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Improving quantum erasing by nonlinear feed-forward

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Zlepšení kvantového smazávání pomocí nelineární dopředné vazby

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Abstract

We introduce an alternative method of continuous variable quantum erasing using a nonlinear feed-forward. We demonstrate the effectiveness of this method by applying it to a single-photon state that interferes with a vacuum state on a Beam splitter causing quantum decoherence. Finally, we compare the effectiveness of the nonlinear method with the linear method showing that the nonlinear method is indeed more effective.

Keywords

CV quantum eraser, Nonlinear feed-forward, Quantum decoherence, Wigner quasiprobability distribution

Abstrakt

Představíme alternativní metodu kvantového vymazávání spojité proměnné použitím nelineární dopředné vazby. Ukážeme effektivitu této metody tím, že ji použijeme na jedno fotonový stav, který interferuje s vakuovým stavem na děliči svazků, což způsobuje kvantovou decoherenci. Nakonec porovnáme efectivitu nelineární metody s lineární metodou a ukážeme že nelineární metoda je doopravdy více efektivní.

Klíčová slova

kvantové smazávání ve spojitých proměnných, nelineární dopředná vazba, kvantová dekoherence, Wingnerova kvazipravděpodobnostní distribuce

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Declaration

I declare that this thesis entitled "Improving quantum erasing by nonlinear feed-forward" has been written and compiled solely by myself under the guidance of doc. Mgr. Petr MAREK Ph.D. by using resources, which are referred to in the list of resources. I agree with the further usage of this document by the Department of Optics.

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Chapter I

Introduction

The quantum eraser experiment and its implications are quite an interesting part of quantum mechanics, an area of physics already abundant with topics that pique our curiosity and test the limits of human understanding. The origin of its inception comes from Young's double-slit experiment. This experiment, normally used to show the wave-like behaviour of light (that being interference), is incredibly interesting when done with quantum systems. The reason is an answer to a simple question: If a single photon went through the double-slit setup, which of the two slits did it pass through? This type of experiment, trying to answer this question, is called the "Which-way" experiment . What makes it so interesting is that when we try to detect this which-way information or are able to determine it, we lose the interference pattern (The measured particle loses its wave-like qualities).

The first which-way experiment that tried to detect this information without causing quantum decoherence was proposed by Albert Einstein [1]. However, Bohr later reported, that this proposal conflicts with the principles of quantum mechanics. [2]. The earlier explanation for this phenomenon was that this decoherence was due to phase changes by way of scattering of the particle caused by the measurement. This idea went through a revaluation when it was found out, that even if we detect this which-way information without inducting random phases (e.g. by way of excitation and deexcitation of a microwave cavity) this measurement still causes quantum decoherence [3]. The difference is that now we measure the wave qualities of our system instead of particle qualities. In other words, instead of measuring the exact position of the particle, we detect the position quadrature of the wave that the particle acts as. This discovery opened up new possibilities. It was now possible to store this information in another quantum system and "erase" it by making it inaccessible. This erasure results in the recovery of the interference pattern making it a way to negate the quantum decoherence caused by measurement [4]. A famous example of research that followed this discovery is Wheeler's delayed-choice experiment [5] that was later realized in different forms by [6] and [7] and showed that we can store the which-way information and erase it at a later time. Quantum erasing can also be realized in a continuous variable setting. This setting is concerned with the general mode of the quantum state of light with many photons, instead of the which-way information of a single photon. The groundwork for this setting is done in [8] and it further continued by showcasing the potential of using this quantum erasure to restore quantum states, that have been disturbed during data storage in quantum information processing [9] and finally by demonstrating this CV quantum eraser experimentally [10]. The quantum erasure described by these papers used a method of linear feed-forward to displace the measured state causing the which-way information to be no longer accessible. This was expanded even more quite recently by the development of new technologies that had made it possible to use nonlinear feed-forward [11]. Our thesis continues from this research by introducing one such method of quantum erasure that utilizes nonlinear feed-forward, which could lead to an even more effective restoration of quantum states.

We started by defining our system and constructing the continuous variable (or CV for short) quantum eraser. This quantum eraser consists of a homodyne detector and a beam splitter, where interference occurs between a single photon state and a vacuum state. We describe this system using the phase-space representation by making use of the Wigner quasiprobability distribution (Wigner function or WF for short). The nonlinear method of quantum erasure uses the displacement operation that shifts the Wigner function. This is the same operation that is used in the case of the linear method. The difference is, that in the case of the nonlinear method, we displace the WF by a value that corresponds to the position of the minimal value of this function. We found this minimal value numerically using software (MatLab). The reason to focus on the minimal value is that we know that the negativity of the Wigner function is a direct result of quantum interference and cannot be simulated classically [12], so we want this negativity to be as high as possible. Finally, we used the negativity as a metric to compare the nonlinear and the linear methods and showed that the nonlinear correction is indeed more effective at restoring quantum qualities.

Chapter II

Theory

2.1 Quantum Harmonic Oscilator

One of the fundamental cornerstones of physics is the Harmonic oscillator. A harmonic oscillator is a system that experiences force proportional to the displacement when displaced from its equilibrium. Our model is based on the electromagnetic oscillator. This oscillator is described by a complex function called spatial-temporal mode which describes the wave's change in space (spatial) and time (temporal). A simple example of this function is a plane wave

$$u(x,t) = u_0 \exp(i(kx - \omega t)), \tag{2.1.1}$$

where u_0 is a polarization vector, ω frequency and $k = \frac{w^2}{c^2}$ wave vector. This mode can be chosen at will as long as it obeys Maxwell's equations. In classical physics, the state of this mode would be described by complex amplitude α given by its magnitude $|\alpha|$ and phase $\arg\alpha$. In quantum mechanics, things get more complicated. The energy of the quantum harmonic oscillator (QHO) is discreetly distributed into energy levels or states

$$\hat{E} = \hbar\omega(\hat{n} + \frac{1}{2}), \tag{2.1.2}$$

where $\hbar$ is Planck's constant, the $\hat{n}$ is a discrete number operator, whose eigenstate corresponds to the energy and number of photons. This quantization of energy arises because any bound system in quantum mechanics can only gain discreet energy levels [13]. For simplicity, we will consider Planck's constant

$$\hbar = 1. \tag{2.1.3}$$

The $\hat{n}$ operator is equal to

$$n = \hat{a}^\dagger \hat{a}, \tag{2.1.4}$$

where $\hat{a}^\dagger$ and $\hat{a}$ are the creation operator and the annihilation operator respectively. They obey the commutation relation

$$[\hat{a}, \hat{a}^\dagger] = 1. \tag{2.1.5}$$

When applied to a state of the QHO the annihilation operator lowers the energy level and the creation operator increases it. Furthermore, The annihilation operator is analogical to complex amplitude in a classical harmonic oscillator. We can therefore describe it similarly using

$$\hat{a}^\dagger = \frac{1}{\sqrt{2}}(\hat{x} + i\hat{p}) \tag{2.1.6}$$

$$\hat{a} = \frac{1}{\sqrt{2}}(\hat{x} - i\hat{p}), \tag{2.1.7}$$

where we described these operators using hermitian and antihermitian parts, called quadratures $\hat{x}$ and $\hat{p}$ respectively. However, since they obey the commutation relation $[\hat{x}, \hat{p}] = i$ we can use them as a sort of position and momentum analogy. When we describe our QHO using this position and momentum, we start operating in a phase space, not too different from the phase space used in classical physics. So, it makes sense to use the phase space formulation of quantum mechanics. To this end, we will use the Wigner quasiprobability distribution.

2.2 Wigner quasiprobability distribution

Another way to analyze the complex amplitude of an electromagnetic oscillator in classical physics is by defining a distribution that describes it statistically. This distribution quantifies the probability of finding a particular value of real and imaginary parts of complex amplitude in their simultaneous measurement. As explained earlier, the real and imaginary parts of complex amplitude can be described as an oscillator's position and momentum. So this distribution also quantifies the probability of finding a concrete pair of $\hat{x}$ and $\hat{p}$. In other words, we get a distribution that calculates the probability of having a particular position and momentum value for a chosen ensemble of particles or a type of phase-space distribution. Analogous to this in quantum mechanics is the Wigner distribution (further abbreviated as the WF-Wigner function). However, as we can deduce this will not be a simple one-to-one analogy as we are limited by the uncertainty principle meaning we cannot simultaneously know the value of two non-commuting variables.

This Wigner function for a mixed state is formally written as

$$W(\hat{x}, \hat{p}) = \frac{1}{\pi} \int_{-\infty}^{\infty} \langle \hat{x} - y | \, \hat{\varrho} \, | \hat{x} + y \rangle \, e^{2i\hat{p}y} \, dy, \tag{2.2.1}$$

where $\hat{\varrho}$ is the density matrix, $\hat{x}$ and $\hat{p}$ are quadratures and y is an auxiliary variable used for integration. As can be deduced from this definition the WF utilizes the fact that if we know the form of the state in x representation we also know its form in p representation by applying a Fourier transform on it. An important characteristic of WFs is that they can attain negative values. This negativity is a direct result of quantum mechanical interference effects that arise when we try to describe nonclassically acting particles using a phase-space representation. This means that these negative values are physically relevant and we can use them as a metric of how ”quantum” a given system is [14]. With this in mind, we define our Wigner function as a quasiprobability distribution or a distribution that has its requirements slightly alleviated. This function is defined in such a way that its marginal distributions correspond to probability densities of the respective observables [15]. So we gain the expectation value in the x-representation by

$$\int_{-\infty}^{\infty} W(\hat{x}, \hat{p})\, d\hat{p} = \langle \hat{x} | \, \hat{\varrho} \, | \hat{x} \rangle \, . \tag{2.2.2}$$

The WF for a particular energy eigenstate $|n\rangle$ will have the following form

$$W_n(\hat{x}, \hat{p}) = \frac{(-1)^n}{\pi} L_n(2(\hat{x}^2 + \hat{p}^2))e^{-(\hat{x}^2+\hat{p}^2)}, \tag{2.2.3}$$

where W_n are the Laguerre polynomials. These polynomials are nontrivial solutions of Laguerre’s differential equations. They are linked to Hermits polynomials (polynomials used to solve the wave function of QHO using Hilbert space operators) through the Wigner-Weyl transform, an invertible mapping between Hilbert space operators and functions in the phase space formulation. For our purposes, we only need the Laguerre polynomial for the ground and first excited state.

$$L_0(\hat{x}) = 1, \tag{2.2.4}$$

$$L_1(\hat{x}) = -\hat{x} + 1. \tag{2.2.5}$$

We use them to derive the WF of the ground state.

$$W_{|0\rangle}(\hat{x}, \hat{p}) = \frac{1}{\pi} e^{-\hat{x}^2 - \hat{p}^2}. \tag{2.2.6}$$

And to derive the WF of the single-photon state.

$$W_{|1\rangle}(\hat{x}, \hat{p}) = \frac{1}{\pi}(2\hat{x}^2 + 2\hat{p}^2 - 1)e^{-\hat{x}^2 - \hat{p}^2}. \tag{2.2.7}$$

2.3 Beam Splitter and Homodyne detector

A beam splitter (BS) is a simple optical device we use in the thesis. It makes two incoming beams of light interfere to produce two emerging beams. This interference is also reversible as we can send the two beams back to the BS, where they produce the original beam. We can therefore describe this device as a two-input, two-output black box. The diagram of this device can be seen in Fig.II.1. The four modes are described by two annihilation operators $\hat{a}_1$ and $\hat{a}_2$ on the input and operators $\hat{a}'_1$ and $\hat{a}'_2$ on the output.

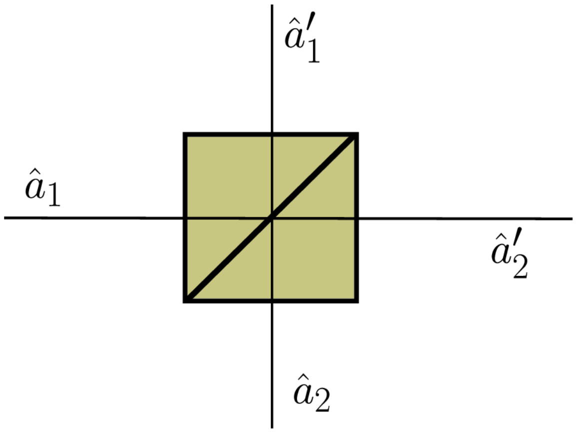

Figure II.1: The diagram of the beam splitter. This optical device always has two input parameters (without comma) and two output parameters (with comma)

The linear transformation (interference) that happens on the BS is then described as

$$\begin{pmatrix} \hat{a}'_1 \\ \hat{a}'_2 \end{pmatrix} = B \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \end{pmatrix}, \tag{2.3.1}$$

where B is the unitary beam splitter matrix. This matrix describes the change of the incoming beams that we can describe as rotations

$$B = \begin{pmatrix} \cos(\Psi/2) & \sin(\Psi/2) \\ -\sin(\Psi/2) & \cos(\Psi/2) \end{pmatrix}. \tag{2.3.2}$$

Finally, we can represent this matrix in terms of transmission t and reflection r coefficients as

$$B = \begin{pmatrix} t & -r \\ r & t \end{pmatrix}, \tag{2.3.3}$$

which satisfies the following relation due to energy conservation

$$t^2 + r^2 = 1. \tag{2.3.4}$$

The interesting characteristic of the BS is that it always acts as a four-port device, even if there is only one beam on the input that gets split into two. This is an essential quantum feature of the BS that makes it so, even if our incoming beam has nothing to interfere with, it interferes with the vacuum state containing vacuum fluctuations. This is caused by the poorly understood spontaneous creation of “virtual particles” in a particle-antiparticle pair that randomly changes the amount of energy in space. [16]. In other words, we can understand this interaction with the vacuum state as a general interaction of our state with its environment which results in some loss.

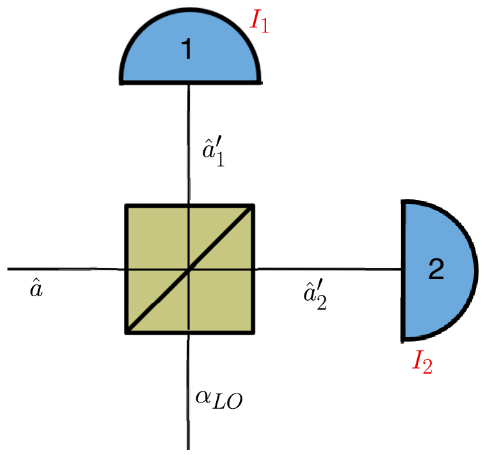

Figure II.2: The diagram of the homodyne detector. The signal on the input is described by $\hat{a}$ and interacts with a coherent wave described by α_{LO} on the BS. We detect the photocurrents I_1 and I_2 on the output

Another optical device used in our measurements is the Homodyne detector seen in Fig.II.2. It is an essential component used to measure the intensity of electromagnetic fields. The homodyne detector is constructed using a balanced BS, where the incoming beam interferes with a local oscillator (coherent laser beam), that we assume to be powerful enough to act classically (we neglect the quantum fluctuations). This causes the beam to split as expected. Next the photocurrents I_1 and I_2 of these two beams are measured using two detectors. These photocurrents are proportional to the photon numbers $\hat{n}_1$ and $\hat{n}_2$ respectively given by

$$I_1 \propto \langle \hat{n}_1 \rangle = \langle \hat{a}_1'^{\dagger} \rangle \langle \hat{a}'_1 \rangle, \; I_2 \propto \langle \hat{n}'_2 \rangle = \langle \hat{a}_2'^{\dagger} \rangle \langle \hat{a}'_2 \rangle. \tag{2.3.5}$$

Where the operators $\hat{a}'_1$ and $\hat{a}'_2$ are given as

$$\hat{a}'_1 = \frac{1}{\sqrt{2}}(\hat{a} - \alpha_{LO}), \; \hat{a}'_2 = \frac{1}{\sqrt{2}}(\hat{a} + \alpha_{LO}), \tag{2.3.6}$$

where α_{LO} denotes the complex amplitude of the local oscillator that we assume acts classically and $\hat{a}$ is the annihilation operator of the measured signal. Furthermore, the difference of the photocurrents $I_{21} = I_2 - I_1$ is proportional to the difference in photon numbers.

$$I_{21} \propto \langle \hat{n}_{21} \rangle = \langle \hat{n}_2 - \hat{n}_1 \rangle = \langle \alpha^*_{LO} \hat{a} \rangle + \langle \alpha_{LO} \hat{a}^{\dagger} \rangle \tag{2.3.7}$$

and so it is also proportional to the quadrature components. If we then set the reference phase of the LO oscillator to 0, the difference of photocurrents I_{21} will be proportional to the position quadrature.

$$I_{21} \propto \langle \hat{n}_{21} \rangle = \sqrt{2} |\alpha_{LO}| \hat{x}. \tag{2.3.8}$$

[17]. Phase $\pi/2$ then leads to the detection of $\hat{p}$. It is appropriate to mention that this

detector measures quadratures in a wave base instead of a particle base by using electromagnetic interactions. This means that measuring a single-photon state this way does not result in the photon acting as a particle.

2.4 Quantum eraser and displacement

In quantum physics we have variables that do not commute, the prime example being position and momentum. When we measure or gain information about one of these variables we inevitably lose information about the other. This is the nature of quantum systems described by the Heisenberg uncertainty relation. The same principle is applicable to all non-commuting variables and measurements. In the case of the CV quantum eraser, we are primarily interested in quadrature operators $\hat{x}$ and $\hat{p}$. However, we can use a binary quantum erasure, which is concerned with measuring one of two non-commuting states, to describe this concept more intuitively. An example of this binary regime can be seen in Young's double-slit experiment. This experiment used originally to show the interference characteristic of waves is a prime example of how measurement destroys the quantum behaviour of systems. In the case of this experiment, the non-commuting part of the uncertainty experiment is the info The diagram of this experiment can be seen in Fig.II.3.

If we send an ensemble of single photons through the double slits we observe that they create interference patterns and thus have the property of waves, a purely quantum behaviour. If we, however, try to find out through which of the two slits the photon went through (a Which-way experiment) the resulting pattern will not be an interference pattern and instead will show a pattern we would expect for a particle. This way the detection can be done in a number of ways as long as one complementary variable is measured. In the case of this double-slit experiment, it has been found that there is a way to measure this which-way information without producing any phase change on the centre of mass of the incoming particle. This is done by using excited atoms that give up its energy before going through a slit. Therefore we can detect this change of energy and gain the which-way information [18]. This is important as it is now possible to erase this which-way information by making it no longer accessible, which will in turn return the desired interference pattern. In this case, the erasure is done by inserting a field detector and it works as follows: Suppose that the state of the photon going through the first slit is $|u\rangle$ and the state of the photon going through the second slit is $|d\rangle$. In that case, the function of the which-way information disregarding detector is to become excited when the slits state (combination of $|u\rangle$ and $|d\rangle$) is a symmetric combination ($|u\rangle + |d\rangle$) and to become deexcited when this state is an asymmetrical combination ($|u\rangle - |d\rangle$). When we analyze two ensembles of particles, one corresponding to the symmetric combination and one corresponding to the antisymmetric combination we will see that the maxima of the interference fringes of one set correspond to the minima of the other set. The interference restoration itself can be done by detecting the phase difference between the symmetrical and asymmetrical slit states. We can then dynamically move the screen (shown in Fig.II.3) in a way that makes the maxima of one set correspond to the maxima of the other set restoring the interference pattern.

We can now see that the purpose of quantum erasure is to make the measured information of the complementary variable not accessible and by doing so to restore its quantum qualities. The reason as to why this can be so beneficial is that by the proper use of this quantum erasure, we can restore quantum states, that have been disturbed due to their interaction with their environment. [9].

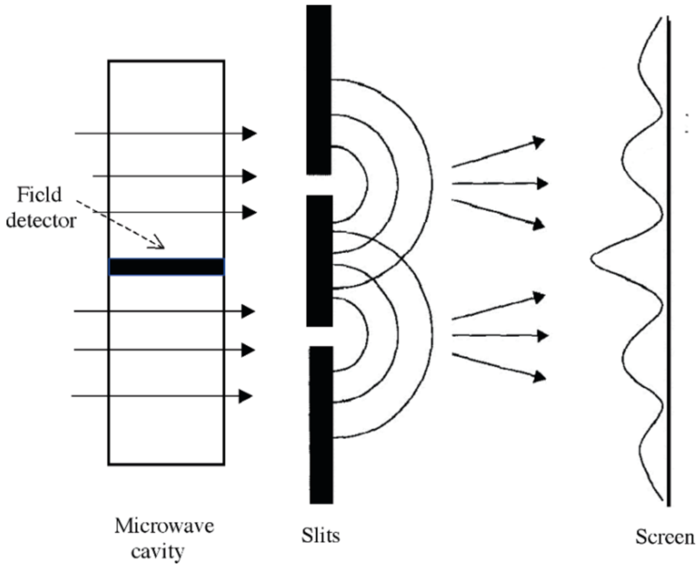

Figure II.3: The diagram of the double-slit experiment. The microwave cavity is used to determine the "which-way" information. The field detector becomes excited or deexcited based on the type of combination of the slit states resulting in quantum erasure and restoration of the interference pattern. [1]

We so far discussed quantum erasure in the binary regime. However, in our thesis, we make use of a quantum eraser in a continuous variable regime. In the case of the CV regime, we look at the complementary pair of amplitude and phase quadrature of light, or in other words, the quadrature that can be described as the operators $\hat{x}$ and $\hat{p}$ of the light. The difference from the binary regime is that now instead of there being only two possible paths (two eigenstates), there is a continuous set of possible paths. The way of gaining this "which-way" information is by encoding the signal information into another state (marker state) using quantum nondemolition entangling coupling [19]. Then the way to erase this information is by applying a local unitary operation on the output state so that the measured quadrature information is no longer accessible. This can be done by using the operation of displacement. The displacement operator is generally described as

$$\hat{D}(\alpha) = \exp(\alpha \hat{a}^\dagger - \alpha^* \hat{a}), \tag{2.4.1}$$

[1] Greenberger, Daniel and Hentschel, Klaus and Weinert, Friedel. Compendium of Quantum Physics, p.547, fig.1

where α is the amount of displacement in optical phase space. So this operation shifts a localized state in phase space by a complex amplitude α. For example, if we split a beam, where on one output we measure the position quadrature q, then applying the displacement operation, dependent on the measured quadrature and scaled with the appropriate gain factor G, on the second output $\hat{x}$ will result in quantum erasure and restoration of quantum qualities.

$$\hat{x}_{\text{fin}} = \hat{x} + Gq, \tag{2.4.2}$$

In this case, the displacement is linearly dependent on the measured quadrature q and so we call this process linear correction where the linear function is given as

$$f_l = Gq. \tag{2.4.3}$$

Chapter III

Method and results

3.1 Defining the model

We consider a single mode of electromagnetic light in the single-photon state, passing through a BS, where it interacts with the second mode in the vacuum state. The reason is that a) a single photon forms a pure and simple quantum system and b) single photon states are an essential resource in CV quantum information theory. The quantum behaviour of this state can be seen from its Wigner function that, as expected, has negative values with the minimal value being $-\frac{1}{\pi}$, which can be seen in Fig.III.1.

When the photon goes through the BS it is not split as a particle. Instead, it creates a superposed state of the photon passing and getting reflected. When we only look at the transmitted mode, the interaction between single-photon state $W_{|1\rangle}(\hat{x},\hat{p})$ and vacuum state $W_{|0\rangle}(\hat{x},\hat{p})$ on the BS is equivalent to loss. Our ultimate goal is to negate this loss by erasing the information lost to the second channel.

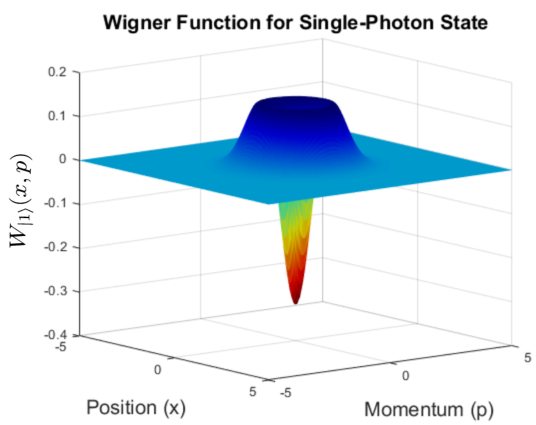

Figure III.1: WF of a single photon state. Its minimal value is located in position W(0,0) and is equal to $-\frac{1}{\pi}$

The diagram of the model describing the CV quantum eraser can be seen in Fig.III.2. With the interaction of the single-photon state and vacuum state on the BS comes some change in quadratures $\hat{x}$ and $\hat{p}$. We denote the quadratures before BS of the single-photon state as $\hat{x}_{\text{in}}$ and $\hat{p}_{\text{in}}$ and of the vacuum state $\hat{x}_0$ and $\hat{p}_0$. After the BS the measured output has quadratures $\hat{x}_{\text{m}}$ and $\hat{p}_{\text{m}}$ and the second output has quadratures $\hat{x}_{\text{out}}$ and $\hat{p}_{\text{out}}$, where we displace the position quadrature to get $\hat{x}_{\text{fin}}$. With this notation, the BS will have the following effect on quadratures $\hat{x}$ and $\hat{p}$.

$$\hat{x}_{\text{out}} = t\hat{x}_{\text{in}} + r\hat{x}_0, \tag{3.1.1}$$

$$\hat{p}_{\text{out}} = t\hat{p}_{\text{in}} + r\hat{p}_0. \tag{3.1.2}$$

Therefore, the WFs of the ground and first excited state will now transform

$$W_{|1\rangle}(\hat{x}_{\text{in}}, \hat{p}_{\text{in}}) \to W_{|1\rangle}(t\hat{x}_{\text{out}} + r\hat{x}_m, t\hat{p}_{\text{out}} + r\hat{p}_m). \tag{3.1.3}$$

$$W_{|0\rangle}(\hat{x}_0, \hat{p}_0) \to W_{|0\rangle}(t\hat{x}_{\text{m}} - r\hat{x}_{\text{out}}, t\hat{p}_m - r\hat{p}_{\text{out}}). \tag{3.1.4}$$

Next, we add a way to detect the shift caused by the interference that we can later use for our correction. We achieve this by adding a homodyne detector that we described in section 2.3. This detector will serve to measure the quadrature shift caused by the vacuum or in other words to measure the value of $\hat{x}_m$ that we then mark as q. Finally, we add a way to displace the outcoming wave that we have not measured. In an experiment, this would have been done with a strong coherent light beam [10].
To summarize, the beam splitter takes the single-photon state and the vacuum state as its inputs and, based on its characteristics described by transmission and reflection coefficients, they interfere to create two outputs. On one of these outputs we measure the quadrature shift of the WF (this output describes the loss on this system) and then by the use of methods of quantum deletion we purposefully displace the second output in such a way that we get a WF as close as possible to the WF of single-photon on the start. From Fig.III.1 we can see that this WF has its minimal value in position $W(0, 0)$ so we aim to get a WF that is as negative as possible in this same position (alternative method not used in this thesis would be to use fidelity).

To describe the displacement done by this system we use the operators shown in Fig.III.2. The quadrature of the marker state x_m is measured by the homodyne detection to yield value denoted as q

$$\hat{q} = t\hat{x}_0 - r\hat{x}_{\text{in}}, \tag{3.1.5}$$

We can now apply the displacement operator on the output signal $\hat{x}_{out}$, which will shift it in regard to the measured position on the marker state $\hat{q}$ scaled by the gain factor G.

$$\hat{x}_{\text{fin}} = \hat{x}_{\text{out}} + Gq, \tag{3.1.6}$$

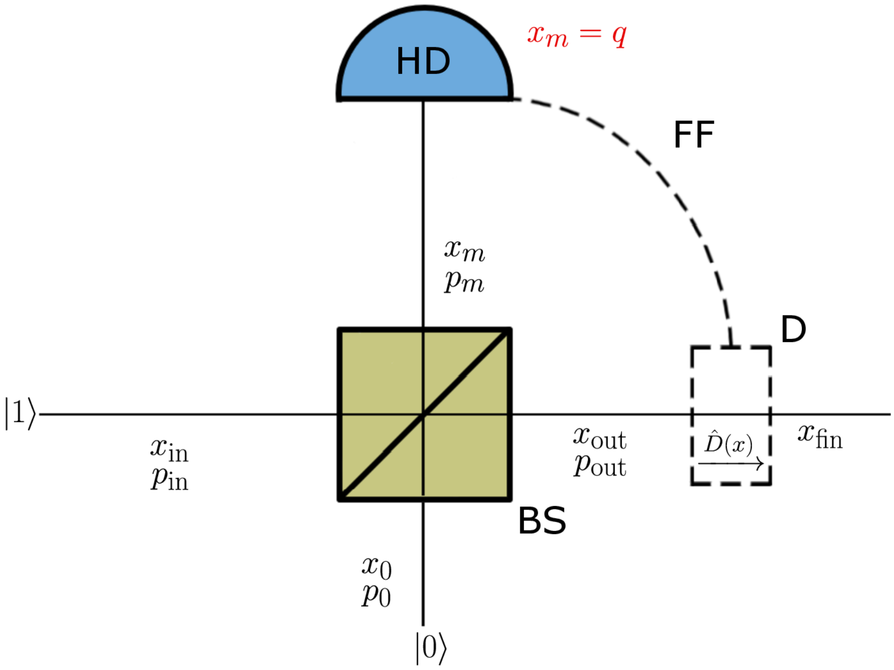

Figure III.2: Schematic depiction of CV quantum erasing. HD is the homodyne detector. FF is the feed-forward and D is displacement where the displacement operation ($\hat{D}(x)$) takes place. The measured position quadrature is marked q

where Gq is the linear correction $f_l(q)$ the same as in equation (2.4.3). We then find the value of the gain factor by substituting equations (3.1.1) and (3.1.5) into (3.1.6) and adjusting appropriately

$$\hat{x}_{\text{fin}} = \hat{x}_{\text{in}}(t - rG) + \hat{x}_0(r + tG). \tag{3.1.7}$$

While generally, a number of different displacements will result in some erasure of the which-way information, since our goal is to minimize the effect of the vacuum state on our system, we choose the gain factor in a way to negate the effect of the vacuum state. As we can see from the equation (3.1.7) this can be achieved by

$$G = -\frac{r}{t}. \tag{3.1.8}$$

By inserting it into equation (2.4.3) we get

$$f_l(q) = -\frac{r}{t}q. \tag{3.1.9}$$

So we measure the position quadrature on the marker state, then based on this measurement we use methods of feed-forward to displace the resulting WF using equation (3.1.8) which will result in the erasure of the which-way information and restoration of the quantum properties [10].

3.2 Conditional WF and its characteristics

With the whole model complete and described using the change of position quadratures, we can now describe it using the WF. This will also describe the loss we get as a result of the interference of single photon and vacuum state. We start by deriving a conditional WF, $W(\hat{x}_{\text{out}}, \hat{p}_{\text{out}}|q)$, that describes the WF dependent on its starting quadratures under the condition that we measure a particular value of q. Essentially, this is the WF on the output is the result of interference of the original single-photon WF with the vacuum state resulting in a shift on its x-axis by q. This conditional function is equivalent to conditional probability and is derived in essentially the same way as

$$W(\hat{x}_{\text{out}}, \hat{p}_{\text{out}}|q) = \frac{W(\hat{x}_{\text{out}}, \hat{p}_{\text{out}}, q)}{P(q)}, \tag{3.2.1}$$

where $P(q)$ is the marginal distribution of variable q gained as

$$P(q) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W(\hat{x}_{\text{out}}, \hat{p}_{\text{out}}, q)\, d\hat{x}_{\text{out}} d\hat{p}_{\text{out}} \tag{3.2.2}$$

and $W(\hat{x}_{\text{out}}, \hat{p}_{\text{out}}, q)$ is the WF, which is a result of interference of single-photon and vacuum states and the measurement of $\hat{x}_m$ as q. We describe it as

$$W(\hat{x}_{\text{out}}, \hat{p}_{\text{out}}, q) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W_{|0\rangle}(t\hat{x}_{\text{m}} - r\hat{x}_{\text{out}}, t\hat{p}_m - r\hat{p}_{\text{out}}) W_{|1\rangle}(t\hat{x}_{\text{out}} + r\hat{x}_m, t\hat{p}_{\text{out}} + r\hat{p}_m)\, d\hat{x}_m, d\hat{p}_m, \tag{3.2.3}$$

where we trace over the quadratures $\hat{x}_m$ and $\hat{p}_m$ on the output of the BS. The Dirac's delta function subsides the $\hat{x}_m$ as the measured value q. We now take relations (3.2.3) and (3.2.2) and input them all into equation (3.2.1) and evaluate the integrals. Since we will now work with only variables $\hat{x}_{\text{out}}$ and $\hat{p}_{\text{out}}$ we relabel them to x and p respectively. The conditional WF that is derived this way looks as:

$$W(x, p|q) = \frac{1}{\pi}\frac{r^2(2q^2+1) + 4rqtx + 2t^2(x^2+p^2) - 1}{-1 + r^2(1+2q^2) + 2t^2} e^{-x^2-p^2}. \tag{3.2.4}$$

With this conditional WF, we can finally look at how this shift caused by interference with the vacuum state looks based on what value of quadrature q we measured. For now, we are only interested in the shift of quadrature x and as such we will analyze our WF only on the x-plane with quadrature momentum being $p = 0$. We also choose the value of the transmission coefficient to be 0.9.

The first thing that we can notice in Fig.III.3 (a) is that the difference between the positive shift and the negative one is just mirroring (it is symmetrical). In (b) we can notice that with bigger values of measured q there is a reduction of negativity of the WF. In the limit of infinity, the minimal value of this conditional WF eventually goes to zero.

We can now analyze the WF on the output where we make no measurements and so do not

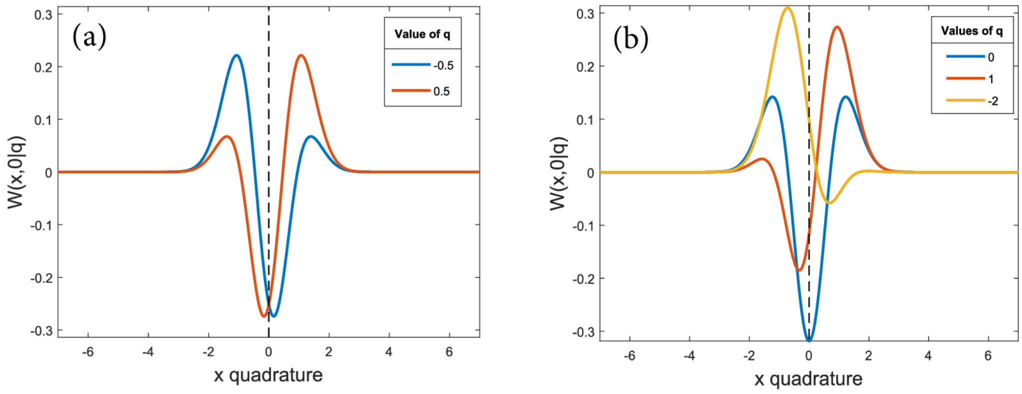

Figure III.3: (a) Figure comparing positive ($q > 0$)and negative shifts ($q < 0$) of the $W(\hat{x}, \hat{p}|q)$ function. (b) Differences based on the increasing value of q of the $W(x, p|q)$ function. The transmission for all of these functions is 0.9

use any corrections. We gain this WF from the conditional one as

$$W(x, p) = \int_{-\infty}^{\infty} W(x, p|q)P(q)\, dq. \tag{3.2.5}$$

And after adjustment

$$W(x, p) = \frac{1}{\pi}(-1 + 2r^2 + t^2(p^2 + x^2))e^{-p^2 - x^2}. \tag{3.2.6}$$

We focus on the value of this WF in position $W(0, 0)$ as this is where we want the most negativity. In Fig.III.4 we can see the resulting dependency of the value in this position based on transmission.

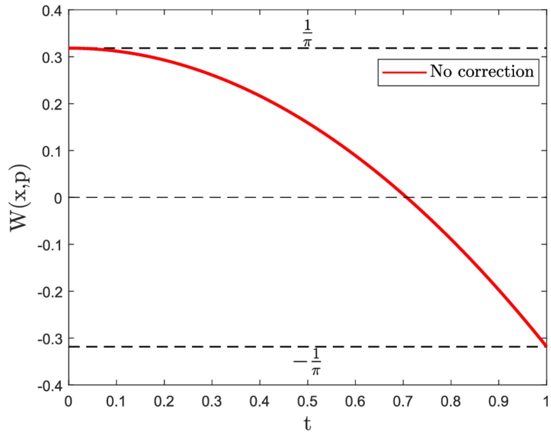

Figure III.4: Dependancy of values in position $W(0,0)$ of the unmeasured WF on transmission t. It shows the amount of loss in relation to the transmission coefficient of the BS

This function describes the loss caused by interference on the BS. When we have a transmission of 0 (and so reflectivity of 1) all we get after the BS is a vacuum state as the photon just gets reflected and we lose all quantum information. So after the BS the value of $W(0,0)$ is $\frac{1}{\pi}$ or the same for the vacuum state. Similarly, for a value of $t = 1$, the photon goes through the BS uninterrupted and as a result, we again get the pure state of a single photon that has its value in $W(0,0)$ equal to $-\frac{1}{\pi}$ and so in this case, there is no loss.

3.3 Corrections

Now that we have examined the resulting WF based on our measurement it is time to try to use the methods of displacement and quantum erasing to correct or shift the resulting WF and with that mitigate the losses caused by the vacuum state. To get the resulting shifted WF we apply the displacement operation on the conditional WF by using equation (3.2.6) resulting in

$$W(x,p) = \int_{-\infty}^{\infty} W(x - f(q), p|q) P(q)\, dq, \tag{3.3.1}$$

where $f(q)$ is our chosen correction. As described earlier the standard way this correction is chosen is (3.1.9), where the dependency on the measured q is linear. We try to improve this method by introducing an alternative method using a nonlinear correction.

Instead of just using the parameters of the BS, as is the case with linear correction, we opted to apply a displacement operation that shifts the WF by a value that corresponds to the position of its minimum. By doing this we effectively take this minimal value and shift it back into our desired position of $W(0,0)$. So if we derive a function $f_n(q)$ that for each measured q finds the position x_{min} where our conditional WF is minimal we can then use this function as a nonlinear correction. We did this numerically by finding the minimal value of the conditional WF based on measured values of q going from $\langle -7, 7\rangle$ with each step being 0.01. In the Fig.3.3 we can see a plot of this function. It is worth reminding that we still assume the value of transmission to be 0.9. As we can see this function is at first glance quite

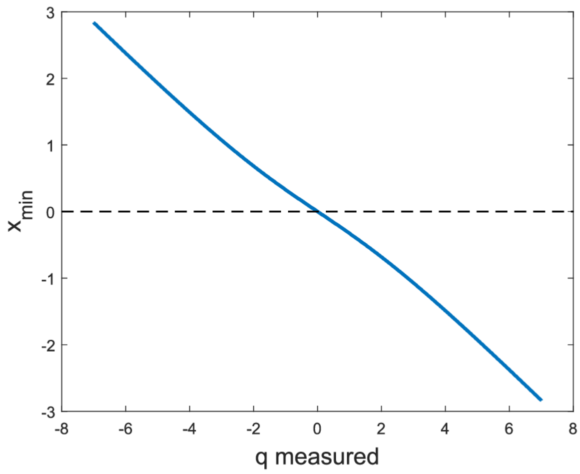

Figure III.5: Dependence of x_{min}(for which $W(x_{min}, 0)$ is minimal) on measured shift in quadrature q for t=0.9

similar to a linear dependence. This similarity makes sense when we consider that the linear correction alone is quite effective and what we are technically doing is merely fine-tuning it. To further show how this function varies for its linear counterpart we plot their value difference based on q. The result of this is in Fig.III.6.

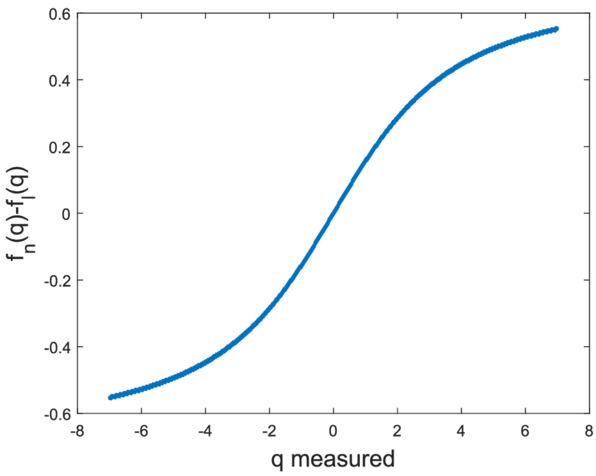

Figure III.6: The difference between nonlinear f_n and linear correction f_l (3.1.9) based on measured quadrature q

It is now worth looking at what these corrections do to our conditional WF $W(x, p|q)$. We will only look at our Winger function on the x-plane because we are not shifting the momentum in any way. The results of these corrections are shown in Fig.III.7 and Fig.III.8

We can see that while the linear correction does shift the conditional WF it overdoes it and

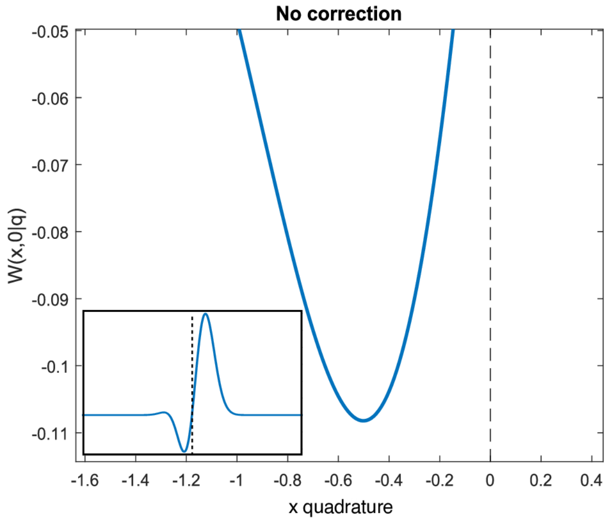

Figure III.7: Wigner function for $q = 1.5$ and $t = 0.9$ with no correction

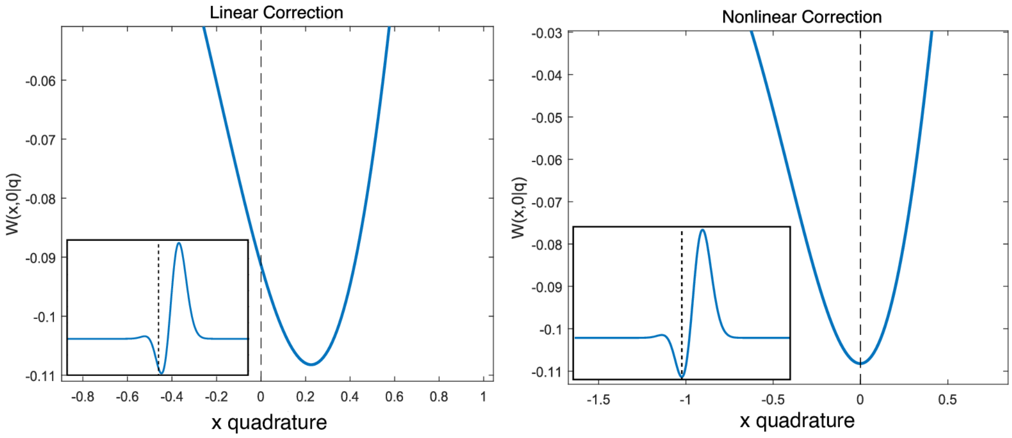

Figure III.8: Comparison of linear and nonlinear correction for $q = 1.5$ and $t = 0.9$

we get a resulting function that has its minimal value slightly off our desired location. However, with our method, since we measure exactly how we need to shift our function we get its minimal value right into the "sweet" spot that is the position of (0,0). However, the real test of the usefulness of our method is to look at the standard WF and see how effective these methods are for values of transmission t going from 0 to 1.

3.4 Generalization to an arbitrary transmission value

The final step of our work is to show how these correcting methods work based on arbitrary values of transmission. For linear correction, this is quite straightforward as we can use the relation (3.3.1) for different values of t. But we can also simplify it even more by evaluating the equation given by (3.3.1) where we assume $x = 0$ and $p = 0$. This value is all we need to compare the effectivity of our corrections, since by just comparing them in relation to transmission, we can see which correction gives us more negative values in this position. We first need to derive this correction for our method as well. That is slightly more complicated than in the case of linear correction since we do not have a formal prescription of our function. We circumvent this by calculating our integral given by relation (3.3.1) numerically, where we are again only interested in the value in position $W(0,0)$, where we for each value of transmission t sum together the average minimal values of our WF (since we already know that our method shifts minimal values into position $W(0,0)$) for all measured positions q that are of course each multiplied by the differential.

The comparison is shown in Fig.III.9.

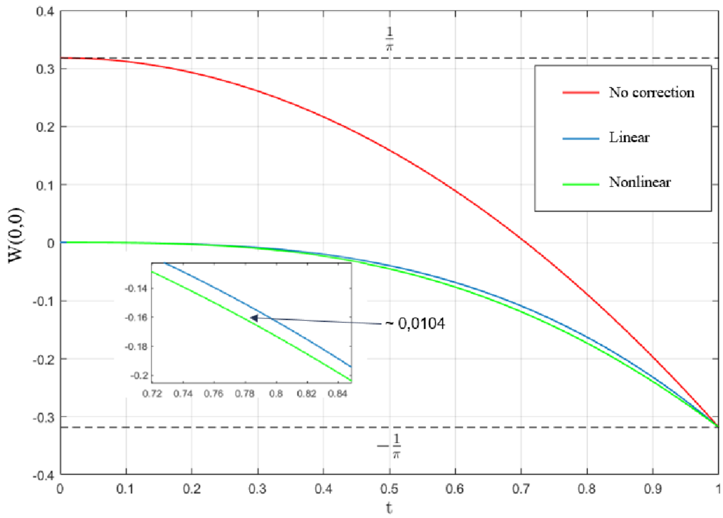

Figure III.9: Comparison of values in W(0,0) based on transmission for linear, nonlinear and no corrections

The crucial part of this graph is the comparison of linear and nonlinear methods. As we can see at worst in the case of transmission going towards zero the final value of the WF will also go to zero and at best with transmission going toward 1 both methods expectedly go towards the value of $-\frac{1}{\pi}$. It is interesting to note that for $t = 0$ the linear correction is not defined while the nonlinear is. This does not necessarily mean that the nonlinear method is better as in this case, we lose all quantum interference anyway. The more interesting behaviour can be seen for t going roughly from 0.4 to 0.9 where the nonlinear method gives us slightly more negative values with the maximum difference being approximately 0.0104 or roughly an 8% improvement. This clearly shows us that the nonlinear method is indeed more efficient than the linear one and so there is a point in examining it more for more complex systems to see just how better it potentially can be.

Chapter IV

Conclusion

In this work, we have shown that adding nonlinear feed-forward to CV quantum erasing enhances our ability to obtain a quantum state with negative Wigner function. This was previously done using a linear correction. This function, dependent on the measured shift of position in phase-space q, when used to displace the WF (this operation being the quantum erasure) resulted in a WF with larger negativity. Our contribution was coming up with an alternative method of correction to improve this mitigation of quantum decoherence. We did this by displacing the WF by the value corresponding to its minimum position. This shifts the minimal value of the WF into the position of $W(0,0)$ or the same position, where the WF of the single-photon state has its minimal value. We defined this function numerically using MatLab by writing an algorithm that found the position of x where the WF was minimal in relation to the measured quadrature q. This function was our nonlinear correction that we then compared to the linear correction used by the standard. For this, we defined a metric to compare the effectiveness of these methods. We have chosen to compare the negativity in position $W(0,0)$ based on different values of transmission. The reason for this is, that our goal was to have a final WF on the output be as close as possible to the WF of the single-photon state on the input and also because the negativity cannot be classically simulated [12]. This WF has its minimal value $-\frac{1}{\pi}$ in position $W_{|1\rangle}(0,0)$. Therefore the closer we are to this negativity in the same position at the output WF, then the better the correction in restoring it to its beginning state. After comparing these two methods we arrived at the conclusion that the nonlinear correction is indeed better at restoring the negativity for the medium values of transmission even for this admittedly simple system. While this difference is quite small, with the biggest difference being 0.0104, we were working with a system, where we can expect that fine-tuning will not have that much of a big impact. The crucial result is that there definitely is a difference and therefore, it is worth trying to use this method on more complex systems. This is also the direction in which our further research will lead. For some next steps, we can use different states. Squeezed vacuum states can replace the vacuum state and we can generalize the single-photon state to multiple photons. Adding more optical devices or using a different detection (measuring $\hat{x}$ and $\hat{p}$

simultaneously) can also be potentially tried and we can also examine a different metric to compare the effectiveness of these corrections (E.g. purity, fidelity). So in conclusion our work has revealed an interesting new path of research that can hopefully with further work lead to some intriguing destinations.

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