BRNO UNIVERSITY OF TECHNOLOGY
VYSOKÉ UČENÍ TECHNICKÉ V BRNĚ

FACULTY OF ELECTRICAL ENGINEERING AND COMMUNICATION
DEPAR. OF CONTROL AND INSTRUMENTATION

FAKULTA ELEKTROTECHNIKY A KOMUNIKAČNÍCH TECHNOLOGIÍ
ÚSTAV AUTOMATIZACE A MĚŘÍCI TECHNIKY

INTEGRATION OF INERTIAL NAVIGATION WITH GLOBAL NAVIGATION SATELLITE SYSTEM

SEMESTRAL PROJECT
SEMESTRÁLNÍ PROJEKT

AUTHOR Bc. IVAN ŠTEFANISKO
AUTOR PRÁCE

Brno 2015

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INTEGRÁCIA INERCIÁLNEHO NAVIGAČNÉHO A GLOBÁLNEHO DRUŽICOVÉHO POLOHOVÉHO SYSTÉMU

SEMESTRAL PROJECT
SEMESTRÁLNÍ PROJEKT

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BRNO 2015

VYSOKÉ UČENÍ TECHNICKÉ V BRNĚ

Fakulta elektrotechniky a komunikačních technologií

Ústav automatizace a měřicí techniky

Diplomová práce

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Kybernetika, automatizace a měření

Student:	Bc. Ivan Štefanisko	***ID:***	138952
Ročník:	2	***Akademický rok:***	2014/2015

NÁZEV TÉMATU:

Integration of inertial navigation with global navigation satellite system

POKYNY PRO VYPRACOVÁNÍ:

1. Design navigation algorithm for inertial navigation system (INS) using accelerometer and gyroscope triad on existing navigation board.
2. Integrate INS with Global Navigation Satellite System (GNSS).
3. Make comparison of created system with existing solutions.

DOPORUČENÁ LITERATURA:

[1] GROVES, Paul D. Principles of GNSS, inertial, and multisensor integrated navigation systems. 2nd Ed. Boston: Artech House, c2008, xvi, 518 p. GNSS technology and applications series. ISBN 978-1608070053.
[2] TITTERTON,H., David a John L. WESTON. Strapdown Inertial Navigation Technology: 2nd edition. United Kingdom: MPG Books Limited, Bodmin, Cornwall, 2004. ISBN 0-86341-358-7.

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Předseda oborové rady

UPOZORNĚNÍ:

ABSTRACT

This paper deals with study of inertial navigation, global navigation satellite system, and their fusion into the one navigation solution. The first part of the work is to calculate the trajectory from accelerometers and gyroscopes measurements. Navigation equations calculate rotation with quaternions and remove gravity sensed by accelerometers. The equation's output is in earth centred fixed navigation frame. Then, inertial navigation errors are discussed and focused to the bias correction. Theory about INS/GNSS integration compares different integration architecture. The Kalman filter is used to obtain navigation solution for attitude, velocity and position with advantages of both systems.

KEYWORDS

Inertial navigation system, inertial measurement unit, accelerometers, gyroscopes, navigation frames, transformation matrix, quaternions, Global navigation satellite system, Kalman filter, INS/GNSS integration.

ABSTRAKT

Táto práca sa zaoberá štúdiou inerciálnej navigácie, globálnym družicovým polohovým systémom a ich integráciou do jedného navigačného riešenia. V prvej časti práce je počítaný výstup inerciálnych rovníc na základe meraní z akcelerometrov a gyroskovpov. Tieto rovnice počítajú rotácie pomocov kvaterniónov a odstraňujú gravitáciu z meraní akcelerometrov. Ďalej sú rozoberané chyby inerciálnej meracej jednotky so zameraním na odstránenie offsetov. V teórii sú rozobraté rôzne metódy integrácii INS a GNSS. Kalmanov filter je použitý pre získanie výsledného navigačného riešenia, ktoré spája výhody oboch systémov. Výsledkom je natočenie, rýchlosť a poloha daného objektu.

KLÍČOVÁ SLOVA

Inerciálny navigačný systém, inerciálna meracia jednotka, akcelerometer, gyroskop, navigačné rámce, transformačná matica, kvaternióny, globálny družicový polohový systém, Kalmanov filter INS/GNSS integrácia.

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DECLARATION

I declare that I have written my semestral project on the theme of "Integration of inertial navigation with global navigation satellite system" independently, under the guidance of the semestral project supervisor and using the technical literature and other sources of information which are all quoted in the project and detailed in the list of literature at the end of the project.

As the author of the semestral project I furthermore declare that, as regards the creation of this semestral project, I have not infringed any copyright. In particular, I have not unlawfully encroached on anyone's personal and/or ownership rights and I am fully aware of the consequences in the case of breaking Regulation § 11 and the following of the Copyright Act No 121/2000 Sb., and of the rights related to intellectual property right and changes in some Acts (Intellectual Property Act) and formulated in later regulations, inclusive of the possible consequences resulting from the provisions of Criminal Act No 40/2009 Sb., Section 2, Head VI, Part 4.

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INTRODUCTION

This project was conducted in the laboratory of ESIEE Paris, France, during the exchange program in 2014/2015. The work continues on an existing embedded navigation board like a part of navigation solution.

Navigation is a huge topic which has become a complex science. For normal users it is about travelling and finding a way between two or more points of interest. The simplest form of navigation is by explanation of direction in oral conversation. (turn right, go to second floor, cross the street,...). Higher navigation form is by the aid of maps with respect to recognition of known objects in our area and moving between them (hills, rivers, valleys, roads,...). This system can be aligned with the earth grid and the user is able to determine his position in reference frame. Reference frames are fundamentals to obtain position in navigation solutions.

In sea navigation, people are used to navigate according to the fixed stars, which define the reference fixed frame. A fixed frame with the knowledge of motion of Earth and the time of observation allow calculate position of the Earth. One of the main principal problems was the lack of accurate time to determine longitude, which died away with the discovery of the chronometer. The latitude is possible to determine from stars if the sky is visible.

An alternative method (called "dead reckoning") to obtain position is to calculate the new position starting in initial position from measurement of speed and direction. All process is to take last known position and update it with the new position calculated on average speed, heading, and time interval between each of them. Speed must be resolved through heading angle to give velocity components North and East. Final position and trajectory is given by the sum of all positions between initial and actual position.

The present equivalent of "dead reckoning" is to use inertial sensors, accelerometers and gyroscopes to sense rotational and translational motion with respect to an inertial reference frame. This sensors reduce mechanical complexity and can be attached directly on the navigation's subject. Their price, size and reliability together with mathematical integration giving inertial navigation system (INS) useful for many applications.

Another navigation solution was developed for military use in United States. It is a system providing services to determine position, velocity, or another navigation information data from satellites. Later, system started to provide navigation data for two sectors, civil and military. It was a step, which leads to research and usage over the world. The term for this system is global navigation satellite system (GNSS). Term GNSS include European, American, Russian and another world satellite systems.

Each navigation solution has some advantages and disadvantages. In dependence of this propositions, new ideas come out. One of them is to combine existing systems and get more precise and/or less expensive navigation solutions.

In this thesis, one of these combined systems is presented and discussed. Integration of INS and GNSS navigation system with Kalman filter is a main topic of all following work, which is divided into the three parts.

The first part presents INS like a separate system for navigation. Structure of inertial measurement unit, inertial sensors and their errors are explained. Description of different navigation frames is discussed. Transformation between navigation frames is described by different methods of direction cosine matrix. Important part of this section is navigation equation algorithm, where inertial navigation solution is created and described in details. In the end of this section embedded navigation board from previous work is presented.

The second part is introduction to GNSS systems. Different types of satellite systems is discussed and main GNSS errors ale presented.

The third part is about INS/GNSS integration architectures. Kalman filter introduction and Kalman filter algorithm are discussed in details.

Last part of this work is the sum of practical work. Simulation of trajectory, measured data from inertial measurement unit and GNSS receiver are printed. Correction of inertial sensors biases are analysed and integration algorithm results are displayed in condition with and without GNSS outage.

1 INERTIAL NAVIGATION SYSTEM

The operation of inertial navigation system comes out from Newton's laws mechanized formula of motion, where he says that object will continue trajectory in a straight line unless it is disturbed by external forces. He also says that these forces will produce an acceleration. With possibilities of the acceleration measurement it could be possible to the calculate velocity and position in time by mathematical integration computing.

Inertial navigation systems usually use multiple accelerometers mounted for each axis separately. Measured forces need to be tracked in the same direction as the accelerometers are pointing. Rotational motion may by sensed by multiple gyroscopes to determine acceleration direction at the same time. These inertial sensors create an inertial measurement unit.

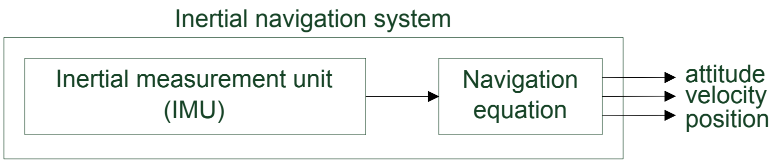

Fig. 1.1: Inertial navigation system block diagram.

1.1 Inertial measurement unit

Inertial measurement unit (IMU) combines multiple accelerometers and gyroscopes. Three accelerometers and tree gyroscopes together can make IMU with six degrees of freedom. Accelerometers measure final forces and gyroscopes measure the angular rate of the IMU body with respect to inertial space in body axes. These sensors are mounted in orthogonal sensitive axes. It also includes temperature sensor, calibration store, clock, power supplies and IMU processor.

1.1.1 IMU classification

IMU's are usually divided into the three global groups of high-, medium- and low-grade inertial sensors, but borders between them can be variable depending on authors and usage. Because of that their are grouped according performance into marine, aviation, intermediate, tactical and automotive categories [1]:

- *Marine grade:* Used in ships, submarine or spacecraft. Cost excess of 1 million EUR and drift of less than 1.8 km per day.
- *Aviation grade:* Used in commercial airlines and military aircraft. According standard navigation units specifying maximum horizontal position drift of 1.5 km in the first hour of operation Cost around 100 000 EUR.

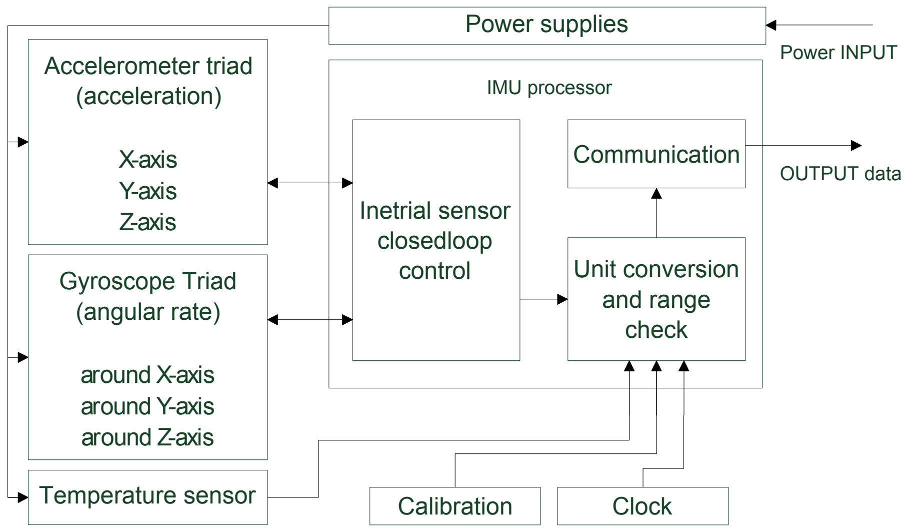

Fig. 1.2: Schematic of an inertial measurement unit. Taken from [1].

- *Intermediate grade:* About one order magnitude less then aviation grade and are used in small aircraft and helicopters. Costs between 20 000 and 50 000 EUR.
- *Tactical grade:* This can provide useful data only for few minutes, usually it is combined with another solution like GPS to obtain positioning system. Used in guided weapons and unmanned air vehicles. Cost between 5 000 and 20 000 EUR.
- *Automotive grade:* They are usually sold like individual sensors than IMU device. Used in pedometers, anti-lock braking system or airbags. Accelerometer cost starts at 1 EUR and gyroscopes start at 10 EUR.

1.1.2 IMU processor

IMUs processor is like a brain of all systems. It makes conversion of inertial sensors output, computes known errors and performs range check to detect sensors. Many IMUs integrate the specific force and angular rate over the sampling interval τ. Output rate is usually between 100 Hz and 1 000 Hz.

1.1.3 Accelerometer

Accelerometer is a sensor which measures specific force (f) along single axis without external reference. It is useful to measures acceleration, heeling, angular rate, vibrations or even gravitation force. Measurement depends on position of the mass inside the accelerometer with respect to it case. When an acceleration force is presented along the sensitive axis, the proof mass will initially continue in previous velocity. In that case, moving force applied to one spring is compressing and the second is stretched. Resultant position of the mass with respect to the case is proportional to the acceleration applied to the case. Gravitation forces are also measured by ac-

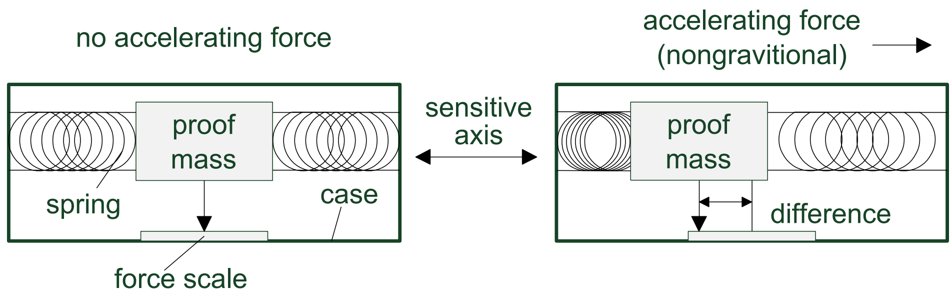

Fig. 1.3: Simple principal model of accelerometer. Taken from [1]

celerometer, because forces are applied to all measurement parts, so there is relative motion of the accelerometer mass with respect to the earth mass. In an orthogonal three axis measurement, we measure specific forces and the gravitation vector which is a sum of gravitation forces. This vector must be compensated to obtain right output values of accelerometer. If accelerometer is placed in Earth's surface, output value for acceleration is equal to opposite gravitation a constant ($g = 9.82\, m/s^{-2}$). An exception: when gravitation forces are not measured by springs. It is the case when spring heading to the Earth's centre with free-fall.

1.1.4 Gyroscope

The gyroscope is a sensor which measure angular rate (ω) along a single sensitive axis without external reference. Most common types of gyroscopes work on spinning-mass, optical, or vibratory principle. Spinning-mass type is the oldest one and operates on the principle of conversation of angular momentum. According the second Newton's law of dynamics, angular momentum of body with respect to inertial space will remain unchanged unless torque is present. The mechanical construct of spinning-mass is mounted in case to allow free rotation about axes. Optical gyroscopes work on a principle that uses sending light beams through optical fiber in opposite direction but with same trajectory. Rotating movement increases time-trajectory of light beam in same direction and decreases in the opposite direction. This physical principle is also know as the Sagnac effect. MEMS gyroscopes often use the vibratory principle which is low-cost and suitable for low-performance devices. Main point is to detect the Coriolis acceleration of the vibration element when gyroscope is rotated. Vibrating element can be for example: beam, pair of beams, string, ring, cylinder or hemisphere[1].

In inertial navigation solutions, gyroscopes measure angular velocities in contrast to attitude angles measured by free gyroscopes, which are typically mounted in gimbaled platforms. Rate gyroscopes sense craft relative to the inertial space. The biggest part of the measured output is angular rate relative to the Earth, then the angular rate measured about spherical Earth and angular rate of the Earth itself. The final output is sum of all these parts. Note, that the sum of these angular rates is necessary to transform them into the same frame.

1.1.5 Inertial sensors errors

Despite of high generation technology of MEMS sensor used for inertial navigation, it is necessary to think about accuracy limit of acceleration and rotation. Error in measurement of angular rate caused by design limitations and constructional deficiencies becomes a quadratic error in velocity and a cubic error in position. Measurement error of accelerometer is integrated twice as part of the mechanization process, where error becomes a linear error in velocity and a quadratic error in position [2]. These errors increase with time very fast and without limitations. Because of that, the system using inertial sensor is not able to give a right value of the position on the output for a long time period without correction. One of the corrections is calibration of these sensors, but in a low-cost system this calibration is not performed. Another possibility is to model these errors or limit it with some another method. For example this method can be integration of INS and GPS. Basic errors of inertial sensors are:

Bias - This error is present in sensor output even if input is zero. Bias may be produced by a variety of effects like residual torques, magnetic field noise, or temperature gradients. Sometimes gyroscope bias may be subjected and referred as the acceleration independent bias, where $g - unit$ is a function contiguous at variety of gravitation constant and its changes trough the Earth position.

Scale-factor - Output gain of sensor may produce ratio changes between measured values. This error is unstable and may be also produced by non-linearity or sensitivity. Typically it is caused by age of sensor or manufacture tolerations.

Cross-coupling - Problem with orthogonality in position (90° between each axis) of the sensor's axis is usually given by manufacturer. This problem is multiplied when using accelerometer and gyroscope triad. It's difficult to place them in the right position.

Noise - Output signal contains noise, which is caused by interference of disturbing electric parts of sensor or another systems.

Previously named errors include all or some of these following components[3]:

- *Fixed or repeatable terms* - Bias component is predictable and it is present each time the sensor is switched on and therefore may be corrected.
- *Temperature induced variations* - Temperature changes can be corrected with suitable calibration.
- *Switch-on* to *switch-on* variation - It is variable on every new run but it is constant during the simple run.
- *In-run variations* - Random biases which change during run, precision depends on type of sensor.

1.2 Coordinate frames

In the simple task of motion modeling in physics we usually work in Earth space, where Earth's rotation is ignored. This is pretending that Earth is an inertial frame without moving. The navigation solution is not able to ignore Earth's rotation. This rotation has a significant impact on navigation computation. Additionally, there are many coordinate frames in which the navigation solution can be represented. Variety of frames is caused by systems used in navigation. Inertial sensors measure their motion in respect with inertial frame. GPS measures position in respect to satellite receivers. But users want to know their position with respect to Earth.

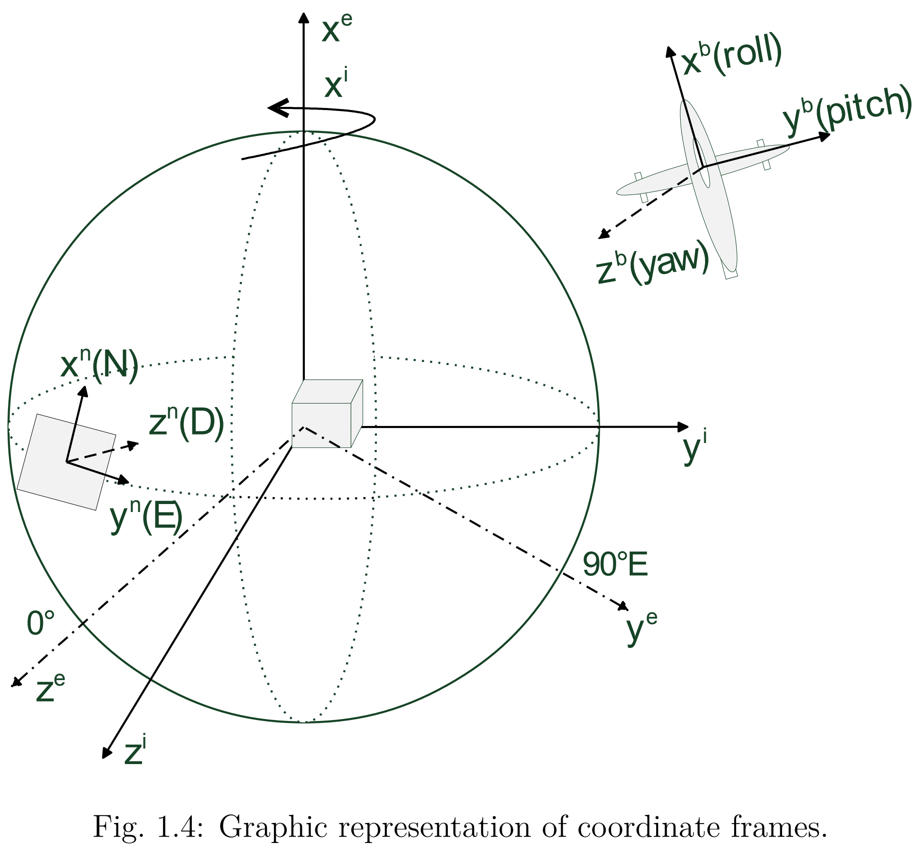

Fig. 1.4: Graphic representation of coordinate frames.

1.2.1 Earth centred inertial frame (ECI)

Talking about inertial frame, this frame does not move and rotate with respect to the rest of Universe. In navigation, we use this frame in combination with Earth.

Denotation of frame is usually by the symbol i. ECI has centre of frame in centre of Earth. The z-axis points along the Earth's axis of rotation from the centre to the north pole. The x-axis and y-axis lie within the equatorial plane. They do not rotate with the Earth, but y-axis always lies 90° ahead of the x-axis in the direction of rotation [1].

1.2.2 Earth centred fixed frame (ECEF)

ECEF frame is similar to ECI frame, but all of axes rotate with respect to Earth's rotation. Denotation of ECEF frame is by symbol e. The z-axis points along the Earth's axis of rotation from centre to the north pole. The x-axis points from centre of mass (Earth) to intersection of equator and meridian. Note that the meridian may be reference or zero. The y-axis completes the right-hand orthogonal set, pointing from the centre of mass to equator with the 90° east offset from x-axis. This frame is important to navigation solutions, because it represent position with respect to the Earth. In this paper ECEF frame representation is used.

1.2.3 Local navigation frame

Local navigation frame is also called geodetic or geographic and is denoted by symbol n. Its centre is in point, where the initial position is defined. Usually it is in the Earth's surface. The z-axis points from this points to the centre Earth with down direction. The x-axis points to the north pole and y-axis point to the east. These axes make together orthogonal set [3].

In some literature, there are different directions defined for this frame. Described orthogonal set is DNE (down, north, east), but there are another types like ENU (east, north, up) or SWD (south, west, down).

This frame is useful when we need to know the position with respect to East, North and Down direction. Solution of this frame is proper set of resolving axes. It is used for navigation equation mechanization but not for the navigation solution. In the end of solution it is transformed to another frame. This frame is not used like navigation solution because it becomes unsuitable for use near the poles. There is a singularity at each pole when north and east axes are not defined.

1.2.4 Body frame

Body frame or vehicle frame, denoted by symbol b, comprises the origin and orientation of the object's body for which a navigation solution is sought. The origin is similar to local navigation frame, but the axes remain fixed with respect to object's body. Generally, the axis are defined as x=forward (the usual direction to go) y=down (the usual direction of gravity) and y=right (completing the orthogonal set). In angular motion we can use phrases like roll for x-axis, pitch for y-axis and yaw for z-axis [1].

1.2.5 Other frames

Previous frames are commonly used in navigation solutions, but there are many other axes representations, which are similar to already described frames or which are not commonly used. It may be a geocentric, tangent, wander azimuth or inertial instrument frame described in [1].

1.3 Transformation matrix

The transformation matrix (3x3) represents tool to describe specific vector transformation between reference (α) and resolving frame (β). Formula to describe vector x_γ transformed from α to β frame is:

$$x_\gamma^\beta = C_\alpha^\beta x_\gamma^\alpha \tag{1.1}$$

where C_α^β is a transformation matrix or often used term, direction cosine matrix (DCM). This DCM is easy to manipulate with reverse rotation or various multiplication. Thus, we can obtain needed transformation solution.

$$C_\beta^\alpha = \left(C_\alpha^\beta\right)^T \tag{1.2}$$

$$C_\gamma^\beta = C_\alpha^\beta C_\gamma^\alpha \tag{1.3}$$

$$C_\alpha^\beta C_\beta^\alpha = I_3 \tag{1.4}$$

Transformation matrix can be computed with Euler angles ϕ (around x-axis, roll) , θ (around y-axis, pitch) and ψ (around z-axis, yaw).

$$C_\beta^\alpha = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} 0 & \cos\psi & \sin\psi \\ 0 & -\sin\psi & \cos\psi \\ 0 & 0 & 1 \end{pmatrix} \tag{1.5}$$

$$= \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\phi \\ (-\cos\phi\sin\psi + \sin\phi\sin\theta\cos\psi) & (\cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi) & \sin\phi\cos\theta \\ (\sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi) & (-\sin\phi\cos\psi + \cos\phi\sin\theta\sin\psi) & \cos\phi\cos\theta \end{bmatrix}$$

The Euler angles rotation has many disadvantages. Property of Euler angles is that rotation $(\phi + \pi, \pi - \theta, \psi + \pi)$ gives the same result as the rotation (ϕ, θ, ψ). Existence of different sequence of rotation around axis (xyz, xzy, yxz,...) can lead to confusedness. Returning to the original orientation is not just simply reversing the sign of Euler angles $(\phi, \theta, \psi) \neq (-\phi, -\theta, -\psi)$. A further problem is a singularity at $\pm 90°$ pitch where roll and yaw become indistinguishable. Because of these difficulties Euler angles are rarely used for three dimensional computation [1].

The solution of rotation angles can be resolved by quaternion attitude representation [3], which is hyper-complex number with four components. Main idea is that a transformation from one frame to another can be effected by a single rotation about vector μ defined with respect to the reference frame.

$$q = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} \cos(\mu/2) \\ (\mu_x/\mu)\sin(\mu/2) \\ (\mu_y/\mu)\sin(\mu/2) \\ (\mu_z/\mu)\sin(\mu/2) \end{bmatrix} \tag{1.6}$$

where μ_x, μ_y, μ_z is the unit vector of the rotation axis and μ is the rotation angle. A quaternion may also be expressed as complex number with a real component a and three imaginary components b, c and d.

$$q = a + ib + jc + kd \tag{1.7}$$

Propagation of quaternion with time accordance with the following equation:

$$\dot{q} = 0.5\, q\, p^{\alpha}_{\beta} \tag{1.8}$$

this equation may be expressed in matrix form as function of the components of vectors q and p.

$$\dot{q} = \begin{bmatrix} \dot{a} \\ \dot{b} \\ \dot{c} \\ \dot{d} \end{bmatrix} = 0.5 \begin{bmatrix} a & -b & -c & -d \\ b & a & -d & c \\ c & d & a & -b \\ d & -c & b & a \end{bmatrix} = \begin{bmatrix} 0 \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \tag{1.9}$$

that is

$$\begin{aligned} \dot{a} &= -0.5(b\omega_x + c\omega_y + d\omega_z) \\ \dot{b} &= 0.5(a\omega_x - d\omega_y + c\omega_z) \\ \dot{c} &= 0.5(d\omega_x + a\omega_y - b\omega_z) \\ \dot{d} &= -0.5(c\omega_x - b\omega_y - a\omega_z) \end{aligned} \tag{1.10}$$

The main advantage of quaternion is that it needs only four differential equations, they have no singularities and are easier for computation. Disadvantage of quaternions is that initial computation is needed, and transformation matrix and Euler angles are not directly available.

the derivative may be used to compute transformation matrix as follows:

$$C^{\alpha}_{\beta} = \begin{bmatrix} (a^2+b^2-c^2-d^2) & 2(bc-ad) & 2(bd+ac) \\ 2(bc+ad) & (a^2-b^2+c^2-d^2) & 2(cd-ab) \\ 2(bd-ac) & 2(cd+ab) & (a^2-b^2-c^2+d^2) \end{bmatrix} \quad (1.11)$$

1.4 Navigation equations in ECEF frame

Figure 1.5 shows how the angular rate ω^b_{ib} and specific force f^b_{ib} measurements update attitude, velocity and position over the integration time interval $t + \tau_i$, expressed in ECEF coordinate frame. Note that the suffixes $(-)$ and $(+)$ represent navigation equation values before and after integration cycle. Following section is written according to [1].

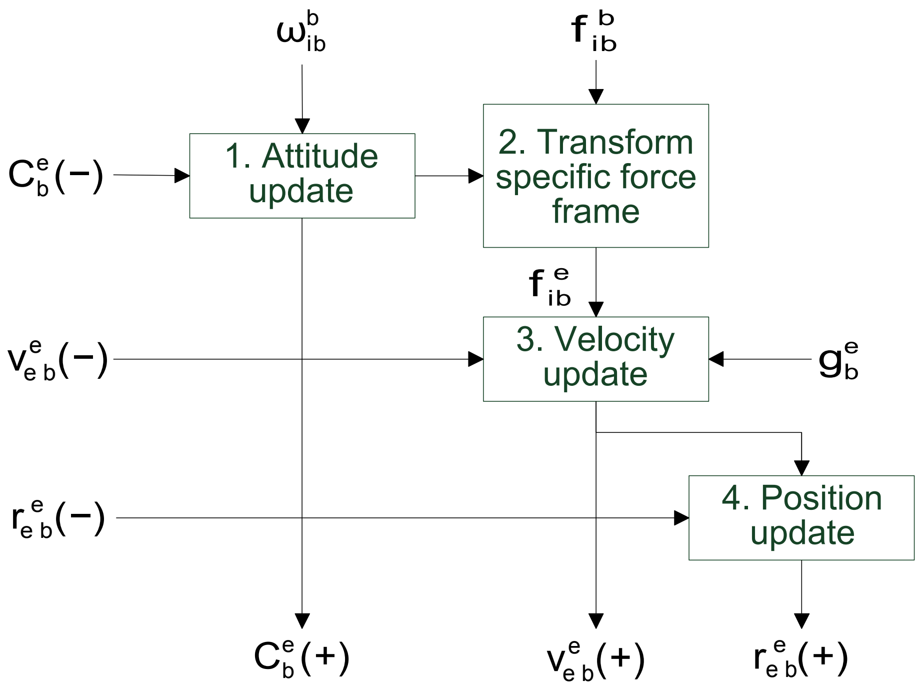

Fig. 1.5: Navigation equation block diagram. Taken from [1]

1.4.1 Attitude update

The attitude update step of ECEF frame navigation equations uses the angular rate measurement ω_{ib}^b, to update the attitude solution, expresses as the body to Earth frame coordinate transformation matrix C_b^e. The time derivation is

$$\begin{aligned} \dot{C}_b^e &= C_b^e \Omega_{eb}^b \\ &= C_b^e \Omega_{ib}^b - \Omega_{ie}^e C_b^e \end{aligned} \tag{1.12}$$

where Ω_{ib}^b is the skew-symmetric matrix of IMU's angular rate measurement and Ω_{ie}^e is the skew-symmetric matrix ot the Earth-rotation vector with constant speed of Earth $\omega_{ie} = 7.27\,10^{-5}\ rad/s$. Thus, the rotation of the Earth must be accounted for in updating the attitude. From Earth rotation vector resolved with respect to space, clockwise about the common z-axis axes is given by

$$\omega_{ie}^i = \omega_{ie}^e = \begin{pmatrix} 0 \\ 0 \\ \omega_{ie} \end{pmatrix} \Rightarrow \Omega_{ie}^e = \begin{pmatrix} 0 & -\omega_{ie} & 0 \\ \omega_{ie} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{1.13}$$

Integrating 1.12 gives

$$C_b^e(t+\tau_i) = C_b^e(t)\left[exp\left(\alpha_{ib}^b \wedge\right)\right] - \left[exp\left(\Omega_{ie}^e \tau_i\right) - I_3\right] C_b^e(t) \tag{1.14}$$

The exponent must be computed as power-series expansion. Applying the small angle approximation by truncating the expansions at first order (higher order precision solution can by found in [1]) and assuming the IMU angular rate measurement is constant over the integration interval (i.e., $\alpha_{ib}^b \approx \omega_{ib}^b \tau_i$) gives

$$C_b^e(+) \approx C_b^e(-)(I_3 + \Omega_{ib}^b \tau_i) - \Omega_{ie}^e C_b^e(-)\tau_i \tag{1.15}$$

where

$$I_3 + \Omega_{ib}^b \tau_i = \begin{pmatrix} 1 & -\omega_{ib,z}^b \tau_i & \omega_{ib,y}^b \tau_i \\ \omega_{ib,z}^b \tau_i & 1 & \omega_{ib,x}^b \tau_i \\ -\omega_{ib,y}^b \tau_i & \omega_{ibxz}^b \tau_i & 1 \end{pmatrix} \tag{1.16}$$

As the Earth rotation rate is very slow compared to the angular rates measured by the IMU, this small angle approximation is always valid for the Earth rate term by the attitude update equation.

Another solution how to compute transformation matrix is by using quaternion formula as described in equations 1.9 and 1.11.

1.4.2 Transform specific force frame

The IMU measures specific force in body frame, therefore transformation to the same frame as attitude step is resolved is necessary. This transformation is given with multiplying of transformation matrix from body to ECEF frame

$$f_{ib}^e(t) = C_b^e(t) f_{ib}^b(t) \tag{1.17}$$

The specific force measurement is an average over time t to $t+\tau_i$, the coordinate transformation matrix should be similarly averaged. Simple implementation is:

$$f_{ib}^e(t) \approx \frac{1}{2}(C_b^e(-) + C_b^e(+)) f_{ib}^b \tag{1.18}$$

1.4.3 Velocity update

For the velocity update step , the reference and resolving frames are the same. Rate of change of velocity resolved in ECEF frame axes incorporates a centrifugal and Coriolis term due to the rotation of the resolving axes.

$$\dot{v_{eb}^e} = f_{ib}^e + g_b^e\left(r_{eb}^e\right) - 2\Omega_{ie}^e v_{eb}^e \tag{1.19}$$

$$f_{ib}^e = a_{ib}^e - \gamma_{ib}^e \tag{1.20}$$

where f_{ib}^e is specific force, difference between applied acceleration a_{ib}^e and specific gravitational force γ_{ib}^e,

Acceleration due to gravity g_b^e is the sum of gravitational and centrifugal acceleration.

$$g_e^b = \gamma_{ie}^e + \omega_{ie}^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} r_{eb}^b \tag{1.21}$$

An analytical solution is complex. However, values for Coriolis term will be much smaller than the specific force and gravity terms and we can neglect them. Then the equation for velocity is

$$v_{eb}^e(+) \approx v_{eb}^e(-) + \left(f_{ib}^e + g_b^e\left(r_{eb}^e(-)\right) - 2\Omega_{ie}^e v_{eb}^e(-)\right)\tau_i \tag{1.22}$$

1.4.4 Gravity

Gravity change with position on the Earth's surface. It is necessary to compute actual value of gravity constant, which is computed on WGS-84 [1] system.

$$g(\varphi) = \gamma \left[1 - \frac{2}{R_0} \left(1 + f + \frac{\omega_{ie}^2 R_0^2 R_p}{\mu} \right) h + \frac{3}{R_0^2} h^2 \right] \tag{1.23}$$

$$\gamma = 9.780327(1 + 0.0053024 sin(\varphi)^2 - 0.0000058 sin(2\varphi)^2) \tag{1.24}$$

Constant	Value
Equatorial radius	$R_0 = 6378137$ $[m]$
Polar radius	$R_p = 6356752.3142$ $[m]$
Earth's gravitational constant	$\mu = 3.986004418 \cdot 10^{14}$ $[m^3 s^{-2}]$
Earth's angular rate	$\omega_{ie} = 7.292158 \cdot 10^{-5}$ $[rads^{-1}]$
Flattening of the ellipsoid	$f = 1/298.257223563$ [-]
Eccentricity of the ellipsoid	$e = 0.0818191908426$ [-]

Tab. 1.1: Constant values of WGS-84 system for computing gravity.

1.4.5 Position update

In the ECEF frame navigation equations, the reference and resolving frames are the same and from knowledge that derivative of position is velocity, and than it can be written as:

$$\dot{r_{eb}^e} = v_{eb}^e \tag{1.25}$$

Integrating this equation and assuming the velocity varies linearly over the integration interval,

$$\begin{aligned} r_{eb}^e(+) &= r_{eb}^e(-) + (v_{eb}^e(-) + v_{eb}^e(+)) \frac{\tau_i}{2} \\ &\approx r_{eb}^e(-) + v_{eb}^e(-)\tau_i + \left(f_{ib}^e + g_b^e\left(r_{eb}^e(-)\right) - 2\Omega_{ie}^e v_{eb}^e(-)\right) \frac{\tau_i^2}{2} \end{aligned} \tag{1.26}$$

[1]WGS84 is an Earth-centered, Earth-fixed terrestrial reference system and geodetic datum. WGS84 is based on a consistent set of constants and model parameters that describe the Earth's size, shape, and gravity and geomagnetic fields. WGS84 is the standard U.S. Department of Defense definition of a global reference system for geospatial information and is the reference system for the Global Positioning System (GPS)[4]

1.5 Embedded navigation board

The embeded navigation board was developed by Laboratory of signals and systems (CNRS) in 2014 for terrestrial vehicles travelling on known routing network. Architecture of the board may be divided into three subsystems.

First subsystem collects all sensors. Inertial measurement unit MPU-9150, barometric altitude meter MLP3115A2 and GNSS receiver NV08C-CSM (supported GLONASS, GPS, GALILEO SBAS systems). Second subsystem is called communication interface, where bluetooth, CAN-bus and USB are used to exchange data with other devices. Last subsystem is main brain of this navigation board. Microcontroller that collects, filters and processes measured data. Type of microcontroller is Atmel AT91SAM7X512 featuring ARM7 architecture. It has 512kB of Flash EEPROM memory, 256kB of SRAM and peripherals like CAN, I2C, USB, DBGU and two UARTs.

Input voltage for this board should be in range between 5V and 15V. Three buttons are included in the assembly of the board, for system reset, bluetooth reset and to erase program. Then there are five LEDs to indicate health of processor, indicate signal which is passing all system circuit to test connectivity and last three LEDs are used to display bluetooth mode. For connectivity there are connectors for external GNSS antenna, usb, JTAG and 9-pin Cannon [5].

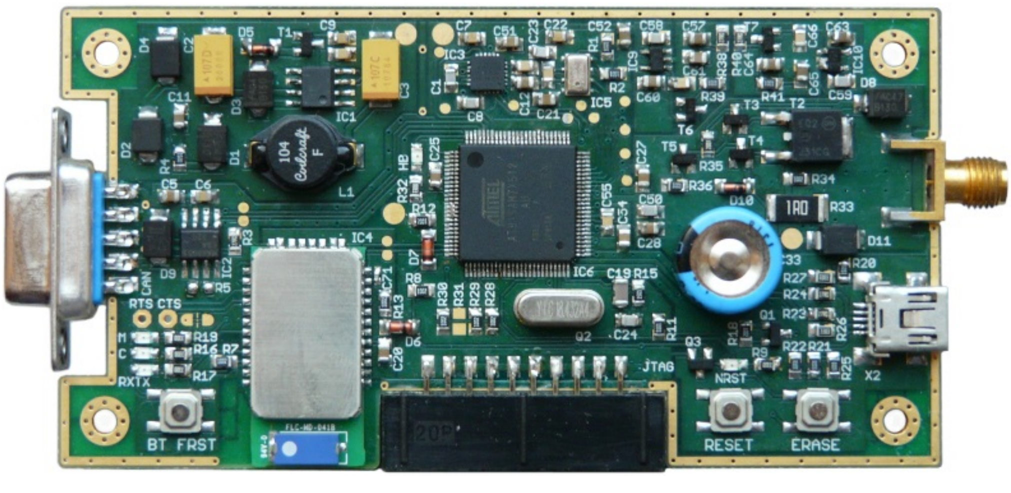

Fig. 1.6: Navigation board. Reprinted with permission from [5].

Configuration of inertial measurement unit (MPU-9150): sampling rate: 166 Hz, gyroscope full-scale range: $\pm 500\,°/\mathrm{s}$, gyroscope sensitivity: $0.0152\,°/\mathrm{s}$, accelerometer full-scale range: $-4\,\mathrm{g}$, accelerometer sensitivity: 122.07 mg.

Configuration of GNSS receiver (NV08C-CSM): update rate: 10 Hz, enabled systems: GPS, GLONASS, SBAS. Maximum acceleration: $10\,\mathrm{m/s^2}$, BINR baudrate: 230.4 kBd. Other parameters and details may be found in [5].

2 SATELLITE NAVIGATION SYSTEM

In early 1960s, several U.S. government organization were interested in developing satellite system with three-dimensional position determination. Their plan was to create system having following attributes: global coverage, continuous, all weather operation, useful in high-dynamic systems and operate with high accuracy. From these time when main criteria were established, many satellites system have been developed. Usual usage of satellite navigation system is in land application, aviation, space guidance or maritime.

2.1 GNSS systems

By definition, a GNSS is a world-wide set of satellite navigation systems (U.S. GPS, European GALILEO, Russian Glonnass, Chinise BeiDou and others). It consist of three major segments [6]:

- The *space segment* consists of GNSS satellites over the Earth surface. They are in orbital planes with nearly circular plane, some inclination angle and some altitude above Earth surface.
- The *control segment* is responsible for monitoring and correcting of the space segment. Consist from several ground stations around equator to measure signals from satellites and collect them in central monitoring network. Master ground station determine orbital model and clock correction parameters for each satellite. These corrected data are transmitted to the satellites and then distribute to the users receivers.
- The *user segment* is set of antennas and receivers. It is very various with aim of use. It can simply determine position of the user or it can also use another signals information with many processing techniques to precise final navigation solution.

2.1.1 GPS

The oldest one is GPS and in the present it is fully operational with all criteria established in the 1960s. The system provides accurate, continuous, world-wide, three-dimensional position and velocity information and in addition function provide propagation of coordinated universal time (UTC). It was reached in full operation in early 1995 after testing of the ground control segment and constellation of satellites.

System nominally consists from 24 satellites located in 6 orbital planes with distribution of four satellites per plane. A worldwide ground control and monitoring support network take care about status, health, navigation updates and other data in satellites. Using the concept of one-way time of arrival ranging, it allows access for unlimited number of users which can passively receive information from GPS. Satellites time synchronization together with GPS time base is managed by highly accurate atomic frequency standards on-board the satellites. Code division multiple access (CDMA) is a technique which use two frequencies L1 (1575.42 MHz) and L2 (1227.60 MHz) to transmit ranging codes and navigation data. Selection of this frequencies is because of low cross-correlation of these numbers with respect to one another. All satellites operate with same frequencies, but transmitted signals are with different ranging code for each one.

Every satellite generate two types of codes. Short code referred as the coarse/acquisition and long code referred as the precision code. The navigation data provides the mean for the receiver to determine the location of the satellite at the time of the signal transmission. The ranging code enables the user's receiver to determine the satellite-to-user range. This technique requires using clock from user's receiver. If the receiver clock were synchronized with satellite clock only three satellites are necessary to obtain three-dimensional location. However, a crystal clock is usually used for various operation to decrease cost, complexity and size of the receiver equipment. Because of that reason, four satellites are required to determine user latitude, longitude, height and receiver clock offset from inertial system time [7].

GPS is a dual-use system which provide separate services for civil and military users. Civilians can use standard positioning services. Only military and selected government users with permission have access to use precise positioning services. This access is secured by cryptography.

2.1.2 GALILEO

GALILEO is a global navigation satellite system developed by the European Union. It is a global positioning service under civilian control and it is inter-operable with GPS and GLONASS, U.S. and Russian navigation systems. GALILEO offers dual frequency standard to deliver real-time positioning accuracy as a time reference system, operate in geodetic coordinate reference frame. Services which are planed in GALILEO [7]:

- An *open* service without direct user charges.
- Combination of value-added data and accuracy in *commercial* service.
- Function *Safety-of-life* with monitoring and notification of safe use.
- *Public regulated* service strictly for government authorized users requiring a higher level of protection.
- *Search and rescue* support.

The fully deployed GALILEO system consists of 30 satellites (27 operational + 3 active spares), positioned in three circular Medium Earth Orbit. First two operational satellites are designed to validate this concept came out in 2011. Two more followed in 2012. This four satellites built effort to become the operational nucleus of the full GALILEO constellation [8].

2.1.3 GLONASS

GLONASS[1] is the Russian counterpart to GPS. It was developed as a military navigation system but like GPS it was designed to provide also civil services. The full GLONASS satellite constellation achieved in 1995. However, financial problem and satellite lifetime about 3 years declined its ability to provide positioning in 1996.

In August 2001, a modernization program was instigated, rebuilding the constellation, introducing new signals, and updating the control segment. The first of the modernized GLONASS-M satellites was launched in 2003, featuring an additional civil signal, higher accuracy clock, and an extended lifetime of 7 year. Newest version of this one is GLONASS-K with lifetime of 10 years. At the time of this writing, full GLONASS satellite constellation was 28 satellites (24 - operational, 2 - under check by the Satellite Prime Contractor, 2 - in flight tests phase) [9]. The Russian are working with the European Union and the U.S. to achieve compatibility between GLONASS, GALILEO and GPS. System use geodetic coordinate frame and time referenced system with frequency division multiple access method FDMA [1],[5],[7].

2.1.4 BeiDou and Compass

The BeiDou system is designed for military and civil users in China and their border areas. The system use a constellation of three satellites in geostationary orbit of China. It is assumed to be using in road, rail, and maritime applications and operates completely independently of GNSS. Previous systems needed to be clock synchronized and needed at least three satellites to determine position. This system using two-way active ranging and just two satellites can be used to determine position. The receiver records a short segment of navigational signal and then transmits to the satellite constellation at the fixed lag. Signal is then send to the control center and computed navigation solution is sent to another users via satellite. Terrain database is used to obtain and correct position from the ranging measurement.Compass, also known as BeiDou-2, is cur- rently developed as China's truly global GNSS system. It is scheduled for full operational capability by 2020 [1],[5],[7].

[1] from Russian language: Global'naya Navigatsionnaya Sputnikovaya Sistema

2.1.5 Local systems

There are another systems like Japanese The Quasi-Zenith Satellite System (QZSS) or Indian Regional Navigation Satellite System (IRNSS). These systems are used locally like BeiDou but with the same principle like GPS, GLONASS and GALILEO. More about this systems can by found in [1].

2.2 GNSS errors

The GNSS measurements are affected by noise and errors due to the propagation of signals through atmospheric layers. These errors are briefly described below.

2.2.1 Clock errors

The main idea for the correct solution with GNSS systems is to use precise clock timing in all GNSS segments. The control segment monitors and fits a polynomial correction of atomic clock for each satellite. This can be modelled by the polynomial coefficients transmitted in the navigation message with respect to a reference time. Another type of clock error is caused due to using non-precise clock in receiver equipments. It is not economic to use atomic clock in user equipment. The receiver clock error can be handled by estimating technique in each time step, subtracting the time information from another satellites or developing receiver bias model and use some estimation method like Kalman filter [6].

2.2.2 Atmospheric errors

To describe atmospheric delay, it may be sufficient to divide atmosphere into two layers. Ionosphere is the dispersive layer with altitude between 50 and 1000 Km. Contains electrons and positively charged molecules and is affected by solar activity, season and time of day. Changes in the level of ionosphere change also reflective index along the path and cause difference in travel time measured by the receiver. Troposphere is non-dispersive layer with altitude to 50 Km. It is composed essentially of electrically neutral particles. The troposphere changes with weather condition as temperature, pressure and humidity. Troposphere causes a delay in both the carrier and code signals. This delay is frequency independent and cannot be cancelled out by using dual frequency measurements. However, can be successfully compensated via models. Tropospheric models depend on empirical models by considering all values of temperature, pressure, relative humidity and mapping function [6],[10].

2.2.3 Signal reflections

Multipath errors are caused by multiple reflections of the signals at the receiver or at the satellite due to multiple paths reaching to the endpoint destination. Reflected signal always reach the endpoint later than direct path but it can still interference correlation function causing the correlation shift to the earlier or later signal point. The best way how to reduce reflecting effects is to count with them in the beginning of design. Position of antenna should be above reflective points (buildings, trees, lakes, vehicle roof). This is not always possible so another solution is needed. Changing antenna's gain, disable receiver settings when satellite is at low elevation or use absorbent materials around antenna can also help decrease reflected signals [10].

Previous sections have considered issues that affect time of propagation, range measurement error due to signal reflections. In addition to those issue, various factors inside antenna, cabling and receiver affect each measurement. It can be signals noise, correlation, quantization or sampling reason. More about GNSS errors can be found in [6].

3 INS/GNSS FUSION

Inertial navigation usually work out at high-bandwidth output from 50 Hz to 200 Hz with low short-term noise. It's implementation giving effective altitude, angular rate and acceleration measurement. Processing by inertial navigation processor with navigation equations we may have solution for position and velocity. However, the accuracy of this solution degrades exponentially with time as the navigation equation integrate inertial errors from measurement. This errors depend on category of grade 1.1.1 which is in use.

GNSS works out at lower output rate then INS, usually it is around 10 Hz with high long-term position accuracy and without measurement of attitude. Errors are limited to a few meters in long time interval. GNSS cannot be relied solution in tunnels or more level crossroad where GNSS signal is object to obstruction and interference.

INS/GNSS integration benefits advantages from the both of systems. Their combination gives a continuous, high-bandwidth, a complete navigation solution with high long- and short-term accuracy. GNSS measurement create borders for inertial solution drift and INS smooth the GNSS solution with possible signal outages. INS/GNSS integration is suitable for many applications in practise with low cost budget. For example in navigation of ships, airlines, aircraft, helicopters, UAVs, [1] small boats, vehicles or personal navigation. [1]

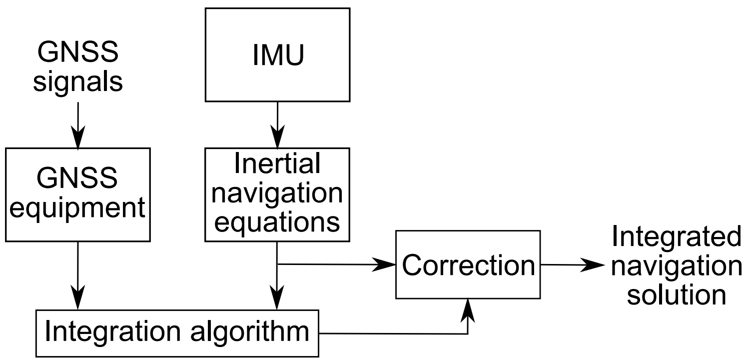

Fig. 3.1: Universal INS/GNSS integration architecture. Taken from [1].

The basic configuration in figure 3.1 shows integration algorithm which compares the inertial navigation solution with the outputs of GNSS user equipment and estimates correction to the inertial position, velocity and attitude. Correction step is usually processed by Kalman filter which forms inertial navigation solution. This architecture is dependent on GNSS signal availability, because solution is produced continuously.

[1] Unmanned aerial vehicle, also called dron. It is aircraft without a human pilot aboard.

3.1 Integration architectures

There are many existing integration structures between INS and GNSS. It is derived form application in different use areas and actual research works. They can be divided into the three main groups of interests. First one is about types and ways how to correct inertial navigation solutions. Second one describes which information are used from GNSS measurement and the last one is about design of aiding algorithm. Borders of these three groups are not strictly defined and can be modelled or coupled with another existing systems to get useful information into the integration architecture. Because of many architectures and their modifications we usually use terms such as loosely coupled, tightly coupled, ultratightly coupled, closely coupled, cascaded and deep to define integration architectures.

3.1.1 Uncoupled system

This is the easiest way how to benefit from of GNSS and INS. Both systems operate independently and providing system redundancy. GNSS is used to reset the INS by using their estimated position and velocity in regular time intervals. INS solution is bounded by errors of GNSS position and velocity. Whilst this approach involves minimal changes to either system, it does not provide the opportunities for performance enhancement and jamming avoidance that are possible with the coupled systems. Uncoupled navigation solution is available only if a good GNSS signals is present. Otherwise, it has all the properties of INS system.[3]

3.1.2 Loosely coupled integration

Figure 3.2 shows a loosely coupled integration algorithm where GNSS is working autonomously, whilst simultaneously providing measurement updates to the inertial navigation solution. The two systems are effectively operated to provide position and velocity information as a input to the integration Kalman filter, which uses these information to estimate INS errors.

Inertial errors are used to correct the inertial system in measurement updates. In some systems it is only position estimated, but it is more usual to use both position and velocity measurement to obtain more robust solution. This is because there are less integration steps between attitude errors and sensor biases. These errors propagate more rapidly as velocity errors and allow more immediate error estimates. However, the use of velocity measurements alone reduces the observability of position errors in the INS as measurement noise is integrated up into the estimates. For these reasons, it is customary to use both GNSS position and velocity updates to aid the inertial system in most integration algorithms of this type.[3]

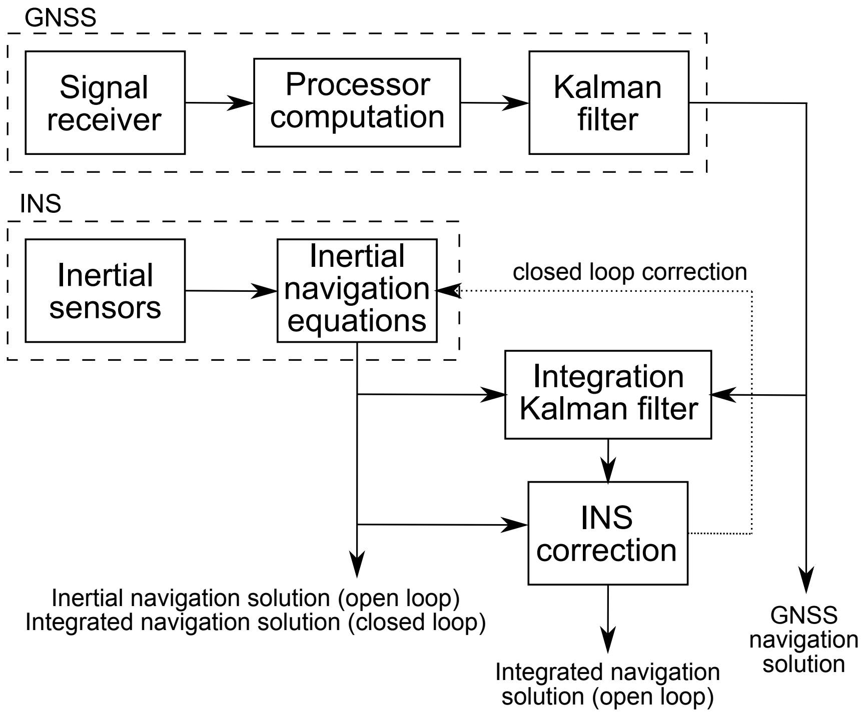

Fig. 3.2: Loosely coupled INS/GNSS integration architecture. Taken from [1].

The main profits of this system are redundancy and simplicity. These advancements allow to use it with any GNSS equipment and INS. Integration can be created like a new system or assembled additionally to the existing solution. In a loosely coupled architecture, it is usual to provide stand-alone GNSS correction in addition to the integrated solution. The extra solution can be useed to duplicate output and enable integrity monitoring of integrated solution. It can also prevent system from a filter failure errors. Where closed-loop INS correction is used, there is duplicity of INS output with an independent inertial open-loop solution.

In loosely coupled INS/GNSS integration algorithm is a important problem. It is caused by cascaded Kalman filters. Output of first Kalman filter is a measurement input to the second, integrated Kalman filter. The errors of Kalman filter outputs are time correlated, whereas Kalman filter measurement errors are assumed to be uncorrelated in time. It caused problem in Kalman state estimate unless filter gain is reduced or the correlated errors are estimated. Correction time of this solution varies in time and can by up to 100 seconds on the position and 20 seconds on the velocity.[1] It seems to bee short interval, but it is long enough to slow down the estimation of the INS errors.

Option of the ideal integration Kalman filter gain and measurement iteration rate is important to correct tuning. Tuning results are dependent on the speed of processing measurement step. If it is too quick, the filter is liable to become unstable. On the other hand if speed of processing measurement step is too slow, the observability of INS errors will be reduced. To obtain stability system the integration Kalman filter bandwidth is always less than GNSS Kalman filter. Note that the bandwidth may vary and measurement update intervals of 10 seconds are common in loosely coupled systems.

Low gain is needed also in case when only GNSS user equipment computes navigation solution without INS. Single-point GNSS navigation solution is much noisier because tracking-loop time interval exceed GNSS time interval.

Another problem is incipient when less than four satellites are available. Request from GNSS navigation solution is to have four or more satellite signals. In a short time period three signal can maintain navigation solution. When there are less then four signals available, the GNSS solution can not be aided with INS. Some of the GNSS user equipment provide additional output with information about satellite geometry and signal availability. Also, the integration filter needs to know the covariance of the GNSS filter output. These covariances varies with satellite geometry and availability quality.[1]

3.1.3 Tightly coupled integration

Tightly coupled integration architecture is an example of centralized integration. There the GNSS Kalman filter is merged together with INS/GNSS integration Kalman filter. GNSS ranging processor produces pseudo-range [2] and pseudo-range rate information as measurement input to the Kalman filter to help estimate the errors in the INS and GNSS system. Same principle as with loosely coupled integration architecture is used to correct integrated navigation solution with inertial navigation solution.

In practical system, either pseudo-range and pseudo-range rate are used rather then separately, because of the observability benefits. Main advance of tightly coupled architecture benefits from using only one Kalman filter instead of two, separately for each system. Using this fusion eliminates problems with cascade Kalman filters. As in previous architecture, also in this one Kalman filter bandwidth must still be kept in range of GNSS tracking loop bandwidths to prevent time-correlated tracking noise from contaminating the state estimates. Covariance of GNSS position and velocity is done implicitly to the integration algorithm due to satellite geometry and availability. System input does not need four satellite signals to keep integration algorithm in function. Even if only one satellite signal is tracked, integration algorithm has input from GNSS measurement data.[1]

[2] distance between a satellite and a navigation satellite receiver

Disadvantage of tightly coupled integration is that stand-alone GNSS solution does not exist in this architecture. On the other hand final integration solution usually give better solution than loosely coupled architecture in terms of robustness and accuracy. If stand-alone solution is requested it can be created in parallel way.

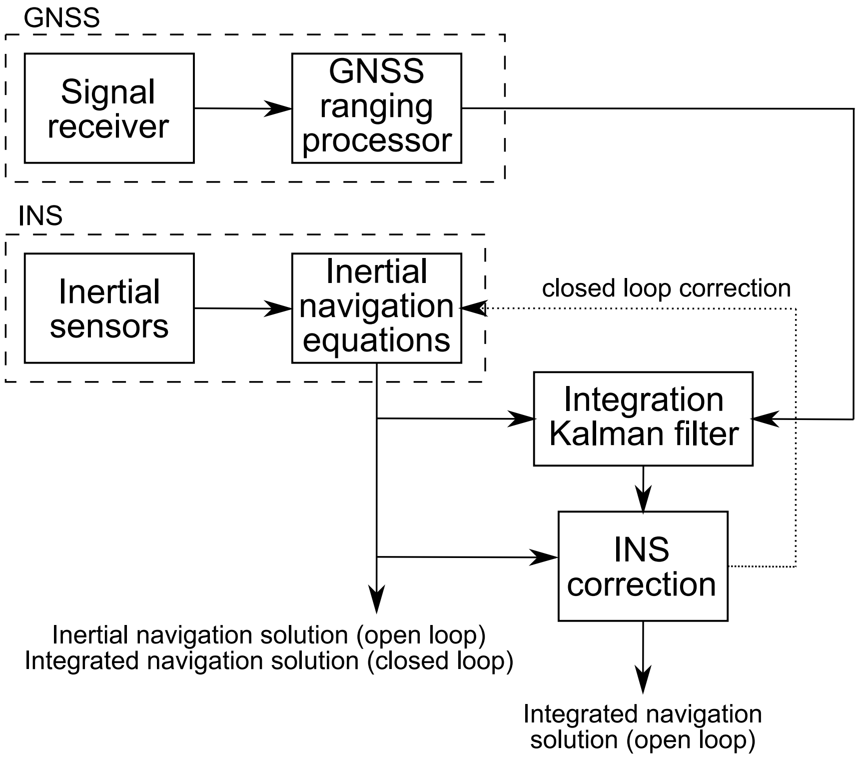

Fig. 3.3: Tightly coupled INS/GNSS integration architecture. Taken from [1].

3.1.4 Deep integration

Deep integration algorithm is a variant of the combined GNSS navigation and tracking. Tracking is a method which smooths noise and enables the navigation processor to be integrated at a lower rate. The information from the baseband signal processing channels, the Is and Qs [3], is filtered by the code and carries tracking loops before being an input to the navigation processor. Each measurement input is derived from a number of successive sets of Is and Qs, where older datasets are adopted with weight constants by the tracking loops to make them less important than newest datasets. Advantage of deep integration algorithm compared with GNSS navigation and tracking is that only the errors in the INS solution need be tracked, as opposed to the absolute dynamics.

[3]in-phase and quadraphase accumulated correlator outputs[1]

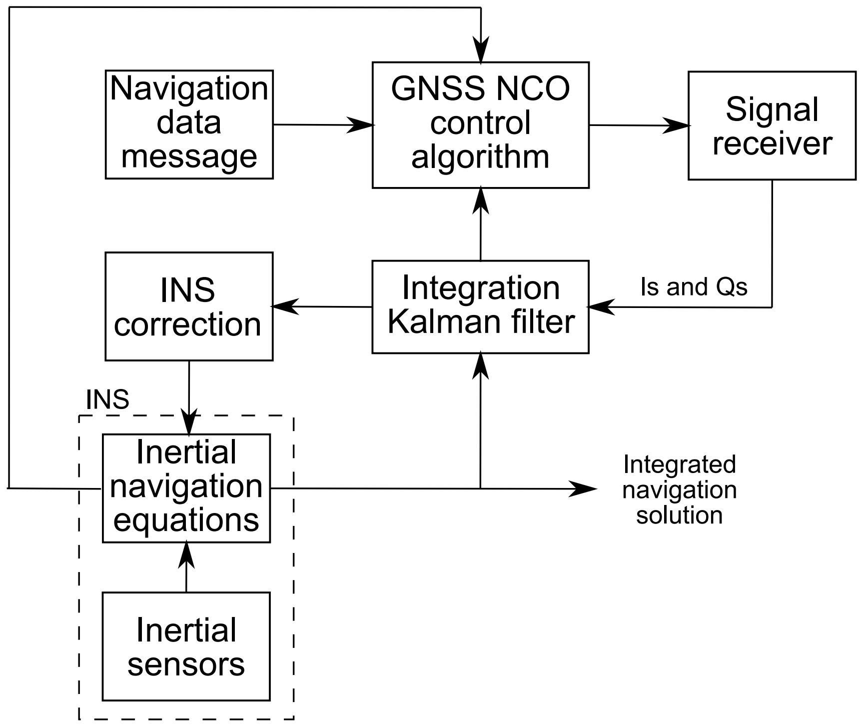

Fig. 3.4: Deep INS/GNSS integration architecture (closed-loop INS correction). Taken from [1].

Figure 3.4 shows the integration architecture with closed-loop INS correction. The code and carrier numerically controlled oscillator (NCO) commands are generated using the corrected inertial navigation solution, the satellite navigation data message estimates (position, velocity, clock, ionosphere and troposphere errors) [1]. Output from the GNSS receiver, Is and Qs, are inputs directly to the integration Kalman filter, where various INS and GNSS errors are estimated. Correction step is same as in other architectures.

Compared with tightly coupled integration, deep integration does not change weight constants of the older datasets, Is and Qs, when pseudo-range and pseudo-range rate output interval is greater than the tracking-loop time constant. It avoids change of Kalman filter gain when it is not needed.

Deep integration can be divided into coherent and non-coherent class. Coherent measurement, Is and Qs, inputs directly to the integration Kalman filter. It is more accurate and avoids discriminator non-linearities and reduces code-tracking noise. Request to iteration at the navigation-data-message rate can impose much higher processing loads. Non-coherent deep integration is more robust because integration inputs, Is and Qs, use discrimination function and can operate even if there is not sufficient signal to noise to track carrier phase.[1]

3.2 Kalman filter

The Kalman filter[1] is a flexible set of mathematical equation, rather then filter. Equations compute recursively means of system to minimize squared errors caused by sensors noise or unpredictable disturbances. The Kalman filter estimates a number of system parameters in combination with s stream of measurements that are subject to noise. It is a real-time algorithm updating estimated parameters with sufficient measurement information to determine the values of the parameters at the time.

The Kalman filter algorithm is a iterative process using deterministic and statistical system properties and the measurements to obtain optimal system estimates. According to this it is necessary to carry more information between each iteration than just the parameters estimates. It maintains parameter's uncertainties and a measure of the correlations between parameter's estimates and their errors. Actual measurement data are derived from new values and together with previous values weighted in average to update estimates (recursively). To do that, initial values must be provided. Terminology of the Kalman filter elements and Kalman filter algorithm steps[1] are described in next paragraphs.

The *state vector* describing a system with the set of parameters. Each state is estimated and may be constant (sensor bias) or time varying (position, velocity).

System state in navigation may include the position, velocity, attitude, GNSS errors, accelerometers errors and gyroscopes errors (clock delay, biases or scale-factors).

Representation of the uncertainties in the Kalman filter's state estimates is by *covariance error matrix*. It also represents correlation between errors in state vector. It is an important information for Kalman filter tuning, because there is not always enough information from measurement. Initial values must be set by user and update is determined from another process.

The *system model* is deterministic for the states and build from known properties of the system. It describe how relation between filter states and covariance error matrix change in time. A state uncertainty should be also included in the system model. It is presented by *system noise covariance matrix*. It is a variation of changes caused from unmeasured dynamics or random noise in an instrument output.

The *measurement vector* is a set of measurement from different sources (systems). In navigation it may be position or/and velocity from GNSS and INS system. Information from this vector is used to derive state vectors estimates. Associated with the measurement vector is *measurement noise covariance matrix* which contains information from noisy sensors measurements. This information may be processed to the system in regular or irregular intervals.

The *measurement model* is a function of the true state vector in the absence of measurement noise and describes how measurement vector varies. Measurement model is deterministic and builds from known properties of the system.

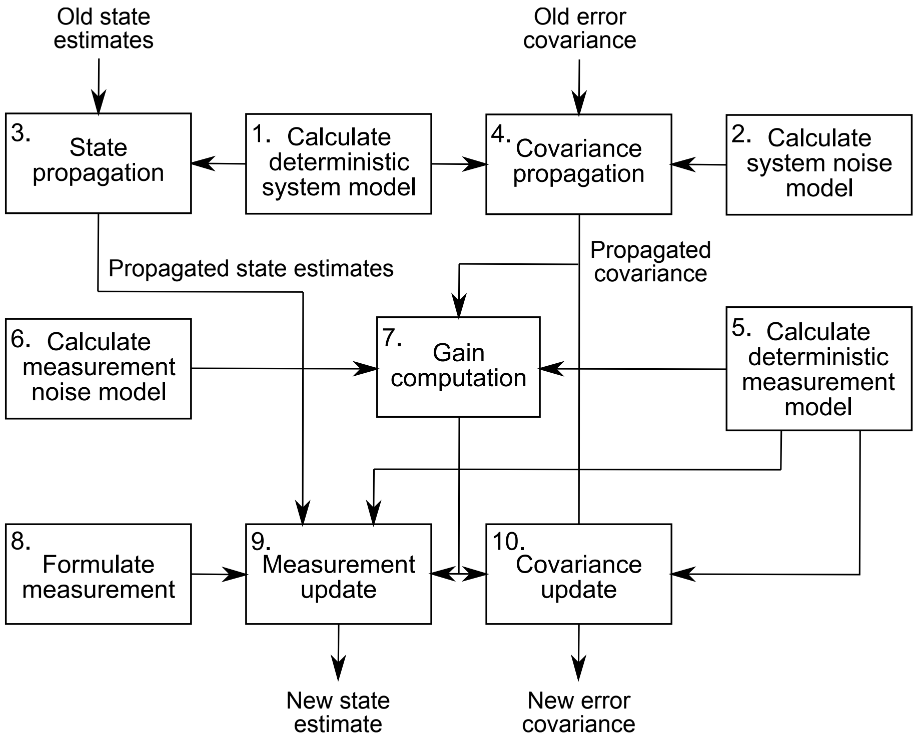

Fig. 3.5: Kalman filter algorithm steps. Taken from [1].

The *Kalman filter algorithm* shown in figure 3.5 consists of ten steps in each iteration cycle. First four steps are the time propagation, or time update, phase and next six steps are the measurement-update phase. It uses the measurement and system model together with measurement vector to maintain optimal estimates of the state vector.

The purpose of the time update phase is to predict forward the state vector estimate and error covariance matrix in time interval between previous and actual measurement data using the known properties of the system. The first two steps calculate the deterministic and noise parts of the system model. The third, state propagation step uses this to actualize state vector estimate. The fourth, covariance propagation completes the corresponding update to the error covariance matrix, increasing the state uncertainty in the system noise.

The measurement update phase, the state vector estimate and error covariance are updated to combine the new measurement information. Steps five and six calculate the deterministic and noise parts of the measurement model. The seventh step, gain computation, calculates the Kalman gain matrix. This is used to optimally weight the correction to the state vector according to the uncertainty of the current state estimates and how noisy the measurements are. The eighth step formulates the measurement vector. The ninth step, the measurement update, updates the state estimates to incorporate the measurement data weighted with the Kalman gain. Finally, the covariance update updates the error covariance matrix to account for the new information that has been incorporated into the state vector estimate from the measurement data.

3.3 The discrete Kalman algorithm

The discrete implementations are more common in use then continuous implementations, due to large part of advantages in digital computation. In this chapter discrete Kalman filter design is presented [11].

Continuous state space model is written as follows:

$$x_i = F_{i-1}x_i + G_{i-1}u_{i-1} + w_{i-1} \tag{3.1}$$

$$y_i = H_i x_i + v_i \tag{3.2}$$

The process noise w_i and v_i are white, zero-mean, uncorrelated and have known covariance matrices Q and R.

According to figure 3.5 the Kalman filter algorithm may be rewritten to steps [1]:

1. Calculate the transition matrix, ϕ_i
2. Calculate the system noise covariance matrix, Q
3. Propagate the state vector estimate from $\hat{x}_{i-1}$ to $\hat{x}_i^-$
4. Propagate the error covariance matrix P_{i-1} to P_i^-
5. Calculate the measurement matrix H
6. Calculate the measurement noise covariance matrix R_i
7. Calculate the Kalman gain matrix K_i
8. Formulate the measurement z_i
9. Update state vector estimate from $\hat{x}_i^-$ to $\hat{x}_i$
10. Update error covariance matrix from P_i^- to P_i

Tab. 3.1: State vector estimate and covariance matrix time propagation.

$\hat{x}_{i-1}$	$\hat{x}_i^-$	$\longrightarrow$	$\hat{x}_i^-$	$\hat{x}_i$
P_{i-1}	P_i^-	$\longrightarrow$	P_i^-	P_i
$i-1$		$time$	i	

Kalman filter states: the vector of position, velocity, attitude, accelerometer and gyroscopes errors. States are referenced and resolved to ECEF frame where state vector become:

$$x = \begin{pmatrix} \delta r^e_{eb} \\ \delta v^e_{eb} \\ \delta \psi^e_{eb} \\ b_a \\ b_g \end{pmatrix} \tag{3.3}$$

This vector is usually initialized with zeros. Correction of navigation equations is given by following equations:

$$\hat{r} = r + \delta r \tag{3.4}$$

$$\hat{v} = v + \delta v \tag{3.5}$$

$$C^e_{eb} = (I_3 - E)C^e_{eb} \tag{3.6}$$

where E is skew-symmetric matrix of attitude errors with simplified denotation of attitude error from $\delta\psi^e_{eb}$ to ϵ and following form:

$$E = \begin{pmatrix} 0 & -\epsilon_D & \epsilon_E \\ \epsilon_D & 0 & -\epsilon_N \\ -\epsilon_E & \epsilon_N & 0 \end{pmatrix} \tag{3.7}$$

System matrix F, which defines how the state vector changes with time as a function of the dynamic of the system modelled by the Kalman filter:

$$F[15 \times 15] = \begin{pmatrix} 0_3 & I_3 & 0_3 & 0_3 & 0_3 \\ 0_3 & -2\Omega^e_{ie} & -S^e_f & C^e_b & 0_3 \\ 0_3 & 0_3 & -\Omega^e_{ie} & 0_3 & C^e_b \\ 0_3 & 0_3 & 0_3 & 0_3 & 0_3 \\ 0_3 & 0_3 & 0_3 & 0_3 & 0_3 \end{pmatrix} \tag{3.8}$$

where S^e_f is the skew symmetric matrix of specific force and Ω^e_{ie} is the skew-symmetric matrix of the Earth-rotation vector (1.13).

Discretization of system matrix F, is usually performed using power-series expansion [1]. In this case power-series expansion of second order:

$$\phi_i = I_{15} + F\tau_i + \frac{1}{2}F\tau_i^2 \tag{3.9}$$

System noise distribution matrix G:

$$G[15 \times 6] = \begin{pmatrix} 0_3 & 0_3 \\ C^e_b & 0_3 \\ 0_3 & C^e_b \\ 0_3 & 0_3 \\ 0_3 & 0_3 \end{pmatrix} \tag{3.10}$$

Measurement observation matrix H defines how the measurement vector varies with the state vector:

$$H[6 \times 15] = \begin{pmatrix} I_3 & 0_3 & 0_3 & 0_3 & 0_3 \\ 0_3 & I_3 & 0_3 & 0_3 & 0_3 \end{pmatrix} \tag{3.11}$$

State covariance matrix P:

$$P[15 \times 15] = \begin{pmatrix} r_{eb}^e & 0_3 & 0_3 & 0_3 & 0_3 \\ 0_3 & v_{eb}^e & 0_3 & 0_3 & 0_3 \\ 0_3 & 0_3 & \psi_{eb}^e & 0_3 & 0_3 \\ 0_3 & 0_3 & 0_3 & b_a & 0_3 \\ 0_3 & 0_3 & 0_3 & 0_3 & b_g \end{pmatrix} \tag{3.12}$$

where initialized values for position, velocity and attitude are part of tuning Kalman's filter.

System noise covariance matrix Q defines how the uncertainties of the state estimate increase in filter model because of noisy sources:

$$Q[6 \times 6] = \begin{pmatrix} \sigma_{ax}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{ay}^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{az}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{gx}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{gy}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{gz}^2 \end{pmatrix} \tag{3.13}$$

contains standard deviations of accelerometer sensors noise $\sigma_{ax}^2, \sigma_{ay}^2, \sigma_{az}^2$ and gyroscope sensors noise $\sigma_{gx}^2, \sigma_{gy}^2, \sigma_{gz}^2$.

Discretization of matrix Q:

$$Q_{d,i} = GQG^T \tau_i \tag{3.14}$$

Measurement noise covariance matrix for GNSS data:

$$R[6 \times 6] = \begin{pmatrix} \sigma_{px}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{py}^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{pz}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{vx}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{vy}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{vz}^2 \end{pmatrix} \tag{3.15}$$

where $\sigma_{px}^2, \sigma_{py}^2, \sigma_{pz}^2,\ \sigma_{vx}^2, \sigma_{vy}^2, \sigma_{vz}^2$ are filled with standard deviations of position and velocity which are given directly by GNSS receiver.

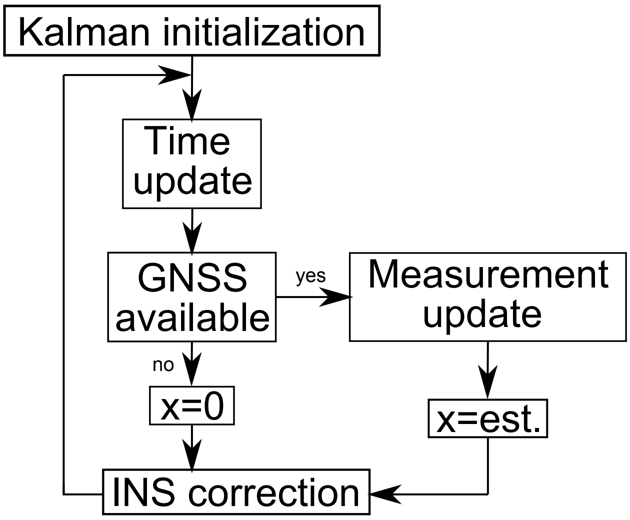

Fig. 3.6: Kalman algorithm diagram for fusion INS and GNSS data.

3.3.1 Time update step ("Prediction")

$$\hat{x}_i^- = \phi_i \hat{x}_{i-1} \tag{3.16}$$

$$P_i^- = \phi_i P_{i-1} \phi_i^T + Q_{d,i} \tag{3.17}$$

3.3.2 Measurement update step ("Correction")

$$K_i = P_i^- H_i^T \left(H_i P_i^- H_i^T + R_i \right)^{-1} \tag{3.18}$$

$$\hat{x}_i = \hat{x}_i^- + K_i \left(z_i - H_k \hat{x}_i^- \right) \tag{3.19}$$

where the measurement vector z_i is the navigation solution difference between a system under calibration and a reference system:

$$z_i = \begin{pmatrix} GNSS_{pos} - INS_{pos} \\ GNSS_{vel} - INS_{vel} \end{pmatrix} \tag{3.20}$$

$$P_i = \left(I_3 - K_i H_i \right) P_i^- \tag{3.21}$$

4 IMPLEMENTATION

4.1 IMU data

Measured data were collected from the Embedded navigation board in developed times. The data collects measurement from IMU (IMU-9150). It contains temperature sensor, three accelerometers, three gyroscopes and timer with period of 0.006 s.

Information obtained from this embedded board are real data from real measurement with all sensors errors, background noise and all presented forces in ECEF frame 1.2.2. Note, that sensors errors grow up quadratically in time and cause this measurement inaccurate. This is a global problem for low-cost IMU.

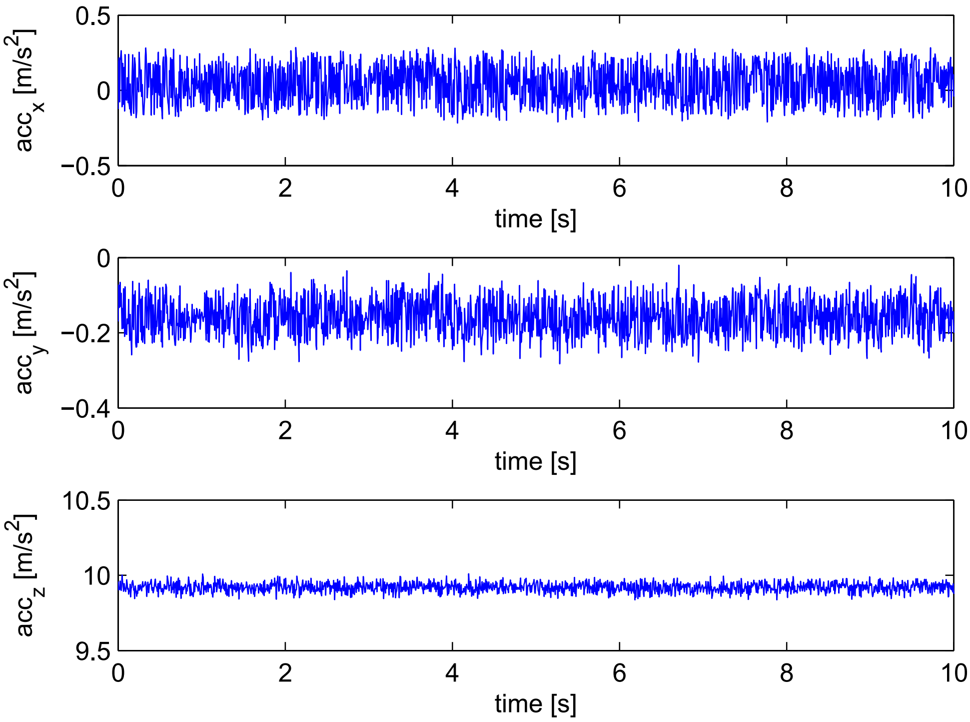

Fig. 4.1: Acceleration output from IMU.

In the figure 4.1 is shown accelerometers measurement from IMU in static measurement situation. There is printed ten second sample period, where ideal measurement of acceleration should be constant zero value without any errors or noise. In real data, errors are presented as described in chapter 1.1.5. Three equivalent accelerometers are used in navigation board however, difference between offset and error variance in each axis is visible. Considerate to the z-axis, offset from zero is much bigger then in x-axis or y-axis. It is caused by gravitational forces with effects on sensors.

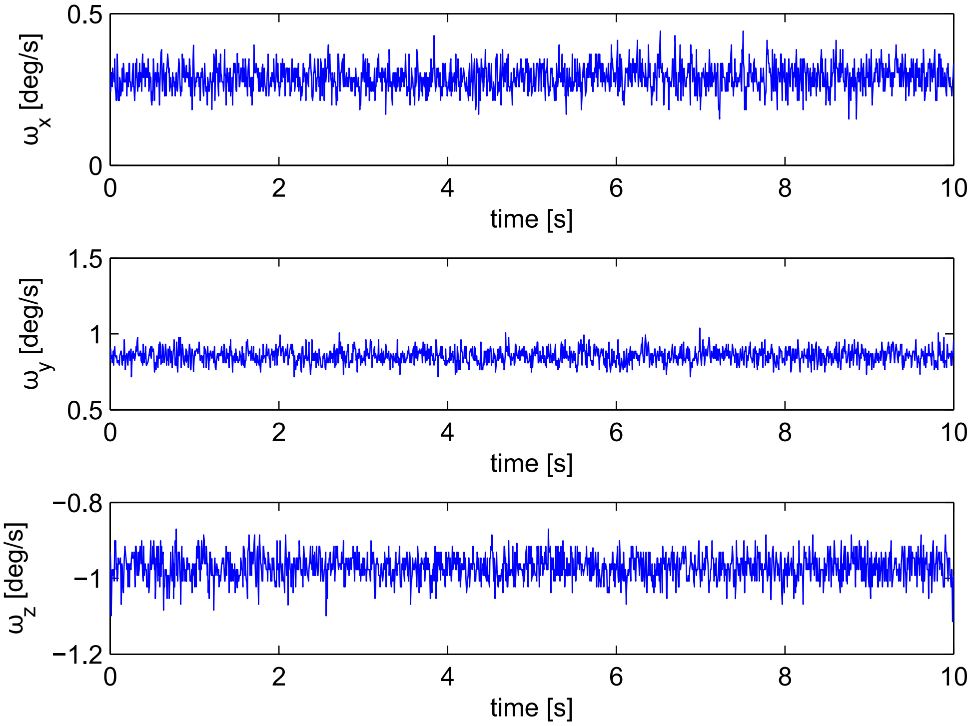

Fig. 4.2: Angular rate output from IMU.

Non-knowledge of real trajectory and initialization position of measured objects leads in developed algorithm to uncertainty in final solution. Reason to know trajectory before navigation equation computation is simple. Navigation equation output is easier to compare with clear (simulated) input trajectory to test correctness of equation's computation. For this reason generator of clear trajectory was made.

In figure 4.2 is a same situation with measurement of angular rate as with acceleration measurement. Main difference is that gyroscopes do not measure gravitational forces.

4.2 Modelling trajectory

The sensor's error in measured data was main reason to make generator for clear trajectory. This generator is created to simulate three accelerometer and three gyroscopes. Final trajectory has six degree of freedom (DOF). Sometimes terms 6DOF can be confusing. In this generator it means moving in x-axis, y-axis and z-axis direction and rotations around each axis. Generator provide clean trajectory without errors and noises from IMU sensors. Gravitational force is also included in this trajectory. It is added to accelerations according to angular rate rotations towards to the Earth centre.

Settings to create trajectory is defined by two matrices. Matrices A (for example: `A = [1 0 0; 0 1 0; 0 1 0]`) to define trajectory points in Cartesian system $[x\,y\,z]$ and matrices B to define rotation angle changes around each axes. (for example: `B = [pi 0 0; pi/2 0 0; pi/4 0 0]`). Function `polyfit()` is used to make this trajectory continuously smoothed and reversal function `polyval()` is used to get exact values for requested time period `Ti` in step `k`. To define length of generated trajectory is used variables `A_step` and `B_step` which is time between defined points. Final notation of all function which return three values of acceleration updated with gravity and three values of angular rate is `function imu = ... gendata( A, A_step, B, B_step, Ti, k )`.

In the picture 4.3 we can see trajectory separately for each axis to show how output from navigation equation if we define position due to matrices `A = [0 ... 0 6; 5 6 4; 10 0 2]` and `B = [0 0 0;0 pi/4 0;0 pi/2 0]`. The rotation around y-axis in figure 4.4 is from 0 to $pi/2$ and x-axis and z-axis is without rotation. This figures are printed in 2D (two dimension).

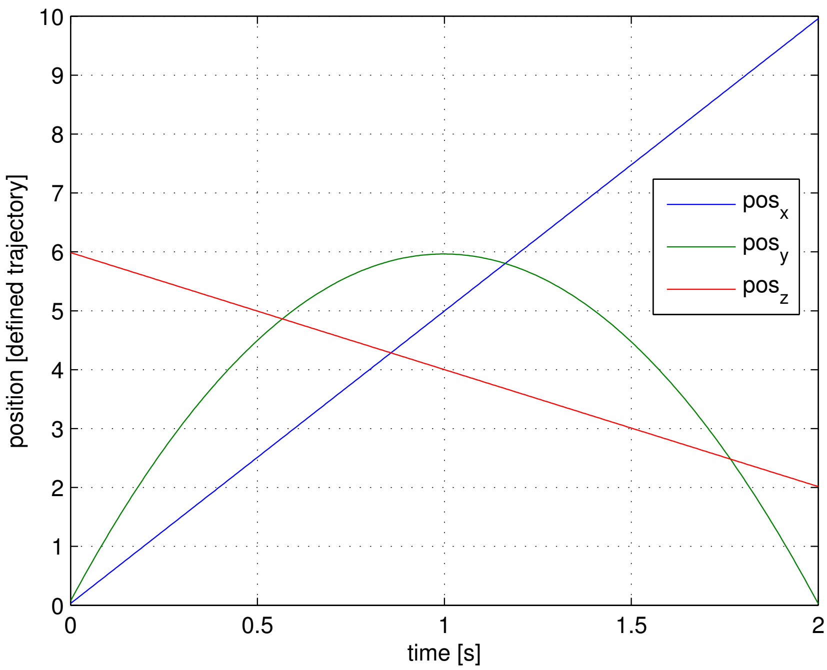

Fig. 4.3: Navigation equation output - simulated position.

The figure 4.5 is printed in 3D (three dimension) and shown three axis move with rotation around y-axis. It is visible that changing rotation around axis has effect to changing end point of generated trajectory. Because of that reason it is difficult to imagine trajectory rotated in all three axis together with different three axis move.

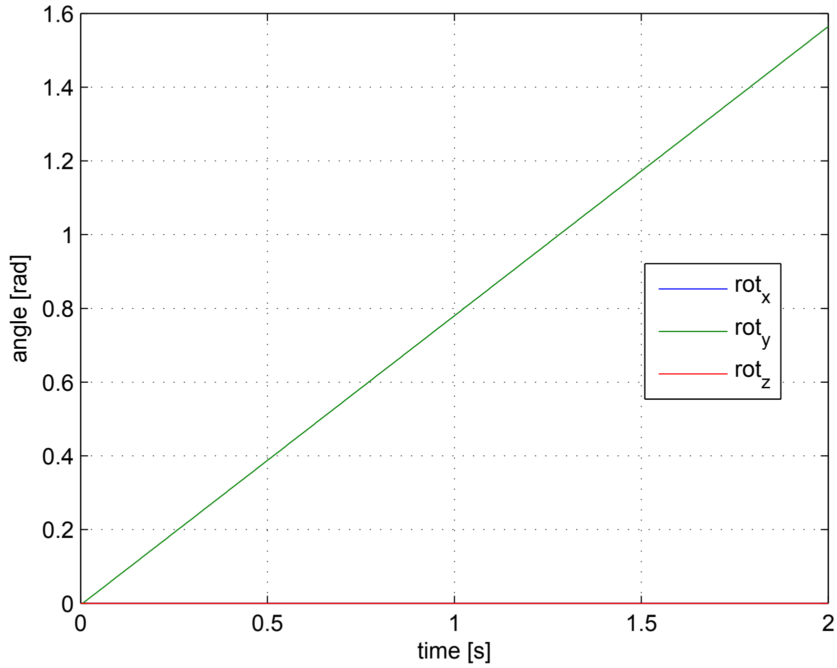

Fig. 4.4: Navigation equation output - simulated rotation.

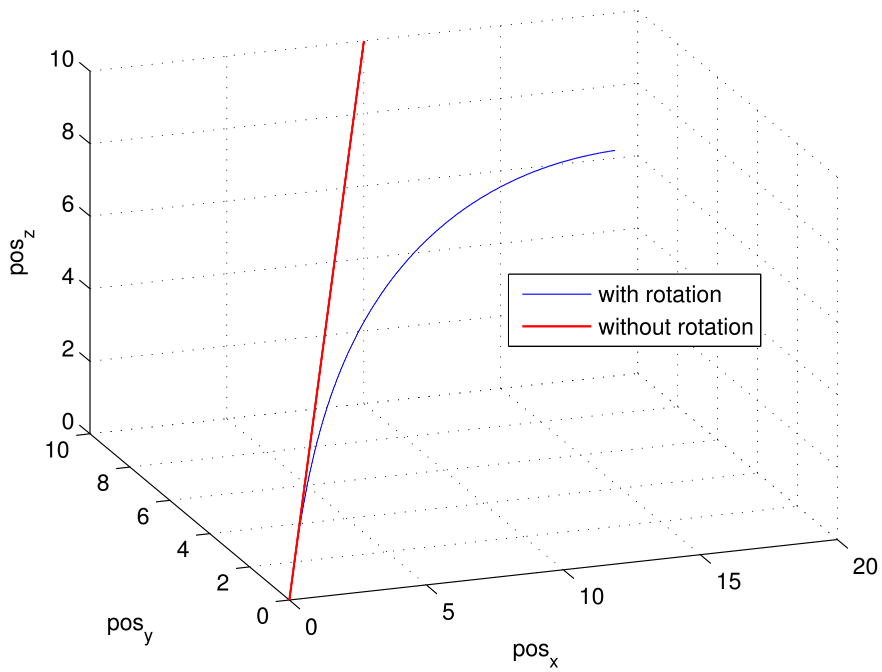

Fig. 4.5: Navigation equation output - compare with/without rotation.

4.3 Navigation equation

Inertial navigation equation computing algorithm was created according to ECEF navigation equations 1.4. Computing is similar for measured and for simulated data. Only difference is that simulations equations does not use centrifugal and Coriolis term due to the rotation of the resolving axes. Final navigation equation solution is derived from IMU measurement and processed by set mathematical equation.

The navigation equation algorithm is divided into continuously repeating steps, which are:

1. Load the angular rate measurement ω_{ib}^{b}
2. Compute the transformation matrix C_b^e
3. Load the acceleration measurement f_{ib}^{b}
4. Transform the specific forces to the ECEF navigation frame f_{ib}^{e}
5. Compute velocity v_{eb}^{e}
6. Subtract gravity and Coriolis forces from the specific forces.
7. Compute position r_{eb}^{e}

The graphical representation of these steps are shown in the figure 1.5.

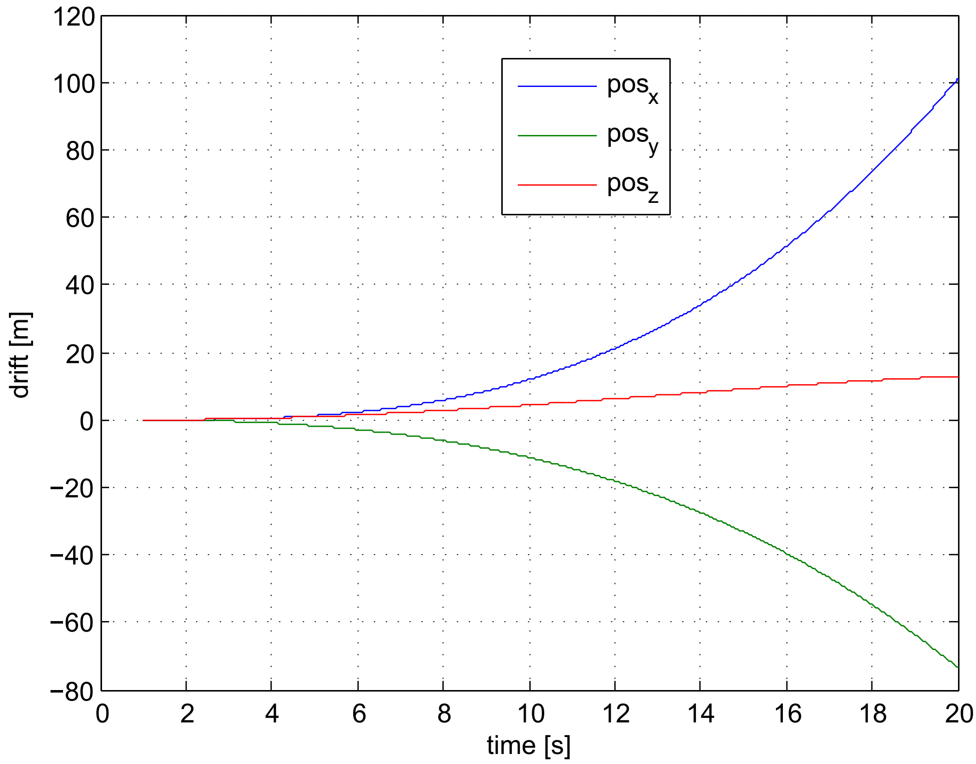

Fig. 4.6: Navigation equation output of position from static measurement with all sources of errors from measurement and navigation equation computing.

Navigation equation output of position in the figure 4.6 is result from navigation equation computation, which come from the real static measurement including inertial errors as described in 1.1.5 and 4.1. There is visible how position error vary in time. The position error increase exponentially with time because of double integration of inertial measurement errors in navigation equation computing. In twenty seconds sample there is approximately 100 m error in x-axis, 70 m error in y-axis and 18 m error in z-axis.

Important part of navigation equation algorithm is initialization process. Each iteration of navigation equation computing uses values from previous step computing. (See block diagram for navigation equations 1.5, values with suffixes (-)) This attitude, velocity and position must be initialized before the beginning of the navigation computing. In real applications this values are usually updated by another navigation system (for example GNSS or more accurately INS). In simulation to make the trajectory these values need to be also updated before beginning ot the computation. Extra function was created to compute velocity initialization from first iterations of trajectory.

4.4 Biases discussion

The most important source of the navigation equation error are biases of accelerometers and gyroscopes. Biases should be constant but really they are not and change over the time. This change in bias is often related to temperature, time and/or mechanical stress on the system.

This analysis focus on the improvement of the navigation equation error when different method are used to cut off these biases. It goes out from theory that, measurement values of the inertial sensors should be zeros when object is in static position without move. In the figure 4.8 are biases simply catted out with precomputed mean value of the biases offset from zero. Another method shown in the figure 4.9 uses 1 s cycle of mean value, where new value is computed from last 1 s measurement period. This technique is sometimes called moving average window.

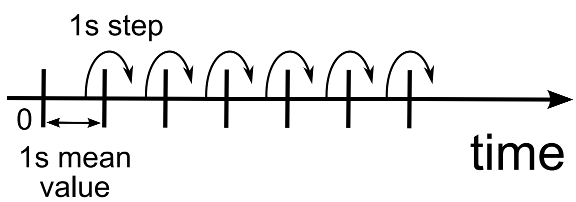

Fig. 4.7: Time shift in bias correction.

The figure 4.8 and figure 4.9 can be compared with figure 4.6, which is without biases correction. There is position error from the navigation equation computing

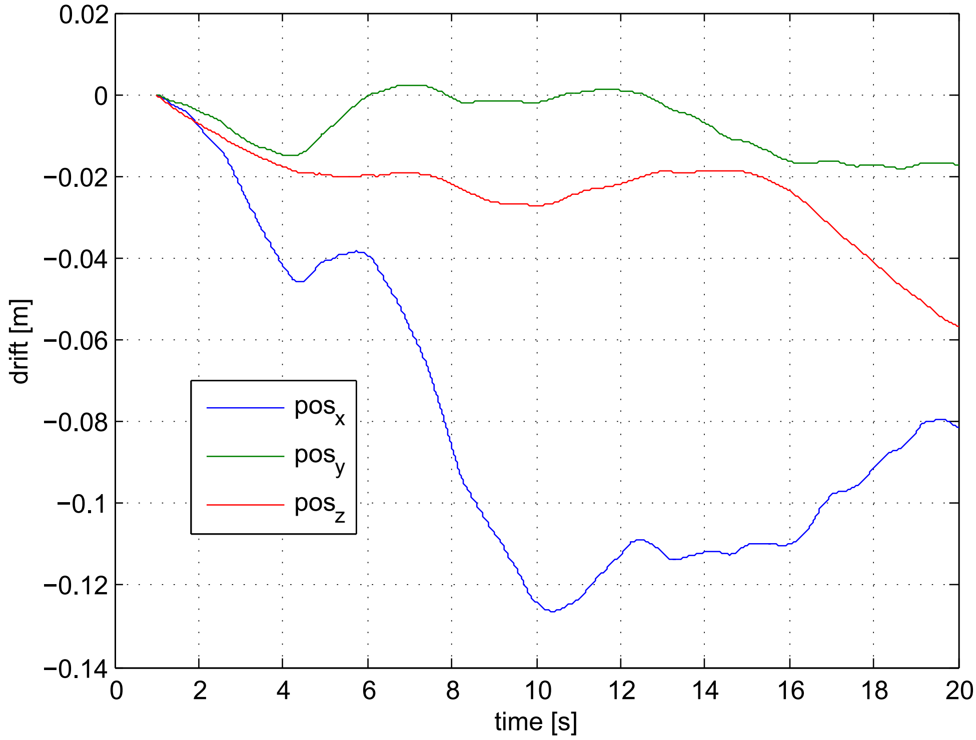

Fig. 4.8: Navigation equation output of position with mean value correction.

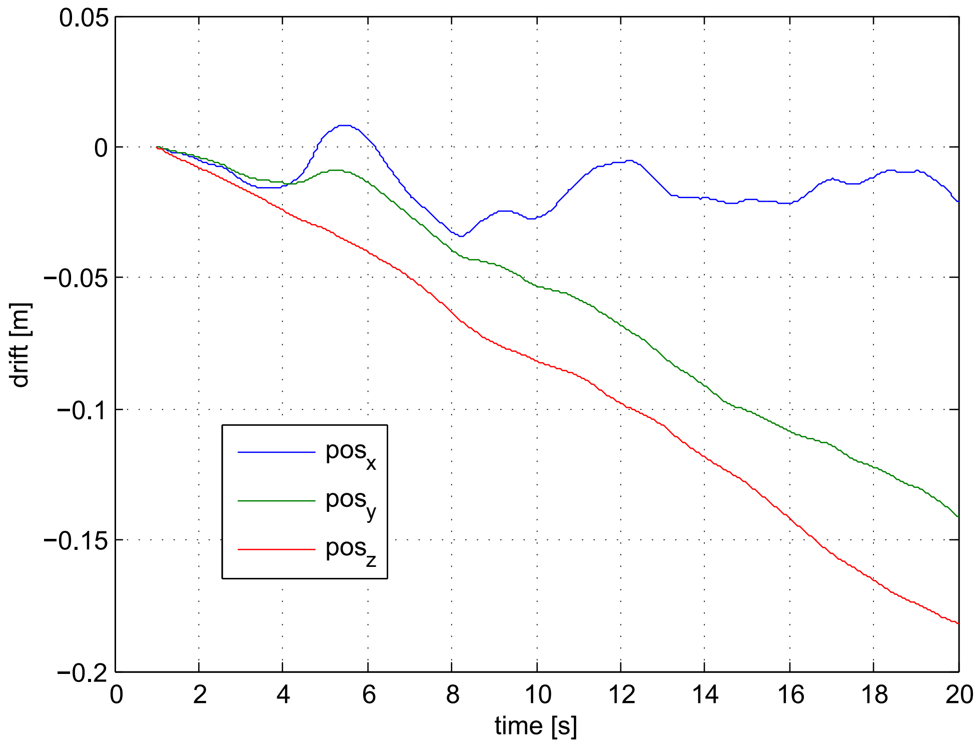

Fig. 4.9: Navigation equation output of position with 1s moving mean correction.

approximately between 18-100 m. When correction with the mean value is applied, error is less then 0.13 m per 20 second sample. With moving average window, error is approximately 0.18 m per 20 second sample. Both of results significantly improve navigation equation output. This is the easiest implementation of biases correction. More sophisticated methods can be used to estimate inertial errors. (For example: Kalman filter based zero-velocity estimator B.)

4.5 GNSS data

Gnss receiver (NV08C-CSM) on navigation board may receive signals from GLONASS, GPS and SBAS navigation systems. SBAS is a low-priority function and their stability is indeterminate. This receiver may also work with GALILEO and BeiDou-2 (after firmware change). In time of collecting GNSS data 22 satellites was available. Navigation message contain information about position, velocity and also heading, which was also used to initialize INS equation. Figure 4.10 and 4.11 shows 20 s sample of position and velocity. Standard deviation of 10 minutes measurement was for position [3.6852, 3.9626,4.3475] m and for velocity [0.0068, 0.0047, 0.0093] m/s.

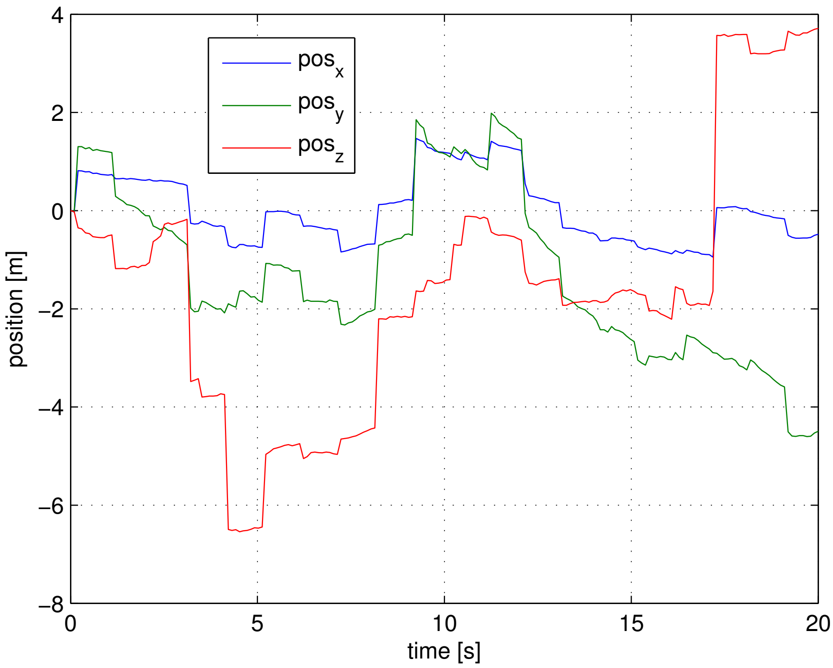

Fig. 4.10: GNSS position output.

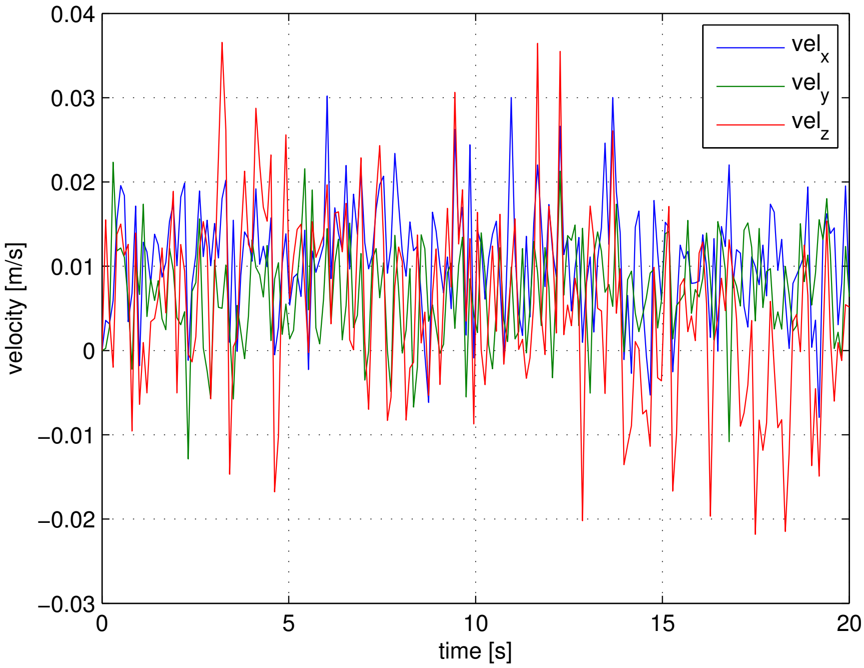

Fig. 4.11: GNSS velocity output.

4.6 Kalman filter tuning

Tuning a Kalman filter parameters is an essential part of INS/GNSS integration. Especially the initial values of covariance matrix, P, measurement noise covariance matrix, Q, and the system noise covariance matrix, Q, are important. Good tuning philosophy is to fix values in matrix P, then Q and in the end vary with matrix R to obtain stable state estimates. Selecting these values too small, the actual errors in the Kalman filter will be much larger than the state uncertainties stored in matrix P. Reversely, if the values are too big, also the uncertainties will be too large.

Another important parameter is the ratio between matrices P and R, which determine Kalman gain, K. If this ratio is too small, system is slow to respond to the changes and states estimates converge to their counterparts more slowly than it is necessary. Reversely, overestimated ratio (P/R) will increase Kalman gain. It may lead to the instability results or biased state estimates due to the measurement noise which has big effect to them [1].

In the ideal case of tuning noise model, results will be with stable state estimates and consistent estimation error. In practice, it is often necessary to tune the filter state uncertainties more larger than the corresponding error standard deviations. This is because Kalman filter's model is just approximation of the real system. Tuning a Kalman filter is kind of compromise between convergence rate and stability.

4.7 INS/GNSS integration results

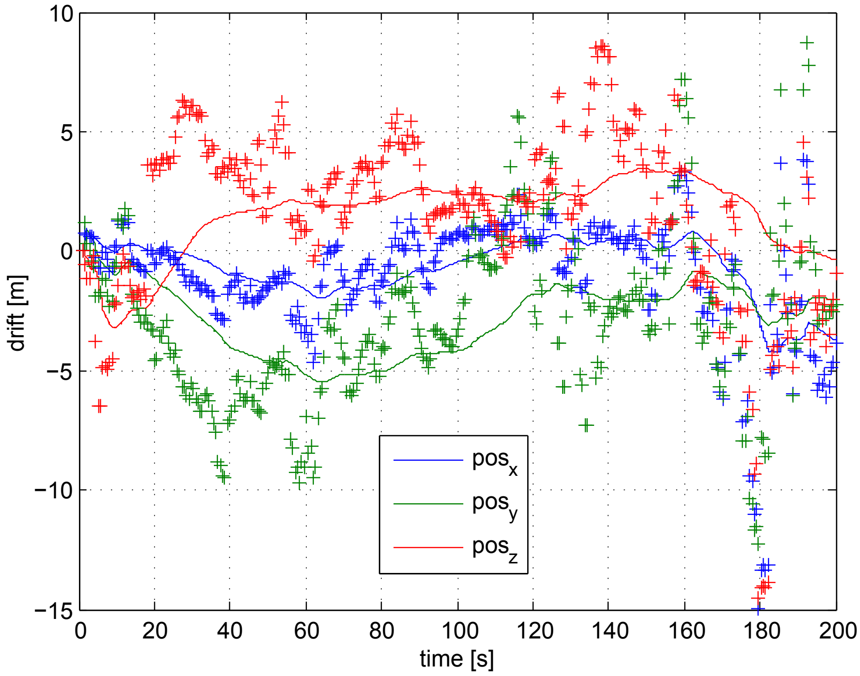

Fig. 4.12: Position error with mean value biases correction.

Note, that values with symbol (+) are measured data from GNSS and continuous line is integrated INS/GNSS navigation solution.

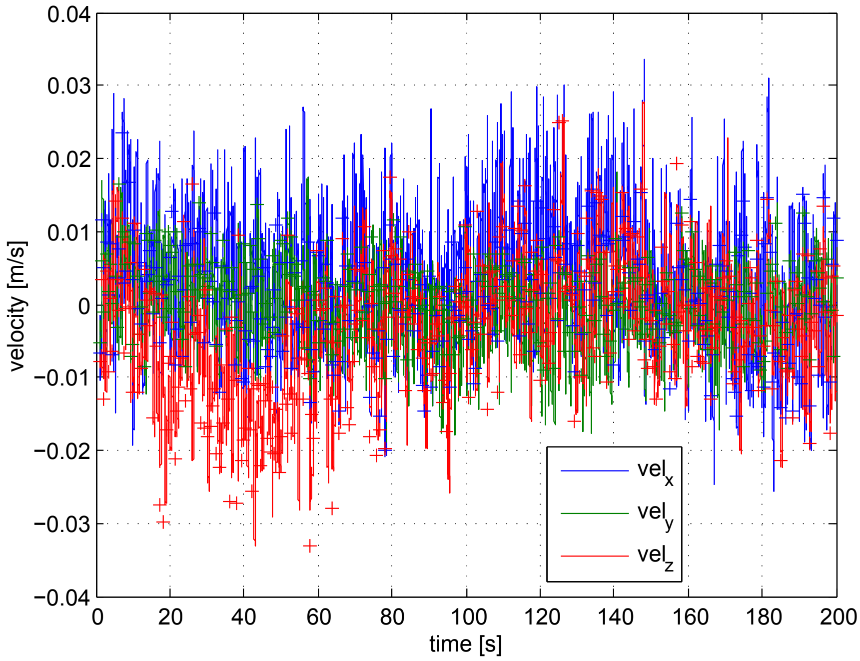

Fig. 4.13: Velocity error with mean value biases correction.

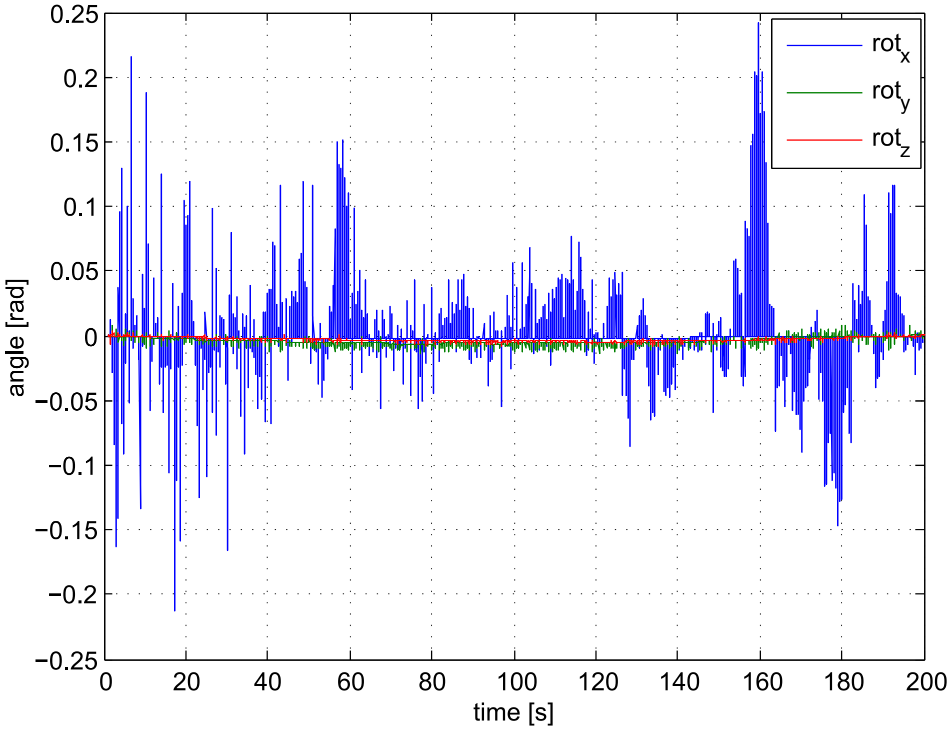

Fig. 4.14: Rotation error with mean value biases correction.

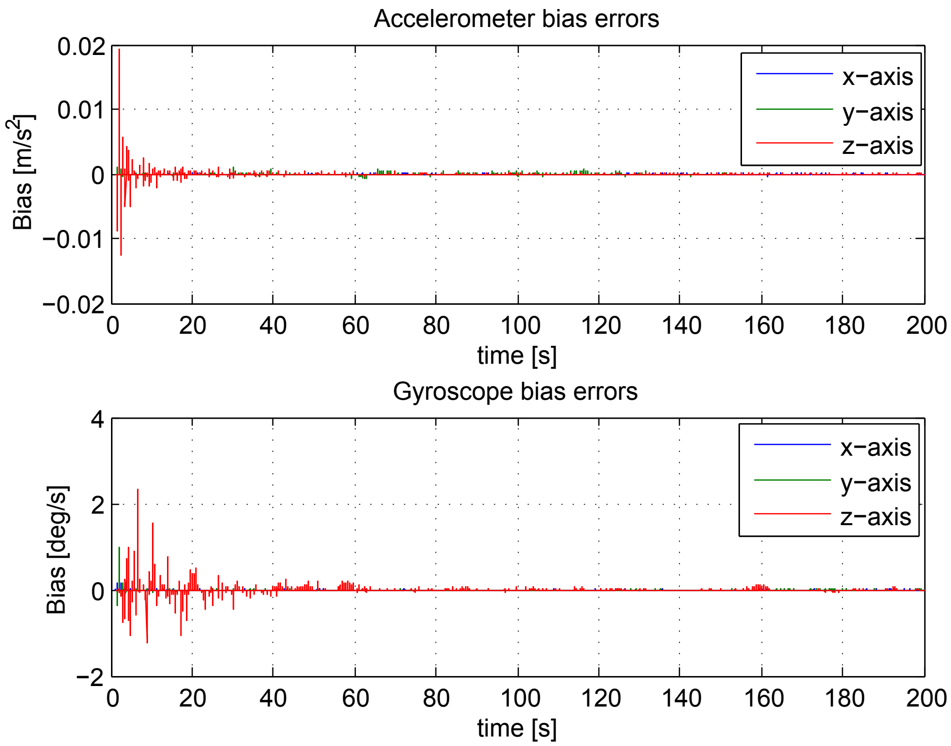

Fig. 4.15: Biases error with mean value biases correction.

4.8 GNSS outage

The following figures shows GNSS outage (between 50s and 90s) and his correction.

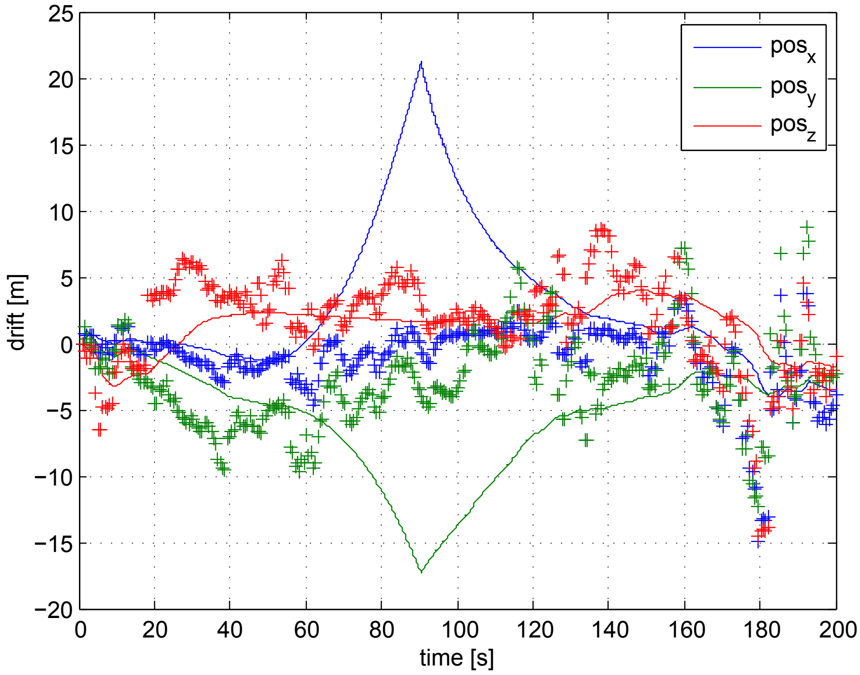

Fig. 4.16: Position error in simulation of lost signal between 50-90s.

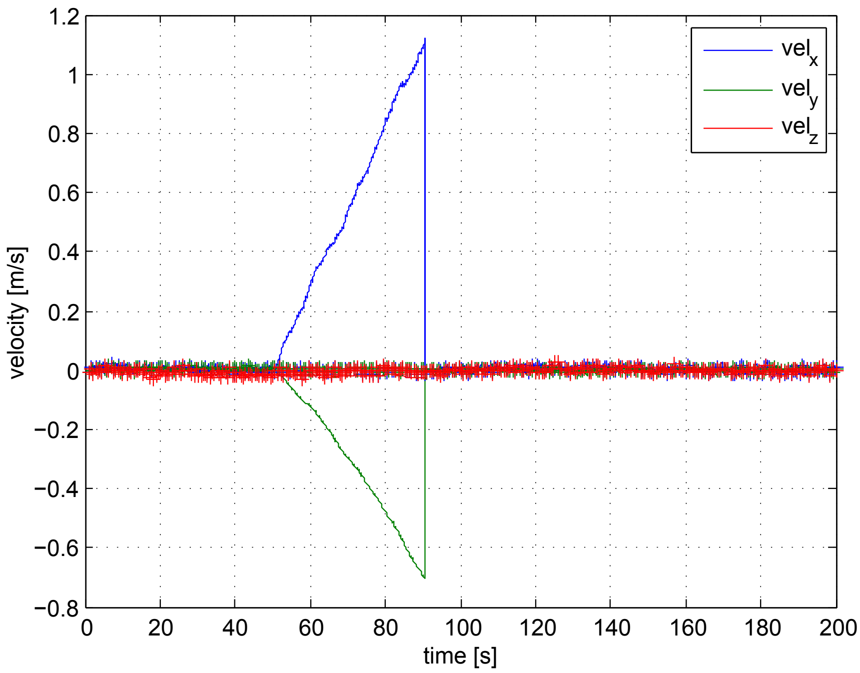

Fig. 4.17: Velocity error in simulation of lost signal between 50-90s.

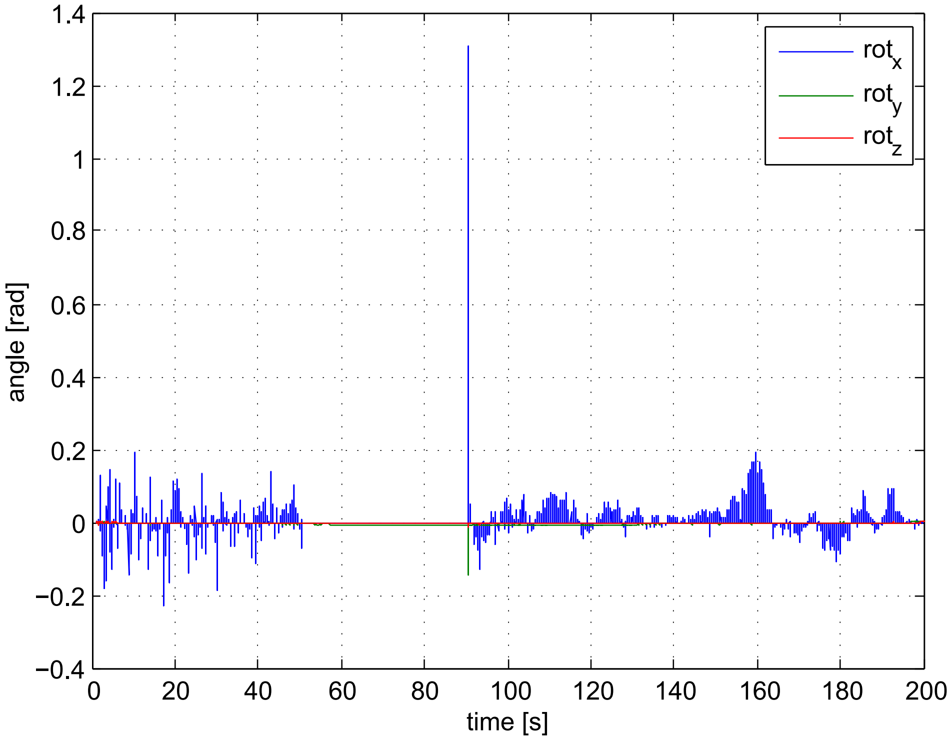

Fig. 4.18: Rotation error in simulation of lost signal between 50-90s.

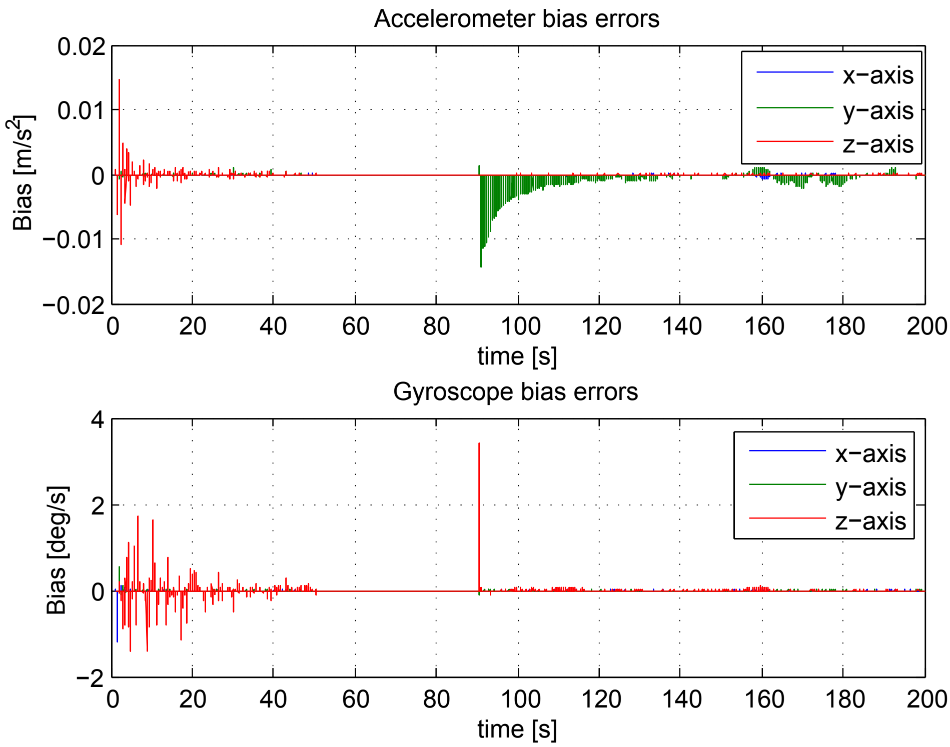

Fig. 4.19: Biases error in simulation of lost signal between 50-90s.

5 CONCLUSION

The main objective of this work was to create a navigation solution, which will combine advantages of INS and GNSS navigation system into the one navigation system. Real measured data from IMU and GNSS were provided to design this system into the specific use for embedded navigation board.

Inertial navigation system uses measured data from IMU (accelerometers and gyroscopes). The processing this data through navigation equation, final position, velocity and attitude can be provided. ECEF navigation frame was chosen like reference frame for this system, because of linearity in Cartesian coordinates. Gravity is a function of positions, therefore re-computation of their value is needed in each step (details in chapter 1.4.4). To test correction of inertial navigation system, simulator with given true trajectory was created (implementation in chapter 4.2). Compared simulated data between input and output of navigation equation confirm validity of inertial solution and its gravity removing. With real measurement data from inertial sensors, errors increase in each step without limitations (see figure 4.6). This is caused because of inertial sensors errors (described in chapter 1.1.5). The biggest effect to this have biases of inertial sensors. Biases analyses were made with different types of corrections (see chapter 4.4). The best result gives method where biases values was calculated from static measurement like mean value. This result confirms that bias leads to be constant which is produced with variety of effects like magnetic field noise or temperature gradients.

GNSS receiver message provide position and velocity data. This values are used in discrete Kalman filter algorithm, which is like a heart of integration of INS and GNSS systems. Tuning of Kalman filter is kind of a compromise between convergence rate and stability. Setting right initial values of covariance matrices: P, Q and R has the straight impact to the estimated values. Correction has to be in each step and with closed-loop feedback, because of low-coast quality of inertial sensors. State vector should be zeroed at every step. Period of INS is 0.006s and period of GNSS is 0.1s, that means that intersection of these values is in each 0.3s of computation. In each intersection the measurement step update of Kalman filter is computed. Final solution of integrated INS/GNSS system is printed in chapter 4.7 and simulated GNSS outage in chapter 4.8. Final solution provide long-term accuracy without exceeding GNSS errors, decreases GNSS peaks, and create less error in case of GNSS outage than single inertial navigation solution.

Areas to improve in are: more detailed model of sensor errors, more precise method of tuning Kalman filter or different modification of it, use different frequencies of both of systems and many sophisticated improvements.

BIBLIOGRAPHY

[1] GROVES, Paul D. *Principles of GNSS, inertial, and multisensor integrated navigation systems*. 2nd Ed. Boston: Artech House, c2008, xvi, 518 p. GNSS technology and applications series. ISBN 978-1608070053.

[2] NOVATEL. *IMU Errors and Their Effects* [online]. Rev.A. 2014, 6 p. Available [2015-05-13]: `http://www.novatel.com/assets/Documents/Bulletins/APN064.pdf`

[3] TITTERTON, David H. and John L. WESTON. *Strapdown Inertial Navigation Technology: 2nd edition*. United Kingdom: MPG Books Limited, Bodmin, Cornwall, 2004. ISBN 0 86341 358 7.

[4] VERHEIJEN, Jos. World Geodetic System 1984. 2013. *QPS Support Portal* [online]. Available [2015-05-13]: `https://confluence.qps.nl/x/xY3HAQ`

[5] KUBIČKA, Matěj. *Road-vehicle navigation* Paris, 2014. Diploma thesis. University of West Bohemia and Esiee Paris. Supervisor: Prof. Hugues Mounier.

[6] FARRELL, Jay A. *Aided navigation GPS with high rate sensors.* New York: McGraw-Hill, 2008. ISBN 00-716-4266-8.

[7] KAPLAN, Elliott D. and Christopher J HEGARTY, *Understanding GPS: principles and applications.* 2006. 2nd ed. Boston: Artech House, xvii, 703 p. ISBN 15-805-3894-0.

[8] ESA. The future - Galileo. 2014. *European GNSS agency* [online]. Available [2015-05-13]: `http://www.esa.int/Our_Activities/Navigation/The_future_-_Galileo/What_is_Galileo`

[9] IAC. Information-analytical centre. *Federal space agency* [online]. Available [2015-05-13]: `https://www.glonass-iac.ru/en/`

[10] KORNHAUSER, Alain L. *Global Navigation Satellite System* [online]. Princeton, NJ, 2006. Available [2015-05-18]: `https://www.princeton.edu/~alaink/Orf467F07/GNSS.pdf`.

[11] TANG, Pham Van and Nguyen Van THANG. 2015. *15-State Extended Kalman Filter Design for INS/GPS Navigation System.* Journal of Automation and Control Engineering. 3(No. 2). Available from: `http://www.joace.org/uploadfile/2014/0801/20140801020118731.pdf`

LIST OF ABBREVIATIONS

ECEF	Earth Center Fixed Frame
IMU	Inertial Measurement Unit
INS	Inertial Navigation System
GPS	Global Positioning System
GNSS	Global Navigation Satellite System
MEMS	Micro Electro Mechanical Sensor
CAN	Controller Area Network
UTS	Coordinated Universal Time
CDMA	Code Division Multiple Access
FDMA	Frequency Division Multiple Access
SBAS	Satellite-based Augmentation System
NCO	Numerically Controlled Oscillator
DGBU	Director General Business Units
UART	Universal Asynchronous Receiver/Transmitter
EEPROM	Electrically Erasable Programmable Read-Only Memory
SRAM	Static Random Access Memory
I2C	Inter-Integrated Circuit
6DOF	Six Degree Of Freedom

LIST OF APPENDICES

A ANOTHER RESULTS FIGURES

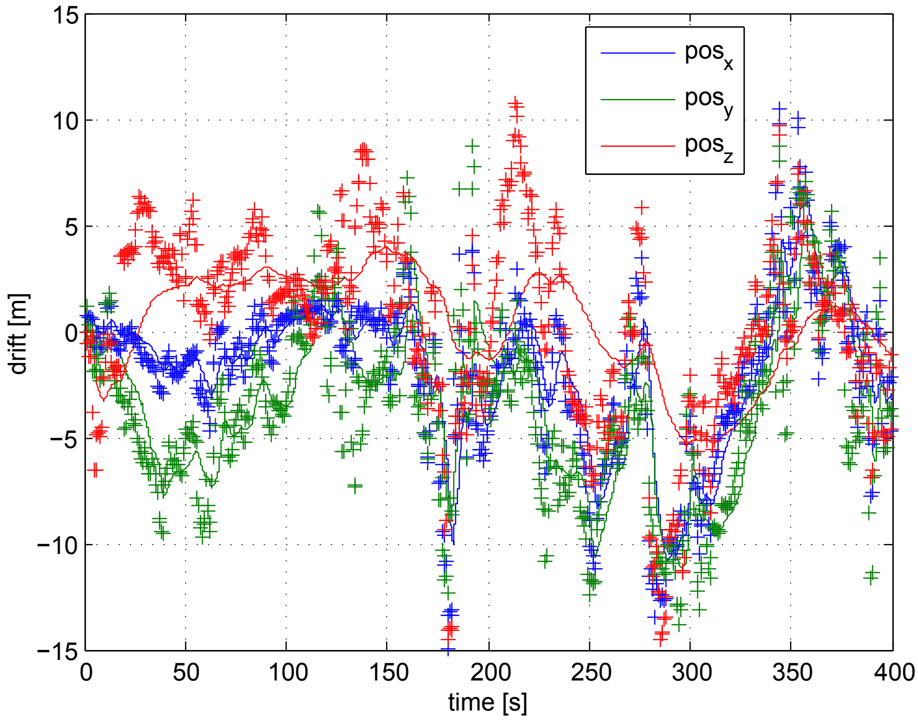

Fig. A.1: Position error with mean value biases correction.

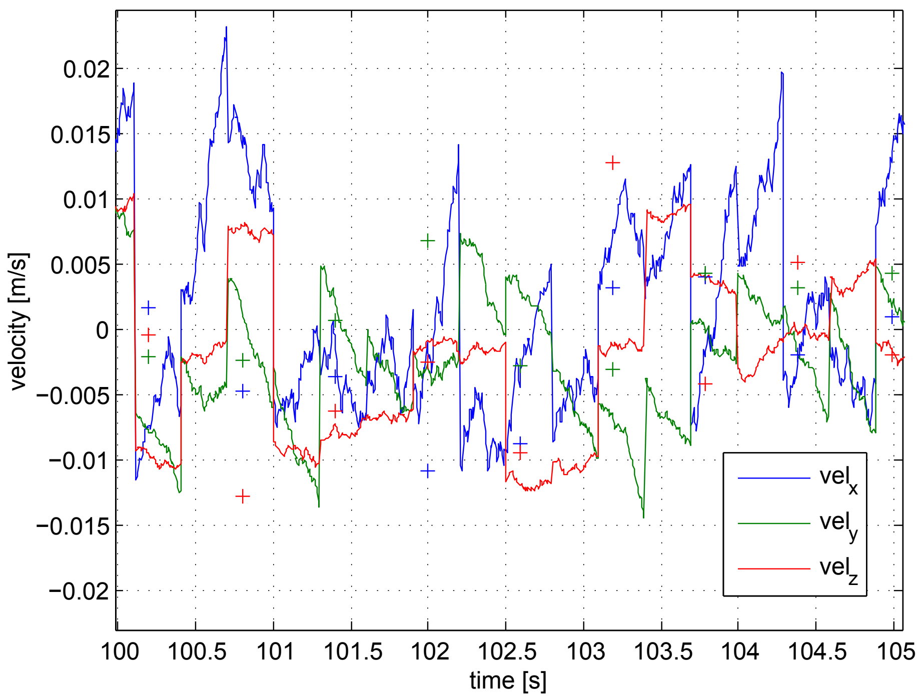

Fig. A.2: Detail of velocity error.

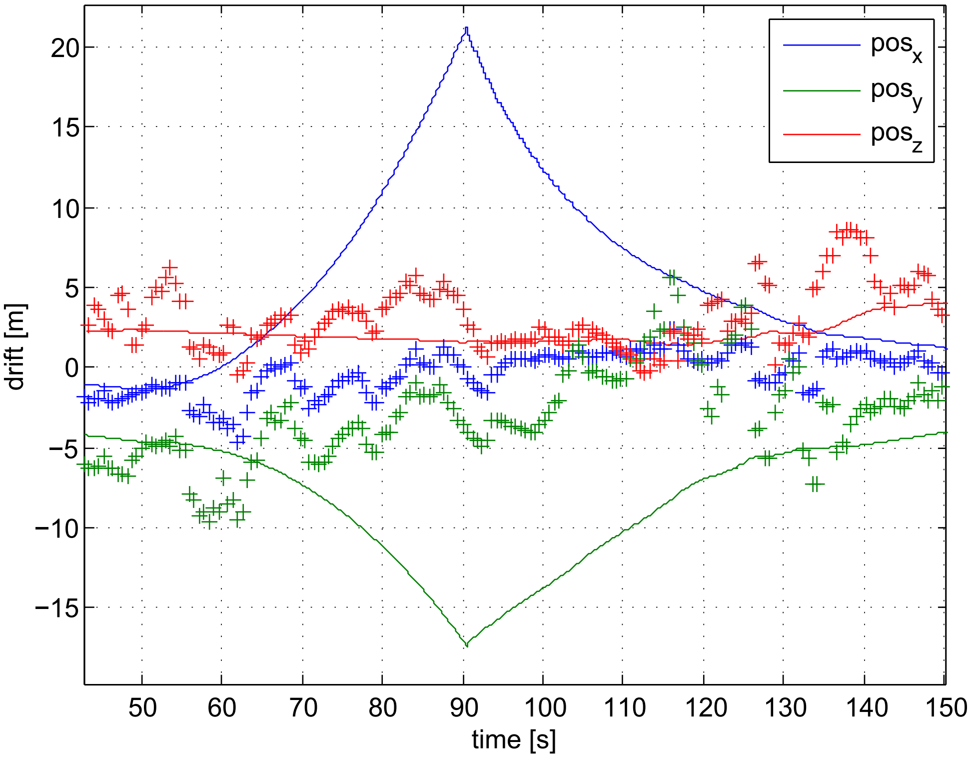

Fig. A.3: Detail of position error with simulated outgage.

B OPEN-SHOE PROJECT

Open-Shoe is a project under the permissive open source Creative Commons Attribution Licence. Hardware and software design is free to use. Implementation and modification into the new or existing system is allowed, even in commercial use. All source codes and information can be found at `www.openshoe.org`.

Part of this project was modified with real inertial measurement from navigation board 1.5. Final output is shown in figure B.1, where accelerometers and gyroscopes errors was estimated. Optionally, biases, scale factor, or both may be estimated (in that case, only biases are estimated.) Kalman filter with ZUPT (Zero Velocity Update) is aided into inertial navigation system.

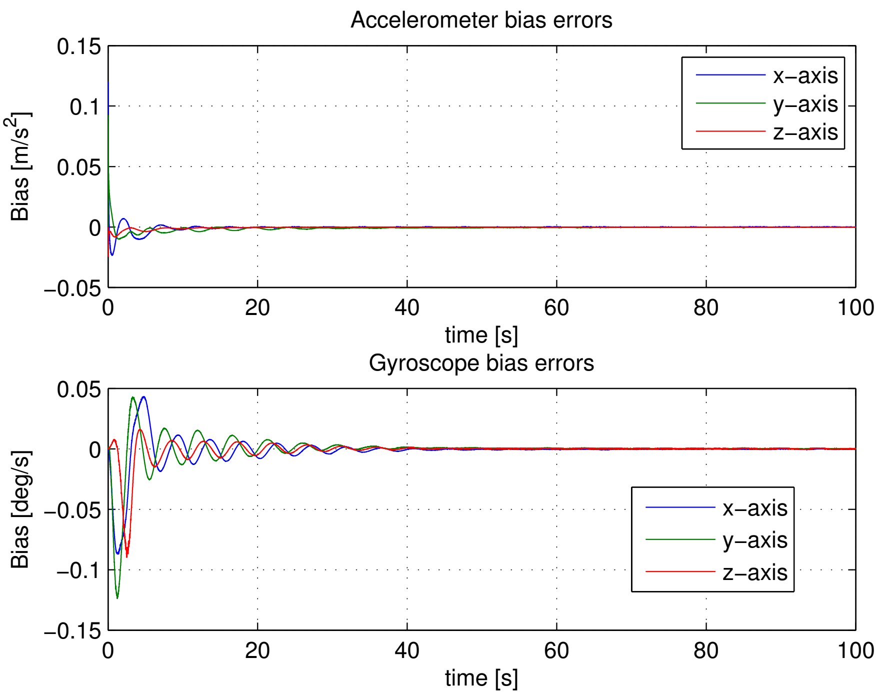

Fig. B.1: ZUPT biases error correction.

C CONTENT OF ATTACHED CD

- Source codes of MATLAB(R2014a) files.
- Source codes of LATEX files.