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FACULTY OF ELECTRICAL ENGINEERING AND COMMUNICATION<br>FAKULTA ELEKTROTECHNIKY<br>A KOMUNIKAČNÍCH TECHNOLOGIÍ

## DEPARTMENT OF THEORETICAL AND EXPERIMENTAL ELECTRICAL ENGINEERING <br> ÚSTAV TEDRETICKÉ A EXPERIMENTÁLNÍ ELEKTROTECHNIKY

## DETECTION OF ABSOLUTE POSITION OF OBJECT BASED ON RELATIVE CHANGE OF PHASE

DETECTION OF ABSOLUTE POSITION OF OBJECT BASED ON RELATIVE CHANGE OF PHASE BACHELOR'S THESIS
BAKALÁŘSKÁ PRÁCE

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## Bachelor's Thesis

Bachelor's study program Electrical Engineering<br>specialization Power Systems and Automation<br>Department of Theoretical and Experimental Electrical Engineering

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## Detection of absolute position of object based on relative change of phase

## INSTRUCTION:

The goal of the bachelors thesis is to present a new concept for the problem of measurement of the absolute position of a certain object, based on the phenomena of phase shift, which occurs when the geometrical distance between the receives is smaller than the wavelength of the incoming signal. The objectives for the work are: verifications of the presented ideas and solutions for the presented problem; verification of the practical feasibility of the chosen solutions; designing a practical system; experimentally testing the capabilities and the limitations of the designed system in order to obtain a comprehensive analysis of the presented concept.

## RECOMMENDED LITERATURE:

1. THE EUROPEAN TABLE OF FREQUENCY ALLOCATIONS AND UTILISATIONS, 2002. [online], [Accessed 30 January 2023]. Available from: https://www.ero.dk/eca-change
2. ANTENNA PATTERNS AND THEIR MEANING, 2007. INDUSTRIAL NETWORKING SOLUTIONS [online], [Accessed 30 January 2023]. Available from: https://www.industrialnetworking.com/pdf/Antenna-Patterns.pdf 3. 433 MHZ RF TRANSMITTER AND RECEIVER MODULE PINOUT, FETURES \& WORKING, 2022. ETECHNOPHILES [online], [Accessed 30 January 2023]. Available from: https://www.etechnophiles.com/433-mhz-rf-transmitter-and-receiver-module-pinout-features-working/

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#### Abstract

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## Abstract

The goal of this work is to present a new method of position detection for autonomous systems. The method suggests usage of classical triangulation technique for which the main input, the angle value, is measured using a new approach, which implies measurement of the phase difference between two wireless receivers physically placed in a specific manner. The mutual placement of the receivers is defined by the selected frequency of the transmitted signal, namely its wavelength. The thesis describes the method itself, its simulated model and some practical issues and limitations.

## Keywords

Wireless, Triangulation, Phase measurement, Reflectivity, Interference, Antenna

## Bibliographic citation

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# Author's Declaration 

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I declare that I have written this paper independently, under the guidance of the advisor and using exclusively the technical references and other sources of information cited in the project and listed in the comprehensive bibliography at the end of the project.

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Brno, May 29, 2023
author's signature

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## INTRODUCTION

The topic of the thesis is related to the area of automatization in autonomous robotic systems, remote control and position detection. The idea of the presented method relies on the physical properties of electromagnetic wave. The change of the intensity of the electromagnetic field can be sensed by the receiver, the antenna. Two antennas placed at different points in space, at the same moment in time, would experience the fields of different intensities. This would result in a phase difference between the sensed signals. If the distance between the antennas is smaller than the wavelength of the signal, then for a selected plane in space, each unique value of phase difference would be proportional to a unique direction of the signal source. Thus, if the phase difference is known, the direction of the signal source can be estimated. Performing the measurement in multiple planes at one point in space, should give the direction line, which can be combined with the same kind of a direction line, obtained at a different space point. This should allow calculation of the position of the signal source with relatively high precision. The main objective of this work is to identify whether the presented approach to position measurement is possible to implement and use in real world applications. For this purpose a theoretical model was created, to simulate and evaluate the process of position calculation with the presented approach. The practical issues related to the real world application were tested with a simplified system model.

Chapter 1 of the thesis includes the definition of a sample problem and discusses some of the currently existing solutions. The detailed explanation of the presented method is included in subchapter 1.3. Chapter 2 focuses on the theoretical model of a system that would utilize the usage of the described concept. Its main part is located in subchapter 2.3, where the simulation created in Matlab is described and subchapter 2.4 includes examples of the results obtained from the simulation execution. The practical part of the work can be found in chapter 3. Subchapters 3.1 and 3.4 include information about the structure of the transmitted signal. Subchapter 3.2 focuses on the impact of the environment reflectivity on the accuracy of the phase measurement. In subchapter 3.3 the influence of the sensitivity of the wiring is discussed. The example of performed phase shift measurement can be found in subchapter 3.5 and subchapter 3.6 includes some information about the accuracy of the presented position detection approach. A discussion about antenna the shape influence of the phase difference is given. Finally, Subchapter 3.8 proposes the layout of a system and gives some suggestions on the hardware for the project.

## 1. PoSition detection in autonomous systems

### 1.1 Problem definition

Let's imagine a single autonomous flying unit - a drone, which operates in a certain area over a long period of time. The main function of this drone is, for example, transportation of some objects between different points in space. Not taking into account how exactly the job is performed, one of the main objectives in such a case would be ensuring that the drone is capable of accurately moving from point A to point $B$, point $B$ to point $C$, point $C$ to point $D$ and so on. An important remark is that all of the above movements have to be done repeatedly over a number of iterations and without any human control.

This is not a trivial task, which may be solved in a variety of ways. Let's try to find a simple algorithm for its solution. If we know the position of two points $A$ and $B$ in a 2D space, we can say that in order to reach point $B$ from point $A$ it is enough to move distance „a" to the right and then distance „b" forward. However, such approach has some problems when it comes to its practical implementation. How do we know that the drone is located at point A? How do we know that the drone has moved exactly distances „"" and „b", not more or less? How do we confirm that after all the calculated movements were made, that the drone has indeed arrived at point $B$ ? We could potentially place some sensors to detect the presence of a drone at both points.

But what happens if after performing all the moves, the sensor at point B does not detect anything? What is the location of the drone in such a case and what are the new movements that have to be done in order to reach point B eventually? Do we need to place a separate sensor at each point in space to know the position of the drone at any moment in time? Of course, using hundreds or thousands of sensors for every point in space is not a clever solution. There should be a more efficient way to solve this dilemma.

### 1.2 Existing solutions

### 1.2.1 Amazon Robotics

One example of a quite similar task was actually solved by Amazon Robotics [1]. Their famous robots are capable of moving precisely around their warehouses. Each of the robots is equipped with a QR reader that reads QR codes located on the floor. Each QR code carries the information about its coordinates, allowing the robots to use it as a reference. Unfortunately, even though such approach has proven itself to be efficient and reliable, it is not suitable in our case. Placing QR codes on the floor is by itself a simple task and Amazon robots are only moving in 2D space. But when we move one dimension up, it gets a bit problematic. Despite being just paint on a floor, those codes
are still physical objects, which have to be supported in some way if we want them to stay at different heights above the floor. Any kind of support for those reference points, a small stand or a thin thread, will get in the way of a drone and would most likely result in its inability to fly at all. If we consider a non-physical way of referencing, for example, visual recognition of the terrain or some types of referencing objects, it is a topic for a different project and will not be discussed in this paper due to its complexity and processing power demands.

### 1.2.2 GPS tracking

Another possible solution is, of course, GPS tracking. The position is determined based on a distance between a certain device and the satellites located on the orbit of the Earth. This distance is calculated by measuring the signal travel time. The main issue with this method of position detection is that for the presented problem it is a bit too inaccurate. In best case scenario, 5-10 m. of uncertainty should be expected, in worse cases more than 20 m . is possible [2]. Considering the fact that typical commercial quadcopter drones do not exceed frame sizes of 2 m . [3], we end up with an error which is few times larger than the size of a drone, which is not appropriate. Moreover, if the drone has to operate in a closed environment, such as a factory, warehouse or greenhouse, GPS would not be usable at all.

### 1.2.3 Distance as a function of signal strength

The last example of position detection techniques is measurement of the distance based on signal strength. For this purpose Bluetooth or Wi-Fi can be used. When any wireless communication is established the strength of the received signal is proportional to the distance between the devices. Therefore, knowing one of these parameters allows estimation of the other. The examples of such systems show relatively good results, reaching the detection accuracy of 1-2 m . [4]. The disadvantage of this approach is the fact that the signal strength can also be affected by many other influences, such as presence or movement of the objects in the surrounding environment. Also, such good results can be achieved at relatively small distances (up to $10-20 \mathrm{~m}$.) but at larger distances the sensitivity of the method drops significantly.

### 1.3 Triangulation and trilateration

Triangulation is a method used to determine the location of a fixed point based on the laws of trigonometry. The laws state that if one side and two angles of a triangle are known, the other two sides and angle of that triangle can be calculated [5]. Another statement which comes from trigonometry is that knowing all sides of a triangle allows calculation of all its angles. The latter is utilized by trilateration technique. There are small differences between the methods, and on the very basic level both can be described as a series of simple geometrical calculations. Trilateration is more commonly
used, as there are many known and tested methods for distance measurement. The examples of such methods were presented in a previous chapter. The aim of this work, however, is not to describe the existing methods, but rather present a new approach for position detection, which is based on triangulation in its pure geometrical meaning. Assuming that one of the variables is known, the side of the triangle, the method for measurement of the remaining two unknowns, the angles, is presented.

### 1.4 Proposed method

As it has been stated earlier, instead of measuring the distance to a signal source, we can measure the direction from which the signal is coming, the angle, and triangulate its position.

Imagine a transmitter located at point S and two receivers R 1 and R 2 (Figure 1.1). If the distance between the receivers is less or equal to the wavelength of the received signal, then depending on the position of the transmitter, we will observe a phase shift between the signals received by the two receivers, which is unique and proportional to a geometrical deflection angle $\alpha$. If the signal comes from point A : both phase shift and angle $\alpha$ are equal to zero, since the distance from point A to each of the receivers is the same. If the signal comes from point B: $\alpha$ is equal to $90^{\circ}$ and the phase shift is equal to its maximum value, depending on the chosen distance between the receivers. If the distance between the receivers is smaller than the wavelength of the signal, the phase difference between the signals should be unique for each potential direction of the source within the shown quadrant of space.


Fig. 1.1
A signal source and 2 receivers
The deflection angle $\alpha$ is proportional to the phase shift between the two signals, therefore if we know what is the phase shift, we can also know in which direction the signal is located. Of course, the presented above model is not sufficient if we want to
work in a 3D space. If we rotate point S around the axis created by the receivers, we will end up with a circle of infinitely many potential points of locations. A transmitter placed at any of the point will create exactly the same phase shift as the same transmitter placed at any other point on a circle.

One more receiver can be added to the system in order to eliminate most of the potential locations. With the configuration shown in Figure 1.2, most of the potential locations of the transmitter can be canceled by introducing 2 more variables. In this case, even though the phase shift between R1 and R2 does not change (considering a rotation around the axis created by R1 and R2), the phase shifts between R1 and R3 and between R2 and R3 do change. Thus, the potential locations have been limited to just two points. These are located on the lines which are perpendicular to the plane created the 3 receivers. In other words, any pair of points that are mirrored with respect to this plane would result in the same phase shift if the transmitter was placed at each of them.


Fig. 1.2
A signal source and 3 receivers
Finally, we can add the last receiver to eliminate one of the two remaining points and form a beacon that would be able to detect accurately the direction of the incoming signal. The resulting configuration consists of four receivers located at the vertices of an equilateral tetrahedron. Adding one more beacon, would allow obtaining two lines, the intersection of which should indicate the position of the source signal.


Fig. 1.3
Two beacons and a signal source
For each beacon the direction line can be expressed by two parameters: the horizontal deflection angle $\alpha$ and vertical deflection angle $\beta$. Both of them are calculated with respect to the geometrical center of the tetrahedron formed by the receivers. Therefore, we have six input parameters: phase shifts between all possible pairs of receivers (edges of the tetrahedron); and 2 output parameters: The deflection angles, which have to be derived from the input parameters.


Fig. 1.4
The deflection angles $\alpha$ and $\beta$

### 1.5 Selecting the frequency

The described above method should be applicable at any frequency of the transmission signal. Therefore, when choosing the frequency we have to consider only the practicality of implementation. At lower frequencies (below $40-50 \mathrm{MHz}$ ), the wavelengths are too big and impractical for building the devices. While at higher frequencies (above 2 GHz ), there is less margin for error when constructing the beacons.

In such a case, it makes sense to choose frequency based on availability of the wireless communication devices. RF-433-MHz is an example of a cheap and accessible communication module [6]. It operates at 433.92 MHz in a range of up to 100 m . (with antenna). This frequency lies within the amateur radiolocation band [7] and can be freely utilized without any restrictions, which makes it a perfect choice for this project. The wavelength of 0.69089 m . is suitable for construction purposes as it gives sufficient space for placement of the components and is small enough to handle.

## 2. Method model

### 2.1 Measuring the phase shift

There are various methods for measurement of the phase shift. For this simulation specifically, taking into account the problem definition, a simple and swift solution is required. Instead of dealing with sampling frequencies and huge amounts of data, a part of the necessary calculation can be done by analogue means.

Because the signals of the same frequency and amplitude are considered, the fact that the mathematical sum of two sinusoidal signals is also a sinusoidal signal can be utilized; its amplitude changes proportionally to the phase shift between the signals. If the phase shift is equal to zero, the amplitude of the resulting signal is higher than the amplitude of the original signals. If the phase shift is $180^{\circ}$ then the amplitude of the resulting signal is zero. Therefore, the geometrical deflection angle can be expressed with either RMS or the maximum value of output signal by the means of calibration.

Unfortunately, there is one huge disadvantage that has to be considered. Using this method results in the inability to distinguish between positive and negative phase shifts. In case of configuration shown on Figure 2.1 (a), the phase shifts from signals located at points S1 and S2 would have the same magnitude and the opposite signs. But since only magnitude is measured with the described method, it is not possible to distinguish between the two points. Figure 2.1 (b) shows why this is the case; the difference between the two output signals is the phase, but here it is not possible to state which one represents point S 1 or S 2 , because in real situation there would be only one of them at a time and there is no reference to compare with. It could potentially be compared with one of the original signals, which makes no sense, since this would create an endless loop of signal comparisons, where the two signals are compared by creating the third, which has to be compared with one of the original signals by creating a fourth signal and so on. In such scenario, it would be better to simply choose a different method, but fortunately, it is not necessary, as this disadvantage can be compensated by some other means, which will be discussed later.


Fig. 2.1 a) Two signal sources and two receivers;
b) Positive and negative phase shifts

### 2.2 Simplified beacon model

The configuration of the beacon presented in chapter 1.4, the 4 receivers located at the vertices of an equilateral tetrahedron, is the theoretically required minimum which should allow accurate position detection. Unfortunately, the attempts to derive the actual dependence between the input and output parameters were not successful. The inability to distinguish between left and right, due to the selected phase shift detection method, did not help either. Therefore, a different configuration was created which is more convenient for use and solves the problem of "left-right" blindness.

Figure 2.2 shows a structure where the receivers are located on the vertices of a square. We can calculate the phase shifts between any pair of receivers to obtain the deflection angle value for both points S1 and S2. But we can also notice an interesting relationship between two specific pairs of receivers: R1-R4 and R2-R3. For point S1, the phase shift R1-R4 is zero, while R2-R3 is maximal. For point S 2 , the situation is opposite. So, by comparing these two phase shifts, we can state whether the signal source is located to the right or to the left with respect to the $y$-axis, or similarly, if it is above or below the x -axis.


Fig. 2.2 Two signal sources and four receivers

The final configuration consists of two such structures placed perpendicularly to each other, one on the XY plane and the other on YZ plane. The former is responsible for measuring the horizontal deflection angle, while the latter for the vertical one. For convenience, phase shift R1-R2 is measured to calculate the horizontal deflection angle and R1-R5 is measured to calculate the vertical deflection angle, since they are placed in parallel with X and Z axes respectively.


Fig. 2.3
Final beacon configuration

### 2.3 Matlab script

In order to prove that the presented concept is valid, the simulation was created using Matlab. The script begins by introducing coordinates of a signal point $S$ as well as the coordinates of the reference beacons A and B.

```
% source of the signal, [x y z] coordinates in centimeters
S = [-1320 4305 -1360];
    initial coordinates of the beacons, [x y z] coordinates in
centimeters
a0 = [-2000 0 0];
b0 = [2000 0 0];
A = placeBeacon (a0, baseDistance);
B = placeBeacon (b0, baseDistance);
```

Function „placeBeacon" positions receivers according to the configuration from Figure 2.3. The „baseDistance" variable represents the selected distance between the receivers.

```
% coordinates of each receiver of a beacon
a1 = A{1,1};
a2 = A{1,2};
a3 = A{1,3};
a4=A{1,4};
a5 = A{1,5};
a6=A{1,6};
```

The distances from the signal source to each of the receivers is calculated:

```
% distances from the signal source to each of the receivers
distanceA1 = norm(S - al);
distanceA2 = norm(S - a2);
distanceA3 = norm(S - a3);
distanceA4 = norm(S - a4);
distanceA5 = norm(S - a5);
distanceA6 = norm(S - a6);
```

Knowing the difference in distances with respect to the reference receiver №1, the relative phase shifts of the signal received be each receiver can be expressed:

```
% converting difference in distance into phase shift for each receiver
shiftA2 = pi*(distanceA2 - distanceA1)/wavelength;
shiftA3 = pi*(distanceA3 - distanceAl)/wavelength;
shiftA4 = pi*(distanceA4 - distanceAl)/wavelength;
shiftA5 = pi*(distanceA5 - distanceA1)/wavelength;
shiftA6 = pi*(distanceA6 - distanceA1)/wavelength;
```

Next the actual signals are simulated with respect to previously calculated phase shifts:

```
% simulating the signal received by each receiver
x = -2*pi:0.01:2*pi;
a1Signal = sin(x); % reference receiver for beacon A
a2Signal = sin(x + shiftA2);
a3Signal = sin(x + shiftA3);
a4Signal = sin(x + shiftA4);
a5Signal = sin(x + shiftA5);
a6Signal = sin(x + shiftA6);
```

The phase shifts between the selected pairs of receivers is calculated:

```
a12difference = max(a1Signal - a2Signal);
a13difference = max(a1Signal - a3Signal);
a14difference = max(a1Signal - a4Signal);
a23difference = max(a3Signal - a2Signal);
a24difference = max(a4Signal - a2Signal);
a34difference = max(a4Signal - a3Signal);
al5difference = max(a1Signal - a5Signal);
a16difference = max(a1Signal - a6Signal);
a45difference = max(a4Signal - a5Signal);
```

The phase shift is expressed as the maximum value of a signal obtained by calculating the mathematical difference between two input signals, which is equal to half of its amplitude. Knowing the phase shifts between the signals (their equivalents in this case), the deflection angles can be calculated:

```
    calculating the deflection angles for each beacon
% angleAalphaTemp - temporary value before the correction
angleAalphaTemp = calibrate(a12difference,baseDistance,portion);
angleAbetaTemp = 90 - calibrate(al5difference,baseDistance,portion);
```

Function „calibrate" calculates the deflection angle by comparing the phase shift with the calibration data, which in its turn is obtained from another function called "generateCalibrationData". In short, generation of calibration data is done by placing two receivers on the X axis of the XY plane and moving the signal source around the center of the axes. First the signal source is moved in fixed steps parallel to the X -axis, then once a certain limit is reached the direction changes and the signal moves parallel to the Y-axis. The resulting path of the signal, together with the axes, creates a square shape. After each step is taken, the phase shift between two receivers is stored into an array together with the calculated geometrical deflection angle, creating the previously mentioned calibration data. Function „calibrate" compares the measured value of the phase shif with the calibration data and outputs the value of a deflection angle which corresponds to the measured phase shift.

The deflection angle value is returned in range from $0^{\circ}$ to $90^{\circ}$, so it has to be corrected for cases when the real deflection angle is higher. This is done according to the logic described by Figure 2.2 by a function called „correctAngle":

```
if (a13difference <= a24difference)
    angleAalpha = correctAngle(1,angleAalphaTemp);
else
    angleAalpha = correctAngle(2,angleAalphaTemp);
end
```

At this point all the necessary information needed to calculate the position of a point S is aquired.

### 2.3.1 Triangulating the transmitter position

The triangulation is done by solving the geometrical structure presented on Figure 2.4 ( AB lies on the X axis, CS ' 1 lies on the Y axis).


Fig. 2.4
Triangulation in 3-dimentional space
Utilizing the fact mentioned in Chapter 1.3 (Figure 1.1): For two receivers, there is an infinite amount of potential positions of the signal source which would result in the same phase shift. These are created by rotating point S around AB (X-axis). Which also means that at least one of them exists on the XY plane; in our case it is point $S^{\prime} 1$. Since the deflection angles for both beacons are known, the [xi, yi] coordinates of $S^{\prime} 1$ can be calculated:

```
% drawing lines for each beacon
tempx1A = a0(1) - 100000*cos(alphaA);
tempy1A = a0(2) - 100000*sin(alphaA);
tempx2A = a0(1) + 100000*cos(alphaA);
tempy2A = a0(2) + 100000*sin(alphaA);
tempx1B = b0(1) - 100000*cos(alphaB);
tempy1B = b0(2) - 100000*sin(alphaB);
tempx2B = b0(1) + 100000*cos(alphaB);
tempy2B = b0(2) + 100000*sin(alphaB);
% the coordinates of the intersection point
[xi,yi] = polyxpoly([tempx1A tempx2A], [tempy1A tempy2A], [tempx1B
tempx2B], [tempy1B tempy2B]);
```

For each beacon two lines are drawn, each going through the respective beacon's center and since two lines are located on the same plane, they will cross at a point S'1, whose coordinates are calculated using Matlab inbuilt function polyxpoly. Once xi and yi are known the triangulation is done by the respective function:

```
results = triangulate(a0 ,b0 , alphaA, alphaB, xi, yi, betaA, betaB);
```

The calculation steps are following:

- Finding coordinates of point C
- $C S=C S^{\prime} 1=r$
- $A S=\frac{C S}{\sin (\alpha A)}$
- $S S^{\prime} 2=h=A S * \sin (\beta A)$
- Plane angle: $\alpha=\sin ^{-1}\left(\frac{h}{r}\right)$
- CS'2 $=r * \cos (\alpha)$

From the steps above the coordinates of the source signal S can be found. X coordinate is equal to xi; Z-coordinate is equal to $h$; Y-coordinate is equal to $r *$ $\cos (\alpha)$.

### 2.4 Simulation results

Once the script is run, the results are displayed in the console and on the 3-dimensional plot. The example of the results can be seen on Figure 2.5:


Fig. 2.5:
The simulation results
In order to observe the response of the simulated system to change in the signal position, the simulation was repeated a number of times for different coordinates of the source signal. The results can be observed in Table 2.1. The simulation results are quite adequate. The displacement error between the actual position of the signal source and the measured one does not exceed one meter for the majority of locations and distances. There are some blind spots, where the accuracy of the measurement drops significantly, but in general it can be concluded that the presented method does indeed work. The observed errors are mostly influenced by the two main parameters: imperfection of the selected triangulation technique and the calibration error. The influence of both of these issues can be potentially reduced, but at this moment in time it does not make much sense, since this simulation does not take into account any of the potential issues which
are typically present in real systems, such as noise or the measurement accuracy. Therefore, improving the simulation performance now would not help with the majority of issues related to the real world applications.

Table 2.1 Simulation results for different positions of the signal source

| Source Signal Position |  |  | Measured Position |  |  | Distance to Source | Displacement Error |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| [ xyz$] \mathrm{cm}$. |  |  | [ xyz$] \mathrm{cm}$. |  |  | cm. | cm . |
| 531 | -665 | 472 | 525 | -696 | 458 | 973 | 34 |
| 731 | -1065 | -680 | 730 | -1075 | -675 | 1460 | 12 |
| 1431 | 1065 | 980 | 1431 | 1087 | 972 | 2035 | 24 |
| 1631 | -1665 | 980 | 1637 | -1711 | 965 | 2528 | 49 |
| -1631 | -1765 | 1480 | -1639 | -1791 | 1559 | 2822 | 83 |
| 2149 | -1915 | 1688 | 2141 | -1950 | 1671 | 3337 | 40 |
| -4149 | 1915 | -4688 | -4160 | 1924 | -4703 | 6547 | 21 |
| -4649 | -5915 | -4988 | -4641 | -5957 | -4985 | 9027 | 43 |
| 6719 | 7955 | 5171 | 6714 | 8012 | 5149 | 11626 | 61 |
| 8234 | 7368 | -7182 | 8205 | 7385 | -7117 | 13178 | 73 |
| -5234 | 12368 | 11182 | -5167 | 12368 | 11171 | 17476 | 68 |
| 100 | 168 | 4182 | 105 | 377 | 4194 | 4187 | 209 |
| 100 | -168 | 9182 | 93 | -614 | 9282 | 9184 | 457 |
| -100 | 200 | 360 | -95 | 223 | 3245 | 424 | 2885 |

## 3. PraCtical Considerations

The mathematical model described in chapter 2 works with the signal source represented by an isotropic source of radiation, a source which radiates equally in all directions in space, and together with the receivers, were described as points in space. In practice, isotropic sources of radiation do not exist [8], moreover both the signal source and the receivers have to be real components that have a physical form and dimensions, which can influence the properties of the components and their mutual interaction.

The most important aspect from a practical point of view is to confirm that the interaction between the transmitter and the receiver in the real world application would be the same as the interaction modeled in the simulation. Otherwise, such a system would not be implementable in practice. In order to better understand what kind of interaction may be expected, we can take a look at how the generated signal behaves in space. Figure 3.1 shows an example spatial distribution of the electric field lines for a free-space wave at some point in time [9]. At points a,b and $c$ the strength of the electric field is the same and the distance between point a and poin c represents the wavelength of the signal. Thus if two receiving antennas were placed at different spots anywhere in between marked points, the field lines surrounding them would have either different direction or intensity or both. The resulting signals induced in the antennas should theoretically have a phase difference with respect to one another which corresponds to their mutual displacement.


Fig. 3.1
Electric field lines for a free-space wave

This gives the hope that the real application is indeed possible. In order to verify the hypothesis, a miniature beacon was built which included the holders for two receiving antennas distanced at approximately 34 cm . apart (half the wavelength of the signal). RF-433-MHz module was used as a transmitter which was controlled by an Arduino Uno board. The phase measurements at the receiver side were taken using Agilent MSO7104B 1 GHz oscilloscope.

### 3.1 Transmitter configuration

RF-433-MHz module uses Amplitude Shift Keying (ASK) for the signal modulation. In the context of digital communications it is a modulation process which imparts to a sinusoid two or more discrete amplitude levels which are related to the number of levels adopted by the digital message. For a binary message sequence there are two levels, one of which is typically zero. Figures 3.2 and 3.3 depict the basic principles of this modulation technique [10]:


Fig. 3.2
ASK: Digital message and modulated signal


Fig. 3.3
Principle of ASK generation
The Arduino code for operating the transmitter control is based on the rc-switch library [11] and requires only one pin to be configured for the output. The library automatically converts the input data into a bit stream which is sent to the corresponding pin on the transmitter chip.

For the purposes of this project, such a way of signal generation is not appropriate. Because the signal is constantly switched on and off, it may create some instability during the phase measurement at the receiver side which in its turn would most likely lead to significant inaccuracies. Fortunately, it is not a big problem since it is possible to generate a signal of a constant output level manually by controlling voltage at the data
pin of the transmitter. However, the pin output which is used by the rc-switch library is reserved and cannot be changed by other means. Therefore a different pin has to be used, which is not very convenient as in this case it is necessary to manually create a bit stream for the data transmission. Maintaining the ability to transmit data is still important and will be discussed further. For now, the solution is to connect two output pins of the Arduino board to the same data pin of the transmitter and use them in turns to generate both the data part of the signal, which is necessary for communication, and the measurement part, which is needed for accurate phase detection. The sample code for the Arduino can be seen in appendix C. Figure 3.4 shows the resulting structure of the modulated signal sampled at the receiver side.


Fig. 3.4 Structure of the transmitted signal

### 3.2 Impact of the environment reflectivity

The experiments have shown some interesting behavior of the signals received by both antennas. First of all, the readings on the oscilloscope were consistent when the measurement conditions remained constant. When the position of the transmitting and receiving antennas did not change, the phases and amplitudes of the signals remained constant, while the change of the transmitter position always resulted in a noticeable change of the readings. This behavior has proven that the difference in the position of the receiving antennas does actually affect the phase difference of the received signals. However, this phase difference did not match any of the theoretical predictions and seemed to be random at a first glance. Moreover, the readings were quite sensitive to some other changes in the surrounding environment.

The most prominent effect was due to movement of any objects including the human body. When there is an object located in between the transmitter and the receiver, it is reasonable to expect variations in the received signal. What is less reasonable to expect is that the presence of some object sufficiently far behind either the transmitter or the receiver will also have a significant impact on the readings. The experiments have shown that even some minor movement can result in a huge change of both amplitude and phase of the received signals. At the same time, it was not possible to identify a clear pattern. Sometimes, significant changes in the surrounding environment had a slight impact on the reading, and other times small changes had a tremendous effect. Further investigation has led to a conclusion than the observed behavior was due to the reflective nature of the surrounding environment.

If we consider a signal source which radiates equally in all directions on a plane, a receiver placed anywhere around it would sense the incoming signal in a simple and predictable way. But if we include some reflective objects in the environment, the interaction of the receiver and its surrounding electromagnetic field can have various forms. Figure 3.5 shows the example of such interaction, the presence of two objects introduces two reflected (indirect) components which are different in amplitudes and phases with respect to the main (direct) component. The difference in amplitude depends on the material type of the object, since different materials have different reflective properties. The difference in phase is a result of different travel distances. These components interfere with each other and the signal at the receiver is proportional to the net sum of all components. The interference can be either constructive (increasing the strength of the received signal) or destructive (decreasing the strength of the received signal), but regardless of its type, for the purposes of the presented method all outcomes are considered to be negative because the phase of the received signal is also affected.


Fig. 3.5
Impact of the reflective environment
In real world cases, especially in enclosed environments, there can be millions of such components, since the waves can be reflected over and over again until a complete fadeout. The majority of these components are too insignificant to have any noticeable impact, but some can be quite substantial. To estimate and visualize the potential influence of the environment reflectivity, a simple simulation was created using Matlab. The code of the simulation is presented in appendix D and the idea behind it is following: we simulate two signals a and $b$, where signal a is the supposed direct component of the transmission that carries useful information and signal b is the actual distorted signal influenced by the reflected components. The calculation of both signals is defined by relation

$$
\left\{\begin{array}{c}
a=\sin (x), x \in[-2 \pi, 2 \pi]  \tag{3.1}\\
b=a+\sum_{0}^{i} \text { significanceFactor } * \text { amplitude } * \sin \left(-\frac{\pi}{2}+\varphi\right)
\end{array},\right.
$$

where „amplitude" represents the strength of the reflected component, its value is generated randomly in range of [0 to 1$] ;{ }_{n} \varphi^{\prime \prime}$ is the phase value generated randomly in range of $[0$ to $\pi]$, so the resulting sinusoid is phase shifted by the angle in range of $\left[-\frac{\pi}{2}\right.$ to $\left.\frac{\pi}{2}\right]$; „"" represents the amount of reflected components; „significanceFactor" is the reflectivity coeficient of the environment, its value represents the degree of reflectivity: lower value represents lower reflectivity.

The script repeats the calculation of the phase difference between signals $a$ and $b$ for the selected number of iteration and outputs its absolute average value, regardless of the sign, as well as the visual representation of the last generated instance of both signals. Since the calculation is done using randomly generated values, each execution of the script results in a different output, but it is still possible to see how much the reflectivity of the environment affects the received signal by looking at the typical outputs of this simulation for the different values of the reflectivity coefficient.

Table $\quad$ 3.1 Simulation of the environment reflectivity impact

|  | Reflectivity coefficient of the environment |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | $1[-]$ | $0.1[-]$ | $0.01[-]$ | $0.001[-]$ |
|  | $114^{\circ}$ | $148^{\circ}$ | $28^{\circ}$ | $1.91^{\circ}$ |
|  | $56^{\circ}$ | $52^{\circ}$ | $17^{\circ}$ | $1.58^{\circ}$ |
|  | $139^{\circ}$ | $104^{\circ}$ | $11^{\circ}$ | $1.21^{\circ}$ |
|  | $64^{\circ}$ | $45^{\circ}$ | $13^{\circ}$ | $0.76^{\circ}$ |
|  | $87^{\circ}$ | $96^{\circ}$ | $7^{\circ}$ | $2.19^{\circ}$ |

The simulation results from table 3.1 show how much impact the high reflectivity of the environment can have on the accuracy of the measurement, making it practically useless for the given purpose. Considering the fact that in our case there are multiple receiving antennas which are placed at different locations, each of them should experience the effects of the reflected components of the transmitted signal in different ways. In practice it would be not possible to identify each separate effect and compensate for it, especially taking into account the fact that the reflection pattern changes together with the change of the transmitter location. Thus, it can be concluded that the presented method is not suitable for environments with high degrees of reflectivity. Potentially, there should exist a possibility to implement some protection against this phenomenon, but even if such protective measures exist, they could not be universal and are highly dependent on the specifics of the given environment. For a better understanding of the reflectivity problem more extensive testing has to be performed within a highly controlled environment.

### 3.3 Wire sensitivity and impact on phase shift

Another interesting phenomenon that has been observed during experiments is the sensitivity of wiring. For the purposes of the presented method it is necessary to measure the phase difference between two receivers, namely between two antennas. However, since the antennas have to be physically placed in different locations in space, the measured signals have to be transferred somehow to a place where they could be processed to obtain the required results. Using wires is the obvious solution, which unfortunately has one major disadvantage: the wires themselves are capable of sensing
the signal the same way as the antennas. Because the wires are also affected by the electromagnetic field coming from the transmitter, the resulting signal received by the processing unit is equal to the sum of the two components: the main component, sensed by the antenna, which carries useful phase information and the parasitic component, sensed by the wiring, which disturbs the resulting signal and changes its phase. This phenomenon can be easily observed by simply connecting the probe to the oscilloscope without any load. The probe wire will act the same way as the antenna and will be able to sense the transmitted signal.


Fig. 3.6
Wiring sensitivity

The higher the sensitivity of a wire, the greater impact it has on the resulting signal. In the initial test with oscilloscope probes have shown that the gain of a probe was in some cases higher than the gain of the antenna connected to the oscilloscope directly. Later tests with the insulated coaxial cables had shown much smaller gain, but still significant enough to cause a noticeable effect on the readings. The experiments have shown that it is necessary to mitigate this impact to its minimum. The solution in this case would be implementing the adequate shielding to block all the radiation coming to the wires and the processing unit. Figure 3.7 shows the difference in the measured signal levels: channel 2 connected to an antenna through a coaxial cable; channel 1 connected to the coaxial cable only.


Fig. 3.7
Signal levels with and without antenna

### 3.4 Interference and multiple access

The decision to work in an amateur radiolocation band is a logical solution for this project as well as for many other projects or already existing systems. This may result in a situation where the designed system is constantly receiving signals from other sources, which would act like a noise, but could not be filtered out since it has the same frequency as the signal from the correct source. In such scenario there is practically nothing that can be done to mitigate the effect of such interference. The only solution is to identify the very fact of a corrupted measurement and repeat it again. This can be done by designing the structure of the transmitted signal in such a way that it is possible to identify the beginning and the end of a transmission, similarly to the structure of a typical internet packet which includes the header, the payload and the trailer [12]. In our case the header of the message may contain the identification information related to the transmitter. The payload includes the stable part of the signal for the phase measurement purposes. The trailer should hold some kind of a checksum corresponding to the identifier. Therefore, if there is any kind of interference in the transmission medium, one of the following outcomes is possible:

| 1. Identifier not recognized | $>$ | No measurement taken | $>$ | Remain in <br> sleeping <br> mode |
| :--- | :--- | :--- | :--- | :--- |
| 2. Identifier recognized, |  |  |  |  |
| checksum not recognized |  |  |  |  |$\gg$ No measurement taken $\quad \gg$| Request new |
| :---: |
| transmission |

Such approach does not solve the interference problem, but at least it allows diminishing the possibility of the wrong measurement and incorrect instruction based on such measurement.

Using the proposed system for the detection of the position of only one transmitter is not very efficient. In the ideal case the position of an infinite amount of devices should be detectable. Of course this is not possible in practice, but it is still possible to increase this number above one. The challenge of multiple access has existed for decades and various methods have been developed and perfected. Among them the most basic ones are:

- Frequency division multiple access (FDMA) technique assigns individual channels of different frequency bands to individual users [13].
- Time division multiple access (TDMA) technique implies sharing a single carrier frequency with several users, where each user makes use of non-overlapping time slots [13].
On its own FDMA is not suitable, since the described method implies usage of one specific frequency which is bound to the physical dimensions of the receiving system. However, it still can be used for the downlink communication (from the beacon to transmitter). Downlink communication is needed for instruction transmission to whatever device whose position is measured. Because the purpose of this communication is the data transmission only, there are no requirements on the type of communication, other than a different carrier frequency, needed to avoid interference with the uplink signal. In its turn, TDMA is the valid option for the uplink communication. In TDMA a data stream is divided into frames which consist of a set of time slots. Such approach allows elimination of the interference between uplink signals coming from different devices.



## Fig. 3.8

Structure of the uplink communication
Combining the two methods should maximize the efficiency of the use of the transmission medium.

### 3.5 Phase shift measurement results

As previously mentioned the environment in which the tests were performed had quite high reflective properties. Because of this, it was possible to take some adequate measurements only in specific conditions, namely when the transmitter was close to the receiving antennas. In such situation the direct component of the transmitted signal was dominant and the effects of the reflected components were showing lesser impact on the readings, allowing observation of the behavior that matched the theoretical predictions.


Fig. 3.9
Measurement outline
The measurement procedure was similar to the calibration procedure described in chapter 2.3. The antennas were connected to the oscilloscope via the created beacon
model. The transmitter was moved around the beacon and its coordinates were noted as well as the phase difference between the two channels on the oscilloscope. Figure 3.9 shows the outline of the performed experimental procedure, while Figure E. 1 shows the one of the steps of the actual procedure.

The value of a deflection angle was calculated with respect to the center point between the two receiving antennas ([44 32]) as $\tan ^{-1} \frac{\Delta x}{\Delta y}$, where $\Delta x=x_{\text {center }}-$ $x_{\text {transmitter }}$ and $\Delta y=y_{\text {center }}-y_{\text {transmitter }}$. The value of theoretical phase shift is defined by relation
phaseShift $=360^{\circ} * \frac{(d 1-d 2)}{\text { wavelength }}$,
where d 1 and d 2 are the distances between the transmitter and each receiving antenna. The results of the measurement are shown in Table 3.2. Figure 3.10 shows the graphical representation of the dependence of the phase shift on the deflection angle, which represents the position of the transmitter, for the chosen distance between the receiving antennas.

Table 3.2 Deflection angle, theoretical and measured phase shift

| Receiver <br> Antenna 1 <br> Position | Receiver <br> Antenna 2 <br> Position | Transmitter position | Deflection angle | Theoretical phase shift value | Measured phase shift |
| :---: | :---: | :---: | :---: | :---: | :---: |
| [ $\mathrm{xy} \mathrm{y}^{\text {cm. }}$ | [ xy$] \mathrm{cm}$. | [ x y$] \mathrm{cm}$. | [ ${ }^{\text {d degrees }}$ ] | [ ${ }^{\text {d }}$ degrees] | [ ${ }^{\circ}$ degrees] |
| [27 32] | [61 32] | [4510] | -2.6 | -6.37 | -3 |
|  |  | [4010] | 10.3 | 25.32 | 40 |
|  |  | [3010] | 32.47 | 82.38 | 90 |
|  |  | [20 10] | 47.49 | 122.56 | 110 |
|  |  | [1010] | 57.09 | 144.56 | 140 |
|  |  | [1020] | 70.56 | 164.57 | 155 |
|  |  | [1030] | 86.63 | 176.76 | 170 |
|  |  | [5010] | -15.26 | -37.68 | -50 |
|  |  | [60 10] | -36.02 | -91.91 | -100 |
|  |  | [7010] | -49.76 | -127.83 | -150 |
|  |  | [70 20] | -65.22 | -154.46 | -160 |
|  |  | [70 30] | -85.6 | -176.26 | -175 |



Fig. 3.10 Dependence of phase shift between two antennas on deflection angle

The difference between the measured and theoretical values of the phase difference were mostly due to the reflective environment. Throughout the experiment the changes in the characteristics of both signals occurred in response to the slightest movements in the background. Moreover, the phase readings taken with the oscilloscope were not stable and fluctuated around values noted in table 3.2, typically plus minus 5 degrees, even when the signals remained visually stable. Hence, the overall accuracy of the measurement was quite low and this it is the main reason why such a small amount of points was measured. The main conclusion that can be made from this data is the fact that the phase difference between the signals does appear and behave according to the expectations presented in the method model. Therefore, it should be possible to base the position detection on this particular phenomenon and obtain some reasonable output. But with the given experimental results it is hard to predict the extent of inaccuracy of such method as it is directly proportional to the accuracy of the phase difference detection. For better understanding of the phenomenon itself and all factors which affect its performance, extensive testing is required. The method has to be tested in highly controlled conditions, where the effect of each harmful influence can be studied separately and in combination with other effects.

### 3.6 Accuracy of position detection as a function of distance

Even though the accuracy of the position detection technique is proportional to the accuracy of the phase shift measurement accuracy, it can change even if the latter remains constant. The ability to calculate the position of a signal source comes from the ability to calculate the geometrical deflection angle and because the measurement of this angle cannot be absolutely errorless, the introduced error, or uncertainty, of the measurement also affects the accuracy of the resulting position estimation. The uncertainty of the position calculation can be expressed as $d * \tan \theta$, where $d$ is the distance to the signal source and $\theta$ is the uncertainty of the geometrical angle measurement. So for a constant uncertainty of $1^{\circ}$, the resulting uncertainty of a position calculation would be equal to 1.7 cm . at a distance of 1 meter, while at the distance of 100 meters it would be equal to 1.7 meters. This property of the method is its main disadvantage, but also its main advantage. Assuming the situations when only a few beacons are used, the effective range of the system is limited and the accuracy of the position estimation decreases proportionally to the increase of the distance to the source. But at the same time, introducing more beacons to the system would allow to increase the effective range and make it limited only by the number of available beacons. Moreover, increasing the density of the beacons in the area will result in more accurate position detection without any other changes required.

### 3.7 Selecting the antenna

The measurement described in chapter 3.5 was performed using a 17.5 cm . custom made monopole antenna. A two mm . diameter copper wire was selected as a material and the length was chosen as a quarter of the wavelength of the signal, which is the optimal length for this type of antenna.

The initial tests were performed with the helical antenna which was included with the transmitter module. The experiments with this antenna type were not successful in terms of finding the relation between the measured phase shift at the receiving antennas and the transmitting one.

In the case of a monopole antenna it was possible to establish some conditions in which the practical measurements were matching the theoretical predictions. But with the helical antenna such conditions were not found at all. It is hard to state whether such results were purely due to the difference in the antenna shape and its radiation pattern or it was due to a difference in the created reflection interference pattern in the tested environment.

Because different types of antennas have different radiation patterns, it is reasonable to assume the resulting reflective interference pattern created in the same environment would also be different. But as it was not possible to identify the exact effect of this interference for the presented testing conditions, it is also not possible to make a clear
distinction between its influence on the phase measurements and the influence of the antenna shape. And because it was not possible to find even one condition in which the practical measurement results would match the theoretical prediction, a conclusion was made that the shape of the antenna does indeed affect the phase of the received signals in some way. Figure 3.11 shows the difference in the typical radiation patterns of the two tested antenna types: Monopole [14] and Helical [15].


Fig. 3.11 Examples of radiation patterns of the tested antennas
It can be clearly seen that the radiation pattern of a helical antenna is not uniform in all directions. Therefore, the interaction of the resulting radiation and the receiving antennas might be also different. In practice it may result in a situation when the intensities of the electromagnetic field surrounding each antenna differ in an unpredictable way and it is hard to state how it would affect the resulting phase difference. Moreover, the shape of the receiving antenna also matters. When the receiving antenna is subjected to the incoming radiation, each of its physical points interacts with the incoming field and the resulting signal sensed by the antenna is equal to the sum of all those interactions. So if the shape of the radiation wavefront is uniform and simple and the shape of the antenna is simple, then the result of the interaction should also be simple and predictable. If either one of them, or both, are of a complex shape, the interaction becomes complex and unpredictable.

The experiments were performed with both types of antennas in all possible combinations, at both the receiver and transmitter sides. Throughout all tests, the results were consistent, just different. In all cases the measured signals changed in response to changing position of the transmitter or change in the environment. If no changes in the environment occurred and the transmitter remained in the same position, the measurements remained constant, regardless of the antenna type.

The recommended antenna shape is, thus, the one with a uniform shape of radiation pattern, such as monopole or dipole.

### 3.8 Beacon design

Figure 3.12 shows the proposed outline for the beacon hardware. The structure corresponds to the design described in chapter 2.2. The need to use 6 antennas and six phase detectors is due to the limitations of available resources, mainly the phase detectors. All previously mentioned phase measurements were performed using the oscilloscope, which was not designed and calibrated for the purposes of this project and did not show good performance. As for the other available options, AD8302 is one of the very few integrated circuits which perfectly suit the needs presented in this paper due to its ability to measure phase difference between two signals at the frequencies up to 2.7 GHz .


Fig. 3.12
Beacon design: electronics
AD8302 includes a phase detector of the multiplier type with precise phase balance driven by the fully limited signals appearing at the outputs of the two logarithmic amplifiers [16]. The output of the phase measurement is a voltage scaled to 10 mV per degree. The accuracy is promised to be independent of signal level over a wide range. But the range of the measurement is limited to 0 to $180^{\circ}$, which is the exact range at which the mathematical model presented in chapter 2.3 was designed to work with. Figure 3.13 shows the dependance of the phase measurement output of the device with respect to the phase difference between two input signals [16].


Fig. 3.13
AD8302: Phase output vs. input phase difference
The characteristics show that the device outputs identical results for negative and positive phase shift values. In other words, it cannot distinguish between them and therefore the solutions described in chapter 2.2 have to be used. Which justifies usage of additional components in the proposed hardware layout. The amplifiers are needed to provide the required minimum signal level at the detector's inputs ( -60 dB ) in cases where the signal is too weak, if the transmitter is too far away for example. And the filters are required to eliminate the potential noise which may have negative a influence on the measurement accuracy.

## 4. CONCLUSION

The concept of position detection based on the measurement of the phase shift between the receivers of a signal has shown to have a complex nature. From the theoretical point of view, the potential of the presented method is great. The results obtained with the mathematical model are promising the accuracy of position detection within the boundaries of 1 meter at distances up to 200 meters. The possibility to expand the area of coverage and improve measurement accuracy by increasing the amount of reference beacons and their density is one of the biggest advantages of the method. On the other hand, the practical limitations are also quite significant. Among them, the sensitivity of the tested system to the reflectivity of the surrounding environment was the biggest issue. The tests had shown that in highly reflective environments, the phase measurement, which is crucial for the method to work, can be compromised to the extent where the method becomes practically useless. This limits the potential application areas and presents the need for some serious protective measures. Without such measures, the possibility of indoor usage seems highly unlikely. The other important requirement is the need for sufficient shielding for the electronics and wiring. It was found that unshielded wiring can sense the transmitted signal together with the antennas and create a negative interference which may affect the measured phase difference between the receiving antennas. The shape of the antenna has also presented some influence on the system response.

The thesis describes the majority of the important factors and solutions required for successful practical implementation of the method. Unfortunately it was not possible to identify the accuracy potential of the method, since the tests performed in laboratory environment were compromised by its reflective nature. The available equipment was not optimal for the purposes presented by this method and the equipment that would allow testing in outdoor conditions, namely the phase detector, was not available at the time. In general, it can be concluded that the proposed solution for the position detection problem has proven to be a viable option, but requires more testing and development.

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## SYMBOLS AND ABBREVIATIONS

| 2D | Two-Dimensional |
| :--- | :--- |
| 3D | Three-Dimensional |
| QR | Quick Response |
| GPS | Global Positioning System |
| MHz | Megahertz |
| GHz | Gigahertz |
| ASK | Amplitude Shift Keying |
| FDMA | Frequency Division Multiple Access |
| TDMA | Time Division Multiple Access |
| dB | Decibels |
| m. | Meters |
| cm. | Centimeters |
| mV | Millivolts |

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## Appendix A - Matlab simulation of the method

```
clc
clearvars
% Run the script to see the results
% defining the maximal possible phase shift (pi/portion)
portion = 2;
% signal parameters: 433.92 MHz - 69.088 cm
wavelength = 69.088;
baseDistance = wavelength/portion;
% source of the signal x.y.z coordinates in centimeters
S = [-1320 4305 -1360];
% initial coordinates of the beacons
a0 = [-2000 0 0];
b0 = [2000 0 0];
A = placeBeacon (a0, baseDistance);
B = placeBeacon (b0, baseDistance);
% coordinates of each receiver of a beacon
a1 = A{1,1};
a2 = A{1,2};
a3 = A{1,3};
a4 = A{1,4};
a5 = A{1,5};
a6 = A{1,6};
b1 = B{1,1};
b2 = B{1,2};
b3 = B{1,3};
b4 = B{1,4};
b5 = B{1,5};
b6 = B{1,6};
% distances from the signal source to each of the receivers
distanceA1 = norm(S - al);
distanceA2 = norm(S - a2);
distanceA3 = norm(s - a3);
distanceA4 = norm(S - a4);
distanceA5 = norm(S - a5);
distanceA6 = norm(S - a6);
distanceB1 = norm(S - b1);
distanceB2 = norm(S - b2);
distanceB3 = norm(s - b3);
distanceB4 = norm(S - b4);
distanceB5 = norm(S - b5);
distanceB6 = norm(S - b6);
% converting difference in distance into phase shift for each receiver
shiftA2 = pi*(distanceA2 - distanceA1)/wavelength;
```

```
shiftA3 = pi*(distanceA3 - distanceAl)/wavelength;
shiftA4 = pi*(distanceA4 - distanceAl)/wavelength;
shiftA5 = pi*(distanceA5 - distanceA1)/wavelength;
shiftA6 = pi*(distanceA6 - distanceA1)/wavelength;
shiftB2 = pi*(distanceB2 - distanceB1)/wavelength;
shiftB3 = pi*(distanceB3 - distanceB1)/wavelength;
shiftB4 = pi*(distanceB4 - distanceBl)/wavelength;
shiftB5 = pi*(distanceB5 - distanceB1)/wavelength;
shiftB6 = pi*(distanceB6 - distanceB1)/wavelength;
% simulating the signal received by each receiver
x = -2*pi:0.01:2*pi;
a1Signal = sin(x); % reference receiver for beacon A
a2Signal = sin(x + shiftA2);
a3Signal = sin(x + shiftA3);
a4Signal = sin(x + shiftA4);
a5Signal = sin(x + shiftA5);
a6Signal = sin(x + shiftA6);
b1Signal = sin(x); % reference receiver for beacon B
b2Signal = sin(x + shiftB2);
b3Signal = sin(x + shiftB3);
b4Signal = sin(x + shiftB4);
b5Signal = sin(x + shiftB5);
b6Signal = sin(x + shiftB6);
% converting phase shift into readable value (maximum value in this
case)
% a12difference - measured difference in signal = phase shift
equivalent
% beacon A, receivers 1 and 2, ...
a12difference = max(a1Signal - a2Signal);
al3difference = max(a1Signal - a3Signal);
al4difference = max(alSignal - a4Signal);
a23difference = max(a3Signal - a2Signal);
a24difference = max(a4Signal - a2Signal);
a34difference = max(a4Signal - a3Signal);
al5difference = max(a1Signal - a5Signal);
al6difference = max(alSignal - a6Signal);
a45difference = max(a4Signal - a5Signal);
%------------------------------------------
% beacon B, receivers 1 and 2, ...
b12difference = max(b1Signal - b2Signal);
b13difference = max(b1Signal - b3Signal);
b14difference = max(b1Signal - b4Signal);
b23difference = max(b3Signal - b2Signal);
b24difference = max(b4Signal - b2Signal);
b34difference = max(b4Signal - b3Signal);
b15difference = max(b1Signal - b5Signal);
b16difference = max(b1Signal - b6Signal);
b45difference = max(b4Signal - b5Signal);
```

```
% calculating the deflection angles for each beacon
% angleAalphaTemp - temporary value before the correction
angleAalphaTemp = calibrate(a12difference,baseDistance,portion);
angleAbetaTemp = 90 - calibrate(a15difference,baseDistance,portion);
angleBalphaTemp = calibrate(b12difference,baseDistance,portion);
angleBbetaTemp = 90 - calibrate(b15difference,baseDistance,portion);
% Because we only measure angle in 0-90 deg. range, we need to correct
the
% value for angles that are higher than 90 deg.
if (al3difference <= a24difference)
    angleAalpha = correctAngle(1,angleAalphaTemp);
else
    angleAalpha = correctAngle(2,angleAalphaTemp);
end
if (b13difference <= b24difference)
    angleBalpha = correctAngle(1,angleBalphaTemp);
else
    angleBalpha = correctAngle(2,angleBalphaTemp);
end
% convertion deg. to radians
alphaA = angleAalpha*pi/180;
alphaB = angleBalpha*pi/180;
betaA = angleAbetaTemp*pi/180;
betaB = angleBbetaTemp*pi/180;
% drawing lines for ech beacon
tempx1A = a0(1) - 100000*cos(alphaA);
tempy1A = a0(2) - 100000*sin(alphaA);
tempx2A = a0(1) + 100000*cos(alphaA);
tempy2A = a0(2) + 100000*sin(alphaA);
tempx1B = b0(1) - 100000*cos(alphaB);
tempy1B = b0(2) - 100000*sin(alphaB);
tempx2B = b0(1) + 100000*cos(alphaB);
tempy2B = b0(2) + 100000*sin(alphaB);
% the coordinates of the intersection point
[xi,yi] = polyxpoly([tempx1A tempx2A], [tempy1A tempy2A], [tempx1B
tempx2B], [tempy1B tempy2B]);
% final triangulation in 3-dimentions
results = triangulate(a0 ,b0 , alphaA, alphaB, xi, yi, betaA, betaB);
xProjection = results(1);
yProjection = results(2);
% correction of z-coordinate (distinguishing up and down, front and
back)
if (xProjection > 0 && yProjection > 0)
    if (a16difference < a45difference)
                zProjection = results(3);
```

```
    else
        zProjection = 0 - results(3);
    end
end
if (xProjection > 0 && yProjection < 0)
    if (al6difference > a45difference)
        zProjection = results(3);
    else
        zProjection = 0 - results(3);
    end
end
if (xProjection < 0 && yProjection > 0)
    if (a16difference < a45difference)
        zProjection = results(3);
    else
        zProjection = 0 - results(3);
    end
end
if (xProjection < 0 && yProjection < 0)
    if (a16difference > a45difference)
        zProjection = results(3);
    else
        zProjection = 0 - results(3);
    end
end
% distance from the begining of the coordinates to the signal source
Sdistance = (norm([0 0 0] - S));
% displacement error
error = norm([xProjection yProjection zProjection] - S);
disp('---------------------');
disp('Actual Position [x y z] (cm.): ');
disp(round(S,0));
disp('Measured Position [x y z] (cm.): ');
disp(round(results,0));
disp('Distance to Source (cm.): ');
disp(round(Sdistance,0));
disp('Displacement Error (cm.): ');
disp(round(error,0));
disp('-----------------------');
figure(5)
v1=[tempx1A, tempy1A,0];
v2=[tempx2A,tempy2A,0];
v=[v2;v1];
plot3(v(:,1),v(:,2),v(:,3),'r')
hold on
c1=[tempx1B,tempy1B,0];
c2=[tempx2B,tempy2B,0];
c=[c2;c1];
```

```
plot3(c(:,1),c(:,2),c(:,3),'b')
plot3(S(1),S(2),S(3),'x')
plot3(a0(1),a0(2),a0(3),'*')
plot3(b0(1),b0(2),b0(3),'*')
plot3(xProjection,yProjection,zProjection,'o')
hold off
legend('','','Actual Position','Beacon A','Beacon B','Measured
Position')
grid on
xlim([-10000 10000])
ylim([-10000 10000])
zlim([-10000 10000])
function triangulation3d = triangulate(A ,B , alphaA, alphaB, pointX,
pointY, betaA, betaB)
xprojection = (A(1) + B(1))/2;
yprojection = (A(2) + B(2))/2;
if (A(1) <= B(1) && pointX >= 0)
    xprojection = A(1) + abs((norm([A(1) A(2)] - [pointX pointY]) *
cos(alphaA))*(cos(asin(abs((A(2) - B(2))) / norm([A(1) A(2)] - [B(1)
B(2)]))))!;
end
if (A(1) <= B(1) && pointX <= 0)
    xprojection = A(1) - abs((norm([A(1) A(2)] - [pointX pointY]) *
cos(alphaA))*(cos(asin(abs((A(2) - B(2))) / norm([A(1) A(2)] - [B(1)
B(2)])))));
end
% {
else
    xprojection = B(1) + abs((norm([A(1) A(2)] - [pointX pointY]) *
cos(afi))*(cos(asin(abs((A(2) - B(2))) / norm([A(1) A(2)] - [B(1)
B(2)]))))!;
end
% }
if (A(2) < B(2))
    yprojection = A(2) + abs(B(2) - A(2)) * (abs(A(1) - B(1)) / abs
(A(1) - xprojection));
else
    yprojection = B(2) + abs(A(2) - B(2)) * (abs(B(1) - A(1)) / abs
(B(1) - xprojection));
end
if (alphaA > pi/2)
    distanceAS = norm([pointX pointY] - [xprojection yprojection])
sin(pi - alphaA);
else
    distanceAS = norm([pointX pointY] - [xprojection yprojection]) /
sin(alphaA);
end
```

```
heightS = (distanceAS * sin(betaA));
zResult = heightS;
planeAngle = asin(heightS / norm([pointX pointY] - [xprojection
yprojection]));
xResult = pointX;
yResult = pointY * cos(planeAngle);
triangulation3d = [xResult yResult zResult];
end
function correctionAngle = correctAngle(quadrant, angle)
if (quadrant == 1) % quadrant 1 = 0-90 deg.
    correctionAngle = angle;
end
if (quadrant == 2) % quadrant 2 = 90-180 deg.
    correctionAngle = 90 + (90 - angle);
end
% quadrants 3 and 4 are not utilized, as angles above 180 deg.
represent
% the same line as those below, for example: 30 deg. deflection angle
% is the same line as 210 deg.
if (quadrant == 3)
        correctionAngle = 180 + angle;
end
if (quadrant == 4)
        correctionAngle = 270 + (90 - angle);
end
end
function deflectionAngle = calibrate(phaseShift, wavelength, portion)
data = generateCalibrationData(wavelength, portion);
deflectionAngle = 0;
for j = 1:1:(length(data) - 1)
    if phaseShift >= data(j,2) && phaseShift <= data(j+1,2)
        deflectionAngle = data(j,1);
    end
end
end
function calibrationData = generateCalibrationData(wavelength,
portion)
i = 0:5:1000;
aa1 = zeros(length(i));
aa2 = zeros(length(i));
shift = zeros(2*length(i));
difference = zeros(2*length(i));
```

```
x11 = -pi:0.01:pi;
calibrationData = zeros(2*length(i),2);
for k = 1:1:length(i)
    calibrationData(k,1) = 180*atan(1000 / norm([i(k) 0 0] - [0 0
0]))/pi;
    calibrationData(length(i)+k,1) = 180*atan(norm([1000 1000-i(k) 0]
- [1000 0 0]) / 1000)/pi;
    aal(k) = abs(norm([i(k) 1000 0] - [0-wavelength/(2*portion) 0 0])
- norm([i(k) 1000 0] - [wavelength/(2*portion) 0 0]));
    aa2(k) = abs(norm([1000 1000-i(k) 0] - [0-wavelength/(2*portion) 0
0]) - norm([1000 1000-i(k) 0] - [wavelength/(2*portion) 0 0]));
    shift(k) = pi*aal(k)/(wavelength);
    shift(length(i)+k) = pi*aa2(k)/(wavelength);
    difference(k) = max(sin(x11) - sin(x11+shift(k)));
    difference(length(i)+k) = max(sin(x11)
sin(xll+shift(length(i)+k)));
    calibrationData(k,2) = difference(k);
    calibrationData(length(i)+k,2) = difference(length(i)+k);
end
end
function coordinates = placeBeacon(a, b) % a - coordinates, b -
wavelength
y1 = a(2) + b/2;
y2 = a(2) + b/2;
y3 = a(2) - b/2;
y4 = a(2) - b/2;
y5 = a(2) + b/2;
y6 = a(2) - b/2;
x1 = a(1) - b/2;
x2 = a(1) + b/2;
x3 = a(1) + b/2;
x4 = a(1) - b/2;
x5 = a(1) - b/2;
x6 = a(1) - b/2;
z1 = a(3) - b/2;
z2 = a(3) - b/2;
z3 = a(3) - b/2;
z4 = a(3) - b/2;
z5 = a(3) + b/2;
z6 = a(3) + b/2;
coordinates = {[x1 yl z1], [x2 y2 z2], [x3 y3 z3], [x4 y4 z4], [x5 y5
z5], [x6 y6 z6]};
end
```


## Appendix B - Miniature beacon structure



Fig. B. 1
Miniature beacon model (a)


Fig. B. 2
Miniature beacon model (b)


Fig. B. 3
Miniature beacon model (c)

## Appendix C - Transmitter code (Arduino)

```
// RF 433MHz
// connecting the necessary library
#include <RCSwitch.h>
// creating a transmitter object from the library
RCSwitch transmitter = RCSwitch();
void setup() {
    // start transmitting on pin D10
    transmitter.enableTransmit(10);
    pinMode(9, OUTPUT);
    data = 100; //some arbitrary data to send
}
void loop() {
    // send the data
    transmitter.send(data, 8);
    // wait data for transmission to end
    delay(50);
    digitalWrite(9, HIGH); // set the digital pin 9 on
    delay(500); // waits
    digitalWrite(9, LOW); // set the digital pin 9 off
    delay(50);
}
```


## Appendix D - Simulation of the reflectivity impact on phase

```
x = -2*pi:0.01:2*pi;
a = sin(x);
b = a;
% simulating the reflectivity nature of the surrounding environment
% recommended range: 0 to 1 (1 = high reflectivity, 0.001 = low
reflectivity)
significanceFactor = 0.01;
phaseShifts = 0;
reflectiveComponents = 50;
numberOfIterations = 100;
for j = 0:1:numberOfIterations
for i = 0:1:reflectiveComponents
    b = b + significanceFactor*randn*(sin(x + (-pi/2 + randn*pi)));
end
% calculating the phase angle of the resulting signal in range of:
% -90 to 270 deg.
if b(1) < b(2)
    angle = -180*asin((1/max(b))*b(1))/pi;
else
    angle = 180 + 180*asin((1/max(b))*b(1))/pi;
end
% recalculating the output range into: -180 to 180 deg.
if angle > 180
    angle = 360 - angle;
end
% saving only the absolute value
phaseShifts = phaseShifts + abs(angle);
end
% the average magnitude of the phase shift of the resulting signal,
with
% respect to the original signal, over the selected number of
iterations
averagePhaseShiftMagnitude = phaseShifts/numberOfIterations
% visualising the original signal
yyaxis left
plot(a);
% visualising the last instance of the reuslting signal
yyaxis right
plot(b)
```


## Appendix E - Phase measurement



Fig. E. 1
Phase measurement procedure example

