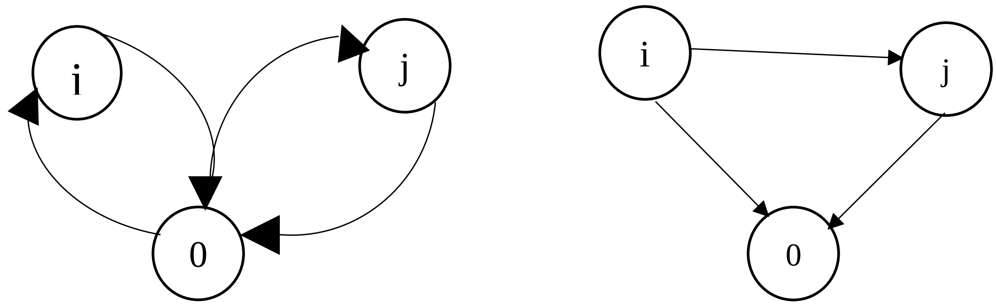
3. Delete the entire column and row in which the cell was located in order to avoid the possibility of repeat stations of the circuit.
4. The procedure is repeated in the table changed table until all rows and columns are deleted of the transport table, and if the maximum difference occurs in two rows, it is necessary to occupy the cell with the most advantageous rate in these rows. (24)

3.3.2.3 The savings algorithm (parallel)

The savings algorithm is the experimental algorithm, therefore it does not present the best optimal result. However, this algorithm proposes a quite good result because that solution deviates not much from the optimal result.

The basic concept of the savings algorithm states through the savings cost which is achieved by merging two routes into one as shown in figure 1, and 0 denote for the depot

Figure 1 - Illustration of savings concept



(a) **(b)**

(6)

At first, customers i and j are travelled on individual routes which can be seen from figure 1 (a). However, there is an alternative to this problem is to travel the two customers on the same route, for example in the sequence $i - j$ which can be seen in figure 1(b). Because the transportation costs are given, the savings that result from driving the route in figure 1(b) instead of the two routes in figure 1(a) can be calculated. Refer to the transportation cost between two given points i and j by C_{ij}, the total transportation cost D_a in figure 1(a):

$$D_a = c_{0j} + c_{i0} + c_{j0} \quad (6)$$

The same as the transportation cost D_b in figure 1(b) is:

$$D_b = c_{0i} + c_{ij} + c_{j0} \quad (6)$$

By merging the two routes one obtains the savings S_{ij}:

$$S_{ij} = D_a - D_b = c_{i0} + c_{0j} - c_{ij} \quad (6)$$

Relatively large values of S_{ij} indicate that it is attractive, with regard to costs, to visit points i and j on the same route such that point j is visited immediately after point i.

The savings algorithm has two main versions parallel version and the other is a sequential version. In the parallel version more than one route may be built at a time, meanwhile in the sequential version just only one route is built at a time.

The first step of the savings algorithm, all pairs of customer points are sorted in descending order of the savings as well as the savings for all pairs of customers are calculated. Afterward, from the top of the sorted list of point pairs, one pair of points is considered at a time. When a pair of points $i - j$ is considered, the two routes that travel i and j are combined (such that j is go to immediately after i on the resulting route), if this can be done without deleting a previously established direct connection between two customer points, and if the total demand on the resulting route does not exceed the vehicle capacity. In the parallel version only requires one pass through the list, meanwhile the sequential version, one must start a new from the top of the list every time a connection is established between a pair of points (since combinations that were not viable so far now may have become viable). (6)

3.3.3 Traveling salesman problem and its application

In real life, Traveling Salesman Problem and its variation can be applied in many other aspects not just only planning the route problem for example: computer science, operational research, engineering, mathematics, electronics, and so forth. (10)

3.3.3.1 Machine scheduling problems

Traveling Salesman Problems is probably well-known for scheduling and machine sequencing. A simple scheduling application can be explained as follow. Given n jobs that needed to be processed sequentially on a machine $\{1,2,\dots n\}$ .Let c_{ij} be the setup cost required for processing task then j immediate processor after task i. Once the tasks are being processed, then the machine is reset to its initial state which is c_{i1} , given j is the last task process so that the total setup cost is minimized. Undoubtedly, finding a permutation π $\{1,2,\dots n\}$ that minimizes $C_{\pi(n)\pi(1)} + \sum_{i=1}^{n-1} C_{\pi(n)\pi(1)}$ to solve the problem. **(**10)

The objective of the sequencing problem is to find order in order to find out the total minimized setup cost which is processed by the tasks. We can see in many practical applications, the tasks usually can be clustered together so as to set up time, if any, the relationship between tasks within a cluster is not too big compared to setup time between tasks in two different clusters. **(**10)

Besides, given the n jobs for the scheduling problem on m machines in the order 1,2,3…, m. There should be no job is allowed to have a waiting time within the processing of two machines. The main goal of this problem is to find out an optimal sequencing of tasks in order to minimize the total completion time. The problem therefore can be described as follow. Make a complete directed graph G on $n+1$ nodes, where $1 \leq j \leq n$ and $n+1$ stand for the start as well as the end of the process. The cost c_{ij} of arc (i,j) in G stand for the additional schedule length if task j replace right after the task i in the workflow. In order to complete the reduction, the value of c_{ij} must be classified. Assume that p_{jk} be the job processing time of task j on the processor k, where $1 \leq j \leq n$ and where $1 \leq k \leq n$. then c_{ij} can be obtained by using the following equations $c_{n+1,i} = \sum_{r=1}^{m} p_{ir}$, where $i = 1,2 \dots . m$. (10)

3.3.3.2 Arc routing problems

Arc Routing Problems often be taken as a mixed windy rural postman problem (MWRPP), and it can be explained as follows. Given $G = (V, A \cup E)$ where components of K are edges with an undirected graph. Given $A \subset A'$ and E ⊂ E'. The arc and edge costs are given to be non-negative. The aim of MWRPP is to find a minimum cost and containing all arc in A' along with all edges in E' which is a closed walk to G. There are some special circumstances related to MWRPP are: windy Chinese postman problem, mixed Chinese postman problem, and so forth. (10)

3.3.4 TSPKOSA:

Under the cooperation of two departments: Department of Systems Engineering and Department of Statistics of Czech University of Life Sciences in Prague, 2010.

TSPKOSA was designed in order to find out the most convenient routes for the Travelling Salesman Problem. The program was created in the Microsoft Visual Basic 6.5 programming language with four selected methods:

- Approximation methods:
    - Nearest Neighbor Algorithm (sequential)
    - Vogel Approximation method
    - Savings algorithm (parallel)
- Optimization method:
    - Branch and Bound (16)

4 Practical Part

4.1 Characteristics of the problem:

With the advantageous location in the middle of Europe, Czech Republic attracts thousands of international tourists to visit the country every year. Given the situation that one student would like to travel around the country during their summer vacation and plan the whole time and trip on their own and suppose all of the destinations are chosen based on the interest of the student. The trip begins at Prague main train station and travels from there to other destinations and finally returns to the starting point after 10 days traveling.

There are 47 destinations included in this journey that needed to be visited apart from the Prague main train station, which is the place of departure as well as the return point of the whole trip. Due to the fact that the trip includes more places than one day can be visited, therefore it is necessary to divide these places into groups, which is 10 days and each day with approximately 5 destinations. The time for sightseeing and break time is given depend on the interest of the student in each destination.

Each day, the student will travel approximately up to 5 destinations with the visiting time vary from 60 – 90 minutes between each location along with the break time already included. In total, the student can travel up to 8 hours per day which is consists of travelling time, sightseeing and also break time. The following days of the trip will be started from the last location of the previous day and continually repeat until the end of the trip.

The total destinations of the trip can be found in the table down below which is not an ideal result to travel for the whole trip.

Table 1 - List of destinations

Number of destinations	**List of destinations**	**Number of destinations**	**List of destinations**
[1]	Prague main train station	[24]	Stromovka park
[2]	Maisel Synagogue museum	[25]	Museum Kampa
[3]	National Library of the Czech Republic	[26]	Troja Palace
[4]	Church of The Most Sacred Heart of Our Lord	[27]	Parukářka park
[5]	Czech Police Museum	[28]	Hloubětín castle
[6]	Academy of Sciences Library of the Czech Republic	[29]	Saints Cyril and Methodius Cathedral
[7]	Obora Hvězda park	[30]	Dolní Počernice Chateau
[8]	Průhonice Castle	[31]	Kbely Aviation Museum
[9]	Church of St. James the Greater	[32]	Ctěnice Castle
[10]	Vrtba Garden	[33]	Čakovice Chateau
[11]	Dancing House	[34]	Ďáblice Chateau
[12]	Czech Museum of Music	[35]	Vinoř Castle
[13]	Prague zoo	[36]	Jenštejn Castle
[14]	Saint Castulus Church	[37]	Chvalský Castle
[15]	Czech National Library of Technology	[38]	Libeň Chateau
[16]	Žižkov TV Tower	[39]	Malešice Chateau
[17]	Botanical garden	[40]	Kunratice Castle
[18]	Letná park	[41]	Petrovice Chateau
[19]	Church of St. Ludmila	[42]	Krč Chateau
[20]	Prague Castle	[43]	Vršovice Castle
[21]	Vyšehrad Castle	[44]	Lochkov Chateau
[22]	St. Nicholas Church	[45]	Černínský Palace
[23]	Municipal Library of Prague	[46]	Záběhlice Monastery Chateau

Source: own processing

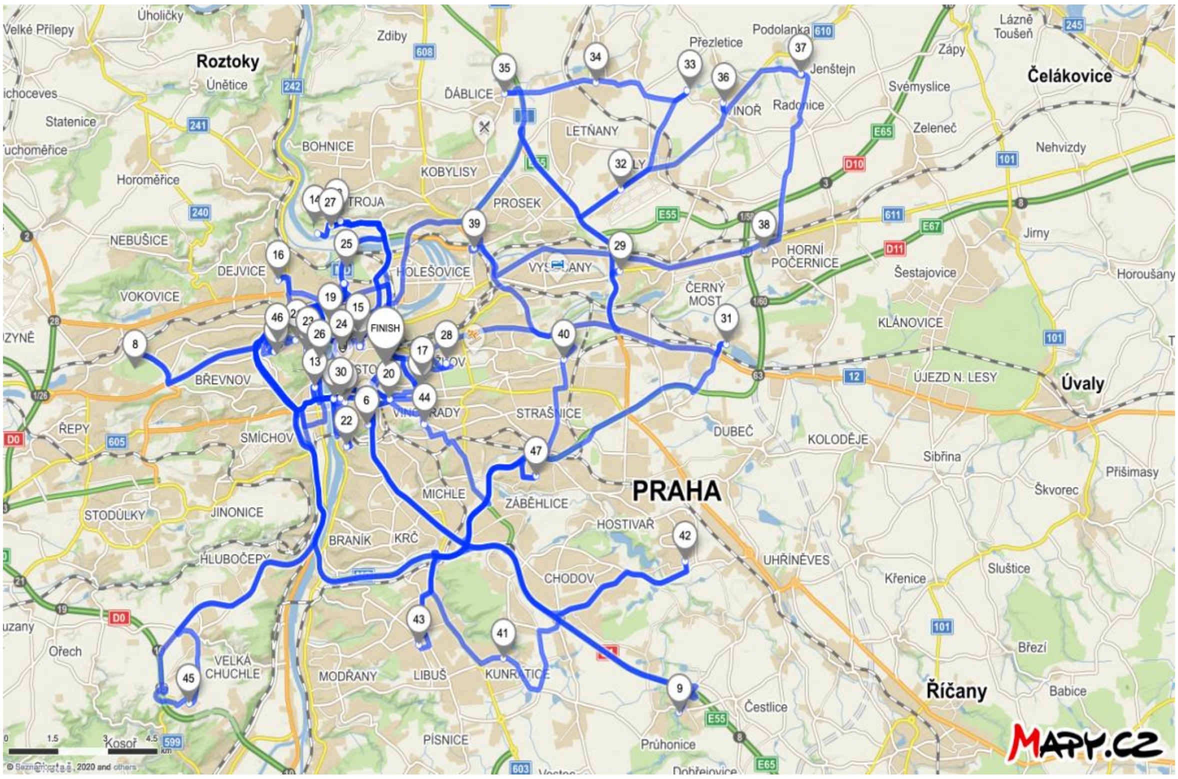
4.2 Starting circuit:

The trip requires a circuit passing through 47 destinations, it is also included the Prague main train station for the starting and finishing point. Assigned each number with the destinations from 1 to 46. Thus, the default circuit in shown as follow:

[1] – [2] – [3] – [4] – [5] – [6] – [7] – [8] – [9] – [10] – [11] – [12] – [13] – [14] – [15] – [16] – [17] – [18] – [19] – [20] – [21] – [22] – [23] – [24] – [25] – [26] – [27] – [28] – [29] – [30] – [31] – [32] – [33] – [34] – [35] – [36] – [37] – [38] – [39] – [40] – [41] – [42] – [43] – [44] – [45] – [46] – [1].

Given the speed of the car is 666,667 meters/minute. The total distance of the circuit is 370 522 meters long and it takes 555,783 minutes to finish the circuit with the break time and sightseeing is already included between each destination. The result is not an optimal version of the whole circuit.

Figure 2 - Default circuit



Source: Mapy.cz 2020

4.3 Optimizing the problems:

In order to minimize the distances, the program TSPKOSA was applied to solve the problem. The Nearest Neighbor algorithm (sequential), The Vogel's Approximation and The Savings algorithm (parallel) are deployed to optimize the circuit. These may not be symmetrical because the route from A to B may not be the same as from B to A due to roundabout, one-way traffic and so on.

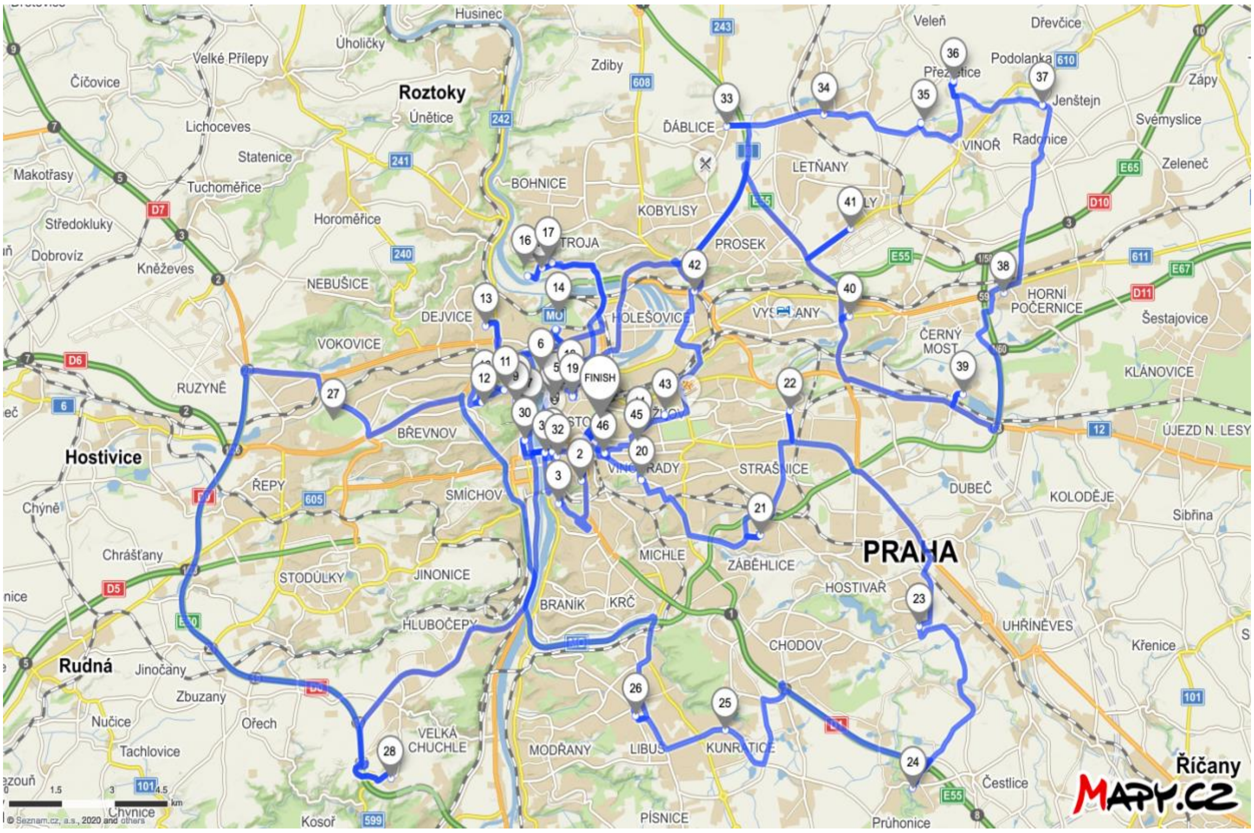
Due to each method proposed diverse alternative solutions to the problem, therefore in the end the result with the lowest outcome will be selected as the goal of this bachelor thesis.

4.3.1 Nearest neighbour algorithm – Distance matrix

The first result obtained from TSPKOSA program, the Nearest Neighbor algorithm on the matrix is used. The result is as follow:
[1] – [5] – [21] – [23] – [2] – [18] – [25] – [23] – [10] – [45] – [20] – [3] – [15] – [24] – [26] – [13] – [17] – [14] – [4] – [43] – [46] – [39] – [41] – [8] – [40] – [42] – [7] – [44] – [29] – [12] – [11] – [6] – [34] – [33] – [32] – [35] – [36] – [37] – [30] – [28] – [31] – [38] – [27] – [16] – [9] – [19] – [1].

The total distance of the circuit is 184 734 meters long and it takes 277,101 minutes to finish the circuit with the break time as well as sightseeing is already included between each destination.

Figure 3 - First calculated routes



Source: Mapy.cz 2020

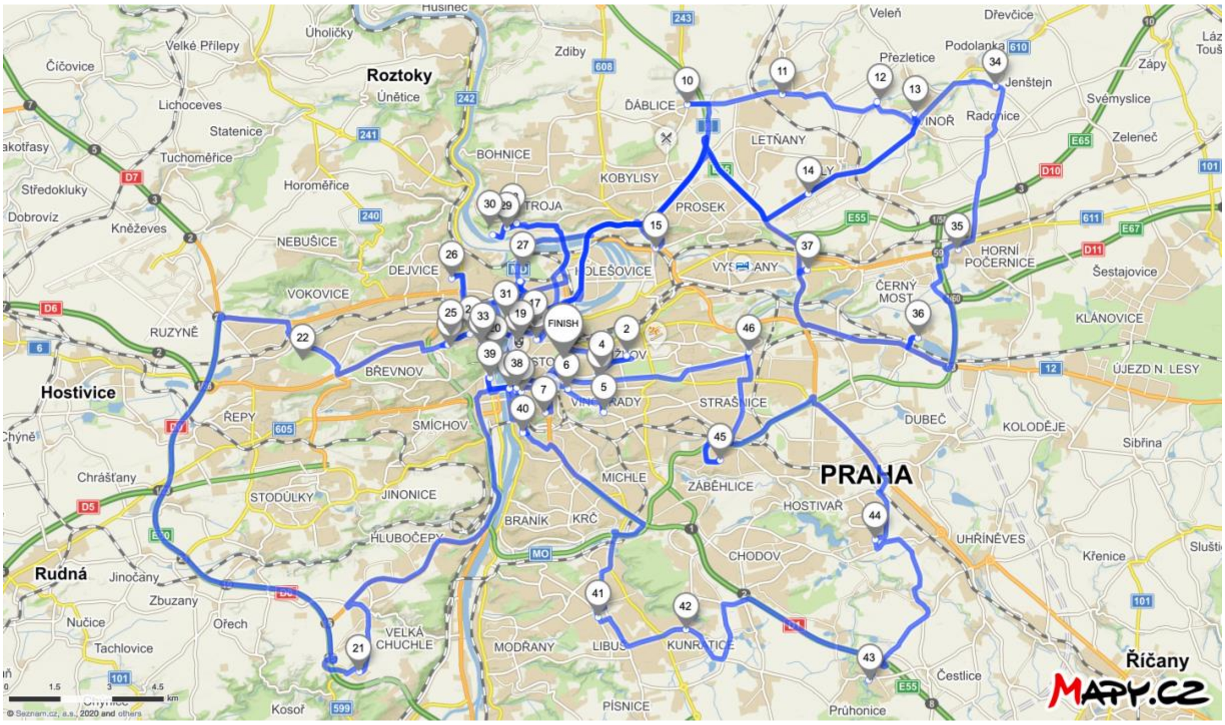
4.3.2 Vogel's approximation algorithm – Distance matrix:

The second result obtained from TSPKOSA program. The Vogel's approximation algorithm on the matrix is used has the following result:

[1] – [27] – [9] – [16] – [43] – [19] – [5] – [11] – [6] – [34] – [33] – [35] – [31] – [38] – [4] – [14] – [23] – [2] – [25] – [44] – [7] – [3] – [20] – [45] – [15] – [24] – [17] – [26] – [13] – [18] – [10] – [22] – [36] – [37] – [30] – [28] – [29] – [12] – [21] – [42] – [40] – [8] – [41] – [46] – [39] – [1].

The total distance of the circuit is 174 714 meters long and it takes 262,071 minutes to finish the circuit with the break time as well as sightseeing is already included between each destination.

Figure 4 - Second calculated routes



Source: Mapy.cz 2020

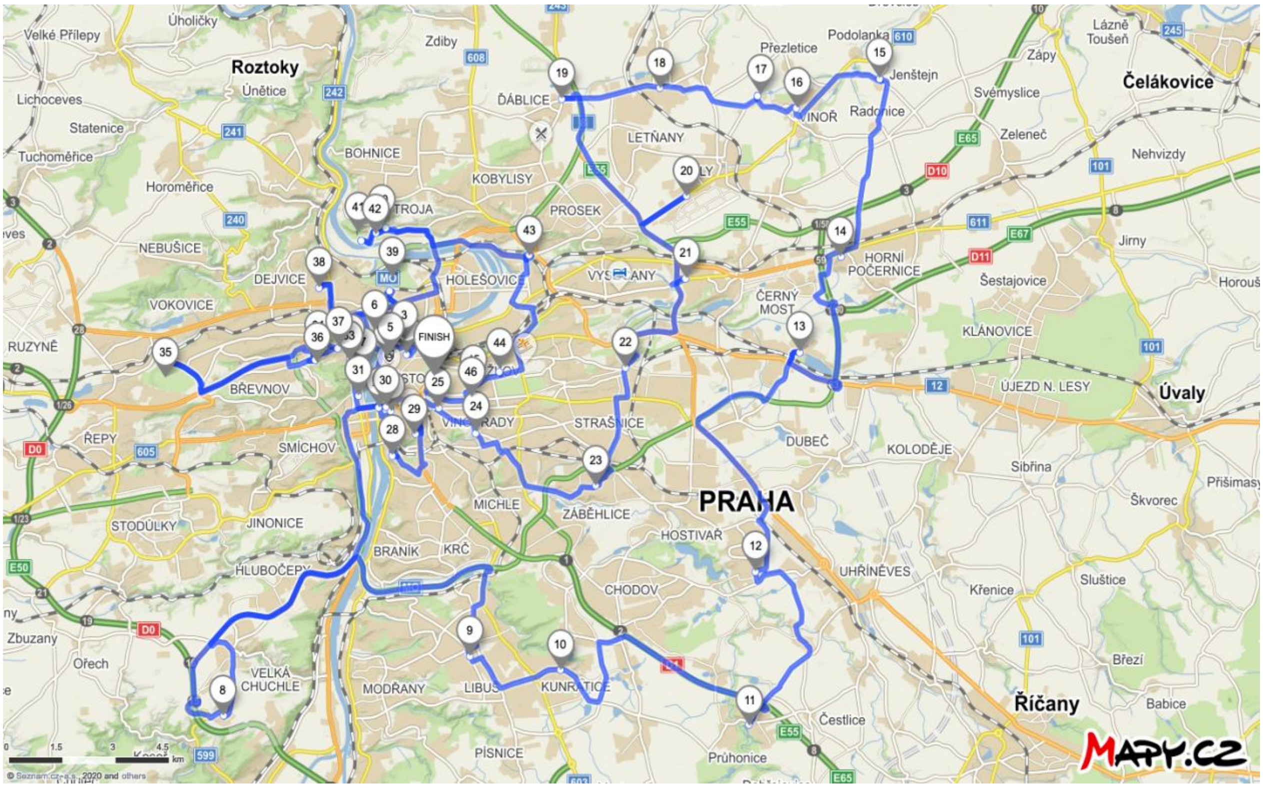
4.3.3 Savings algorithm (parallel) – Distance matrix:

The second result obtained from TSPKOSA program. The savings algorithm on the matrix is used has the following result:

[1] – [4] – [14] – [23] – [2] – [18] – [25] – [44] – [42] – [40] – [8] – [41] – [30] – [37] – [36] – [35] – [32] – [33] – [34] – [31] – [28] – [39] – [46] – [43] – [19] – [11] – [6] – [21] – [5] – [29] – [12] – [22] – [10] – [45] – [7] – [3] – [20] – [15] – [24] – [17] – [13] – [26] – [38] – [27] – [16] – [9] – [1].

The total distance of the circuit is 161 266 meters long and it takes 241,899 minutes to finish the circuit with the break time and sightseeing is already included between each destination.

Figure 5 - Third calculated routes



Source: Mapy.cz 2020

4.4 Solving the problems:

According to the results given above by the program TSPKOSA. The best result for traveling is the Savings method with the total distance of the circuit is 161 266 meters long and it takes 241,899 minutes.

Due to the fact that the trip includes more places than one day can visit. Therefore, it is necessary to divide these places into groups which are 10 days and each day with approximately 5 destinations.

However, as the trip contains several days to travel, therefore the first destination of the following days will not be counted as the place needed to visit because it is already be visited the day before.

Besides, the time to sightseeing and break time was included together and vary from 60 – 90 minutes which is based on the interest of the student in each destination. The speed of the car was chosen to be 666,667 meters/minute along with the distance numbers in meter was obtained from the Savings algorithm.

The results of the problems between each day are calculated as follow:

4.4.1 Day one travel:

Table 2 - *Day one travel results obtained from the Savings method*

	v = 666,667 (meters/minute)		
Day 1	**Spending time (minutes)**	**Travelling time $t = \frac{s}{v}$ (minutes)**	**s (meters)**
[1]	-	-	-
[4]	90	4,333	2 889
[14]	90	1,051	701
[23]	90	2,349	1 566
[2]	60	0,393	262
[18]	60	1,851	1 234
Total time	**399,978 (minutes)**		

Source: own processing of data, results are obtained by the program TSPKOSA.

Chosen the starting point of the trip is from destination [1]. When traveling in sequence by the obtained the result of the Nearest Neighbor problem from destination [1] to [18], the total distance is calculated as follow:

$$2\ 889 + 701 + 1\ 566 + 262 + 1\ 234 = 6\ 652 \text{ (meters)}$$

Given the speed of the vehicle is 666,667 meters/minute.

In order to find the travelling time, the formula: $\boldsymbol{t = \frac{s}{v}}$ applied in sequence from destination [18] to destination [8], the total travelling time is calculated as follow:

$$4{,}333 + 1{,}051 + 2{,}349 + 0{,}393 + 1{,}851 = 9{,}978 \text{ (minutes).}$$

Moreover, the total result of the spend time in one place from destination [18] to destination [8] is calculated as follow:

$$90 + 90 + 90 + 60 + 60 = 390 \text{ (minutes).}$$

Together, the total time that needed to spend on day one is:

$$9{,}978\ +\ 390 = 399{,}978\ \text{ (minutes)}$$

The routes of day one are shown as follow:

[Prague main train station] – [Church of The Most Sacred Heart of Our Lord] – [Saint Castulus Church] – [Municipal Library of Prague] – [Maisel Synagogue museum] – [Letná park].

Figure 6 - Day one travel scenario

Source: Mapy.cz 2020

4.4.2 Day two travel:

Table 3 - *Day two travel results obtained from the Savings method*

	v = 666,667 (meters/minute)		
Day 2	**Spending time (minutes)**	**Travelling time:** $t = \frac{s}{v}$ **(minutes)**	**s (meters)**
[18]	-	-	-
[25]	90	1,525	1 070
[44]	60	18,450	12 300
[42]	60	19,125	12 750
[40]	90	4,838	3 225
[8]	60	10,925	7 283
Total time	**414,864 (minutes)**		

Source: own processing of data, results are obtained by the program TSPKOSA.

The starting point of day two begins with the last destination from day one which is destination [18]. When traveling in sequence by the obtained the result of the Nearest Neighbor problem from destination [18] to [8], the total distance is calculated as follow:

$$1\,017 + 12\,300\ + 12\,750 + 3\,225 + 7\,283 = 36\,575 \text{ (meters).}$$

Given the speed of the vehicle is 666,667 meters/minute.

In order to find the travelling time, the formula: $\boldsymbol{t} = \frac{\boldsymbol{s}}{\boldsymbol{v}}$ is applied in sequence from destination [18] to destination [8], the total travelling time is calculated as follow:

$$1{,}526 + 18{,}450 + 19{,}125 + 4{,}838 + 10{,}925\ = 54{,}863 \text{ (minutes).}$$

Moreover, the total result of the spend time in one place from destination [18] to destination [8] is calculated as follow:

$$90\ + 60\ + 60\ +\ 90 + 60\ \ = 360 \text{ (minutes).}$$

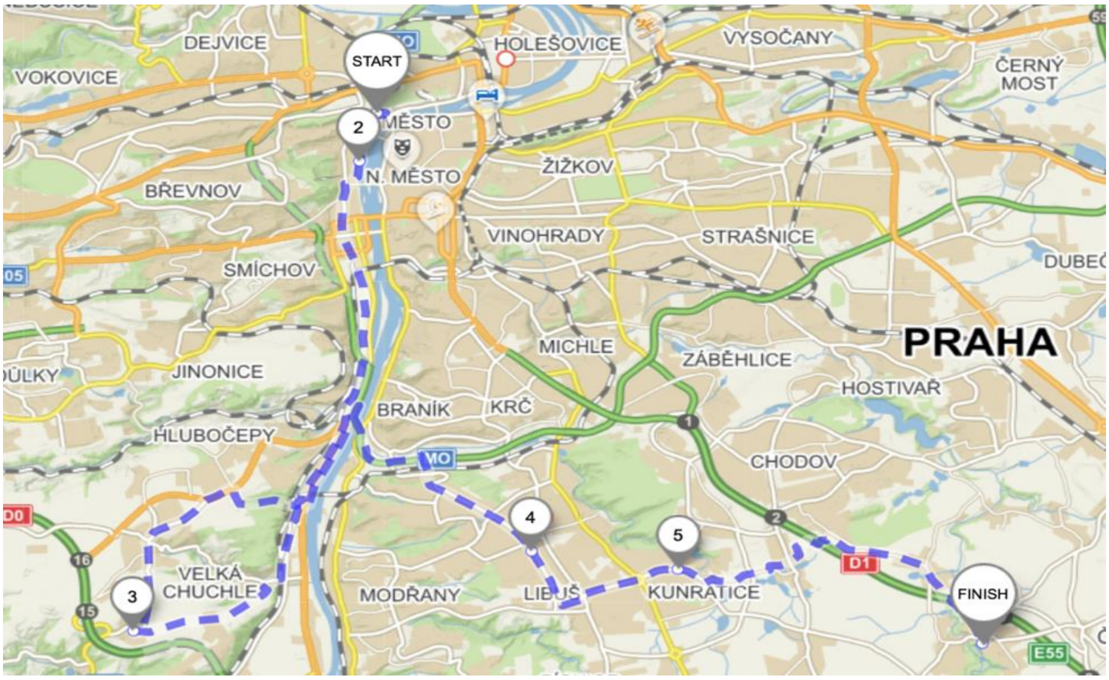
Together, the total time that needed to spend on day two is:

$$54{,}864\ +\ 360 = 414{,}864 \text{ (minutes)}$$

The routes of day two are shown as follow:

[Letná park] – [Museum Kampa] – [Lochkov Chateau] – [Krč Chateau] – [Kunratice Castle] – [Průhonice Castle].

Figure 7 - Day two travel scenario



Source: Mapy.cz 2020

4.4.3 Day three travel:

Table 4 - Day three travel results obtained from the Savings method

	v = 666,667 (meters/minute)		
Day 3	**Spending time (minutes)**	**Travelling time: $t = \frac{s}{v}$ (minutes)**	**s (meters)**
[8]	-	-	-
[41]	90	10,398	6 932
[30]	60	15,421	10281
[37]	90	7,44	4 963
[36]	60	8,100	5 400
[35]	90	4,573	3 049
Total time	**435,937 (minutes)**		

Source: own processing of data, results are obtained by the program TSPKOSA.

The starting point of day three begins with the last destination from the day two which is destination [8]. When travelling in sequence by the obtained the result of the Nearest Neighbor problem from destination [8] to [35], the total distance is calculated as follow:

$$6\,932 + 10\,281 + 4\,963 + 5\,400 + 3\,049 \; = 30\,625 \text{ (meters).}$$

Given the speed of the vehicle is 666,667 meter/minute.

In order to find the travelling time, the formula: $\boldsymbol{t} = \frac{\boldsymbol{s}}{\boldsymbol{v}}$ is applied in sequence from destination [8] to destination [35], the total travelling time is calculated as follow:

$$10{,}398 \; + 15{,}421 + 7{,}444 + 8{,}100 + 4{,}573 \; = 45{,}937 \text{ (minutes).}$$

Moreover, the total result of the spend time in one place from destination [8] to destination [35] is calculated as follow:

$$90 + 60 + 90 + 60 + 90 \; = 390 \text{ (minutes).}$$

Together, the total time that needed to spend on day three is:

$$45{,}937 \; + \; 390 = \; 435{,}937 \text{ (minutes)}$$

The routes of day three is shown as follow:

[Průhonice Castle] – [Petrovice Chateau] – [Dolní Počernice Chateau] – [Chvalský Castle] – [Jenštejn Castle]- [Vinoř Castle].

Figure 8 - Day three travel scenario

Source: Mapy.cz 2020

4.4.4 Day four travel:

Table 5 - Day four travel results obtained from the Savings method

	v = 666,667 (meters/minute)		
Day 4	**Spending time (minutes)**	**Travelling time: $t = \frac{s}{v}$ (minutes)**	**s (meters)**
[35]	-	-	-
[32]	90	2,811	1 874
[33]	60	4,714	3 143
[34]	60	4,266	2 844
[31]	90	9,283	6 189
[28]	90	5,838	3 892
Total time	**416,913 (minutes)**		

Source: own processing of data, results are obtained by the program TSPKOSA.

The starting point of day four begins with the last destination from day three which is destination [35]. When traveling in order by the obtained the result of the Nearest Neighbor problem from destination [35] to [28], the total distance is calculated as follow:

$$1\,874 + 3\,143 + 2\,844 + 6\,189 + 3\,892 \ = 17\,942 \text{ (meters).}$$

Given the speed of the vehicle is 666,667 meters/minute.

In order to find the travelling time, the formula: $\boldsymbol{t} = \frac{\boldsymbol{s}}{\boldsymbol{v}}$ is applied in sequence from destination [35] to destination [28], the total travelling time is calculated as follow:

$$2{,}811 \ + 4{,}714 + 4{,}266 + 9{,}283 + 5{,}838 = 26{,}913 \text{ (minutes).}$$

Moreover, the total result of the spend time in one place from destination [35] to destination [28] is calculated as follow:

$$90 + 60 + 60 + 90 + 90 = 390 \text{ (minutes).}$$

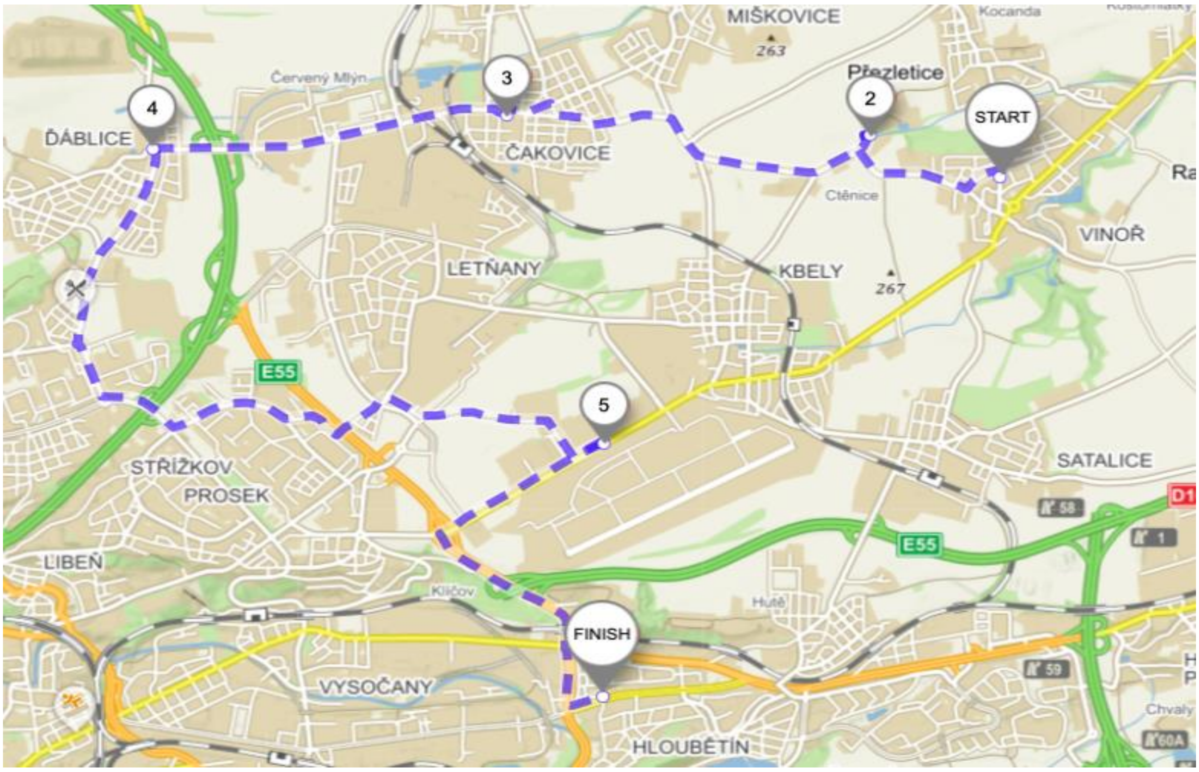
Together, the total time that needed to spend on day four is:

$$26{,}913 \ \ + \ 390 = \ 416{,}913 \text{ (minutes)}$$

The routes of day four are shown as follow:

[Vinoř Castle] – [Ctěnice Castle] – [Čakovice Chateau] – [Ďáblice Chateau] – [Kbely Aviation Museum] – [Hloubětín castle].

Figure 9 - Day four travel scenario



Source: Mapy.cz 2020

6.4.5 Day five travel:

Table 6 - Day five travel results obtained from the Savings method

	v = 666,667 (meters/minute)		
Day 5	**Spending time (minutes)**	**Travelling time: $t = \frac{s}{v}$ (minutes)**	**s (meters)**
[28]	-	-	-
[39]	90	6,144	4 096
[46]	60	6,235	4 157
[43]	90	8,037	5 358
[19]	60	3,546	2 364
[11]	90	2,994	1 996
Total time	**416,956 (minutes)**		

Source: own processing of data, results are obtained by the program TSPKOSA.

The starting point of day five begins with the last destination from day four which is destination [28]. When travelling in order by the obtained the result of the Nearest Neighbor problem from destination [28] to [11], the total distance is calculated as follow:

$$4\ 096 + 4\ 157 + 5\ 358\ + 2\ 364\ + 1\ 996 = 17\ 971 \text{ (meters).}$$

Given the speed of the vehicle is 666,667 meters/minute.

In order to find the travelling time, the formula: $\boldsymbol{t} = \frac{\boldsymbol{s}}{\boldsymbol{v}}$ is applied in sequence from destination [28] to destination [11], the total travelling time is calculated as follow:

$$6{,}144 + 6{,}235 + 8{,}037\ + 3{,}546 + 2{,}994 = 26{,}956 \text{ (minutes).}$$

Moreover, the total result of the spend time in one place from destination [28] to destination [11] is calculated as follow:

$$90 + 60 + 90 + 60 + 90 = 390 \text{ (minutes).}$$

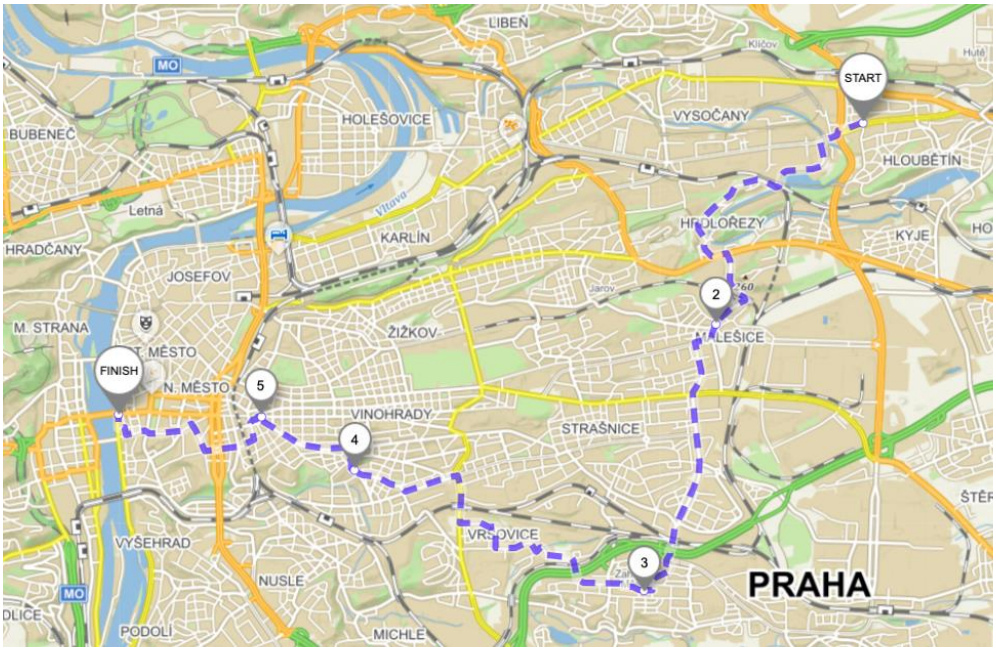
Together, the total time that needed to spend on day five is:

$$26{,}956\ +\ 390 =\ 416{,}956 \text{ (minutes)}$$

The routes of day five are shown as follow:

[Hloubětín castle] – [Malešice Chateau] – [Záběhlice Monastery Chateau] – [Vršovice Castle] – [Church of St. Ludmila] – [Dancing House].

Figure 10 - Day five travel scenario



Source: Mapy.cz 2020

4.4.6 Day six travel:

Table 7 - Day six travel results obtained from the Savings method

	v = 666,667 (meters/minute)		
Day 6	**Spending time (minutes)**	**Travelling time: $t = \frac{s}{v}$ (minutes)**	**s (meters)**
[11]	-	-	-
[6]	60	0,699	466
[21]	90	2,610	1 740
[5]	90	4,089	2 726
[29]	60	2,893	1 929
[12]	90	1,783	1 189
Total time	**402,075 (minutes)**		

Source: own processing of data, results are obtained by the program TSPKOSA.

The starting point of day six begins with the last destination from day five which is destination [11]. When traveling in order by the obtained the result of the Nearest Neighbor problem from destination [11] to [12], the total distance is calculated as follow:

$$466 + 1\ 740 + 2\ 726 + 1\ 929 + 1\ 189 = 8\ 050 \text{ (meters).}$$

Given the speed of the vehicle is 666,667 meters/minute.

In order to find the travelling time, the formula: $\boldsymbol{t} = \frac{s}{v}$ is applied in sequence from destination [11] to destination [12], the total travelling time is calculated as follow:

$$0{,}699 + 2{,}610 + 4{,}089 + 2{,}893 + 1{,}783 = 12{,}075 \text{ (minutes).}$$

Moreover, the total result of the spend time in one place from destination [11] to destination [12] is calculated as follow:

$$60 + 90 + 90 + 60 + 90 = 390 \text{ (minutes).}$$

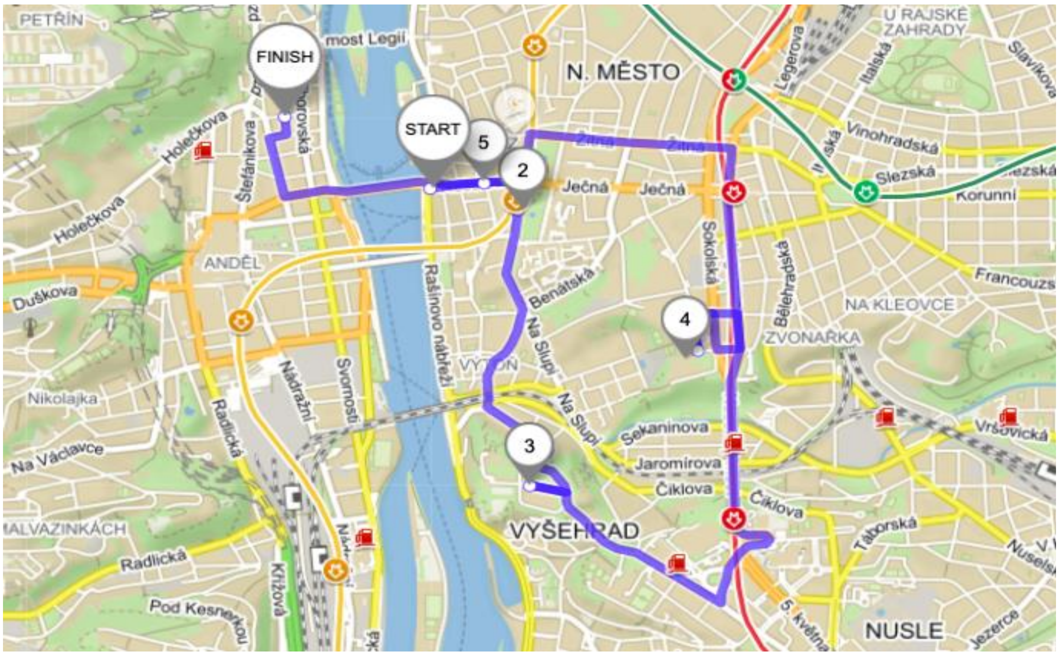
Together, the total time that needed to spend on day six is:

$$12{,}075\ +\ 390 =\ 402{,}075 \text{ (minutes).}$$

The routes of day six are shown as follow:

[Academy of Sciences Library of the Czech Republic] – [Vyšehrad Castle] – [Czech Police Museum] – [Saints Cyril and Methodius Cathedral] – [Czech Museum of Music].

Figure 11 - Day six travel scenario



Source: Mapy.cz 2020

4.4.7 Day seven travel:

Table 8 - Day seven travel results obtained from the Savings method

	v = 666,667 (meters/minute)		
Day 7	**Spending time (minutes)**	**Travelling time: $t = \frac{s}{v}$ (minutes)**	**s (meters)**
[12]	-	-	-
[22]	90	4,779	1 593
[10]	60	3,306	1 102
[45]	90	7,38	2 460
[7]	60	16,503	5 501
Total time	**331,968 (minutes)**		

Source: own processing of data, results are obtained by the program TSPKOSA.

The starting point of day seven begins with the last destination from day six which is destination [11]. When traveling in order by the obtained the result of the Nearest Neighbor problem from destination [12] to [7], the total distance is calculated as follow:

$$1\ 593\ + 1\ 102 + 2\ 460 + 5\ 501 = 10\ 656 \text{ (meters).}$$

Given the speed of the vehicle is 666,667 meters/minute.

In order to find the travelling time, the formula: $\boldsymbol{t} = \frac{s}{v}$ is applied in sequence from destination [12] to destination [7], the total travelling time is calculated as follow:

$$4{,}779 + 3{,}306 + 7{,}38\ + 16{,}503 = 31{,}968 \text{ (minutes).}$$

Moreover, the total result of the spend time in one place from destination [12] to destination [7] is calculated as follow:

$$90 + 60 + 90\ + 60 = 300 \text{ (minutes).}$$

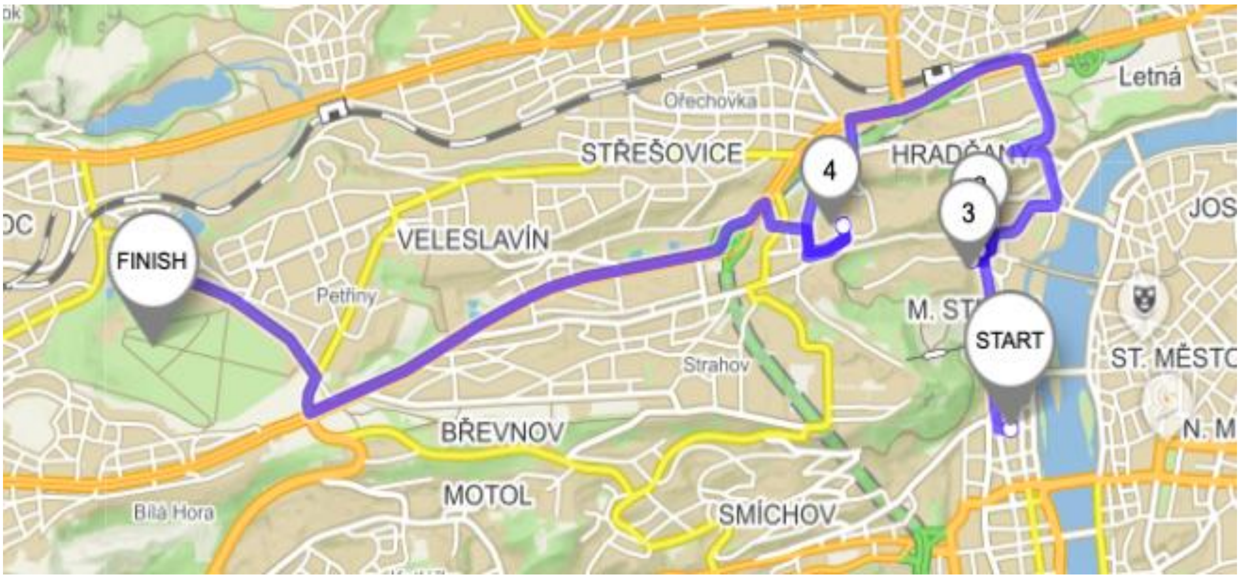
Together, the total time that needed to spend on day seven:

$$31{,}968\ +\ 300 =\ 331{,}968 \text{ (minutes).}$$

The routes of day seven are shown as follow:

[Czech Museum of Music] – [St. Nicholas Church] – [Vrtba Garden] – [Černínský Palace] – [Obora Hvězda park].

Figure 12 - Day seven travel scenario



Source: Mapy.cz 2020

4.4.8 Day eight travel:

Table 9 - Day eight travel results obtained from the Savings method

	v = 666,667 (meters/minute)		
Day 8	**Spending time (minutes)**	**Travelling time: $t = \frac{s}{v}$ (minutes)**	**s (meters)**
[7]	-	-	-
[3]	90	7,528	5 019
[20]	90	1,750	1 167
[15]	60	3,870	2 580
[24]	60	6,006	4 004
Total time	**319,155 (minutes)**		

Source: own processing of data, results are obtained by the program TSPKOSA.

The starting point of day eight begins with the last destination from day seven which is destination [7]. When traveling in sequence by the obtained the result of the Nearest Neighbor problem from destination [7] to [24], the total distance is calculated as follow:

$$5\ 019 + 1\ 167\ + 2\ 580\ + 4\ 004 = 12\ 770 \text{ (meters).}$$

Given the speed of the vehicle is 666,667 meters/minute.

In order to find the travelling time, the formula: $\boldsymbol{t} = \frac{\boldsymbol{s}}{\boldsymbol{v}}$ is applied in sequence from destination [7] to destination [24], the total travelling time is calculated as follow:

$$7{,}528 + 1{,}750 + 3{,}870 + 6{,}006 = 19{,}155 \text{ (minutes).}$$

Moreover, the total result of the spend time in one place from destination [7] to destination [24] is calculated as follow:

$$60 + 90 + 90 + 60 = 300 \text{ (minutes).}$$

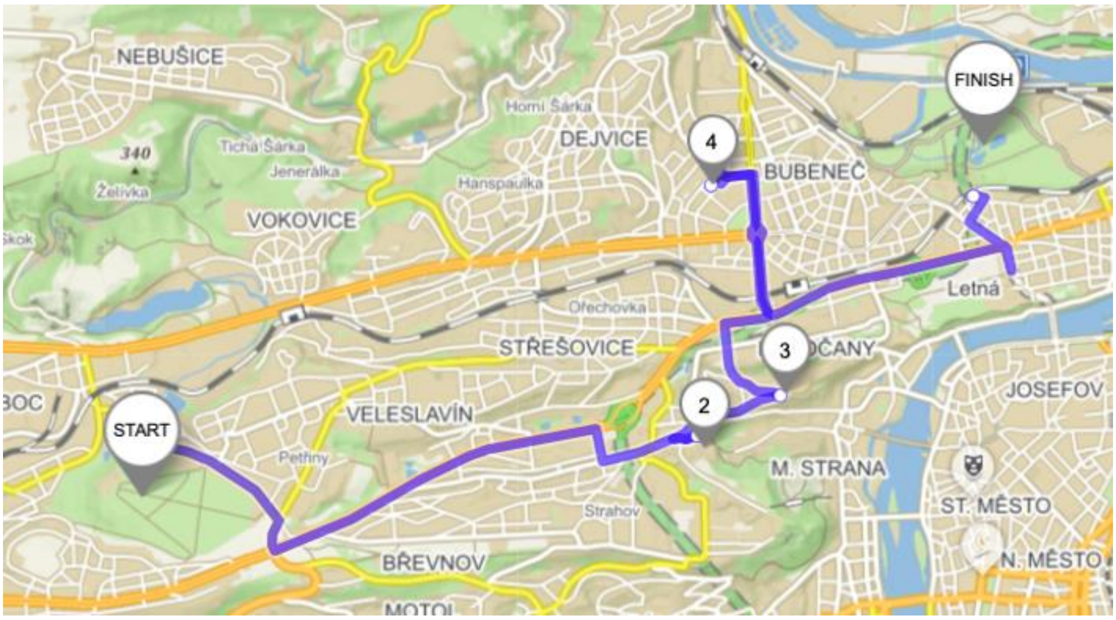
Together, the total time that needed to spend on day eight is:

$$19{,}155\ +\ 300 =\ 319{,}155 \text{(minutes)}$$

The routes of day eight are shown as follow:

[Obora Hvězda park] – [National Library of the Czech Republic] – [Prague Castle] – [Czech National Library of Technology] – [Stromovka park].

Figure 13 - Day eight travel scenario



Source: Mapy.cz 2020

4.4.9 Day nine travel:

Table 10 - Day nine travel results obtained from the Savings method

	v = 666,667 (meters/minute)		
Day 9	**Spending time (minutes)**	**Travelling time: $t = \frac{s}{v}$ (minutes)**	**s (meters)**
[24]	-	-	-
[17]	60	6,204	4 136
[13]	90	2,544	1 696
[26]	90	1,041	694
[38]	60	7,657	5 105
Total time	**317,446 (minutes)**		

Source: own processing of data, results are obtained by the program TSPKOSA.

The starting point of day nine begins with the last destination from day eight which is destination [24]. When traveling in sequence by the obtained the result of the Nearest Neighbor problem from destination [74] to [38], the total distance is calculated as follow:

$$4\,136\ + 1\,696 + 694 + 5\,105 = 11\,631 \text{ (meters).}$$

Given the speed of the vehicle is 666,667 meters/minute.

In order to find the travelling time, the formula: $\boldsymbol{t} = \frac{s}{v}$ is applied in sequence from destination [24] to destination [38], the total travelling time is calculated as follow:

$$6{,}204\ + 2{,}544 + 1{,}041\ + 7{,}657 = 17{,}446 \text{ (minutes).}$$

Moreover, the total result of the spend time in one place from destination [24] to destination [38] is calculated as follow:

$$60 + 90 + 90\ + 60 = 300 \text{ (minutes).}$$

Together, the total time that needed to spend on day nine is:

$$17{,}446\ +\ 300 =\ 317{,}446\ \text{ (minutes)}$$

The routes of day nine are shown as follow:

[Stromovka park] – [Botanical garden] – [Prague zoo] – [Troja Palace] – [Libeň Chateau]

Figure 14 - Day nine travel scenario

Source: Mapy.cz 2020

4.4.10 Day ten travel:

Table 11 - Day ten travel results obtained from the Savings method

	v = 666,667 (meters/minute)		
Day 10	**Spending time (minutes)**	**Travelling time: $t = \frac{s}{v}$ (minutes)**	**s (meters)**
[38]	-	-	-
[27]	60	7,303	4 869
[16]	90	1,422	948
[9]	90	0,595	397
[1]	-	3,270	2 180
Total time	**252,591 (minutes)**		

Source: own processing of data, results are obtained by the program TSPKOSA.

The starting point of day ten begins with the last destination from day nine which is destination [38]. When traveling in sequence by the obtained the result of the Nearest Neighbor problem from destination [38] to [1], the total distance is calculated as follow:

$$4\ 869 + 948 + 397\ + 2\ 180 = 8\ 394 \text{ (meters)}$$

Given the speed of the vehicle is 666,667 meters/minute.

In order to find the travelling time, the formula: $\boldsymbol{t} = \frac{s}{v}$ is applied in sequence from destination [38] to destination [1], the total traveling time is calculated as follows:

$$7{,}303 + 1{,}422 + 0{,}595\ + 3{,}270 = 12{,}591 \text{ (minutes).}$$

However, the total spend time on each place of the last day is not apply for the final destination of the day ten which is [1] – the starting point as well as the ending point. As soon as, the student returns the initial point, they will return home and finish ten days excursion in the Czech Republic.

Therefore, the total result of the spend time in one place from destination [38] to destination [1] is calculated as follow:

$$60 + 90 + 90 = 240 \text{ (minutes).}$$

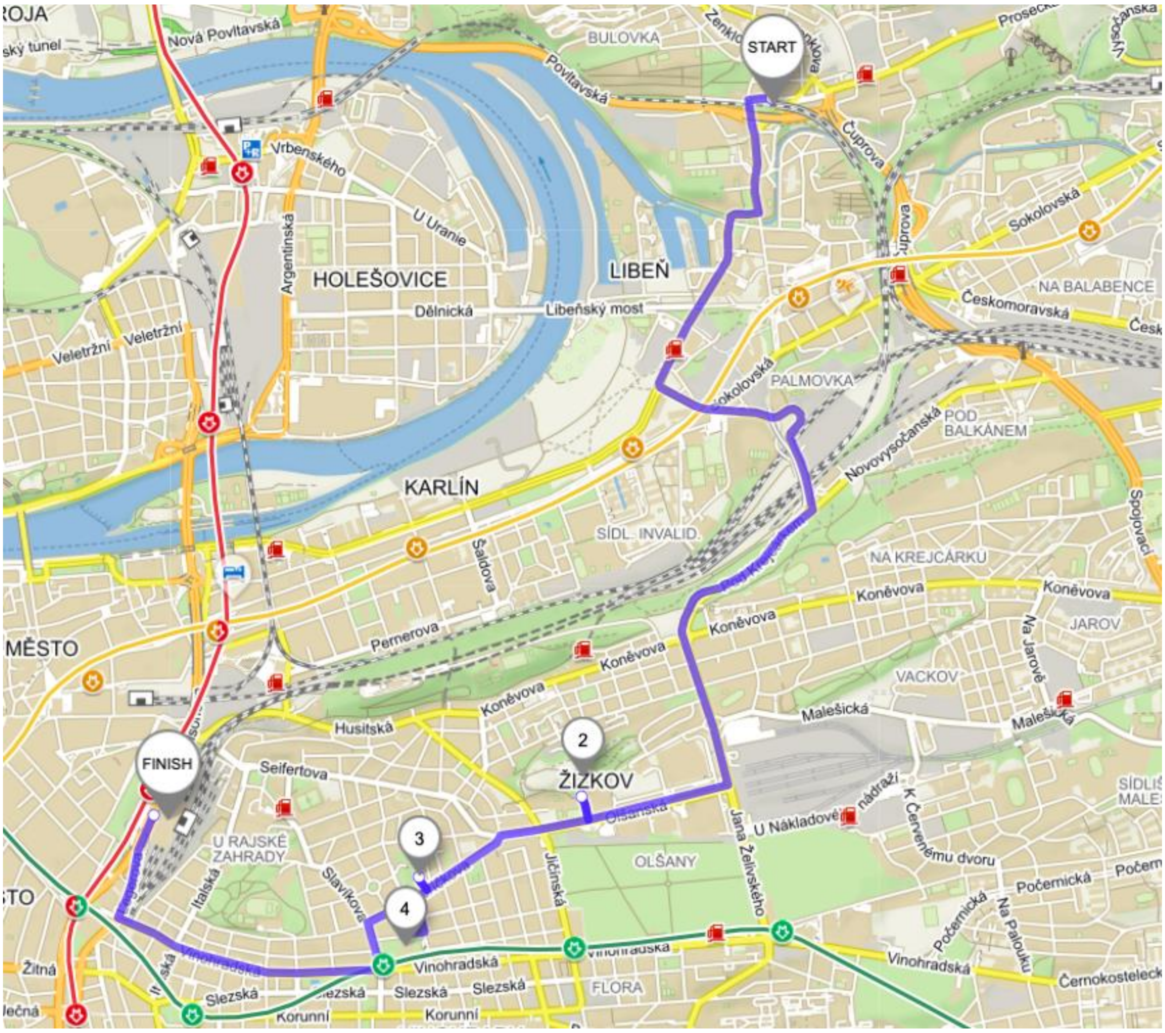
Together, the total time that needed to spend on day ten is:

$$12{,}591 + 240 = 252{,}591 \text{ (minutes)}$$

The routes of day ten are shown as follow:

[Libeň Chateau] – [Parukářka park] – [Žižkov TV Tower] – [Church of St. James the Greater] – [Prague main train station].

Figure 15 - Day ten travel scenario



Source: Mapy.cz 2020

5 Results and Discussion:

5.1 Evaluation of the initial routes and the optimization results:

In order to find out the best solution, three approximation algorithms were deployed to solve the problems. Given the result was obtained through the TSPKOSA program. The circuit obtained by the Nearest Neighbor algorithm lasts 277,101 minutes with 184 734 meters long. Besides, the circuit obtained by the Vogel's approximation algorithm lasts 262,071 minutes with 174 714 meters long. Meanwhile, the circuit obtained by the Savings algorithm lasts 241,899 minutes with 161 266 meters long.

Therefore, the most favorable route is a circuit has the result from the Savings algorithm with 161 266 meters and 241,899 minutes long which is shorten the journey time of the circuit by 209 256 meters and 313,831 minutes long.

Table 12 – Distances and times comparison of the default circuit and alternative circuit of the whole trip

Algorithm	Distance (meters)	Time Travelling (minutes)	Distance Improvement (meters)	Time Improvement (minutes)
Nearest Neighbor Algorithm (sequential)	184 734	277,101	185 788	278,682
Vogel's Approximation Algorithm	174 714	262,071	195 808	293,712
Savings Algorithm (parallel)	161 266	241,899	209 256	313,831
Default circuit	370 522	555,783		

Source: own processing of data, results are obtained by the program TSPKOSA.

6 Conclusion

The main objective of this work is to find out the nearest routes for one student who would like to travel around the Czech Republic during their holidays. Assume all the destinations are chosen based on the interest of the student. In order to solve the problem, the program TSPKOSA was introduced along with three algorithms: Nearest Neighbour algorithm (sequential), Vogel's approximation algorithm, Savings algorithm (parallel) which helped to figure out the problems based on the distance matrix.

In this case, the optimal solution was found by the Nearest Neighbor algorithm (parallel) from the distance matrix with 161 266 meters and 241,899 minutes long which is shorten the journey time of the circuit by 209 256 meters and 313,884 minutes long compare to the default circuit.

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8 Appendix

Table 13 - Default distance matrix of the circuit

Destinations	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	0	3 787	8 231	2 889	2 508	2 549	11 179	16 820	2 205	5 624	2 839	3 763	6 250	2 685	6 299	2 293	5 980	3 251	2 145	5 784	4 045	4 521	3 600	5 586
2	4 040	0	4 285	1 407	3 713	2 328	8 063	18 025	5 758	3 433	1 861	2 394	7 053	1 474	3 904	4 907	6 783	1 234	3 794	3 149	3 815	1 886	348	3 812
3	7 346	4 496	0	5 360	8 458	6 594	4 998	22 872	9 064	5 270	6 127	4 943	8 679	5 156	3 260	8 213	8 409	3 660	8 539	1 167	7 743	4 168	4 228	4 273
4	3 458	1 804	5 003	0	4 570	4 611	9 670	18 882	5 176	3 868	2 846	3 379	6 450	701	4 791	4 325	6 180	1 495	4 651	4 028	4 800	2 765	1 617	4 078
5	2 152	3 545	7 180	5 041	0	1 903	10 093	14 839	3 399	5 062	2 461	3 118	8 402	4 837	8 451	3 508	8 132	5 403	1 572	7 670	2 590	4 600	3 300	7 738
6	2 146	2 800	6 435	5 035	1 881	0	9 349	16 193	3 393	4 317	1 896	2 373	8 396	4 831	7 666	3 502	8 125	3 394	1 962	6 925	1 740	3 855	2 532	8 657
7	11 035	8 153	5 019	8 686	10 593	9 208	0	26 472	12 753	9 347	8 742	7 728	12 004	8 482	7 013	11 902	11 734	7 318	10 674	5 543	10 528	7 825	7 885	7 598
8	16 156	18 122	23 404	19 046	15 021	16 479	26 318	0	17 403	22 214	16 456	20 273	22 406	18 841	24 635	17 512	22 136	19 408	15 576	23 894	15 675	21 751	17 853	25 626
9	2 180	4 940	9 385	4 043	3 538	3 578	12 332	17 849	0	6 777	3 868	4 793	7 403	3 839	7 453	387	7 133	4 405	1 919	6 938	5 074	5 675	4 754	6 740
10	5 814	2 690	6 546	4 259	4 949	3 564	8 925	19 261	6 460	0	3 097	2 249	9 113	4 055	4 799	6 681	8 842	2 129	5 030	2 777	4 884	1 514	2 421	4 706
11	2 116	1 787	5 071	2 659	1 852	466	7 984	16 163	3 363	2 953	0	1 009	11 698	2 727	6 301	3 472	11 428	2 381	1 933	5 561	2 217	2 490	1 519	7 292
12	3 269	2 120	4 791	2 992	3 004	1 619	7 170	17 316	4 516	2 055	1 152	0	11 526	3 060	6 129	4 625	11 256	2 207	3 085	2 856	2 939	1 593	1 852	7 120
13	7 334	8 465	9 679	8 303	8 446	8 486	12 626	29 698	9 052	9 240	8 777	13 020	0	8 098	8 313	8 201	1 696	6 184	8 527	10 203	9 982	8 137	8 197	7 329
14	3 335	1 752	6 600	839	4 448	4 488	9 547	18 759	5 053	3 816	2 794	3 327	6 328	0	4 668	4 202	6 057	1 444	4 529	3 977	5 984	2 714	1 566	3 955
15	7 192	4 649	3 116	4 843	8 304	7 957	6 513	24 235	8 910	5 424	7 490	7 557	8 410	4 639	0	8 059	8 140	3 814	8 385	4 739	9 037	4 321	4 381	4 004
16	2 242	5 302	9 747	4 405	3 599	3 640	12 694	14 912	397	7 139	3 930	4 855	7 765	4 201	7 815	0	7 495	4 767	2 129	7 300	5 136	6 037	5 116	7 102
17	7 063	8 195	9 408	8 032	8 176	8 216	12 356	29 428	8 781	8 969	8 506	12 750	1 696	7 828	8 042	7 930	0	5 914	8 256	9 932	9 712	7 867	7 927	7 059
18	3 686	1 486	3 508	2 130	4 798	2 866	7 286	19 110	5 404	2 372	2 400	2 645	7 441	1 926	3 127	4 553	7 170	0	4 879	2 533	4 354	1 270	1 218	3 035
19	1 383	3 348	6 983	4 272	1 665	1 706	9 897	15 977	1 958	4 865	1 996	2 921	7 633	4 068	7 682	2 067	7 363	4 634	0	7 473	3 202	5 904	3 080	6 969
20	6 666	3 816	1 890	4 680	7 778	5 196	5 593	23 298	8 384	4 590	4 730	6 190	7 999	4 476	2 580	7 533	7 729	2 980	7 859	0	6 684	3 487	3 548	3 593
21	3 861	4 564	7 848	6 750	2 726	3 264	10 762	14 619	5 108	5 730	2 777	3 786	10 111	6 546	9 079	5 217	9 840	7 112	3 281	8 338	0	5 268	4 296	10 070
22	4 794	1 670	4 160	3 239	4 075	2 690	8 051	18 387	6 512	1 102	2 223	1 375	8 093	3 035	3 779	5 661	7 822	1 109	4 156	1 757	4 010	0	1 401	3 686
23	3 993	262	4 239	1 135	3 359	1 974	8 017	17 671	5 711	3 079	1 508	2 040	7 007	1 202	3 857	4 860	6 736	1 187	3 440	3 103	3 462	1 840	0	3 765
24	7 334	7 493	8 935	6 585	8 446	8 487	11 882	22 758	9 052	8 496	8 777	12 276	4 406	6 381	7 569	8 201	4 136	6 536	8 527	9 458	9 983	7 393	7 306	0
25	4 761	1 636	4 127	3 205	5 873	3 017	7 905	20 185	6 479	2 830	2 550	3 083	8 059	3 001	3 746	5 628	7 789	1 075	5 954	2 991	4 505	1 728	1 368	3 653
26	6 640	7 772	8 985	7 609	7 752	7 793	11 932	29 004	8 358	8 546	8 083	12 326	694	7 405	7 619	7 507	1 002	5 491	7 833	9 509	9 289	7 443	7 503	6 635
27	3 109	5 359	9 803	4 461	4 221	4 262	12 751	15 039	1 281	7 196	4 552	5 477	7 822	4 257	7 871	948	7 551	4 823	3 212	7 356	5 758	6 093	5 172	7 158
28	8 890	9 928	14 814	9 031	15 890	10 043	17 761	20 080	7 999	16 642	17 325	21 022	10 482	8 826	13 448	7 666	10 211	14 270	16 445	15 337	16 543	15 540	9 741	12 464
29	3 409	2 711	5 251	3 584	3 145	1 760	8 165	17 457	4 656	3 133	1 293	1 189	11 879	3 651	6 482	4 766	11 609	3 305	3 226	5 741	3 080	2 671	2 443	7 473
30	16 881	18 846	24 009	13 625	15 746	17 204	26 923	19 937	11 088	22 819	17 181	20 878	14 798	13 421	17 764	10 636	14 528	15 790	16 301	24 499	16 400	17 588	18 578	16 780
31	14 190	14 350	17 147	13 442	15 303	15 343	20 094	23 293	10 975	15 765	15 633	20 489	12 815	13 238	15 781	10 641	12 545	13 393	15 384	17 671	19 756	14 662	14 163	14 798
32	15 710	15 869	18 667	14 962	16 822	16 863	21 614	31 637	14 584	17 285	17 153	22 008	14 335	14 757	17 301	14 250	14 065	14 912	16 903	19 191	18 359	16 182	15 683	16 317
33	12 627	12 786	15 584	11 879	13 740	13 780	18 531	27 889	14 345	14 202	14 070	18 925	11 252	11 675	14 218	13 494	10 982	11 829	13 820	16 108	15 276	13 099	12 600	13 234
34	11 302	11 462	14 259	10 554	12 415	12 455	17 206	26 513	13 020	12 877	12 745	17 601	9 927	10 350	12 893	12 169	9 657	10 505	12 496	14 783	13 951	11 774	11 275	11 910
35	18 430	18 589	21 387	17 682	19 543	19 583	24 334	31 489	15 215	20 005	19 873	24 728	17 055	17 478	20 021	14 881	16 785	17 632	19 623	21 911	27 952	18 902	18 403	19 037
36	20 873	21 032	23 830	20 124	26 688	28 146	26 777	30 879	22 030	22 447	28 123	31 821	19 498	19 920	22 464	17 324	19 227	20 075	27 244	24 353	27 342	21 345	20 845	21 480
37	21 662	14 521	22 029	18 324	20 527	21 985	24 976	17 600	12 592	20 647	21 962	25 659	17 697	18 119	20 663	12 259	17 427	18 274	21 082	22 553	21 181	19 544	14 334	19 679
38	6 275	6 179	10 687	5 281	7 388	7 428	13 634	21 699	5 931	8 016	7 718	8 643	6 355	5 077	9 321	5 597	6 085	5 643	7 469	11 211	8 924	6 913	5 992	8 337
39	6 470	10 165	15 080	9 268	7 828	7 868	18 027	16 100	4 832	14 641	8 158	9 083	10 748	9 063	13 714	4 905	10 478	9 630	6 236	15 604	12 563	13 538	9 978	12 731
40	11 831	13 796	16 568	14 720	10 696	9 700	19 481	7 283	9 848	15 377	13 185	13 437	23 196	14 516	17 798	9 957	22 925	15 082	11 251	17 058	11 349	14 915	13 528	18 789
41	14 668	16 634	21 916	13 100	13 533	14 991	24 830	6 885	15 915	20 726	14 968	18 785	19 703	17 353	23 147	16 024	19 433	17 920	14 088	22 406	14 187	20 263	16 365	24 138
42	10 778	12 948	14 561	13 667	9 643	11 101	17 474	13 859	12 025	13 370	11 177	11 430	21 188	13 463	15 791	12 134	20 918	14 030	10 198	15 051	10 297	12 908	12 680	16 782
43	3 754	6 556	9 840	6 644	4 492	4 077	12 754	13 469	2 686	7 722	4 769	5 778	10 004	6 439	10 054	2 706	9 734	7 006	2 364	10 330	4 648	7 260	6 288	9 341
44	13 377	13 032	14 644	18 285	13 113	11 728	17 174	29 492	14 624	13 454	11 261	11 513	21 272	18 080	15 875	14 733	21 002	16 698	13 194	15 134	12 174	12 992	12 764	16 866
45	7 138	4 288	1 558	5 153	8 250	6 340	5 501	22 618	8 856	5 062	5 874	5 617	8 471	4 948	3 052	8 005	8 201	3 452	8 331	573	7 420	3 960	4 020	4 065
46	10 487	12 452	17 615	13 376	9 352	10 810	20 529	13 543	5 577	16 425	10 787	14 484	16 737	13 172	18 846	5 686	16 466	13 738	9 907	18 105	7 300	15 962	12 184	19 837
47	0	3 787	8 231	2 889	2 508	2 549	11 179	16 820	2 205	5 624	2 839	3 763	6 250	2 685	6 299	2 293	5 980	3 251	2 145	5 784	4 045	4 521	3 600	5 586

Destinations	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47
1	4 269	5 900	2 761	8 717	2 575	11 108	12 420	15 115	11 997	10 412	14 400	19 101	13 218	5 183	6 169	12 093	15 269	9 600	3 400	15 199	5 467	11 356	0
2	1 380	6 360	5 375	9 766	2 480	14 002	13 374	16 069	12 951	11 366	17 628	20 055	14 267	6 232	8 782	13 298	16 475	10 200	5 287	14 217	2 831	12 562	4 040
3	4 028	7 985	8 681	13 859	6 029	24 064	15 489	18 185	15 067	13 481	19 743	22 170	20 368	9 121	14 103	16 468	21 321	13 460	10 032	15 568	849	17 197	7 346
4	2 513	5 756	4 793	9 184	4 637	13 420	12 770	15 466	12 348	10 763	17 025	19 452	17 650	6 602	8 200	14 155	13 200	12 211	6 144	19 438	3 710	7 900	3 458
5	4 285	7 708	4 913	16 715	1 929	16 243	14 572	17 267	14 149	12 564	18 826	27 350	21 388	7 335	12 446	10 113	13 289	8 169	3 065	18 777	7 352	9 376	2 152
6	3 540	7 702	4 907	10 863	1 184	13 300	14 566	17 261	14 143	1 200	18 820	18 400	22 742	7 329	7 663	11 467	14 643	9 523	3 455	13 923	6 607	10 730	2 146
7	7 685	11 311	12 370	17 185	9 589	20 500	18 814	21 510	18 392	16 807	23 068	25 496	23 694	12 446	17 428	20 068	24 922	17 060	12 167	18 294	5 225	20 798	11 035
8	20 425	21 712	15 371	20 217	16 505	19 745	23 348	26 957	27 605	26 496	31 506	30 852	24 890	21 339	15 948	7 323	6 932	11 582	14 247	21 400	23 576	12 877	16 156
9	5 422	6 710	1 534	8 029	3 604	11 037	10 994	16 269	13 151	11 566	15 248	17 675	12 530	6 644	5 189	13 123	12 999	11 179	2 109	16 229	6 620	5 400	2 180
10	2 221	8 419	7 149	11 541	3 549	22 925	15 922	18 618	15 500	13 915	20 177	22 604	16 042	9 554	10 557	15 328	17 710	12 320	6 523	14 428	2 460	16 057	5 814
11	2 527	11 005	4 878	10 700	1 154	17 568	14 536	17 232	14 114	12 528	16 300	28 675	22 712	7 299	7 633	11 437	14 613	10 336	3 200	12 444	5 243	10 700	2 116
12	2 300	10 833	6 030	16 706	2 297	20 980	18 336	21 032	17 914	16 329	22 590	25 017	23 215	11 968	8 785	13 383	15 765	9 300	4 578	12 483	2 538	14 113	3 269
13	7 997	694	10 892	10 537	8 512	14 832	12 167	14 863	11 745	10 160	16 421	18 848	17 046	5 799	10 781	23 294	19 580	20 286	10 020	22 394	9 885	24 024	7 334
14	2 461	5 634	4 670	9 062	4 514	13 297	12 648	15 343	12 225	10 640	14 500	19 329	17 527	5 300	8 078	14 033	17 209	12 089	6 022	13 600	3 659	13 296	3 335
15	4 181	7 717	7 400	13 591	8 371	17 885	15 220	17 916	14 200	13 213	19 474	19 700	20 100	8 852	13 834	17 831	22 684	14 823	9 878	16 100	4 421	18 560	6 100
16	5 784	7 072	1 211	7 706	3 666	10 713	10 671	14 280	13 513	11 928	14 300	17 352	12 207	6 321	4 865	10 180	13 055	10 184	1 912	19 960	6 982	5 681	2 242
17	7 727	1 002	10 622	10 200	8 242	14 561	11 897	14 592	11 474	9 889	16 151	18 578	16 776	5 528	10 511	23 024	19 309	20 016	9 750	22 124	9 614	23 753	7 063
18	1 017	6 747	5 021	9 412	4 864	13 648	14 251	16 946	13 828	12 243	18 505	20 932	13 913	5 878	8 428	14 383	17 559	16 354	6 372	18 462	2 215	13 646	3 400
19	5 652	6 939	3 357	10 100	1 732	17 381	13 803	16 498	13 380	11 795	18 057	20 484	14 601	6 566	6 228	11 251	14 427	8 300	2 078	14 356	7 155	10 514	1 383
20	3 347	7 305	8 001	13 179	5 349	17 473	13 100	17 504	14 386	12 801	19 063	21 490	19 688	8 441	13 423	16 894	21 748	13 887	9 352	15 995	605	17 624	6 666
21	5 304	9 417	6 622	16 495	3 463	16 023	16 280	18 976	15 858	14 273	17 600	27 130	21 168	9 044	12 226	9 892	13 068	7 948	4 774	13 989	8 020	9 155	3 861
22	1 201	7 399	6 129	10 521	2 675	22 051	12 100	17 598	14 480	12 895	19 157	1 900	15 022	7 200	9 537	14 454	16 836	11 446	5 649	13 554	1 440	15 183	4 794
23	1 333	6 400	5 328	9 720	1 700	13 955	13 327	16 023	12 905	11 320	17 581	20 008	14 220	6 186	8 736	12 945	16 121	11 755	4 933	13 863	2 785	12 208	3 993
24	7 253	3 712	9 941	9 586	8 513	13 881	11 216	13 911	10 793	9 208	15 470	17 897	16 095	4 848	9 830	22 550	18 629	19 542	10 020	21 650	9 141	16 953	7 334
25	0	7 366	6 096	10 487	3 169	17 534	14 869	17 565	14 300	12 862	19 123	21 551	14 988	8 501	9 504	15 459	18 635	16 973	4 900	12 300	2 673	14 722	4 761
26	7 303	0	7 800	9 844	7 819	14 138	11 473	14 169	11 051	9 466	15 727	18 155	16 353	5 105	10 087	22 600	18 886	19 593	9 326	21 700	9 191	23 330	6 640
27	5 841	7 128	0	8 858	4 288	10 370	10 429	14 039	13 569	11 984	14 683	15 900	11 965	6 079	5 065	10 307	12 681	10 311	2 853	20 087	7 038	5 808	3 109
28	15 287	9 788	6 938	0	10 069	5 836	3 736	7 345	7 993	6 884	7 990	12 610	5 669	5 144	4 096	15 348	10 293	15 078	10 816	21 900	15 020	8 617	8 890
29	3 451	11 185	6 171	17 059	0	21 121	18 689	21 384	18 266	16 681	22 943	25 370	14 700	12 320	8 926	13 524	15 906	10 516	4 719	12 624	5 423	7 800	1 900
30	17 448	14 104	9 908	5 500	17 230	0	8 749	10 721	13 006	11 897	9 680	9 790	4 963	9 460	6 228	15 204	9 070	14 934	11 000	24 710	24 181	8 473	16 881
31	14 410	12 121	8 900	3 892	15 369	9 048	0	3 602	6 243	6 007	4 247	6 674	7 231	5 000	7 308	18 560	13 505	18 290	11 500	28 066	17 353	11 829	14 190
32	15 930	13 641	13 523	7 500	16 889	11 012	3 601	0	3 143	5 927	1 710	4 407	6 642	8 517	11 300	26 904	17 114	26 634	16 054	31 382	18 873	20 173	14 600
33	12 847	10 558	13 033	8 488	13 806	13 644	4 775	3 143	0	2 844	4 633	7 330	9 565	7 881	11 904	23 156	18 101	21 300	15 314	28 300	15 790	16 425	12 627
34	11 522	9 233	11 708	7 112	12 481	12 268	6 189	5 929	2 811	0	7 419	10 116	11 068	5 200	10 528	21 780	16 725	21 510	13 989	26 975	14 465	15 049	11 302
35	18 650	16 361	14 153	8 132	19 609	9 676	4 232	1 874	4 797	7 581	0	3 071	5 306	9 148	11 548	26 756	17 745	26 486	16 684	36 262	21 593	14 500	18 430
36	21 092	18 804	16 596	11 431	28 172	12 451	6 675	4 316	7 239	10 023	3 049	0	5 430	11 591	16 102	26 147	21 387	25 876	22 983	35 653	24 036	19 416	20 873
37	19 292	17 003	11 531	5 198	22 011	4 973	7 740	6 354	9 277	10 888	5 313	5 400	0	9 760	8 688	19 985	15 226	19 715	16 822	29 491	22 235	13 254	13 700
38	6 900	5 661	4 869	5 416	7 454	9 809	5 300	8 876	7 840	6 255	9 520	11 948	9 917	0	5 759	16 973	14 557	21 294	7 400	23 402	10 893	12 882	6 275
39	13 398	10 054	4 177	4 716	7 894	6 228	7 847	11 456	12 104	10 995	12 101	16 721	8 758	5 410	0	11 367	8 938	11 097	5 653	20 873	15 286	4 157	6 500
40	16 099	22 502	10 318	16 576	12 180	16 104	17 300	23 316	23 964	22 855	27 865	27 211	21 249	17 014	12 307	0	7 200	3 542	9 193	17 621	16 740	9 237	11 831
41	18 937	19 010	12 427	10 339	15 017	10 281	13 470	17 079	17 727	16 618	17 724	21 388	14 840	14 365	8 992	7 442	0	10 895	11 793	24 151	22 088	6 762	14 668
42	16 981	20 495	10 871	15 524	11 127	15 052	18 654	22 264	22 912	21 803	26 812	26 159	20 196	21 630	11 254	3 225	12 728	0	8 168	19 216	14 733	8 184	10 778
43	8 023	9 310	4 176	13 507	4 103	13 034	16 637	20 247	15 752	14 166	24 795	24 142	18 179	8 239	5 894	8 736	11 912	8 740	0	18 517	10 012	4 669	3 754
44	17 065	20 578	19 174	22 000	11 500	23 354	26 957	30 777	26 000	26 074	35 115	34 462	28 499	21 713	19 557	18 048	24 471	12 750	18 050	0	14 816	16 487	13 377
45	3 820	7 778	8 473	13 651	4 800	15 700	15 281	17 977	14 859	11 900	19 535	21 963	16 300	8 913	13 895	16 214	21 068	13 206	9 824	15 314	0	16 944	7 138
46	14 756	16 043	5 508	9 116	10 836	8 500	12 246	15 856	16 503	15 394	20 404	19 751	13 788	9 668	4 258	8 810	6 692	8 540	5 358	18 316	17 787	0	7 100
47	4 269	5 556	2 761	8 717	2 575	11 108	12 420	15 115	11 997	10 412	16 674	19 101	13 218	5 183	6 169	12 093	15 269	10 149	3 638	15 199	5 467	11 356	0

Source: TSPKOSA program

Table 14 - The sequence of default circuit and the best algorithm Savings method in day one

Day one					
Default circuit			**Savings method**		
Sequences	**Distances**	**Time**	**Sequences**	**Distances**	**Time**
[1]	-	-	**[1]**	-	-
[2]	3 787	5,680	**[4]**	2 889	4,333
[3]	4 285	6,427	**[14]**	701	1,051
[4]	5 360	8,040	**[23]**	1 566	2,349
[5]	4 570	6,855	**[2]**	262	0,393
[6]	1 903	2,854	**[18]**	1 234	1,851
Total	**19 905**	**29,857**		**6 652**	**9,978**

Table 15 - The sequence of default circuit and the best algorithm Savings method in day two

Day two					
Default circuit			**Savings method**		
Sequences	**Distances**	**Time**	**Sequences**	**Distances**	**Time**
[6]	-	-	**[18]**	-	-
[7]	9 349	14,023	**[25]**	1 070	1,525
[8]	26 472	39,708	**[44]**	12 300	18,450
[9]	17 403	26,104	**[42]**	12 750	19,125
[10]	6 777	10,165	**[40]**	3 225	4,838
[11]	3 097	4,645	**[8]**	7 283	10,925
Total	**63 098**	**97,647**		**36 575**	**54,863**

Table 16 - The sequence of default circuit and the best algorithm Savings method in day three

Day three					
Default circuit			**Savings method**		
Sequences	**Distances**	**Time**	**Sequences**	**Distances**	**Time**
[11]	-	-	**[8]**	-	-
[12]	1 009	1,513	**[41]**	6 932	10,398
[13]	11 526	17,289	**[30]**	10281	15,421
[14]	8 098	12,147	**[37]**	4 963	7,44
[15]	4 668	7,002	**[36]**	5 400	8,100
[16]	8 059	12,088	**[35]**	3 049	4,573
Total	**33 360**	**50,040**		**30 625**	**435,937**

Table 17 - The sequence of default circuit and the best algorithm Savings method in day four

Day four					
Default circuit			**Savings method**		
Sequences	Distances	**Time**	**Sequences**	**Distances**	**Time**
[16]	-	-	**[35]**	-	-
[17]	7 495	11,242	**[32]**	1 874	2,811
[18]	5 914	8,871	**[33]**	3 143	4,714
[19]	4 879	7,318	**[34]**	2 844	4,266
[20]	7 473	11,209	**[31]**	6 189	9,283
[21]	6 684	10,026	**[28]**	3 892	5,838
Total	**32 445**	**48,667**		**17 942**	**26,913**

Table 18 - The sequence of default circuit and the best algorithm Savings method in day five

Day five					
Default circuit			**Savings method**		
Sequences	**Distances**	**Time**	**Sequences**	**Distances**	**Time**
[21]	-	-	**[28]**	-	-
[22]	5 268	7,903	**[39]**	4 096	6,144
[23]	1 401	2,101	**[46]**	4 157	6,235
[24]	3 765	5,674	**[43]**	5 358	8,037
[25]	7 253	10,879	**[19]**	2 364	3,546
[26]	7 366	11,049	**[11]**	1 996	2,994
Total	**25 053**	**37,579**		**17 971**	**26,956**

Table 19 - The sequence of default circuit and the best algorithm Savings method in day six

Day six					
Default circuit			**Savings method**		
Sequences	**Distances**	**Time**	**Sequences**	**Distances**	**Time**
[26]	-	-	**[11]**	-	-
[27]	7 800	11,700	**[6]**	466	0,699
[28]	8 858	13,287	**[21]**	1 740	2,610
[29]	10 069	15,103	**[5]**	2 726	4,089
[30]	21 121	31,681	**[29]**	1 929	2,893
[31]	8 749	13,123	**[12]**	1 189	1,783
Total	**56 597**	**84,895**		**8 050**	**12,075**

Table 20 - The sequence of default circuit and the best algorithm Savings method in day seven

Day seven					
Default circuit			**Savings method**		
Sequences	**Distances**	**Time**	**Sequences**	**Distances**	**Time**
[31]	-	-	**[12]**	-	-
[32]	3 602	5,403	**[22]**	1 593	4,779
[33]	3 143	4,714	**[10]**	1 102	3,306
[34]	2 844	4,266	**[45]**	2 460	7,38
[35]	7 419	11,128	**[7]**	5 501	16,503
Total	**17 008**	**25,512**		**10 656**	**31,968**

Table 21 - The sequence of default circuit and the best algorithm Savings method in day eight

Day eight					
Default circuit			**Savings method**		
Sequences	**Distances**	**Time**	**Sequences**	**Distances**	**Time**
[35]	-	-	**[7]**	-	-
[36]	3 071	4,606	**[3]**	5 019	7,528
[37]	5 430	8,145	**[20]**	1 167	1,750
[38]	9 760	14,640	**[15]**	2 580	3,870
[39]	5 759	8,638	**[24]**	4 004	6,006
Total	**24 020**	**36,030**		**12 770**	**19,155**

Table 22 - The sequence of default circuit and the best algorithm Savings method in day nine

Day nine					
Default circuit			**Savings method**		
Sequences	**Distances**	**Time**	**Sequences**	**Distances**	**Time**
[39]	-	-	**[24]**	-	-
[40]	11 367	17,050	**[17]**	4 136	6,204
[41]	7 200	10,800	**[13]**	1 696	2,544
[42]	10 895	16,342	**[26]**	694	1,041
[43]	8 168	12,252	**[38]**	5 105	7,657
Total	**37 630**	**56,445**		**17,446**	**11 631**

Table 23 - The sequence of default circuit and the best algorithm Savings method in day ten

Day ten					
Default circuit			**Savings method**		
Sequences	**Distances**	**Time**	**Sequences**	**Distances**	**Time**
[43]	-	-	**[38]**	-	-
[44]	18 517	27,775	**[27]**	4 869	7,303
[45]	14 816	22,224	**[16]**	948	1,422
[46]	16 944	25,416	**[9]**	397	0,595
[47]	7 100	10,650	**[1]**	2 180	3,270
Total	**57 377**	**86,065**		**8 394**	**12,591**