Advanced identification of quantum properties of the light

summary of Ph. D. thesis by

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The thesis is available at the study department of the Faculty of Science, Palacký University at the address 17. listopadu 12, 771 46 Olomouc, Czech Republic. It is also possible to download the thesis at the web site stag.upol.cz. The defence will take place on .... at the Department of Optics, Faculty of Science, Palacký Univesity Olomouc.

Declaration

The thesis is based on scientific work that was done with co-authors of publications [T1–T6]. I hereby declare that this thesis is my own work, which was done under guidance of my supervisor prof. Radim Filip, Ph.D. and that, to the best of my knowledge and belief, all the resources are properly cited. I agree with the further usage of the thesis according to the requirements of Palacký University and the Department of Optics.

Abstract

This report provides results of the thesis with the title Advanced identification of quantum properties of the light. It explores nonclassicality and the quantum non-Gaussianity, which represent key properties useful for the quantum technologies. The thesis derive criteria of both the quantum aspects and gives a comprehensive analysis of realistic states that exhibit them. It also involves results of experiments proving feasibility of the presented theory.

Keywords

quantum optics, photon statistics, nonclassicality, quantum non-Gaussianity

Acknowledgement

I want to express my warm gratitude to my supervisor Radim Filip for his guidance, assistance and fruitful discussions, which opened me the scientific word and, also, for his care and support, which helped me to overcome difficulties. My thanks belong to Lukáš Slodička for his pieces of advice, enthusiasm and cordial attitude to the science. I want to thank Ivo Straka and Miroslav Ježek for their help and their advice. I am grateful to all other colleagues for the irreproducible atmosphere of friendship, which they have constituted. Last but not least, I want to express my thanks to my family for their unfailing kindness and support.

Thank you!

Lukáš

Contents

1 Introduction

Since photons are bosons, many of them can have identical properties, and therefore they can be treated collectively by a single wave function. This collective behavior is so significant for the light that the classical optics investigates only the wave features and ignore the corpuscular aspects completely. A fundamental property of the waves that classical optics examines is the coherence [B1]. A row of experiments, including the Young double slit experiment [B2] or the Arago white spot in the center of a shadow threw by a circular object [1], explored the coherence. The physics developed in the twentieth century showed that the wave description of light is incomplete. First signals appeared in the thermodynamics where an idea to quantize the electromagnetic field enabled clarification of the spectral properties of the thermal radiation [B1]. It opened a path leading to an explanation of corpuscular aspects of light that had been hidden so far by laws of nature.

The corpuscular aspects say that the energy of a wave function of light gains an integer multiple of a small unit of the energy corresponding to an energy of a single photon. The coherent aspects, known for bright beams, are kept even when light is so weak that it comprises only a single-photon having the indivisible unit of the energy. All interference experiments from the classical optics can be repeated with a single-photon and the interference fringes remain visible for many repetitions [B2]. It strikes our intuitive concept of nature where objects are either indivisible particles or waves that can always split and spread to the whole space. A photon picks the scenario of its behaving according to an experiment and performed detection.

A theory explaining the classical theory of coherence by terms of the quantum optics was established by R. Glauber and E. C. G. Sudarshan in 1963. They identified a narrow class of quantum states of light that behave like a classical wave when a detector measures the intensity of these states [2, 3]. Their theory is broadly used for distinction of attractive states going beyond the classical solution of Maxwell's equations. The quantum non-Gaussianity has recently appeared as a more demanding reference for quantum aspects, which light can possess [4]. The quantum non-Gaussianity inspects whether light overcomes both the classical theory of coherence and even the linear dynamics in the quantum optics, which is used to generate squeezed states of light [5]. The thesis related to this report provides a comprehensive analysis of both the nonclassicality and the quantum non-Gaussianity in the context of currently developing quantum technologies.

2 Methods

Fundamental states of light

The quantum optics stems from replacing the measurable quantities in Maxwell's equations by operators. These equations applied on light confined in a virtual box yields Hamiltonian [B2]

$$H = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \left(a_{\mathbf{k}}^{\dagger} a_{\mathbf{k}} + \frac{1}{2} \right), \tag{1}$$

where $\mathbf{k}$ is the wave vector distinguishing the modes, $\omega_{\mathbf{k}}$ is the frequency of oscillations, $a_{\mathbf{k}}$ is the annihilation operator and $a_{\mathbf{k}}^{\dagger}$ is the creation operator. These operators obey the commutation relation

$$\left[a_{\mathbf{k}_1}, a_{\mathbf{k}_2}^{\dagger} \right] = \delta_{\mathbf{k}_1,\mathbf{k}_2}. \tag{2}$$

Further, we can avoid the dependence of these quantities on $\mathbf{k}$ because only a single-mode light is considered in the following. Let us introduce the Fock states $|n\rangle$ that correspond to the eigenstates of the Hamiltonian, i. e.

$$H|n\rangle = E_n|n\rangle, \tag{3}$$

where E_n is the energy of the Fock state $|n\rangle$ with n being an integer. The creation and annihilation operator in the Hamiltonian are not hermitian operators, and therefore they do not represent any measurement. Nevertheless, they are important for building the quantum theory of light. They affect the Fock states according to [B3]

$$\begin{aligned} a|n\rangle &= \sqrt{n}|n-1\rangle \\ a^{\dagger}|n\rangle &= \sqrt{n+1}|n+1\rangle. \end{aligned} \tag{4}$$

The relations show the creation operator increases the energy by a single unit and the annihilation operator reduces the energy by a single unit.

The Fock states are appropriate for description of corpuscular aspects of light. For description of the wave features, it is convenient to introduce the coherent state $|\alpha\rangle$ that is defined as [B3]

$$a|\alpha\rangle = \alpha|\alpha\rangle. \tag{5}$$

The expansion in the Fock state basis yields

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle\langle n|. \tag{6}$$

Since the coherent states behave similarly as coherent waves in classical theory of the electromagnetic field, they are used for explanation of classical theory of coherence.

Representation of light

Detectors responding on the light in quantum optics measure quantities depending on ordering of the creation operator $a^\dagger$ and annihilation operator a. An interaction of the light with a detector dictates which ordering describes a measurement properly. For convenient evaluation of measured results, a representation of light inherent to a given ordering is useful. Common detectors register light by absorbing photons. It leads to a normal ordering when all creation operators are on the left from the annihilation operators [6]. Besides that, the homodyne detection technique measures always symmetrical combination of the creation and annihilation operators. For a formal representation of light in those ordering, we introduce two characteristic functions [B2]

$$\begin{aligned} \chi_s(\beta, \beta^*) &= \mathrm{Tr}\left[\rho e^{\beta a^\dagger - \beta^* a}\right] \\ \chi_n(\beta, \beta^*) &= \mathrm{Tr}\left[\rho e^{\beta a^\dagger} e^{-\beta^* a}\right], \end{aligned} \tag{7}$$

where ρ is a density matrix. They are defined in symmetric and normal orderings. Their inverse Fourier transformation gives rise to

$$\begin{aligned} W(\alpha, \alpha^*) &= \frac{1}{4\pi^2} \int \chi_s(\beta, \beta^*) e^{-\beta\alpha^* + \beta^*\alpha} \mathrm{d}^2\beta \\ P(\alpha, \alpha^*) &= \frac{1}{4\pi^2} \int \chi_n(\beta, \beta^*) e^{-\beta\alpha^* + \beta^*\alpha} \mathrm{d}^2\beta. \end{aligned} \tag{8}$$

The function $W(\alpha, \alpha^*)$ is called the Wigner function and it quantifies results of the homodyne detection. The value in the origin corresponds to the mean value of the parity operator, i.e.

$$W(0,0) = \frac{1}{2\pi}\langle(-1)^{a^\dagger a}\rangle, \tag{9}$$

which indicates the Wigner function obtains negative values for some states.

The following function P in (8) allows us to represent of any density matrix by [3]

$$\rho = \int P(\alpha)|\alpha\rangle\langle\alpha|\mathrm{d}^2\alpha. \tag{10}$$

However, the formal definition in (8) does not guarantee the function P is an ordinary function. Some states acquire the function P that is negative or even more singular than the Dirac's delta function. It classifies states of light according to behaving of the function P. Quantum states possessing the function P corresponding to a density probability function are sufficient for explaining the classical theory of coherence.

Nonclassicality

The classical theory of coherence investigates impacts of the stochastic processes on the coherence of light. Used detectors respond to the intensity of the light in a good approximation. A degree of coherence is quantified by correlation functions [B1]. Clarifying the classical coherence by the quantum optics follows from substituting the amplitudes of the coherent waves in the correlation functions by the annihilation and creation operators that are set in the normal ordering. It gives rise to a series of the correlation functions [2]

$$g^{(n)}(\tau_1, ..., \tau_{n-1}) = \frac{1}{\langle a^\dagger(0)a(0)\rangle^n} \langle a^\dagger(\tau_{n-1}) \times ... \times a^\dagger(0)a(\tau_{n-1}) \times ... \times a(0)\rangle, \tag{11}$$

which behave as the correlation functions in the classical theory for some class of quantum states. All the correlation functions equal one for the coherent states (5), which indicates the coherent state approximates a classical coherent wave in this detection. Therefore, the function P in (10) corresponding to a density probability function describes stochastic processes affecting the coherence. States with such a function P represent quantum analogues of classical waves, and therefore these states are called classical. Contrary, the quantum states with the function P not being the density probability function are beyond the classical description and are denoted as nonclassical states.

The second-order correlation function $g^{(2)}(\tau)$ was historically used the most for recognition of the nonclassical states [7, 8]. All the classical states exhibit super-Poissonian statistics meaning that $g^{(2)}(0) > 1$ [9]. Therefore, violation of the inequality discloses the nonclassicality. This phenomenon is called sub-Poissonian statistics. Also, antibunching that is associated with growing $g^{(2)}(\tau)$ for $\tau > 0$ reveals the nonclassical states [9].

Detectors that enable observation of the nonclassicality are single-photon avalanche diodes (SPADs). A SPAD is sensitive even to single-photon states but it does not allow us to recognize a number of arriving photons [10]. Therefore, the detection of the nonclassicality restricts employing $g^{(2)}(\tau)$ only for a limit of very weak states since the SPADs measure the moments of the annihilation and creation operators only approximately [11]. Measuring nonclassicality of any possible states of light requires criteria imposing conditions only on responses of employed detectors.

Basic unitary operators and quantum non-Gaussianity

Unitary operators transforming a state to another states describes the evolution of light in closed systems. Let us introduce simplest examples of such operators. Classical driving of an optical mode underlies evolution that describes the operator [B4]

$$D(\alpha) = e^{\alpha^* a - \alpha a^\dagger}. \tag{12}$$

The operator is called a displacement operator. When it acts on the vacuum, it produces the coherent state, i.e.

$$|\alpha\rangle = D(\alpha)|0\rangle. \tag{13}$$

It represents a definition of the coherent state equivalent to (5). Another unitary operator obtains a form [B4]

$$S(\xi) = e^{\xi\left(a^\dagger\right)^2 - \xi^* a^2}. \tag{14}$$

It stems from a driving Hamiltonian $H = i\left[g\left(a^\dagger\right)^2 - g^* a^2 b^\dagger\right]$, where b and $b^\dagger$ are operators associated with a pumping light. When the pumping is a strong undepleted classical beam, these operators can be substituted by amplitudes β and β^* and the Hamiltonian gets a linear form yielding the unitary operator (14). The operator can reduce the quantum noise of the canonical coordinate $X = a + a^\dagger$ or the canonical momentum $P = i(a + a^\dagger)$ below the quantum noise of the vacuum. Thus, the operator (14) is called squeezing. It is used to identify a new class of states given by [5]

$$|\alpha, \xi\rangle = S(-\xi)D(\alpha)|0\rangle. \tag{15}$$

In analogy with the nonclassicality, the mixtures of the states $|\alpha, \xi\rangle$ constitute a new quantum aspect called quantum non - Gaussianity. It is identified by inequality

$$\rho \neq \int P(\alpha, \xi)|\alpha, \xi\rangle\langle\alpha, \xi| \mathrm{d}^2\alpha \mathrm{d}^2\xi, \tag{16}$$

where the function $P(\alpha, \xi)$ is some density probability function of its arguments. The quantum non-Gaussianity is a more demanding condition than nonclassicality because only some nonclassical states are quantum non-Gaussian.

The last introduced unitary operator describes interference of light on a beam-splitter or interference in linear optical couplers in fiber optics. The operator obtains a form [B4]

$$U_{BS} = e^{\kappa a_1 a_2^\dagger - \kappa^* a_1^\dagger a_2}, \quad (17)$$

where the subscripts differentiate the interacting modes. It allows us to determine interference of two coherent states by

$$\begin{aligned} U_{BS}(T)|\alpha\rangle|\beta\rangle &= |\sqrt{T}\alpha - \sqrt{1-T}\beta\rangle \\ &\otimes |\sqrt{1-T}\alpha + \sqrt{T}\beta\rangle. \end{aligned} \quad (18)$$

It shows the amplitudes of the coherent states transform as coherent waves in classical optics. Transformation of Fock states can be expressed by

$$\begin{aligned} U_{BS}(T)|m\rangle|n\rangle = \frac{1}{\sqrt{n!m!}}(\sqrt{T}a_1 - \sqrt{1-T}a_2)^m \\ (\sqrt{1-T}a_1 + \sqrt{T}a_2)^n|0\rangle|0\rangle. \end{aligned} \quad (19)$$

When one of the modes is occupied by the vacuum, the photons in the second mode are split according to the binomial law as classical particles. Some detection networks use this phenomenon for an estimation of a number of photons.

Measuring statistical properties of light

A sequence of BSs enables partial estimation of photon number distribution when SPADs measures the outgoing spatial modes. Fig. 1 depicts possible schemes. The distribution of clicks P_n of those detectors is given by a convolution

$$P_n = \sum_{k=n}^{\infty} R_{n,k}\rho_k, \quad (20)$$

where $\rho_k = \langle k|\rho|k\rangle$ is a probability of k arriving photons and $R_{n,k}$ is a response function of the detector on a Fock state $|k\rangle$, which is given

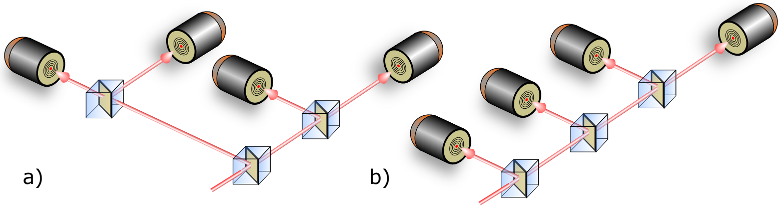

Figure 1: Examples of networks splitting a state of light among several emerging modes detected by SPAD. The network can have a tree structure (*a*) or split the light successively by a series of BSs (*b*).

by [11]

$$R_{n,k} = 1 + \sum_{l=0}^{n} \binom{n}{l} (-1)^l \left(1 - \frac{l}{N}\right)^k \tag{21}$$

with N being a number of spatial modes measured by SPADs. Alternatively, the Wigner function enables us to calculate the click distribution in the network as well. It is expressed by an overlap of the Wigner function of the split state W_s and the Wigner function of the detector response W_n [12], i. e.

$$\begin{aligned} P_n &= \int W_n(x_1, p_1, ..., x_n, p_n) \\ &\times W_s(x_1, p_1, ..., x_n, p_n) \mathrm{d}x_1 \mathrm{d}p_1 ... \mathrm{d}x_n \mathrm{d}p_n. \end{aligned} \tag{22}$$

This way of calculation is convenient especially for states with Gaussian Wigner function because the integral leads to an analytic expression. Other states exhibit click statistics, which are better expressed by (21). Both approaches allow us to quantify the detector response for coherent states, Fock states and all Gaussian states that are split on the network. These calculations give formulas that are important for a theory exposing quantum aspects of the light.

3 Nonclassicality

This section is based on the publications by L. Lachman *et al.* [T1] and by P. Obšil and *et al.* [T3].

The concept of the nonclassicality is broadly exploited for evaluation of quantum light. It differentiates the light with desirable aspects from classical waves. A workhorse for a generation of the nonclassical light have been parametric processes for the last three decades, which enable radiation of the sub-Poissonian light by heralding [10, O1]. Currently, platforms exploiting ions, molecules or solid state sources are being developed intensively. Their advantage is a level structure emitting exactly a single-photon [O2]. However, background noise often deteriorates them and the collection of light is very low in many experiments [13]. Moreover, they are often fabricated in clusters behaving as several independent emitters [O3]. Diverse criteria capture the nonclassicality inherent to such sources. They expose the nonclassicality by homodyne detection [14], by a photon-number resolving detector [15, O4, O5] or by a multi-channel detector [16]. We will investigate a systematic approach giving ab-initio criteria for any detection scheme. The criteria do not suffer by any involved approximation and as such they are reliable for an arbitrary state.

A basic detection scheme utilizes simple splitting of light on a BS and two SPADs for measuring the light as shown in Fig. 2. The scheme is used for the approximate detection of the second-order correlation function $g^{(2)}$ as well [7]. It was firstly exploited by Hanbury - Brown and Twiss for measuring intensity correlation of light from a star [17]. Let us abbreviate their names by HBT when referring to the scheme in the following. For our purpose, we introduce probabilities of no-click of both SPADs P_{00} and no-click of ith SPAD $P_{0,i}$. The criterion follows from considering the linear combination

$$F_a(\rho) = P_0 + aP_{00}, \tag{23}$$

where a is a free parameter. The functional (23) can be optimized over

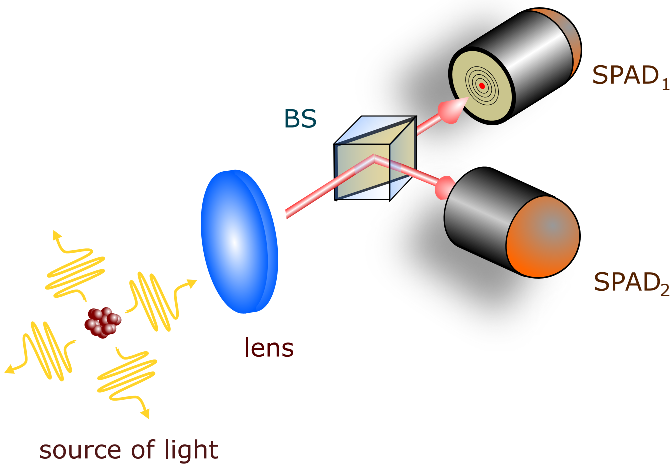

Figure 2: An experimental scheme for detection of the nonclassicality. A BS split the light from an ensemble of single-photon emitters and SPADs measure the reflected and transmitted modes.

all classical states resulting in function $F(a)$, which discloses the nonclassicality when $\exists a : P_0 + aP_{00} > F(a)$. This condition can be simplified to $P_{0,1} > P_{00}^T$ or $P_{0,2} > P_{00}^{1-T}$ (according to a used SPAD for measuring P_0) with T being the transmission of the BS. Their combination yields

$$\frac{P_{0,1}P_{0,2}}{P_{00}} - 1 > 0. \tag{24}$$

It imposes a condition that is reliable for an arbitrary state. It can be also formulated equivalently for click probabilities, which works out

$$\frac{P_{s,1}P_{s,2}}{P_c} < 1, \tag{25}$$

where $P_{s,i} = 1 - P_{0,i}$ is a click probability of ith SPAD and $P_c = 1 - P_{0,1} - P_{0,2} + P_{00}$ denotes a probability that clicks of both SPADs coincide. Although both conditions are equivalent their left sides represent different parameters useful for evaluation of the nonclassicality manifested in this layout.

A relevant model of a measured state shows behaving of those conditions in experiments. The model takes into account a density matrix

$$\rho = [\eta|1\rangle\langle 1| + (1-\eta)|0\rangle\langle 0|]^{\otimes N} \otimes \rho_{\bar{n}} \tag{26}$$

describing light radiated from an ensemble of N single-photon emitters that Poissonian noise $\rho_{\bar{n}}$ with a mean number of photons $\bar{n}$ deteriorates. The parameter η stands for the efficiency of emission and detection. The SPADs respond on the state by

$$\begin{aligned} P_0 &= (1-\eta/2)^N e^{-\bar{n}/2} \\ P_{00} &= (1-\eta)^N e^{-\bar{n}}. \end{aligned} \tag{27}$$

Putting the relations into the conditions (24) and (25) reveals the state is nonclassical for arbitrary background noise if the emitters contribute with $\eta > 0$. The parameter $d = P_0^2/P_{00} - 1$ does not depend on the background noise and it grows with a number of emitters. Contrary, the parameter $\alpha = P_c/P_s^2$ is insensitive to optical losses in a limit of weak states with $N\eta \ll 1$ and $\bar{n} \ll 1$ but it converges to one for both a high number of emitters N and high $\bar{n}$. Movement of some states in d-α plot is depicted in Fig. 3.

The only limiting factor in measuring nonclassicality of the state (26) are experimental error bars. They are quantified by a variance of the parameters in finite measurement. The variance of the parameter d works out

$$\begin{aligned} \mathrm{var}(d) &= \frac{V_c}{P_{00}^2}\left(\frac{\sin\phi}{2\sqrt{P_{00}}} + \cos\phi\right)^2 \\ &+ \frac{V_a}{P_{00}^2}\left(\frac{\cos\phi}{2\sqrt{P_{00}}} - \sin\phi\right)^2 + \frac{d^2}{P_{00}}V_{00}, \end{aligned} \tag{28}$$

where $\phi = \arctan 1/2$, V_c is the variance of P_c and V_a is the variance of an auxiliary quantity $P_0 + P_{00}$. The expected error bars achieved

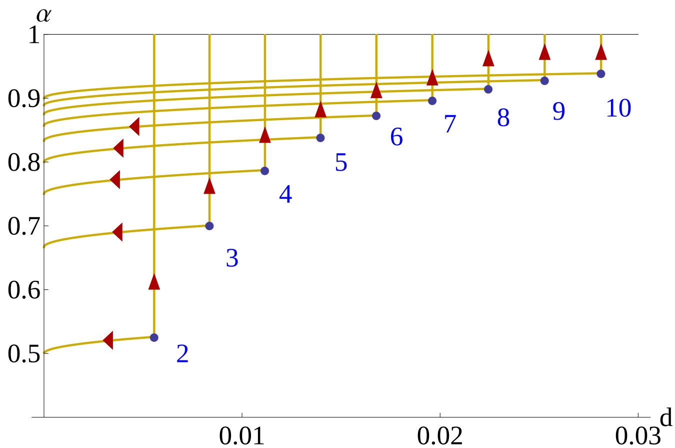

Figure 3: Figure demonstrates evaluation of a state (26) by the parameters α and d. Blue points correspond to states having $\eta = 0.1$ and no deteriorating background noise. The numbers represent numbers of single-photon emitters of these states. The vertical lines indicate how the background noise influences the parameters and the vertical lines show effects of losses.

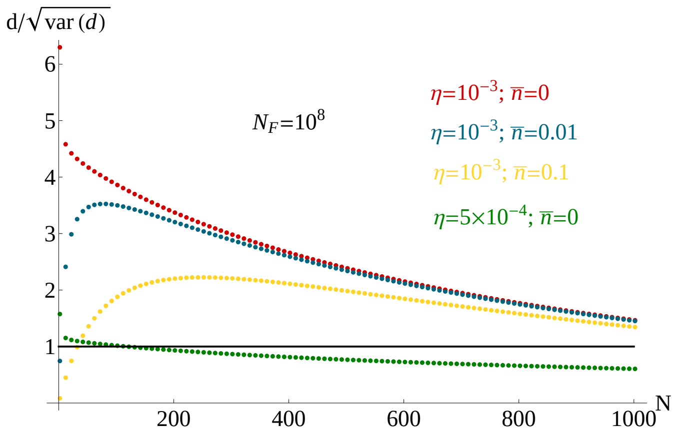

Figure 4: Expected behaving of the ratio $d/\sqrt{\mathrm{var}(d)}$ for the state (26) with growing number of contributing emitters N. The colors distinguish the parameters in the model. A number of experimental runs is 10^8. When the ratio is above one, the measured state is likely in the nonclassical region.

in measurement of the considered state are shown in Fig. 4. It demonstrates that the larger number of emitters prolongs measurement time that is necessary for achieving sufficient ratio between the parameter d and its error bars.

Those theoretical predictions were verified by an experiment where light came from an ensemble of ions in the Paul trap. The ions interacted with pumping beams and emitted a single-photon due to a transition between addressed energy levels. A lens collected the emission from the ions and aimed the light at a BS with two SPADs as Fig. 2 depicts. The recorded data was evaluated by the parameter d for pulsed and continuous pumping regimes. The nonclassicality was observed on light emitted from hundreds of ions. It certified the predicted dependence of the parameter d on the number of emitters. We fitted the experimental outputs by our model for a deeper analysis of the results. The fit considers the collection efficiency depends on a position of an

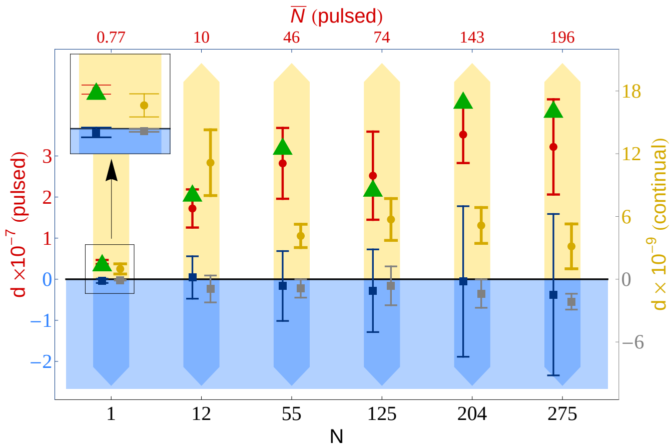

Figure 5: Figure presents measured parameter d for a crystal having n ions. Whereas the red points correspond to the parameter measured in pulsed regime the yellow points were achieved in the continual regime. The green triangles show predicted parameter d in the pulsed regime. Measurement of the scattered laser light certificated the threshold $d = 0$. The blue and gray points present the achieved results.

ion. The background noise was neglected in the fit. Fig. 5 presents the experimental results together with the calculated fit.

In summary, we derived a criterion, which can disclose the nonclassicality of multi-photon light reliably. The criterion is formulated by a condition imposed on the parameters α or d. We suggested using these parameters for advanced evaluation of sources emitting nonclassical light due to their convenient behaving for light emitted from an ensemble of single-photon emitters. The nonclassicality of such a sources was detected on light radiated from up to 275 emitters.

4 Advanced tests of nonclassicality

The section outlines the publication by L. Lachman and R. Filip [T6].

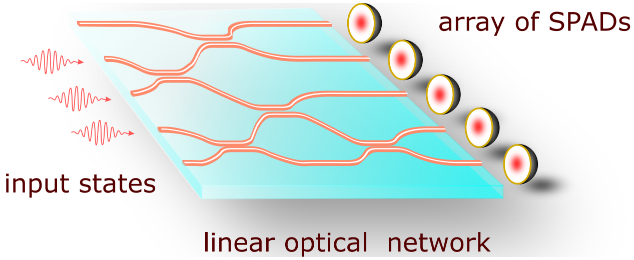

Figure 6: A general scheme where nonclassicality manifests itself. It is realized by a linear optical network that guides several states of light towards an array of SPADs.

The nonclassicality always manifests itself relative to detection. It stimulates for exploring nonclassicality in layouts different to the HBT layout. Mach-Zehnder interferometer represents a next possibility allowing us to test first-order coherence and nonclassicality simultaneously. Another extension is detection of a famous Hong - Ou - Mandel effect [18] when the interference of two indistinguishable photons on a BS cancels cases when these photons emerge separately from the BS. Generally, a scheme constitutes a linear optical network where n impinging single-photon states are split, interfere among themselves and emerge in $m \geq n$ modes [19]. Such linear optical networks have application in the quantum protocols [20–23]. Furthermore, emerging integrated optics allows their fabrication on a chip where the parameters of the network can be driven electrically [24, 25]. A formulation of a library of criteria for such layouts provides new insights into nonclassical manifestation. A new aspect of the criteria is sensitivity to the Poissonian background noise, which nonclassicality in the HBT layout does not exhibit. A row of layouts giving gradually more demanding criteria establishes a hierarchy if some conditions are arbitrarily lenient and some are arbitrarily strict. Such hierarchy enables an operational comparison of the single-photon states.

A general scheme enabling the detection is depicted in Fig. 6. It transforms the annihilation operators $\boldsymbol{a} = (a_1, ..., a_m)$ of the input modes by

$$\boldsymbol{a}_o = \boldsymbol{U}\boldsymbol{a}_i, \tag{29}$$

where $\boldsymbol{U}$ is $m \times m$ unitary matrix characterizing the linear-optical network and $\boldsymbol{a}_o$ stands for a vector of the annihilation operators for the outgoing modes. Detection of nonclassicality can be realized when single-photon states input $n < m$ modes in the layout. A density matrix of a realistic single-photon state approaches $\rho_\eta \otimes \rho_{\bar{n}}$. It is composed of the attenuated Fock state $\rho_\eta = \eta|1\rangle\langle 1| + (1-\eta)|0\rangle\langle 0|$ deteriorated by the background noise $\rho_{\bar{n}}$ with the Poissonian statistics. It encourages to derive a general library of nonclassical criteria from linear functionals with forms

$$P_n + aP_{n+1}, \tag{30}$$

where P_n refers to a probability that a selected group of n SPADs registers n clicks and P_{n+1} means that at least $n+1$ SPADs give a positive response. The criteria exclude all classical states

$$\sum_{\omega_1,\dots,\omega_n} \int P_{\omega_1,\dots,\omega_n}(\alpha_{\omega,1},\dots,\alpha_{\omega,n}) |\alpha_{\omega,1}\rangle_{1,\omega}\langle\alpha_{\omega,1}| \otimes \\ \dots \otimes |\alpha_{\omega,n}\rangle_{n,\omega}\langle\alpha_{\omega,n}| \mathrm{d}^2\alpha_{\omega,1} \dots \mathrm{d}^2\alpha_{\omega,n}, \tag{31}$$

where $P_\omega(\alpha_{\omega,1},\dots,\alpha_{\omega,n})$ is a density probability function, the subscripts $1,\dots,n$ distinguish spatial modes and ω indexes all the remaining degrees of freedom. The optimizing of (30) over classical states gives rise to a criterion derived exactly for a specific layout. Because the optimizing is done over classical states with any degree of coherence, the criteria can be applied to states showing coherent properties and even to incoherent states, which do not interfere in the layout. The criteria always reveal nonclassicality of the ideal single-photon states without noise. The noise contributions affect the nonclassicality diversely according to the detection scheme.

A simple splitting of light in HBT scheme does not allow us to derive criteria that are sensitive to background noise in the state $\rho_\eta \otimes \rho_{\bar{n}}$. A natural extension leads to layouts where more BSs split light as in

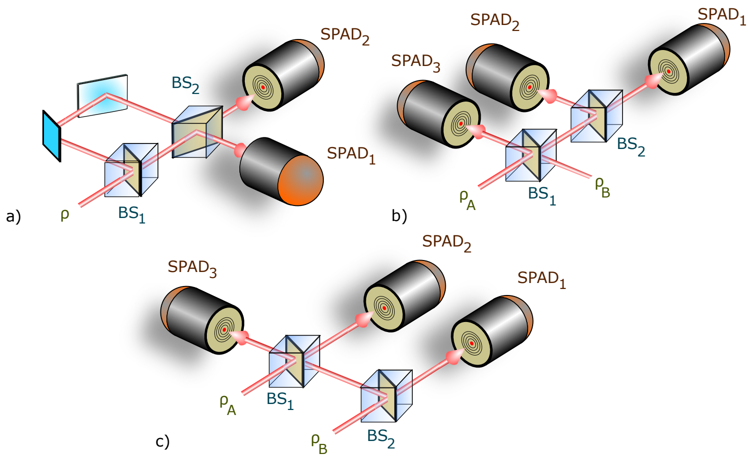

Figure 7: Examples of interfering layouts where nonclassical manifestation imposes nontrivial conditions on realistic single-photon states. It can be achieved in the Mach-Zehnder interferometer (a) for states that propagate incoherently or with partial coherence at most. Demanded networks for two copy states can be a modified version for the Hong-Ou-Mandel test (b) or a layout with two BSs and three SPADs (c).

Fig. 1. In a conceivable formulation, a success corresponds to cases when n SPADs register a signal and error means that $n+1$ SPADs click simultaneously. However, it still does not establish the criteria giving some non-trivial conditions on the state $\rho_\eta \otimes \rho_{\bar{n}}$. It appeared that the demanded criteria can be formulated only for networks in which interference between photons occurs. Fig. 7 presents examples of such networks.

Mach-Zehnder interferometer (MZI) depicted in Fig. 7a) allows tests of the first-order coherence. When light exhibits the maximal first-order coherence, recognized by the maximal visibility, the MZI acts as a BS and the analysis of the nonclassicality is the same as for the HBT test. If light shows the partial first-order coherence, the MZI manifests the nonclassicality differently to the HBT layout. The nonclassicality in this scheme excludes all possible mixtures of coherent states with any

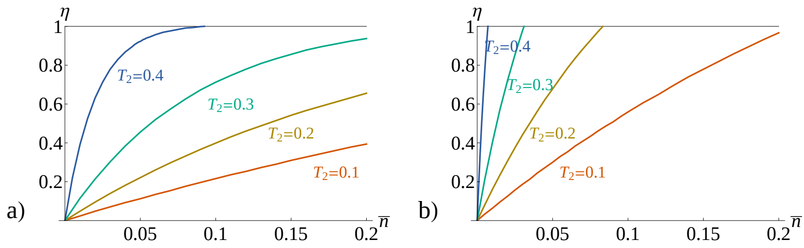

Figure 8: A hierarchy of conditions in the Mach-Zehnder interferometer for the state $\rho_\eta \otimes \rho_{\bar{n}}$ for $T_1 = 1/2$ and different settings of T_2. Whereas the noise $\rho_{\bar{n}}$ is assumed to be always incoherent, the state ρ_η is coherent (a) or incoherent (b). In both cases, the nonclassicality is manifested if the parameters η and $\bar{n}$ are above thresholds depicted by the solid lines.

first-order coherence. The criteria are derived from a linear functional

$$P_1 + aP_2, \tag{32}$$

where P_1 is a probability of a click of SPAD_1 in Fig. 7a) and P_2 refers to a probability of simultaneous click of both SPADs. Optimizing (32) over the classical states is equivalent to getting an optimum over single-mode coherent states $|\alpha\rangle_\omega$. The MZI splits the coherent state $|\alpha\rangle_\omega$ as a BS with the transmittance T and the reflectance R, which are given by

$$\begin{aligned} T &= T_1T_2 + R_1R_2 - 2\cos\Delta\phi\sqrt{T_1T_2R_1R_2} \\ R &= T_1R_2 + R_1T_2 + 2\cos\Delta\phi\sqrt{T_1T_2R_1R_2}, \end{aligned} \tag{33}$$

where $T_{1,2}$ are the transmittances of the beam-splitters $\text{BS}_{1,2}$, $R_{1,2}$ are their reflectances and $\Delta\phi$ corresponds to a phase shift given by propagation between the two paths in the interferometer. A resulted criterion is solvable only numerically but an experimentally significant corner with a very low probability $P_2 \ll 1$ offers an approximate con-

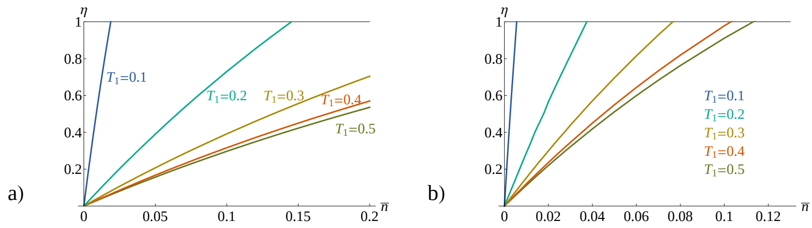

Figure 9: Nonclassical conditions recognizing nonclassicality in the layout depicted in Fig. 7b). The conditions are imposed on parameters η and $\bar{n}$ in the model state $(\rho_\eta \otimes \rho_{\bar{n}})^{\otimes 2}$ for the case when the states ρ_η are indistinguishable *a*) and distinguishable *b*). The lines correspond to thresholds that have to be surpassed to achieve nonclassicality. The colors distinct different transmittances T_1 of BS_1. The transmittance of BS_2 is fixed to the value 0.5.

dition

$$P_1 > \sqrt{\frac{R}{1-R} P_2}, \tag{34}$$

where $R = T_1 R_2 + R_1 T_2 + 2\sqrt{T_1 T_2 R_1 R_2}$. The criterion (34) goes beyond the HBT test when an inspected state does not propagate coherently. Fig. 8 demonstrates nonclassicality of the single-photon state $\rho_\eta \otimes \rho_{\bar{n}}$ in this layout. The figure shows the condition (34) is becoming strict arbitrarily when the transmittances are approaching $T_2 = 1 - T_1$ because in that case, the optimal coherent state exhibits no error events.

Interfering networks where two optical signals interfere mutually are different to both the HBT test and to the MZI. The layout where the Hong - Ou - Mandel dip is measured [18] can be extended for a test of photon indivisibility as depicted in Fig. 7b). The functional possesses a form

$$P_2 + a P_3, \tag{35}$$

where the success probability P_2 means that clicks of $SPAD_1$ and $SPAD_2$ coincide and error probability P_3 corresponds to simultaneous click of all three SPADs. Because the interference of two single-photon states on a BS is independent of their relative phases [26], the refused classical states are assumed to be phase randomized, i. e. the nonclassicality means

$$\rho \neq \sum_{\omega_1,\omega_2} \int P_{\omega_1,\omega_2}(|\alpha|_{\omega_1}, |\beta|_{\omega_2}) \rho_{\omega_1,1}(|\alpha|_{\omega_1}) \otimes \rho_{\omega_2,2}(|\beta|_{\omega_2}) \mathrm{d}|\alpha|_{\omega_1} \mathrm{d}|\beta|_{\omega_2}, \tag{36}$$

where $P_{\omega_1,\omega_2}(|\alpha|_{\omega_1}, |\beta|_{\omega_2})$ is a density probability function and $\rho_{\omega_i,i}$ occupies the ith spatial mode, oscillates with a frequency ω_i and obeys the Poissonian statistics, i. e.

$$\rho_{\omega_i,i}(|\alpha|) = e^{-|\alpha|^2} \sum_{n=0}^{\infty} |\alpha|^{2n}/n!|n\rangle\langle n|. \tag{37}$$

Optimizing over the state (36) gives rise to a condition, which ideal single-photon states $\rho_\eta \otimes \rho_\eta$ satisfy regardless of their indistinguishability. The realistic states with noise pass the criterion only when the error probability is surpassed sufficiently. Fig. 9 shows the condition on the parameters η and $\bar{n}$ required by nonclassicality. Although the transmittances of the BS_1 and BS_2 alter the nonclassical condition, they can not establish an arbitrarily tolerant condition, and therefore the layout does not enable a formulation of the hierarchy. To do that, the network can be arranged according to Fig. 7 c). The criteria involve a success probability corresponding to clicks of $SPAD_1$ and $SPAD_2$ and an error probability quantifying clicks of all three detectors. Fig. 10 presents the conditions establishing the hierarchy. Their demanding increases with growing transmission of both BSs.

In summary, we explored nonclassical manifestation in linear optical networks. A hierarchy of nonclassical criteria sensitive to noise was formulated for the Mach-Zehnder interferometer and a layout where two single-photon states interfere. According to setting of the transmission of BSs in those networks, the criteria get less or more demanding. Such nonclassical manifestation goes beyond the HBT test and any other splitting scheme where the nonclassicality is observable regard-

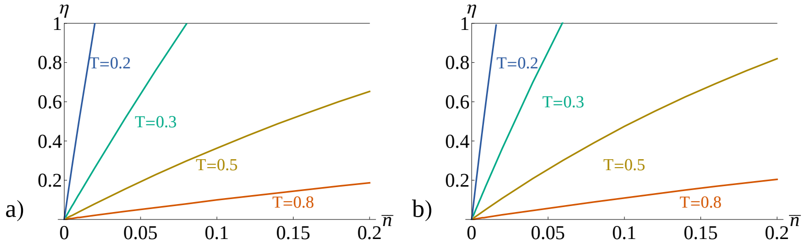

Figure 10: A predicted nonclassical thresholds in the layout in Fig. 7c). The hierarchy for two copies of the state $\rho_\eta \otimes \rho_{\bar{n}}$ is resolved for transmittances of BSs $T_1 = T_2 = T$. The solid lines represent thresholds above which the states manifest the nonclassicality. The states ρ_η are assumed indistinguishable (a) or distinguishable (b).

less of the background noise.

5 Quantum non-Gaussianity

The section provides results of the publications by L. Lachman *et al.* [T2] and by I. Straka *et al.* [T4]

Nonlinearity is a very attractive aspect in the quantum optics. It is represented by processes beyond the processes driven by Hamiltonian mostly quadratic in the annihilation and creation operators. The nonlinearity is inherent only to states with a non-Gaussian Wigner function, which was broadly explored [27–29]. However, some mixtures of coherent states also possess the non-Gaussianity [30]. For this substantial reason, this concept has to be upgraded. An unambiguous recognition of the nonlinearity has to refuse all stochastic mixtures of Gaussian states, i.e.

$$\rho \neq \int P(\xi,\alpha) D(\alpha) S(\xi) |0\rangle\langle 0| S^\dagger(\xi) D^\dagger(\alpha) \mathrm{d}^2\alpha \mathrm{d}^2\xi, \tag{38}$$

where $P(\xi, \alpha)$ is a density probability function. The introduced quantum property (38) is called quantum non-Gaussianity. Since the quantum non-Gaussianity refuses all classical states and the squeezed states, it represents a new benchmark for an evaluation of the quantum aspects.

The negativity of the Wigner function is a possible evidence of the quantum non-Gaussianity, which all the Fock states exhibit. However, the negativity is too challenging for photonic systems that the losses affects because it disappears when the losses exceed fifty percentages. It stimulates for exploring criteria disclosing the quantum non-Gaussianity of states with the positive Wigner function, especially in the early stage of many experimental platforms. The criteria exploit the homodyne measurement [31], combine the homodyne measurement with the intensity detection [30, 32] or utilize the heterodyne detection [33]. A different criterion involves only outputs of a photon-number-resolving detector [4]. It compares a response of the detector on a single-photon with a response on multiphoton contribution and uncover the quantum non-Gaussianity of the attenuated Fock state $\eta|1\rangle\langle 1| + (1-\eta)|0\rangle\langle 0|$. It was modified for the HBT layout [O6] and achieved experimentally [34, 35, O7]. It was also explored concerning the security of the quantum key distribution [36] and single photon-phonon-photon transfer [37]. The following step aims to disclose the quantum non-Gaussianity of all the attenuated Fock states. Such a theory goes before the current experiments in optics where only multiplexing of many spatial or temporal modes can simulate the statistical behaving of the high Fock states so far.

The quantum non-Gaussianity of multiphoton light can recognize a multi-channel detector depicted in Fig. 11. The detector responds to a state propagating through the network by a sequence of clicks of the SPADs. Let us consider n clicks of n SPADs as success events and $n+1$ clicks as error events due to an expected response to the Fock state $|n\rangle$. The linear functional

$$F_a(\rho) = P_n + aP_{n+1} \tag{39}$$

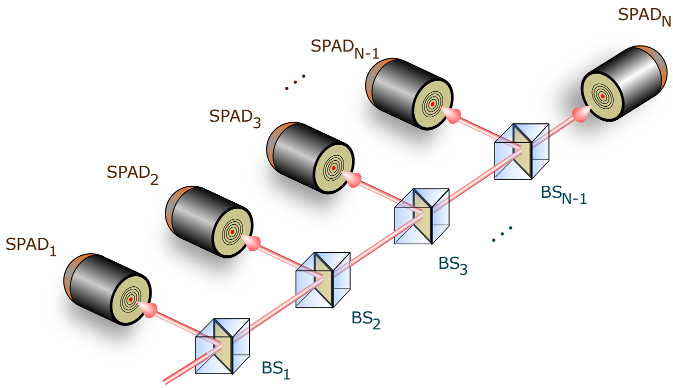

Figure 11: Quantum non-Gaussian light is recognized in a multi-channel detector that splits incoming light by an array of BSs. Outgoing light is measured by SPADs. If the detector contains N SPADs a criterion incorporating a probability of a success event corresponding to simultaneous clicks of $N-1$ selected SPADs and a probability of error event meaning that all N SPADs register a click can be tested.

yields the criteria after optimizing over all mixtures of Gaussian states. The optimizing is done over a pure state $|\alpha, \xi\rangle$ defined in (15). After exclusion the parameter a, the criteria impose conditions on the measured probabilities of the success and error. The Fig. 12 shows the resulted conditions in $log - log$ plot. The numerical thresholds can be approximated in a limit of weak states by conditions

$$P_n^{n+2} > H_n^4(x) \left[\frac{P_{n+1}}{2(n+1)^3} \right]^n. \qquad (40)$$

where x obtains the greatest value among those satisfying $H_{n+1}(x) = 0$. Inequalities (40) represents the most rough approximations, which have to be used carefully, because they are below the exact thresholds and therefore they can lead to a false positive. On the other hand, they illustrate sensitivity of the quantum non-Gaussianity to imperfections in realistic states.

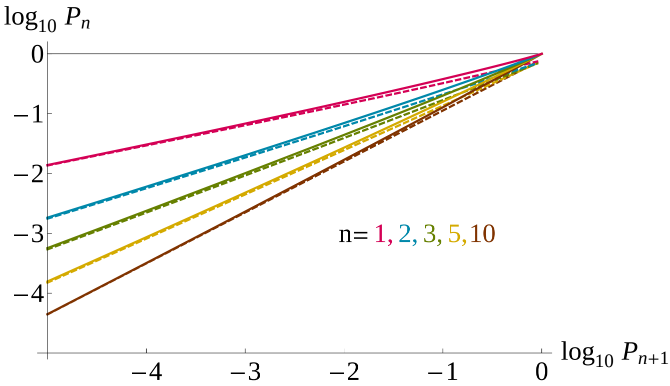

Figure 12: Hierarchy of criteria revealing quantum non-Gaussianity in $log - log$ plot. The solid lines represent thresholds that have to be exceeded to achieve the quantum non-Gaussianity. The colors distinguish a particular definition of success and error probabilities. The dashed lines depict an analytic approximate thresholds that are appropriate for states with a low error probability.

The criteria can be applied to revealing the quantum non-Gaussianity of multiphoton light with a density matrix approaching

$$\rho_{\eta,M} = [\eta|1\rangle\langle 1| + (1-\eta)|0\rangle\langle 0|]^{\otimes M}\,, \tag{41}$$

where η is a product of emission and detection efficiency and the M denotes a number of emitters. Since the states $\rho_{\eta,M}$ are restricted sharply in a number of photons, their quantum non-Gaussianity is always observable by a criterion where a number of SPADs measuring the success events equals to a number of emitting single-photon states. When a number of SPADs giving success is lower than a number of single-photon emitters, the criterion imposes a condition on the parameter η as shown in Fig. 13. Thus, the observation of the quantum non-Gaussianity requires a complex detector that reveals the truncation of photon statistics. Considering the background noise and losses leads to a more realistic model where the quantum non-Gaussianity can be number of emitters

threshold on η

1	2	3	4	5	6	7	8	9	10	order of criterion (number of detectors detecting the succes events)
0.00	0.30	0.42	0.58	0.52	0.56	0.58	0.60	0.61	0.61	1
	0.00	0.27	0.40	0.46	0.51	0.54	0.57	0.58	0.60	2
		0.00	0.26	0.38	0.45	0.50	0.53	0.56	0.58	3
			0.00	0.24	0.36	0.44	0.49	0.52	0.55	4
				0.00	0.22	0.35	0.43	0.48	0.51	5
					0.00	0.20	0.33	0.41	0.47	6
						0.00	0.19	0.31	0.40	7
							0.00	0.15	0.30	8
								0.00	0.13	9
									0.00	10

Figure 13: The table states minimal efficiencies η in an ideal state $\rho_{\eta,M}$ required for the detection of quantum non-Gaussianity when a criterion with the success probability quantifying clicks of n SPADs is employed.

lost. The state obtains a form $\rho_{\eta,M} \otimes \rho_{\bar{n}}$ with $\rho_{\bar{n}}$ representing noise having Poissonian distribution of photons with a mean number of photons $\bar{n}$. The losses decrease the parameters η and $\bar{n}$ by the factor T. In the approximation of states with strongly suppressed noise, the quantum non-Gaussianity imposes a condition

$$T > \frac{M\bar{n}H_N^{4/M}(x)}{2\eta^2(M+1)(M!)^{2/M}}. \tag{42}$$

It shows, the robustness of the quantum non-Gaussianity is inversely proportional to the mean number of photons of the noise. This methodology substantially improves the robustness of the detection to losses in comparison with utilizing the negativity of the Wigner function for the detection of quantum non-Gaussianity.

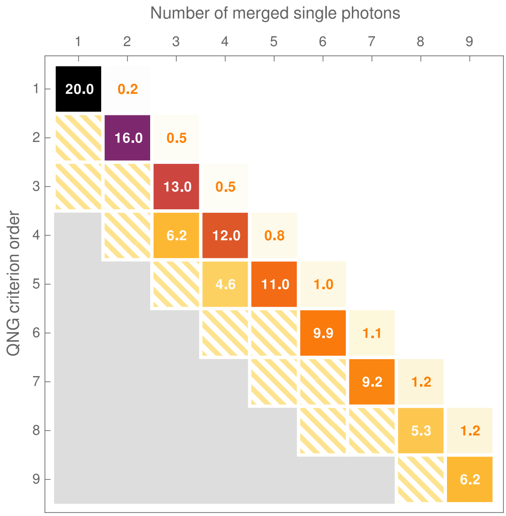

Figure 14: The table presents the robustness of the quantum non-Gaussianity against optical losses when the hierarchy of criteria is exploited [T4]. The horizontal axis quantifies how many heralded states were merged and the vertical one shows the employed criterion. The solid boxes correspond to cases when the quantum non-Gaussianity was recognized. The numbers in these boxes stand for maximal attenuation in decibels that preserves the quantum non-Gaussianity. The orange stripes below the diagonal identify inconclusive cases when error bars cross the thresholds. The gray region stands for situations when no data was acquired. The white region above the diagonal represents combinations when the criteria fail in the recognition.

The quantum non-Gaussianity was recognized experimentally using spontaneous parametric down-conversion in a periodically poled KTP crystal. The multiphoton light was simulated by taking n successive time windows, where the trigger detector registered a signal. Therefore, the light was produced by multiplexing of temporal modes. The total number of SPADs in the realized detector was ten. It rendered to test the criteria from functional (39) up to $n = 9$. The criteria applied on the state revealed the quantum non-Gaussianity of a state with the mean number of photons up to five despite detection losses. Relevant information associated with the quantum non - Gaussianity is its robustness against optical losses. The table in Fig. 14 summarizes which criteria revealed the quantum non-Gaussianity of merged heralded states together with predicted robustness against losses in decibels.

In summary, the quantum non-Gaussianity of multiphoton light was explored. The hierarchy of conditions is derived for a multi-channel detector, where the incoming light is split equally among many SPADs. Observation of that property on ideal states without the noise is possible only by a sufficiently complex network of BSs. The quantum non-Gaussianity of more realistic states deteriorated by the background noise can be lost. The criteria impose stricter conditions than the nonclassicality does, and therefore the quantum non-Gaussianity can be exploited for a tighter identification of quantum features, which light can possess. Simultaneously, the detection is more tolerant of losses than the negativity of the Wigner function. The theory was supported by an experimental test where light with a mean number of photons up to five exhibited the quantum non-Gaussianity.

6 Genuine n-photon quantum non-Gaussianity

The section summarizes the publication by L. Lachman *et al.* [T5].

The individual Fock states differ themselves in topology of the negative regions in the Wigner function. Since the Gaussian operations preserve the topology, it stimulates to introduce a hierarchy of quantum attributes that classifies states approaching the Fock states and that is preserved from influence of the squeezing or displacement operators. These requirements are fulfilled by a n-order property of a pure state

$$|\psi\rangle \neq S(\xi)D(\alpha)|\widetilde{\psi}_{n-1}\rangle, \tag{43}$$

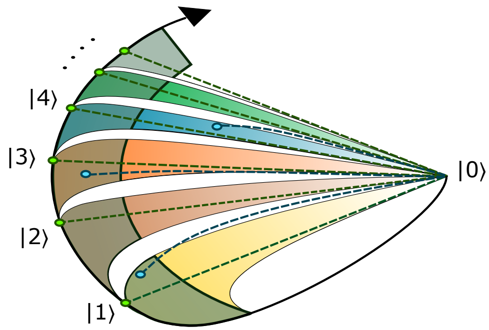

Figure 15: A scheme demonstrating the genuine n-photon quantum non-Gaussianity. The white area stands for the mixtures of Gaussian states. All color regions correspond to states beyond those mixtures. Different colors represent a hierarchy of new quantum properties, which classify the quantum non-Gaussian states with negative (opaque region) and even positive Wigner function. Each property is inherent to a respective Fock state and cannot be achieved by the lower Fock states and their superposition.

where $|\widetilde{\psi}_{n-1}\rangle = \sum_{k=0}^{n-1} c_k|k\rangle$ corresponds to some superposition of Fock states up to $|n-1\rangle$. Light with a density matrix possesses this quantum aspects when it is not any statistical mixture of the right side in (43). Such property is called genuine n-photon quantum non-Gaussianity and it has never been discussed in the literature before. Fig. 15 illustrates a scheme for these properties.

Detection of the genuine n-photon quantum non-Gaussianity utilizes the same layout that is exploited for the recognition of the quantum non-Gaussianity of the multiphoton light. Also, the criteria involve the same probabilities of the success P_n and the error P_{n+1}. However, the imposed conditions are different since the thresholds cover a broader class of states. The resolved thresholds can be approximated by

$$\begin{aligned} P_{n+1} &\approx \frac{nn!(2+n)^2}{55296(n+1)^{n-1}} t^3 \left[384 + t(896 + 307n + 99n^2)\right] \\ P_n &\approx \frac{nn!}{12(n+1)^n} t \left[6 + t(6 + 12n + n^2)\right], \end{aligned} \tag{44}$$

where t parametrizes the thresholds. Fig. 16 depicts the exactly resolved thresholds for the second and third order and compare them with an approximate solution (44). The figure also shows results of a Monte-Carlo simulation that verifies the thresholds. It was performed by generating randomly squeezing, displacement and the core state $|\widetilde{\psi}_{n-1}\rangle$ in 10^6 (2nd order) and 10^8 (3rd order) cycles. Importantly, the thresholds cover even multimode states where the core state $|\widetilde{\psi}_{n-1}\rangle$ obeys a condition

$$\langle m_1| \otimes ... \otimes \langle m_M|\widetilde{\psi}_{n-1}\rangle \neq 0 \tag{45}$$

and the excluded states involve all Gaussian modulation of the state $|\widetilde{\psi}_{n-1}\rangle$. It justifies to apply the thresholds to measurement where the multiphoton light is prepared by merging single-photon states. Fig. 16 shows the experimental results that achieve the genuine two and three-photon quantum non-Gaussianity. It also presents robustness of these properties against the losses and the background noise contributions. The limiting factors for achieving the following genuine fourth-photon quantum non-Gaussianity appeared the dark counts of the detector and measurement time, which would take several months.

In summary, we introduced a hierarchy of quantum attributes that classifies the quantum non-Gaussian light according to its approach to the Fock states in the multi-channel detector. Although the quantum properties are motivated by behaving of the negative regions of the Wigner function, the derived criteria tolerate even significant losses. The experimental feasibility of the hierarchy was manifested by an experiment where multiplexing heralded single-photon states simulated a photon statistics of the Fock states. This experimental test revealed

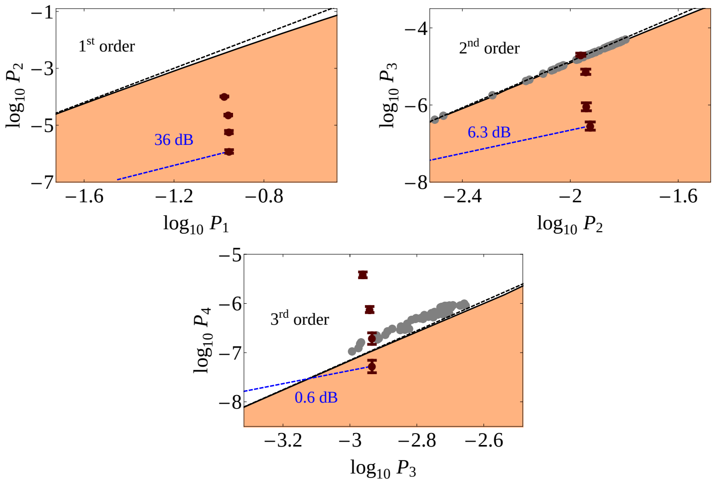

Figure 16: Threshold of the genuine n-photon quantum non-Gaussianity up to order three is compared with experimentally achieved data and results of a Monte-Carlo simulation. The states surpassing the criteria are in the orange regions. The solid black lines in the boundary of the orange regions are thresholds derived exactly, the dashed black lines are the approximate solutions (44). The gray points correspond to results of a Monte-Carlo simulation. Brown points represent the measured states. The sequences of the brown points in the vertical direction demonstrate an impact of background noise on the measured states. The dashed blue lines predict a movement of the states when they are affected by attenuation. The theoretical robustness is shown above the blue dashed lines.

the genuine n-photon quantum non-Gaussianity up to the order three.

7 Conclusion

The nonclassicality and quantum non-Gaussianity were investigated with focus on their manifestation by multiphoton states of the light. Criteria revealing these two properties stemmed from the methodology introduced in [4]. Firstly, we acquired a criterion revealing the nonclassicality in the HBT layout and applied it to a relevant model considering a cluster of single-photon emitters radiating light under background noise with the Poissonian statistics. The criterion always recognizes the nonclassicality of the state regardless of a number of single-photon emitters or an amount of the Poissonian background noise deteriorating the source. The only limiting factor is the experimental time needed for sufficient suppression of error bars. Thus, the nonclassicality represents a test that is feasible for a broad group of experimental platforms.

Layouts, where the nonclassicality is examined, were extended from the HBT layout to advanced networks. It enables recognition of different nonclassical aspects that the light can possess. The explored detection schemes involve the Mach-Zehnder interferometer and two layouts where two states of light interfere. Obeying the criteria by realistic states was investigated for a model considering an ideal single-photon state that background noise deteriorates. The criteria impose non-trivial condition on parameters of the model state. A layout establishes a hierarchy of criteria if it formulates an arbitrary lenient or strict condition on the background noise according to settings of parameters in the layout. Such hierarchies were formulated and analysed.

Further, the quantum non-Gaussianity of the multiphoton light was explored on a network splitting light among many spatial modes. We derived a sequence of conditions recognizing this quantum property and exploited them for revealing the quantum non-Gaussianity of the light emitted from an ensemble of single-photon emitters influenced by Poissonian background noise. In realistic situations, the quantum non-Gaussianity gets lost due to losses and background noise. The quantum non-Gaussianity represents an appropriate test when observation of the nonclassicality is too easy but the negativity of the Wigner function cannot be achieved due to the losses. The feasibility of the quantum non-Gaussianity for realistic states was verified by measuring the property on a state produced by multiplexing heralded states radiated from the spontaneous parametric down-conversion process.

The quantum non-Gaussianity was utilized for discrimination of quantum features that classify the Fock states. These features establish an ordered hierarchy called genuine n-photon quantum non-Gaussianity, where n denotes the lowest Fock state that possesses the quantum property. Criteria recognizing the genuine n-photon quantum non-Gaussianity were derived and achieved experimentally up to order three. The photon statistics of the Fock states was simulated by multiplexing heralded states radiated from the spontaneous parametric down-conversion process.

The following research aims to explore these quantum aspects in more platforms. Firstly, impacts of coherence on the nonclassicality are going to be investigated more deeply. Simultaneously, we are going to explore the quantum non-Gaussianity of motional states of ions and analyse the quantum non-Gaussianity of light emitted from the cavity.

8 Shrnutí v Českém jazyce

Tato dizertační práce se zabývá nástroji umožňujícími rozeznat neklasičnost a kvantovou ne-Gaussovost světla. Tyto dvě kvantové vlastnosti se projevují u řady důležitých kvantových stavů světla, které jsou klíčové pro řadu aplikací kvantové fotoniky a hrají tedy stimulující roli pro moderní kvantové technologie.

Dizertační práce začíná stručným úvodem do kvantové optiky, kde jsou popsány pojmy a postupy důležité pro text práce. Následují kapitoly pojednávající o samotném vědeckém výzkumu. Ten začíná odvozením kritéria neklasičnosti pro experimentální schéma, kde světlo prochází děličem a následně je měřeno dvěma lavinovými foto-diodami. Toto schéma umožňuje měřit $g^{(2)}$ funkci pouze za aproximativních předpokladů, které fotonové světlo mající více fotonů nesplňuje. Odvozené kritérium neklasičnosti obsahuje pravděpodobnosti kliků, které jsou přímo měřitelné, a tím umožňuje rozpoznat věrohodně neklasičnost světla emitovaného z velkého souboru jedno-fotonových emitorů i v případě, že je světlo znehodnoceno šumem.

Neklasičnost je kvantová vlastnost, kterou nelze oddělit od detekce. Proto se může neklasičnost na jiných detekčních schématech projevovat jinak. Předmětem následující kapitoly je neklasičnost manifestovaná na Mach-Zehnderově interferomtru a dalších dvou schématech, kde dvě kopie jedno-fotonových stavů interferují. Oproti předchozí situaci, tato nová kritéria dávají podmínku na množství šumu. Tato podmínka lze měnit nastavením parametrů v detekčním schématu. Pokud se v závislosti na těchto parametrech stane libovolně náročnou či libovolně tolerantní, příslušná kritéria tvoří hierarchii, která umožňuje operacionalisticky porovnat dva realistické jedno-fotonové stavy.

Další kapitoly se zabývají kvantovou ne-Gaussovostí. Ta představuje další stěžejní kvantovou vlastnost, která vylučuje nejen klasické stavy, ale i všechny směsi Gaussovských stavů, kam patři i stlačené stavy světla. V práci byla odvozena kritéria rozpoznávající kvantovou ne-Gaussovost světla s velkým středním počtem fotonů i pro stavy s pozitivní Wignerovou funkcí. Kritéria formulují podmínky na odezvu detektoru tvořeného několika lavinovými foto-diodami. Tato kritéria je možné uplatnit na stavy blízké Fockovým stavům, které jsou ale znehodnoceny šumem pozadí i optickými ztrátami. Tyto vlastnosti činí zavedená kritéria vhodnými kandidáty pro evaluaci kvantové ne-Gaussovosti realistických stavů světla tvořeného více fotony, což bylo prověřeno experimentálně. Koncept kvantové ne-Gaussovosti byl rozšířen na n-fotonovou kvantovou ne-Gaussovost, která tvoří hierarchii s řádem n. Tento řád určuje, kdy přípravu stavů blízkých Fockovému stavu $|n\rangle$ nelze chápat jako působení Gaussovských operací na superpozici nižších Fockových stavů. Pro rozpoznání těchto vlastností byla odvozena kritéria měřená na stejném schématu, který detekuje kvantovou ne-Gaussovost.

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