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UTILIZATION OF GEOGRAPHICALLY WEIGHTED REGRESSION (GWR) IN FORESTRY MODELING

DIPLOMA THESIS

2014/2015 **Ing. María Quirós Segovia**

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Utilization of Geographically Weighted Regression (GWR) in Forestry Modeling

ABSTRACT

The diploma thesis is focused on the application of the Geographically Weighted Regression (GWR) in forestry models. This is a prospective method for coping with spatially heterogeneous data. In forestry, this method has been used previously in small areas with good results, but in this diploma thesis it is applied to a bigger area in the Region of Murcia, Spain. Main goal of the thesis is to evaluate GWR for developing of large scale height-diameter model based on data of National Forest Inventory of Spain. Final model is compared with local height-diameter model on validation plots. The obtained results are very different according to the level of input data and GWR calibration type. The best result is obtained with individual tree data and with fixed kernel calibration. In order to improve quality of GWR calibration several suggestion were made, such as change weighted functions.

GWR method is highly promising because in the case of need a particular model for some large area, there is possible to make sufficiently precise model at any point of the area of interest without need of any additional measurements in particular forest stand.

Key words: Spatial analysis, Spatial heterogeneity, Geographically Weighted Regressions.

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Využití geograficky vážené regrese (GWR) v lesnických modelech

ABSTRAKT

Diplomová práce je zaměřena na použití geograficky vážené regrese (GWR) v lesnických modelech. Je to perspektivní metoda pro analýzu prostorově heterogenních dat, přičemž v lesnictví byla tato metoda již používaná dříve v menších oblastech s dobrými výsledky. V této práci je metoda aplikována na větším území v regionu Murcia ve Španělsku. Hlavním cílem práce je ověření možnosti použití GWR pro odvození velkoplošně platného modelu výškové funkce na základě dat Národní invetarizace lesů Španělska. Výsledný model je pro validační plochy porovnán s lokálními výškovými funkcemi. Získané výsledky se liší pro různé úrovně vstupních dat a různé způsoby nastavení a kalibrace GWR. Nejlepší výsledky vykazuje model založený na individuálních stromových datech a kalibračním jádru s pevnou šířkou. V rámci práce byly take navrženy možné změny a náměty pro další výzkum, které pravděpodobně přispějí ke zlepšení kvality GWR modelu.

Metoda GWR se jeví jako vysoce perspektivní, zvláště pro velkoplošné modely, kdy při správném provedení umožníodvodit dostatečně přesný model výškové funkce v jakémkoliv bodě zájmové oblasti bez nutnosti přímého měření výšky a tloušťky v daném porostu.

Klíčová slova: prostorová analýza, prostorová heterogenita, geograficky vážená regrese, GWR.

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LIST OF ABBREVIATIONS

AIC: Aikake Information Criterion
ANOVA: Analysis of Variance
Appr.: Approximately
CART: Classification and Regression Trees
Cca.: Circa
COK: Cokriging
CV: Cross-Validation
DBH: Diameter at Breast Height
Dg: quadratic mean diameter
ED: European Datum
e.g.: Exempli gratia
GAM: Generalized Additive Model
Gha: Basal area per hectare
Hdom: Dominant Height
Hmed: Average Height
Ht: Total Height
IDW: Inverse Distance Weighting
i.e.: id est
incG: Increment of Basal area
incV: increment of Volume
km: kilometre
LiDAR: Laser Illuminated Detection And
LME: Linear Mixed-Effects model
LMM: Linear Mixed Models
GAM: Generalized Additive Models
GAWR: Geographically and Attitudinal
GLLMM: Geographically Local Linear Mixed
GIS: Geographical Information Systems
GLM: Generalized Lineal Models
GLS: Generalized Least Squares with a non-
GWR: Geographically Weighted Regression
m: meters
MCD: Minimum Covariance Determinant
MLP: Multi-Layer Perceptron
mm: millimetres
Nha: Number of Trees per Hectare
NPP: Net Primary Production
OLS: Ordinary Least Squares
OK: Ordinary Kriging
QGIS: Quantum GIS
QB: Quickbird
R2: Coefficient of determination
RBF: Radial Basis Function
RMSE: Root mean standard error model
SANOVA: Specific Analysis of Variance
SDI: Stand Density Index
SH%: Spatial Heterogeneity Percent
USA: United States of America
UTM: Universal Transverse Mercator
Vha: Volume per hectare
VIF: Variance inflation factor

I. INTRODUCTION

Traditional regressions are actually very well known. This contains several methods for modelling relationships among one dependent variable and one or more independent variables. There exists a big diversity of model types, such as linear regression, multiple linear regression, non-linear regression etc. The main task of the regression is to get quantitative relationships between variables.

Traditional linear regressions models with standard estimation techniques make a number of assumptions about the predictor variables, the response variables and their relationship. These assumptions are as follows (Nau, 2014):

- Linearity and additivity of the relationship between dependent and independent variables.
- Normality of the error distribution.
- Homoscedasticity (constant variance) of the errors.
- Statistical independence of the errors, this assumes that the errors of the response variables are uncorrelated with each other.
- Lack of multicollinearity in the predictors.

Sadly, not all data fulfil these conditions. In fact, in most of the real data (forestry data included) we can find that the data is structured. This data is structured mostly because of the following reasons:

- Spatial variability, data varies from place to place and depends of environmental conditions.
- Temporal variability, data varies according with the time.
- Hierarchical structure, data belonging to one level of measurement takes part into more general data level (e.g. tree→stand→forest). The main problem with structured data is that data is correlated, so variables are not independent. A linear regression cannot correctly explain real datasets and obtained results and conclusions could be probably wrong.

There are several methods to solve this problem, for example models based on heteroskedasticity specification, linear mixed models, generalized additive models, classification and regression trees and finally parametric instability models, e.g. geographically weighted regression (GWR) among others. The origin of GWR was in the area of geography and sustainable development but also has been applied in other areas as economy with very good results. In the forestry sciences we count with little experience with it, especially in the case of its applications in large areas. GWR attempts to capture spatial variation by calibrating a multiple regression model that allows different relationships between variables to exist at different points in space (Zhang, et al., 2004). GWR have several advantages when it is compared with other methods, for instance, it is widely applicable to almost any form of spatial data and it uses geographic information as well as attribute information among others.

The spatial modelling made by GWR describes the influence of the geographical position of the trees in their state of competition, their grow potential and the impact of the management activities. This made this regression a powerful tool for understand and improve the current management processes of some area.

II. OBJECTIVES

The diploma thesis is focused on the application of the Geographically Weighted Regression (GWR). Because it is a prospective method for coping with spatially heterogeneous data that are typical in forestry. In this field of study, this method has been used previously in small areas with good results. The objectives of this diploma thesis is to evaluate this method for large scale study. For this purpose we are going to use data of *Pinus halepensis Mill.* pure forests for selected area of Spain. As a model, we are going to use height curve, it means height-diameter relationship, because this model is very suitable from methodological point of view because of its simplicity of application to the data.

The probable positive result in the application of geographically weighted regression in large areas would be very important under the practical point of view, especially in very large countries as Spain because it can make using of the forestry models highly more effective. The spatial modelling made by GWR, besides modelling the height diameter relationship itself, would be able to describe the influence of the geographical position of the trees in their growth potential. The methodological conclusions based on relatively simple height-diameter model can be applied in a further research for tree growth and volume models.

The main objectives of the thesis are as follows:

1. A review concerning the spatial heterogeneity in forestry modelling and possibilities how to cope with it.
2. Application of geographically weighted regression in large area
3. Solution of basic methodological problems in its application (especially bandwidth and kernel type selection, calibration of the model on training data set, etc.).
4. Application of GWR on height-diameter model of validation plots and comparison GWR model with local models from area of interest.
5. Applicable recommendations and conclusions concerning both statistical comparison with traditional methods and practical usability.

III. LITERATURE REVIEW

1. SPATIAL ANALYSIS

According to Fotheringham and Rogerson (2009), spatial analysis is a general term to describe a technique that uses locational information in order to better understand the processes generating the observed attribute values. Spatial analysis extracts or creates new information from spatial data and try to determine how spatial patterns are generated by one or several processes (Fortin and Dale, 2005).

This spatial analysis is quite different from the traditional or non-spatial analysis. Firstly, the traditional techniques developed for non-spatial data, are not valid for spatial data, because spatial data have unique properties and problems that demand a different set of statistical techniques and modelling approaches. The main problem and difference in both analysis is related with the location at which the regression is undertaken, this function may vary over the space in different locations. Subsequently, statistical analyses for spatial data have to deal with two potential types of local variation: the local relationship being measured in attribute space and the local relationship being measured in geographical space (Fortin and Dale, 2005), this means that spatial analysis deal with attribute information as well as geographical information.

Furthermore, we can find different kinds of spatial analysis:

- Descriptive spatial statistics that are similar to the descriptive traditional statistics are used for a variety of purposes in geography, particularly in quantitative data analyses involving Geographic Information Systems.

- Spatial pattern analysis that try to identify if the data follow some pattern or have some structure. It can be used for a lot of purposes, for instance, in criminology, ecology or medicine.

- Spatial analysis that try to find and measure spatial relationships between variables involved in the study.

Spatial analysis has become one of the most rapidly growing field in ecology and forestry sciences. This popularity is related with three factors: (1) a growing awareness among scientist that it is important to include spatial structure in ecological thinking; (2) the alteration of landscapes around us at an increasing rate, which requires a constant re-evaluation of their spatial heterogeneity; (3) and the availability of software designed to perform spatial analyses (Fortin and Dale, 2005).

1.1. Local vs Global Statistics

When an analysis in the real worlds is made, it is logical to think that same stimulus makes different reaction depending of the location, type of soil, climate etc. Consequently, this natural reactions cannot be explained by the simple "global" models. The global models summarizes the characteristics of the spatial pattern over the whole study area, but in the other hand local statistics makes explicit the differences in the pattern observed among parts on the study area

(Fortin and Dale, 2005). This global models do not work with heterogeneity of spatial data (they dealing with them as they would be homogenous) and it often produce conclusions that would be probably biased.

Several differences exist between global and local models, which can be summarize in the following ones (Table 1). (1) Global statistics are typically single-valued, local statistics are multi-valued (different values of the statistic can occur in different locations in the study region). Consequently, (2) global statistics are non-mappable - they cannot be analysed with GIS. Local statistics are GIS friendly and can be mapped and examined within a GIS. (3) Local statistics are therefore spatial statistics but global statistics are spatially limited. (4) Finally, by their nature local statistics underline differences across space but global statistics lead into thinking that all parts of the study region can be represented by a single value or some existing pattern. This shows that local statistics are useful in searching for exceptions or what are known as local hot spots (Fotheringham, et al., 2002).

Table 1 - Main properties of global regression VS local regression. (Fotheringham, et al., 2002)

Global	**Local**
Summarize data for whole region	Local disaggregation of global statistic
Single-valued statistic	Multi-valued statistic
Non-mappable	Mappable
GIS-unfriendly	GIS-friendly
Spatially limited	Spatial
Emphasize similarities across space	Emphasize differences across space
Search for regularities or laws	Search for exceptions or local hot spots
Example; classic regression	Example: GWR

1.2. Spatial Autocorrelation and Spatial Heterogeneity

In spatial modelling there are two phenomena that cannot be ignored, spatial heterogeneity and autocorrelation. Both concepts are certainly linked and they will be explained in the next part.

1.2.1.Spatial Autocorrelation

When a spatial analysis is implemented, is crucial to determine whether the data lack independence: and if so, what is the nature of the spatial dependence. We trust on independence to allow us to make trustworthy interpretations and predictions (Fortin and Dale, 2005). For this, it is very important the concept of spatial autocorrelation.

Cliff and Ord (1973) define spatial autocorrelation as: '*If the presence of some quantity in a sampling unit (e.g., a county) makes its presence in neighbouring sampling units (e.g., adjacent counties) more or less likely, we say that the phenomenon exhibits spatial autocorrelation*'. This type of autocorrelation tests the Tobler's first law of geography, '*everything is related to everything, but near things are more related than distant things*' (Tobler, 1970). It implies that the relationship among the values of a given variable is a function of the spatial distances between them or their locations in space. Hence, the notion of spatial dependence implies that there is a lack of independence between data from nearby locations (Fortin and Dale, 2005).

There are mainly two causes of autocorrelation which can be named in two categories, ‘spurious’ and ‘real’. Spurious autocorrelation is an artefact of experimental design that use to happen when samples have not been randomly chosen but can occur as a result of some other aspect of the experimental design. In other hand, real autocorrelation can be defined as caused by the interaction of a response variable with itself or with independent variables due to some inherent characteristic of the variables (Zuur, 2008). The first case is called univariate spatial autocorrelation and the second is known as multivariate spatial autocorrelation.

For measure and detect spatial autocorrelation, there is common to use the Morans’s I coefficient, developed by Patrick Alfred Pierce Moran (Moran, 1950). This coefficient has similar design as Pearson correlation index. Its values oscillate between 1 and -1, where the 1 means perfect positive autocorrelation (perfect correlation), the -1 value means perfect negative autocorrelation (perfect dispersion) and 0 incomes a completely random spatial pattern that is the most desirable (see Figure 1).

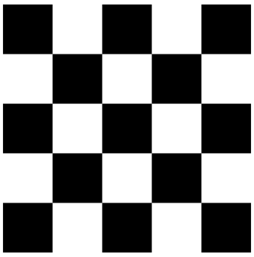


Figure 1 - the white and black squares are perfectly dispersed so Moran's I would be −1. If the white squares were stacked to one half of the board and the black squares to the other, Moran's I would be close to +1. A random arrangement of square colours would give Moran's I a value that is close to 0. Source: Wikipedia, 2014.

1.2.2. Spatial Heterogeneity

When a non-spatial regression was made, generally the heterogeneity of the data never was assumed. The relationships modelled were supposed to be the same everywhere within the study area. Talking about spatial data it cannot be assumed the homogeneity of the data, this condition is known as spatial heterogeneity (Charlton and Fotheringham, 2003).

Spatial heterogeneity refers to the uneven distribution of a feature, event, or relationship across a region (Anselin, 2010), and is defined as structural instability in the form of systematically varying model parameters or different response functions (Anselin, et al., 1988). It is related to locations in space, missing variables, and functional misspecification (Anselin 1988).

In forestry, spatial heterogeneity is hypothesized as one of the major drivers of biological diversity (Wiens, 1976). Spatial heterogeneity results from the spatial interactions between a number of biotic and abiotic factors and the differential responses of organisms to these factors (Milne 1991). It may have significant influences on many ecosystem processes at multiple spatial scales (Turner 1989). The spatial heterogeneity of vegetation patterns (i.e., landscape heterogeneity) is a structural property of landscapes (Li and Reynolds 1994) that can be defined by the complexity and variability of ecological systems’ properties in space. The spatial heterogeneity of a tree variable (growth, total height, diameter...etc.) in a forest stand results from the complex historical and environmental mosaic imposed by competition and systematic environmental heterogeneity. This implies a spatio-temporal heterogeneity. Forest researchers have realized that spatial pattern of tree locations strongly affect (1) competition among neighboring trees, (2) size variability and distribution, (3) growth and mortality and (4) crown structure (Zhang, et al., 2004). Ignoring spatial heterogeneity in forest modeling, causes biased parameters estimates, misleading significance test, and sub optimal prediction (Anselin, et al., 1988).

There are several reason why we expect measurements of relationships to vary over space. An obvious reason in related with the sampling variation. This variation is uninteresting in that it relates to a statistical artefact and not to any underlying spatial process (Fotheringham, et al., 2002). A second possible cause is that the model from which the relationships are estimated is a gross misspecification of reality and that one or more relevant variables are either omitted from the model or are represented by an incorrect functional form, mapping local statistics is useful in order to understand more clearly the nature of model misspecification (Fotheringham, et al., 2002). And the last cause is that some relationships are intrinsically different across the space, this is what is called real heterogeneity (Danlin and Yehua, 2013).

As Anselin recommends (Anselin 1988), there are three powerful reasons why heterogeneity should be studied: (I) the structure undelaying the spatial instability is geographic, therefore the localization of the point is essential for define the form or specify this variability. (II) With spatial data, the heterogeneity occurs in conjunction with the autocorrelation problem. Then, traditional analysis and habitual heteroskedasticity contrasts can be biased in a spatial context. (III) In a regression model, both effects of autocorrelation and spatial heterogeneity can be fully equivalent. For example, a "cluster" or residual spatial clustering (seen in very nearby locations) with extreme values could be interpreted as a problem of spatial heterogeneity (heteroskedasticity groups or "groupwise"), or as an effect of spatial autocorrelation. Finally, traditional regression model can distinguish different specifications for spatial heterogeneity effect, manifested as heteroskedastic or structural parametric instability (Chasco, 2004).

Consequently for cope with spatial heterogeneity two options exist (Figure 2), (I) to specify spatial heteroskedasticity or (II) to use parametric instability models, and this one is the most important from our point of view.

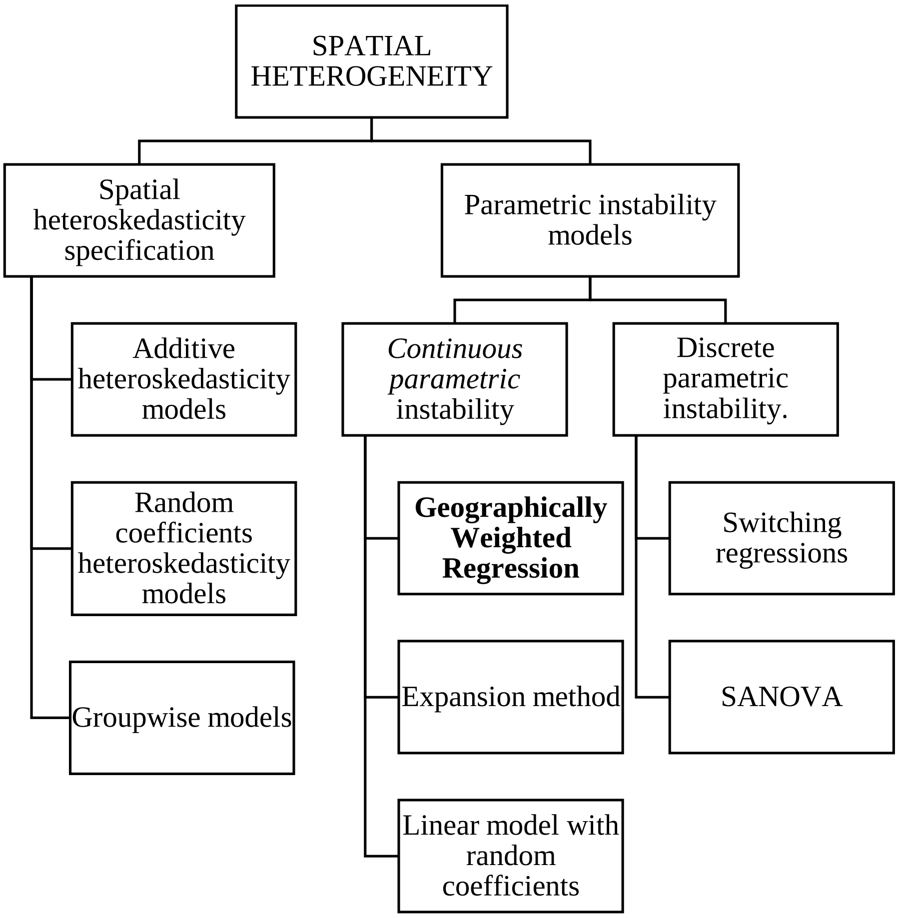


Figure 2 - Methods for handle spatial heterogeneity.

1.2.2.1. Spatial Heteroskedasticity Specification

Heteroscedasticity refers to the circumstance in which the variability of a variable is unequal across the range of values of a second variable that predicts it. The causes of the existence of heteroskedasticity in spatial regression model, according to Chasco (2004), would be: (I) Using data from irregular spatial units, i.e. with different area or territorial extension (countries, regions, provinces ...). (II) Treatment of geographic units in which a phenomenon is distributed irregularly in space. (III) Oversight of relevant variables or other model misspecification, which occur in the random disturbance term non-constant variance. (IV) Some causes of spatial heterogeneity can also cause the appearance of spatial autocorrelation, the contrast of both effects could be necessary in this cases.

Three models exist that are able to specify the heteroskedasticity of the data: (1) additive heteroskedasticity models, (2) random coefficients heteroskedasticity models and (3) groupwise models (Chasco, 2004).

1.2.2.2. Parametric Instability Models

As its name shows, parametric instability models refers to the lack of stability on the space (spatial variability) of a variable (Chasco, 2004). In this situations, as both the functional form and the parameters of the regression may differ in every geographic location being therefore not homogeneous throughout the sample data. In this group of models we can find two kind of models with *continuous parametric instability* and *discrete parametric instability*.

- Continuous parametric instability models: In this case the coefficients associated with the explanatory variables may take a different value, or for each sample observation, as in the linear model with random coefficients (Hildreth-Houck, 1968) or as an expansion variable. Which may or may not be spatial, as in the model called *expansion method* (Casseti, 1997) or the *geographically weighted regression*, (Fotheringham, et al., 2002), that will be explained deeply in this thesis.

- Discrete parametric instability models: this is a specific case of parametric instability that prevents an overall estimation of different coefficients for the entire data sample (N) by dividing it in a limited number (n) of different structures (where n < N), overcoming the problem of incidental parameters (lack of degrees of freedom) and obtaining efficient estimates. There are two distinct specifications for discrete parametric instability, the *space ANOVA model* (SANOVA) (Griffith, 1992), applicable rather in a context of univariate exploratory analysis, and *model changing regressions* ("switching regressions") (Quandt, 1958).

As conclusion, spatial heterogeneity occurs when it is used spatial data. This effect of spatial heterogeneity usually occurs in conjunction with the effect of spatial autocorrelation. Then, the traditional statistical analysis is no longer adequate for study this data (i.e. models commonly used use to be biased in a spatial analysis) and is necessary different statistical tools for solve this problem.

1.2.2.3. Other Models for Deal with Spatial Heterogeneity

- Linear mixed models

Linear Mixed Models (LMM) are statistical models for continuous outcome variables in which the residuals are normally distributed but may not be independent or have not constant variance. Study designs leading to data sets that may be appropriately analyzed using LMMs include (1) studies with clustered data (autocorrelation) and (2) longitudinal or repeated-measures studies, in which subjects are measured repeatedly over time or under different conditions (West et al., 2007). This model works in an intermediate way between local and global models. LMM gives better results than Ordinary Least Squares (OLS), because LMM takes into account spatial and temporal dependence.

A linear mixed model can be expressed as:

$$\boldsymbol{Y} = \boldsymbol{X\beta} + \boldsymbol{Z\gamma} + \boldsymbol{\varepsilon} \quad (1)$$

where **Y** is a vector of the observed response variable, **X** is a known model matrix including a column of 1 (for intercept), $\boldsymbol{\beta}$ is a vector of unknown fixed-effects parameters, **Z** is a known design matrix, $\boldsymbol{\gamma}$ is a vector of unknown random effects parameters, and $\boldsymbol{\varepsilon}$ is a vector of unobserved random errors. LMM can be used to model spatial correlation among observations in data through R =Var(ε) such that:

$$\boldsymbol{R} = \boldsymbol{Cov}(\boldsymbol{\varepsilon_i}, \boldsymbol{\varepsilon_j}) = \boldsymbol{\sigma^2 \mathrm{f}(d_{ij})} \quad (2)$$

Where d_{ij} is the distance between locations i and j. Different functions f(d_{ij}) are available including spherical, exponential, Gaussian, power, etc. (Zhang et al., 2005).

- Generalized additive models

Generalized additive models (GAM) were originally developed by Trevor Hastie and Robert Tibshirani (1990) to blend properties of generalized linear models (GLM) with additive models. GLM emphasizes estimation and inference for the parameters of the model but GAM focuses on exploring data non-parametrically. The strength of GAM is its ability to deal with highly non-linear and non-monotonic relationships between the response variable and the set of explanatory variables (Zhang et al., 2005). GAM can be expressed by:

$$\boldsymbol{Y} = \boldsymbol{S_0} + \sum_{\boldsymbol{g=1}}^{\boldsymbol{p}} \boldsymbol{S_g}(\boldsymbol{X_g}) + \boldsymbol{\varepsilon} \quad (3)$$

Where $\boldsymbol{S_0}$ is the intercept, and $\boldsymbol{S_g(X_g)}$ is a nonparametric smoothing function for the g^{th} independent variable $\boldsymbol{X}$.

The only underlying assumption is that the smoothing functions in GAM are additive. This additive restriction allows us to interpret a GAM model in a similar way as a traditional linear regression model.

- Classification and Regression trees (CART)

It is a non-parametric method that builds classification and regression trees for predicting continuous dependent variables (regression) and categorical predictor variables (classification) - see Figures 3 and 4. The CART methodology was introduced in 1984 by Leo Breiman, Jerome Friedman, Richard Olshen and Charles Stone as an umbrella term to refer to the following types of decision trees:

- Classification Trees: where the target variable is categorical and the tree is used to identify the "class" within which a target variable would likely fall into.

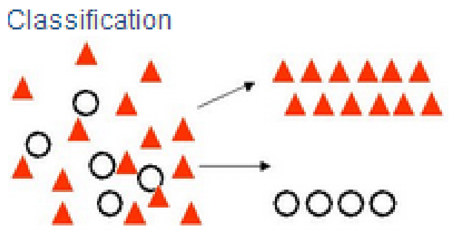


Figure 3 - Example of classification trees. (Rao, 2013)

- Regression Trees: where the target variable is continuous and tree is used to predict its value.

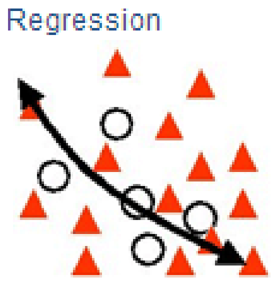


Figure 4 - Example of Regression tree. (Rao, 2013)

The CART algorithm is structured as a sequence of questions, the answers to which determine what the next question, if any should be. The result of these questions is a tree like structure where the ends are terminal nodes at which point there are no more questions (Rao, 2013).

2. INTRODUCTION TO GEOGRAPHICALLY WEIGHTED REGRESSION

The geographically weighted regression (GWR) is a kind of spatial analysis for explore spatial non-stationary or spatial heterogeneous processes (Brunsdon, et al., 1996). The underlying idea of GWR is that parameters may be estimated anywhere in the study area given a dependent variable and a set of one or more independent variables which have been measured at places whose location is known (Charlton and Fotheringham, 2009). GWR extends OLS linear regression models by accounting for spatial structure and estimates a separate model and local parameter estimates for each geographic location in the data based on a 'local' subset of the data using a differential weighting scheme (Mattews and Yang, 2012). This means that is possible to say that GWR is similar to a 'spatial microscope' in reference to the ability to measure and visualize variations in relationships that are unobservable in non-spatial, global models (Mattews and Yang, 2012).

2.1. Theoretical Background

The following information has been taken mainly from two books, "*Geographically Weighted Regression: the analysis of spatially varying relationships*" (Fotheringham, et al., 2002) and "*Comparison of bandwidth selection in application of geographically weighted regression: a case study" (Guo, et al., 2008).*

Considering the global regression model:

$$\boldsymbol{y_i = \beta_0 + \sum_k \beta_k x_{ik} + \varepsilon_i} \qquad (4)$$

GWR extends this traditional regression (2) by allowing local parameters, instead global, to be estimated so the model is rewritten as:

$$\boldsymbol{y_i = \beta_0(u_i v_i) + \sum_k \beta_k(u_i v_i) x_{ik} + \varepsilon_i} \qquad (5)$$

Where $(\boldsymbol{u_i v_i})$ denotes the coordinates of the ith point in the space and $\boldsymbol{\beta_k(u_i v_i)}$ is a realization of the continuous function $\boldsymbol{\beta_k(uv)}$ at point *i*. That is, is allowed there to be a continuous surface of parameter values, and measurements of this surface are taken at certain point to denote the spatial variability of the surface. Is important to notice that equation (4) is special case of (5) in which the parameters are assumed to be spatially invariant. Thus the GWR equation (5) recognizes that spatial variation in relationships might exist and provides a way in which they can be measured.

Very important part of GWR is its calibration. We need to calibrate GWR function for each independent variable $\boldsymbol{X}$ and at each geographic location *i*. The estimation procedure of GWR is as follows:

1. Draw a circle of given bandwidth, *h* around one particular location *i* (centre).
2. Calculate a weight for each proximal observation according to the distance between the neighbour and the centre.
3. Estimate the model coefficients using weighted least squares regression, such that:

$$\hat{\boldsymbol{\beta}}_{\boldsymbol{i}} = (\boldsymbol{X}^{\boldsymbol{T}}\boldsymbol{W}_{\boldsymbol{i}}\boldsymbol{X})^{-1}\boldsymbol{X}^{\boldsymbol{T}}\boldsymbol{W}_{\boldsymbol{i}}\boldsymbol{Y} \qquad (6)$$

Were $\boldsymbol{W}_i$ is a geographical weight matrix for the center *i*, such that $\boldsymbol{W}_{\boldsymbol{i}} = \boldsymbol{f}(\boldsymbol{d}_{\boldsymbol{i}}, \boldsymbol{h})$, where ***f()*** is a spatial kernel function, $\boldsymbol{d}_{\boldsymbol{i}}$ is a distance vector between the centre *i* and all neighbours, and *h* is a bandwidth or decay parameter (see Figure 5).

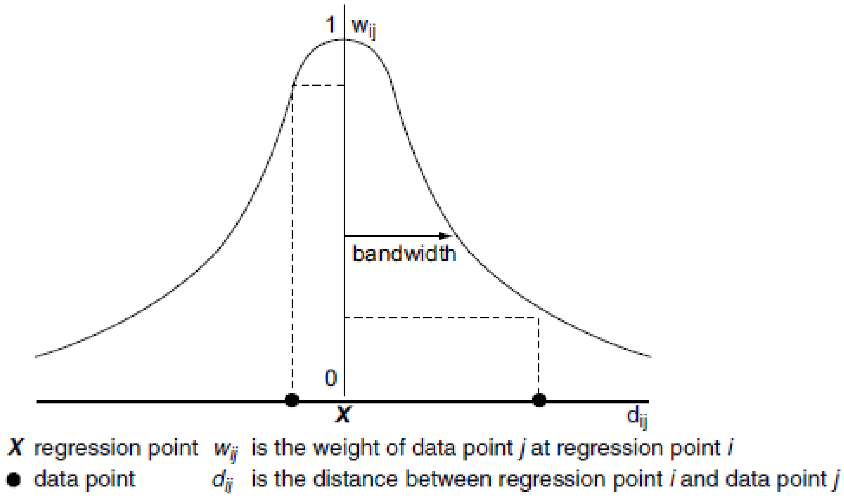


Figure 5 - A spatial kernel. (Fotheringham, et al., 2002)

In the basic GWR, two kernel functions exist for accomplish this calibration, the fixed spatial kernel function and the adaptive spatial kernel function. In general, geographically weighted regression calibration uses regions described around regression points *i* and all the points in this regions were used to calibrate a model. Each data point is weighted by its distance from the regression point and data that is closer are weighted more heavily than are data points farther away. For example, for a given data point, the maximum weight is given when it shares the same location as the regression point. This weight decrease as the distance between the two points increases. The full regression model is calibrated locally by the moving the regression point across the region. For each location the calibration is different.

2.1.1.Fixed Kernel Function

A fixed spatial kernel function can be used to define the geographical weight matrix $\boldsymbol{W}_{\boldsymbol{i}}$, such as the Gaussian distance- decay kernel function:

$$w_{ik} = e^{-(\frac{d_{ik}}{h})^2} \quad (7)$$

The fixed kernel function assumes that the bandwidth at each centre *i* is a constant across the study area (see Figure 6). If the locations *i* and *k* are the same (d_{ik}=0), $w_{ik} = 1$**;** whereas $\boldsymbol{w_{ik}}$ decreases according to a Gaussian curve as $\boldsymbol{d_{ik}}$ increases. Nevertheless, the weights are nonzero for all data points, no matter how far they are from the centre *i*.

A potential problem with the fixed kernel function is that for some locations in the study area there are only few data points available to calibrate the model if the data is sparse around the centre location, that is 'weak data' problem. In this case the local models might be calibrated on very few data points, this may cause large standard error and resulting surfaces that are under smoothed. And even can be impossible if there are insufficient data.

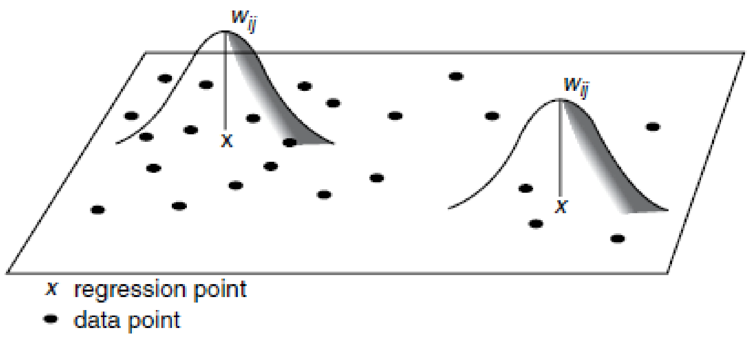


Figure 6 - GWR with fixed kernel (Fotheringham, et al., 2002).

2.1.2. Adaptive Kernel Function

To reduce the problem with the 'weak data' the spatial kernels in GWR can be made to adapt themselves in size to variation in the density of the data so that the kernels have larger bandwidths where the data is sparse and have smaller bandwidths where the data is plentiful. A commonly used adaptive kernel function is a bisquare distance decay kernel function:

$$w_{ik} = \left[1 - \left(\frac{d_{ik}}{h_i}\right)^2\right]^2 \; when \; d_{ik} \leq \; h_i$$

$$(8)$$

$$w_{ik} = 0 \; when \; d_{ik} > \; h_i$$

Weight is $w_{ik} = 1$ at the center **I** and $w_{ik} = 0$ when the distance equals the bandwidth. When the distance is greater than the bandwidth, the weight is 0. The bandwidth is selected such that the number of observation with nonzero weights is the same at each location *i* across the study area. Consequently, the adaptive kernel function adapts itself in the size to the variation in the density of data. It has larger bandwidths where the data is sparse and smaller ones where the data is denser (see Figure 7).

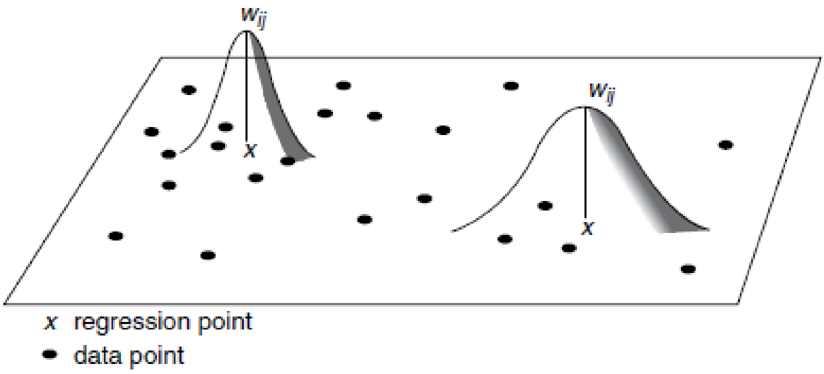


Figure 7 - GWR with adaptive kernel. (Fotheringham, et al., 2002)

2.1.3. Determining the Bandwidth

Once selected the proper kernel function, there are three possible strategies for determining the bandwidth. (1) Predefined bandwidth, (2) a technique based on a least squares cross-validation (CV) (equation 9) and (3) a method that minimizes the Aikake information criterion (AIC) for fitting the regression model.

Cross-validation method (CV) minimizes the squared error:

$$\boldsymbol{CV} = \sum_{\boldsymbol{i}} [\boldsymbol{y_i} - \boldsymbol{y_{\neq i}}(\boldsymbol{h})]^{\boldsymbol{2}} \quad (9)$$

where $\boldsymbol{y_{\neq i}}(\boldsymbol{h})$ is the fitted value of $\boldsymbol{y}_i$ with the observation for location i omitted from the estimation process.

Finally, there are some important facts about the kernel selection studied by Guo (2008) that should be mentioned.

- GWR models with smaller bandwidths fit the data better, yielded smaller model residuals across tree sizes, significantly reduced spatial autocorrelation and heterogeneity for model residuals, and generated better spatial patterns for model residuals; however, smaller bandwidth sizes produced a high level of coefficient variability.
- GWR models based on the fixed spatial kernel function produced smoother spatial distributions for the model coefficients than those based on the adaptive kernel function.
- The GWR cross-validation or Akaike's information criterion (AIC) optimization process may not produce an ''optimal'' bandwidth for model fitting and performance. It was evident that the selection of spatial kernel function and bandwidth has a strong impact on the descriptive and predictive power of GWR models.

Finally, GWR results can be mapped in a visualization tool such as GIS to explore spatial heterogeneity or non-stationarity across the study area (Zhang, et al., 2004).

3. SUMMARY OF GEOGRAPHICALLY WEIGHTED REGRESSION PROPERTIES IN COMPARISON WITH OTHER TYPES OF SPATIAL ANALYSIS

Geographically Weighted Regression method have several and very powerful gains over other spatial analysis methodologies. Mainly for testing the heterogeneity of the study area, for the ability of working with spatial and attribute data, and other practical and technical advantages that will be explained next, we think that GWR method is appropriate for working with forestry data, which are entirely heterogeneous according with their natural behavior.

This advantages has been already mentioned along this document, but as a summary the most important are the following ones:

(1) Let us move from a global perspective to local analysis of the problem obtaining better detail and precision (Duque, et al., 2011).

(2) Facilitates the exploration of the spatial structure of the model, this means, measure the degree of spatial dependence (autocorrelation) in the model - if be positive or negative, or detect data clusters (Anselin, 1988). Similarly, the spatial heterogeneity is tested in this method and it is supposed that everything is related, especially things that are closer (Danlin and Yehua, 2013).

(3) It is possible calculate local indexes for each spatial unit based on the values of a set of neighbouring observations. This lets to know how locally the regression combine the variables for achieve a "specific fit" in a specific location (Fotheringham, et al., 2002).

(4) The disaggregation of the global R-square coefficient in local coefficients and the analysis of its geographical distribution allows the recognition of what point in the independent variables have greater or worse explanatory power (Fotheringham, et al., 2002).

(5) GWR is widely applicable to almost any form of spatial data (Danlin and Yehua, 2013).

(6) GWR is an advance spatial technique because it uses geographic information as well as attribute information.

(7) Employs a spatial weighting function with the assumption that near places are more similar than distant ones, it means that the geographical position really matters in the moment of calculate the regression.

(8) Residuals from GWR are generally much lower and usually much less spatially dependent (Zhang, et al., 2004; Zhang and Shi, 2004) compared with other methods, especially ordinary least square, linear mixed model and generalized additive model.

(9) Is very easy to use GWR together with Geographical Information Systems (GIS). It is really easy to make big variety of maps with the results of the analysis (R-squared, dependent and independent variables, coefficients…) (Mennins, 2006).

(10) Is possible to generate interpolated surfaces to know the continuous spatial distribution of the parameters and apply the principles of "spatial prediction" to find the values of missing observations (Páez, 2006).

IV. MATERIALS AND METHODS

4. MATERIALS

4.1. Overview of the Study Area

4.1.1.Location

The Region of Murcia (see Figure 8) is an Autonomous Community of Spain located in the southeast of the state, between Andalusia, Castile-La Mancha, Valencian Community, and on the Mediterranean coast. The Region of Murcia is bordered by the provinces of Almeria and Granada (Andalusia); the province of Albacete (Castile–La Mancha), which was historically connected to Murcia until 1980; province of Alicante (the Valencian Community); and the Mediterranean Sea. The community measures 11,313 km² and has a population of 1.4 million.

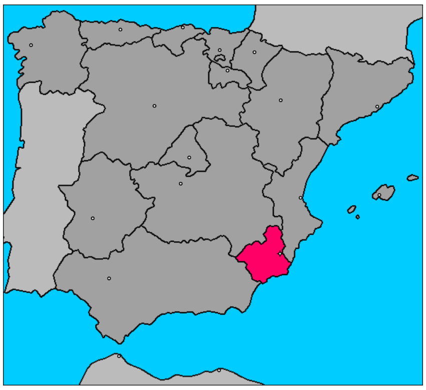


Figure 8 Region of Murcia Location.

4.1.2.Climate

The region of Murcia in south east part of Spain is in a zone of subtropical climate. Its orographic distribution makes difficult maritime Atlantic influences; this way it presents a clear influence of Mediterranean see in all the climatic particulars.

There are two seasons with defined characteristics (summer and winter) and other two transitional seasons (autumn and spring). There is an absence of cold season, temperatures below 0 are really strange (see Figure 9). The warm season is from June to October, and there are common heat waves (tropical sub-Saharan air) where is possible see white skies and very high temperatures. The precipitation rate is very low during the year. All the region is under 700mm per year.

Wind is one of the most important climatic factors in the region, due to the transfer of atmospheric action centres governing weather and climate throughout the year on the Peninsula. The barrier effect of the Betic Cordilleras favours the direction of the south west.

Murcia region can be divided in five climate regions:

- Zone I: Areas above 800m and limited with Granada and Albacete. It has cold season with temperatures under 7ºC during 5-7 months. Precipitation rate is around 500mm, with a dry season of 4-6months. According to H. Walter and H. Lieth (1967), it is climate area IV.

- Zone II: Areas above 600m and limited with Albacete and Alicante. Is transition area.
- Zone III: Areas between 400-800m and areas near Segura River. The cold season is 3-5 months with average temperature 1-7ºC. Warm season of 2-3months with average temperature of 25º-27ºC. The climate area is III and IV.
- Zone IV: Areas under 400m and closer to sea. Not real cold season, the average of cold months are around 8-11ºC, and warm season average temperature is 26º-28ºC. Climate area III.
- Zone V: Area around coast (200m-0m). There are not cold season at all, the average temperature in the coldest month is 13ºC this is due to the coast influence. There are dry period of 11-12 months and the precipitation rate per year is 150-200mm. Climate area III.

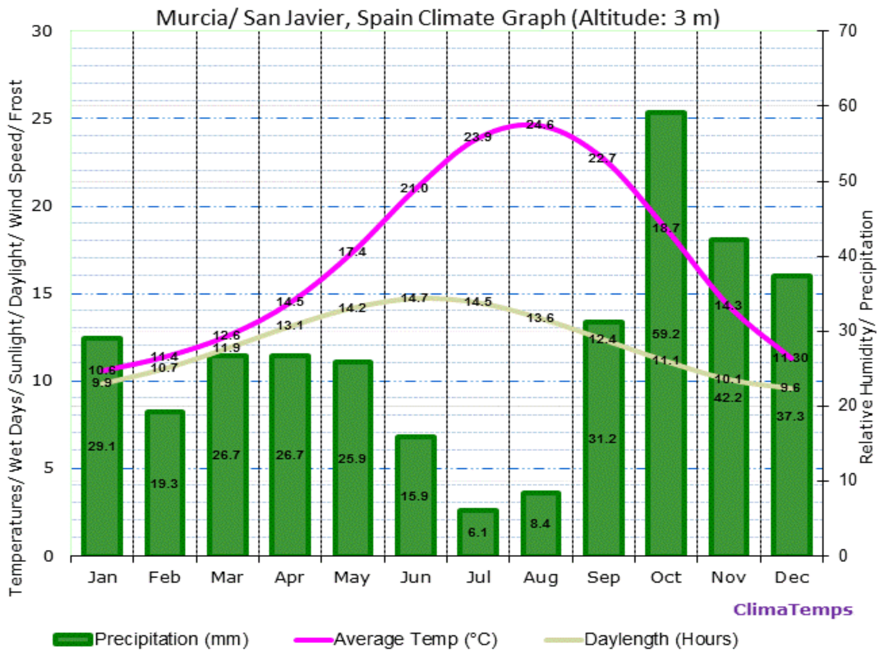


Figure 9 - Provincia de Murcia Climate, (San Javier observatory) España. Instituto Nacional de Meteorología [www.inm.es]. Years 1971-2000.

4.1.3. Geology, Edaphology and Hydrology

The region is located in the eastern part of the Betics range mountains and it is influenced by their orography. These are chains of Alpine folding, affected by regional scale faults and remaining activity from the Upper Miocene. Betics mountain ranges are divided as well in the Prebetic, Subbetic and Penibetic mountain ranges (Figure 10).

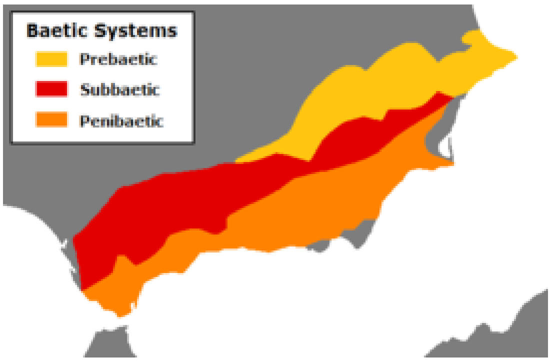


Figure 10 - Baetic Systems. (Wikipedia, 2014)

Traditionally it has been considered that the peak of Revolcadores, in the range of the same name, was the highest point in the Region of Murcia, with a height of 2027 meters; but in measurements of the most recent maps of the SNIG (National Service of Geographic Information of Spain), Revolcadores appears with a height of 1999 m, and the mountain, Los Obispos ("The Bishops"), located slightly further north, is higher (2015 m). Approximately 27% of the Murcian territory can be described as mountainous, 38% as intra-mountainous depressions and running valleys, and the remaining 35% as flat lands and plateaux.

The edaphology of Murcia region presents unevolved soils, with low amount of horizons and not easily differentiated. The amount of organic matter generally is not big. Nitrogen in soils has an organic origin, the levels of nitrogen are similar than levels of organic matter. The concentration of available phosphorus in soils is usually low. The cation exchange capacity of the soil use to be in average way (Region de Murcia, 2014). It is possible to find six types of soils: (1) Uninvolved soils, (2) developed soils on Quaternary calcareous sediments, (3) Developed soils on consolidated limestone, (4) Alluvial soils, (5) Soils over siliceous materials and (6) Saline soils.

If we talk about hydrology, the hydrographic network of the region is made up of the Segura River and its effluents. The river Mundo, it is the one that contributes to the Segura with the greatest volume. Other rivers are Alhárabe and its affluent, the Benamor; Mula River, Guadalentín, Sangonera and Reguerón (which is born upper before town of Lorca). Due to the water supplying incapacity of the Segura river basin, contributions to this river basin are made from the basin of the Tajo River, by means of the Tajo-Segura transvasement.

The greatest natural lake of Spain can be found in the region: the Mar Menor (Small Sea) lagoon. It is a salt water lagoon, adjacent to the Mediterranean Sea. Its special ecological and natural characteristics make the Mar Menor a unique natural place and the largest saltwater lake in Europe. With a semi-circular shape, it is separated from the Mediterranean Sea by a sand strip of 22 km in length and between 100 and 1200 m wide, known as La Manga del Mar Menor (the Minor Sea's Sandbar). The lagoon has been designated by the United Nations as a Specially Protected Zone of Importance for the Mediterranean. Its coastal perimeter accounts for 73 km of coast in which beaches follow one another with crystal clear shallow water (the maximum depth does not exceed 7m). The lake has an area of 170 square kilometres.

4.1.4.Flora and Fauna

The climatic conditions of Murcia region, makes flora highly variable around the region. In inner mountain areas, use to rain more and the flora is similar to other places of the Betics Mountains. There are the presence of *Pinus halepensis Mill. (Figure 11)* and *Pinus pinaster* Aiton. and *Pinus* nigra J.F. Arnold *subsp. clusiana* (Clemente) Rivas-Mart., *Quercus ilex L subs ballota*, *Quercus rotundifolia* Lam., *Juniperus oxycedrus* L., *Juniperus communis* L. subsp. *hemisphaeric*a (C. Presl) Nyman, *Arbutus unedo* L. and finally some relict areas of *Quercus suber L.* In humid areas of the north part can be found species of *Sorbus aria* (L.) Crantz, *Fraxinus angustifolia* Vahl. and *Quecus faginea* Lam.

Figure 11 - Pinus halepensis cone. (Wikipedia, 2014)

Some inner ranges suffered high deforestation along centuries, this made problematic flooding processes along the valleys. This made that Spanish foresters start in this areas a fast process of forestation, the first forestation was made in Sierra Espuña in the end of XIX century. In this moment started a big recovery of forests in all Murcia region, but mainly in the center area. In areas closer to Almeria province, in Andalusia, there is a low pluviometric average. In this area we can find semiarid areas, in terms of flora it is possible to find a big amount of endemism and afro-endemism. Some examples of this species are *Juniperus phoenicea* L., *Tetraclinis articulata* (Vahl) Mast., *Chamaerops humilis* L., *Rhamnus hispanorum* Gand., *Anthemis chrysantha* J.Gay, *Thymus moroderi* Pau ex Martínez, *Periploca angustifolia* Labill., *Astragalus nitidiflorus* Jiménez Pau. In this semiarid area the most common plant is the *Stipa tenacissima* L. More information about current situation of vegetation in Murcia is in the Figure 12.

The different kinds of climatic conditions in Murcia makes this region very rich also in fauna species, with some endemic species. Firstly, we can find raptors like *Aquila chrysaetos* L., *Aquila fasciata* V., *Hieraaetus pennatus* Gmelin., *Circaetus gallicus* G., *Falco peregrinnus* T. and *Bubo bubo* L. In some ranges like 'Carrascoy' and 'El Valle' exists a big communities of *Bubo bubo*. In higher ranges “Sierra de Mojantes” and “Sierra del Gigante” it is possible to find *Gyps fulvus* H. About mammals is possible to find *Capra pyrenaica* S., and *Cervus elaphus*. In the 70’s it was introduced as game specie the *Ammotragus lervia* P., this specie currently is extended in some mountains of south Spain. Also are important species *Sus scrofa* L., *Felix silvestris* S., *Meles meles* L., *Vulpes vulpes* L. There are native species like *Sciurus vulgaris hoffmani* Valverde. In the river areas around Segura and some inflowing as river Alhárabe recently we can find *Lutra lutra* L.

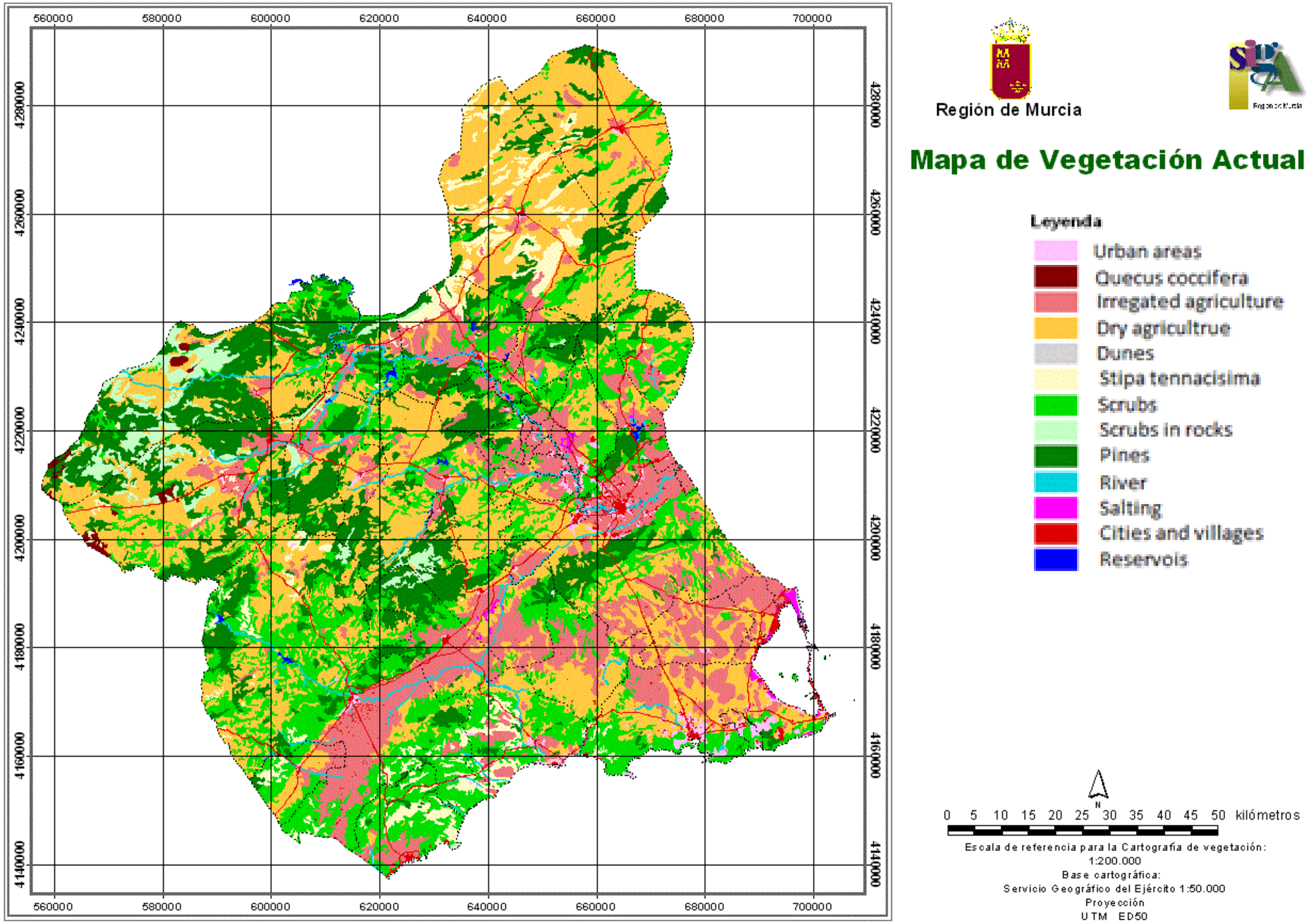


Figure 12 - Actual vegetation map. Source (Region of Murcia, 2014)

4.2. Data Sources and Operational Tools

4.2.1. National Forestry Inventory of Spain

In this research the data were obtained of the Third National Forestry Inventory of Spain. It has been elaborated between 1997 and 2007. In Spain National Forest Inventories are made every 10 years, which means that actually the Fourth National Inventory is under construction. In the Third Inventory are included all the forest systems: the wooded, as it was done in the previous Inventories, and the un-wooded species, like bush, brushwood and herbaceous (Villaescusa Sanz, 1997).

This inventory was made with the objective of show and evaluate the state of the forest in Spain though 100 state index and evolution, for instance, area, tree and bush species, growing, distribution and soil features. Also include data of biodiversity, forest health, silviculture, recreational importance and sustainable management (MAGRAMA, 2014).

- Design of the Forest National Inventory

The design of the inventory was made in order to be accurate for measure very different kinds of forest (even age, uneven age, dense or spread forest, industrial reforestations...). For this reasons, it was decided measure in plots with variable radio, which is a cheap method to get better estimations of forest variables (Schreuder et al., 1993). IFN uses circular plots composed by other subplots of 5, 10, 15 and 25 meters of radio and in every plot the tree diameter that can be measured is different (Figure 13). These plots are distributed in UTM grids of 1km aside (MAGRAMA, 2014) and the plot's centre is in the vertex of the grid.

The data collection is according with the specie, localization with polar coordinates, perpendicular diameters of trees at breast height (1.3 meters) (DBH), total height of tree (Ht), quality and other parameters.

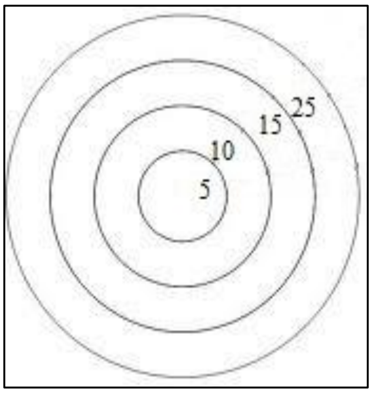


Figure 13 - Spanish National Inventory design. (Aguirre, 2012)

Tree distance to the centre of the plot	**DBH to measure**
≤ 5 m	75mm ≤ d < 125 mm
≤ 10 m	125 mm ≤ d < 225 mm
≤ 15 m	225 mm ≤ d < 425 mm
≤ 25 m	425 mm ≤ d

- Data used in the thesis

For this study, pure stands of *Pinus halepensis Mill.* were used. This pine can reach 20 m height, its crown is sparse and irregular. It is very resistant to the drought (minimum water requirements of 250 mm of water per year) and we can find it from heights around 1300m to the sea level. This pine is widespread around all the region but the best forests are in the northeast (Valle del Ebro, Iberian range and Pre-Pyrenees ranges). This tree has been traditionally used for resins and in natural medicine. The quality of the wood is not good for processing (e.g. construction, furniture), so the wood is used for firewood.

National Forest Inventory provides us basic data of the measured trees (height, diameter, coordinates, species...). Some of this data has been provided by professor Sonia Condés of the Polytechnic University of Madrid, consultant of this thesis.

We have a sample of 5352 *Pinus halepensis's* plots in Spain, with the stand variables: number of trees per ha (Nha), basal area per ha (Gha), quadratic mean diameter (dg), top height (Hdom), mean height (Hmed), Reineke stand density index (SDI), volume per ha (Vha), increments of basal area (incG) and volume (incV), and finally, coordinates of plots which are in European Datum 1950 (ED-50). And all *Pinus halepensis* in this plots are measured with the tree variables: diameter at breast height (DBH), total height (Ht), and individual coordinates of trees etc.

4.2.2. R Statistic Programming Language

The statistic programming language R is a freeware for statistical computing, analysis and graphics. R was created by Ross Ihaka and Robert Gentleman at the University of Auckland, New Zealand, and is currently developed by the R Development Core Team. R is an implementation of the S programming language combined with lexical scoping semantics inspired by Scheme. S was created by John Chambers while at Bell Labs. There are some important differences, but much of the code written for S runs unaltered (Wikipedia, 2015).

R provides a wide variety of statistical (linear and nonlinear modelling, classical statistical tests, time-series analysis, classification, clustering etc.) and graphical techniques (R Development Core Team, 2011). R can be easily extensible thanks to the "packages", which are collections of R functions, data, and compiled code in a well-defined format. The directory where packages are stored is called the library. R comes with a standard set of packages. Others are available for download and installation (Kabacoff, 2014). The most important packages used in this study: car, lmtest, nlstools, nls2, spgwr, sp, nortest.

4.2.3. Geographical Information System

A geographic information system (GIS) is a computer system designed to capture, store, manipulate, analyse, manage, and present all types of spatial or geographical data. The most used and famed GIS is ArcGIS, it is a geographic information system for working with maps and geographic information. It is used for creating and using maps; compiling geographic data; analysing mapped information; sharing and discovering geographic information; using maps and geographic information in a range of applications; and managing geographic information in a database (ESRI, 2014).

As alternative to ArcGIS is QGIS that is a cross-platform free and open source desktop geographic information systems application providing data viewing, editing, and analysis capabilities (Sherman, 2014). This program has the advantage to be free.

In this diploma thesis has been used QGIS. The used tools and modules were: (1) interpolation tool for rastering GWR results, (2) Geospatial Simulation plugin for transform raster to polygons, and (3) the vector tool intersect for obtain the final desired GWR parameters.

5. METHODS

In this diploma thesis, we want to evaluate whether National Forest Inventory data from relatively large area can be used for sufficiently precise height-diameter model for selected point of area of interest. Furthermore, because there are two levels of data ((I) average data of plots and (II) data of individual trees), another objective is evaluate if the model derived from average plot data can be sufficiently precise in comparison with model derived from individual trees data. Both models were compared with local height curve computed from all trees measured on respective plot.

This will be made in two ways.

1. Applying plot average information (dg, Hmed, coordinates of the centre of plot) in GWR and comparing the resulting model with local models derived from direct measured information of trees (DBH, Ht).
2. Applying measured information of individual trees (DBH, Ht, coordinates of individual trees) in GWR and comparing the resulting model with local models of height curve based on individual trees.

5.1. Analysis Procedure

The Province of Murcia was selected because it has suitable properties for such type of study – area is large enough (11 313 km²) and there is a relative high number of plots with pure stands of *Pinus halepensis*. Positions of individual plots can be seen on Figure 14.

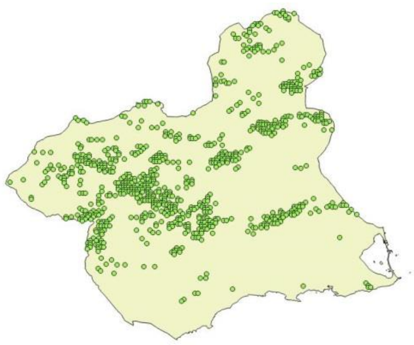


Figure 14 - Sampled plots in Murcia Region

The procedure of the analysis consists of following steps:

1) **Area Selection**.
2) **Sample selection**. Two different samples were randomly selected.
   a) Training data set (appr. 2/3 of all plots) – this set of data were used for estimating of model parameters
   b) Validation data set - this set of data were used for validation of final model
3) **Height-diameter function. -** We must select suitable height function that can be converted into linear form because of easy implementation in GWR.
4) **GWR model fitting**
   a) Finding of the best type and bandwidth of GWR kernel

b) Application of GWR model for the whole area

c) Extracting of model parameters for validation plots

5) Comparison of GWR model vs local height-diameter models

a) Application of height-diameter models with parameters derived from GWR on validation plots

b) Computation of height-diameter local models with parameters derived from individual trees of respective plots

c) Comparison of both models

6) Evaluation of the results for the whole area

This step will be explained in detail in the following text.

5.2. *Area Selection*

In Murcia, we count with 835 plots with pure *Pinus halepensis and* 8358 trees were measured on the plots.

A summary of the data can be found in the Table 2.

Table 2 - Summary statistics, Murcia sample

	Nº trees/ha	**dg(cm)**	**G(m^2/ha)**	**H(m)**
Average	255.160	17.560	4.690	6.030
Standard deviation	272.770	6.520	4.300	1.850
Minimum	5.090	7.500	0.390	2.000
Maximum	1750.700	54.000	26.120	14.570

According to the Table 2 it is possible to observe that forests are not very dense and the number of trees per ha is very different from one plot to another, this shows very irregular forest. This high variability across the area is very challenging for large scale GWR model and it is probable that parameter estimation will be complicated. Spatial distribution of selected variables is described in Figure 15.

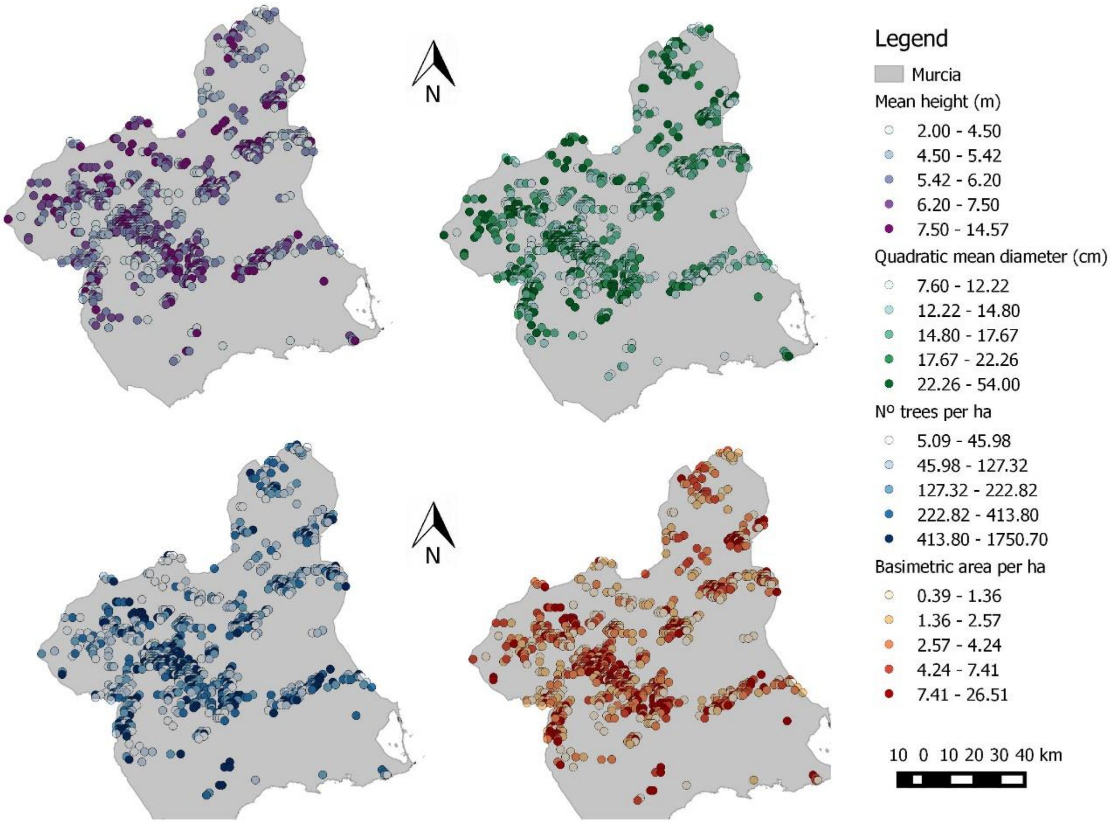


Figure 15 - Murcia's average forestry characteristics

5.3. Sample Selection

Within the Murcia region, two thirds of the plots was used as training data and the other third as the validation data set. These plots were chosen randomly without replacement.

For each training sample plot we were used both average variables (the quadratic mean diameter (dg) and the mean height (Hm) and variables for individual trees (total height (Ht) and the diameter at breast height (DBH)) for comparison of two types of GWR models.

In the other hand, the *validation data set* is composed by the individual trees that compose that plots (measured diameter at breast height and total height). This set will be used for validate the final model.

Originally, we wanted to work with complete Murcia region, but the big amount of data in Murcia makes problematic to work with all the trees, it caused unacceptably long time of computations (several days in some cases for individual calibration or estimation of model). In order to avoid that problem, we had to select smaller region, preferably composed by a regular grid of plots and with a big number of trees for accomplish the analysis.

As a result, Espuña, Cambrón, Burete Lavia and Quipar ranges has been selected (Figure 16). Inside the mountain ranges previously selected we can find 234 plots with 2931 trees.

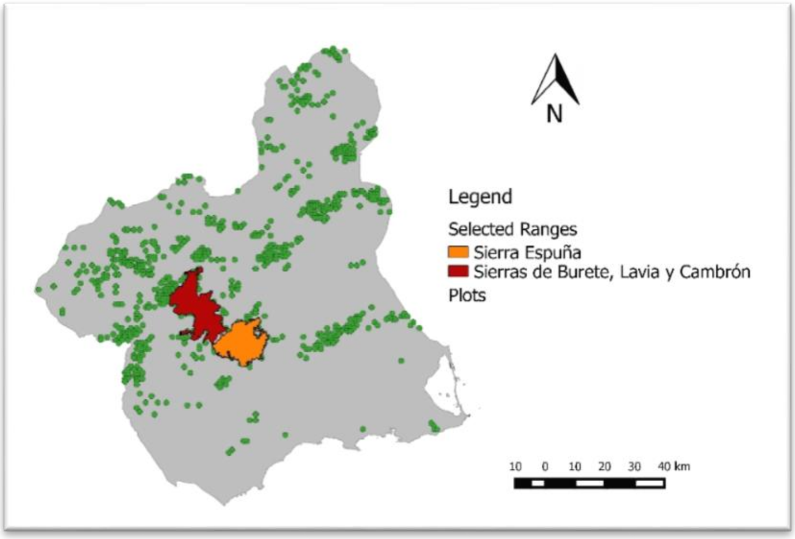


Figure 16 - Selected area for the study

Within this area, two samples were made. Two thirds of the data (156 plots and 1697 trees) have formed 'training data set' and the other third (78 plots) were used as the validation data set (Figure 17). Sampling was made randomly without replacement.

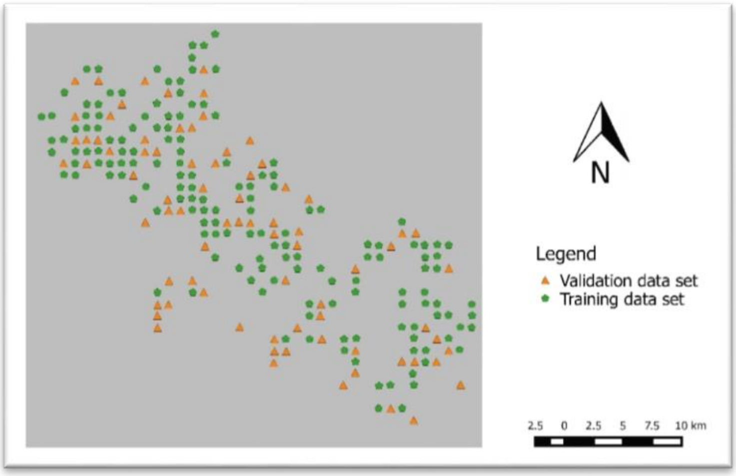


Figure 17 - Validation and training data sets

5.3.1. Statistical Characteristics - Training Data Set

In the Table 3, there is a statistical comparison among both training data sets. We can see that diameter values have higher variability in comparison with the height that remains with low standard deviation.

Table 3 - Training data set

	TRAINING DATA SET			
	Average values of plots		**Individual trees**	
	Height	**Diameter**	**Height**	**Diameter**
Average	6.450	18.180	8.570	21.390
Standard deviation	1.840	6.130	2.580	8.310
Minimum	3.000	8.000	2.000	7.500
First quantile	5.160	13.340	6.700	15.050
Median	6.260	16.670	8.500	19.850
Third quantile	7.500	21.220	10.000	25.700
Maximum	14.100	35.300	21.500	55.700

According to Figure 18 and 19, we can see that both average data and tree data is slightly asymmetric but there are no very distant extremes. The degree of asymmetry is higher in the case of DBH.

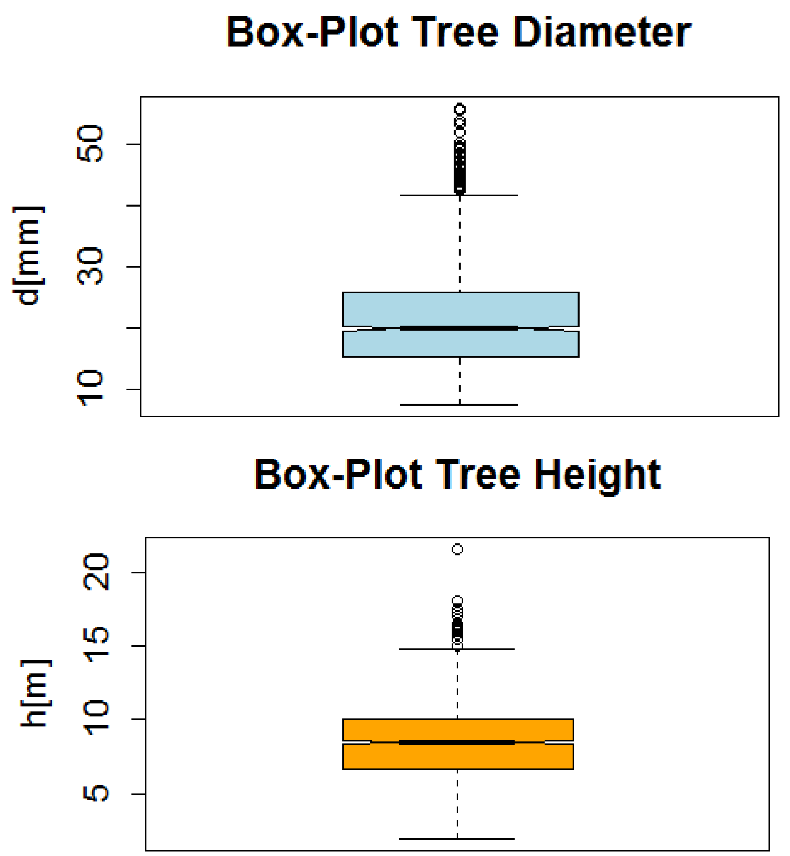


Figure 18 - Training data set Box-Plots for individual trees

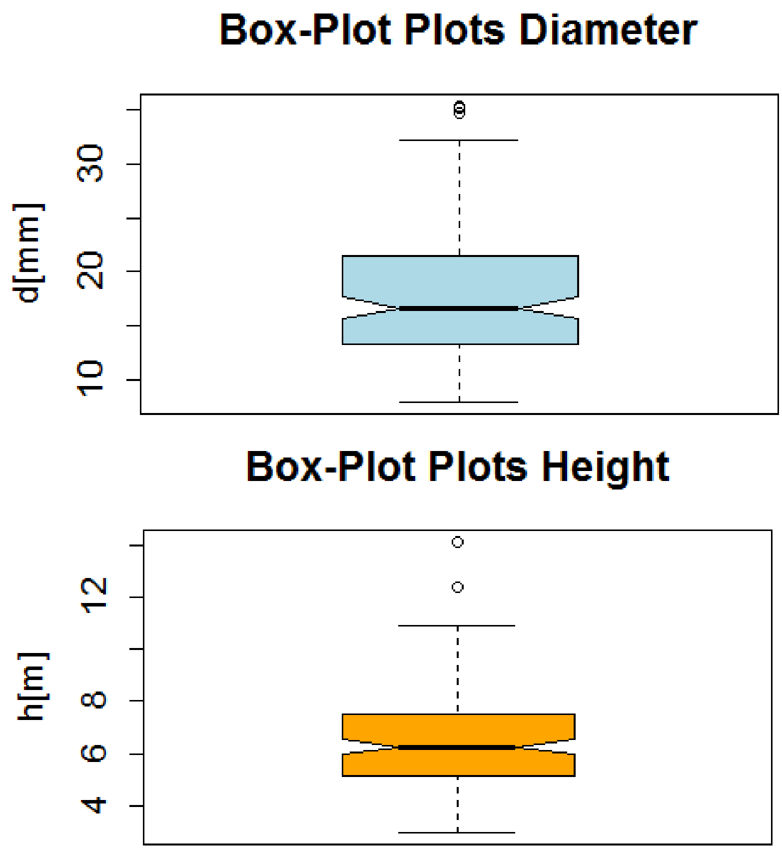


Figure 19 - Training data set Box-Plots for plots

5.3.2. Statistical Characteristics - Validation Data Set

The validation statistical characteristics (Table 4) are similar to the training data set relative to trees. It is logical, because both data sets come from the same population by random sampling.

Table 4 - Validation data set

	VALIDATION DATA SET	
	Height	**Diameter**
Average	8.010	19.950
Standard deviation	2.070	7.400
Minimum	2.400	7.550
First quantile	6.500	14.300
Median	8.000	18.900
Third quantile	9.200	24.400
Maximum	15.500	65.200

In the Figure 20, we can see that in general the validation data set follows same pattern that training tree data set.

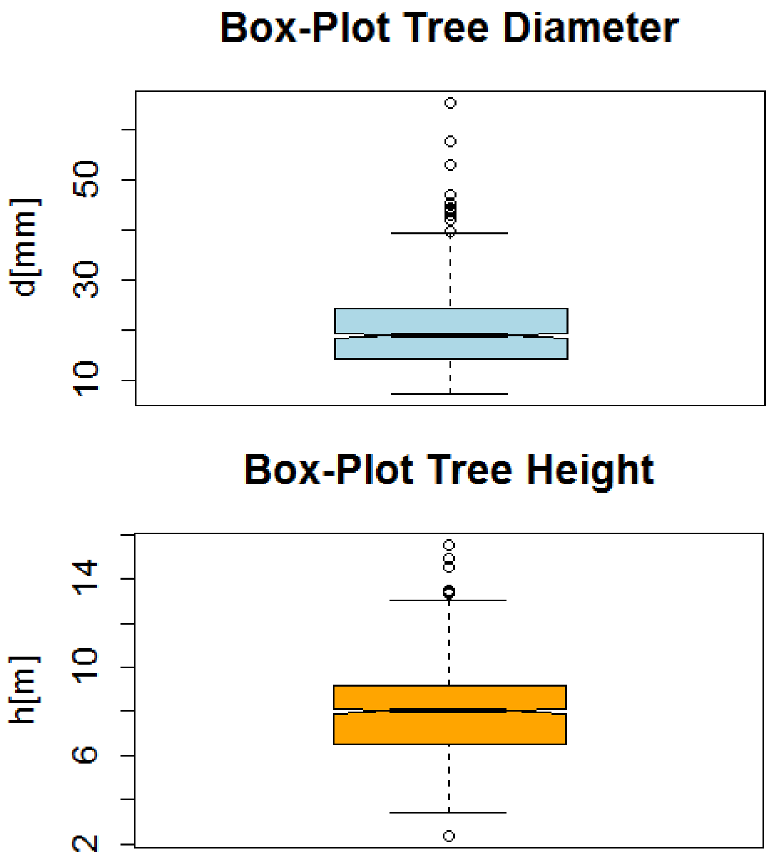


Figure 20 - Validation data set Box-Plot

5.4. Height- Diameter Function

In this study Petterson (Petterson, 1955) height-diameter formula was used

$$h = 1.3 + \frac{1}{\left(a+\frac{b}{d}\right)^3} \qquad (10)$$

This equation (10) has been selected because it was used with very good results in many previous studies (eg. Adamec, 2014, Drápela, 2011), it was also recommended by respected yield science textbooks (eg. Pretzsch, 2009) and it is possible to linearize it. It was applied for obtain OLS coefficients and GWR coefficients.

The way how to linearize this formula is the following one:

$$\frac{1}{\sqrt[3]{h-1.3}} = \mathrm{a} + \mathrm{b} * \frac{1}{d} \qquad (11)$$

Equation (10) would be easily computed in R programing language by nonlinear models, but for correct application in GWR is needed to linearize it.

If we take $H = \frac{1}{\sqrt[3]{h-1.3}}$ and $= \frac{1}{d}$, we can assume equation (11) as H=a + b*D, which is linear regression that can be easily implemented in GWR. It was checked that models of linearizable equation and nonlinear equation do not differ, just make easy the operation in GWR. For all tests we used 95% significance level.

5.5. *GWR model fitting*

5.5.1. Finding the optimal bandwidth. GWR method overview

The first step for correct implementation of GWR method is the selection of suitable kernel and its bandwidth. Kernel type tells us whether we should define our bandwidth based on distance (fixed) or number of neighbours (adaptive). As it was said in the literature review we count with three options to calibrate the GWR model, (1) adaptive kernel with and bisquare weighted function, (2) fixed kernel with a Gaussian weighted function and (3) predefined bandwidth where the user choose the bandwidth. According to Luo Guo's article *(*Guo, et al., 2008*)* GWR model made by adaptive kernel gives smaller bandwidths, fits data better and has smaller model residuals with reduced spatial autocorrelation and heterogeneity. On the other hand, model based on fixed spatial kernels produces smoother spatial distributions.

In this study all the possibilities were checked in order to obtain the best possible result. In the case of predefined bandwidths, we select values of 1km and 500m. The reason for this selection is that 1km is the distance between inventory plots and 500m is the half of this distance.

5.5.1.1. Calibrations

- Adaptive kernel

It was used the adaptive kernel with the bisquare distance decay kernel function (equation 8) (Fotheringham, et al., 2002). The bandwidth was optimized by the method which minimize AIC.

- Fixed kernel

It was used the fixed spatial kernel with Gaussian distance- decay kernel function (equation 7) (Fotheringham, et al., 2002). The bandwidth was optimized by the technique based on a least squares CV that minimizes the squared error (equation 9).

- Predefined bandwidths

It was used two predefined bandwidths in accordance with the National Forest Inventory grid of 1000 m (distance between grid points) and 500 m.

All kernel types and bandwidth functions are summarised in Figure 21.

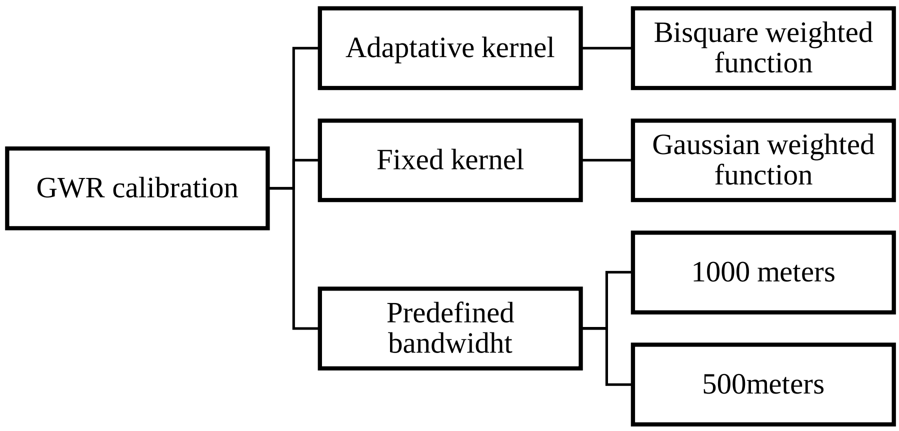


Figure 21 - Kernel and bandwidth function types for the GWR calibration

5.5.2.GWR application to analysis area

All above mentioned types of GWR (adaptive, fixed kernel and different bandwidths) were applied both to Plots training data set and to Tree training data set.

Results are presented both in graphical and numerical outputs. Graphical output is represented by map of interpolated parameters which represents the values of the parameters around all the selected area. Numerical output consists of the following values:

- Coefficients of the regression.
- The Akaike information criterion (AIC) - which is a measure of the relative quality of a statistical model for a given set of data.
- The residual sum of squares (RSS) - it is a measure of the discrepancy between the data and an estimation model.
- R squared - a number that indicates how well data fit a statistical model.

5.5.3. Extracting model parameters

GWR has the advantage that we can estimate the value of parameter for any point of the area of interest. Therefore we can compare model obtained by GWR with the model obtained by local regression. The desirable result is that both models should be as close as possible.

We used GIS tools for extracting parameter values for validation plot positions. For make this comparison we count with powerful tools as the GIS. This tool will be used as follows:

- Interpolation of the coefficients obtained in GWR analysis were made (156 plots and 1697 trees). The result was four raster layers, two with coefficients relative to GWR applied to plots and likewise, two relative to GWR applied to tree data set.
- The four interpolation rasters were transformed to polygon (vector) by the tool called 'Geospatial simulation' in QGIS.
- Validation data set (vector) and polygon layer were intersected. In this way, the values of "GWR" coefficients were obtained for all validation plots.

5.6. Compare GWR and Local Regression Results

When parameters of GWR models are known for all validation plots we can compute and visualised all height-diameter models based on these parameters.

Local Petterson function was applied on validation plots (78 overall). Because some of validation plots have only a few trees and local model would be very unreliable, we used for comparison with GWR models only plots with 10 and more trees.

The comparison of models consists of two parts.

a) Curves comparison: both curves relative to local regression and GWR regression of both training data sets were computed and visualised in plot. In total there are five curves in every image: Petterson local regression, curve made by Adaptive GWR parameters, curve made by Fixed GWR parameters, curve made by 1000 meters bandwidth parameters and curve made by 500 meters bandwidth parameters. In this way we can visually compare the similarity or dissimilarity of local model (it is considered as the most precise model) with all types of GWR models.

b) Residual analysis and regression diagnostics in the Table 5 there are all the formulas that were applied for the numerical evaluation of residuals and selected regression diagnostics. Results of this analysis were presented as the average number of these values in all validating plots.

Table 5 - Residual formulas. Legend: e, value of residuals; n, sample size; m, number of parameters; y_i measured value of individual i; $\hat{y}_i$, model value of individual i; $\overline{y_i}$ average value of all individuals I; $\hat{y}_{GWR}$ model value of GWR model, $\hat{y}_{PET}$ model value of local regression model

Mean value of residuals	$\bar{e}_i = \frac{\sum e_i}{n}$
Standard deviation of residuals	$\sigma_{e_i} = \sqrt{\frac{\sum(e_i - \bar{e}_i)^2}{n}}$
Standard error of residuals	$SE_{e_i} = \frac{\sigma_{e_i}}{\sqrt{n}}$
Aikake information criterion	$AIC = n * \ln\left(\frac{\sum {e_i}^2}{n}\right) + 2 * m$
Coefficient of determination	$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \overline{y_i})^2}$
Root mean standard error model	$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n-m}}$
Mean value of deviation between GWR and local models	$\Delta_i = \frac{\sum_{i=1}^{n}\lvert(\hat{y}_{GWR} - \hat{y}_{PET})\rvert}{n}$

- Mean value of residuals: the difference between the observed value of the dependent variable and the predicted value is called the residual. The average of residuals is the mean value of residuals. The ideal value of this criterion is zero.
- Standard deviation of residuals: is a measure that is used to quantify the amount of variation or dispersion of the residuals. The smaller value the better.
- Standard error of residuals: is the standard deviation of the sampling distribution.
- Aikake information criterion: is a measure of the relative quality of a statistical model for a given set of data. If a single model is to be selected as the best, then this should be the one with the lowest *AIC.*
- Coefficient of determination: is a number that indicates how well data fit a statistical model. It will give some information about the goodness of fit of a model. In regression, the R^2 coefficient of determination is a statistical measure of how well the

regression line approximates the real data points. An R^2 of 1 indicates that the regression line perfectly fits the data.

- Root mean standard error model (RMSE): refer to the amount by which the values predicted by an estimator differ from the quantities being estimated.
- Mean value of deviation between GWR and local models: It explains the existent difference between the GWR and local models.

V. RESULTS

6. *GWR APPLIED TO PLOTS TRAINING DATA SET*

In the Figures 22, 23, 24 and 25, there are the results of interpolation of parameters of four studied GWR types along with the training plots. Table summarises basic regression statistics.

In this situation fixed kernel gives the worst result and adaptive kernel is slightly better.

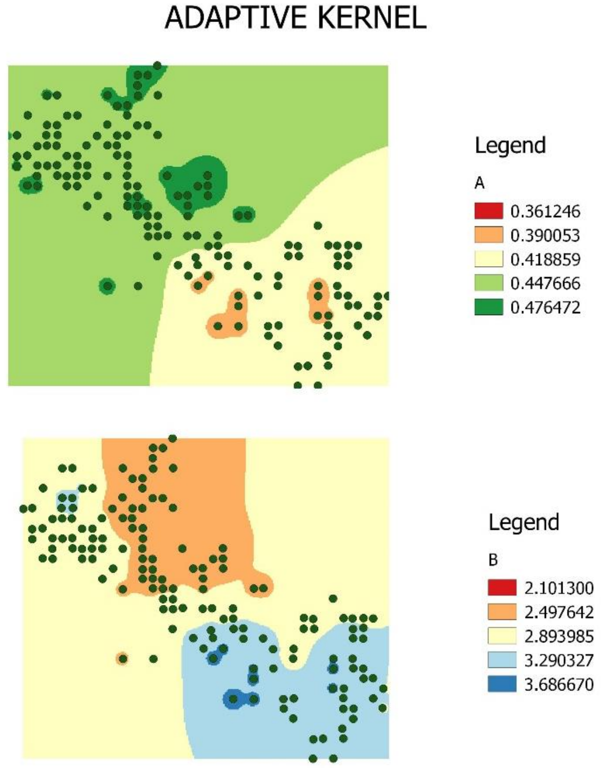


Figure 22 - Adaptive Kernel parameters

Kernel function: gwr.bisquare

Adaptive quantile: 0.301 (about 47 of 156 data points)

Summary of GWR coefficient estimates at data points

	Min	1st Q	Median	3rd Q	Max
A	0.361	0.407	0.434	0.448	0.477
B	2.012	2.417	2.74	2.988	3.689

Number of data points: 156

AIC : -483.591

Residual sum of squares: 0.373

Global R^2: 0.545

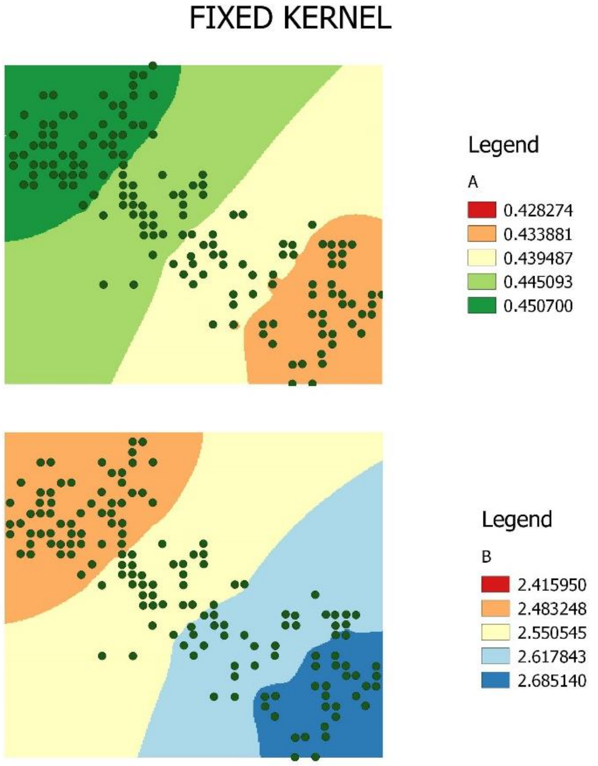


Figure 23 - Fixed Kernel parameters

Kernel function: gwr.Gauss

Fixed bandwidth: 19048.03 m

Summary of GWR coefficient estimates at data points:

	Min	1st Q	Median	3rd Q	Max
A	0.428	0.433	0.441	0.447	0.451
B	2.416	2.451	2.515	2.609	2.685

Number of data points: 156

AIC : -467.821

Residual sum of squares: 0.447

Global R^2: 0.454

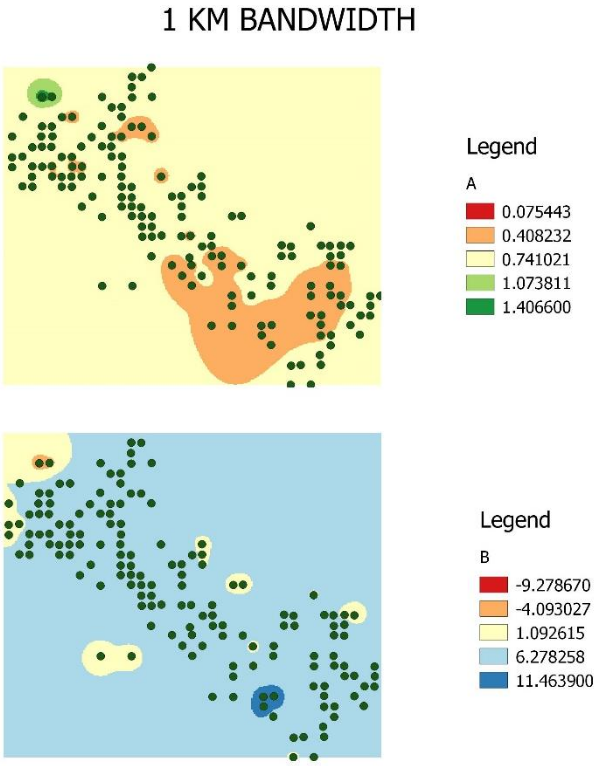


Figure 24 - 1Km bandwidth parameters

Kernel function: gwr.Gauss

Fixed bandwidth: 1000 m

Summary of GWR coefficient estimates at data points:

	Min	1st Q	Median	3rd Q	Max
A	0.075	0.390	0.449	0.493	1.413
B	-9.356	1.572	2.456	3.242	11.460

Number of data points: 156

AIC : -594.442

Residual sum of squares: 0.110

Global R^2: 0.866

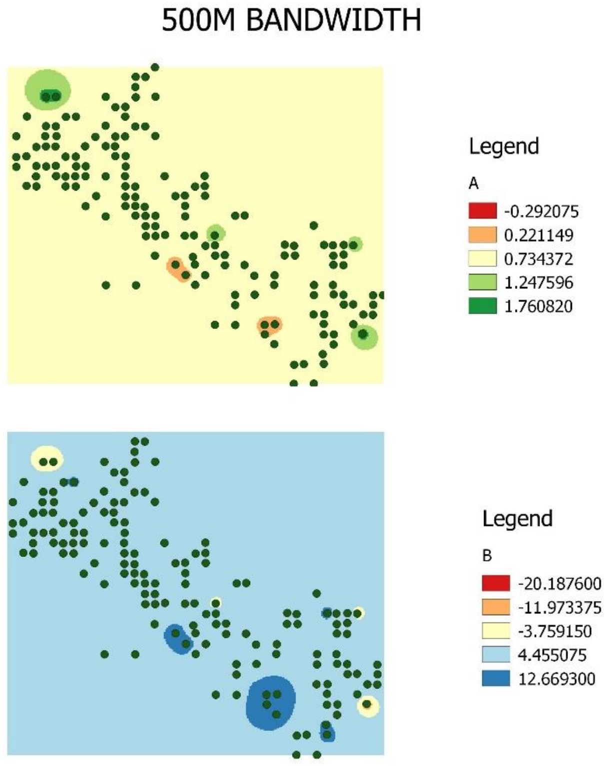


Figure 25 - 500 meters bandwidth parameters

Kernel function: gwr.Gauss

Fixed bandwidth: 500 m

Summary of GWR coefficient estimates at data points:

	Min	1st Q	Median	3rd Q	Max
A	-0.302	0.366	0.449	0.561	1.761
B	-20.20	0.462	2.118	3.688	12.670

Number of data points: 156

AIC : -933.583

Residual sum of squares: 0.009

Global R^2: 0.989

7. GWR APPLIED TO TREES TRAINING DATA SET

In the Figures 26, 27, 28 and 29, there are the results of GWR to the tree training data set.

In this case adaptive kernel and fixed kernel have similar results, maybe slightly better in the case of fixed kernel. Is important to remark that bandwidth obtained by fixed kernel (366.97m) is smaller than the predefined bandwidths.

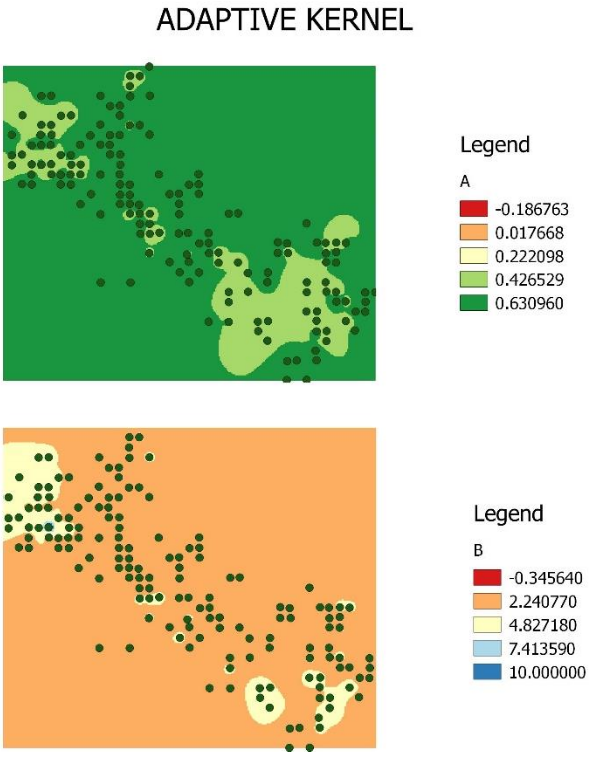


Figure 26 - Adaptive Kernel parameters

Kernel function: gwr.bisquare

Adaptive quantile: 0.012 (about 20 of 1697 data points)

Summary of GWR coefficient estimates at data points:

	Min	1st Q	Median	3rd Q	Max
A	-0.205	0.406	0.438	0.467	0.638
B	-0.495	1.142	1.660	2.355	22.440

Number of data points: 1697

AIC : -6870.908

Residual sum of squares: 1.447

Global R^2: 0.835

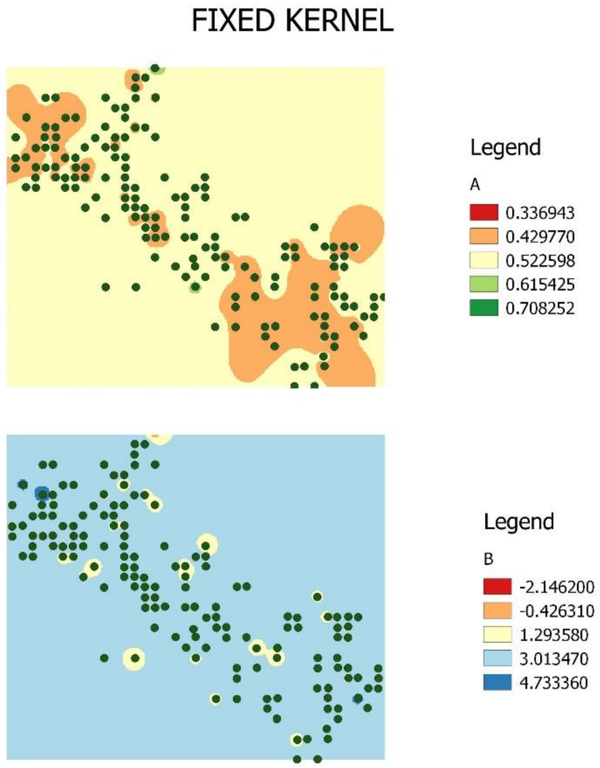


Figure 27 - Fixed Kernel parameters

Kernel function: gwr.Gauss

Fixed bandwidth: 366.9758 m

Summary of GWR coefficient estimates at data points:

	Min	1st Q	Median	3rd Q	Max
A	0.355	0.404	0.435	0.460	0.710
B	-2.446	1.159	1.673	2.331	4.752

Number of data points: 1697

AIC : -6881.24

Residual sum of squares: 1.466

Global R^2: 0.832

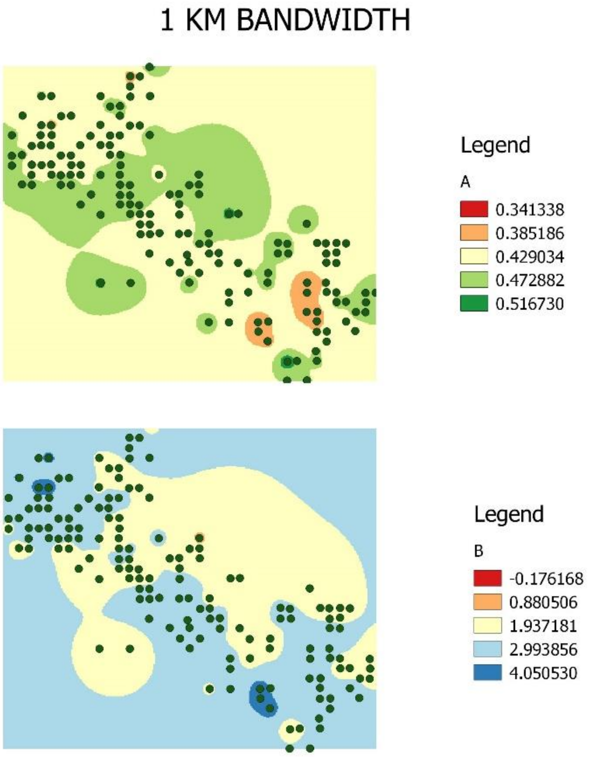


Figure 28 - 1Km bandwidth parameters

Kernel function: gwr.Gauss

Fixed bandwidth: 1000 m

Summary of GWR coefficient estimates at data points:

	Min	1st Q	Median	3rd Q	Max
A	0.341	0.404	0.422	0.441	0.517
B	-0.225	1.605	2.023	2.489	4.065

Number of data points: 1697

AIC (GWR p. 96, eq. 4.22): -5923.227

Residual sum of squares: 2.832

Global R^2: 0.676

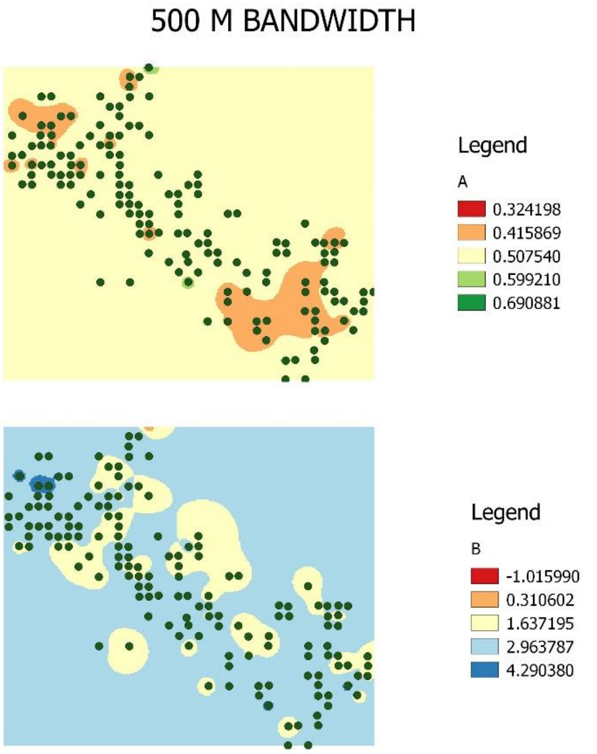


Figure 29 - 500meters bandwidth parameters

Kernel function: gwr.Gauss

Fixed bandwidth: 500 m

Summary of GWR coefficient estimates at data points:

	Min	1st Q	Median	3rd Q	Max
A	0.324	0.405	0.427	0.454	0.693
B	-1.09	1.261	1.719	2.421	4.312

Number of data points: 1697

AIC : -6675.755

Residual sum of squares: 1.703

Global R^2: 0.805

8. VALIDATION DATA SET

There are 20 plots which fulfil the requirements of (1) have 10 or more trees and (2) linear regression is significant. Parameters of local height curves are in Table 6.

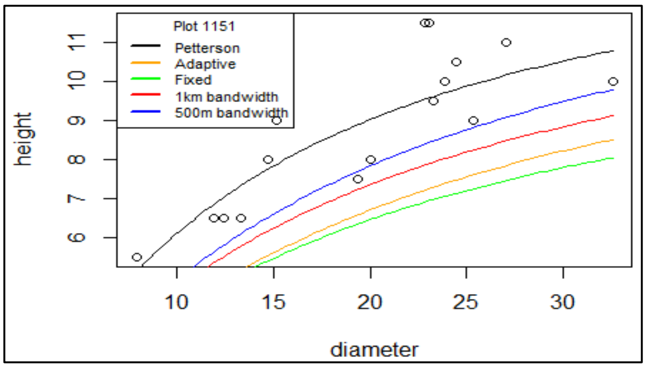
Table 6 - Validation tested points

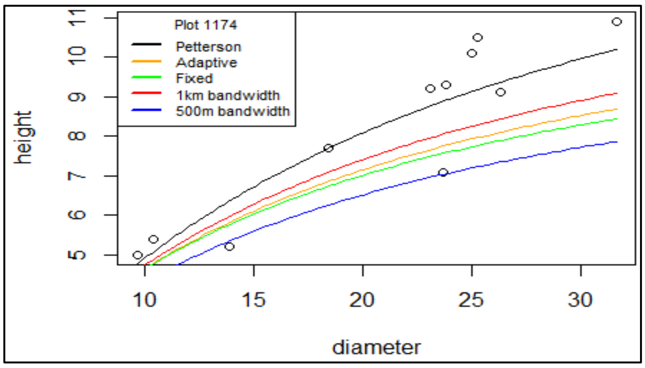
plot	Nº trees	A	B	R^2
1151	17	1.738	0.419	0.770
1174	12	2.467	0.405	0.826
1191	11	2.229	0.423	0.811
1196	17	3.829	0.415	0.675
1202	32	1.144	0.448	0.373
1215	12	1.749	0.497	0.786
1455	15	2.198	0.396	0.769
1468	13	0.994	0.452	0.693
1477	23	3.155	0.363	0.529
1511	10	2.718	0.403	0.779
1524	10	2.460	0.381	0.812
1527	26	2.245	0.377	0.565
1533	15	1.515	0.387	0.607
1555	14	0.930	0.440	0.595
1598	13	1.517	0.437	0.738
1648	16	1.629	0.446	0.558
1660	11	2.163	0.403	0.673
1666	17	2.608	0.410	0.886
1667	30	1.501	0.430	0.687
1781	10	2.643	0.400	0.962

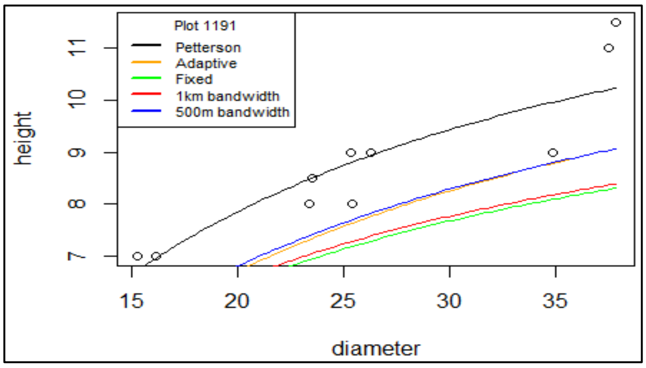
9. COMPARISON OF GWR MODELS WITH LOCAL HEIGHT CURVES FOR SELECTED VALIDATION PLOTS

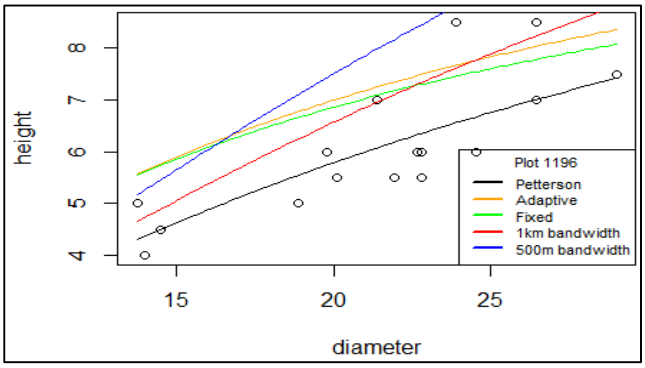
In this section, we compared local height curves (they are considered as the best possible model) with different GWR regression curves obtained in the 20 selected plots (listed in Table 6). The first part of graphical comparison is done with GWR models based on average plot values, which parameters values are in the Appendix, Table 8. The second part count with GWR models based on individual trees, which parameters values are in the Appendix, Table 9.

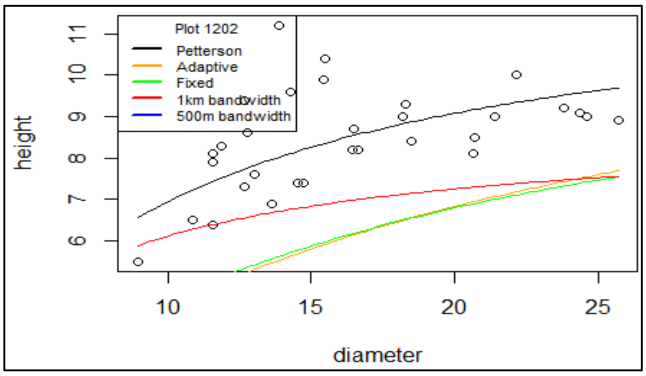
9.1. Local Petterson parameters vs Plots training GWR

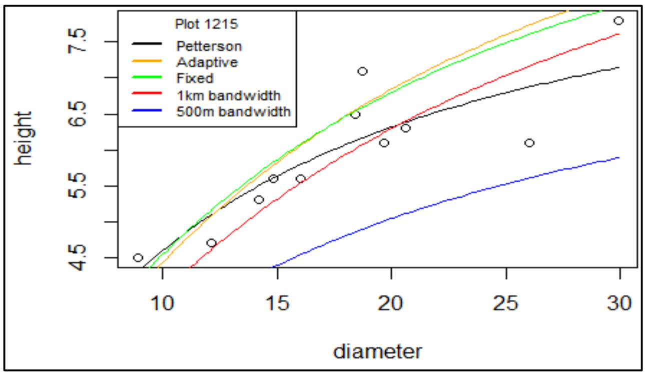


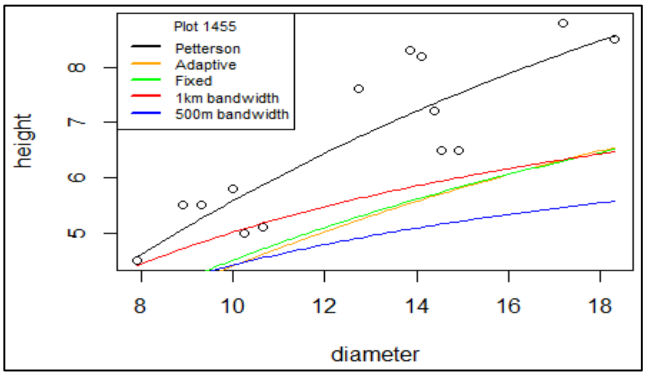


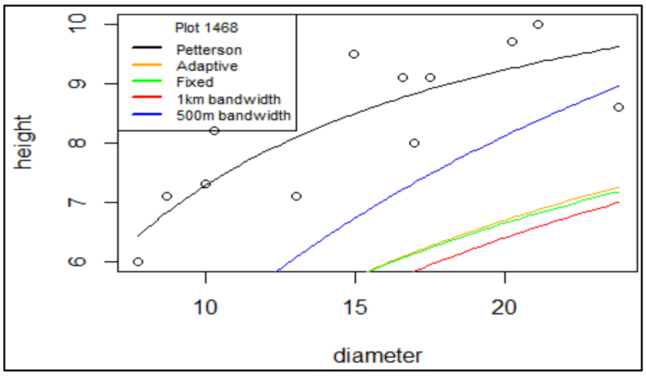


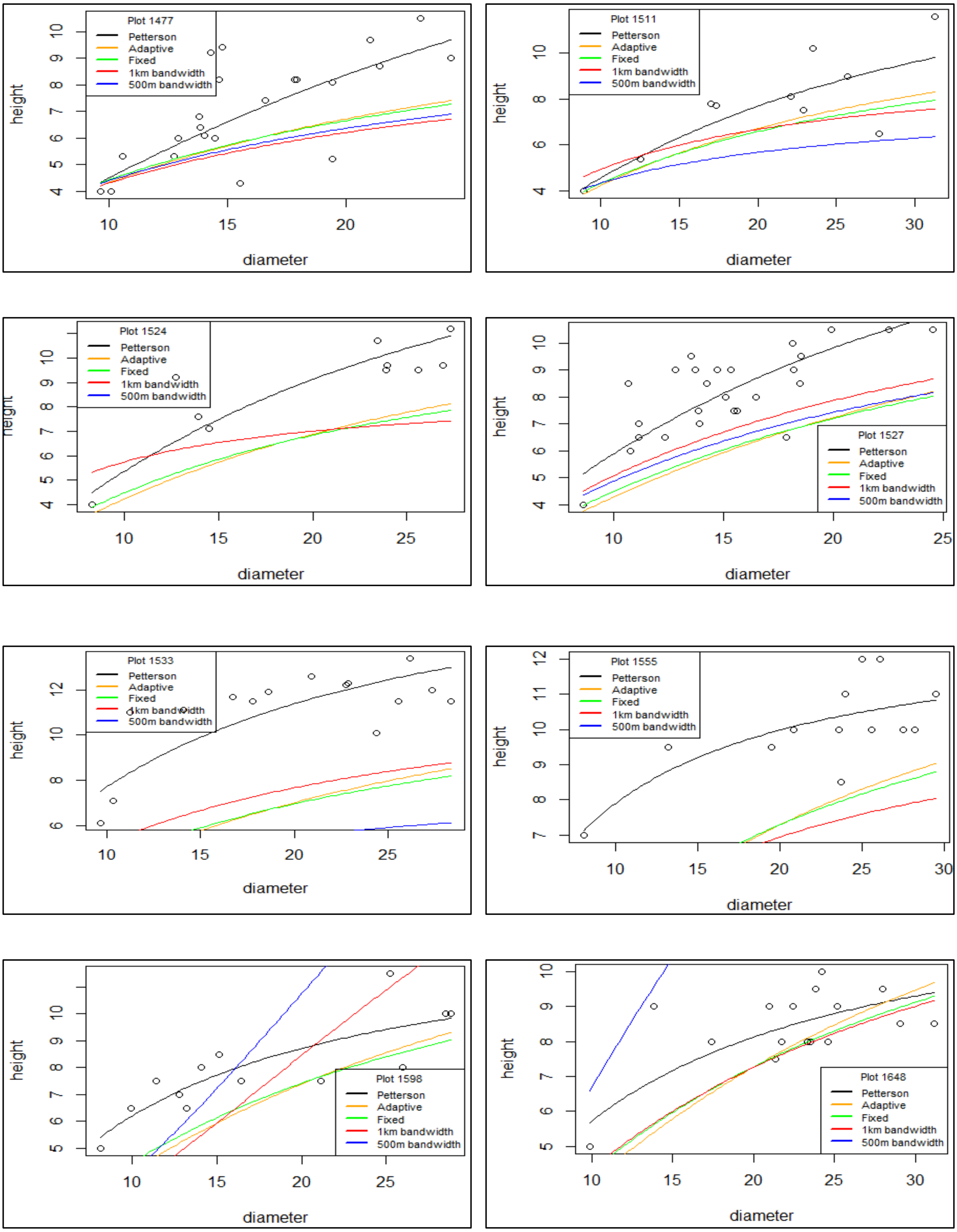











Plot 1477
Plot 1511
Plot 1524
Plot 1527
Plot 1533
Plot 1555
Plot 1598
Plot 1648
Petterson
Adaptive
Fixed
1km bandwidth
500m bandwidth
height
diameter

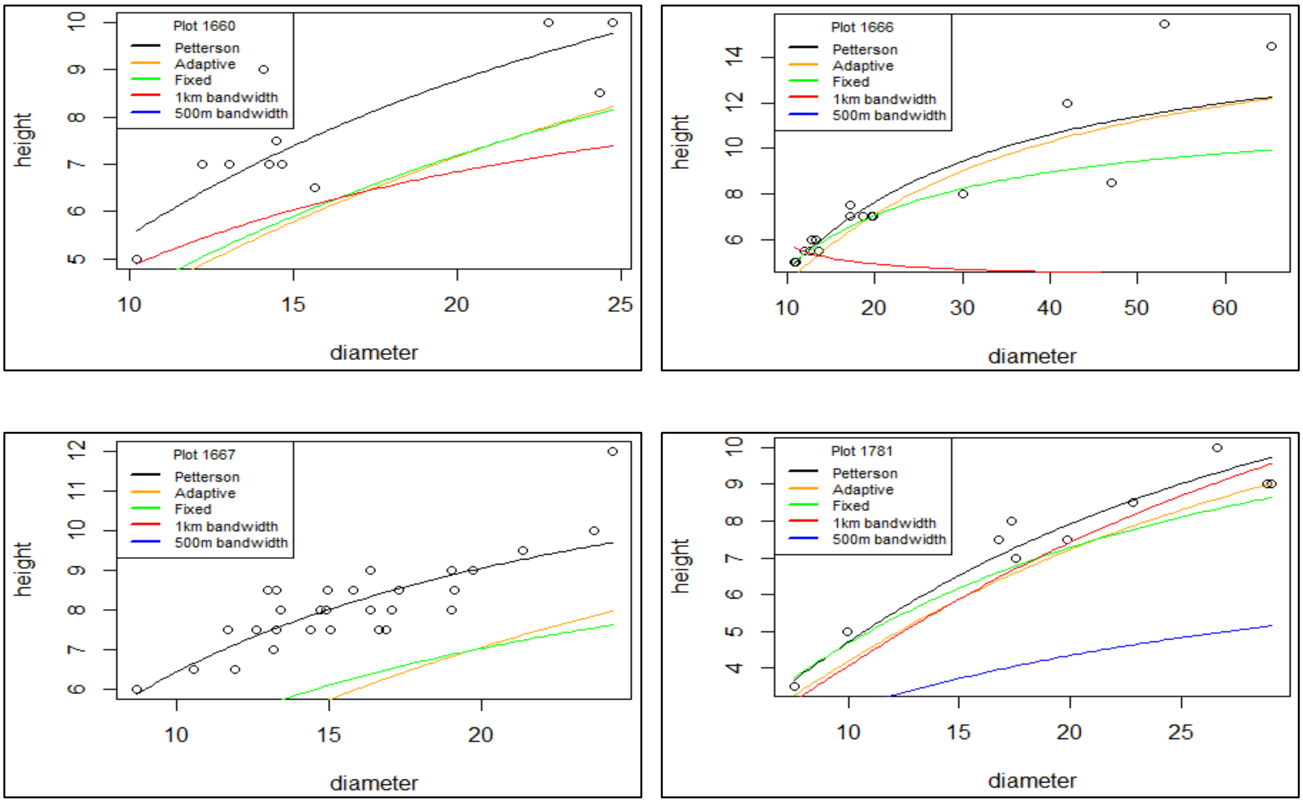
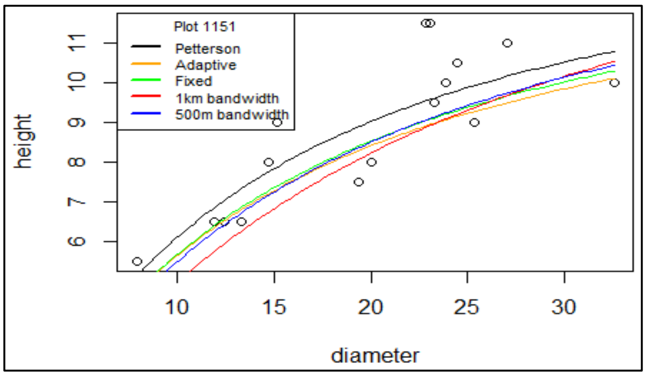


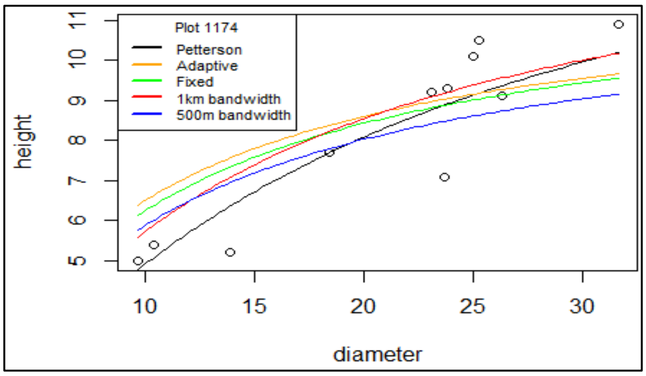
Figure 30 - Curves comparing different GWR curves and Local Petterson parameters (Plots).

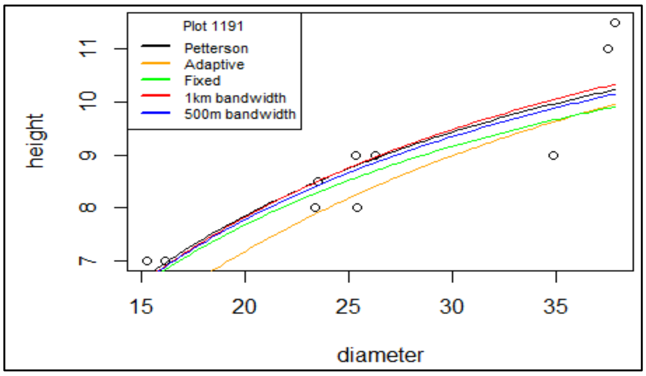
We can see in Figure 30 that almost all the curves have some kind of offset from the Petterson curve.

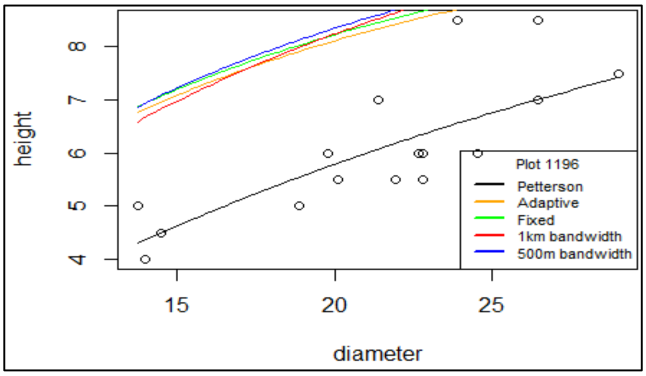
In order to find some explanation, we can compare two groups of validation plots (1) the most isolated plots from the surrounding training plots and (2) the plots that are surrounded by training plots. The most isolated plots are the number 1202, 1215, 1648 and 1660 with no plots nearby or just one plot around them; and the most surrounded plots are 1191, 1455, 1477 and 1667 with at least four plots around them. The logical thinking would lead us to think that results in the isolated plots would be worse than in the surrounded plots, but in Figure 29 we can see that results in both cases are unsatisfactory. It is clear that GWR model based only on average values has insufficient precision.

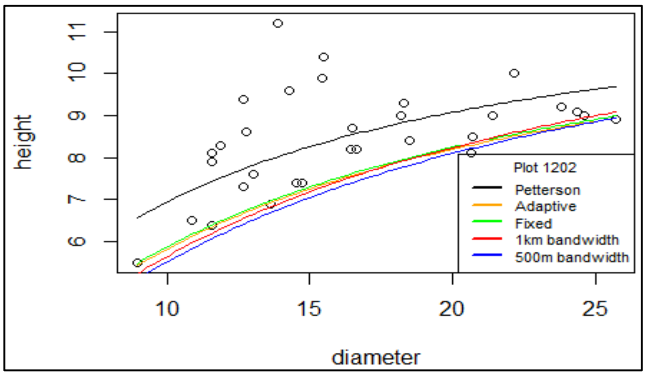
9.2. Local Petterson Parameters Vs Trees Training GWR

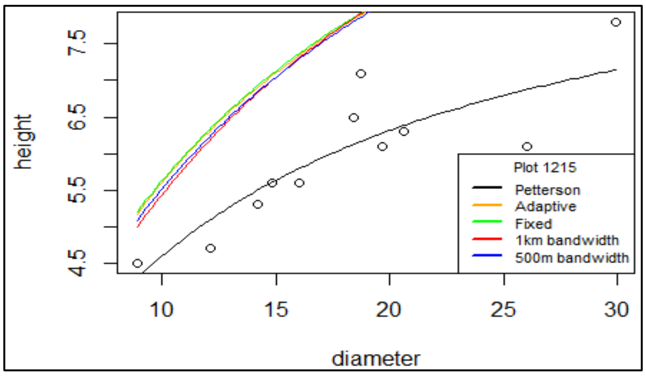


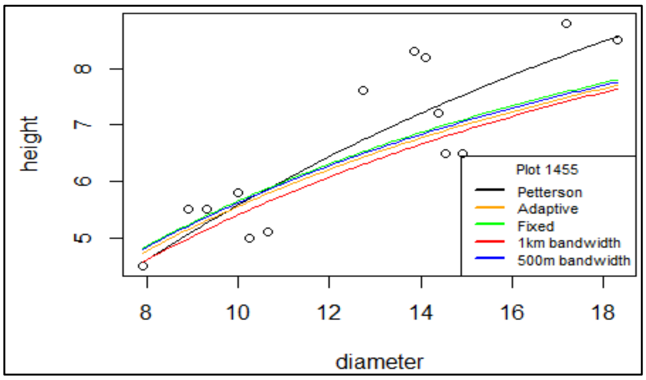


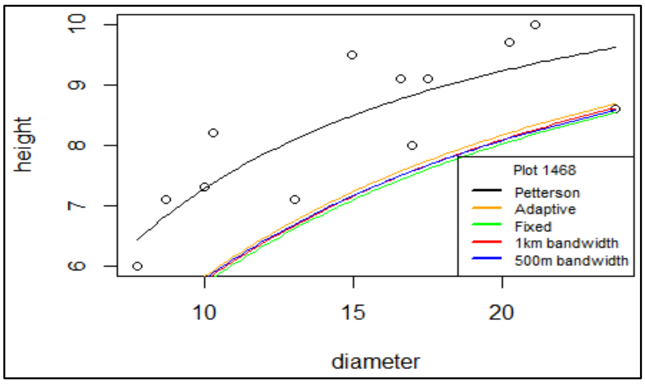


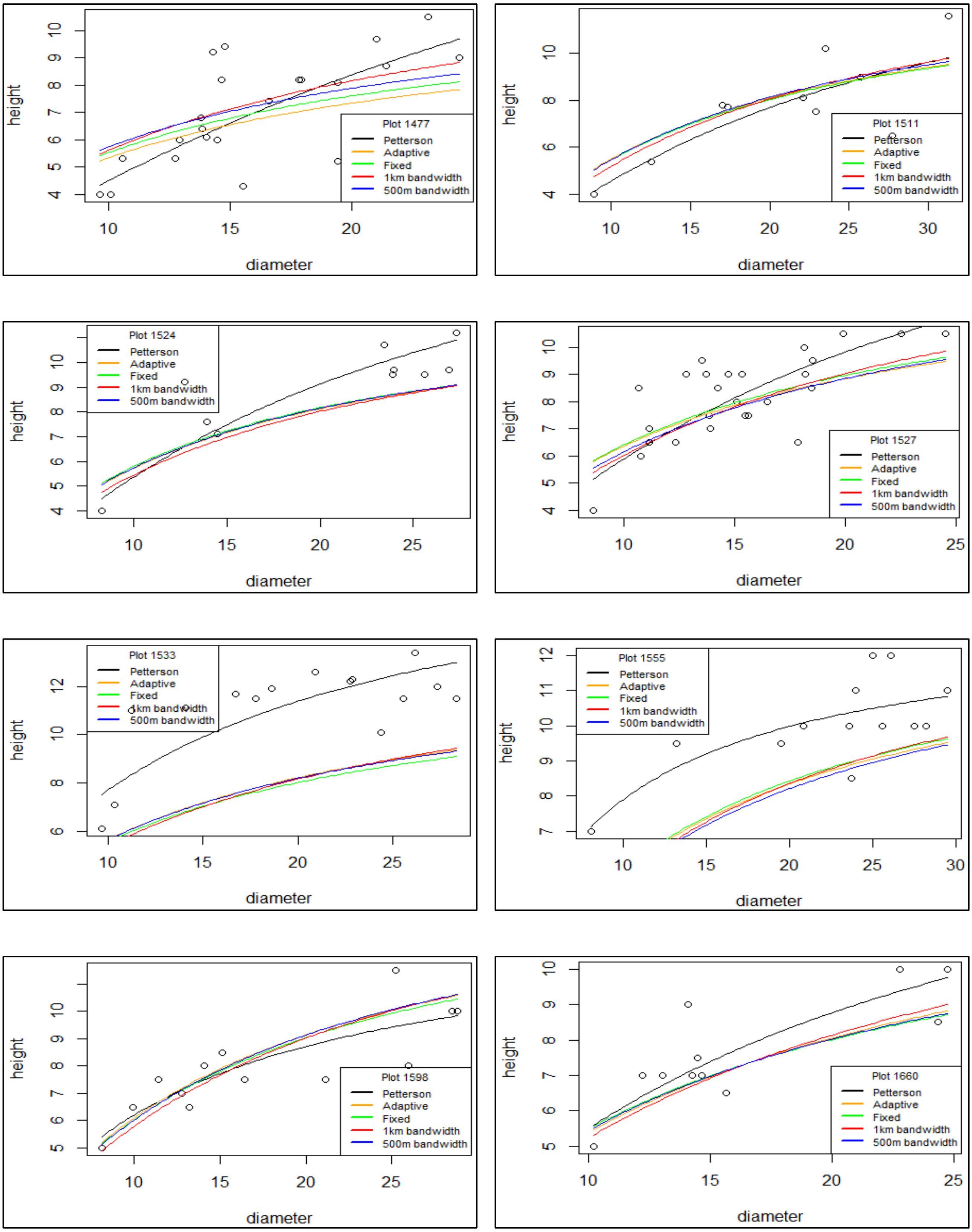











Plot 1477
Plot 1511
Plot 1524
Plot 1527
Plot 1533
Plot 1555
Plot 1598
Plot 1660
Petterson
Adaptive
Fixed
1km bandwidth
500m bandwidth
height
diameter

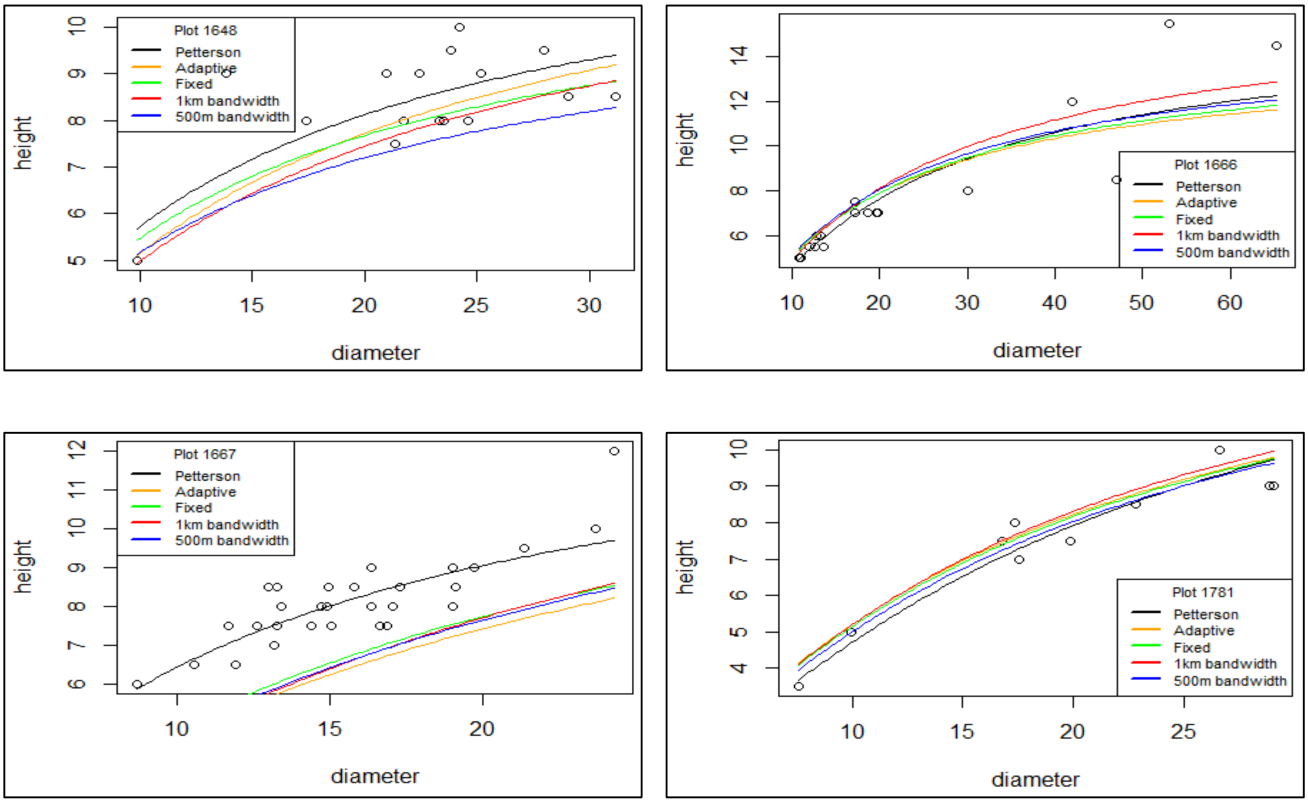


Figure 31 - Curves comparing different GWR curves and Local Petterson parameters (Trees).

In the Figure 31 we can see that the results have been clearly improved in comparison with average plot models (Figure 30). Some curves still have the displacement but in other cases they follow almost exactly Petterson local regression.

The comparisons between plots locations shows that the plots which gives best results (1781, 1666, 1598 and 1191) are surrounded by very different amount of plots (2, 2, 3 and 6 respectively). Likewise, the worst results are in plots 1533, 1555 and 1667, this plots are surrounded correspondingly by 4, 2 and 4 plots. Therefore, there is no relation among location of plots and obtained GWR results.

If GWR types of curves are compared, we can see that almost all calibrations are very similar among each other. For example in the plots 1196, 1468 1555 and 1667 it is possible to see that adaptive kernel, fixed kernel and both bandwidths are almost the same kind of curve, and they shift systematically to the same area. This could mean that the GWR regression gives correct shape of the curves but they are displaced from the local regression. The reason of this shift should be examined in further research.

9.3. Residual Analysis and Regression Diagnostics

Table 7 - Residual Analysis. Legend: Mean of Residuals; RMSE, Root Mean Standard Error Model; Δi, Mean value of deviation between GWR and local models; LL Δi. Lower Limit of Confidence Interval; UL Δi, Upper Limit of Confidence Interval; Sdev, Standard Deviation; Sdev Error, Standard Deviation Error; R^2, Coefficient of Determination; AIC, Aikake Information Criterion.

Model	Criterion								
	Mean residuals	RMSE	Δi	LL Δi	UL Δi	Sdev	Sdev Error	Coefficient of determination	AIC
Local regression	-0.072	0.986				0.944	0.066	0.645	0.740
For average plot values:									
Adaptive	-1.536	2.145	1.546	1.078	2.013	0.998	0.223	0.148	24.929
Fixed (19 km)	-1.574	2.181	1.579	1.121	2.037	1.024	0.229	0.109	25.049
Bandwidth 1 km	-1.566	2.280	1.639	1.176	2.101	1.233	0.276	0.125	25.708
Bandwidth 500 m	-2.273	3.917	3.148	2.218	4.079	1.486	0.332	0.019	41.195
For individual tree values:									
Adaptive	-0.419	1.551	0.940	0.606	1.273	0.996	0.223	0.359	14.430
Fixed (366.98 m)	-0.385	1.526	0.907	0.552	1.263	0.992	0.222	0.375	13.410
Bandwidth 1 Km	-0.382	1.539	0.947	0.600	1.294	0.970	0.217	0.367	14.050
Bandwidth 500 m	-0.414	1.549	0.946	0.595	1.298	0.989	0.221	0.365	14.090

The Table 7 shows important information that confirms the graphical comparison in Figures 30 and 31. The original data used in the calculation of Table 7 can be found in the Appendix, Tables 10 to 20:

- Mean residuals: This criteria shows that GWR curves for average plot values are in generally far from the real values of the trees. This results confirm the presence of big displacements observed in Figure 30. It is observed that results obtained in GWR curves for tree values are much better.

- RMSE: This result is, again, another hint that confirm the offsets observed in the figure 30. Very high value there is in the case of the 500 m bandwidth. But it is logical result if we take a look into the Figure 30 we can see that there are some plots where 500m bandwidth curve is not even in the frame, this means that the deviation is big. The case of the GWR curves for tree values, the variability is still big but it improves from the previous ones, this also is visible in the Figure 31.

- Δi: In this point we can see the big difference that exists between local regression and GWR applied in average plot values. If we compare this criteria together with confidence interval we can see that the average plot values has deviation about 1.5-1.6 meters and the CI width is around 1 meter. In other hand, individual tree values have a deviation around 0.9 meter with CI width of 0.6 meters. The most important result is that GWR curves for tree values are significantly closer to the local regression with smaller confidence intervals.

- Sdev and Sdev error: it shows that GWR curves for average plot values are more dispersed than the GWR curves for tree values.

- Coefficient of determination: This coefficient is clearly better for individual tree values but not as good as in the local regression. In average plot values are evidently worse in comparison with tree values, fact that is expected according with previous results.

- AIC: We can see that best AIC in case of GWR curves for average plot data is more or less similar except in the case of 500m bandwidth. In case of GWR curves for tree data AIC stays more or less constant but the best one is for fixed kernel, width a bandwidth of 366.98 meters. If we take a look into Figure 30 this curve (colour green) is the one which is closest to local regression curve in most of the plots.

In this diploma thesis, has been used the basic GWR configuration for its calibration, it means that bisquare weighted function for adaptive kernel and Gaussian function for fixed kernel has been used (Fotheringham, et al., 2002). This calibration was applied for the average data and the tree data. The results of this calibration are different in both types of data: (1) average data set shows poor results with high values of mean residuals, RMSE, standard deviation and very low coefficient of determination. (2) On the other hand, tree data shows better results in all the criteria, but still it is not completely satisfactory because results are little bit farther of obtained local regression. Furthermore, in GWR basic calibration we can observe (Table 7) that results are very different according with the kind of selected kernels. In general, the application of the fixed kernel give good overall results with better criteria values compared with adaptive kernel. However, talking about predefined bandwidths it is possible to see that when smaller bandwidth are selected, the model decrease in its quality giving bad results in criteria values and with the existence of abnormal extreme values that are visible in the minimum and maximum of the regression summary displayed, for instance, in Figures 24 and 25. It is important to say that a lower bandwidth selection limit exists at which the weighted function cannot be computed. In the case of average plot training data set this limit is 388 meters and in case of tree data set is 270 meters.

VI. DISCUSSION

GWR was able to work correctly in large areas in many applications, like economy and civil engineering. For example, GWR was applied in the calculation of the hedonic pricing, this is the modeling to estimate the extent to which factor (size, appearance, features, condition, accessibility to schools, level of water, air pollution, value of other homes…) affects the price of houses, in the city of London (Fotheringham, et al., 2002) or in the estimation of the Metro traffic in Madrid (Puebla, et at., 2012), both with good results.

In forestry, there exist some remote sensing applications made in large areas with very different results about GWR performance. For instance, it was obtained a net primary production (NPP) regression model based on the GWR and compared with OLS and lag model (Wang, et al., 2005), the conclusion of this comparison is that GWR made a significant improvement in model performance of GWR over OLS and the spatial lag model. In the other hand, we can find a study in which is used LiDAR technology for modelling tree diameter from airborne laser scanning derived variables. In this study there was compared GWR, OLS, generalized least squares with a non-null correlation structure (GLS) and linear mixed-effects model (LME); the conclusion exposed that GWR displays no improvement in this modelling (Salas, et al.m 2010). As final example, we can find one study about GWR that was used to estimate forest canopy height using high spatial resolution Quickbird (QB) images (Cheng, et al., 2012). It was examined four spatial analysis techniques: OLS, inverse distance weighting (IDW), ordinary kriging (OK) and cokriging (COK) and compared their performance with GWR. Conclusion shows better GWR performance than other spatial techniques.

In the field of forestry and ecology, there exist some large scale researches that provides interesting information about GWR applied to large areas, these studies are framed mainly in the fields of forest fires, afforestation and less commonly in forest management. Some examples are: (1) Mexican researchers analyzed the causing factors behind deforestation in the state of Mexico by the application of GWR (Pineda-Jaimes, et al., 2010), the conclusion was that the GWR application is especially effective in explaining the spatial heterogeneity in deforestation processes from one region to another within the state territory. (2) About wildfires it was explored the spatial patterns of fire density in Southern Europe (Oliveira, et al., 2014), this relationship was investigated with GWR and it was compared with OLS. These results agree with previous studies, which showed the potential of GWR for modelling fire occurrence at a large scale, and regarding model performance GWR showed an improvement over OLS. (3) Finally, Douglas fir site index (SI) was modelled in a large area (206 000 ha) using ecological data (climate, soil, soil parent material, SI) (Kimsey, et al., 2008) with two forms of regressions: standard multiple linear regression (MLR) and GWR. The conclusions show again that GWR model is able to provide better results in large areas.

GWR also has been applied with forestry data (height, DBH…) but in this case the studies have proven to be clearly effective just in small forest areas (Zhang, et al., 2004, Zhang & Shi, 2004). For instance, in the previous cited researches it was applied GWR together with OLS in areas around 0.5 ha, with the aim of knowing what model gives better results. The conclusions were similar in both studies, it was determined that GWR improves significantly the model over global models. Similarly, other studies (Zhang, et al., 2005) confirm that in small areas (around 5 ha) GWR provides better results than other kinds of regressions such as

OLS, GWR, linear mixed model (LMM), generalized additive model (GAM), multi-layer perceptron (MLP) neural network and radial basis function (RBF) neural network.

Most of the studies above repeat the same pattern in their design, in general it was applied the basic GWR calibration in some area and then this results are compared with other kinds of regression, especially global regressions. The basic GWR calibration is based on fixed kernel with Gaussian distance decay weighted function and adaptive kernel with bisquare distance decay kernel function. Also there are some cases in what it is used the predefined bandwidth, in order to facilitate comparison (Zhang, et al., 2012). Exists certain issues in the bandwidth calibration, because occur some cases where the study area is very small for spatial analysis and is not possible to compute the kernel calibration (Zhang, et al., 2005). About data set division, in most studies there are no selection of data in terms of validation and training data set. However, in some researches GWR is applied in all the study area and then are selected some plots or points within the analyzed area for check the results (Zhang, et al., 2005), or studied area is divided in training data set (60 % of points) and validation data set (40 % of points), but in this last case a GWR modification is used, namely geographically weighted logistic regression (Rodrigues, et al., 2014). In most of the cases, when there are not selected points for testing the results, the global models or other kind of models are compared with GWR by some other criteria like: Moran coefficient, criteria used for measure spatial autocorrelation; Z-Values, that are the number of standard deviations an observation or datum is above the mean; F-Test, used for comparing statistical models in order to identify the model that best fits the population from which the data were sampled; and spatial heterogeneity percent (SH%), where large SH values indicate high levels of heterogeneity or low levels of spatial randomness; Variance inflation factor (VIF), which quantifies how much an estimated regression coefficient increases due to multicollinearity (Zhang, et al., 2004., Zhang, et al., 2005, Zhang, et al., 2008, Zhang, et al., 2009., Cheng, et al., 2012, Oliveira, et al., 2014).

All of previous studies, independently of the used method, agree that GWR is a useful tool that provides much more information on spatial relationships to assist in model development and understanding of spatial processes.

In this way, we count with important hints and good hopes about the workability of GWR in large areas with the application of Forest Inventory data. One of the objectives of this project was to check the behavior of GWR applied in large areas, the studied area is around 871.80 km^2. We wanted to compare two levels of data from the Third National Spanish Forest Inventory (MAGRAMA, 2014) – average data from individual plots and data about individual trees. However, the obtained results have not been as satisfactory as we would have expected, and especially wrong result was obtained in case of the average data from individual plots.

According with our current results, it is evident that basic GWR configuration is not suitable for obtain optimal results, but there are several possibilities how to improve the current calibration. Several different calibration options exists (Figure 32). For instance, it is possible to change weighted function in the kernel to: (1) exponential, (2) box-car and (3) tri-cube (Binbin, et al., 2015). Where (1) exponential kernel is continuous function of the distance between two observation points, in our case Gaussian weighted functions has several similitudes with exponential function (Gollini, et al., 2015). The (2) box-car kernel is a simple discontinuous function that excludes observations that are further than some distance ‘b’ from the GW model calibration point. This kernel allows for efficient computation, since only a subset of the observation points need to be included in fitting the local model at each geographically weighted model calibration point. This can be particularly useful when handling large data sets. (3) The tri-cube kernel is discontinuous function that makes it similar to bisquare function (Gollini, et al., 2015), giving null weights to observations with a distance greater than a specific point. However unlike a box-car kernel, they provide weights that decrease as the distance between observation/calibration points increase.

Also, apart from changing weighted functions is possible to apply some extensions to GWR, these extensions are very promising because our data is very heterogeneous and mostly of this methods cope with some kind of non-constant variability:

- Robust GWR (Fotheringham, et al., 2002). This kind of GWR is made in order to reduce the effect of anomalous observations or outliers on its outputs. To provide a robust GWR, each local covariance matrix is estimated using the robust minimum covariance determinant (MCD) estimator (Gollini, et al., 2015).

- Heteroscedastic GWR (Fotheringham, et al., 2002). In the basic GWR, although the regression coefficients vary geographically, the variance of error term is assumed as fixed. In this method there is introduced that variance may also vary geographically. This method could be very interesting for our investigation because of the big data variability across the study area.

- Mixed GWR models (Fotheringham, et al., 2002). In some situations not every regression coefficient in a model varies geographically, in others the degree of variation for some coefficients might be insignificant. It therefore may be helpful to consider mixed GWR models in which some coefficients are global (they do not vary over space). The remaining coefficients are termed local and these are expected to be functions of geographical location, as in the basic GWR model.

- Geographically Weighted Generalized Linear Models (GWGLM) (Fotheringham, et al., 2002), local form of generalized linear models that assumes that the data follow a Poisson or Binomial distribution.

Other options in GWR calibration are based in the GWR modification itself. For instance, with the application of some modifications by incorporating attitudinal effects into the spatial weighting function, this method was named ‘Geographically and Attitudinal Weighted Regression’ (GAWR) (Propastin, 2012). Another modification example is related with the incorporation of spatial dependence among neighboring observations at each location in the study area by modelling local variograms, it was called ‘Geographically Local Linear Mixed Models’ (GLLMM) (Zhang & Lu, 2012). Finally, in the study of wildfires occurrence in Spain GWR modified into GW linear model and GW logistic model was used (Martínez-Fernández, et al., 2013). The calibration options and modifications are so wide and are in constant evolution.

Finally, it would be interesting change the initial height-diameter equation. The regression equation that has been selected in this diploma thesis is the Petterson function. This function usually gives good results, however it is possible that for the shape of Spanish forest this equation may be insufficient. Spanish forest are very heterogeneous, hence the formula to model them should include for example, density data. Likewise, it is possible that GWR model gives better results when they are included more parameters (ESRI, 2014). It would be interesting apply generalized functions in the next research of this topic. There are many possible generalized functions for modelling height-diameter relationship in what we can use tree and stand variables. For instance, López-Sánchez et al., 2003 analyzed the 26 generalized height-diameter equations for modelling of *Pinus radiata* in Galicia, Spain and they grouped them into the following categories:

Group 1: Low sampling effort models, including those models which need diameter measurements and knowledge of age in some cases.

Group 2: Medium sampling effort models, including models which need measurements of diameter and of a sample of tree heights.

Group 3: High sampling effort models, including models which need the knowledge or measurements of stand age as well.

The problem with these functions is in their more difficult application in GWR model because they are not easily linearizable. Consequently, in this case we must develop some another strategy how to apply generalized functions to GWR.

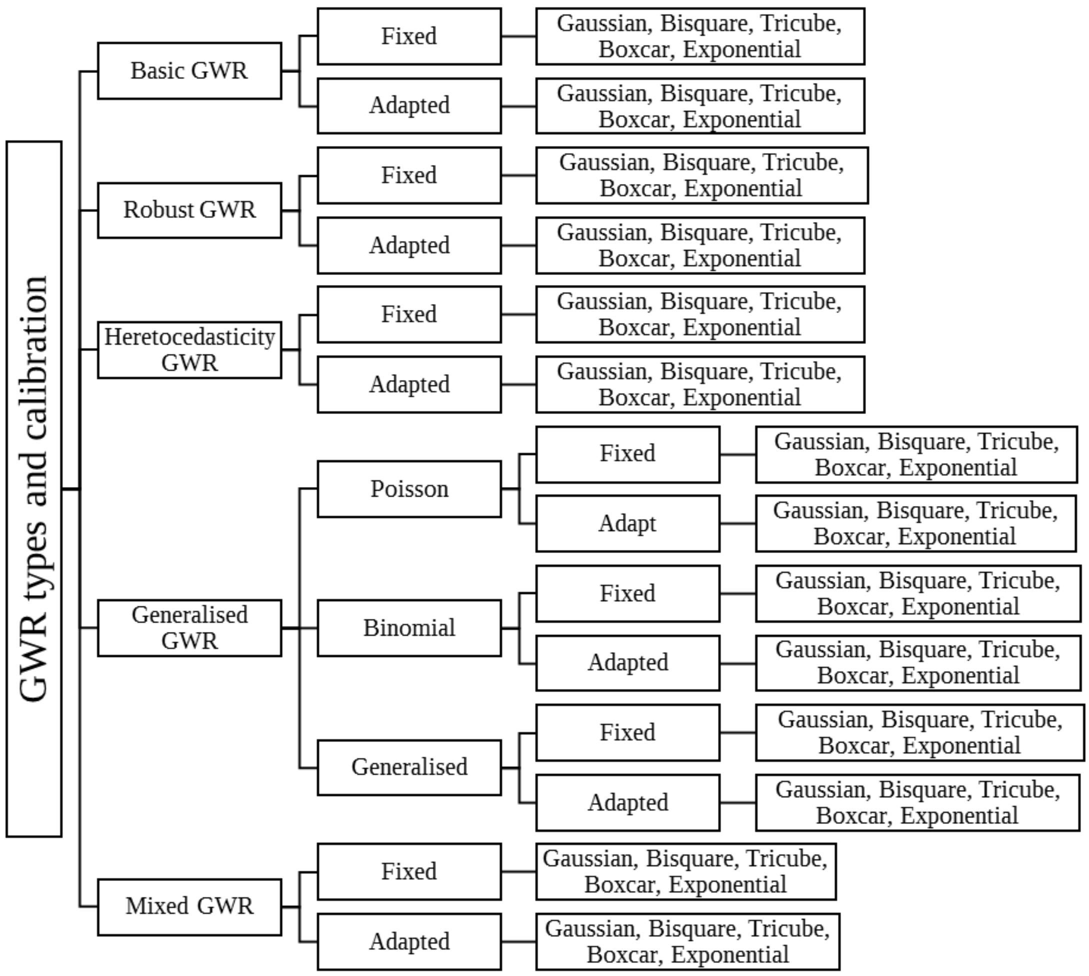


Figure 32 - GWR calibration option

VII. CONCLUSIONS

In this diploma thesis it was tried to deal with spatial heterogeneity of forestry data through geographically weighted regression. The results of this study are promising, but different for two used data levels:

- In the case of **average data** of plots we can see that results are completely unsatisfactory. The results are completely far from the local regressions. It seems that average data (eg. stand average data in practice) are too rough for practically applicable height-diameter model for large area. A possible solution for this, is just work with the average tree data. With this kind of data it is possible investigate about the growing and volumes, as well as increment of this growing and volumes in Forest Inventory plots.

- In case of **tree data** we obtain relative good results, being the fixed kernel the one which gives best results. Therefore, it is likely to assume that GWR is promising to obtain the optimal desired results with some calibration or modelling method changes. The average deviation of current GWR model from local model is about 0.9 m (with confidence interval appr. 0.6 – 1.2 m). Desired deviation that can be used in practice with good results should be cca 0.5 m (with confidence interval about 0.3 – 0.7 m, at least below 1 m).

From the methodological point of view, there are several possibilities how to improve the model. These changes would focus in: (1) select more suitable height-diameter equation based on generalized height-diameter functions, and (2) make more precise GWR calibration. The calibration options themselves are wide but there are some GWR extensions based on coping with the heterogeneity of the data that can be useful for our interest (details are in the chapter “Discussion”). Also both methodological and practical comparison with other prospective methods (like ‘Classification and Regression trees’, ‘Linear mixed models’ and ‘Generalized additive models’) can be useful and interesting.

GWR is highly promising method because it gives the opportunity that in the case of need a particular model for some area, there is possible to make sufficiently precise height-diameter curve without any additional measurements to the existing Forest Inventories. Therefore, this method needs very detailed data, for instance, it is needed each specific geographical location and spatial distribution of the information, as well as attribute data information about all desired parameters. This makes GWR method very demanding in the sampling and also in the needed powerful equipment for calculate the regression.

The above mentioned methodological suggestions and possible improvements of the method will be developed and evaluated in my Ph.D. study on the same topic.

VIII. SUMMARY

The diploma thesis is focused on the application of the Geographically Weighted Regression (GWR). This is a prospective method for coping with spatially heterogeneous data, which is the kind of data that we can find in the forests. GWR method has been used previously in small forest areas with good results. The objectives of this diploma thesis is to evaluate and check GWR behaviour for a large scale study. For this purpose we used data of *Pinus halepensis Mill.* pure forests in the Spanish Region of Murcia. Height-diameter model was selected as an example of model frequently used in forestry, because it is very suitable from methodological point of view because of its simplicity of application to the data.

Within Murcia Region, a suitable study area was selected, characterized by a regular sample plot spatial distribution. Inside this region, two random samples were made, the 'training data set' and the 'validation data set'. Training data set is composed by two levels of data, plot average data and individual tree data; and validation data set is composed just by individual tree data. In order to apply GWR to the selected data, Petterson height-diameter equation was selected for its good quality and relatively easy application in GWR.

In training data set, GWR was implemented with the basic recommended GWR calibration (fixed kernel together with Gaussian weighted function and adaptive kernel together with Bisquare function), plus two predefined bandwidths (1km and 500 m). Local regressions were implemented in the suitable validation plots (with 10 measured trees at least). Finally, at selected validation plots, GWR parameters were extracted by suitable GIS tools and resulting GWR height-diameter models were compared with local models based on all measured trees of respective plot. GWR and local models were compared by residual analysis and regression diagnostics tools.

The comparison results are clearly divisible by type of analysed data. Plot average data gives bad results in all the analysed points (average deviation from local model was about 1,5 m, with confidence interval about 1-2 m) . In the other hand, individual tree data, in general, gives more satisfactory results, being closer to local models (average deviation from local model was about 0,9 m, with confidence interval about 0,6 -1,2 m). However, is noticeable the need to improve GWR calibration quality for obtain the optimal result (average deviation from local model should be about 0,5 m, with upper boundary of confidence interval less than 1 m).

Suggested improvements of the GWR calibration are focused on two aspects. The first one is using more suitable weighted function in the calibration, there exist several possible functions, for instance Tricube, Boxcar or Exponential. The second possible improvement is using of GWR extension that cope with highly heterogeneous data, such as Robust, Heterocedastic, Generalised or Mixed GWR; these extensions are very promising because they deal with some kind of non-constant variability.

Finally, another option that can improve the GWR quality is the change of the height-diameter Petterson equation for another more suitable equation based on generalized height-diameter functions. It is possible that this kind of functions would be able to deal with the spatial heterogeneity that exist in Spanish Mediterranean forests in a better way.

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X. APPENDIX

10. R CODE

```
library(nlstools)
library(nls2)
library(nortest)
library(car)
library(lmtest)
```

- ***Petterson local formula***

```
#h~1.3 + (1/(a+(b/diameter))^3) tree by tree
start <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/valiP.csv",
sep=";", dec=".")
#start <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/species_24_coor
ds_polar/24_all_out.csv", sep=";", dec=".")
attach(start)
h<- height
d<-diameter
#Peterson lineal
H= 1/(h-1.3)^(1/3)
D<-1/d
Peters<-lm(H~D)
summary(Peters)
anova(Peters)
out<-summary(Peters)
confint(Peters)
plot(Peters)
res<-residuals(Peters)
lillie.test(res) #no normal
durbinWatsonTest(res) #no correlated
AIC(Peters)
```

- ***Petterson GWR***

```
plots<-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/valiP.csv",
sep=";", dec=".")
library(spgwr)
library(sp)
attach(plots)
map = SpatialPointsDataFrame(data=start, coords= cbind(CX_H30,CY_H30))

#Fixed kernel calculation
#Gauss-cross-validation
bandA<-gwr.sel(H~D, data=map)
gwrA<-gwr(H~D, data=map, bandwidth=bandA,hatmatrix=T, se.fit=T)
gwrA

#Adapted kernel calculation
#Bisquare-AIC
Bisq_AD_AIC<-gwr.sel(H~D, data=map, adapt=T,
gweight=gwr.bisquare,method = "AIC", verbose = TRUE, longlat=NULL,
RMSE=FALSE, tol=.Machine$double.eps^0.25, show.error.messages = FALSE)
```

```
Bisqu_AD_AIC_gwr<-gwr(H~D, data=map, adapt=Bisq_AD_AIC,hatmatrix=T,
se.fit=T, gweight = gwr.bisquare)
Bisqu_AD_AIC_gwr

#1 km bandwidth calculation
#Bisquare-AIC
gwrB<-gwr(H~D, data=map, bandwidth=1000,hatmatrix=T, se.fit=T)
gwrB

#500 m bandwidth calculation
#Bisquare-AIC
gwrB<-gwr(H~D, data=map, bandwidth=500,hatmatrix=T, se.fit=T)
gwrB
```

- ***Curves Plots vs Local Regression (Petterson)***

```
#Plot n° 1151
start3 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/V1151.csv", sep=";", dec=".")
attach(start3)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.41902+1.73792 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.434711+2.701881 /x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.450459+2.56751/x)^3), add=TRUE,col="green") #Fixed
curve(1.3+(1/(0.43273604+2.319682 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.4190656+2.31722 /x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1151',"topleft",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)

#Plot n° 1174
start4 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1174.csv", sep=";", dec=".")
attach(start4)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.40478+2.4672 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.441815+2.261727 /x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.4504086+2.1794 /x)^3), add=TRUE,col="green") #Fixed
curve(1.3+(1/(0.43139446+2.311824 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.46094277+2.31182 /x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1174',"topleft",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)

#Plot n° 1191
start7 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1191.csv", sep=";", dec=".")
```

```
attach(start7)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.42307+2.22909 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.432269+2.752624 /x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.458488+2.427429 /x)^3), add=TRUE,col="green") #Fixed
curve(1.3+(1/(0.45880453+2.3411 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.43646272+2.5987987 /x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1191',"topleft",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot n° 1196

```
start9 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1196.csv", sep=";", dec=".")
attach(start9)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.41489+3.82912 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.435216+2.496689 /x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.448562+2.321209 /x)^3), add=TRUE,col="green") #Fixed
curve(1.3+(1/(0.36822739+4.1315299 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.33919546+4.1019285 /x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1196',"bottomright",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot n° 1202

```
start10 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1202.csv", sep=";", dec=".")
attach(start10)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.4475+1.1437 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.445705+2.396964 /x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.459891+2.135036 /x)^3), add=TRUE,col="green") #Fixed
curve(1.3+(1/(0.5111695+0.8107863 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.64521706+0.94664653 /x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1202',"topleft",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot n° 1215

```
start12 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1215.csv", sep=";", dec=".")
attach(start12)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
```

```
curve(1.3+(1/(0.49675+1.74904 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.446581+2.36908 /x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.4572283+2.184688 /x)^3), add=TRUE,col="green") #Fixed
curve(1.3+(1/(0.45285087+2.643429 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.51661324+2.5489603 /x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1215',"topleft",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot n° 1455

```
start13 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1455.csv", sep=";", dec=".")
attach(start13)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.3963+2.1981 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.4427+2.43219 /x)^3), add=TRUE,col="orange") #Adaptive
curve(1.3+(1/(0.45494+2.2343299 /x)^3), add=TRUE,col="green") #Fixed
curve(1.3+(1/(0.49723301+1.487969 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.53426256+1.5033928 /x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1455',"topleft",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot n° 1468

```
start16 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1468.csv", sep=";", dec=".")
attach(start16)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.45172+0.9936 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.454889+2.30014 /x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.461077+2.212158 /x)^3), add=TRUE,col="green") #Fixed
curve(1.3+(1/(0.45023518+2.60985 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.4025015+2.4939588 /x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1468',"topleft",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot n° 1477

```
start17 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1477.csv", sep=";", dec=".")
attach(start17)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.3631+3.15528 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.44725+2.43812 /x)^3), add=TRUE,col="orange") #Adaptive
curve(1.3+(1/(0.45944+2.24098 /x)^3), add=TRUE,col="green") #Fixed
curve(1.3+(1/(0.48407964+2.08492 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.47791439+2.0829614 /x)^3),add=TRUE,col="blue") #500m
```

```
legend(title='Plot 1477',"topleft",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot 1511

```
start20 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1511.csv", sep=";", dec=".")
attach(start20)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.40292+2.71806 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.439642+2.594797 /x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.45658401+2.35388 /x)^3), add=TRUE,col="green") #Fixed
curve(1.3+(1/(0.49115155+1.593816 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.53202745+1.5871263 /x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1511',"topleft",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot 1524

```
start22 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1524.csv", sep=";", dec=".")
attach(start22)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.38065+2.46048 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.428184+2.70688 /x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.45019628+2.299784 /x)^3), add=TRUE,col="green") #Fixed
curve(1.3+(1/(0.51110544+0.9706428 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.79693978+0.98158809 /x)^3),add=TRUE,col="blue") #500m
#curve(1.3+(1/(0.78620205-1.1405355/x)^3),add=TRUE,col="violet") #388m
legend(title='Plot 1524',"topleft",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot 1527

```
start23 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1527.csv", sep=";", dec=".")
attach(start23)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.3773+2.2447 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.409362+2.852913 /x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.42682768+2.523297 /x)^3), add=TRUE,col="green") #Fixed
curve(1.3+(1/(0.42588919+2.16222 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.43876482+2.1529716 /x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1527',"bottomright",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot 1533

```
start24 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1533.csv", sep=";", dec=".")
attach(start24)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.38709+1.5151 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.417151+2.853721 /x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.44041852+2.42105 /x)^3), add=TRUE,col="green") #Fixed
curve(1.3+(1/(0.44441696+1.9060955 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.52498635+1.9060955 /x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1533',"topleft",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot 1555

```
start25 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1555.csv", sep=";", dec=".")
attach(start25)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.44002+0.93015 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.410985+2.788965 /x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.42839681+2.439593 /x)^3), add=TRUE,col="green") #Fixed
curve(1.3+(1/(0.46104902+2.0101134 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.80511484+2.0101134 /x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1555',"topleft",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot 1598

```
start27 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1598.csv", sep=";", dec=".")
attach(start27)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.4369+1.51652 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.3917304+3.1196247/x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.41382609+2.658969 /x)^3), add=TRUE,col="green") #Fixed
curve(1.3+(1/(0.27760987+4.8266982 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.23557085+4.7515349 /x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1598',"bottomright",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot 1648

```
start29 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1648.csv", sep=";", dec=".")
```

```
attach(start29)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.44571+1.62916 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.3868865+3.288398 /x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.40941561+2.8312022 /x)^3), add=TRUE,col="green")
#Fixed
curve(1.3+(1/(0.41570921+2.7156847 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.29831487+2.7151616 /x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1648',"bottomright",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)

#Plot 1660
start30 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1660.csv", sep=";", dec=".")
attach(start30)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.40342+2.163 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.3996343+3.101009 /x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.41183224+2.8391931 /x)^3), add=TRUE,col="green")
#Fixed
curve(1.3+(1/(0.47383604+1.8238681 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.66925232+1.7930344 /x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1660',"topleft",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)

#Plot 1666
start31 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1666.csv", sep=";", dec=".")
attach(start31)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.41014+2.60797 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.4041607+3.060545 /x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.45596309+2.05049 /x)^3), add=TRUE,col="green") #Fixed
curve(1.3+(1/(0.69590184-0.88811265 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.79270163-0.88811265 /x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1666',"topleft",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)

#Plot 1667
start32 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1667.csv", sep=";", dec=".")
attach(start32)
h<-height
```

```
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.43019+1.50054 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.40560477+3.0434651 /x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.45648572+2.0457464 /x)^3), add=TRUE,col="green")
#Fixed
curve(1.3+(1/(0.67738605-0.41408126 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.87263822-0.23019408 /x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1667',"topleft",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot 1781

```
start33 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1781.csv", sep=";", dec=".")
attach(start33)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.40038+2.64309 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.4028729+2.996512 /x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.43432961+2.3311978 /x)^3), add=TRUE,col="green")
#Fixed
curve(1.3+(1/(0.38003307+3.3344083 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.52364313+3.3344083 /x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1781',"topleft",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

- ***Curves Trees vs local Petterson***

#Plot 1151

```
start3 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/V1151.csv", sep=";", dec=".")
attach(start3)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.41902+1.73792 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.42724577+1.8565383 /x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.42302678+1.8852055/x)^3), add=TRUE,col="green") #Fixed
curve(1.3+(1/(0.40100915+2.4699093 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.41536399+2.0513537 /x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1151',"topleft",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot 1174

```
start4 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1174.csv", sep=";", dec=".")
attach(start4)
```

```
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.40478+2.4672 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.45380431+1.2310749 /x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.45230982+ 1.3451496/x)^3), add=TRUE,col="green")
#Fixed
curve(1.3+(1/(0.42486352+1.8383351 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.45723842+1.4488124/x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1174',"topleft",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot 1187

```
start5 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1187.csv", sep=";", dec=".")
attach(start5)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/((0.38193+2.68613 /x)^3)), add=TRUE,col="black")
#Petterson
curve(1.3+(1/((0.45376848+1.117626 /x)^3)), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.44075716+1.4338303/x)^3), add=TRUE,col="green") #Fixed
curve(1.3+(1/(0.41316598+2.0220193 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.42901412+1.7120061 /x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1187',"topleft",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot 1191

```
start7 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1191.csv", sep=";", dec=".")
attach(start7)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.42307+2.22909 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.41188752+2.8469041 /x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.43025968+2.1816946/x)^3), add=TRUE,col="green") #Fixed
curve(1.3+(1/(0.41845423+2.3373027 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.4240043+2.2525501 /x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1191',"topleft",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot 1196

```
start9 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1196.csv", sep=";", dec=".")
attach(start9)
h<-height
d<-diameter
```

```
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.41489+3.82912 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.43900264+1.7721102/x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.43771839+1.7383301/x)^3), add=TRUE,col="green") #Fixed
curve(1.3+(1/(0.41493189+2.1877471 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.42799641+1.8728016 /x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1196',"bottomright",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot 1202

```
start10 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1202.csv", sep=";", dec=".")
attach(start10)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.4475+1.1437 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.4460641+1.5854774/x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.44569024+1.56827 /x)^3), add=TRUE,col="green") #Fixed
curve(1.3+(1/(0.43550696+1.774532 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.43670611+1.8232702 /x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1202',"bottomright",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot 1215

```
start12 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1215.csv", sep=";", dec=".")
attach(start12)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.49675+1.74904 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.43941702+1.7548859/x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.43942092+1.7428753 /x)^3), add=TRUE,col="green")
#Fixed
curve(1.3+(1/(0.42907945+1.9394386/x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.43670611+1.8232702 /x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1215',"bottomright",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot 1455

```
start13 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1455.csv", sep=";", dec=".")
attach(start13)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.3963+2.1981 /x)^3), add=TRUE,col="black") #Petterson
```

```
curve(1.3+(1/(0.44398075+1.7320823 /x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.44313654+1.6965262 /x)^3), add=TRUE,col="green")
#Fixed
curve(1.3+(1/(0.43876759+1.8602901/x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.44431856+1.6976159 /x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1455',"bottomright",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot 1468

```
start16 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1468.csv", sep=";", dec=".")
attach(start16)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.45172+0.9936 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.44683822+1.5829411 /x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.44902797+1.6152754 /x)^3), add=TRUE,col="green")
#Fixed
curve(1.3+(1/(0.44748814+1.608336 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.44998584+1.5663694/x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1468',"bottomright",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot 1477

```
start17 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1477.csv", sep=";", dec=".")
attach(start17)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.3631+3.15528 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.47015152+1.578027 /x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.46412782+1.5427659 /x)^3), add=TRUE,col="green")
#Fixed
curve(1.3+(1/(0.43801047+1.7655199/x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.45852328+1.4997644 /x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1477',"bottomright",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot 1511

```
start20 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1511.csv", sep=";", dec=".")
attach(start20)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.40292+2.71806 /x)^3), add=TRUE,col="black") #Petterson
```

```
curve(1.3+(1/(0.43632981+1.8376018/x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.43694763+1.8501165 /x)^3), add=TRUE,col="green")
#Fixed
curve(1.3+(1/(0.42166756+2.1462104 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.43259811+1.889839/x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1511',"bottomright",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot 1524

```
start22 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1524.csv", sep=";", dec=".")
attach(start22)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.38065+2.46048 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.44527166+1.6397589 /x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.44641821+1.5903641 /x)^3), add=TRUE,col="green")
#Fixed
curve(1.3+(1/(0.43760595+1.8479355/x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.44474732+1.6362684/x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1524',"topleft",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot 1527

```
start23 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1527.csv", sep=";", dec=".")
attach(start23)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.3773+2.2447 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.43726257+1.451449 /x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.43335128+1.4739929/x)^3), add=TRUE,col="green") #Fixed
curve(1.3+(1/(0.41536579+1.8048409/x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.42881637+1.6184497/x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1527',"bottomright",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot 1533

```
start24 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1533.csv", sep=";", dec=".")
attach(start24)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.38709+1.5151 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.43690102+1.7596909 /x)^3), add=TRUE,col="orange")
#Adaptive
```

```
curve(1.3+(1/(0.44355362+1.7317658/x)^3), add=TRUE,col="green") #Fixed
curve(1.3+(1/(0.42751475+1.9813983 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.43798761+1.7526127/x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1533',"topleft",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot 1555

```
start25 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1555.csv", sep=";", dec=".")
attach(start25)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.44002+0.93015 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.4403088+1.6239772 /x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.437975+1.6300924 /x)^3), add=TRUE,col="green") #Fixed
curve(1.3+(1/(0.43120479+1.8006116 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.43717904+1.754497/x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1555',"topleft",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot 1598

```
start27 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1598.csv", sep=";", dec=".")
attach(start27)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.4369+1.51652 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.41375696+1.7925799/x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.41563524+1.8012457/x)^3), add=TRUE,col="green") #Fixed
curve(1.3+(1/(0.40575553+2.0038413 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.41109678+1.8509747/x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1598',"bottomright",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot 1660

```
start30 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1660.csv", sep=";", dec=".")
attach(start30)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.40342+2.163 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.4324257+1.9290773 /x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.43988133+1.8039612/x)^3), add=TRUE,col="green") #Fixed
curve(1.3+(1/(0.42037046+2.1298904 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.43716172+1.8505755/x)^3),add=TRUE,col="blue") #500m
```

```
legend(title='Plot 1660',"bottomright",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot 1648

```
start29 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1648.csv", sep=";", dec=".")
attach(start29)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.44571+1.62916 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.43863539+1.984595/x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.45778777+1.6278768/x)^3), add=TRUE,col="green") #Fixed
curve(1.3+(1/(0.44483963+2.0213166/x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.47004587+1.665568/x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1648',"topleft",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot 1666

```
start31 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1666.csv", sep=";", dec=".")
attach(start31)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.41014+2.60797 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.42617671+2.1488279/x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.42232201+2.2148975/x)^3), add=TRUE,col="green") #Fixed
curve(1.3+(1/(0.40401351+2.4784541 /x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.41897464+2.2087962/x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1666',"bottomright",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot 1667

```
start32 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1667.csv", sep=";", dec=".")
attach(start32)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.43019+1.50054 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.42482001+2.4346516/x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.42175064+2.3092389/x)^3), add=TRUE,col="green") #Fixed
curve(1.3+(1/(0.40942222+2.576566/x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.41922925+2.4149062/x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1667',"topleft",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

#Plot 1781

```
start33 <-
read.csv("C:/Users/mariA/Documents/R/datos/Phalepensis/Inventos/sIERRA
/ValiTrees/1781.csv", sep=";", dec=".")
attach(start33)
h<-height
d<-diameter
plot(h~d, xlab='diameter', ylab="height")
curve(1.3+(1/(0.40038+2.64309 /x)^3), add=TRUE,col="black") #Petterson
curve(1.3+(1/(0.41385572+2.2144794/x)^3), add=TRUE,col="orange")
#Adaptive
curve(1.3+(1/(0.41335765+2.2532911/x)^3), add=TRUE,col="green") #Fixed
curve(1.3+(1/(0.40883907+2.2713785/x)^3),add=TRUE,col="red") #1km
curve(1.3+(1/(0.41246948+2.3442321/x)^3),add=TRUE,col="blue") #500m
legend(title='Plot 1781',"bottomright",
c("Petterson","Adaptive",'Fixed','1km bandwidth','500m bandwidth'),
col = c('black','orange','green','red','blue'),lwd=2, cex = 0.7)
```

11. PARAMETERS TABLE

Table 8 Parameters table average data plot

Plot	n	DepP	IndepP	R^2	DepAd	IndepAd	DepFix	IndepFix	Dep1km	Indep1km	Dep500m	Indep500m	x	y
1151	17	1.738	0.419	0.770	2.702	0.435	2.568	0.450	2.320	0.433	2.317	0.419	604988	4211030
1174	12	2.467	0.405	0.826	2.262	0.442	2.179	0.450	2.312	0.431	2.312	0.461	611974	4210011
1191	11	2.229	0.423	0.811	2.753	0.432	2.427	0.458	2.341	0.459	2.599	0.436	601953	4207998
1196	17	3.829	0.415	0.675	2.497	0.435	2.321	0.449	4.132	0.368	4.102	0.339	606948	4208000
1202	32	1.144	0.448	0.373	2.397	0.446	2.135	0.460	0.811	0.511	0.947	0.645	615964	4207937
1215	12	1.749	0.497	0.786	2.369	0.447	2.185	0.457	2.643	0.453	2.549	0.517	613981	4206982
1455	15	2.198	0.396	0.769	2.432	0.443	2.234	0.455	1.488	0.497	1.503	0.534	610968	4205960
1468	13	0.994	0.452	0.693	2.300	0.455	2.212	0.461	2.610	0.450	2.494	0.403	616027	4204976
1477	23	3.155	0.363	0.529	2.438	0.447	2.241	0.459	2.085	0.484	2.083	0.478	611943	4203872
1511	10	2.718	0.403	0.779	2.595	0.440	2.354	0.457	1.594	0.491	1.587	0.532	615000	4201000
1524	10	2.460	0.381	0.812	2.707	0.428	2.300	0.450	0.971	0.511	0.982	0.797	620165	4200165
1527	26	2.245	0.377	0.565	2.853	0.409	2.523	0.427	2.162	0.426	2.153	0.439	630190	4200042
1533	15	1.515	0.387	0.607	2.854	0.417	2.421	0.440	1.906	0.444	1.906	0.525	620018	4199003
1555	14	0.930	0.440	0.595	2.789	0.411	2.440	0.428	2.010	0.461	2.010	0.805	625025	4196977
1598	13	1.517	0.437	0.738	3.120	0.392	2.659	0.414	4.827	0.278	4.752	0.236	622058	4193972
1648	16	1.629	0.446	0.558	3.288	0.387	2.831	0.409	2.716	0.416	2.715	0.298	619079	4189940
1660	11	2.163	0.402	0.672	3.101	0.3996	2.839	0.412	1.824	0.474	1.793	0.669	617998	4189940
1666	17	2.608	0.410	0.886	3.061	0.404	2.050	0.456	-0.888	0.696	-0.888	0.793	628994	4189064
1667	30	1.501	0.430	0.687	3.043	0.406	2.046	0.456	-0.414	0.677	-0.230	0.873	630086	4189039
1781	10	2.643	0.400	0.962	2.997	0.403	2.331	0.434	3.334	0.380	3.334	0.524	632000	4191000

Table 9 - Parameters table tree data

Plot	n	DepP	IndepP	R2p	DepAd	IndepAd	DepFix	IndepFix	Dep1km	Indep1km	Dep500m	Indep500m	x	y
1127	13	0.893	0.468	0.563	1.873	0.437	1.832	0.441	2.164	0.418	1.971	0.431	612002	4213978
1151	17	1.738	0.419	0.770	1.857	0.427	1.885	0.423	2.470	0.401	2.051	0.415	604988	4211030
1174	12	2.467	0.405	0.826	1.231	0.454	1.345	0.452	1.838	0.425	1.449	0.457	611974	4210011
1187	11	2.686	0.382	0.678	1.118	0.454	1.434	0.441	2.022	0.413	1.712	0.429	609958	4208987
1191	11	2.229	0.423	0.811	2.847	0.412	2.182	0.430	2.337	0.418	2.253	0.424	601953	4207998
1196	17	3.829	0.415	0.675	1.772	0.439	1.738	0.438	2.188	0.415	1.873	0.428	606948	4208000
1202	32	1.144	0.448	0.373	1.585	0.446	1.568	0.446	1.775	0.436	1.612	0.442	615964	4207937
1215	12	1.749	0.497	0.786	1.755	0.439	1.743	0.439	1.939	0.429	1.823	0.437	613981	4206982
1455	15	2.198	0.396	0.769	1.732	0.444	1.697	0.443	1.860	0.439	1.698	0.444	610968	4205960
1468	13	0.994	0.452	0.693	1.583	0.447	1.615	0.449	1.608	0.447	1.566	0.450	616027	4204976
1477	23	3.155	0.363	0.529	1.578	0.470	1.543	0.464	1.766	0.438	1.500	0.459	611943	4203872
1511	10	2.718	0.403	0.779	1.838	0.436	1.850	0.437	2.146	0.422	1.890	0.433	615000	4201000
1524	10	2.460	0.381	0.812	1.640	0.445	1.590	0.446	1.848	0.438	1.636	0.445	620165	4200165
1527	26	2.245	0.377	0.565	1.451	0.437	1.474	0.433	1.805	0.415	1.618	0.429	630190	4200042
1533	15	1.515	0.387	0.607	1.760	0.437	1.732	0.444	1.981	0.428	1.753	0.438	620018	4199003
1555	14	0.930	0.440	0.595	1.624	0.440	1.630	0.438	1.801	0.431	1.754	0.437	625025	4196977
1598	13	1.517	0.437	0.738	1.793	0.414	1.801	0.416	2.004	0.406	1.851	0.411	622058	4193972
1648	16	1.629	0.446	0.558	1.985	0.439	1.628	0.458	2.021	0.445	1.666	0.470	619079	4189940
1660	11	2.163	0.403	0.673	1.929	0.432	1.804	0.440	2.130	0.420	1.851	0.437	617998	4188940
1666	17	2.608	0.410	0.886	2.149	0.426	2.215	0.422	2.478	0.404	2.209	0.419	628994	4189064
1667	30	1.501	0.430	0.687	2.435	0.425	2.309	0.422	2.577	0.409	2.415	0.419	630086	4189039
1781	10	2.643	0.400	0.962	2.214	0.414	2.253	0.413	2.271	0.409	2.344	0.412	632000	4191000

12. RESUDUALS TABLES

12.1. Petterson Local Regression

Table 10 - Regression residual coefficients

Plot	average_p	rmse_p	std_p	stde_p	aic_p
1151	-0.107	0.960	0.903	0.082	0.887
1174	-0.064	1.013	0.971	0.069	2.195
1191	-0.031	0.511	0.483	0.044	-12.994
1196	-0.041	0.721	0.679	0.068	-4.768
1202	0.009	0.576	0.543	0.054	-9.265
1215	-0.057	0.920	0.871	0.079	-0.033
1455	-0.095	1.066	1.026	0.064	3.923
1468	-0.057	0.921	0.888	0.055	-0.770
1477	-0.085	1.067	1.046	0.034	5.979
1511	-0.022	1.166	1.099	0.110	4.847
1524	-0.058	0.738	0.707	0.050	-6.650
1527	-0.091	1.111	1.085	0.042	7.406
1533	-0.092	1.400	1.345	0.090	11.943
1555	-0.040	0.737	0.702	0.058	-5.504
1598	-0.041	0.680	0.666	0.022	-21.248
1648	-0.162	1.468	1.411	0.083	14.916
1660	-0.081	0.838	0.805	0.050	-3.790
1666	-0.130	1.407	1.319	0.132	8.591
1667	-0.004	1.009	0.966	0.074	2.074
1781	-0.201	1.411	1.361	0.062	17.052

12.2. Plot Curves

Table 11 - Adaptive kernel residual coefficients.

Plot	average_ad	rmse_ad	mean_gwr_pred_ad	std_ad	stde_ad	aic_ad
1174	-1.066	1.635	0.950	1.075	0.098	12.607
1555	-2.449	2.883	2.387	1.101	0.079	31.486
1215	0.337	0.708	0.421	0.571	0.052	-5.807
1191	-1.185	1.506	1.148	0.674	0.067	9.952
1781	-0.646	0.924	0.668	0.543	0.054	0.184
1660	-1.625	2.022	0.086	0.880	0.080	17.278
1151	-2.335	2.723	2.241	1.049	0.066	33.916
1648	-0.637	1.387	0.616	1.167	0.073	12.324
1202	-2.470	2.790	2.364	1.105	0.036	65.543
1524	-2.162	2.660	2.126	1.047	0.105	21.339
1455	-1.524	1.846	1.475	0.802	0.057	19.001
1527	-2.253	2.586	2.173	1.068	0.041	51.315
1533	-4.318	4.846	4.232	1.353	0.090	49.196
1468	-2.731	3.092	2.679	0.747	0.062	28.907
1667	-2.245	2.413	2.210	0.638	0.021	54.773
1666	-0.622	1.542	0.465	1.348	0.079	16.596
1196	1.073	1.432	1.157	0.828	0.052	13.359
1511	-1.129	1.927	0.997	1.372	0.137	14.887
1598	-1.481	2.002	1.472	1.139	0.088	19.876
1477	-1.246	1.975	1.048	1.445	0.066	31.841

Table 12 - 1km bandwidth residual coefficients

Plot	average1km	rmse_1km	mean_gwr_pred_1km	std_1km	stde_1km	aic_1km
1174	-0.786	1.379	0.671	1.016	0.092	8.865
1555	-2.997	3.393	2.934	0.979	0.070	36.051
1215	-0.174	0.607	0.279	0.546	0.050	-9.178
1191	-1.563	1.932	1.526	0.776	0.078	14.939
1781	-0.441	0.847	0.463	0.650	0.065	-1.547
1660	-1.425	1.874	0.559	0.963	0.088	15.611
1151	-1.716	2.128	1.622	1.042	0.065	26.032
1648	-0.776	1.343	0.707	1.021	0.064	11.312
1202	-1.564	1.942	1.459	1.056	0.034	43.075
1524	-1.983	2.728	2.114	1.498	0.150	21.840
1455	-1.128	1.569	1.079	0.950	0.068	14.449
1527	-1.534	1.934	1.454	1.071	0.041	36.230
1533	-3.659	4.163	3.573	1.324	0.088	44.643
1468	-3.039	3.422	2.987	0.757	0.063	31.339
1667	-3.162	3.505	3.128	1.231	0.041	77.182
1666	-2.758	4.683	2.769	3.532	0.208	54.363
1196	0.805	1.212	0.889	0.825	0.052	8.029
1511	-1.192	2.131	1.168	1.567	0.157	16.897
1598	-0.663	2.541	2.022	2.333	0.179	26.075
1477	-1.569	2.269	1.371	1.525	0.069	37.949

Table 13 - 500 m bandwidth residual coefficients

Plot	average500m	rmse500m	meangwr_pred_500m	std_500m	stde_500m	aic_500m
1174	-1.689	2.242	1.574	1.176	0.107	19.552
1555	-7.506	8.215	7.444	1.275	0.091	60.809
1215	-1.273	1.499	1.236	0.488	0.044	10.694
1191	-1.118	1.448	1.081	0.689	0.069	9.174
1781	-3.354	3.929	3.377	1.103	0.110	29.134
1660	-4.318	4.976	2.607	1.332	0.121	37.095
1151	-1.248	1.702	1.153	1.022	0.064	18.883
1648	6.262	7.197	6.330	2.554	0.160	65.023
1202	-4.315	4.607	4.209	1.129	0.036	96.637
1524	-5.906	6.925	5.870	1.968	0.197	40.472
1455	-1.824	2.248	1.775	1.041	0.074	24.526
1527	-1.870	2.241	1.791	1.087	0.042	43.876
1533	-5.743	6.362	5.657	1.498	0.100	57.362
1468	-1.774	2.200	1.722	0.983	0.082	20.737
1667	-5.231	5.542	5.197	1.158	0.039	104.669
1666	-4.043	5.545	3.886	3.385	0.199	60.112
1196	1.831	2.188	1.915	0.944	0.059	26.914
1511	-2.178	3.027	2.046	1.695	0.169	23.920
1598	1.275	4.089	2.851	3.683	0.283	38.445
1477	-1.440	2.163	1.242	1.512	0.069	35.859

Table 14 - Fixed kernel residual coefficients

Plot	average_fix	rmse_fix	mean_gwr_pred_fix	std_fix	stde_fix	aic_fix
1151	-2.553	2.951	2.459	1.085	0.068	36.496
1174	-1.190	1.769	1.074	1.123	0.102	14.345
1191	-1.638	2.004	1.600	0.767	0.077	15.667
1196	0.894	1.300	0.978	0.851	0.053	10.258
1202	-2.421	2.731	2.315	1.075	0.035	64.229
1215	0.343	0.677	0.400	0.532	0.048	-6.796

1455	-1.460	1.797	1.411	0.827	0.059	18.252
1468	-2.735	3.094	2.683	0.735	0.061	28.920
1477	-1.239	1.989	1.041	1.470	0.067	32.160
1511	-1.281	2.078	1.149	1.420	0.142	16.401
1524	-2.214	2.735	2.177	1.098	0.110	21.893
1527	-2.173	2.515	2.093	1.077	0.041	49.872
1533	-4.355	4.879	4.269	1.336	0.089	49.402
1555	-2.517	2.927	2.455	1.042	0.074	31.913
1598	-1.391	1.859	1.382	1.035	0.080	17.954
1648	-0.707	1.327	0.639	1.053	0.066	10.914
1660	-1.486	1.882	0.225	0.872	0.079	15.706
1666	-0.849	2.112	0.715	1.848	0.109	27.285
1667	-1.934	2.119	1.899	0.682	0.023	46.973
1781	-0.578	0.876	0.615	0.558	0.056	-0.869

Table 15 - Coefficient of Determination

Plot	R_ad	R_fix	R_1km	R_500km
1151	0	0	0	0.269
1174	0.496	0.409	0.641	0.052
1191	0	0	0	0.063
1196	0	0.076	0.196	0
1202	0	0	0	0
1215	0.527	0.568	0.652	0
1455	0	0	0	0
1468	0	0	0	0
1477	0.026	0.012	0	0
1511	0.321	0.209	0.169	0
1524	0	0	0	0
1527	0	0	0	0
1533	0	0	0	0
1555	0	0	0	0
1598	0	0	0	0
1648	0	0	0	0
1660	0	0	0	0
1666	0.781	0.59	0	0
1667	0	0	0	0
1781	0.802	0.822	0.834	0

12.3. Tree Curves

Table 16 - 500m bandwidht residuals

Plot	average500m	rmse500m	meangwr_pred_500m	std_500m	stde_500m	aic_500m
1174	-0.224	1.336	0.592	1.245	0.113	8.166
1555	-1.708	2.122	1.645	1.006	0.072	22.904
1215	1.560	1.840	1.592	0.607	0.055	15.206
1191	-0.121	0.733	0.080	0.679	0.068	-4.445
1781	0.111	0.548	0.146	0.503	0.050	-10.255
1660	-0.544	1.110	0.487	0.885	0.080	4.088
1151	-0.611	1.241	0.516	1.020	0.064	8.778
1648	-1.017	1.411	0.960	0.868	0.054	12.874
1202	-0.980	1.498	0.895	1.085	0.035	26.987
1524	-0.949	1.562	1.045	1.081	0.108	10.690
1455	-0.293	0.881	0.314	0.790	0.056	-1.694
1527	-0.483	1.205	0.471	1.073	0.041	11.617

1533	-3.163	3.660	3.071	1.312	0.087	40.780
1468	-1.357	1.677	1.318	0.739	0.062	14.217
1667	-1.584	1.767	1.544	0.646	0.022	36.076
1666	0.173	1.579	0.371	1.518	0.089	17.396
1196	2.403	2.709	2.484	0.832	0.052	33.753
1511	0.243	1.464	0.405	1.356	0.136	9.389
1598	0.241	1.122	0.337	1.045	0.080	4.823
1477	0.029	1.524	0.657	1.487	0.068	20.455

Table 17 - Adaptive kernel residual coefficients

Plot	average_ad	rmse_ad	mean_gwr_pred_ad	std_ad	stde_ad	aic_ad
1174	0.343	1.398	0.564	1.276	0.116	9.159
1555	-1.595	2.011	1.532	0.995	0.071	21.397
1215	1.600	1.875	1.631	0.589	0.054	15.616
1191	-0.564	0.935	0.523	0.651	0.065	0.432
1781	0.299	0.624	0.290	0.497	0.050	-7.656
1660	-0.548	1.104	0.491	0.876	0.080	3.977
1151	-0.686	1.299	0.591	1.036	0.065	10.232
1648	-0.410	1.050	0.353	0.922	0.058	3.417
1202	-1.057	1.546	0.972	1.076	0.035	28.959
1524	-0.976	1.584	1.063	1.083	0.108	10.965
1455	-0.355	0.906	0.340	0.789	0.056	-0.907
1527	-0.388	1.184	0.468	1.090	0.042	10.684
1533	-3.136	3.633	3.044	1.311	0.087	40.554
1468	-1.295	1.621	1.255	0.748	0.062	13.408
1667	-1.768	1.944	1.727	0.646	0.022	41.823
1666	0.004	1.636	0.382	1.584	0.093	18.606
1196	2.148	2.459	2.229	0.852	0.053	30.661
1511	0.198	1.467	0.397	1.368	0.137	9.439
1598	0.270	1.114	0.328	1.029	0.079	4.629
1477	-0.464	1.622	0.610	1.510	0.069	23.196

Table 18 - Fixed kernel residual coefficients

Plot	average_fix	rmse_fix	mean_gwr_pred_fix	std_fix	stde_fix	aic_fix
1174	0.169	1.320	0.485	1.239	0.113	7.897
1555	-1.507	1.932	1.443	1.000	0.071	20.280
1215	1.624	1.899	1.655	0.588	0.053	15.900
1191	-0.271	0.799	0.231	0.697	0.070	-2.713
1781	0.245	0.596	0.236	0.499	0.050	-8.589
1660	-0.539	1.112	0.482	0.891	0.081	4.133
1151	-0.578	1.230	0.483	1.028	0.064	8.501
1648	-0.524	1.064	0.467	0.874	0.055	3.855
1202	-1.008	1.511	0.923	1.076	0.035	27.520
1524	-0.923	1.550	1.044	1.091	0.109	10.539
1455	-0.260	0.866	0.299	0.787	0.056	-2.192
1527	-0.299	1.148	0.411	1.083	0.042	9.099
1533	-3.328	3.828	3.236	1.320	0.088	42.128
1468	-1.428	1.748	1.389	0.744	0.062	15.221
1667	-1.468	1.656	1.428	0.647	0.022	32.212
1666	0.035	1.594	0.334	1.543	0.091	17.717
1196	2.264	2.575	2.345	0.851	0.053	32.135
1511	0.151	1.462	0.373	1.369	0.137	9.364
1598	0.175	1.083	0.270	1.021	0.079	3.902
1477	-0.226	1.553	0.617	1.498	0.068	21.283

Table 19 - 1 km bandwidht residual coefficients

Plot	averagE1km	rmse1km	mean_gwr_pred_1km	std_1km	stde_1km	aic_1km
1174	0.297	1.115	0.406	1.011	0.092	4.193
1555	-1.542	1.977	1.478	1.023	0.073	20.920
1215	1.599	1.899	1.630	0.659	0.060	15.903
1191	-0.021	0.708	0.046	0.667	0.067	-5.139
1781	0.379	0.683	0.370	0.505	0.051	-5.859
1660	-0.557	1.100	0.500	0.864	0.079	3.884
1151	-0.873	1.417	0.778	1.030	0.064	13.018
1648	-0.700	1.203	0.643	0.910	0.057	7.776
1202	-1.096	1.599	1.012	1.109	0.036	31.039
1524	-1.086	1.645	1.115	1.046	0.105	11.720
1455	-0.480	0.959	0.422	0.775	0.055	0.666
1527	-0.441	1.173	0.387	1.058	0.041	10.224
1533	-3.212	3.709	3.120	1.312	0.087	41.174
1468	-1.368	1.691	1.328	0.748	0.062	14.420
1667	-1.595	1.777	1.554	0.646	0.022	36.435
1666	0.286	1.468	0.448	1.391	0.082	14.931
1196	2.329	2.629	2.411	0.814	0.051	32.793
1511	0.185	1.425	0.320	1.330	0.133	8.857
1598	0.106	1.135	0.390	1.081	0.083	5.131
1477	0.147	1.472	0.582	1.428	0.065	18.906

Table 20 - Coefficient of Detemination

Plot	R_ad	R_fix	R_1km	R_500km
1151	0.574	0.618	0.493	0.611
1174	0.631	0.671	0.765	0.663
1191	0.405	0.49	0.639	0.555
1196	0	0	0	0
1202	0	0	0	0
1215	0	0	0	0
1455	0.642	0.673	0.599	0.661
1468	0	0	0	0
1477	0.342	0.397	0.459	0.419
1511	0.606	0.609	0.628	0.608
1524	0.492	0.513	0.452	0.506
1527	0.454	0.486	0.463	0.434
1533	0	0	0	0
1555	0	0	0	0
1598	0.618	0.639	0.603	0.612
1648	0.213	0.191	0	0
1660	0.536	0.529	0.539	0.531
1666	0.754	0.766	0.802	0.771
1667	0	0	0	0
1781	0.91	0.918	0.892	0.93