Czech University of Life Sciences Prague

Faculty of Economics and Management

Department of Economics

Bachelor Thesis

Application of mathematical model in consumer decision making in product selection

Ilyas Akhmetov

CZECH UNIVERSITY OF LIFE SCIENCES PRAGUE

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Ilyas Akhmetov

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Application of mathematical model in consumer decision making in product selection

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Objectives of thesis

Perform theoretical research and description of methods of multi-criteria analysis.
Identify the most appropriate multi-criteria analysis methods to use in consumer product selection.
Collect real data and survey consumers of the store “alza.cz” in Prague.
Chose the most appropriate product using the methods selected and the data obtained.
Analyze the results and make general conclusion towards possible application of the multi-criterion analysis by customers.

Methodology

The thesis consists of two parts: theoretical and practical. The theoretical part is based on the study and analysis of information taken from literary sources. This part describes the concepts of multi-criteria analysis, and explains more fully the methods used in MCDA:

- Methods of determining weight coefficients of criteria;

- Multi-criteria decision-making methods for selecting the best alternatives.

In the practical part, the work is conducted to collect and analyze the necessary information from real sources, select and substantiate the most suitable methods of multi-criteria analysis for calculation and describe the situation of consumer decision-making on the selection of the best product using selected methods of multi-criteria analysis.

The proposed extent of the thesis

35-40 pages

Keywords

Multi criteria analysis, decision making, customer

Recommended information sources

Kahraman, C. (2008). Fuzzy multi-criteria decision making. New York, NY: Springer.
ŠUBRT, T. *Ekonomicko-matematické metody.* Plzeň: Vydavatelství a nakladatelství Aleš Čeněk, s.r.o., 2015. ISBN 978-80-7380-563-0.
Thomas L. Saaty Decision Making Method of analysis of hierarchies: Int. J. Services Sciences, Vol. 1, No. 1, 2008.- 83 pp.

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The Bachelor Thesis Supervisor

doc. Ing. Ludmila Dömeová, CSc.

Supervising department

Department of Systems Engineering

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doc. Ing. Tomáš Šubrt, Ph.D.

Head of department

Electronic approval: 16. 11. 2020

Ing. Martin Pelikán, Ph.D.

Dean

Prague on 26. 11. 2020

Declaration

I declare that I have worked on my bachelor thesis titled "Application of mathematical model in consumer decision making in product selection" by myself and I have used only the sources mentioned at the end of the thesis. As the author of the bachelor thesis, I declare that the thesis does not break copyrights of any their person.

In Prague on 15.03.2021 ______________________________

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Application of mathematical model in consumer decision making in product selection

Abstract

The bachelor`s thesis is devoted to the study of the possibility of using multi-criterion analysis methods to help the consumer when choosing from the many offered products, the best option technically and economically advantageous, using the example of the “alza.cz” household appliances store in Prague.

The theoretical part of the dissertation describes the basic concepts of multi-criteria analysis of decision-making, methods for determining the best alternatives and deciding on the choice of goods by the consumer.

In the beginning to the practical part, describes the store and the choice problems facing the consumer. Based on the methods that were presented in the theoretical part, the necessary calculations were made. After obtaining the results, it is concluded that a consumer can use multi-criterion analysis methods when choosing a product.

All calculations are based on data taken from the real store and from consumers of this store by the survey method.

Keywords: Multi criteria analysis, decision making, customer.

Aplikace matematického modelu při rozhodování spotřebitele při výběru produktu

Abstrakt

Bakalářská práce je věnována studiu možnosti využití vícekriteriálních analytických metod na pomoc spotřebiteli při výběru z mnoha nabízených produktů, nejlépe technicky a ekonomicky nejvýhodnější variantou, na příkladu „alza.cz“ prodejna domácích spotřebičů v Praze.

Teoretická část disertační práce popisuje základní pojmy multikriteriální analýzy rozhodování, metody stanovení nejlepších alternativ a rozhodování o volbě zboží spotřebitelem.

Na začátku praktické části je popsán obchod a problémy s výběrem, s nimiž se spotřebitel potýká. Na základě metod uvedených v teoretické části byly provedeny potřebné výpočty. Po získání výsledků se dospělo k závěru, že spotřebitel může při výběru produktu použít metody vícekriteriální analýzy.

Všechny výpočty jsou založeny na datech získaných ze skutečného obchodu a od spotřebitelů tohoto obchodu metodou průzkumu.

Klíčová slova: multikriteriální analýza, rozhodování, zákazník.

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1. Introduction

Humanity has faced the problem of choice since the most primitive times. Where to go hunting? Where to build a house? What to plant for a good harvest? What product is profitable to buy? And in each case, men wanted to make the best, most correct, most profitable choice. But this did not always work out.

Over time, the problem of choice has become even more relevant and influenced human life in many ways. For example, managers of various levels and ranks are forced to engage in the selection of personnel of their units, choose one or another strategic line of behavior, and make specific economic decisions. Specialists in various fields of science and technology, who are engaged in the development of various kinds of devices and appliances, design certain structures, new models and types of cars, airplanes, etc., strive constantly to choose the best engineering or design solution. Bank employees choose objects for investment, economists of enterprises and firms plan an optimal economic program, etc. [1]

The complexity of the choice has increased with the development of society. A large number of possible options with a variety of different criteria have made it difficult to make the right choice and influenced the quality of the decision more and more. Therefore, it became necessary to study this topic, and thus, decision theory began to emerge. Decision theory studies the problems of making the best choice. It can help you learn how to make more informed choices by making effective use of the information you have about preferences. This theory helps to avoid making obviously bad decisions and take into account the possible negative consequences of ill-considered choices. An extremely wide and important class of choice problems from a practical point of view is made up of multicriteria problems, in which the quality of the decision made is assessed by several criteria simultaneously.

2. Objectives and Methodology

2.1 Objectives

The main goal: Analyze the possibility of using multi-criteria analysis as a tool to help theconsumer select an item.

Goals:

- Perform theoretical research and description of methods of multi-criteria analysis.
- Identify the most appropriate multi-criteria analysis methods to use in consumerproduct selection.
- Collect real data and survey consumers of the store "alza.cz" in Prague.
- Chose the most appropriate product using the methods selected and the data obtained.
- Analyze the results and make general conclusion towards possible application ofthe multi-criterion analysis by customers.

2.2 Methodology

The thesis consists of two parts: theoretical and practical. The theoretical part is based on the study and analysis of information taken from literary sources. This part describes the concepts of multi-criteria analysis, and explains more fully the methods used in MCDA:

- Methods of determining weight coefficients of criteria;
- Multi-criteria decision-making methods for selecting the best alternatives.

In the practical part, the work is conducted to collect and analyze the necessary information from real sources, select and substantiate the most suitable methods of multi-criteria analysis for calculation and describe the situation of consumer decision-making on the selection of the best product using selected methods of multi-criteria analysis

3. Literature Review

3.1 Multiple Criteria Decision Analysis (MCDA)

MCDA is the general field of study, which includes decision making in the presence of two or more conflicting objectives, and/or decision analysis processes involving two or more attributes [2].

There are several types of decision-making problems:

• Well-structured problems

Significant relationships between key characteristics can be quantified (e.g., operations research tasks);

• Unstructured problems

They are characterized by the fact that their description is dominated by qualitative factors that are difficult to formalize, and the quantitative relationships between these factors are usually not defined;

• Poorly structured problems

They combine quantitative and qualitative dependencies. However, insufficiently defined aspects of the problem tend to dominate. [3]

The general objective of MCDA is to assist a decision maker or a group of decision makers to choose the best alternative from a range of alternatives in an environment of conflicting and competing criteria.

Several methods of solving MCDA problems have been proposed over the years:

• By the end of World War II: Operations research tasks. Consulting firms (RAND, SEMA, etc.);

• 1950s: The Cost-Effectiveness Method;

• 1960s and early 70s: Human-machine procedures (HMP): STEM method;

• 1970s - 90s: Electre, VAR, AHP, MAUT methods, etc.

MCDA methods differ, however, in the way the idea of multiple criteria is considered, the application and computation of weights, the mathematical algorithm utilised, the model to describe the system of preferences of the individual facing decision- making, the level of uncertainty embedded in the data set and the ability for stakeholders to participate in the process [4].

3.2 Multiple Criteria Decision-Making Problem.

A decision-making problem arises when there are several options for actions (alternatives) to achieve a given or desired result. In this case, it is required to choose the best alternative in a certain sense.

Each of the compared alternatives is characterized by some indicators. Some of these indicators act as criteria for selecting an alternative from many others. It is believed that the criteria are the same for all alternatives and their number is the same for all alternatives. If the alternatives are evaluated by m criteria, where m> 1, then such a decision-making problem is called a multicriteria problem.

3.2.1 Alternatives

First of all, a set of alternatives which are used to make a choice should be provided for a multi-objective problem. We denote it by X and call it the set of possible solutions (alternatives). The minimum number of elements in this set is two (in order to really have a choice). There are no upper restrictions on the number of possible alternatives; their number can be either finite or infinite.

There are several types of alternatives:

- Dominant and dominated alternative.

Alternative A1 is dominant in relation to alternative A2 if according to all its assessment criteria alternative A1 is not worse than alternative A2, and at least according to one criterion A1 is better. In this case, alternative A2 is called dominated.

- Ideal and basal alternative.

Ideal is the best alternative by all criteria. Usually, there is no such ideal alternative, because it would automatically be the most effective alternative. The basal alternative is the opposite of the ideal alternative. An imperfect alternative is the worst alternative by all criteria. Also, there is usually no ideal alternative.

- Optimal alternative

The optimal alternative is the most effective solution of all the alternative options, selected according to some optimization criterion. The optimal alternative, which is the best alternative from the point of view of a given criterion of optimality and certain constraints, exists in a situation where only one solution is acceptable.

3.2.2 Criteria

The criteria are indicators which constitute the basis of assessment of alternatives interms of the possibility of achieving the set goal. A set of criteria is often used for assessment.

The following requirements are imposed on the criteria:

• completeness (the set of criteria should ensure the adequacy of the assessment ofthe achievement of the decision goal);

• operationality (the criterion has a clear, unambiguous formulation);

• decomposability (the possibility of structuring the system of criteria);

• sufficiency (no redundancy);

• minimality (the set of criteria should be the minimum necessary for theassessment);

• measurability (each criterion should provide a quantitative or qualitativeassessment of the degree of achievement of the goal).

The alternatives which measure the efficiency with a single quantitative criterion (income, profit, costs, etc.) are the most convenient for analysis. The only criterion used to evaluate alternatives is called a scalar criterion, and the set of criteria which characterize alternatives is called a vector criterion.

3.2.3 Decision Maker

People can play different roles in the decision-making process. Undoubtedly, the most important role in this process is assigned to the person who actually makes the choice of the best option, the best solution. Such a person is called a decision-maker (abbreviated as DM).

In order for the chosen solution to be optimal, a decision maker must be a specialist, who is a professional in the problem under consideration. If the decision-maker does not have enough knowledge, experts, who are professionals in a particular field, can be involved in the selection process. They can be consulted for assessments and recommendations.

Sometimes, several people united in a group whose members have equal rights (jury, commission) can act as a decision maker. The most important objective of the activities of such a group is to reach agreement when working out joint decisions.

3.3 Edgeworth – Pareto Set

The concepts of dominant and dominated alternative are presented in section 3.2.1of this work. Now let us examine an example.

Table 1 shows the values of two most important criteria characterizing investment projects: profit and the amount of capital investments for six projects.

Table 1. Characteristics of Investment Projects

Alternative	A1 Project 1	A2 Project 2	A3 Project 3	A4 Project 4	A5 Project 5	A6 Project 6
Profit, million CZK	26	18	23	27	20	16
Cap. investments, million CZK	10	9	10	12	7	6

Pairwise comparison of the alternatives shows that alternative A1 (Project 1) dominates alternative A3 (Project 3), and alternative A5 (Project 5) dominates alternative A2 (Project 2). Alternatives A1 and A5 are dominant, while alternatives A2 and A3 are dominated. These projects should be excluded from consideration. Each of the other projects is in some sense better than the other remaining, and in some sense worse: either it gives more profit, but requires large capital investments, or vice versa. Alternatives 1, 4, 5, 6 are not in dominance relations and belong to the Edgeworth-Pareto set.

Let us introduce the following definition: alternatives belong to the Edgeworth- Pareto set (E-P), if each of them is superior to any other by some of the criteria. The Edgeworth-Pareto set is named after the scientists who first paid attention to alternatives which are not inferior to each other according to criterion estimates, that is, alternatives which are not in dominance relations. Alternatives which belong to the E-P set are usually called incomparable. They really cannot be compared directly on the basis of criterion assessments. But if a decision has to be made, then a comparison of alternatives which belong to the E-P set is possible on the basis of additional information. [5]

3.4 Decision-Making Process

One should not think that decision-making is a one-time act. Very often this is a rather long and painful process. G. Simon identifies its three stages as follows: searching for information, searching for and finding alternatives, and choosing the best alternative. [6]

The first stage involves collecting all the information available at the time of making a decision, such as factual data, the experts' opinions. Where possible, mathematical models are built; sociological surveys are conducted; influential active groups' perspective to the problem is determined. The second stage is associated with the definition of what can and cannot be done in the existing situation, i.e., with the definition of solutions (alternatives). Finally, the third stage includes comparing alternatives and choosing the best solution (or options).

The most attention is traditionally paid to the last stage of the three stages of the decision-making process listed above. Recognition of the importance of seeking information and generating multiple alternatives is followed by the understanding that these stages are not highly formalized. The ways of going through the stages depend not only on the content of the decision-making problem, but also on the experience, habits, personal style of the decision maker and his environment.

Note that the selection of a small number of alternatives from what often happens to be an indefinite number of possible options for action during the second stage requires a comprehensive analysis of these options. At the same time, the analysis of a large number of options can be rather crude, but should, if possible, include all such options. At the third stage, on the contrary, it is required to carefully analyze and compare only a small number of alternatives that have already been formulated in an explicit form. This view of the decision-making process is used in the development of decision support methods. It is necessary to consider many criteria during all the stages reviewed above.

Let us formulate the required actions for all the stages of the decision-making process more specifically.

Stage 1.

• Determine the persons who are involved in the process (decision makers, experts, analysts).

• Determine the area of study and the purpose of the analysis to be performed.

- Collect the information concerning all possible solutions, determine the number of these solutions and draw up a list of them.
- Define the criteria for evaluating the alternatives.
- If necessary, conduct various types of surveys to collect the information requiredfrom persons who can influence the result.

Stage 2.

- Determine ways and methods of analysis for a large, sometimes indefinite, number of alternatives which allow to reduce their number.
- Collect additional information if necessary.
- Conduct a comprehensive analysis of the options using the chosen methods andtechniques and reduce the large number of alternatives to a relatively small one.

Stage 3.

- Analyze the methods of assessment of the small number of alternatives and selectthe most appropriate method for the given decision-making process.
- Collect additional information if necessary.
- Use the chosen method to evaluate the alternatives.
- Select several or a single best alternative on the basis of the alternatives assessment results depending on the goals set.

3.4.1 Ways to narrow down (reduce) a large number of alternatives.

In accordance with the selection procedure outlined above, the multi-criteria selection begins with the identification of a variety of alternative solutions available for analysis.

That is, the decision maker studying the problem's area of research collects all the information required about possible solutions and determines the set of options for further analysis.

The number of alternative solutions can be infinite or extremely large and immeasurable for the decision maker, so it becomes necessary to narrow down this number. Thus, criteria restrictions are introduced.

The set of alternative solutions is narrowed down to a set of acceptable solutions based on the consideration of restrictions. Acceptable or allowable solutions are those that satisfy multiple restrictions. For instance, only those applicants who have data concerning their education, work experience and other characteristics that meet the formulated restrictions can be acceptable candidates for a certain position. The procedure for obtaining a set of acceptable solutions from the initial set can be carried out by logical thinking or formally, depending on the degree of information formalization. For example, if there is an automated procedure that contains information about the alternatives, you can formulate a request and get a list of alternatives that satisfy the restrictions listed in the request, i.e., get many acceptable solutions. Meeting restrictions is a necessary condition for the choice of solutions. Therefore, the final decision is found only in the set of acceptable solutions. It follows that, it is sufficient to consider only the set of acceptable solutions for the further selection process. In practice, the process of narrowing down the set of solutions to an acceptable one begins at the stage of forming the initial set. The process of narrowing down is often carried out in an implicit, hidden form, so it goes unnoticed. The use of a computer to obtain an acceptable set of solutions usually requires clear formulation of the restrictions.

A subset of effective solutions is further determined from the set of acceptable solutions.

The analysis of preferences is carried out to narrow down the set of acceptable solutions to a set of effective solutions. It is performed using the Edgeworth-Pareto principle. The determination of effective solutions in the multi-criteria choice problem comes down to comparing the options for solutions among themselves applying each indicator and using the "not worse" vector ratio. Those solutions that are incomparable in all respects are determined by consistently eliminating the ineffective solutions when comparing pairs of solutions with each other. Thus, the effective solutions are found. A solution is said to be effective if there is no better one. The set of effective solutions in the literature is also called the Edgeworth-Pareto set, the set of non-dominated decisions. All effective solutions are incomparable with each other, i.e., it cannot be said which one is preferable. In particular cases, the set of effective solutions can contain only one solution or coincide with the set of acceptable solutions. In the first case, the only solution is optimal, and in the second case, the narrowing down of the acceptable set did not occur.

If the set of effective solutions contains more than one solution, the problem of choosing the final solution arises.

3.5 Methods of Alternative Assessment

In the process of making a decision in a multi-criteria environment, a decision-maker at a certain stage is faced with a problem when he has to make a choice from several incomparable alternatives. That is, the decision maker cannot visually determine which alternative is preferable. And thus, in order to determine the best solution, a certain mathematical model is required - the task of multi-criteria assessment.

Tools required to solve the problems of multi-criteria assessment are methods for assessing alternatives. The purpose of developing such methods is to support decision makers in the face of emerging choice problems.

Gradually, the number of different methods has increased. Currently, several dozen methods for solving multicriteria problems and many approaches to the classification of these methods have been developed.

Here are some of them, classified taking into account the use of various criteria in the calculations:

• Methods based on quantitative measurements.

Multi-Attribute Utility Theory (MAUT) [Keeney, Rife, 1981], [Köksalan et al., 2003]. Ideal point method TOPSIS [Hwang, Yoon, 1981]. Simple Additive Weighting (SAW);

• Methods based on qualitative measurements whose results are translated into quantitative form. Analytic Hierarchy Process (AHP) [Saaty, 1993]. Methods based on the theory of fuzzy sets [Zadeh, 1976];

• Methods based on quantitative measurements, but using multiple indicators when comparing alternatives. Electra group of methods (ELECTRE) [Roy, 1996];

• Methods based on qualitative measurements, without any transition to quantitative variables. Verbal decision analysis (VDA) [Larichev, 1986-2006], [Petrovsky, 2009];

At the current development stage of the multi-criteria decision-making methods, the most popular are:

- WSM (Weighted Sum Method - Simple Additive Weighting (SAW);
- AHP (Analysis of Hierarchies, Saaty Method);
- ANP (Analysis of Hierarchies taking into account interrelations);

- ELECTRE (Outranking - ranking method);
- PROMETHEE (Outranking - ranking method);
- Hellwig's (Method of proximity to a positive ideal);
- TOPSIS (Method of closeness to the double ideal);
- VIKOR (method of proximity to the ideal, taking into account the factor of regret);
- SMART, COPRAS, OWA, MACBETH, etc.

In this paper, we shall consider some of the methods based on quantitative measurements. TOPSIS, SAW, ELECTRE.

3.6 SAW Method

In decision theory the weighted sum model (WSM), also called weighted linear combination (WLC) or simple additive weighting (SAW) is the best known and simplest method of multicriteria decision analysis (MCDA).

SAW refers to direct methods for quantitative decision analysis. It is also one of the expert methods that allows you to make a managerial solution to a problem using a quantitative scoring of many of its alternatives based on specially selected choice criteria and scales. The (SAW) method allows working with a large number of criteria of a complex tree-like hierarchical structure and an unlimited number of compared objects.

Calculation procedure using the SAW method:

1. Construct matrix X where x_{ij} is the value of the criterion c_i for the alternative a_j:where

$C = \{c_1, c_2, \ldots, c_m\}$ is a set of evaluated criteria,

$A = \{a_1, a_2, \ldots, a_m\}$ is a set of potential alternatives.

$$X = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{pmatrix} \quad (3.1)$$

2. Find the normalized values of the matrix.

To normalize the criterion assessment matrix, first determine the maximum and minimumvalues of the criteria x_{ij} of the initial matrix of criteria values $X = \{ x_{ij} \}$.

To find the normalized values of the matrix, we use the following formulas:

$$p_{ij} = \frac{x_{ij} - x_j^{min}}{x_j^{max} - x_j^{min}}$$, if the criteria are maximizable; (3.2)

$$p_{ij} = \frac{x_j^{max} - x_{ij}}{x_j^{max} - x_j^{min}}$$, if the criteria are minimizable. (3.3)

The matrix of normalized values of the criteria takes the following form:

$$P = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ p_{m1} & p_{m2} & \cdots & p_{mn} \end{pmatrix}, \ i = 1,m ; \ j = 1,n \qquad (3.4)$$

3. For a more objective result, calculate weight coefficients $w_i \in [0,1]$ for each criterion.

All values of the matrix (3.4) are multiplied by the corresponding weight coefficients by the formula:

$$\tilde{p} = w_i p_{ij} \qquad (3.5)$$

As a result, we obtain the following matrix $\tilde{P} = (\tilde{P}_{ij})$

4. Next is the calculation for the rating:

$$r(a_j) = \sum_{i=1}^{m} \tilde{p}_{ij} \ , \ i = \overline{1, m} \ , \ j = \overline{1, n} \qquad (3.6)$$

The vector of functions of rating estimates of alternatives a_j takes the form:

$$R = \{r(a_1), r(a_2), \ldots, r(a_n)\} \qquad (3.7)$$

5. Find the optimal solution, which is determined by the highest value of the rating of alternatives:

$$r^* = \max_{a_j \in A} r(a_j) \qquad (3.8)$$

3.7 TOPSIS Method

TOPSIS is a technique that was originally developed by Ching-Lai Hwang and Yoon in1981, followed by Yoon in 1987, and Hwang, Lai and Liu in 1993.

TOPSIS is a compensating aggregation method that is based on the concept that the chosenalternative should have the smallest geometric distance from the positive ideal solution(PIS) and the largest geometric distance from the negative ideal solution (NIS). An ideallypositive decision is a vector of maximum values of the matrix of weighted alternatives.The ideal negative decision, on the other hand, is a vector of minimum values. [8] Calculation procedure using the TOPSIS method:

1. Let $C=\{c_i\}$ be the set of evaluated criteria, $A=\{a_j\}$ be the set of alternatives, on

2. whose basis the matrix of criteria values $X = (x_{ij})$ is constructed.

To obtain a matrix of normalized values of the criteria $P = (p_{ij})$, convert thecriteria intodimensionless form by the formula:

$$p_{ij} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{n}(x_{ij}^2)}} \quad (3.9)$$

3. Then construct a matrix of criteria weighted values using the weight coefficients w_i.

$$\tilde{p} = w_i p_{ij} \quad (3.10)$$

4. The matrix of normalized weighted values can be represented as:

$$\tilde{P} = (w_i p_{ij}) = (\tilde{p}_{ij}) \quad (3.11)$$

5. Find the ideally positive and ideally negative solutions.

$$A^+ = (max\,(\tilde{p}_{11}),\ldots,max\,(\tilde{p}_{1n})) = (\tilde{p}_1^+,\ldots,\tilde{p}_n^+) \quad (3.12)$$

$$A^- = (min\,(\tilde{p}_{11}),\ldots,min\,(\tilde{p}_{1n}) = (\tilde{p}_1^-,\ldots,\tilde{p}_n^-) \quad (3.13)$$

6. Determine the distances from alternatives to ideal-positive and ideal-negativesolutions.

$$S_j^+ = \sqrt{\sum_{j=1}^{n}(\tilde{p}_1^+ - \tilde{p}_{ij})^2}\,, j = 1,\overline{n} \quad (3.14)$$

$$S_j^- = \sqrt{\sum_{j=1}^{n}(\tilde{p}_1^- - \tilde{p}_{ij})^2}, j = 1, \bar{n} \quad (3.15)$$

7. Calculate the relative proximity and the relative range to the ideally positive andideally negative solutions using the formula:

$$P_j^+ = \frac{S_j^-}{S_j^+ + S_j^-} \quad (3.16)$$

8. Select the alternative whose value will be closer to 1.

3.8 ELECTRE Method

The ELECTRE method is the first of the outranking methods. It was proposed by the French scientist B. Roy in the 1970s and gave rise to a whole line of research in the field of methods of supporting the choice from a finite number of alternatives. The essence of the method is to use more sensitive indicators of comparability in the calculations - the indices of agreement and disagreement [9]. In the ELECTRE method, instead of a utility function, a rule is constructed in the form of a binary relation, which allows you to select a subset of alternatives from the initial population. Alternatives are specified by the values of their indicators (selection criteria), which can be obtained in various ways, including using mathematical modeling. More complex versions of the ELECTRE method have been developed, which are called ELECTRE II and ELECTRE III

Calculation procedure using the ELECTRE method:

1. Let $C=\{c_i\}$ be the set of evaluated criteria, $A=\{a_j\}$ be the set of alternatives. The value of *i* criterion of *j* alternative is presented as x_{ij}.

2. Assign weight coefficients w_1, w_2, w_3 ....w_m for each criterion.

3. At the next stage, determine the indices of agreement and disagreement. To calculate the goodness of fit indices, divide the set of criteria into three groups for each pair of alternatives:

$I^+_{jk} = \{i \in I \mid x_{ji} > x_{ki}\}$ is a set of criteria where *j* alternative is better than *k*;

$I^=_{jk} = \{i \in I \mid x_{ji} = x_{ki}\}$ is a set of criteria where *j* alternative is equal to *k*;

$I^-_{jk} = \{i \in I \mid x_{ji} < x_{ki}\}$ is a set of criteria where *j* alternative is worse than *k* [10].

Determine the consensus index by the formula:

$$C_{jk} = \frac{\sum_{i \in I+, I=} Wi}{\sum_{i=1}^{m} Wi} \tag{3.17}$$

Find the unconformity index for each pair of alternatives by the formula:

$$d_{jk} = \max_{i \in I^{-}_{jk}} \left| \frac{l_k^i - l_j^i}{L} \right| \tag{3.18}$$

where l_k^i, l_j^i are estimates of a pair of alternatives;

L is the maximum value of the point scale.

4. At the last stage, build a permissive rule. Construct a binary relation on the basis of the numbers $p \in (0;1]$ and $q \in [0;1)$; *j* alternative is considered better than *k* when $c_{ik} \geq p$ and $d_{ik} \leq q$[10].

3.9 Methods for Determining the Weights of Criteria

As we have considered above, the application of the Edgeworth-Pareto principle makes it possible to exclude obviously unacceptable solutions, i.e., those that can never be chosen if the choice is made "wisely" enough, from the set of all possible ones. After such an exclusion, a set remains, which is called the Pareto set or the area of compromises. As a rule, it is quite broad and in the decision-making process the question inevitably arises of what kind of possible solution to choose among the Pareto-optimal ones? When solving practical multicriteria problems, this issue is the most difficult and least worked out to date. Generally, no specialist in decision-making can give a reasonable answer to the question posed having only a set of possible solutions and a set of criteria (i.e., those remaining within the framework of the model of a multicriteria problem) since the implementation of a compromise (the choice of one or another Pareto optimal solution) is possible only by expanding the choice model by attracting additional information about the decision maker's preference relation. [1]

The main type of additional information which has to be dealt with most often when solving applied multicriteria problems is information about the relative importance of the criteria. Therefore, many of the existing approaches to solving multicriteria problems use this information, most frequently in the form of the so-called coefficients of relative importance of criteria (criteria weights).

We can say that criteria weights are the most delicate issue in the criteria analysis problem. The weighting coefficients should qualitatively reflect the importance of the relevant particular criteria. They must satisfy the following condition - the sum of all criteria weights is equal to 1.

Most often, weights are assigned based on an intuitive understanding of the relative importance of criteria. However, research shows that a person (expert) is not able to directly assign correct numerical weights to criteria. Special procedures are required to obtain the weights.

Let's consider some methods for calculating the weighting coefficients of the criteria:

3.9.1 Expert Assessments

The main idea of expert methods is to use the intelligence of people, their ability to seek and find solutions to weakly formalized problems. A number of methods for conducting an examination have been developed in expert assessments theory. The most effective ones are the methods of ranking and assigning points.

3.9.2 Point Assigning Method

This method is based on the procedure when experts assess the importance of a particular criterion on a scale [0-10]. At the same time, it is allowed to evaluate the importance with fractional values or assign the same value from the selected scale to several criteria. Let h_{ik} denote the score of expert *i* for criterion *k*, then

$$r_{ik} = \frac{h_{ik}}{\sum_{k=1}^{m} h_{ik}},$$ where $\sum_{k=1}^{m} h_{ik}$ is the sum of row

(3.19)

r_{ik} is the weight calculated for criterion *k* by expert *i*. Hence, given that $r_i = \sum_{j=1}^{L} r_{ji}$, we get

$$\lambda_i = \frac{r_i}{\sum_{i=1}^{m} r_i}$$ [11]. (3.20)

3.10 Ranking Method

The ranking method is as follows: let a group of experts L who are qualified specialists in the field where the decision is to be made carry out the examination. The ranking method is based on the fact that each expert is asked to arrange particular criteria of the designed object in the order of their importance. Number 1 denotes the most important particular criterion, number 2 - the next most important particular criterion, etc. These ranks are transformed in such a way that rank 1 - gets score m (the number of particular criteria), rank 2 - score m-1, etc. up to rank m, which is assigned a score of 1. Let us denote the resulting scores by r_{ik} - where *i* is the number of expert *i*, k is the number of criterion *k*. Then the results of the survey of experts can be summarized in Table 2.

Table 2. Criteria Table

Experts	Criteria			
1	r_{11}	r_{12}	. . .	r_{1m}
2	r_{21}	r_{22}		r_{2m}
. . .	. . .	. . .	. . .	. . .
L	r_{L1}	r_{L2}	. . .	r_{Lm}
∑ of scores	r1	r2	. . .	rm

$$r_i = \sum_{j=1}^{L} r_{ji}, \ i=1,2, \ldots,m. \quad (3.21)$$

Row (L+1) contains the sums of criteria scores obtained from the experts. Then the weight coefficients can be determined as follows:

$$\lambda_i = \frac{r_i}{\sum_{i=1}^{m} r_i} \text{-(i=1,2, . . . , m)} \quad (3.22)$$

- is the formula for calculating weights coefficients λ_i using the ranking method. [12]

3.11 Formal Methods for Determining Weights

Let us consider some methods and numerical techniques that make it possible to determine the values of λi weight coefficients using information about the quality of the values of particular optimality criteria.

Method 1. Calculate the coefficient of relative spread for each particular criterion of optimality $Fi(X) > 0, i = 1,2,\ldots,m$ by the following formula:

$$\delta_i = \frac{F_i^+ - F_i^-}{F_i^+} = 1 - \frac{F_i^-}{F_i^+} \qquad (3.23)$$

where $F_i^- = \min_{X \in D} F_i(X), \quad F_i^+ = \max_{X \in D} F_i(X)$, which determines the maximum possible deviation according to particular criterion i. The weight coefficients λ_i get the greatest value for those criteria whose relative spread in the range of estimates is the most significant

$$\lambda_i = \frac{\delta_i}{\sum_{k=1}^{m} \delta_k} \quad (i = 1, \dots, m) \qquad (3.24)$$

Method 2.

Let all $F_i^- \neq 0, i = 1, 2, \dots, s$, then the coefficients are considered

$$\beta_i(X) = \frac{F_i(X) - F_i^-}{F_i^-} \qquad (3.25)$$

which characterize the deviation of a particular criterion of optimality from its smallest value.

Suppose that the importance of optimality criterion i depends on the fulfillment of thefollowing inequality: $\beta_i(x) \leq \xi_i$ (1)

Here, values ξ_i are set by the decision maker on the condition that the more important thecriterion is, the smaller value ξ_i is chosen.

Let R_i^* be the largest radius of the ball built around the minimum point X_i^* of optimality criterion *i*, inside which points $x \in d(X_i^* R_i^*)$ (a ball of radius R_i^* centered at X_i^*)satisfy the condition (1).

Then $R_i^* = \max\limits_{X \in D} \left[\sum_{k=1}^{n} (x_k - x_k^*)^2 \right]$, under $\beta_i(\vec{x}) = \frac{F_i(X) - F_i^-}{F_i^-} \le \xi_i$ condition (3.26)

Now it is obvious that the larger the radius of the ball R_i^*, where the relative deviation of *i* criterion from its minimum value does not exceed ξ_i, the smaller value of λi weight coefficient should be chosen:

$$\lambda_i = \frac{\frac{1}{R_i^*}}{\sum_{i=1}^{s} \frac{1}{R_i^*}} \quad i = 1, \ldots, m$$

(3.27)

3.12 Methods using pairwise comparison of criteria

3.12.1 Method of pairwise comparison of criteria based on floating preference which forms the basis of the method of analysis of hierarchies (Saaty method)

This method is applied when only one decision maker evaluates the problem. Saaty method is one of the most used methods of calculation of criterion weights. This method determines the inconsistency of the pairwise comparison matrix. Saaty method can be divided into two steps. The first step is determination of preferences between each pair of criteria and the second step is determination of criterion weights. The advantage of Saaty method is that decision maker can express their preferences verbally rather than numerically. [13] [14]

Usually for evaluation are used a nine-point scale scheme with values 1, 3, 5, 7, 9, but it is also possible to use intermediate values 2, 4, 6, 8. Even values are used to more accurately determine preferences.

1 – equal importance

3 – moderate importance

5 – strong importance

7 – very strong importance9 – extreme importance

2, 4, 6, 8 – intermediate values

Decision maker compares all the pairs of criteria and writes preferences into the Saatymatrix $C = (c_{ij})$.

$$C = \begin{pmatrix} 1 & C_{12} & \cdots & C_{1n} \\ {}^{1}/_{C_{12}} & 1 & \cdots & C_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ {}^{1}/_{C_{1n}} & {}^{1}/_{C_{2n}} & \cdots & 1 \end{pmatrix} \quad (3.28)$$

Elements of matrix c_{ij} are represented by preference value of i-th criterion against j-th criterion. The Saaty matrix is always a square matrix n x n. If the value of i-row and jcolumn is equal, then this preference is written as $c_{ij} = 1$. Otherwise, if j-th criterion is more preferable than i-th criterion, then the preference value is equal to the inverted value. There are several methods of determination of the weights; most frequently used weighted geometric average of the Saaty matrix.

$$R_i = \sqrt[n]{\prod_{j=1}^{n} C_{ij}} \quad (3.29)$$

After normalizing averages, weights calculated by normalizing the R_i value.

$$w_i = \frac{R_i}{\sum_i^n R_i} \quad (3.30)$$

When filling out matrices of pairwise comparisons, a person can make mistakes. One of the possible mistakes is the violation of transitivity: from $c_{ij} \succ c_{jk}$, $c_{jk} \succ c_{is}$ may not follow $c_{ij} \succ c_{is}$ .Secondly, there may be violations of the consistency of numerical judgments.

It is known that the consistency of a positive inverse symmetric matrix is equivalent to the requirement that its maximum eigenvalue λmax be equal to n. Note that the inequality λmax ≥ n is always true. The closer λmax is to n (the number of objects or types of actions in the matrix), the more consistent is the result. λmax is calculated by the formula:

$$\lambda_{max} = \sum_{i=1}^{n} \lambda_i \qquad \lambda_i \quad \sum_{i=1}^{n} C_{ij} W_{ij} \quad (3.31)$$

One can also estimate the deviation from consistency by the difference λmax - n, divided by (n −1), this value will be called the consistency index (CI).

$$I_s = \frac{\lambda_{max} - n}{n-1} \quad (3.32)$$

The consistency index of the inversely symmetric matrix generated randomly on a scale from 1 to 9 with the corresponding inverse values of the elements is called the random index (RI).

Below are the matrix order (the first line) and the average RI (the second line):

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0,00	0,00	0,58	0,90	1,12	1,24	1,32	1,41	1,45	1,49	1,51	1,48	1,56	1,57	1,59

How bad the consistency for a particular problem is can be estimated by comparing the value of the quantity (CI) we obtained with its value from randomly selected judgments and the corresponding inverse values of a matrix of the same size (SI).

The ratio of the CI to the average SI for a matrix of the same order is called the consistency relation (CR). The CR value, which is less than or equal to 0.10 is considered acceptable. [14]

4. Practical Part

This part is devoted to the practical application of the theory.

The materials used in this work are from the website of Alza.cz online store in Prague (Czech Republic) and data is from the results of an online survey of the customers living in Prague.

Two methods of calculation described in the theoretical part of this work have been used herein.

4.1 Introduction to the Practical Part

An ordinary person most often acts as a buyer during his lifetime, that is, one who chooses and purchases various goods. The quantity and variety of goods has been increasing significantly with the development of production. The variety of products with different characteristics has led to the fact that when choosing a product, the buyer has to compare different options with one other and choose the best one.

When choosing a product, an ordinary person, without knowing it, has begun to perform actions related to the stages of the decision-making process. (Section 3.4 of the practical part of this work).

Thus, during the first stage, a person performs the following:

- sets a goal to acquire a product. In order to do it, this person has to choose the most optimaloption from a variety of different options, i.e., the best one by his standards;

-finds out possible options for a given product, i.e., determines a set of alternatives forconsideration;

- studies all sorts of characteristics of the product, that is, analyzes the alternatives;

- defines the criteria for evaluating the choice.At the second stage, the person:

- narrows down the number of possible alternatives to a certain number, applying the restrictionsassigned by him to the goods (for example, price, color, size, etc.);

- compares the remaining alternatives according to the criteria defined by him, discards the unsuitable ones, the worst in all indicators, criteria, that is, conducts an analysis of preferences using the Edgeworth-Pareto principle.

However, at the third stage, the buyer faces a problem. When he is left with several options that are not comparable with each by various criteria, the problem of choosing the best product arises, that is, it becomes difficult to make a choice visually, without any calculations. This is when the methods used in multi-criteria analysis come to the buyer's rescue, allowing him to choose the best product from the remaining number of goods to be considered.

Let us examine all these actions and apply the MCA methods for calculations in a specific example of choosing a washing machine for the home by an ordinary buyer. The goods have been selected at Alza.cz Store in Prague.

4.1 Description of Alza.cz Store

Alza.cz a.s. is an e-commerce store operating in the Czech Republic, Slovakia (Alza.sk), and since 2014, in other European Union countries (Alza.de, Alza.at, Alza.hu, Alza.co.uk, Alzashop.com). It's one of the largest online consumer electronics retailers in Central Europe with a strong stance in the toys, hobby, media and entertainment, perfume, watches and beauty segments too. It operates a chain of brick-and-mortar stores bearing the same name (Alza).

The company was founded in 1994 under the name Alzasoft.

In 1994, the company founder Aleš Zavoral acquired his trade license for commercial activity. In the spring 1998, the company's first website was created, although it did not yet serve as an e-commerce site. In the summer of 1998, Alzasoft rented out a small store on Dělnická street in Prague, Czech Republic. In 2000, it moved to a bigger space on Jateční streetin Prague. That is when its website also became an e-commerce store. In August 2002, the store and warehouses were flooded, but this did not jeopardise the company much.

The join-stock company Alzasoft a.s. emerged on 1 January 2004. It had 35 employees during that time. Alzasoft opened a branch in Slovakia (and gradually in Bratislava, Nitra, Trenčín and Trnava). The company warehouses located in the Prague Market in Holešovice were also expanded to a total of 1600m². In the year 2006, the company went through rebranding and was renamed Alza.cz. In 2008, the company created its own company mascot as the main face of most of its advertisements – Alza Alien.

In the summer of 2010, Alza opened a new logistics centre in Prague - Horní Počernice. In October 2015, Alza launched its Alza Premium club program. With an annual fee, Alza Premium members can receive free shipping, music, movies, special delivery services and an extended returns period.

In 2016, Alza.cz launched its Hungary e-shop, and it started to deliver purchase orders to customers even on Saturdays and Sundays. Alza also opened up its store hours during the weekend. In December 2016, Alza surpassed two of its records: it sold over one billion CZK in one week, and it hit a record of over 200 million CZK in purchase orders in one day.

In 2017, Alza.cz started selling electric cars, such as Tesla and other brands, and took part in cryptocurrency by giving the option to pay for purchases using Bitcoin. In regards to its European expansion, in the same year it opened its first store in Austria, a showroom in Hungary, and it rented out the new warehouse P3 Prague D8 in Zdiby. It also created a new B2C E- Commerce subject at the University of Economics, Prague.

Now the company has 49 stores and 78 AlzaBoxes, which are found throughout the Czech Republic, Slovakia, Hungary and Austria. Showrooms can be found in Prague, Bratislava and Budapest. The company Alza.cz is the largest e-shop in the Czech Republic with an annual turnover of over €820 million.

The online platform is located at https://www.alza.cz/ and has a very user-friendly navigation and interface. On the website, the buyers can choose the language that is convenient for them. In addition to the English and Czech languages, the Croatian, Hungarian, German, Slovak, Slovenian, Polish, Romanian languages are available.

The assortment of goods is huge, so the Czechs and residents of the other EU countries prefer to visit Alza.cz website to shopping offline, for example at IKEA, H&M or other hypermarkets.

4.2 Decision making process

The store's website https://www.alza.cz has been used to make a choice of washing machines options and perform all the actions required to reduce their number taking into account the established restrictions [15].

518 washing machines of various brands, types and modifications, with various technical characteristics and price indicators were offered for sale on the website when the possible optionswere being considered.

4.3 Criteria for evaluating alternatives

In order to evaluate the alternatives, the main criteria, which will be applied to choose the best option, have been determined according to the information taken from the store's website.

Price (CZK).

Price is the amount of money a customer will spend to purchase a product. This criterion is the type of minimization.

Washing machine size. The simplest and most obvious parameter is the size of the washing machine. The depth of the washing machine determines the maximum amount of laundry that can be loaded into it. In termsof depth, washing machines are divided into two types: standard - more than 50 cm deep and narrow - less than 50 cm.

Loading method.

There are two ways of loading laundry - frontal or vertical.Energy efficiency class.

The higher the class, the more efficiently the appliance spends the consumed energy. This parameter is one of the main indicators of the quality of household appliances. To facilitate the consumer's choice, experts have developed a letter classification system for washing machines. Most of the washing machines on the market have energy class A or B. Manufacturers are constantly improving technology, appliances are becoming more efficient. Before 2010 only A efficiency was available, today it is divided into A +, A ++ and A +++. WMs with such markingsconsume 10%, 30% and 50% less, respectively.

Maximum drum rotation speed (rpm).

The spin quality of the laundry is determined by the maximum available drum rotation speed. Formost household washing machines, it is typically in 800-1200 rpm range. For most household tasks, 1000 rpm is sufficient, and higher values usually mean that you are going to wash a lot of laundry for a large family.

Noise level (dB).

The noise level of a washing machine is expressed in decibels (dB), and in general, the higher thenumber, the louder the washing machine. Therefore, the lower the better.

Noise spectrum: 30 dB (whisper), 50 dB (office silence), 60 dB (regular call), 70 dB (TV broadcast), 80 dB (vacuum cleaners).

Number of automatic programs (item).

Depending on the needs of different users, it may be important to have some special programs andadditional features. Therefore, the more the number of automatic programs, the more attractive themodel of the washing machine being selected.

Loading capacity (kg).

The maximum amount of laundry in one work cycle. This parameter should be considered depending on the number of people who will use the washing machine. Recommended capacity for one or two people: up to 5 kg; for a family: 7 - 8 kg.

4.3.1 Narrowing down the set of alternative solutions using criteria constraints.

We determine the criteria which will be used to introduce the restrictions out of the selected criteria for evaluating alternatives. First of all, those which allow us to divide all the alternatives into twooptions. They are:

- the size of the washing machine;
- the method of loading;
- energy efficiency class.

We also introduce restrictions on the price criterion.

An online survey was conducted via the Internet in order to determine the parameters of restrictionsaccording to these criteria among a potential audience of buyers living in Prague. The following form was used for the survey:

Questionnaire No. 1

Specify your preference when buying an automatic washing machine

1. Maximum amount you can allocate (spend) on the purchase of a washing machine (write in the box, in CZK) ☐

2. Your preference when choosing a washing machine(check the box)

a) size (depth)

- standard more than 50 cm. ☐

- narrow up to 50 cm. ☐

b) laundry loading method

- top loading ☐

- front loading ☐

c) energy efficiency class

- the most economical A +++ ☐
- less economical from B to A ++ ☐

The survey results summarized in the 3rd Table

Table 3. Results of survey No. 1

Customer 's number in the survey	Maximum amount to purchase a washing machine (CZK)	Washing machine size(depth)	Loading method	Energy efficienc yclass
Customer 1	3000	narrow	front loading	A^{+++}
Customer 2	10000	standard	front loading	A^{+++}
Customer 3	5000	narrow	front loading	$B – A^{++}$
Customer 4	8000	standard	front loading	A^{+++}
Customer 5	9000	standard	front loading	A^{+++}
Customer 6	5000	narrow	front loading	A^{+++}
Customer 7	3000	narrow	top loading	A^{+++}
Customer 8	10000	standard	front loading	A^{+++}
Customer 9	12000	standard	front loading	A^{+++}
Customer 10	7500	standard	top loading	A^{+++}
Customer 11	5000	standard	top loading	A^{+++}
Customer 12	10000	standard	front loading	A^{+++}
Customer 13	5000	standard	front loading	A^{+++}
Customer 14	13500	standard	front loading	$B – A^{++}$
Customer 15	8000	narrow	front loading	A^{+++}
Customer 16	4000	standard	front loading	A^{+++}
Customer 17	7000	standard	front loading	A^{+++}

Customer 18	15000	standard	front loading	A^{+++}
Customer 19	14000	standard	top loading	A^{+++}
Customer 20	9000	standard	top loading	$B – A^{++}$
Customer 21	4000	standard	front loading	A^{+++}
Customer 22	10000	standard	front loading	A^{+++}
Customer 23	16000	standard	top loading	A^{+++}
Customer 24	9000	standard	front loading	A^{+++}
Customer 25	11000	standard	front loading	A^{+++}
Customer 26	8000	standard	top loading	A^{+++}
Customer 27	12000	standard	front loading	A^{+++}
Customer 28	7000	standard	front loading	A^{+++}
Customer 29	5000	standard	front loading	A^{+++}
Customer 30	9000	standard	front loading	A^{+++}
The resultof the survey	**8500**	**standard**	**front loading**	$\mathbf{A^{+++}}$

(Source: own processing)

Based on the results of the survey, the following parameters of the criteria were determined:

- price - no more than CZK 8500;
- washing machine size - standard;
- method of loading - front loading;
- energy efficiency class - class A +++.

We have introduced all these restrictions on the store website. As a result, the number of possiblealternatives has decreased to 22 units. Thus, an admissible set of options meeting the established requirements has been determined out of the entire variety of options for washing machines.

4.3.2 Narrowing down the remaining set of feasible solutions using the Edgeworth-Pareto principle.

In order to further analyze the remaining 22 alternatives, let us define the criteria for assessment and tabulate all the alternatives and parameters according to the criteria. Our evaluation criteria are:

- price (CZK) - minimization criteria;maximum drum rotation speed (rpm) - maximization criteria;

- noise level (dB) - minimization criteria;

- number of automatic programs (items) - maximization criteria;

- laundry capacity (kg) - maximization criteria.

Table 4. Summary table of alternatives

№ of alternatives	Brand of washing machine	Price (CZK)	Maximum drum rotation speed (rpm)	Noise level (dB)	Number of automatic programs (items)	Laundry capacity (kg)
1	**ECG EWF 1064**	5499	1000	58	15	6
2	**WHIRLPOOL FWF71253W**	6499	1200	63	14	7
3	**GORENJE WEI84CPS**	7590	1400	53	16	8
4	**SAMSUNG WW70K5210UW/ZE**	7999	1200	54	14	7
5	**SAMSUNG WW70J5446DW/ZE**	8499	1400	51	14	7
6	**GORENJE WE723**	6990	1200	57	16	7
7	**INDESIT BWA 71283X W EU**	6490	1200	51	16	7

8	**CANDY BWM 148PH7 1-S**	7999	1400	51	9	8
9	**BEKO WRE6632ZWBW**	6989	1200	56	14	6
10	**GORENJE WA84CS**	7999	1400	53	14	8
11	**CANDY RO 1284DWMCE/1-S**	7199	1200	51	16	8
12	**CANDY CSO 14105TE/1-S**	6589	1400	60	16	10
13	**CANDY CS 1482DE/1-S**	6390	1400	61	16	8
14	**TOSHIBA TW-BJ80S2PL**	6989	1200	56	16	7
15	**WHIRLPOOL FFL 6238 W EE**	6666	1200	51	14	6
16	**ECG EWF 1472**	7299	1400	58	15	7
17	**TOSHIBA TW-BJ90S2PL**	7999	1200	56	15	8
18	**WHIRLPOOL FFS 7238 B EE**	7999	1200	51	14	7
19	**CANDY RO 1285DWMCE/1-S**	7490	1200	61	16	8
20	**HISENSE WFHV8012**	7990	1200	58	15	8
21	**ETA 355290000**	7999	1200	58	15	8
22	**SHARP ES GFB7143W3**	7490	1400	58	15	7

(Source: data has been taken from the website of Alza.cz online store, https://www.alza.cz)

Now let's analyze the preferences using the Edgeworth-Pareto principle (Section 3.3 of the Theoretical Part).

Alternative 3 dominates alternative 4, that is, according to all indicators, it is better or the same. Similarly, A3 dominates A10, A7 dominates A2, A7 dominates A6, A7 dominates A9, A7 dominates A14, A7 dominates A15, A7 dominates A18, A11 dominates A17, A11 dominates A20,A11 dominates A21, A13 dominates A19, A16 dominates A22.

We are left with Alternatives 1, 3, 5, 7, 8, 11, 12, 13, 16, which are not in the dominance relationand belong to the Edgeworth-Pareto set.

4.3.4 Determining the best alternative

The remaining 9 alternatives are not comparable. They really cannot be compared directly on the basis of criterion assessments. Therefore, in order to select the best solution, it is necessary to usethe MCA methods.

4.3.4.1 Selection of methods for evaluating alternatives

In order to determine the best alternative from the remaining ones, let us calculate using two methods - SAW and TOPSIS.

The use of two methods allows us to compare the results obtained and make sure that the study iscorrect.

Both methods are the most common and easiest to use.

In our case all the specified criteria for alternatives have a quantitative characteristic which makesit possible to painlessly apply these methods for calculations. Indeed, there is often a subjective opinion of the decision maker when translating qualitative characteristics into quantitative ones which can negatively affect the final result of the selection.

When using the other methods, for example, pairwise comparison methods, a sufficiently large number of alternatives and criteria would significantly increase the amount of time spent on making the necessary calculations. These methods allow us to make the necessary calculations infairly short time.

4.3.4.2 Selecting a method for determining criteria weights

To determine the weight coefficients, we will use information about preferences. This informationis collected among the potential audience of buyers. To collect the necessary data, we will use theSCORE method. The SCORE method is a model of efficiently organized data collection.

An online survey was conducted among the potential audience of buyers living in Prague. The respondents participating in the survey were 30 people. Of these:

- students-14 people (47%);

- working in various fields of activity-16 people (53%);

- family – 10 people (33%);

- age: 21 – 5 persons, ' 22 – 2 pers, 23 – 4 pers, 24 year – 3 pers, 25 years old – 6 pers, 27 years –

4 pers, 28 years – 3 pers, 31 – 3 people.

The online survey was conducted via the Internet, using the following programs and socialnetworks:

Google questionnaire – Questionnaire) - 20 people were interviewed;Facebook-10 people were interviewed.

The following form has been used for the survey:

Questionnaire number 2

Give points to the 5 criteria according to their importance to you when choosing a washingmachine

The higher the score, the more important this criterion is for you.The sum of all points must be 100.

CRITE RIA	EXAMPL E	YOUR VARIANT
1. Price.	19	
2. Drum rotation speed when spinningthe laundry.	14	
3. Noise level.	17	
4. Number of automatic programs.	30	

5. Maximum load weight.	20	
Total points	100	

The survey results have been tabulated in table 5:

Table 5. Survey results No. 2

Customer' snumber in the survey	C 1	C 2	C 3	C 4	C 5
	Price	Maximum drum rotation speed (rpm)	Noise level(dB)	Number of automatic programs (items)	Laundry capacity (kg)
Customer 1	30	15	12	15	28
Customer 2	35	10	25	10	20
Customer 3	46	16	14	10	14
Customer 4	30	10	25	10	25
Customer 5	35	25	10	15	15
Customer 6	20	25	10	15	30
Customer 7	30	10	10	20	30
Customer 8	60	5	15	10	10
Customer 9	20	23	25	20	12
Customer 10	20	16	28	4	32
Customer 11	35	25	10	15	15

Customer 12	30	10	25	10	25
Customer 13	40	10	10	20	20
Customer 14	30	15	15	20	20
Customer 15	20	12	30	8	30
Customer 16	40	10	20	10	20
Customer 17	25	15	25	15	20
Customer 18	20	10	25	15	30
Customer 19	30	15	20	10	25
Customer 20	35	15	20	10	20
Customer 21	25	20	15	10	30
Customer 22	45	10	15	10	20
Customer 23	30	10	20	10	30
Customer 24	35	18	15	12	20
Customer 25	28	13	22	12	25
Customer 26	34	16	10	20	20
Customer 27	30	10	25	20	15
Customer 28	20	25	10	15	30
Customer 29	24	18	20	16	22
Customer 30	40	15	20	15	10
$\frac{1}{n}\sum_{i}^{n} x$	31,4	14, 9	18,2	13, 4	22, 1

(Source: own processing)

According to the survey results, it has been determined that the most important criterion is the price of the washing machine. Further, the criteria have been distributed by their importance as follows, in descending order:

- capacity of loading;
- noise level;
- maximum drum rotation speed;
- the number of automatic programs.

We bring these values to the format of weight coefficients whose sum is equal to one. (Table 6)

Table 6. Criteria character

Criteria	C 1	C 2	C 3	C 4	C 5	Sum
–	Price	Maximum drum rotation speed (rpm)	Nois e level (dB)	Number of automati c progra ms (items)	Laundry capacity (kg)	
Criteria characte r	0,314	0,14 9	0,18 2	0,13 4	0,22 1	1,0

(Source: own processing)

4.3.4.3 Assessment of alternatives using the SAW method

For further calculations use the scale criteria obtained by SCORE method (Table 6).Use matrix (Table 7) to apply the weighted sum method.

Table 7. Matrix for applying a method

	C 1	**C 2**	**C 3**	**C 4**	**C 5**
A1	5499	1000	58	15	6
A3	7590	1400	53	16	8
A5	8499	1400	51	14	7
A7	6490	1200	51	16	7
A8	7999	1400	51	9	8
A11	7199	1200	51	16	8
A12	6589	1400	60	16	10
A13	6390	1400	61	16	8
A16	7299	1400	58	15	7
Criteria weights	min	max	min	max	max
Criteria characte r	0,31 4	0,14 9	0,18 2	0,134	0,22 1

(Source: own processing)

The next step will be to find standardized assessments of the criteria. For this purpose, the maximum and minimum estimates of the criteria are determined.

Table 8. Ideal and basal alternatives

	C 1	C 2	C 3	C 4	C 5
max	8499	1400	61	16	10
min	5499	1000	51	9	6
$X_j^{max} - X_j^{min}$	3000	400	10	7	4

(Source: own processing)

The criteria in our case are cost or benefit, so the formula (3.2, 3.3) is used to normalize theestimates.

Table 9. Normalized matrix

	C 1	C 2	C 3	C 4	C 5
A1	1	0	0,3	0,857	0
A3	0,303	1	0,8	1	0,5
A5	0	1	1	0,714	0,25
A7	0,670	0,5	1	1	0,25
A8	0,167	1	1	0	0,5
A11	0,433	0,5	1	1	0,5
A12	0,637	1	0,1	1	1
A13	0,703	1	0	1	0,5
A16	0,400	1	0,3	0,857	0,25

(Source: own processing)

The normalized criteria values are then multiplied by the weight factors

Table 10. Normalized matrix with weight factors

	C 1	C 2	C 3	C 4	C 5
A1	0,314	0	0,055	0,115	0
A3	0,095	0,149	0,146	0,134	0,110
A5	0	0,149	0,182	0,096	0,055
A7	0,210	0,075	0,182	0,134	0,055
A8	0,052	0,149	0,182	0	0,110
A11	0,136	0,075	0,182	0,134	0,110
A12	0,200	0,149	0,018	0,134	0,221
A13	0,221	0,149	0	0,134	0,110
A16	0,126	0,149	0,055	0,115	0,055

(Source: own processing)

and the value of the aggregated trade-off function for each of the variants is determined. For this purpose we use formula (3.6).

We define the rating of alternatives. (Table 11)

Table 11. Order of alternatives (SAW)

	Tradeoff	Rank
A1	0,484	8
A3	0,634	4
A5	0,482	9
A7	0,656	2
A8	0,493	7
A11	0,637	3
A12	0,722	1
A13	0,614	5
A16	0,500	6

(Source: own processing)

As a result of applying the method SAW, it has been determined that the best solution is the alternative A12. That is, for a buyer who wants to buy a washing machine in the Alza.cz store in Prague, the best option, according to the criteria set by him, will be a CANDY CSO 14105TE / 1-S washing machine.

4.4 Conclusion

The conducted studies of the possibility of using MCA methods to help the buyer in choosing a product have shown:

1. The person acting as the buyer performs actions related to the stages of the decision-making process. At the same time, the buyer goes through several stages of the decision-making process, often without realizing it.

2. At the stage of making a final decision about choosing the best alternative, studies have shown that the use of multi-criteria analysis methods can lead to better results, MCDM can be used to structure and map the decision-making process, and the user must clearly define their priorities and preferences.

3. To make a final decision, it is possible to use various methods of MCA, but in the consumer environment, when choosing a product, the simplest, fastest and not difficult to calculate methodis necessary.

4. Comparing the calculation methods, I found that the calculations performed by the TOPSIS and ELECTRE methods are the most complex in mathematical terms, time-consuming and difficult to use for the average consumer. Many MCDM methods are not very intuitive and are too complex for the client, not even readable for the company owner.

5. From the simpler methods, I chose SAW, and according to my results, this method is suitable for this purpose.

6. The SAW method can be recommended as part of the customer support system included in the eshop.

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