Czech University of Life Sciences Prague Faculty of Economics and Management Department of Systems Engineering



Bachelor Thesis

Multicriteria decision making in company practice

Oryna Kosenko

© 2022 CZU Prague

CZECH UNIVERSITY OF LIFE SCIENCES PRAGUE

Faculty of Economics and Management

BACHELOR THESIS ASSIGNMENT

Oryna Kosenko

Economics and Management Economics and Management

Thesis title

Multicriteria decision making in company practice

Objectives of thesis

The thesis is focused on selecting an optimal provider of raw materials for a company by the means of multicriteria analysis methods for decision making. Selecting a suitable method will serve for suppliers' offers review in sight of the given company's criteria and expectations.

Methodology

The first part of work will consist of the theoretical description of methods along with its utilization and comparison based on specialized literature of several authors. Finding an appropriate multi-criterial analysis method will be followed by practical part concentrating on selection of optimal choice of raw materials provider for the company. The application of the selected methods will comprise of explanation of the goal and criteria setting. An importance of individual criteria will be deterimined with the help of company representatives. The computation of efficient alternatives will be described in detail and followed by the discussion on sensitivity analysis of the results. Recommendations will be provided in the end.

The proposed extent of the thesis

40 pages

Keywords

multi-criteria analysis, criteria, TOPSIS, Saaty method, supplier, polyurethane

Recommended information sources

HAMMOND, J S. – KEENEY, R L. – RAIFFA, H. Smart choices : a practical guide to making better decisions. Boston, Mass.: Harvard Business School Press, 1999. ISBN 0875848575.

Saaty T. L. The Analytic Hierarchy Process: Planning Setting Priorities. N. Y. : McGraw Hill Text, 1980

ŠUBRT, T. *Ekonomicko-matematické metody.* Plzeň: Vydavatelství a nakladatelství Aleš Čeněk, 2011. ISBN 978-80-7380-345-2.

TZENG, G. – WANG, H F. – WEN, P. *Multiple criteria decision making*. NEW YORK: SPRINGER, 1995. ISBN 3-540-94297-1.

Expected date of thesis defence 2021/22 SS – FEM

The Bachelor Thesis Supervisor

Ing. Robert Hlavatý, Ph.D.

Supervising department Department of Systems Engineering

Electronic approval: 24. 11. 2021

doc. Ing. Tomáš Šubrt, Ph.D.

Head of department

Electronic approval: 30. 11. 2021

Ing. Martin Pelikán, Ph.D. Dean

Prague on 28. 03. 2022

Declaration

I declare that I have worked on my bachelor thesis titled "Multicriteria decision making in company practice" by myself and I have used only the sources mentioned at the end of the thesis. As the author of the bachelor thesis, I declare that the thesis does not break any copyrights.

In Prague on 28.03.2022

Acknowledgement

I would like to thank Ing. Robert Hlavatý, Ph.D. for his valuable knowledge, constructive advice, and guidance. I also express the deepest gratitude to my parents for their support and patience throughout my studies.

Multicriteria decision making in company practice

Abstract

This bachelor thesis is focused on application of multicriteria decision making on example of the company «VITLUS» LLC PCC based in Ukraine. The company produces rubber and polyurethane components. Due to the decision to expand production and increase diversity of polyurethane-based products the company faced a challenge to select an optimal supplier of raw materials. The main idea of multicriterial decision making will be presented in the theoretical part, including description of methods applicable for the designated problem and all the necessary elements of the multicriteria analysis model and its specificities. The practical part will identify the company's decision problem and characteristics of the desired demand for the raw material. Firstly, the company will provide the list of suppliers and the dataset of performance on various criteria of the individual offers. The suggested method for identifying the most optimal decision is TOPSIS, the weights of the individual criteria will be determined using the Saaty's method. This decision will be recommended to the company. The method will also present all solutions from the perspective of optimality from best to worst.

Keywords: multicriteria analysis, criteria, TOPSIS, Saaty method, supplier, polyurethane.

Vícekriteriální rozhodování v podnikové praxi

Abstrakt

Tato bakalářská práce je zaměřena na aplikaci vícekriteriálního rozhodování na příkladu společnosti «VITLUS» LLC PCC se sídlem na Ukrajině. Společnost vyrábí pryžové a polyuretanové komponenty. Vzhledem k rozhodnutí rozšířit výrobu a zvýšit rozmanitost produktů na bázi polyuretanu byla společnost postavena před výzvou vybrat optimálního dodavatele surovin. V teoretické části bude představena hlavní myšlenka vícekriteriálního rozhodování, včetně popisu metod použitelných pro zadaný problém a všech nezbytných prvků modelu vícekriteriální analýzy a její specifik. V praktické části bude identifikován rozhodovací problém firmy a bude popsaná charakteristika požadované poptávky po surovině. Nejprve společnost poskytne seznam dodavatelů a datový soubor plnění kritérií jednotlivých nabídek. Navrhovaná metoda pro identifikaci nejoptimálnějšího rozhodnutí je TOPSIS, váhy jednotlivých kritérií budou určeny pomocí Saatyho metody. Toto doporučení bude společnosti předáno. Metoda také představí všechna řešení z pohledu optimality od nejlepšího k nejhoršímu.

Klíčová slova: vícekriteriální analýza variant, TOPSIS, Saatyho metoda, dodavatel, polyuretan

1	Intro	roduction	9
	1.1	Objectives	10
	1.2	Methodology	10
2	Liter	erature Review	
	2.1	Multicriteria Decision Analysis	
	2.1.	1.1 Steps of MCDA process	11
	2.1.	1.2 MCDA modeling	12
	2.2	5 5	
	2.2.	1	
	2.2.	2.2 Point allocation method	15
	2.2.	2.3 Saaty's method	16
	2.3	Methods for assessing alternatives	18
	2.3	3.1 TOPSIS	18
3	Prac	ctical Part	
	3.1	Company's scope of business	20
	3.2	Need for a solution	
	3.3	Alternatives	
	3.4	Criteria	
	3.5	Criteria weights	
	3.6	TOPSIS	
4	Resu	sults and Discussion	
	4.1	Sensitivity analysis	
С	onclu	usion	
5	Refe	erences	
6	List	t of tables and figures	
Ĵ	6.1	List of tables	
	6.2	List of figures	

1 Introduction

Through the history humanity was facing a problem of decision making. One way or another, almost all human life consists of choice, and this is reflected in history and culture. With continuous development and improvement of various technological, socio and economic factors the problem of decision making has been turning more complex and multifaceted. Decision making is now a matter of expert consultation. To solve this problem, many researchers and authors design theories applied to numerous instances, but every time one theory is developed to solve a problem, some questions remain unanswered.

In the book by Hammond, Keeney and Raiffa (1999), authors outline that people who do not know the issue of multicriteria decision making make intuitive decisions in day-to-day activities. This approach is fine if one decides on short-term problems that are not costly or the decision is reversible. In other cases, where the decision affects a person's lifetime, has a long-term impact, or involves large costs, there are situations in which we must carefully consider all possible consequences. For more complex and at the same time more serious decisions that have a long-term impact, it is necessary to think more deeply, obtain sufficient information about the situation and, if necessary, consider consulting an expert. Complex situations need to be assessed according to the factors that affect them. We call these factors criteria and select them logically according to the purpose of the decision.

This bachelor thesis deals with procedures that help decision making in solving complicated tasks with a certain number of alternatives and criteria with aim to find an optimal solution. The operation of the selected methodology is shown on a concrete example of the company «VITLUS» LLC PCC dealing with selection of the best option among raw-materials providers. The task contains a sufficient number of alternatives that we compare based on a number of parameters. This work is divided into two parts. Firstly, the theoretical part deals with the methodology and then in the practical part, the methodology is verified by using it on the specific example. The first part contains key concepts, methods for determining the weights of criteria and the key part about the method of determining the evaluation of alternatives. Secondly, the practical part contains a summary of information obtained from the company's representative, relevant data concerning the company's scope of work and the background causing the need of applying multicriteria decision analysis. The aim of this work is to introduce the concept of multicriteria decision analysis to the reader, explain its principles and to show the procedure in detail.

1.1 Objectives

This thesis is focused on selecting an optimal provider of raw materials for a company by the means of multicriteria analysis methods for decision making. Selecting a suitable method will serve for suppliers' offers review in sight of the given company's criteria and expectations.

1.2 Methodology

The first part of work will consist of the theoretical description of methods along with its utilization and comparison based on specialized literature of several authors. Finding an appropriate multicriterial analysis method will be followed by practical part concentrating on selection of optimal choice of raw materials provider for the company. The application of the selected methods will comprise of explanation of the goal and criteria setting. The importance of individual criteria will be determined with the help of company representatives. The computation of efficient alternatives will be described in detail and followed by the discussion on sensitivity analysis of the results. Recommendations will be provided in the end.

2 Literature Review

This part of the bachelor thesis will describe main characteristics of decision-making processes with detailed explanation of its steps and calculation.

2.1 Multicriteria Decision Analysis

Multicriteria Decision Analysis (MCDA) derived from operations research with origins of such disciplines as mathematics, economics, and psychology. When making decisions people commonly consider two basic parameters - cost and quality, assuming that the higher the quality of a product, the higher its cost, and vice versa. In the book by Tzeng, Wang and Wen (1995), authors presume that by looking at decision-making problems differently than the usual ways, we can discover a better structure and solution to the problems. We can indeed enhance our decision-making quality by letting go of the habitual constraints of the problems. Thus, we are able to expand and enrich our domain of thinking, find better solutions, and make better decisions. In the work presented by Belton and Stewart (2001), authors make a similar observation that while making a choice one can approach intuitive way of deciding, this way, however, may be judged for missing the "overall picture", considering numerous aspects forming the problem of a choice. In short, MCDA exempts "the facilitator/analyst and decision maker from the technical implementation details, allowing them to focus on the fundamental value judgments", therefore, multiple criteria must be clearly evaluated and then combined for the purpose of ranking or choosing between given options.

According to a book by Šubrt et. al. (2011), which will serve to acquaint the reader with technical details further in the work, an important aspect of making a choice is the decision-maker's involvement in the outcome. The decision-maker should always proceed as objectively as possible.

2.1.1 Steps of MCDA process

The aim for the decision maker is to be as objective as possible when making decisions by means of various procedures and methods. In order to bring the decision making closer to MCDA model, Hansen and Devlin (2019) split the procedure to the detailed series of steps in Table 1:

Table 1: Steps of MCDA process

Step	Description
Structuring the decision problem	Identify objectives, alternatives, decision
	makers and output required
Specifying criteria	Specify criteria for the decision that are
	relevant to decision maker
Measuring alternatives'	Gather information about alternatives'
performance	performance on the criteria
Scoring alternatives on the	Convert performance measures into scores,
criteria	representing each alternative's degree of
	achievement on the criteria
Weighting the criteria	Determine weights for the criteria, representing
	their relative importance to decision maker
Applying scores and weights to	Multiply alternatives' scores on the criteria by
rank alternatives	weights and sum to get "total scores" by which
	the alternatives are ranked
Supporting decision-making	Use MCDA outputs, including sensitivity
	analysis, to support decision-making – i.e.,
	ranking or selecting alternatives (depending on
	the application)

Source: Hansen and Devlin (2019)

2.1.2 MCDA modeling

The whole decision-making process is part of human life, and in some cases, decision making is so complex that it is appropriate or even necessary to use a mathematical model. In most cases, this is a decision where a significant number of important criteria need to be considered. In most cases, each specific criterion has a different solution claiming to be the best. The purpose of the MCDA models is characterized by Šubrt et. al. (2011) as either to find the best solution according to all considered aspects, or to exclude inefficient alternatives, or to arrange and rank set of alternatives. Below more components and its types of the decision-making process will be described.

Decision-making goal

We understand this goal as a state that we are trying to achieve with a solution. The goal may serve to find an alternative which is evaluated as best as possible according to all criteria, or find an efficient alternative, or sort alternatives from the best to the worst, or eliminate ineffective alternatives.

Subject and object

Subject and object of the decision making are decision maker and the matter of decision, respectively.

Alternatives

Alternatives are possible choices, or more precisely feasible decision-making options that are not logical nonsense. They are evaluated according to individual criteria, from which the decision maker selects the most suitable one. More details on criteria and its types will be further formulated. Alternatives all together form a set of solutions to a given problem.

Types of alternatives

According to Brožová, Houška and Šubrt (2003) individual alternatives may have the below specified properties.

Dominant – this alternative is evaluated better by all criteria rather than the dominated alternative. Sometimes, however, it is not possible to identify those.

Non-dominated – this alternative is also called as Pareto alternative. It is not dominated by any other alternative according to a certain criterion, thought it may have a cost of worse scoring according to another criterion. If such a compensatory affect is acceptable the decision maker can choose one of non-dominated alternatives.

Ideal and negative-ideal alternatives – these alternatives often do not really exist. It consists of all the criteria that achieve the best and the worst possible scores in all criteria respectively.

Efficient alternative must be a non-dominated alternative and it is recommended as desired decision. Selection of the efficient alternative may depend on the method applied in a problem that is being solved.

Evaluating all the alternatives allows us to sort them. When all criteria values are converted to the quantitative form, we can construct so called decision matrix Y(1.1), where the element y_{ij} expresses evaluation of the *i*-th alternative according to the *j*-th criterion.

$$Y = \begin{pmatrix} y_{11} & y_{12} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & y_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ y_{m1} & y_{m2} & \cdots & y_{mn} \end{pmatrix}$$
(1.1)

Criteria

We identify a type of alternative according to given criteria - aspects of evaluation of alternatives. This is a rule we use to compare the alternatives. In the book by Šubrt et. al. (2011) authors divided criteria into two groups. The first classifies a criterion by quantifiability:

-quantitative criteria can be measured objectively and there is no need to modify them to work with them.

- qualitative criteria - these criteria can be evaluated verbally. The values of quantitative criteria represent objectively measurable data therefore these criteria are also referred to as objective. In contrast, it is not possible to measure value of qualitative criteria objectively. Very often these are the values that decision maker estimated only subjectively. For their use in models, it is necessary to convert them into a quantitative form.

The second group simply divide criteria to maximization and minimization. For maximization criteria, it is assumed that the best alternative reaches the highest value according to this criterion. On the contrary, the lowest values for the minimization criteria indicate the best alternative.

Criteria requirements

The selected set of criteria should meet certain requirements. Fotr and Švecová (2010) specifically focus on completeness, operationality, redundancy, minimum scope, and independence.

- 1. Completeness criteria should reflect all properties of a certain alternative that are essential and important for the decision maker. It should assess and evaluate all direct and indirect consequences of alternatives, its positive and negative consequences. If the set of criteria meets this requirement, it can be considered complete.
- 2. Redundancy there should be no overlapping or duplicating criteria as it can significantly affect the decision-making process, meaning that a choice of an aspect must be such that it enters only once into the evaluation of solution variants.
- 3. Operationality all criteria must be applicable to any considered alternative, in other words, each criterion must be measurable (quantitatively or qualitatively).
- 4. Mutual independence of preferences the assessed score of a certain criterion should not depend on knowing a score of another criterion
- 5. Quantity only reasonable number of criteria needs to be considered, this simplifies final evaluation of alternatives, avoiding overloading of the model.

Criterion weight

When solving a problem, it is essential to know if one criterion is preferred over another. Expressing the importance of a criterion compared to other criteria is called criterion preference. Setting preferences is a very challenging task and usually results from subjective opinion of the decision-maker.

In the study by Hansen and Delvin (2019), authors claim that "decision makers usually handle the trade – offs between objectives by evaluating the alternatives under consideration based on the explicit weighting of criteria relevant to the overarching decision – in order to, depending on the application, rank (or prioritize) or choose between the alternatives".

Criteria weighting, according to Fotr and Švecová (2010), is a numerical expression of importance of the given criteria considering the goal of decision-making. As the criterion becomes more important, its weight increases and vice versa. When comparing the weights of a set of criteria, which can be obtained by different methods, the weights are usually normalized to obtain a value from the interval <0; 1>, which implies a sum of obtained weights must be equal 1.

2.2 Methods for determining criteria weights

Most methods of decision-making require individual criteria comparison and indication of their importance. Based on the information provided by the decision maker we can distinguish between different methods for determining the weights of the criteria:

- Decision maker cannot determine the preferences between the criteria. In this trivial case, all criteria are assigned the same weight.

$$w_i = \frac{1}{n}, i = 1, \dots n$$
 (1.2)

- Decision maker is able to determine the order of importance of the criteria, in such case sequence method is often used.

- Decision maker is able to determine not only the order but also the degree of importance of the criteria, in such case methods to be considered are point allocation method or Saaty's method.

2.2.1 Sequence method

According to Šubrt et. al. (2011), the sequence method is usually used to determine the weights of the criteria, when their significance is assessed by several experts. These criteria are ranked from most important to least important. The most important criterion is evaluated by *n* points, where *n* is the number of criteria, the next criterion receives *n*-1 points, etc. The least important criterion is then evaluated as 1. The weights of all criteria are then determined by the number of points of a certain criterion divided by the total number of points. It follows that the total sum of the weights of all criteria equals to one. If *i*-th criterion has a relative importance $b_i \in \{1, ..., n\}$ then its resulting in normalized weight w_i obtained by following calculation:

$$w_i = \frac{b_i}{\sum_{i}^{n} b_i}, i = 1, \dots, n$$
 (1.3)

2.2.2 Point allocation method

This is one of the simplest methods for determining the weights of criteria by priority of criteria, decision maker can evaluate the importance of the criteria quantitatively, according to a pre-selected scoring scale, for example a decision maker is asked to award b_i points to a criterion under consideration that the sum of the weights of all criteria should be 100. The more points a criterion gets, the higher its relative importance. In the book by Friebelová and Klicnarová (2007) authors state that the advantage of this method over the sequence method is that the same number of points can be allocated for equally important criteria. In this scenario the method is easy to normalize by means of the expression 1.3 However, the weights obtained with the point allocation method are not very accurate and considering that the allocation is strictly based on a decision maker's subjective judgements it becomes more complicated when there are more than five criteria.

2.2.3 Saaty's method

Saaty's method is considered to be more sophisticated approach, and therefore it is also one of the most used methods for estimating the weights of criteria. It was developed in the 1970's as part of the Analytic Hierarchy Process (AHP). The simplicity of use and broad application have led to the wider use of multicriteria decision making. It is a method of quantitative pairwise comparison, for which values of the scale 1, 3, 5, 7, 9 are used, it is also possible to use intermediate values - 2, 4, 6, 8. The individual degrees of preference are listed in the following table, and we can also find the evaluation scale directly in the work of Saaty (1990).

Table 2: Scales for expressing preferences in the Saaty method (1990)

1	equal importance <i>i</i> and <i>j</i>
3	moderate importance <i>i</i> over <i>j</i>
5	strong importance <i>i</i> over <i>j</i>
7	very strong importance <i>i</i> over <i>j</i>
9	extreme importance <i>i</i> over <i>j</i>
00	$a_{\rm optr}$ (1000)

Source: Saaty (1990)

The evaluation is always performed by only one decision maker that compares two criteria and writes the magnitude of the preference of the *i*-th criterion to the *j*-th criterion in the Saaty's matrix represented as below.

If the value of *i*-row and *j* column is equal, then this preference is written as Cij = 1 and vice versa if *j*-th criterion is more preferable than *i*-th criterion, then the preference value is equal to the inverted value as following $C_{ij}=1/3$, $C_{ij}=1/5$.

Saaty's matrix is a square matrix of type $m \times n$, and only the value 1 occurs on the diagonal because each criterion is equivalent to itself.

When using the Saaty's matrix, it is also necessary to calculate its consistency. The consistency ratio (*CR*) is used to assess the rationality of entering the weights of the criteria. This parameter is generally used with a requirement of *CR* < 0.1. *CR* is defined as follows:

$$CR = \frac{CI}{RI} \tag{1.5}$$

Where *RI* is a random index value for different number of criteria developed by Saaty (2008), these are represented in the below tables:

n	1	2	3	4	5	6	7	8
RI	0	0	0,52	0,89	1,11	1,25	1,35	1,4
Source: Saaty (1990)								

Table 3: The random index value *RI* (part 1)

Table 4: The random index value *RI* (part 2)

n	9	10	11	12	13	14	15
RI	1,45	1,49	1,52	1,54	1,56	1,58	1,59
Source: Saaty (1990)							

And *CI* is a consistency index calculated by the below expression:

$$CI = \frac{l_{max} - n}{n - 1} \tag{1.6}$$

Where l_{max} is maximum eigenvalue of the matrix and n is a number of criteria.

A fully consistent matrix where CI = 0, is almost impossible to obtain in real situations. It can be shown that the *CI* consistency index is always non-negative. At the same time, the lower is *CI* value, the more a comparison itself is considered consistent and therefore more credible. Many authors, such as Lamata and Pelaez (2002) however, criticizes this approach because of its unsatisfactory results for a higher number of criteria and other methods of consistency verification. Nevertheless, Saaty's method continues to be widely accepted and used. A different view of the problem of consistency verification is then presented by Stoklasa, Jandova and Talašová (2002), where it is proposed that the plausibility of the pairwise comparison be assessed based on the so-called weak consistency.

If we finally reach the conclusion that the matrix *C* is sufficiently consistent for further calculations, we can move on to the very determination of weights of the individual criteria. There are many ways to do this, and for a closer comparison you can refer to the work of Ishizaka and Lusti (2006), I will mention one of the most used approaches which is a standardized geometric mean calculated with the below equation:

$$b_i = \sqrt[n]{\prod_{j=1}^n C_{ij}} \tag{1.7}$$

For the *i*-th criterion we calculate b_i as the geometric mean of the elements of the *i*-th row of the matrix *C*, then we normalize it according to the expression below and obtain the resulting weight w_i :

$$w_i = \frac{b_i}{\sum_{i=1}^n b_i} \tag{1.8}$$

2.3 Methods for assessing alternatives

There are many methods that can be used to evaluate alternatives. They may differ quite a bit from their approach to the very concept of efficient alternative, and thus also in the way of construction of criteria functions. Methods also vary by their computational complexity and usability for different MCDA problems.

2.3.1 TOPSIS

This work will mainly focus on the Technique for Order Preferences by Similarity to an Ideal Solution (TOPSIS) method due to its simplicity of use for a high number of alternatives and criteria. The approach was developed by Hwang and Yoon in 1981. As can be read from the name the basis of this method lays in assessment of alternatives according to their distance from the ideal and negative-ideal alternatives. The requirements for utilization of the method are knowing weights of criteria and quantified differences between criterion weights values (e.g., one criterion is twice important as another criterion).

TOPSIS procedure

According to Šubrt (2011):

First, we create a normalized criterial matrix $R = (r_{ij})$ from the original criterion matrix Y_{ij} , where i = 1, 2, ..., n and j = 1, 2, ..., m. We can do this according to the formula originally proposed by Hwang and Yoon such as follows:

$$r_{ij} = \frac{y_{ij}}{\sqrt{\sum_{i=1}^{n} y_{ij}^2}} \tag{1.9}$$

Then we calculate normalized weighted values of the criteria matrix $V = (v_{ij})$ from the given matrix $R = (r_{ij})$ and $v_j, j = 1, ..., k$, which is a value of *j*-th criterion:

$$V_{ij} = w_j r_{ij} \tag{1.10}$$

From the matrix *W* we identify ideal alternative *H* where $h_j = max_j(w_{ij})$ and negativeideal alternative *D*, where $d_j = min_j(w_{ij})$

Afterwards we identify the distance d_i^+ from ideal and the distance d_i^- from negative-ideal alternatives respectively as per the following:

$$d_i^+ = \sqrt{\sum_{j=1}^k (v_{ij} - h_j)^2}$$
(1.11)

$$d_i^- = \sqrt{\sum_{j=1}^k (v_{ij} - d_j)^2}$$
(1.12)

The last calculation step is to determine the relative indicator of the distance of alternatives from the negative-ideal alternative denoted c_i as:

$$c_i = \frac{d_i^-}{d_i^+ + d_i^-}$$
(1.13)

Resulted values of the indicator are on the interval (0,1), while c_i acquires the left extreme value for negative-ideal alternative and the right extreme value for the ideal alternative.

In order to select the efficient alternative, we sort alternatives in descending order of value of c_i , the alternative with the highest value is considered to be the final solution of the problem.

3 Practical Part

In this chapter, the theory of problem solving will be demonstrated in practice, in particular selection of a suitable supplier of the raw materials for «VITLUS» LLC PCC (hereinafter "Company"). Company specializes on rubber products manufacture of various types of complexity, overall size, types of combination of parts and products from rubber and polyurethanes.

3.1 Company's scope of business

According to the company's website (Vitlus, 2020) the company's main specifications are:

- design and manufacture systems of wearproof slurry pipeline;

- design and manufacture hydrocyclone installations, in single and battery versions, with the calculation of technological parameters based on the initial data and technological requirements provided by the customer;

- design and manufacture rubber parts subject to multidirectional, variable or complex loads;

- intensify the grinding process by installing and introducing effective designs of rubber lining for mills of the 2nd and subsequent stages.

3.2 Need for a solution

Due to production expansion of polyurethan-based products, company initiated a search of one supplier of 3-component polyurethane system (hereinafter "polyurethane") to avoid extra costs on storage and delivery from several different suppliers. Company's procurement department carried out negotiations with prospective suppliers offering suitable quantities, price, and characteristics of polyurethane. Due to relatively high number of requirements for the offered deals the company made a decision to rely on a mathematical model for selecting the optimal solution. The individual criteria of the model were determined and evaluated by one competent expert (company's engineering technologist) who is competent to solve this decision-making problem.

3.3 Alternatives

- 1. Era Polymers Pty Ltd (a_1)
- 2. Covestro AG (a_2)
- 3. The Dow Chemical Company (a_3)
- 4. LANXESS Deutschland GmbH (*a*₄)
- 5. BASF SE (*a*₅)
- 6. COIM GROUP SRL (a_6)

3.4 Criteria

The following criteria of the model were determined and evaluated by one competent expert:

1. Price per kilogram (c_1)

Purchase price is one of the basic criteria. In this model this is a minimization criterion. The price will be in Ukrainian hryvnia per kilogram (UAH).

2. Delivery service (c_2)

This criterion corresponds to a supplier's delivery service availability. Company prefers the ordered material to be delivered by supplier itself, hence this is a maximization criterion.

3. Delivery price (c_3)

Minimization criterion expressed in hryvnia per kilogram (UAH).

4. Delivery time (c_4)

Minimization criterion.

5. Drying time (c_5)

Maximization criterion shown in minutes (min.), even though it is not obvious at first glance, the drying time plays important role and depends on the size of a product. Containers that are intended for heating and processing polyurethane before it is poured into a casting form have a particular volume. Sometimes it is impossible to prepare enough of material for casting and for this reason the preparation needs to be performed in several sets. Of course, if a product is small, the faster material is solidified the better, however when working on product of bigger size it is crucial that structure of the first layer remains unchanged before the second layer is casted. It guarantees the product's homogeneity and hence its durability.

6. Δ Drying time, where Δ is range of possible drying time (*c*₆)

This is a maximization criterion as the drying time range can be controlled during production, which allows to use the selected polyurethane to produce wider range of products more effectively.

7. Compatibility with catalyst (c_7)

By adding the catalyst to polyurethane, it is possible to control drying time. This is a maximization criterion.

8. Tensile strength (c_8)

According to Ismail, Khulbe and Matsuura (2019), tensile strength "is defined as the ability of a material to resist a force that tends to pull it apart. This is maximization criterion measured in kilonewton per meter (symbolized as kN/m). This is a maximization criterion.

9. Elongation at break (*c*₉)

According to Fu and Fan (2017), this characteristic can be explained as a capability of the material to resist changing of its shape without the appearance of cracks. This parameter is measured in percentage (%). This is a maximization criterion.

10. Number of hardness options (c_{10})

Hardness describes the material's resistance to scratches and indentations. The degree of hardness of polyurethane is determined by the Shore scale. For example, polyurethane with hardness Shore 40A is softer and more elastic than polyurethane with hardness Shore 95A. The hardness of polyurethane is controlled on site by the proportion of components of the material, hence this is a maximization criterion.

11. Abrasion Loss (c11)

Abrasion resistance can be described as ability of polyurethane to withstand mechanical action such as rubbing, scraping or erosion that progressively tends to remove material from its surface. Abrasion loss is measured in cubic millimeters (mm³). This is a minimization criterium.

Tables 5 and 6 show individual alternatives evaluated according to all criteria.

	Price	Delivery service	Delivery price	Delivery time	Drying time	Δ Drying time
a_1	275 UAH	Yes	2 UAH	3 days	30	10
a_2	240 UAH	No	20 UAH	30 days	60	30
<i>a</i> ₃	255 UAH	Yes	3 UAH	3 days	40	25
<i>a</i> ₄	260 UAH	No	24 UAH	30 days	40	10
<i>a</i> 5	250 UAH	Yes	1 UAH	4 days	7	3
<i>a</i> ₆	280 UAH	Yes	2 UAH	5 days	40	10

Table 5: Criterial matrix (part 1)

Source: own processing

	Compatibility	Tensile	Elongation	Number of hardness	Abrasion
	with catalyst	Strength		options	Loss
a_1	Yes	35.2 kN/m	690 %	9	44
a_2	Yes	60 kN/m	670 %	8	22
<i>a</i> ₃	No	38 kN/m	650 %	11	21
<i>a</i> 4	No	61 kN/m	827 %	10	45
<i>a</i> 5	Yes	36 kN/m	680 %	9	26
<i>a</i> ₆	yes	32 kN/m	630 %	9	25

Table 6: Criterial matrix (part 2)

Source: own processing

In order to use the TOPSIS method for the calculation, it is necessary to convert qualitative evaluation to quantitative values. Since there are only two criteria with evaluation in verbal form (listed in the above table as Delivery service (c_7) and Compatibility with catalyst (c_7)), the expert was also asked to allocate point scales for these two criteria in quantitative values. Quantitative evaluations were selected as follows: Delivery service: yes = 5 and no = 1 and Compatibility with catalyst: yes = 10 and no = 1. Quantitative evaluation of variants is given in the table below.

Table 7: Criterial matrix – quantitative evaluation

	<i>C</i> 1	<i>C</i> ₂	С3	<i>C4</i>	C5	С6	С7	C8	С9	C10	<i>C</i> 11
a_1											
	275	5	2	3	30	10	10	35.2	690	9	44
a_2											
	240	1	20	30	60	30	10	60	670	8	22
<i>a</i> ₃											
	255	5	3	3	40	25	1	38	650	11	21
<i>a</i> ₄											
	260	1	24	30	40	10	1	61	827	10	45
<i>a</i> 5											
	250	5	1	4	7	3	10	36	680	9	26
<i>a</i> ₆											
	280	5	1	5	40	10	10	32	630	9	25

Source: own processing

3.5 Criteria weights

The weights of the criteria were obtained through Saaty's method which derives the weights of the criteria as a normalized geometric mean, the principle of which was described in detail in the theoretical part of the work. This method was chosen since the weights were selected by only one expert from the firm. The expert was required to make a pairwise comparison of the individual criteria by means of comparing two criteria against each other and assigning a numerical value from the range 1... 9 to the preferred criterion and the inversed value to the less preferred criterion according to the formula (1.4). This pairwise comparison was recorded in the table below:

	c_1	<i>C</i> ₂	C3	<i>C</i> 4	C5	C6	<i>C</i> 7	C8	С9	<i>C</i> 10	<i>c</i> ₁₁
<i>C</i> 1	1.00	5.00	4.00	2.00	0.33	0.25	0.50	0.33	0.25	0.20	0.17
<i>C</i> ₂	0.20	1.00	0.50	0.50	2.00	0.50	0.50	0.25	0.25	0.14	0.14
C3	0.25	2.00	1.00	2.00	0.33	0.25	0.33	0.25	0.33	0.14	0.14
<i>C</i> 4	0.50	2.00	0.50	1.00	0.33	0.20	0.25	0.20	0.20	0.14	0.17
C5	3.00	0.50	3.00	3.00	1.00	0.25	2.00	0.33	0.33	0.20	0.25
C ₆	4.00	2.00	4.00	5.00	4.00	1.00	0.50	0.25	0.33	0.17	0.20
<i>C</i> 7	2.00	2.00	3.00	4.00	0.50	2.00	1.00	0.33	0.50	0.20	0.20
C8	3.00	4.00	4.00	5.00	3.00	4.00	3.00	1.00	3.00	0.33	0.50
<i>C</i> 9	4.00	4.00	3.00	5.00	3.00	3.00	2.00	0.33	1.00	0.33	0.33
C10	5.00	7.00	7.00	7.00	5.00	6.00	5.00	3.00	3.00	1.00	2.00
<i>C</i> 11	6.00	7.00	7.00	6.00	4.00	5.00	5.00	2.00	3.00	0.50	1.00

Table 8: Saaty's matrix

Source: own calculation

The consistency ration explained earlier in the theoretical part (formula 1.5) has resulted in 0.99, which complies with the requirement CR < 0.1, meaning that the Saaty's matrix is sufficiently consistent.

In the next step of the Saaty's method we will calculate geometric means (b_i) of the rows (formula 1.7). Then these values will be used in formula 1.8. The resulted values represent criteria weights (w_i) that will be used further in TOPSIS calculation. The weights are shown in the following table:

Table 9: Criteria weights

	b_i	Wi
<i>C</i> 1	0.61343	0.039706
<i>C</i> ₂	0.388726	0.025161
С3	0.401764	0.026005
<i>C</i> 4	0.343493	0.022233
C5	0.740582	0.047936
<i>C</i> ₆	0.990227	0.064095
<i>C</i> 7	0.900776	0.058305
<i>C</i> 8	2.138463	0.138418
С9	1.581919	0.102394
<i>C</i> 10	4.03614	0.26125
<i>C</i> 11	3.313821	0.214496

Source: own calculation

The criterium C_{10} indicating the number of hardness options of the polyurethan has obtained the highest weight (0.26125), which is subsequently followed by the weight (0.214496) of the criterium C_{11} (Abrasion Resistance) and then C_8 (Tensile Strength) with the weight (0.138418). The lowest weight has resulted in the criterion Delivery time C_4 - (0.022233).

The visual interpretation demonstrated in the Figure 1.

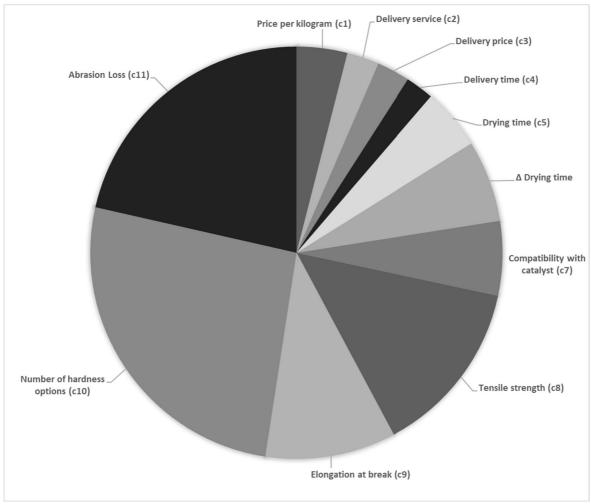


Figure 1: Pie chart of criteria weights

Source: own processing

From the weights of the criteria determined in this way, it is clear that the decision maker gives higher preference to those criteria that reflect technical characteristics which stand for variety of end products and their quality. The firm then sacrifices the weights of delivery parameters and price of the raw material itself.

3.6 TOPSIS

With the weights determined in the previous paragraph, we will perform a calculation using the TOPSIS method whose general procedure was given in the theoretical part. For this reason, the above-mentioned general formulas will not be repeated. The solution will indicate the optimal result of the decision-making process, namely a suitable supplier of raw materials.

As the first step, we will form a normalized criterial matrix $R = (r_{ij})$ according to the expression 1.9.

At this step, all characteristics are reduced to dimensionless values.

	<i>C</i> 1	<i>C</i> ₂	С3	<i>C</i> 4	C5	C6	С7	C8	С9	C10	<i>C</i> 11
a	0.431	0.495	0.063	0.069	0.310	0.233	0.498	0.317	0.405	0.391	0.559
1	19	074	532	58	269	507	755	078	831	675	387
a	0.376	0.099	0.635	0.695	0.620	0.700	0.498	0.540	0.394	0.348	0.279
2	311	015	321	795	538	522	755	474	068	155	694
a	0.399	0.495	0.095	0.069	0.413	0.583	0.049	0.342	0.382	0.478	0.266
3	831	074	298	58	692	768	875	3	305	714	98
a	0.407	0.099	0.762	0.695	0.413	0.233	0.049	0.549	0.486	0.435	0.572
4	671	015	385	795	692	507	875	482	409	194	101
a	0.391	0.495	0.031	0.092	0.072	0.070	0.498	0.324	0.399	0.391	0.330
5	991	074	766	773	396	052	755	284	95	675	547
a	0.439	0.495	0.031	0.115	0.413	0.233	0.498	0.288	0.370	0.391	0.317
6	03	074	766	966	692	507	755	253	542	675	834

Table 10: Normalized criterial matrix

Source: own calculation

In the next step the values will be multiplied by the relevant criterion weights presented in the Table 9.

According to the formula 1.10, each *j*-*th* column of the normalized criterion matrix is multiplied by the relevant weight. Results in the below table represent normalized weighted matrix.

Table 11: Normalized	weighted matrix
----------------------	-----------------

	<i>C</i> 1	<i>C</i> ₂	С3	<i>C4</i>	C5	C6	С7	C8	С9	C10	<i>C</i> 11
a_1	0.0171	0.010	0.001	0.001	0.014	0.014	0.000	0.042	0.041	0.100	0.110
	0.0171	0.012	0.001	0.001	0.014	0.014	0.029	0.043	0.041	0.102	0.119
	208	457	652	547	873	967	08	889	555	325	986
a_2											
	0.0149	0.002	0.016	0.015	0.029	0.044	0.029	0.074	0.040	0.090	0.059
	4179	491	522	47	746	9	08	811	35	956	993
<i>a</i> ₃											
	0.0158	0.012	0.002	0.001	0.019	0.037	0.002	0.047	0.039	0.125	0.057
	7565	457	478	547	831	417	908	38	146	064	266
a_4											
	0.0161	0.002	0.019	0.015	0.019	0.014	0.002	0.076	0.049	0.113	0.122
	8694	491	826	47	831	967	908	058	805	694	713
<i>a</i> ₅											
	0.0155	0.012	0.000	0.002	0.003	0.004	0.029	0.044	0.040	0.102	0.070
	6437	457	826	063	47	49	08	887	952	325	901
a_6											
	0.0174	0.012	0.000	0.002	0.019	0.014	0.029	0.039	0.037	0.102	0.068
	3209	457	826	578	831	967	08	899	941	325	174
	3209	457	826		831			899	941	325	174

Source: own calculation

In the following steps it is required to calculate the distances of all variants from the ideal alternative h using the expression 1.11 and also the negative-ideal alternative d (expression 1.12). Therefore, it is necessary to determine these alternatives.

 Table 12: Ideal and negative-ideal alternatives (part 1)

	<i>C</i> ₁	<i>C</i> ₂	Сз	<i>C</i> 4	C5	C6	
h	0.014941793	0.012456708	0.000826084	0.001547	0.02974621	0.044900015	
d	0.017432092	0.002491342	0.019826006	0.01546995	0.003470391	0.004490002	
Source: own calculation							

Table 13: Ideal and negative-ideal alternatives (part 2)

	<i>C</i> 7	C8	С9	C10	<i>C</i> 11			
h	0.029079966	0.076058035	0.049805357	0.125063905	0.057266205			
d	0.002907997	0.039899297	0.037941203	0.090955568	0.122713297			
Courses over calculation								

Source: own calculation

And finally, we apply the expression 1.11 and 1,12 allowing us to calculate distances of individual alternatives from the ideal alternative and distances from the negative-ideal alternative.

Table 14: Distances of individual alternatives from ideal (h) and negative-ideal (d) alternatives

	h	d
a_1	0.081709381	0.041400172
a_2	0.042441478	0.090465118
<i>a</i> ₃	0.04217729	0.086327702
<i>a</i> ₄	0.082143673	0.048417392
<i>a</i> ₅	0.063851647	0.064623833
<i>a</i> ₆	0.055550305	0.069229248

Source: own calculation

Then we calculate the relative indicator c_i of the distance from individual alternatives to the negative-ideal alternative using the expression 1.13.

Table 15: Relative indicator c_i

	Ci
a_1	0.336287239
a_2	0.680666881
<i>a</i> ₃	0.671784814
<i>a</i> ₄	0.370840969
<i>a</i> ₅	0.503005189
a_6	0.554812439

Source: own calculation

The results for selection of the optimal solution are represented in the Table 16 as an arrangement of c_i indicator in descending order.

Table 16: Arrangement of alternatives is descending order according to the c_i indicator

	C_i
a_2	0.680666881
<i>a</i> ₃	0.671784814
<i>a</i> ₆	0.554812439
<i>a</i> 5	0.503005189
<i>a</i> ₄	0.370840969
a_1	0.336287239
~	1 1 .

Source: own calculation

4 **Results and Discussion**

Based on the above arrangement of alternatives, TOPSIS method has resulted in suggesting the alternative Covestro AG (a_2) as efficient alternative with c_i value 0.68, meaning that this alternative has the longest distance from the negative-ideal alternative. This alternative has the lowest price, the second-best value for the criterion Abrasion loss (c_{11}) however, a_2 is closely followed by the alternative a_3 with index 0.67 which can also be recommended as a solution, though when looking closely to the criterial matrix captured in Table 5, 6, we can see that a_3 has worse performance on couple of significant criteria describing technical characteristics, such as compatibility with catalyst and tensile strength. Nevertheless, the alternative The Dow Chemical Company (a_3) can be recommended as an alternative solution, having the best performance on the criteria c_{10} and c_{11} - the most important technical parameters.

4.1 Sensitivity analysis

For enhancement's sake the offered solution has been extended by adding the sensitivity analysis of a criterion weights to analyze the degree of stability of the resulted ranks. The company's preference towards the criterion c_{10} (Number of hardness options) was indicated in the chapter 3.5. The highest value of the weight is explained by the company's main strategy to expand the variety of products they plan to offer. The analysis will display to what extent this criterion impacts the solution of MCDA model.

The below table demonstrates how TOPSIS results are affected by gradual change of c_{10} weight. The table embodies the original c_{10} weight 0.26125 and artificial weights. The first artificial weight is 0.1, this value is gradually increased by adding 0.1 until the final value 0.9, while weights of remaining criteria are calculated with maintenance of the original weight ratio.

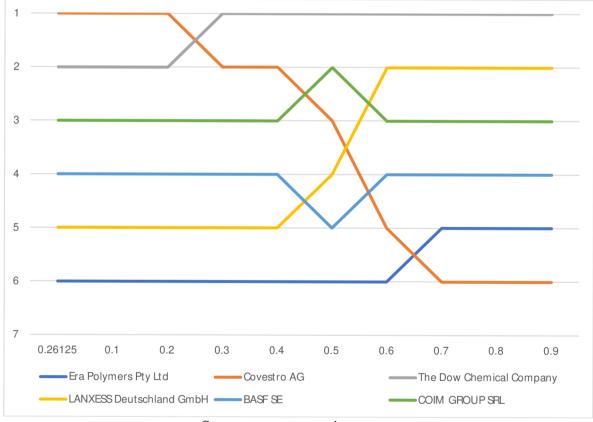
weight	0.26125	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
				alter	matives	rankin	g			
Era Polymers Pty										
Ltd	6	6	6	6	6	6	6	5	5	5
Covestro AG	1	1	1	2	2	3	5	6	6	6
The Dow Chemical										
Company	2	2	2	1	1	1	1	1	1	1
LANXESS										
Deutschland										
GmbH	5	5	5	5	5	4	2	2	2	2
BASF SE	4	4	4	4	4	5	4	4	4	4
COIM GROUP										
SRL	3	3	3	3	3	2	3	3	3	3
Source: own processing										

Table 17: Sensitivity analysis: change of alternatives ranking

Source: own processing

The results are captured in the Figure 2 for better representation.

Figure 2: Sensitivity analysis graph: change of alternatives ranking



Source: own processing

We can clearly see that with the first slight increase of the c_{10} weight from the original value 0.26125 to 0.3 the ranking of the alternative Covestro AG (a_2) has dropped to the second place, after reaching the weight 0.4 the rank was dramatically dropping and had made it to the worst rank on the weight 0.7. And vice versa, the alternative The Dow Chemical Company (a_3) has taken the leading position with the value of the c_{10} weight 0.3 and stabilized. There is also a notable lift of the alternative LANXESS Deutschland GmbH after reaching the weight 0.4.

Conclusion

In accordance with set objectives, the submitted work deals mainly with the tasks of multicriteria analysis of alternatives and detailed description of TOPSIS as the selected method to use. The theoretical part includes citing different authors that cover problem of the decision making. The described method was applied on the specific example of the company «VITLUS» LLC PCC challenged by selecting an optimal raw materials supplier in order to expand production and produce high-quality polyurethane components and at the same time avoid extra costs like in case of supplying different kinds of polyurethane from several suppliers.

Based on the necessary findings from the theoretical part, the practical part was concentrating on the specific example explained during regular consultations with engineering technologist – representative of the company.

The main goal of the multicriteria analysis model was to identify the best supply offer which would comply with the list of requirements from technical perspective and would be optimal costs-wise. For this reason, the company's representative set of 11 criteria and provided the list of given offers with measured performance of individual alternatives on each criterion. Based on this information the criterial matrix was formulated which gave the reader an overview of the supply offers the company needed to select from. The representative also conducted the pairwise comparison of the criteria and identified a degree of importance of an individual criterion over another one. This allowed to determine criteria weights by means of Saaty's method. Then it was followed by the research on the pairwise comparisons' consistency index.

Final results were obtained by TOPSIS, by which the company was suggested to select the offer of Covestro AG based on the highest value of the indicator representing distance of the Covestro AG offer from the negative-ideal alternative.

Sensitivity analysis was conducted in order to demonstrate how the criterion Number of hardness options affected the results of the MCDA model. The demonstrated graph identified that the offer of Covestro AG stably loses its rank with gradual increase the criterion weight, and vice versa, the second best-rated offer of the supplier The Dow Chemical Company obtains and keeps the best rank. The findings were shared with the company to internally familiarize with interpretation and provide feedback on the recommended solution.

The company made a decision to accept the offer from Covestro AG based on the technical characteristics of the factory where the production takes place.

5 References

BELTON, V., & STEWART, T. (2001). Multiple criteria decision analysis: An integrated approach. Norwell, MA: Kluwer Academic Publishers.

BROŽOVÁ, Helena, ŠUBRT, Tomáš a HOUŠKA, Milan. Modely pro vícekriteriální rozhodování. 1. vyd. Praha: Credit, 2003. 172 s. ISBN 80-213-1019-7.

FOTR, Jiří a Lenka ŠVECOVÁ. *Manažerské rozhodování: postupy, metody a nástroje*. 2., přeprac. vyd. Praha: Ekopress, 2010. ISBN 978-80-86929-59-0.

FRIEBELOVÁ, Jana a Jana KLICNAROVÁ. Rozhodovací modely pro ekonomy. 1. vyd. České Budějovice: Jihočeská univerzita v Českých Budějovicích, 2007. 135 s. ISBN 9788073940355.

FU, Feng, and Mizi FAN. Advanced High Strength Natural Fibre Composites in Construction. Elsevier Science, 2017. ISBN 978-0-08-100411-1

HAMMOND, J S. -- KEENEY, R L. -- RAIFFA, H. *Smart choices: a practical guide to making better decisions*. Boston, Mass.: Harvard Business School Press, 1999. ISBN 0875848575.

HANSEN, P., & DEVLIN, N. Multi-Criteria Decision Analysis (MCDA) in Healthcare Decision-Making. Oxford Research Encyclopedia of Economics and Finance. Retrieved 29 Dec. 2021, from

https://oxfordre.com/economics/view/10.1093/acrefore/9780190625979.001.0001/acrefore-9780190625979-e-98.

HWANG C. L., YOON K. Multiple Attribute Decision Making Methods and Applications. A State of the Art Survey. New York: Springer Verlag, Berlin, Heidelberg, 51 1981. 259 p.

ISHIZAKA, Alessio a Markus LUSTI. How to derive priorities in AHP: a comparative study. Central European Journal of Operations Research, 2006, 14(4), 387–400.

ISMAIL, Ahmad Fauzi, KHULBE, Kailash Chandra, MATSUURA, Takeshi. Reverse Osmosis. Elsevier, ISBN 9780128114681.

LAMATA, Maria Teresa a Jose Ignacio PELAEZ. A method for improving the consistency of judgements. International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, 2002, 10(06), 677–686.

SAATY, Thomas L. How to make a decision: The analytic hierarchy process. European Journal of Operational Research. 1990, 1990(48), 9-26. ISSN 0377-2217. SAATY, Thomas L. Relative measurement and its generalization in decision making. Revista de la Real Academia de Ciencias Exactas, Físicas y Naturales. Serie A. Matemáticas. 2008, 2008(102), 251–318.

STOKLASA, Jan, Vera JANDOVA a Jana TALAŠOVÁ. Weak consistency in Saaty's AHP-evaluating creative work outcomes of Czech art colleges. Neural network world, 2013, 23(1), 61.

ŠUBRT, Tomáš a kol. Ekonomicko-matematické metody. Plzeň: Vydavatelství a nakladatelství Aleš Čeněk, 2011, 351 s. ISBN 978-80-7380-345-2.

TZENG, G. -- WANG, H F. -- WEN, P. Multiple criteria decision making. NEW YORK: SPRINGER, 1995. ISBN 3-540-94297-1.

VITLUS | PRODUCTS. (2020). PRODUCTION-COMMERCIAL COMPANY "VITLUS." Retrieved March 16, 2022, from http://www.vitlus.net/

6 List of tables and figures

6.1 List of tables

Table 1: Steps of MCDA process	.12
Table 2: Scales for expressing preferences in the Saaty method (1990)	
Table 3: The random index value <i>RI</i> (part 1)	
Table 4: The random index value <i>RI</i> (part 2)	
Table 5: Criterial matrix (part 1)	
Table 6: Criterial matrix (part 2)	
Table 7: Criterial matrix – quantitative evaluation	
Table 8: Saaty's matrix	
Table 9: Criteria weights	
Table 10: Normalized criterial matrix	
Table 11: Normalized weighted matrix	
Table 12: Ideal and negative-ideal alternatives (part 1)	
Table 13: Ideal and negative-ideal alternatives (part 2)	
Table 14: Distances of individual alternatives from ideal (h) and negative-ideal (d)	
alternatives	28
Table 15: Relative indicator <i>c</i> _i	
Table 16: Arrangement of alternatives is descending order according to the c_i indicator	29
Table 17: Sensitivity analysis: change of alternatives ranking	

6.2 List of figures

Figure 1: Pie chart of criteria weights	
Figure 2: Sensitivity analysis graph: change of alternatives ranking	