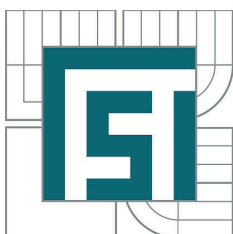# VYSOKÉ UČENÍ TECHNICKÉ V BRNĚ
BRNO UNIVERSITY OF TECHNOLOGY

FAKULTA STROJNÍHO INŽENÝRSTVÍ
ÚSTAV MECHANIKY TĚLES, MECHATRONIKY A BIOMECHANIKY

FACULTY OF MECHANICAL ENGINEERING
INSTITUTE OF SOLID MECHANICS, MECHATRONICS
AND BIOMECHANICS

APLIKACE ALGORITMŮ PRO ODHAD MODÁLNÍCH PARAMETRŮ PŘI EXPERIMENTÁLNÍ MODÁLNÍ ANALÝZE A STUDIE JEJICH CITLIVOSTI NA OKRAJOVÉ PODMÍNKY
APPLICATION OF ALGORITHMS FOR ESTIMATING MODAL PARAMETERS FOR EXPERIMENTAL MODAL ANALYSIS AND SENSITIVITY TO THE BOUNDARY CONDITION STUDY

DIPLOMOVÁ PRÁCE
MASTER'S THESIS

AUTOR PRÁCE   Bc. MATÚŠ JAKUŠ
AUTHOR

VEDOUCÍ PRÁCE   Ing. PETR LOŠÁK, Ph.D.
SUPERVISOR

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Master's Thesis Assignment

Institut:	Institute of Solid Mechanics, Mechatronics and Biomechanics
Student:	**Bc. Matúš Jakuš**
Degree programm:	Applied Sciences in Engineering
Branch:	Mechatronics
Supervisor:	**Ing. Petr Lošák, Ph.D.**
Academic year:	2015/16

As provided for by the Act No. 111/98 Coll. on higher education institutions and the BUT Study and Examination Regulations, the director of the Institute hereby assigns the following topic of Master's Thesis:

Creation and Application of Algorithms for Estimating Modal Parameters in Time Domain and Sensitivity to the Boundary Condition Study

Brief description:

Knowledge of modal parameters of system is very important. It provides information about dynamic behavior of structures which is helpful in matter of avoiding vibrations. There is lot of algorithms which allow evaluation of values of modal parameters using measurements on structure. This master thesis deals with deep examination of these algorithms and their application on measured data. Part of thesis will be dedicated to study of impact of boundary conditions applied to measured object. Conclusion will include advantages, disadvantages, possibilities and limitations of various algorithms.

Master's Thesis goals:

1) Research - Experimental modal analysis: Description, application - Algorithms used for experimental modal analysis
2) Development of algorithms for modal parameter estimation
3) Verification of algorithm functionality on real structure and verification of sensitivity to boundary conditions
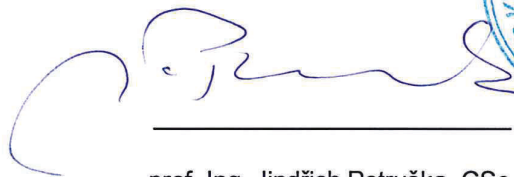4) Summary and conclusion - Possibilities and limitations of various algorithms

Bibliography:

Ewins, D. J. (1986): Modal Testing: Theory and Practic, Research Studies Press, LTD., England.

Ewins, D. J., Rao, S. S. (2002): Encyklopedia of Vibration, Academic Press, London

Students are required to submit the thesis within the deadlines stated in the schedule of the academic year 2015/16.

In Brno, 30. 11. 2015

prof. Ing. Jindřich Petruška, CSc.
Director of the Institute

doc. Ing. Jaroslav Katolický, Ph.D.
FME Dean

Abstrakt

Cieľom tejto diplomovej práce je predstavenie Experimentálnej modálnej analýzy a štúdium a použitie algoritmov pre výpočet modálnych parametrov z odmeraných vibrácií pri Experimentálnej modálnej analýze. Bodom záujmu sú predovšetkým algoritmy, pracujúce s odmeranými dátami v časovej oblasti. Diplomová práca sa zaoberá programovaním algoritmu ITD a jeho implementáciou pre Experimentálnu modálnu analýzu. Ďalšou časťou práce je štúdium citlivosti algoritmu na okrajové podmienky meranej sústavy pri výpočte modálnych parametrov a štúdium možnosti využitia algoritmu pri Operačnej modálnej analýze.

Summary

The aim of this diploma thesis is introduction of Experimental modal analysis and research and application of algorithms for modal parameter estimation using vibrations measured during Experimentl modal analysis. The point of interest is mainly algorithms working with time-domain measured data. Diploma thesis deals with programming of algorithm ITD and its implementation for Experimental modal analysis. Other part of thesis studies sensitivity to the boundary conditions of measured structure during modal parameter estimation and thesis also verifies the algorithm functionality during Operational modal analysis.

Kľúčové slová

Experimentálna modálna analýza, modálne vlastnosti, metódy výpočtu modálnych parametrov, Complex exponential, Least-Squares complex exponential, Ibrahim time domain

Keywords

Experimental modal analysis, modal parameters, mothods of modal parameter estimation, Complex exponential, Least-Squares complex exponential, Ibrahim time domain

JAKUŠ, M. *Aplikace algoritmů pro odhad modálních parametrů při experimentální modální analýze a studie jejich citlivosti na okrajové podmínky.* Brno: Vysoké učení technické v Brně, Fakulta strojního inženýrství, 2016. 63 s. Vedoucí diplomové práce Ing. Petr Lošák, Ph.D..

I declare that I have written my master's thesis on the theme of "Application of algorithms for estimating modal parameters for experimental modal analysis and sensitivity to the boundary condition study" independently, under the guidance of the master's thesis supervisor and using the technical literature and other sources of information which are all quoted in the thesis and detailed in the list of literature at the end of the thesis.

Bc. Matúš Jakuš

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1. INTRODUCTION

Historically after realization of theory background of modal analysis and applying concept of matrices the development of modal analysis went hand in hand with the development of computer technologies. Significant step forward in history of modal analysis came in 1965 when Fast Fourier Transformation was presented by J. W. Cooly and J. W. Tukey. This allowed application of experimental techniques in modal analysis using frequency domain measured data. The main principle of Experimental or Operational modal analysis is system identification from measured vibrations and that is strongly dependent on development of electrical engineering and signal processing as well.

Development of modal analysis in general these days is rapidly moving forward. Application of experimental modal analysis is usually related with getting modal parameters of the real structure and modification and improvement of mathematical model used for future simulations. It has wide range of applications in engineering and science. The first application in real industry which also meant significant improvement for modal analysis was in aeronautical engineering. Modal analysis is also being used in astronautical, automotive, civil engineering and it also found its place in bioengineering or music industry.

Modal analysis can be divided into two main categories. The first is modal analysis by analytical finite element method which calculates modal parameters of perfectly designed structure on computer with precisely defined material characteristics. The second approach is experimental or operational modal analysis of real structure which works with measured vibrations of system excited by force or harmonic or random excitation or combination of both of them. Experimental modal analysis and also Finite element method are both powerful tools because of the fast development of control techniques and improvement in the field of computer science.

Another division into to two various groups could be applied inside of experimental modal analysis and the first group is methods working with frequency domain data and the other one is methods which use time domain data. Algorithms working with time domain data were designed to improve disadvantages of methods from the first group. Another effective usage of time domain methods is in Operational modal analysis (OMA) thanks to its property to calculate modal parameters without having an informations about excitation. OMA is similar to EMA but the difference is that vibrations of structures in OMA are measured during operation of machine or structure in real conditions. That means vibrations are often produced by unknown or hardly measurable excitation.

In general, success of Experimental or Operational modal analysis requires 3 basic knowledges and skills. The first one is understanding of theoretical background, second is accurate measurement of vibrations and the third one is precise data processing.

Theoretical part of this thesis consists of description of experimental modal analysis and possible applications of algorithms used for evaluation of modal parameters from measured frequency domain data and also time domain data. This thesis is focused to study algorithms working with time domain data and to program, simulate and test Ibrahim time domain algorithm. Thesis also includes comparison with frequency domain methods and describes possibility of using algorithms for Operational modal analysis. Practical part includes verification of Ibrahim time domain algorithm using perfect data obtained from Transient structural analysis in Ansys and study of usage possibilities of this algorithm for calculation of modal parameters on real structures.

In general it is not possible to simulate cantilever with perfect constraint between structure and base and that is why algorithms in Experimental analysis need to operate in "not perfect" conditions. Thesis also includes study of algorithm sensitivity to the boundary conditions while Ibrahim time domain method deals with plate mounted to the base in three different ways and reveals how is algorithm capable of calculations with vibrational data obtained during measurements in these various conditions.

2. PROBLEM DEFINITION

The real structures in the real world are usually complicated systems. To do computer simulations, tests, development and improvements of these structures, it is necessary to estimate mathematical model, representing them. Mathematical model can be formated by using modal parameters.

Experimental modal analysis (EMA) is process where studied (real) structure is excited in laboratory conditions (for example by hit from hammer) where the main goal is to get modal parameters from measured vibrations in various points of structure (experimental data). In this process real structure can be imagined as a black box and to find the mathematical representation of this black box, algorithms working with experimental input/output data are used. Algorithms can operate with frequency domain or time domain inputs. Frequency domain methods have their advantages and also disadvantages, mainly related with accuracy of measured frequency domain data. To avoid disadvantages of these methods there is second approach developing with accordance to development of controlling systems. The other approach is called time domain algorithms.

Definition of the main subject of this thesis is evaluation of modal parameters using time domain methods within experimental modal analysis. The main goal is to program and verify algorithm working with time domain data and check the sensitivity of algorithm to boundary conditions of measured structure.

3. EXPERIMENTAL MODAL ANALYSIS

3.1. Description

Experimental modal analysis is process where internal dynamic properties such as natural frequencies, damping factors and mode shapes of system are derived using experimental data (for example vibrations excited by hit from hammer). Dynamic properties are called modal data and it is possible to create mathematical model (modal model) of system based on these data. Experimental, measured data are input data for EMA and usually they are represented by impulse response function, frequency response function or output displacements of structure caused by harmonic or random excitation.

The principle of modal analysis has strong theoretical background. The main concept is based on the fact, that vibration response of linear, time-invariant dynamic system could be expressed as a linear combination of simple harmonic motions, called natural modes of vibration. These modes are determined totally by its physical properties such as mass, stiffness and damping of structure and their spatial distribution. Every mode is described by modal parameters - natural frequency, modal damping factor and mode shape. Using other words experimental modal analysis creates the modal model from measured data or measured free vibration response data from real manufactured structure. [2]

3.2. Application

The main goal of EMA is to create modal model of dynamic system, which portrays dynamic properties of studied structure. Therefore, application of experimental modal analysis is closely related to application of modal model. Next part of thesis is devoted to description of modal model possible applications. [2]

Troubleshooting

Troubleshooting is searching for a undetermined problem in certain task plus application of improvements in order to increase efficiency of whole real process. This is the most popular use of experimental modal analysis thanks to its application as a tool, which is able to look inside of a dynamic system, only through outsource data (natural frequency, damping ratio, mode shapes). These data often uncover the roots of dynamical problems in real life. [2]

Correlation of experimental data and finite element model

Mathematical model of structure created by finite element method is usually different in comparison to model derived from measured experimental data. Real data of structure vibrations don't include all vibration modes and they describe dynamic properties of system more accurate in exact range of frequencies. Due to this fact, another application of experimental analysis is to update finite element model with experimentally gained knowledge [2]. The main goal in correlation of model extracted from experimental data and finite element model is verification of theoretical model for future calculations, for example calculations of responses for various loads. [1]

Structural modification

Term structural modification stands for mass, stiffness or damping changes in structure. Extracted modal model can be used for simulations and predictions of situations where these dynamic properties are slightly changed. For example if we already have modal model and we are aware of new mass addition in some point of structure, it is possible to derive new modal model without another complete analysis. This approach is usually useful in early stage of design preparations or in cases where engineers need to evaluate possible improvements of dynamic properties. [2]

Sensitivity analysis

Sensitivity analysis is looking for the most efficient way of physical parameters change in order to get different modal parameters. For example it means how shifted natural frequency would be if the mass in exact position of structure was changed. This analysis is useful for engineers who are trying to reach specific modal properties. [2]

Active response control

Active response control is complicated operation which requires accurate info about real structure in real time. Experimental modal analysis is suitable analysis for this application due to its real system identification option. If there is accurate modal model and suitable filter in controlling loop, it is possible to calculate proper reaction to the signal and tame the selected vibrational modes. [2]

Damage detection

The principle of using modal analysis as a tool for detection of invisible damage is comparison of modal properties obtained in various time of structure lifetime. At the beginning of operational existence, the baseline is created and "correct" modal parameters are evaluated. In the future experimental modal analysis is applied regularly on the structure and calculated parameters are compared. [2]

Reduction of mathematical model

Mathematical model of structure could be uselessly complicated for future finite element method analysis and simulations. It is necessary to make this model more simple but accordingly accurate. Experimental modal analysis can be used for this application. In case the interest in structure simulations is in lower frequency range, reduced model can be evaluated with help of informations from modal analysis. [2]

Prediction of response to force

One of the application of experimental modal analysis is prediction of vibrations to force. This application requires accurate measurement of the force during test measurement. For example the airplane can be tested under testing conditions while some forces effects wings and lately it can be predicted how would wings react for different force applied during regular or extreme maneuvers.

Force identification

Sometimes it is very hard to measure force inside of some structure but it is necessary to calculate this force precisely to avoid possible problems in different parts of structure. If we have measured vibrations and if we also somehow know modal model of the structure, it is possible to use experimental modal analysis as a tool for calculation of the force which excited measured vibrations. [2]

3.3. Practical applications of modal analysis

Practical application of experimental modal analysis is very closely connected to development of experimental techniques. This section is written to bring overview and potential of modal analysis in field of practical implementation. Three main divisions of application in industry are aeronautical, automotive, mechanical, civil and lately audio engineering.

3.3.1. Aeronautical engineering

Development of airplanes, military aircrafts and spacecrafts or satellites led to development of modal analysis in general. In this field of engineering all requirements on the list of priorities are moved behind property weight. Aircrafts are pushed to the edge when it comes to be as light as possible with certain dynamic properties. This thesis is written with intention of deep motivation for using various analysis together for optimization of Modal Analysis. Connection of Finite element method with Experimental/Operational modal analysis or connection between frequency and time domain algorithms and possibility of using them together. First articles about correlation between Finite element methods with Experimental modal analysis are from 1980's and the history began in aeronautical industry.

3.3.2. Automotive engineering

Study and improvement of dynamic properties of cars is highly required these days. Typical simple application of Experimental modal analysis is Modal analysis of body, chassis or some sub-connected structure inside of whole system. More refined application is sensitivity analysis and optimization of structure for increasing comfort, materials lifetime and improving driving experience. New techniques, for example adaptive suspension are being developed in close relation to Modal analysis especially in the phase of testing.

The troubleshooting, described in section 3.2 plays the key role in study of vehicle noise and vibration harshness (NVH). Problems with noise either outside or inside the cabin are related with modal properties of body or structure. For example resonance of the door in certain speed of complicated structure could be hidden in modal analysis using Finite element method but it could be predicted easily during tests with usage of Experimental modal analysis.

To avoid all the problems or to improve dynamic properties Experimental modal analysis is being often used in automotive industry especially in early phases of designing, when awareness of possible problems is welcome and changes are relatively cheap.

3.3.3. Other practical applications

Modal testing was originally developed for aerospace industry but it found its place in various engineering fields. The next practical implementation is in civil engineering when prediction of response of construction for various excitations, for example strong wind, waves or earthquakes, could be beneficial addition to design phase. Experimental modal analysis is nowadays being used in audio industry as well, while moving the borders of quality inside of speakers or musical instruments.

3.4. Algorithms for EMA

In general algorithms for EMA are divided into two categories. The first group consists of frequency domain algorithms, which process signals transformed using Fourier transformation into frequency domain signals (FRF). It is very common these days to extract modal parameters from frequency domain signals which can be displayed in various forms. Frequency domain algorithms are based on the theory to fit the curve of FRF data using predefined mathematical model. Basic theory about errors resulting from these methods is described in subsection 3.4.1. The imperfection of this class of methods led scientists to start searching for different approaches, which could process signals directly from time domain. The second group is called time domain algorithms.

3.4.1. Frequency domain algorithms

As mentioned earlier Frequency domain methods use curve fitting of FRF data using predefined mathematical model of measured construction to extract modal parameters. This method is sometimes called gray box approach due to the fact that inputs and outputs of the system are known and some parameters of mathematical model are predicted before curve fitting. Predicted parameters are amount of degrees of freedom of structure, damping type and amount of vibrational modes from frequency range data were measured.

Precision

The precision of analysis is complex problem. It should be based upon the fact how precisely is FRF curve fitted. In other words more precise FRF means more precise curve fitting. The success of curve fitting is evaluated as a comparison of computed error (difference between FRF and fitted curve) with desired small error. This theory is correct only if correct predicted parameters of structure were included inside of a gray box approach.

Amount of vibrational modes

The third predicted parameter is very important to evaluate but not the easiest one. During modal analysis (frequency domain) the information about amount of vibrational modes in frequency range of FRF data is required. Suitable analysis for this prediction is to use FRF in form where frequency is represented on one axis of graph and modulus on the second. This representation highlights resonance peaks.

The first very often problem in reality which should be kept in mind is that not every mode is shown on FRF. The second one is that not every peak necessarily represents the

mode we are interested in (usually caused by noise in signal). This problem is more often that one could see from the first sight and the reason is that constructions or systems usually include parts that are meant to be the same (on drawing). However in reality it is not possible to create them 100% identically. That puts two modes on FRF very close to each other and makes them look like a single mode.

To avoid this situation it could be helpful to transform FRF into dB scale (figure 3.1) which cause the vibrational modes are on the FRF "more visible". As an addition Phase plot could be used because theoretically every vibrational mode switches the phase by 180°. However this plot is very sensitive to noise. Different option is to check how are FRF data displayed on Nyquist plot. Circular part of the plot stands for mode shape and straight line represents false resonance peak. Combination of all these approaches should theoretically lead to the most accurate results of prediction.
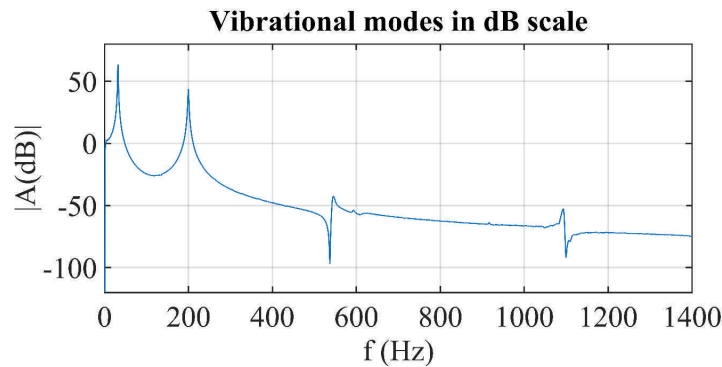
Figure 3.1: FRF in dB scale

SDoF methods

All single degree of freedom methods splits FRF data and process the part including one vibrational mode (resonance peak and its near vicinity). Effect from the rest of vibrational modes is either neglected (Peak picking method) or it is including in calculation (Circle fit method) which has significant influence on precision of results. Principles of these two commonly used methods and their advantages and disadvantages are described below.

SDoF methods - Peak picking method

Peak picking method (half-power method) is very the very simplest frequency domain method. This approach absolutely doesn't include effects from other vibrational modes in FRF and it uses FRF data as the data from simple SDoF system. Procedure of extracting modal parameters consists of three main steps: evaluation of natural frequency, damping ratio and modal constants (mode shapes). Natural frequency is simply taken from the peak on the modulus-frequency graph. Next procedure is evaluation of damping ratio as follows [5]:

$$\eta_r = \frac{\omega_b^2 - \omega_a^2}{2 \cdot \omega_r^2} \approx \frac{\omega_b - \omega_a}{\omega_r}$$

or

$$\zeta_r = \frac{\omega_b^2 - \omega_a^2}{4 \cdot \omega_r^2} \approx \frac{\omega_b - \omega_a}{2 \cdot \omega_r}$$

where η_r represents damping ratio, ω_r is natural frequency (frequency in the peak) and ω_b, ω_a stands for frequencies taken from the peak in height of amplitude $\frac{\alpha_{max}}{\sqrt{2}}$, where α_{max} is maximum amplitude of the peak. Points ω_b, ω_a are also called "half-power points". ζ_r represents viscous damping ratio. In other words damping ratio in peak picking method is simply calculated from the width of resonance peak in the height of half-power points. For perfect understanding of SDoF FRF methods, let's assume we have data from vibrations in 5 points of simple plate. Also let's assume we measured data in frequency range which we expect to contain 4 vibrational modes (3rd predicted parameter). It means there are 4 peaks in every measured FRF.

Last piece of puzzle is to obtain modal constant from every point of measured vibrations for these 4 vibrational modes. All 5 modal constants for the same vibrational mode (mode with the same natural frequency) creates the mode shape or in other words these constants creates the representation how is plate vibrating. Constants could be calculated using formula where amplitude of peak depends on modal constant:

$$\alpha_{max} = \frac{A_r}{\eta_r \cdot \omega_r^2}$$

where A_r is modal constant.

$$A_r = \alpha_{max} \cdot \eta_r \cdot \omega_r^2$$

Advantages of this method come out of the fact that it is very simple to understand and fast. Practical use is for educational purpose as it gives very simple and easily understandable background of modal analysis for future deepen studies. However method is highly dependent on measured FRF data which is very hard to measure accurately. Damping is calculated only from half-power points and method is not noise-proof. Method is useful for lightly damped systems with well separated vibrational modes.

SDoF methods - Circle fit method

The principle of Circle fit method is based upon the circularity of the Nyquist plot which is simply another form of representation of FRF data. Circle fit method is the most often used SDoF modal analysis method thanks to its simplicity and significantly improved accuracy compared to Peak picking method. Amplitude of the FRF $\alpha(\omega)$ in the Nyquist plot is split into Real and Imaginary part of complex number and curve fitting model can be described as follows:

$$\Big(Re(\alpha)\Big) + \Big(Im(\alpha) + \frac{1}{2 \cdot h}\Big)^2 = \Big(\frac{1}{2 \cdot h}\Big)^2$$

where h is general parameter of curve fitting model. This is useful property because as it is mentioned in following text, this approach can include influence of other vibrational modes in separated SDoF peak. Explanation how Circle fit method includes other vibrational modes while estimating results from Nyquist plot comes out of the properties of the complex numbers. Amplitude of FRF of N-DoF system could be described as:

$$\alpha_{jk}(\omega) = \sum_{r=1}^{N} \frac{{}_rA_{jk}}{\omega_r^2 - \omega^2}$$

r-th mode (SDoF peak) can be separated:

$$\alpha_{jk}(\omega) = \frac{{}_rA_{jk}}{\omega_r^2 - \omega^2} + \sum_{s=1;s\neq1}^{N} \frac{{}_sA_{jk}}{\omega_s^2 - \omega^2}$$

It is possible now to replace the final part with sum with complex constant B_{jk}:

$$\alpha_{jk}(\omega) = \frac{{}_rA_{jk}}{\omega_r^2 - \omega^2 + j \cdot \eta_r \cdot \omega_r^2} + B_{jk}$$

Complex constant B_{jk} represents influence of other vibrational modes. Constant doesn't change the circularity of Nyquist plot but it moves the circle further from origin. Derivation of modal parameters comes in the same order that during Peak picking method (natural frequency, damping ratio, modal constant) but accuracy is significantly increased.

3.4.2. Time domain algorithms

Time domain methods uses response data with time represented on first axis and amplitude on the second. In comparison to FRF methods there are some advantages and

disadvantages. First of all FRF data are not "clean" data. They are results of transformation of measured vibrations in time with certain time history. That brings the first theoretical advantage, accuracy.

While using FRF data it is possible to increase accuracy by using informations about known excitation. However that is not always easy to measure. As long as the experiment is being measured in real conditions, outside laboratory, it requires expensive equipment to measure this kind of excitation. Time domain methods are powerful tool if the only excitation is from ambient sources because they can operate with only output responses. When it comes to disadvantages in comparison to FRF methods, time domain algorithms require more significant different in noise/signal. This diploma thesis includes close look into the most popular time domain modal parameter estimation methods - CE, LSCE and ITD.

Complex exponential method (CE)

Complex exponential algorithm operates with impulse response function data from 1 trajectory signal, measured in exact position of structure. IRF has form as:

$$x_i(t) = \sum_{k=1}^{2n} \psi_{ik} e^{s_k t}, \quad i = 1, ..., n \tag{3.1}$$

where:

$x_i(t)$ - impulse response function (displacement) in point i of structure
ψ - constitute desired mode shapes
s_k - provides the natural frequency and the damping ratio

For describing theory background of CE, we assume that impulse responses are sampled with length $2n$ and we get equation (3.2) [11], [4]:

$$x_i(t_0) = \sum_{k=1}^{2n} \psi_{ik}, \quad x_i(t_1) = \sum_{k=1}^{2n} \psi_{ik} e^{s_k \Delta t}, ..., \quad x_i(t_{2n}) = \sum_{k=1}^{2n} \psi_{ik} e^{s_k 2n \Delta t} \tag{3.2}$$

or

$$x_i(t_0) = \sum_{k=1}^{2n} \psi_{ik}, \quad x_i(t_1) = \sum_{k=1}^{2n} \psi_{ik} V_k, ..., \quad x_i(t_{2n}) = \sum_{k=1}^{2n} \psi_{ik} V_k^{2n} \tag{3.3}$$

where $V_k = e^{s_k \Delta t}$.

Left side of equation (3.3) is known, right side (V_k and ψ_{ik}) is unknown. To solve this problem, CE algorithm uses Prony's technique developed by Gaspard Riche de Prony in

1795. Prony's method says, that if V_k is expected to be in complex conjugate pairs form, then there exists polynomial in form:

$$\beta_0 + \beta_1 V_k + \beta_2 V_k^2 + ... + \beta_{2n} V_k^{2n} = 0 \tag{3.4}$$

where β represents real coefficients. If both sides of equation (3.3) are multiplied with real coefficients $\beta_0, ..., \beta_{2n}$, we will get:

$$\sum_{j=0}^{2n} \beta_j x_i(t_j) = \sum_{j=0}^{2n} \left(\beta_j \sum_{k=1}^{2n} \psi_{ik} V_k^j \right) = \sum_{k=1}^{2n} \left(\psi_{ik} \sum_{j=0}^{2n} \beta_j V_k^j \right) \tag{3.5}$$

and comparing equations (3.4) and (3.5), we can deduce following formula:

$$\sum_{j=0}^{2n} \beta_j x_i(t_j) = 0 \tag{3.6}$$

which in matrix form looks like:

$$
\begin{bmatrix}
x_i(t_0) & x_i(t_1) & \cdots & x_i(t_{2n}) \\
x_i(t_1) & x_i(t_2) & \cdots & x_i(t_{2n+1}) \\
\vdots & \vdots & \ddots & \vdots \\
x_i(t_{2n-1}) & x_i(t_{2n}) & \cdots & x_i(t_{4n-1})
\end{bmatrix}
\begin{bmatrix}
\beta_0 \\
\beta_1 \\
\vdots \\
\beta_{2n}
\end{bmatrix}
= 0
$$

If β_{2n} is set as 1, the rest of β vector can be calculated as a ratio to referenced β_{2n}:

$$
\begin{bmatrix}
x_i(t_0) & x_i(t_1) & \cdots & x_i(t_{2n-1}) \\
x_i(t_1) & x_i(t_2) & \cdots & x_i(t_{2n}) \\
\vdots & \vdots & \ddots & \vdots \\
x_i(t_{2n-1}) & x_i(t_{2n}) & \cdots & x_i(t_{4n-2})
\end{bmatrix}
\begin{bmatrix}
\beta_0 \\
\beta_1 \\
\vdots \\
\beta_{2n-1}
\end{bmatrix}
= -
\begin{bmatrix}
x_i(t_{2n}) \\
x_i(t_{2n+1}) \\
\vdots \\
x_i(t_{4n-1})
\end{bmatrix}
$$

After evaluating β coefficients, it is possible to get roots of polynomial from equation (3.4) and that provides us with complex conjugate pairs V_k. CE method counts using

only single response from one sensor, located in exact position of structure. The modal residues in this position can be solved using equation (3.7).

$$
\begin{bmatrix}
1 & 1 \cdots & 1 \\
V_1 & V_2 & \cdots & V_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
V_1^{2n-1} & V_2^{2n-1} & \cdots & V_{2n}^{2n-1}
\end{bmatrix}
\begin{bmatrix}
\psi_{i1} \\
\psi_{i2} \\
\vdots \\
\psi_{i2n}
\end{bmatrix}
=
\begin{bmatrix}
x_i(t_0) \\
x_i(t_1) \\
\vdots \\
x_i(t_{2n-1})
\end{bmatrix}
\tag{3.7}
$$

Mode shapes can be evaluated by combining modal residues from various single output responses.

Least-Squares complex exponential method

Least-Squares complex exponential method (LSCE) is extension of CE method. While CE algorithm operates with only single output responses, LSCE uses multiple signals at once. Form of modified matrices are shown in equation 3.8.

$$
\begin{bmatrix}
1 & 1 \cdots & 1 \\
V_1 & V_2 & \cdots & V_{2n} \\
V_1^2 & V_2^2 & \cdots & V_{2n}^2 \\
\vdots & \vdots & \ddots & \vdots \\
V_1^{2n-1} & V_2^{2n-1} & \cdots & V_{2n}^{2n-1}
\end{bmatrix}
\begin{bmatrix}
\psi_{11} & \psi_{21} & \cdots & \psi_{2n1} \\
\psi_{12} & \psi_{22} & \cdots & \psi_{2n2} \\
\vdots & \vdots & \ddots & \vdots \\
\psi_{12n} & \psi_{22n} & \cdots & \psi_{2n2n}
\end{bmatrix}
$$

$$
=
\begin{bmatrix}
x_1(t_0) & x_2(t_0) & \cdots & x_{2n}(t_0) \\
x_1(t_1) & x_2(t_1) & \cdots & x_{2n}(t_1) \\
\vdots & \vdots & \ddots & \vdots \\
x_1(t_{2n-1}) & x_2(t_{2n-1}) & \cdots & x_{2n}(t_{2n-1})
\end{bmatrix}
$$

or

$$
V\Phi^T = \check{X}
\tag{3.8}
$$

where Φ is complex mode shape matrix.

Ibrahim time domain method (ITD)

History of ITD method dates back in 1973, when the algorithm was introduced by Mr. Ibrahim [3]. To estimate modal parameters this method operates with Impulse response data, gained as a displacement of the structure in various positions. Theoretical back-

ground of this algorithm is based on equation 3.9, which is equation of motion for an n-degree of freedom vibration system [11].

$$M\ddot{x} + C\dot{x} + Kx = 0 \tag{3.9}$$

where M, C, K are mass, damping and stiffness matrices with dimensions n x n. Solution to this motion equation can be expressed in the same form as formula (3.1):

$$x_i(t) = \sum_{k=1}^{2n} \psi_{ik} e^{s_k t}, \quad i = 1, ..., n \tag{3.10}$$

where as was mentioned earlier $x_i(t)$ represents trajectory of i-th structure point in time t, ψ constitute desired mode shapes and s_k includes informations about natural frequency and damping ratio. The original ITD was in article [3] described using signals of length $2n$ (n is the system's degree of freedom). For example if the structure with 5 degrees of freedom is determined, algorithm takes only 10 samples from signal. Due to this fact it is highly recommended to sample signal carefully. Method was later improved in order to usage of longer periods of signal than $2n$. This improvement is described later in equation 3.17. To continue with description of ITD algorithm from equation 3.10 we get:

$$
\begin{bmatrix}
x_1(t_1) & x_1(t_2) & \cdots & x_1(t_{2n}) \\
x_2(t_1) & x_2(t_2) & \cdots & x_2(t_{2n}) \\
\vdots & \vdots & \ddots & \vdots \\
x_n(t_1) & x_n(t_2) & \cdots & x_n(t_{2n})
\end{bmatrix}
$$

$$
=
\begin{bmatrix}
\psi_{11} & \psi_{12} & \cdots & \psi_{12n} \\
\psi_{21} & \psi_{22} & \cdots & \psi_{22n} \\
\vdots & \vdots & \ddots & \vdots \\
\psi_{n1} & \psi_{n2} & \cdots & \psi_{n2n}
\end{bmatrix}
\begin{bmatrix}
e^{s_1 t_1} & e^{s_1 t_2} & \cdots & e^{s_1 t_{2n}} \\
e^{s_2 t_1} & e^{s_2 t_2} & \cdots & e^{s_2 t_{2n}} \\
\vdots & \vdots & \ddots & \vdots \\
e^{s_{2n} t_1} & e^{s_{2n} t_2} & \cdots & e^{s_{2n} t_{2n}}
\end{bmatrix}
$$

or simply

$$X = \tilde{\Psi} e^{st} \tag{3.11}$$

where X represents trajectory in time from n - displacement sensors. Then we use same output signals but shifted by certain sampling time interval (τ in equation 3.12) and later we again use this output signals but shifted by 2τ (3.13).

$$\begin{bmatrix} x_1(t_1+\tau) & x_1(t_2+\tau) & \cdots & x_1(t_{2n+\tau}) \\ x_2(t_1+\tau) & x_2(t_2+\tau) & \cdots & x_2(t_{2n+\tau}) \\ \vdots & \vdots & \ddots & \vdots \\ x_n(t_1+\tau) & x_n(t_2+\tau) & \cdots & x_n(t_{2n+\tau}) \end{bmatrix}$$

$$= \begin{bmatrix} \psi_{11} & \psi_{12} & \cdots & \psi_{12n} \\ \psi_{21} & \psi_{22} & \cdots & \psi_{22n} \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{n1} & \psi_{n2} & \cdots & \psi_{n2n} \end{bmatrix} \Lambda \begin{bmatrix} e^{s_1 t_1} & e^{s_1 t_2} & \cdots & e^{s_1 t_{2n}} \\ e^{s_2 t_1} & e^{s_2 t_2} & \cdots & e^{s_2 t_{2n}} \\ \vdots & \vdots & \ddots & \vdots \\ e^{s_{2n} t_1} & e^{s_{2n} t_2} & \cdots & e^{s_{2n} t_{2n}} \end{bmatrix}$$

or

$$X_\tau = \tilde{\Psi}\Lambda e^{st} \tag{3.12}$$

where:

$$\Lambda = \begin{bmatrix} e^{s_1 \Delta t} & 0 & \cdots & 0 \\ 0 & e^{s_2 \Delta t} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{s_{2n} \Delta t} \end{bmatrix}$$

$$X_{2\tau} = \tilde{\Psi}\Lambda^2 e^{st} \tag{3.13}$$

Algorithm continues with combination of equations 3.11 - 3.13 as follows:

$$\begin{bmatrix} X \\ X_\tau \end{bmatrix} = \begin{bmatrix} \tilde{\Psi} \\ \tilde{\Psi}\Lambda \end{bmatrix} e^{st}, \quad \bar{X} = \Phi e^{st} \tag{3.14}$$

and

$$\begin{bmatrix} X_\tau \\ X_{2\tau} \end{bmatrix} = \begin{bmatrix} \tilde{\Psi}\Lambda \\ \tilde{\Psi}\Lambda^2 \end{bmatrix} e^{st}, \quad \hat{X} = \Phi\Lambda e^{st} \tag{3.15}$$

It is possible to eliminate e^{st} by modifying equations 3.14 and 3.15 into formula:

$$\hat{X}\bar{X}^{-1}\Phi = \Phi\Lambda \qquad (3.16)$$

where $\hat{X}\bar{X}^{-1}$ are known matrices which describe dynamical properties of structure, upper half of matrix Φ is the mode shape matrix and Λ contains natural frequencies and damping ratios. Equation 3.16 can be solved using eigenvalue decomposition and that calculates all modal parameters. As was mentioned earlier in this section, it is possible to manipulate equation 3.16 and use time periods of signal longer than $2n$ (equation 3.17) [11].

$$(\hat{X}\bar{X}^T)(\bar{X}\bar{X}^T)^{-1}\Phi = \Phi\Lambda \qquad (3.17)$$

or

$$\bar{A}\Phi = \Phi\Lambda$$

Natural frequency and damping ratio of r-th mode is calculated from diagonal matrix Λ using following formulas:

$$f_r = \left(\frac{1}{\Delta}\sqrt{ln(\Lambda_r)ln(\Lambda'_r)}\right)/(2\pi) \qquad (3.18)$$

$$\zeta_r = \frac{-ln(\Lambda_r\Lambda'_r)}{4\pi f_r\Delta} \qquad (3.19)$$

where Λ'_r represents complex conjugate number to Λ_r. Elements inside of matrix Λ are in form:

$$\Lambda_r = e^{s_r\Delta t} = \alpha + j\cdot\beta$$

Knowing this knowledge there is also another mathematical approach how to calculate natural frequency and damping ratio of r-th mode during analysis by Ibrahim time domain method. This approach is described in following equations 3.20 - 3.23 [2].

$$n_r = -\frac{1}{2\cdot\tau}\cdot ln(\alpha_r^2 + \beta_r^2) \qquad (3.20)$$

$$\omega_{dr} = \frac{1}{\tau} tan^{-1} \frac{\beta_r}{\alpha_r} \tag{3.21}$$

$$\omega_r = \sqrt{\omega_{dr}^2 + n_r^2} \tag{3.22}$$

$$\zeta_r = \frac{n_r}{\omega_r} \tag{3.23}$$

3.4.3. Time domain algorithms for OMA

Experimental modal analysis have some limitations. First of all to evaluate modal parameters during EMA the measured data are FRF or IRF. That is very hard to measure in the field test or for large structures. Secondly EMA is processed in laboratory and operation conditions in industry may significantly differ. Another example of limitation of EMA is simulation of boundary conditions in laboratory. This limitations led to development of Operational modal analysis (OMA) in early 1990's mainly within civil engineering during modal analysis of buildings, towers, bridges. Lately OMA became popular in aerospace and mechanical engineering because of its advantages as cheap and fast measuring or unnecessary of simulation of boundary conditions [10].

Operational modal analysis (OMA) is analysis of structure in operational conditions. That means it was created respectively to situations where system is excited by random noise. In reality this random noise often includes harmonic vibrations. Modal parameters should be computed without knowing excitation which brings to the scene time domain methods. These methods are valuable in this case because of there improved accuracy in comparison with frequency domain methods in cases when excitation is unknown. Algorithms work only with data of responses and basic assumptions, for example that mode shapes are expected to have maximum at the actual natural frequency etc. [9]

OMA is analysis when structure is excited by itself or by outer random forces during operation and that is the main source of vibrations. As mentioned earlier there are often presented harmonic excitations for example source of excitation could be rotation of wheels in car or impact from waves to the ship in the ocean. It is important to deal with these harmonic frequencies and process them because on results from analysis they show as vibrational modes with very low damping.

For automatic filtration of these modes is necessary condition good separation of these harmonic excitation from natural frequencies of structures. Time domain methods in OMA assume that harmonic frequencies from excitation are known. They could be estimated from nature of machine or environment or from pre-processing of measured vibrations [7]. Limitation of time domain methods still occurs when excitation of structure is white noise, which is random noise with constant power spectral density.

4. PROBLEM ANALYSIS

The main subject of this thesis is to study time domain methods used for evaluation of modal parameters within experimental modal analysis. The reason why time domain methods are interesting are inaccuracies in commonly used frequency domain methods while testing real structures. Uncertainties in frequency domain algorithms are mainly caused by complicated measurement of precise frequency response function which is the most important part in this type of analysis. By using time domain methods these uncertainties should be theoretically overcome and based upon theory part of this thesis these methods appear to be suitable addition within Experimental modal analysis.

Advantage of time domain methods is that they avoid Fast Fourier Transformation which is very important part of preciseness in frequency domain methods. However it is important to be aware of the fact that time domain methods also have their limitations and boundaries. The first problem is necessity of having prime knowledge and assumptions about studied system. These assumptions are later huge help in searching the correct vibrational modes in structure. Another limitations is noise inside the measured data which are inputs into time domain algorithms. Based upon the research in theory part it is expected that noise could cause inaccuracies in modal parameter estimation. This thesis deals with Ibrahim time domain (ITD) method and verifies functionality of this algorithm in various possible conditions and that should reveal advantages and disadvantages of using this specific approach within Experimental modal analysis.

This thesis also includes comparison of evaluated modal parameters by time domain algorithm with modal analysis results from finite element method. Designing and analysis in FEM method was proceeded using Ansys Workbench (release 15 and 16.2) with student license where calculations of structural physics are limited by size of 32000 nodes/element. To create algorithm was used software Mathworks - Matlab 2014b.

During practical part of thesis, the tested structure was cantilever made of steel in laboratory of Mechatronics department in Brno University of Technology. To measure acceleration in different points was used accelerometer PCB Piezotronics, model 352C03 with sensitivity 10.20 mV/g (1.040 $mV/m/s^2$). Quality of the measurements of this kind strongly depends on quality of cable transferring data from sensor to data acquisition card.

5. VERIFICATION OF ITD ALGORITHM FUNCTIONALITY

The first task in practical part of this diploma thesis was to program algorithm using Matlab and evaluate structure's modal parameters from "perfect" output data (displacement) obtained from transient analysis in Ansys Workbench 15.0 and later 16.2 (with academic license). In this context word perfect means that output data are without noise. First section in this chapter verifies algorithm for data obtained from simulation of vibrations of simple beam. This section also includes verification of algorithm for using in Operational modal analysis and calculation of damping. The results were compared with modal analysis results calculated in Ansys Workbench as a representative of Finite element methods. Next section verifies algorithm on plate (cantilever) to test possibility of finding torsion modes inside. Last section is verification of algorithm functionality on real structure and verification of sensitivity to boundary conditions. Real measured structure was 500 mm long plate made of steel. Sensitivity to boundary conditions was verified by evaluation and comparison of results for three different measurements. During the first measurement was cantilever attached to the basement as strong as possible (figure 5.6). In our case the first measurement stands for the best conditions for evaluation of modal parameters during experimental modal analysis. The second measurement was cantilever connected to the basement with torque approximately 10 Nm on every screw. Last measurement was set with piece of plastic block stuck inside of constraint (figure 5.11). The last condition was expected to be the worst case scenario from described three examples for evaluation of modal parameters.

It was not possible to hit the structure every time with the same force while testing. Important part of verification was verification of possibility to use only one accelerometer for whole process of measurement and re-calculate vibrations with respect to hammer hit force. This task was simulate in Transient structural analysis while force of the simulated hit was changed for every measurement in various points of structure. Vibrations were later modified to vibrations expected from normalized force. It was tested that it is sufficiently precise to re-calculate vibrations with linear relationship between displacement data and force. In other words if the force maximum was 5 times stronger than chosen normalized value, then normalized displacement in selected point was 5 times amplified. One approach (sufficient enough) of solving this issue is to use maximum of measured force and the other approach is to use area under the curve of measured hammer hit force in time. This part was necessarily applied in pre-processing part of the code, calculating modal parameters using ITD method.

5.1. Verification using perfect output data (beam)

5.1.1. Ansys Workbench - beam designing and analysis

This part of thesis deals with simple structure, beam (figure 5.1). To obtain output data for testing of future algorithm, Ansys Workbench was used.

Engineering data

The best way to simulate time-variant behavior of 5 DOF beam structure is transient structural tool. In Ansys Workbench this analysis requires lot of computational time and time step (frequency of sampling) is limited. Generally, the time step should be less than $\frac{1}{20 \cdot f_{max}}$, where f_{max} is the highest frequency interested. This fact led to the setup of a new "Test material", which is more "mellow" than steel. Characteristics of new material are shown in table 5.1.

Table 5.1: Test material characteristics

Density	3000 kg/m^3
Isotropic Elasticity	
- Young's Modulus	$3 \cdot 10^8 \ Pa$
- Poisson's Ratio	0.3

Geometry

Structure was designed in Ansys Design Modeler. To create geometry of beam with 5 degrees of freedom following steps were taken:

1. Placement of 6 points to the locations of specified coordinations (50 mm distance between points). This step was taken mainly for possibility to get transient analysis in each point
2. Connection of points using line (Concept - Lines from points, Operation Add Frozen)
3. Modeling of cross section (Concept - Cross Section). Cross section is rectangle with dimensions 3 x 20 mm
4. Creation of Part (Line Body). Cross section is rectangle prepared in former step

Modal analysis

Outputs of modal analysis which are the points of our interest are mode shapes and natural frequencies when these shapes are excited. First thing to set is material of examined structure in Material properties of line body (Details - Material - Assignment). Next step is addition of boundary condition. Fixed support in one end of the structure was chosen. Results of modal analysis by Ansys are revealed on figure 6.1 and table 6.1.
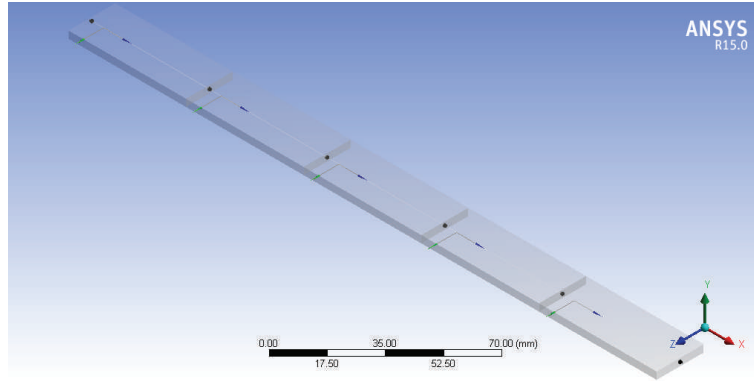
Figure 5.1: Beam - geometry

Transient structural analysis

Transient analysis was executed for testing ITD algorithm. Result of transient structural analysis is displacement in desired points (in our cas in 5 points) of structure while structure is excited by hammer (force) and vibrating for certain time. These results were prepared as an inputs into the algorithm of ITD method. Time step was set as low as possible due to the interest in as many as possible mode shapes while testing the algorithm. Whole process is set in Analysis Settings. Number of steps was set to 3. The first two steps represents the hit by hammer and last one is vibration. Auto Time Stepping was Off and Time Step was set to $5 \cdot 10^{-4}$ s. In this part of setup was was added force which stands for hammer hit. Time settings of Transient Analysis is shown in table 5.2.

Table 5.2: Transient Structural Analysis - Time Settings

Step	End Time [s]	Force [N]
$1st$	0.1 s	-0.2 N
$2nd$	0.1006 s	0 N
$3rd$	5 s	0 N

To measure displacement of our 5 point in time Solution option "Directional Deformation" was added. It is necessary to create Directional Deformation for every point of interest and set up correct geometry (Vertex). Orientation option in Details was chosen same as orientation of force (hammer hit).

5.1.2. Algorithm ITD

Program was composed of 4 main parts:

1. Data Read - reads data from folder
2. Pre-processing - crops data of vibration
3. ITD - main algorithm ITD with calculation of natural frequencies
4. Data Plot - plots mode shapes

ITD Algorithm was created exactly in accordance to article [11] which is described in chapter 3 in section 3.4.

Pre-processing

Pre-processing or data cropping was important part of program because algorithms in experimental modal analysis are sensitive to input data. In general the longer vibration time was used the better results were evaluated. Other important factor was time-sampling. Experience in tuning algorithm has shown that it is good to calculate time-step in accordance to number of mode shapes we are interested in. In this test was used time-step from Transient Structural Analysis $5 \cdot 10^{-4}$ s. Example of pre-processed data which were used for the test of ITD algorithm (data from 3rd sensor) is shown on figure 5.2.
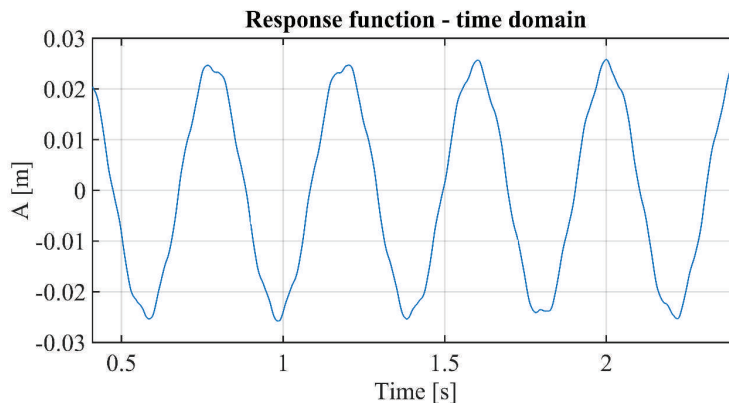

Figure 5.2: Pre-processed data for ITD

ITD

First step in algorithm is filling matrix of displacements and other two matrices ($X, X_\tau, X_{2\tau}$ is equal to X_1, X_2, X_3 in program) where displacement is shifted by selected shift time (delta Δ). Program continues with combining shifted displacement vectors and creating matrices $\bar{X}, \hat{X}$ as described in section 3.4.2 - Ibrahim time domain method (ITD).

Next step is eigenvalue decomposition, modified for signals containing more than $2n$ components (n is amount of measurement points on the structure). Result of eigenvalue decomposition are two matrices. The first one includes mode shapes inside and the second one is diagonal matrix which contains informations about natural frequencies and damping ratios of single modes. ITD function is shown in listing 5.1.2.

```
% function ITD
%
% outputs:   MS - Modal shape matrix
%            D - f_r matrix
% inputs:    X - trajectory matrix from n-channel sensor
%            N - amount of loaded samples
%            d - delta (shift in ITD)

function [ MS, D ] = ITD( X, N, d )

% size of input matrix
sizeX = size(X);

% cycle while: shifting input displacement data
i = 1;
while i <= sizeX(1,1)
    X_1(i, :) = X(i, 1 : N);
    X_2(i, :) = X(i, 1+d : N+d);
    X_3(i, :) = X(i, 1+2*d : N+2*d);
    i = i + 1;
end

% X_bar, X_hat
X_bar = [X_1; X_2];
X_hat = [X_2; X_3];

% ITD eigenvalue decomposition for time periods longer then 2n
A_bar = (X_hat * X_bar') * (X_bar * X_bar')^-1;

[V, D] = eig(A_bar);    % V - mode shapes
                        % D - natural frequencies, damping

% size of matrix V
sizeV = size(V);

% matrix of mode shapes
MS = V(1 : sizeV(1, 1)/2, :);

end
```

<div align="center">Listing 5.1: ITD method in Matlab</div>

Natural frequencies were calculated using formula 3.18. Please notify that valuable D in cycle stands for elements of complex conjugate pairs in matrix D.

```
% natural frequency calculation

% size of input matrix
sizeX = size(X);

% cycle for: through lines in X
% T = 0.0005 - sampling period from transient structural analysis (Ansys)

for i = 1:sizeX(1,1)
    f(1, i) = 1/(d*T) * sqrt(log(D(i*2-1, i*2-1)) * log(D(i*2, i*2))) / (2*pi);
    % D - complex conjugate pairs of elements in D matrix;
end
```

<div align="center">Listing 5.2: Natural frequency calculation</div>

Final part of the algorithm is plotting results. Mode shapes are included in columns of matrix MS while two columns next each other are complex conjugate pairs. Correct mode shape pairs are usually next to each other inside matrix, however it is necessary to search for them because frequency range of measured data doesn't have to correspond with amount of DoF's we are expecting to find. To avoid this imperfection it is beneficial to predict correct frequency range of data and amount od DoF's. For example if we are looking for min. 4 mode shapes it is good help to use theoretical Finite element method at the beginning and predict frequency range where these 4 modes has their natural frequencies. Additionally it is good to measure vibrations in sufficient amount of points. In general sufficient amount means 2 times more than the highest mode we are looking for (in our example 8).

5.1.3. Operational modal analysis with ITD

Operational modal analysis is modal analysis outside of laboratory conditions, while excitation is powered by measured machine itself or by ambient source. Theoretical background of time domain methods for estimating modal parameters is more suitable for operational modal analysis in comparison to frequency domain methods. The reason is they can work without informations about excitation more accurately than frequency domain algorithms [7]. Condition for this analysis is that frequency of harmonic excitation has to be different from natural frequencies of the structure.

This section of thesis includes testing of the algorithm with input data from transient structural analysis in Ansys Workbench. Transient structural analysis was slightly updated from the one used in former testing. Measurement time step was set the same ($5 \cdot 10^{-4}$ s), simulation took time of 4 seconds. Excitation was set as a force (0.1 N), constantly hitting the end of the beam every constant time period (0.2 s) during the measurement.

Output of transient structural analysis is data set for 5 points on beam. Example of vibrations (cropped data - input to algorithm) of the structure in 3rd (middle) point are shown on the figure 5.3. Amplitudes of vibrations of structure are significantly lower than amplitudes in earlier described section simulating EMA but that should not cause any differences in operational modal analysis proceeded by ITD algorithm.
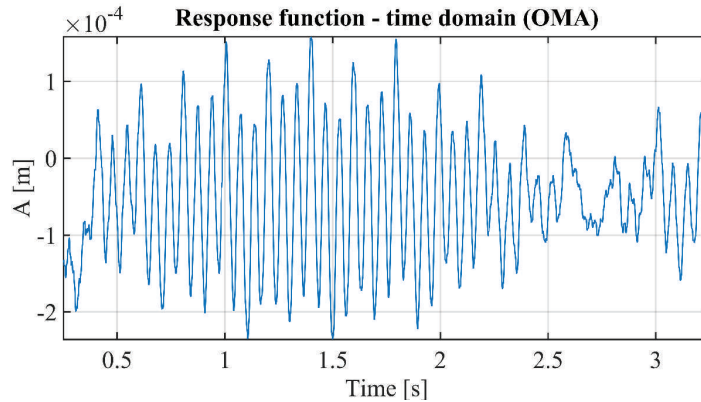

Figure 5.3: Pre-processed data for ITD (OMA)

5.1.4. Damping

In reference to theory part of this thesis in chapter 3, section 3.4.2, it is possible to calculate damping ratio during analysis with Ibrahim time domain method. The equation used in algorithm is shown in mentioned section as an equation 3.19. Source code implemented inside of ITD algorithm is shown below. Valuable D is part of complex conjugate pair inside of matrix D.

```
% damping ratio calculation

% size of input matrix
sizeX = size(X);

% cycle for: through lines in X
% T = 1/25600 − sampling period from transient structural analysis (Ansys)

for i = 1:sizeX(1,1)
    da(1, i) = −log(D(i*2−1, i*2−1) * D(i*2, i*2))/(2*(d*T)* 1/(d*T) * sqrt(log(D(i*2−1, i*2−1)) *
    * log(D(i*2, i*2))));
    % D − complex conjugate pairs of elements in D matrix;
end
```

Listing 5.3: Damping ratio calculation

For testing calculation of damping ratio, new measurement was created. Tested structure was beam made of pre-defined Structural steel. Sampling period was set to $1/25600 \; s$ and measurement took time $1 \; s$. As an excitation was chosen hammer hit of force $5 \; N$ at the beginning time of measurement in the opposite side of cantilevered side of the beam. To obtain damped vibrations from transient structural analysis it is necessary to add damping in damping control option in Analysis settings. In case of this testing scenario, stiffness coefficient was defined by direct input and the value 0.0002 was chosen. Lightly damped vibrations are shown on the figure 5.4. All results are shown in chapter 6, section 6.1.
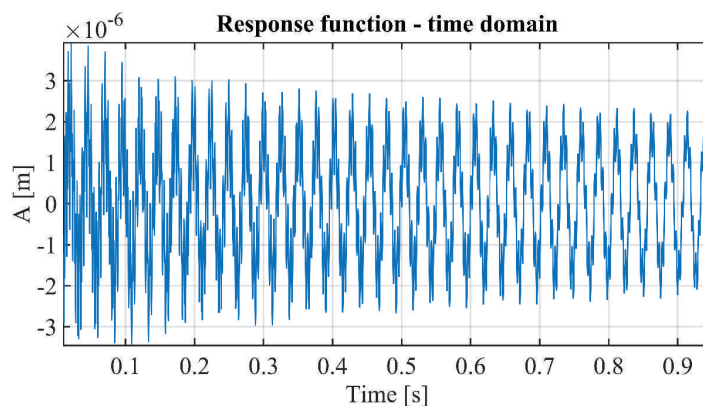

Figure 5.4: Damped time-domain data for ITD

5.2. Verification using perfect output data (plate)

This part of testing algorithm deals with cantilever (plate). Main idea was to measure vibrations in 2 rows of the plate and apply algorithm for both rows separately, as it was data from simple beam. It was expected to gain 2 rows of mode shapes and 2 sets of data, showing natural frequencies. Although these frequencies are very similar, the final natural frequency was calculated as an average of those two. The final mode shape is presented as a surface between 2 rows of the "beam" mode shapes. The goal of this section was to compare results from Finite element method and Experimental modal analysis with data in time domain.

5.2.1. Ansys Workbench - plate designing and analysis

Ansys Workbench, as a representation of Finite element approach was used for evaluating test displacements (vibrations) in Transient Structural Analysis. Obtained data were time domain data while vibrating took time approximately 5 seconds. Simulated plate was supported (fixed support) on one side (cantilevered surface) and hammer hit was set to the opposite corner for excitation of torsion.

Engineering data

The algorithm was tested on the plate made of the same material as was used for the test in section 5.1. Properties of material are shown in tab. 5.1. The "Test Material" is more mellow to ensure small time step in transient analysis. Material is showing signs of slower vibrations.

Geometry

Geometry of the plate was designed with the focus on the easy possibility of obtaining displacement data of vibrations in 2 rows. Sketch was drawn as a rectangle with construction points with specified spacing. Through these points were created slices (Create - Slice) which divided the plate into few bodies (figure 5.5). This allows easy way to choose the points where is the interest of measurement of vibrations in Transient Structural Analysis. The width of plate was set to 100 mm to ensure torsion excitation during testing phase.
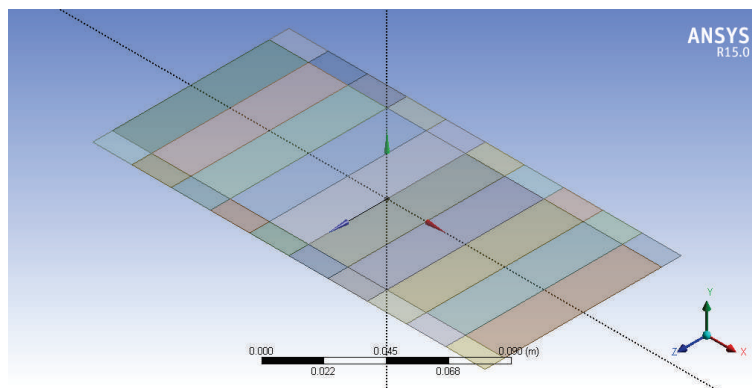

Figure 5.5: Plate - geometry

Modal analysis

Mode shapes and natural frequencies are shown on figure 6.3 and tab. 6.3. In this scenario the size of the element in mesh is not very important because in every row we are measuring vibrations only in 5 points. With regard to speed of transient structural analysis and sufficient accuracy of vibrations, size of the element inside mesh was set as square with dimension 10 mm. First, second, third and fourth total deformation was added to solution for evaluation of 4 vibrational modes.

5.2.2. Algorithm ITD

Algorithm ITD was the same algorithm as the one used in section 5.1 applied for 2 rows separately. Inputs to the algorithm were 2 rows of vibrations from Transient Structural Analysis in Ansys and outputs were 2 rows of mode shapes and 2 sets of natural frequencies. Final plots were presented as a surface between these 2 rows of mode shapes and final natural frequency was calculated as an average of values from each row. All results are presented in chapter 6, section 6.2.

5.3. Verification using real output data

Theory says it is not easy to use time domain methods for evaluating modal parameters while data includes noise inside. Methods are sensitive and that makes the verification of algorithm in this section big challenge. Experimental measured structure was simple plate made of steel (figure 5.6).

To measure vibrations was used accelerometer PCB Piezotronics, model 352C03 with sensitivity 10.20 mV/g (1.040 $mV/m/s^2$). Sensor was mounted to plate by Petro wax (adhesive type of connection) which is the most practical (easily removable) way for connecting accelerometers in room temperature conditions. This type of connection is in general capable of transmission of frequencies lower than 500 Hz. Frequency range, sent from steel plate to accelerometer through wax, strongly depends on temperature around connection, surface of measured structure and thickness of adhesive between structure and sensor.

During experimental modal analysis sensor was placed at 10 different points along the measured cantilever in the middle (with respect to studied structure as a beam). The point number one was set nearest to the cantilevered side and spacing between single points was 50 mm.

Hammer, capable of measurement of hitting force, hit the structure in the 9th point during whole measurement. Tip of the hammer was chosen from silicon material to ensure the excitation of equal or higher frequencies than 500 Hz, limited by adhesive connection of accelerometer.

To transfer and save data from accelerometer was used software LabVIEW by National instruments company. Measurement in selected point started by the trigger which measured and compared the value of the force in the hammer. Vibrations of duration 1 s with sampling frequency 25600 Hz were saved during single measurements.

Figure 5.6: Testing structure

5.3.1. FEM (Ansys) modal analysis

To create baseline for comparison modal parameters obtained by Ibrahim time domain method, FEM modal analysis was created for measured real structure. Geometry of can-
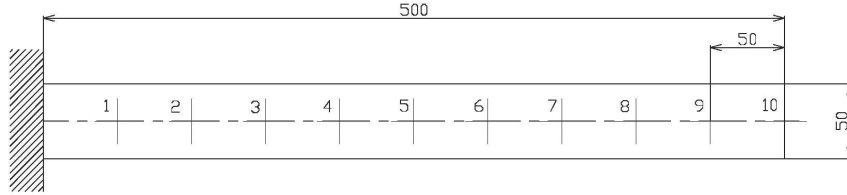
Figure 5.7: Testing structure - dimensions

tilever was designed as a beam with rectangular cross-section of dimensions $500x50x10\ mm$ (figure 5.7). Material of plate was set as pre-defined structural steel with properties shown in table 5.3. As a constraint was set fixed support on one side.

As mentioned earlier in this section adhesive (wax) connection between cantilever and accelerometer transfers frequencies lower than $500\ Hz$. Higher frequencies from measurement on real structure were expected to be damped and this expectation was later shown as correct. With respect to this statement modal analysis was evaluated and calculated natural frequencies (tab. 6.4) revealed that our modes of interest should be first two modes shown on structure.

Table 5.3: Plate material characteristics

Density	7850 kg/m^3
Isotropic Elasticity	
- Young's Modulus	$2 \cdot 10^{11}\ Pa$
- Poisson's Ratio	0.3

5.3.2. ITD modal analysis

Following subsections deal with first two vibrational modes and compare influence of various constraints setups for evaluation of modal parameters using Ibrahim time domain algorithm for modal analysis on real experimental structure. ITD method was tested for various conditions of fixation to the base.

The first fixation (Constraint 1) stands for the best possible fixation between jaws of the gripper in laboratory. The second constraint (Constraint 2) represents the conditions when the jaws were holding the structure with torque $10\ Nm$ in screws of gripper. During the last measurement the experimental structure was placed inside the jaws with piece of plastic block and screws were tighten as strong as possible (Constraint 3).

During experimental testing and setting up the algorithm, several problems occurred. Hammer hit point was chosen as the 9-th point on the structure but it wasn't possible to hit the middle of plate during 9-th measurement, when accelerometer was placed in this point. Accelerometers are very sensitive to any kind of vibrations and due to this fact it was important to hit further from the point where sensor was situated. This problem led to removal of 9-th measurement in testing the algorithm with constraint condition 1.

Different eye-catching problem was transformation of acceleration data into displacement data which were inputs to algorithm ITD. Measured values of acceleration did not

vibrate around zero value, however they were shaking around value slightly above zero. The possible reason of this behavior could be that if accelerometer measures vibrations upright to Earth surface, measured value is affected by gravitational acceleration. Solution of this problem could be solved by correction of acceleration or velocity data by creation of fitted curve, followed by subtraction of fitted data from measured values. Both approaches were tested and correction of velocity data was proved as more accurate.

Constraint 1

Constraint 1 was the closest possible to ideal cantilever constraint during experimental measurement. Cantilever was mounted between jaws of the gripper and screws were tightened as hard as possible. This part of measurement should tested the algorithm in the best possible conditions of simulating cantilever.

First of all acceleration data were transformed into displacement data by integration to velocity followed by second integration to displacement. Displacement data were double-checked by making frequency response function in dB scale and that has shown peaks. These peaks should be located in frequencies close to frequencies obtained from modal analysis in Ansys (tab. 6.4). Transformed FRF data are shown on figure 5.8.
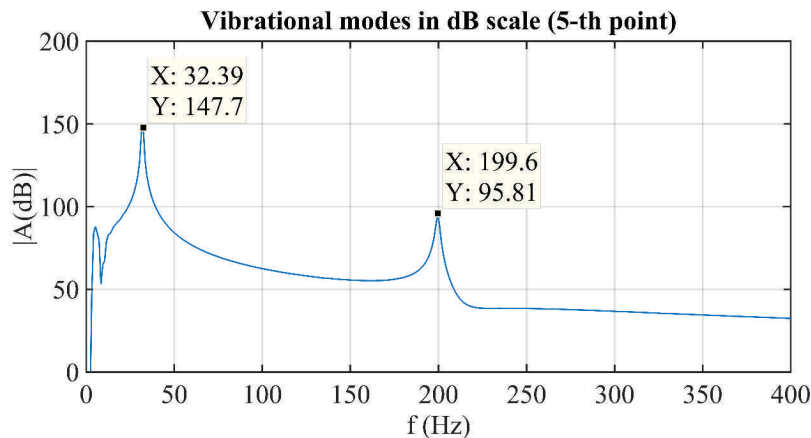

Figure 5.8: Real structure - FRF in dB scale (Constraint 1)

Respectively to theory part in section 3.4, subsection 3.4.2, the output of ITD algorithm is mode shape matrix (Φ) and matrix Λ, which contains informations about natural frequencies and damping ratios of single modes. Matrix of mode shapes calculated by ITD algorithm obtains $2n$ columns while n represents amount of measurement points. Mode shapes are plotted from these columns. Two columns next to each other are complex conjugate pairs. Algorithm described in listing 5.1.2 counts natural frequencies in for loop for every column pair. The best way of looking for correct modes in mode shape matrix was to make first awareness about natural frequencies (using ITD algorithm), compare these frequencies with the ones from modal analysis using FEM (Ansys) and plot the related columns from the matrix of mode shapes evaluated by ITD method.

Constraint 2

Constraint 2 was very similar to first one with only difference, that all four screws of the gripper were released and tightened to base with torque approximately 10 Nm. That represented different condition and to obtain basic informations about vibrations frequency response function was calculated. FRF is displayed on figure 5.9. Natural frequencies of first two vibrational modes are slightly moved to lower frequencies.

During measurement with second constraint the experimental testing followed same process as testing with constraint 1 condition. Hammer hit the structure in the same point (9-th) but measured acceleration data from 9-th point were precise enough to be kept involved in algorithm calculation.
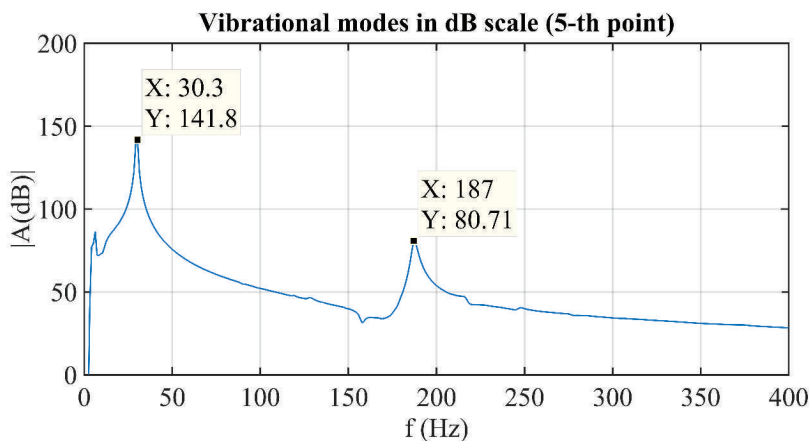

Figure 5.9: Real structure - FRF in dB scale (Constraint 2)

Constraint 3

Process of measurement was the same for the third condition. Constraint 3 represented situation of different cantilever which was mounted to base with piece of plastic block between jaws of the gripper. Four screws were tightened as hard as possible and accelerations of various points on structure excited by hammer hit were measured. 9-th point vibrations did not cause any troubles to modal parameter estimation this time as well. Found vibrational modes are shown on frequency response function on figure 5.12.
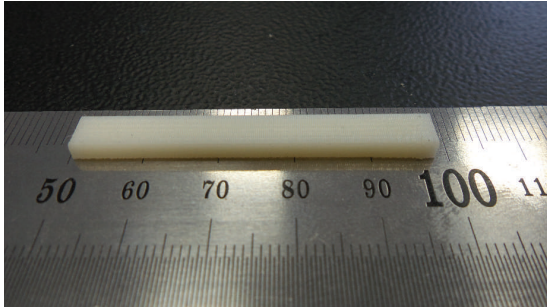
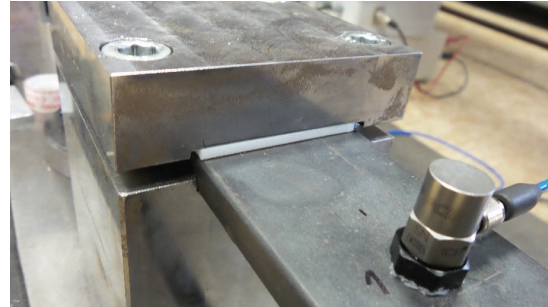Figure 5.10: Real structure - Constraint 3, plastic block

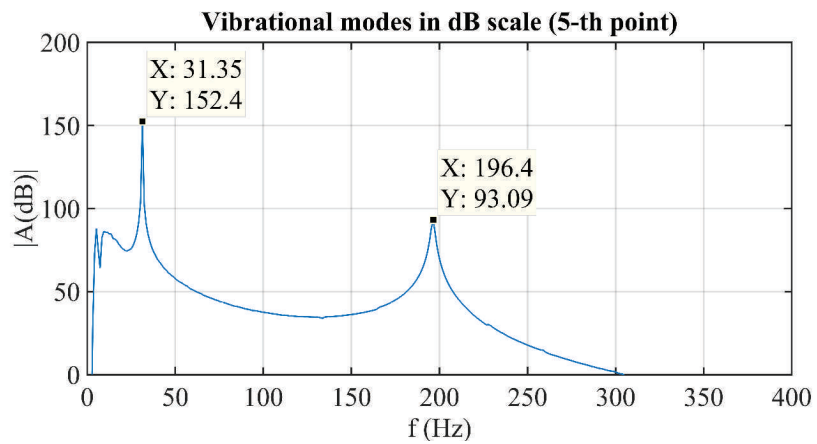Figure 5.11: Real structure - Constraint 3

Figure 5.12: Real structure - FRF in dB scale (Constraint 3)

6. RESULTS OF ITD ALGORITHM TESTING

6.1. Modal Analysis (beam)

The results of algorithm calculation were mode shapes (figure 6.1) and natural frequencies (tab. 6.1). Figures prove the algorithm functionality as evaluated modes are same as modes found by FEM.

Theoretical prediction, mentioned above in subsection 5.1.2 is very helpful at the beginning before measurement but it doesn't fit every case. In this testing of algorithm, 5 measurement points were used and 4 vibrational modes were found (prediction was to find 2 modes). The reason was that data used in this part of experiment are perfect and statement described in theoretical part is set for general situations. However amount of found vibrational modes in measurement on real structure depends on numerous conditions, for example properties of used accelerometer, type of connection between sensor and structure and quality of cable between PC and sensor.

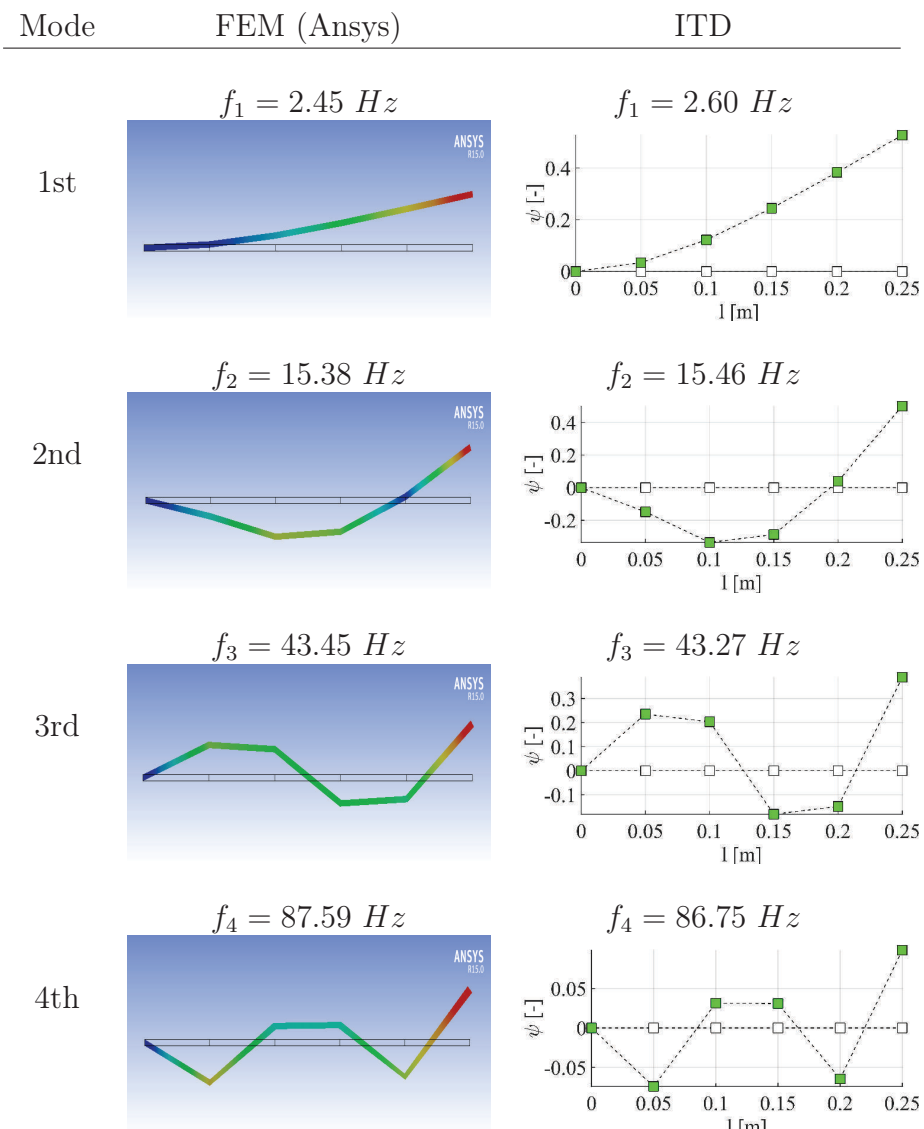| Mode	FEM (Ansys)	ITD

Figure 6.1: Beam - modal analysis (FEM vs. ITD)

Table 6.1: Natural frequencies comparison - ITD vs. Ansys

Mode shape	ITD [Hz]	FEM (Ansys) [Hz]
$1st$	$2.60~Hz$	$2.45~Hz$
$2nd$	$15.46~Hz$	$15.38~Hz$
$3rd$	$43.27~Hz$	$43.45~Hz$
$4th$	$86.75~Hz$	$87.59~Hz$

6.1.1. Operational Modal Analysis ITD (beam)

Computed modal parameters (by ITD) are shown on figure 6.2. Mode shapes and natural frequencies are identical with those computed by Ansys in section 6.1 and also with results from Experimental modal analysis which used ITD. As a conclusion the ITD (represent of time domain methods) is successful modal analysis tool for Experimental and Operational modal analysis. This conclusion is theoretical for now because data used as inputs were perfect, without any noise included. This subsection could be the first motivation for future studies.
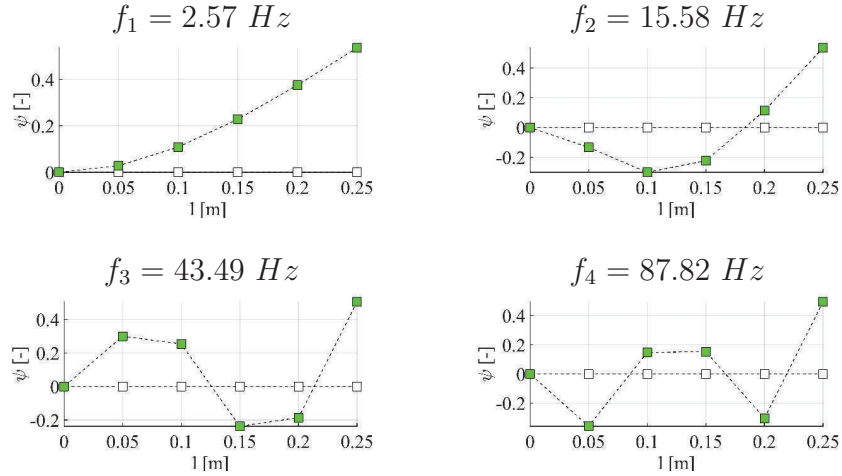
Figure 6.2: Beam - Operational modal analysis (ITD)

6.1.2. Damping by ITD

Table 6.2 shows comparison of damping ratio computed by ITD method and damping ratios of each mode computed by Ansys, representative of finite element methods. Calculation of damping ratio, used in ITD algorithm, was described in article [11]. However results tend to be sensitive to starting point in time-domain pre-processed data. Preciseness of this parameter is higher when using bigger amount of input time-domain data.

Table 6.2: Damping ratio - ITD vs. FEM

Mode shape	ITD [-]	FEM (Ansys) [-]
$1st$	$6.9 \cdot 10^{-5}$	$7.8 \cdot 10^{-5}$
$2nd$	$4.4 \cdot 10^{-4}$	$4.9 \cdot 10^{-4}$
$3rd$	$1.3 \cdot 10^{-3}$	$1.4 \cdot 10^{-3}$
$4th$	$2.8 \cdot 10^{-3}$	$2.8 \cdot 10^{-3}$

6.2. Modal Analysis (Plate)

Results from Finite element method (Ansys) and Experimental modal analysis with time domain data ITD are similar what confirms algorithm functionality. Mode shapes are shown on figure 6.3. Natural frequencies for single modes (tab. 6.3) were established as an average of natural frequencies of the first and the second row of measured transient structural data.

Mode	FEM (Ansys)	ITD
1st	$f_1 = 5.24\ Hz$	$f_1 = 5.26\ Hz$
2nd	$f_2 = 22.41\ Hz$	$f_2 = 23.34\ Hz$
3rd	$f_3 = 32.77\ Hz$	$f_3 = 33.78\ Hz$
4th	$f_4 = 73.16\ Hz$	$f_4 = 74.03\ Hz$

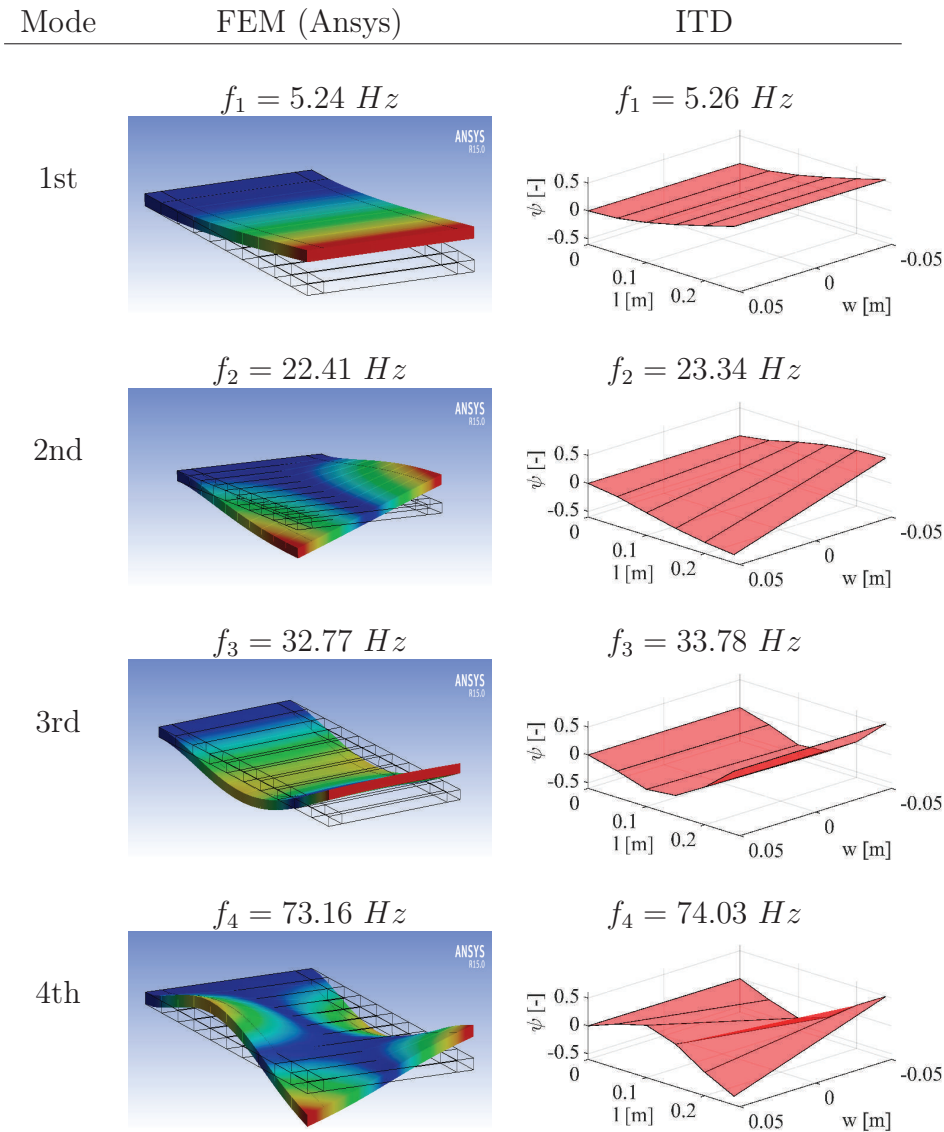Figure 6.3: Plate - modal analysis (FEM vs. ITD)

Table 6.3: Natural frequencies comparison - ITD vs. Ansys

Mode shape	ITD [Hz]	FEM (Ansys) [Hz]
1st	5.26 Hz	5.24 Hz
2nd	23.34 Hz	22.41 Hz
3rd	33.78 Hz	32.77 Hz
4th	74.03 Hz	73.16 Hz

6.3. Modal Analysis (real structure - beam)

Results of modal analysis for real structure from program Ansys Workbench are presented on figure 6.4 and in table 6.4.
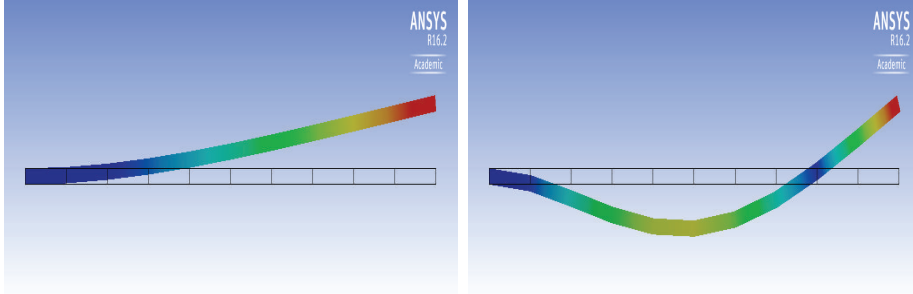

Figure 6.4: FEM modal analysis - 1st, 2nd mode shape

Table 6.4: Natural frequencies - real structure (FEM)

Mode shape	Natural frequency (FEM) [Hz]
$1st$	$32.6\ Hz$
$2nd$	$203.97\ Hz$

6.3.1. Constraint 1

Modal parameters obtained by ITD method are shown on figure 6.5 and in table 6.5. Circles on the plot show exact original points of mode shapes calculated by algorithm. These points includes inaccuracies caused mainly by noise included in acceleration data and following transformation of acceleration data into displacement, which are inputs to ITD algorithm. Another source of errors was inaccuracies in placement of sensor and hammer hit. To create view of the mode shapes there is also shown adapted mode shape curve.

This curve was created as an regression curve between original mode shape points. From the measurement with constraint 1 condition, 9-th point of experimental data was removed. These acceleration data show significant signs of error caused by the fact, that 9-th point was chosen as a point of hammer hit but during this measurement the point was occupied by acceleration sensor. 9-th point on figure 6.5 was calculated as an average between points 8 and 10.
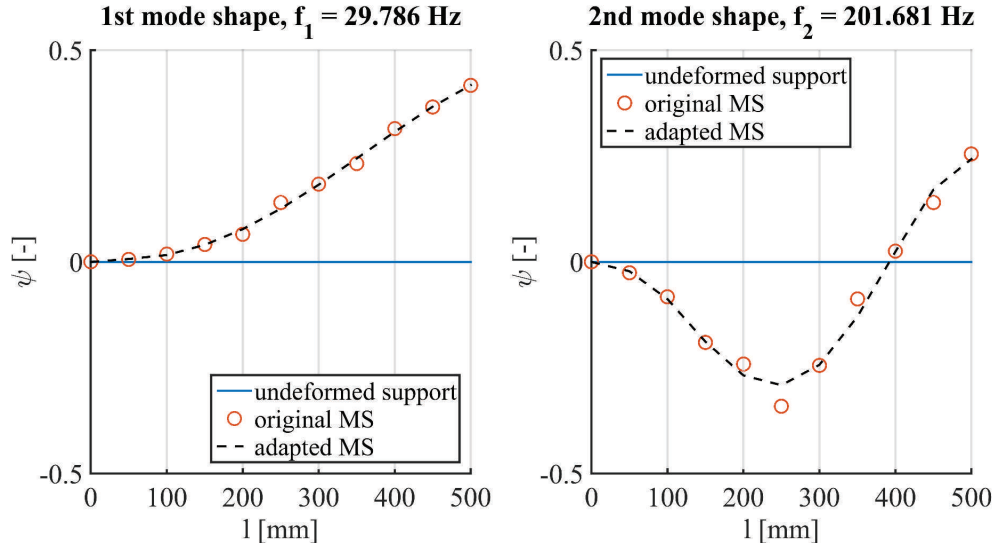
Figure 6.5: Real structure - modal analysis ITD (Constraint 1)

Table 6.5: Natural frequencies - real structure (ITD - Constraint 1)

Mode shape	Natural frequency (ITD) [Hz]
$1st$	$29.79\ Hz$
$2nd$	$201.68\ Hz$

6.3.2. Constraint 2

During testing with constraint 2, cantilever was mounted between jaws of the gripper with torque on screws approximately $10\ Nm$. This condition did not effect algorithm functionality, however it had impact to modal parameters of whole system. The studied plate was vibrating with natural frequencies shown on FRF presented on figure 5.9. The same result was obtained by ITD algorithm and shown on table 6.6. Mode shapes were calculated same as mode shapes evaluated by FEM (Ansys) and are presented on figure 6.6.

Table 6.6: Natural frequency - real structure (ITD - Constraint 2)

Mode shape	Natural frequency (ITD) [Hz]
$1st$	$29.68\ Hz$
$2nd$	$189.08\ Hz$
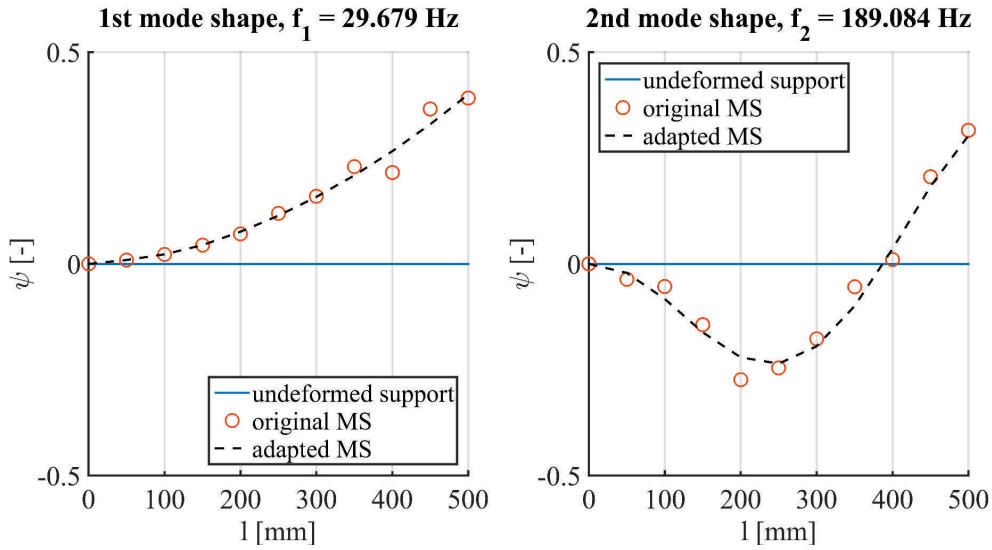
Figure 6.6: Real structure - modal analysis ITD (Constraint 2)

6.3.3. Constraint 3

The third constraint represented non-ideal cantilever with piece of plastic block inserted into system between base and the plate (figures 5.10 and 5.11). As results shown on figure 6.7 and in table 6.7 present, this situation did not effect algorithm functionality.
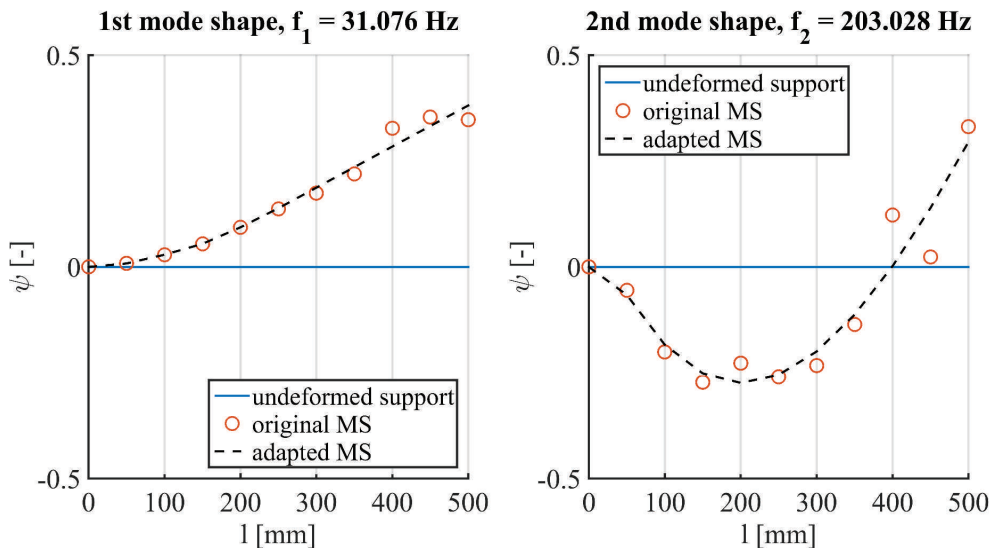
Figure 6.7: Real structure - modal analysis ITD (Constraint 3)

Table 6.7: Natural frequencies - real structure (ITD - Constraint 3)

Mode shape	Natural frequency (ITD) [Hz]
$1st$	$31.08\ Hz$
$2nd$	$203.03\ Hz$

7. Conclusion

The most important conclusion from the research part of this thesis is possibilities and limitations of described methods for modal parameters estimation.

Frequency domain algorithms are widely used these days within Experimental modal analysis and they have two major limitations. First is that accuracy of evaluated modal parameters is closely related to accuracy of measured frequency response function which doesn't represent "clean" response data but it stands for data modified to frequency range what brings inaccuracies into calculation. The second limitation is necessity of having assumptions about studied structure (for example frequency range of searched modes) which helps to find correct modes or to realize incorrect modes.

Second group of mentioned algorithms are Time domain methods. Accuracy of measured data is more controllable and algorithms are getting "cleaner" data as an input before computation. In theory part were described three methods: Complex exponential (CE), Least-Squares complex exponential (LSCE) and Ibrahim time domain (ITD). Methods CE and LSCE requires more computational power in case of using higher amount of input data (vibrations) because of Prony's method included in algorithm. ITD method can be used for bigger time-domain data and for evaluation of modal parameters uses different mathematical approach, eigenvalue decomposition.

Within practical part of this thesis, Ibrahim time domain method was programmed and its functionality was tested on different case scenarios. The first scenario testing structure was beam and experimental modal analysis was simulated in Ansys Workbench - Transient structural analysis. During this part of thesis was also tested possibility of modal parameter estimation in operational modal analysis and calculation of damping ratios. The second scenario represented modal analysis of plate which tested algorithm in case where was excited also torsion. All these testing cases verified the algorithm and results has shown that mode shapes and natural frequencies were evaluated correctly. The longer time domain data inputs were included in calculation, the more precise results were obtained. After testing algorithm using data from Ansys simulations, ITD method was also used in real experimental analysis (testing structure was cantilever) and during this part was tested sensitivity of algorithm to boundary conditions of system. Three different constraints were chosen and algorithm found correct modes in every case. Boundary conditions, tested within experimental modal analysis on real structure didn't influence algorithm functionality, however they changed modal parameters of whole system and this conclusion was confirmed by obtained frequency response functions from measured vibrations.

While processing Experimental analysis, it is very important to have assumptions about tested structure. To get these assumptions it is valuable to use combinations of different approaches in modal testing, for example Finite element method as a beneficial addition to Experimental modal analysis while testing real system.

7.1. Future research

Possible future research of time domain algorithms for modal parameter estimation is in operational modal analysis. Time domain methods are suitable for this task because of their ability to estimate modal parameters without knowing excitation. This thesis

7.1. FUTURE RESEARCH

presents demonstration of possibility to process modal analysis using time domain method. Thesis verifies algorithm in simple scenario where operational modal analysis is simulated in Ansys Workbench. Future research could be focused on deepen this verification and testing more case scenarios.

Future improvement could be found also in programming automatized application with user friendly environment for experimental modal analysis or addition of time domain algorithms into application created within thesis [8] and this could led to comparison of accuracy between frequency and time domain methods during Experimental modal analysis.

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8. LIST OF SYMBOLS AND PHYSICAL CONSTANTS

α	Receptance
Δ	Sampling time step [s]
η_r	Damping ratio (r-th mode)
Λ	Natural frequency and damping ratio matrix
ψ	Mode shape constant
Φ	Mode shape matrix
τ	Time step [s]
ζ_r	Viscous damping ratio (r-th mode)
A_r	Modal constant (r-th mode)
C	Damping matrix
CE	Complex exponential
EMA	Experimental modal analysis
f_r	Natural frequency [Hz]
FEM	Finite element method
FRF	Frequency response function
ITD	Ibrahim time domain
IRF	Impulse response function
K	Stiffness matrix
LSCE	Least squares complex exponential
M	Mass matrix
MDoF	Multi degree of freedom
MS	Mode shape
SDoF	Single degree of freedom
t	time [s]
X	Displacement matrix

x_i	Displacement in i-th point of structure
X_τ	Displacement shifted matrix
$X_{2\tau}$	Displacement 2-times shifted matrix
$\hat{X}$	Special ITD matrix of displacements
$\bar{X}$	Special ITD matrix of displacements

A. APPENDICES

A.1. Matlab scripts

```matlab
% Program description :
% program calculates modal paramters from vibrational data obtained during
% Experimental modal analysis .

% Parts of program :
% 1. Data read
% 2. Data pre−processing − Normalization of acceleration data to same force
%                          (normDataF)
%                        − Cropping data (cropData)
%                        − Transformation of acceleration to displacement
%                          (accToDis)
%
% 3. ITD                 − Modal shape matrix
%                        − f_r
% 6. plot
%
% Author : Matus Jakus

clc ; clear all ; close all ;

%% time
Fs = 25600;        % Sampling frequency
T = 1/Fs ;         % Sampling period
t = 0:T:1−T;       % Time vector

%% data read
% read data from folders
% t2 = time
% K = hammer
% A = acceleration
[t2 , K, A] = readDataV1 ;

%% data pre−processing
% normalization of A data (max F)
An = normDataF(K, A);

% crop data and time vector
% n_s = start point
% N = amount of samples
n_s = 600;
N = 24500;
An_c = cropData(An, n_s , N);
t_c = cropData(t , n_s , N);

% displacement data
[Dn_u] = accToDis(t_c , An_c);

%% ITD
% [MS, D] = ITD(Dn, N, d);
% N = amount of loaded samples
% d = delta (shift in ITD)
d = 1;

% Ms − matrix of mode shapes
% D − complex conjugate pairs of elements in D matrix ;
[MS, D] = ITD(Dn_u, N−2*d, d);

%% natural frequency calculation

% size of input matrix
sizeX = size(Dn_u);

for i = 1:sizeX(1,1)
    f(1, i) = 1/(d*T) * sqrt(log(D(i*2−1, i*2−1)) * log(D(i*2, i*2))) / (2*pi);
    % D − complex conjugate pairs of elements in D matrix ;
end


%% fft
point = 5;
[f_ , P1] = fft_(Dn_u, Fs , point);

% plot fft
figure()
plot(f_,P1)
title(sprintf('Vibrational_modes_in_dB_scale_(%.0f−th_point)', point))
xlabel('f_(Hz)')
ylabel('|A(dB)|')
axis([0 1400 −120 80])
grid on

%% plot
finalPlotV1(MS, 20, 16, f); % 20, 16 − positions of modes in MS matrix
```

Listing A.1: Parameter estimation program structure

```
% function ITD
%
% outputs:   MS - Modal shape matrix
%            D - f_r matrix
% inputs:    X - trajectory matrix from n-channel sensor
%            N - amount of loaded samples
%            d - delta (shift in ITD)

function [ MS, D ] = ITD( X, N, d )

% size of input matrix
sizeX = size(X);

% cycle while: shifting input displacement data
i = 1;
while i <= sizeX(1,1)
    X_1(i, :) = X(i, 1 : N);
    X_2(i, :) = X(i, 1+d : N+d);
    X_3(i, :) = X(i, 1+2*d : N+2*d);
    i = i + 1;
end

% X_bar, X_hat
X_bar = [X_1; X_2];
X_hat = [X_2; X_3];

% ITD eigenvalue decomposition for time periods longer then 2n
A_bar = (X_hat * X_bar') * (X_bar * X_bar')^-1;

[V, D] = eig(A_bar);      % V - mode shapes
                          % D - natural frequencies, damping

% size of matrix V
sizeV = size(V);

% matrix of mode shapes
MS = V(1 : sizeV(1, 1)/2, :);

end
```

Listing A.2: ITD method in Matlab

```
% natural frequency calculation

% size of input matrix
sizeX = size(X);

% cycle for: through lines in X
% T = 0.0005 - sampling period from transient structural analysis (Ansys)

for i = 1:sizeX(1,1)
    f(1, i) = 1/(d*T) * sqrt(log(D(i*2-1, i*2-1)) * log(D(i*2, i*2))) / (2*pi);
    % D - complex conjugate pairs of elements in D matrix;
end
```

Listing A.3: Natural frequency calculation

```
% damping ratio calculation

% size of input matrix
sizeX = size(X);

% cycle for: through lines in X
% T = 1/25600 - sampling period from transient structural analysis (Ansys)

for i = 1:sizeX(1,1)
    da(1, i) = -log(D(i*2-1, i*2-1) * D(i*2, i*2))/(2*(d*T)* 1/(d*T) * sqrt(log(D(i*2-1, i*2-1)) *
    * log(D(i*2, i*2))));
    % D - complex conjugate pairs of elements in D matrix;
end
```
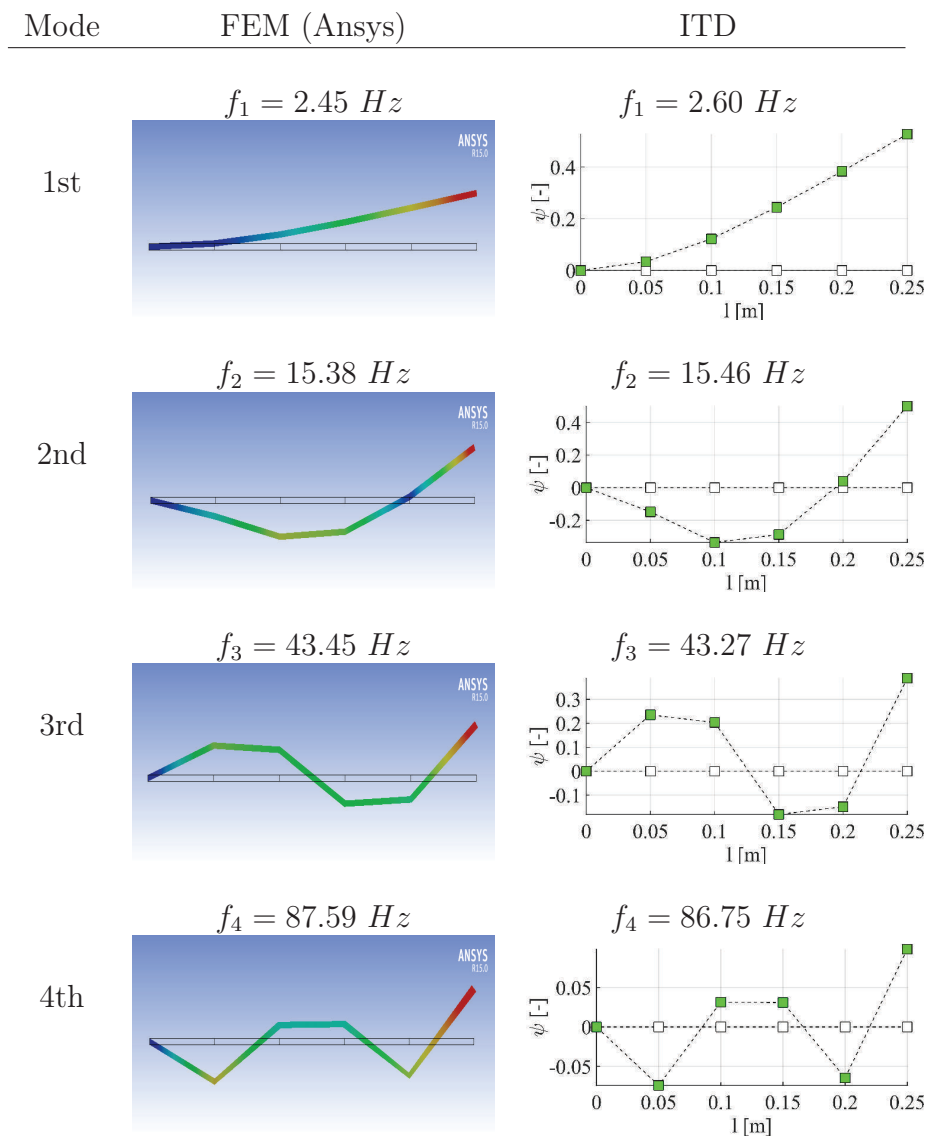
Listing A.4: Damping ratio calculation

A.2. Test case - beam

A.2.1. Comparison FEM vs. ITD

Mode	FEM (Ansys)	ITD
1st	$f_1 = 2.45 \ Hz$	$f_1 = 2.60 \ Hz$
2nd	$f_2 = 15.38 \ Hz$	$f_2 = 15.46 \ Hz$
3rd	$f_3 = 43.45 \ Hz$	$f_3 = 43.27 \ Hz$
4th	$f_4 = 87.59 \ Hz$	$f_4 = 86.75 \ Hz$

Operational modal analysis with ITD

Mode	FEM (Ansys)	ITD
1st	$f_1 = 2.45\ Hz$	$f_1 = 2.57\ Hz$
2nd	$f_2 = 15.38\ Hz$	$f_2 = 15.56\ Hz$
3rd	$f_3 = 43.45\ Hz$	$f_3 = 43.49\ Hz$
4th	$f_4 = 87.59\ Hz$	$f_4 = 87.82\ Hz$
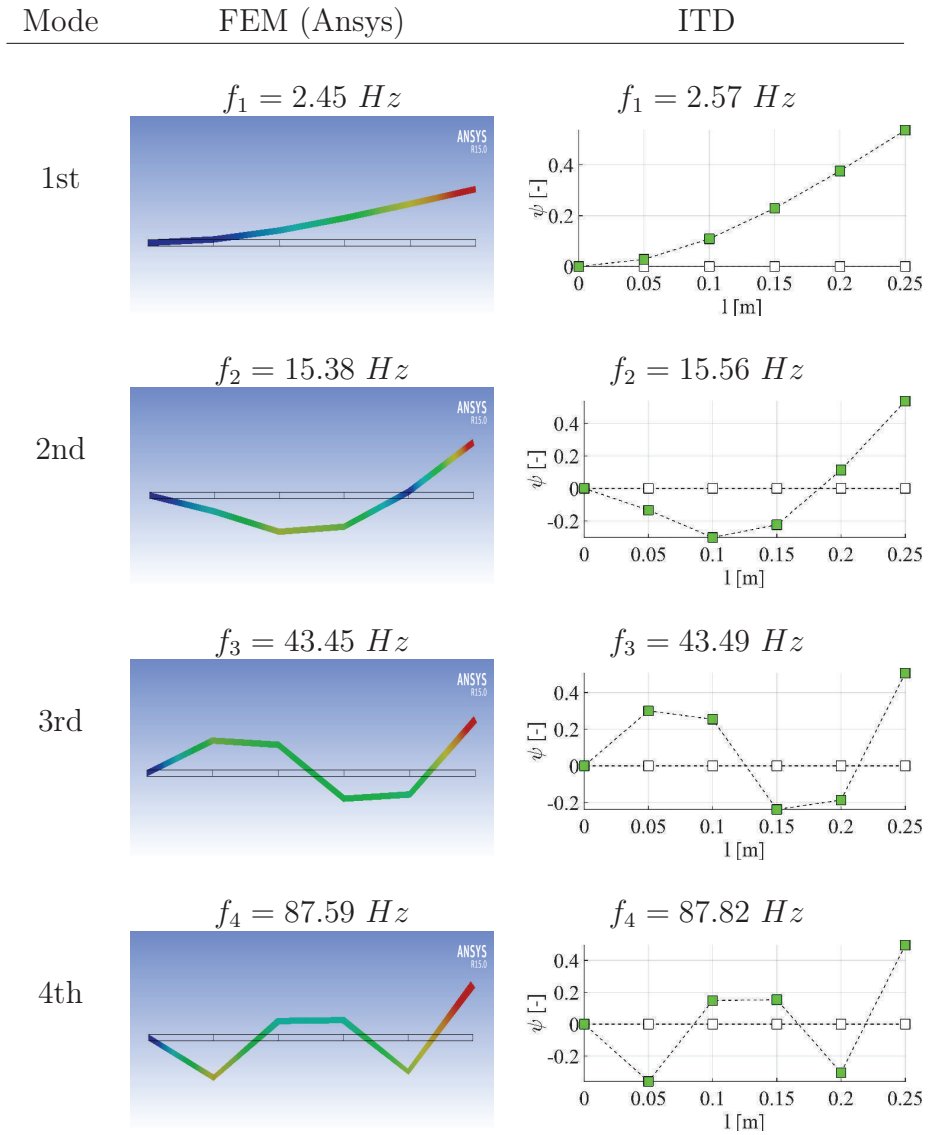
A.2. TEST CASE - BEAM

Evaluation of damping ratio

Mode	FEM (Ansys)	ITD

1st — $f_1 = 39.14\ Hz$ (FEM), $f_1 = 39.14\ Hz$ (ITD)

2nd — $f_2 = 245.46\ Hz$ (FEM), $f_2 = 245.37\ Hz$ (ITD)

3rd — $f_3 = 693.45\ Hz$ (FEM), $f_3 = 691.37\ Hz$ (ITD)

4th — $f_4 = 1398.1\ Hz$ (FEM), $f_4 = 1381.33\ Hz$ (ITD)
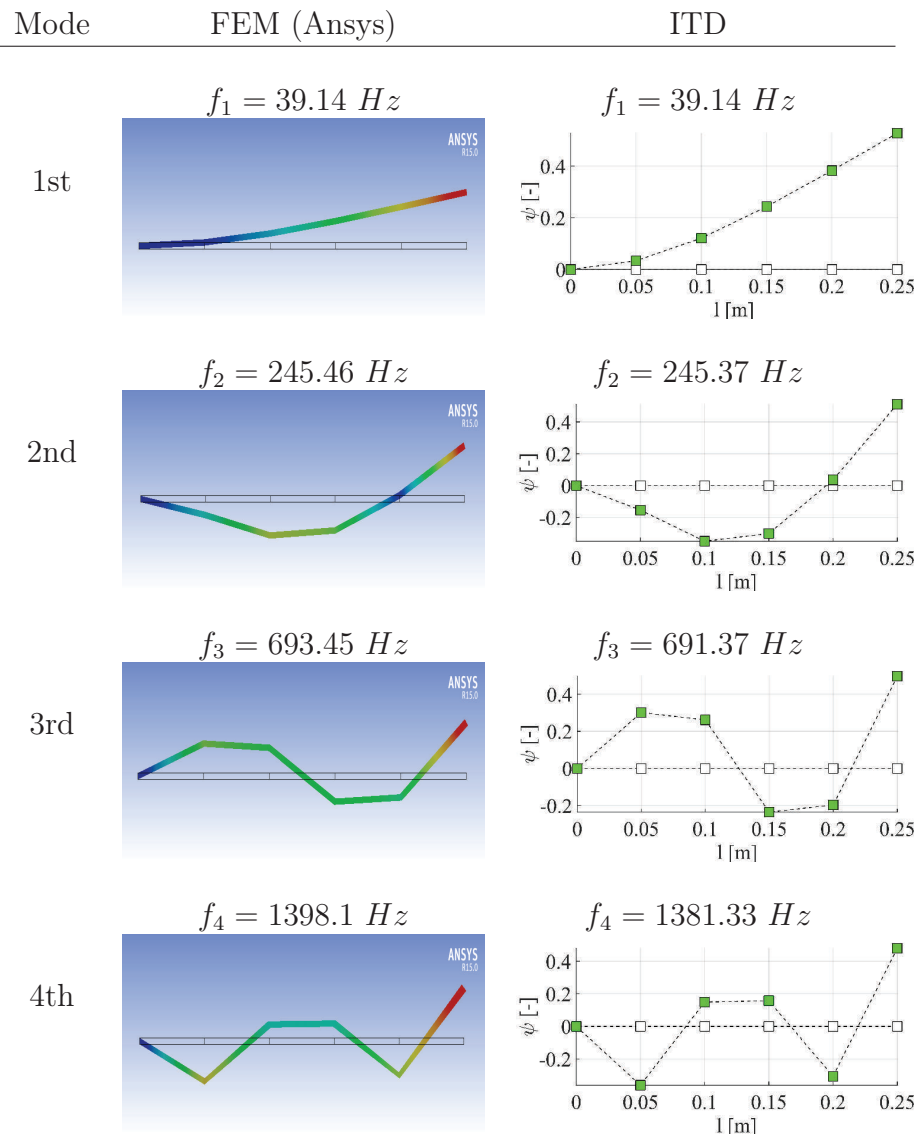
Table A.1: Damping ratio - ITD vs. FEM

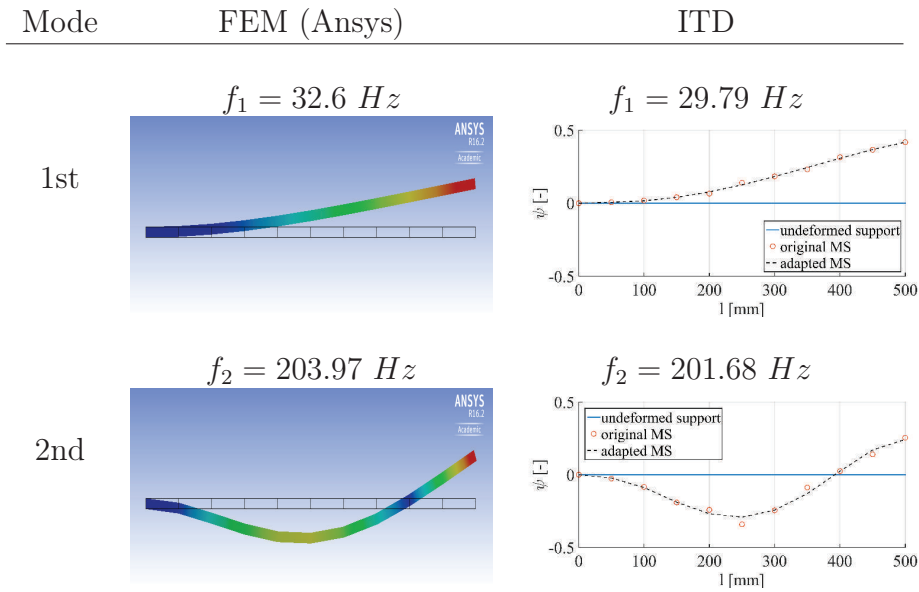| Mode shape	ITD [-]	FEM (Ansys) [-]
$1st$	$6.9 \cdot 10^{-5}$	$7.8 \cdot 10^{-5}$
$2nd$	$4.4 \cdot 10^{-4}$	$4.9 \cdot 10^{-4}$
$3rd$	$1.3 \cdot 10^{-3}$	$1.4 \cdot 10^{-3}$
$4th$	$2.8 \cdot 10^{-3}$	$2.8 \cdot 10^{-3}$

A.3. Test case - plate
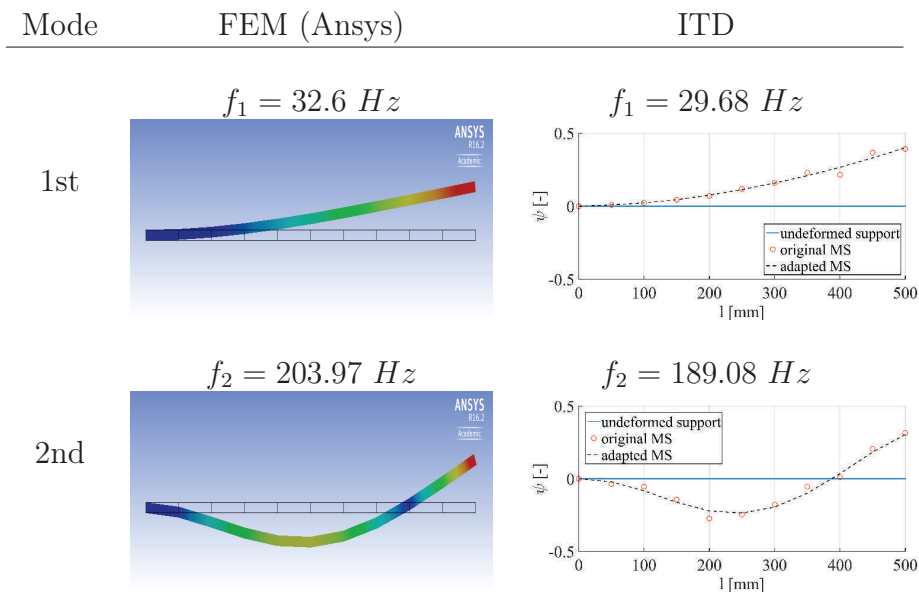
A.3.1. Comparison FEM vs. ITD

Mode	FEM (Ansys)	ITD
1st	$f_1 = 5.24\ Hz$	$f_1 = 5.26\ Hz$
2nd	$f_2 = 22.41\ Hz$	$f_2 = 23.34\ Hz$
3rd	$f_3 = 32.77\ Hz$	$f_3 = 33.78\ Hz$
4th	$f_4 = 73.16\ Hz$	$f_4 = 74.03\ Hz$
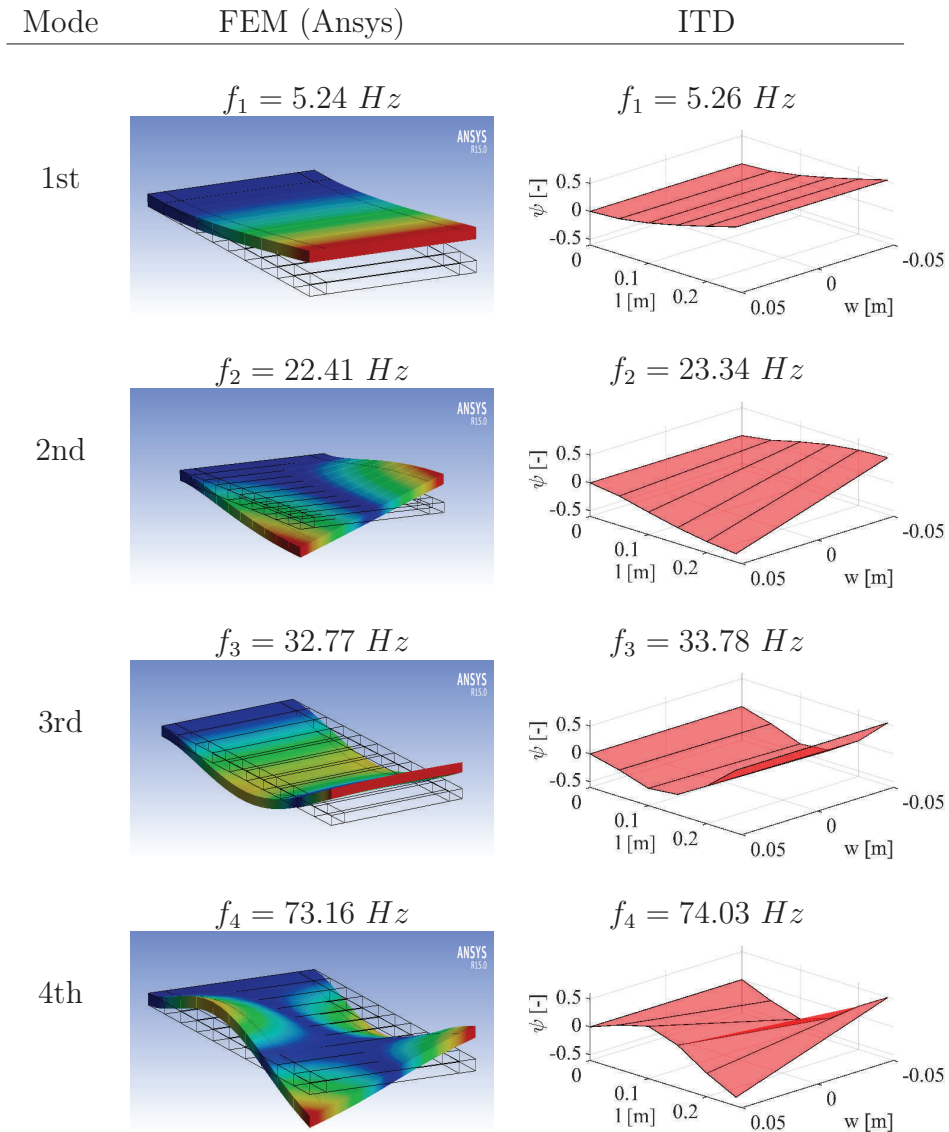
A.4. Test case - real structure

A.4.1. Constraint 1

A.4.2. Constraint 2

A.4.3. Constraint 3

Mode	FEM (Ansys)	ITD
1st	$f_1 = 32.6\ Hz$	$f_1 = 31.08\ Hz$
2nd	$f_2 = 203.97\ Hz$	$f_2 = 203.03\ Hz$