Czech University of Life Sciences Prague

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Bachelor Thesis

Multi-criteria decision making for personal investments

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Faculty of Economics and Management

BACHELOR THESIS ASSIGNMENT

Olha Tsipoviaz

Economics
and
Management

Thesis title

Multi-criteria decision making for personal investments

Objectives of thesis

The main objective of the thesis is to determine the best real estate for personal investment based on the criteria provided, using the methods of multi-criteria analysis.

Methodology

This work consists of two parts: theoretical and practical. The first part of the work will consist of a theoretical description of the methods and their comparison. The practical part is dedicated to the selection of the efficient real estate using the methods described in the theoretical part. The application of the selected methods will consist in identifying alternatives and setting criteria (by which the best property option will be determined)

The proposed extent of the thesis

30-40

Keywords

Multiple criteria decision making, TOPSIS, weighted sum approach, Saaty's method ;

Recommended information sources

Köksalan, M., Wallenius, J. and Zionts, S., 2011. Multiple criteria decision making: from early history to the 21st century. Hackensack, N.J.: World scientific.

Parreiras, R., Pedrycz, W. And Ekel, P. Fuzzy multicriteria decision-making : models, methods and applications..

Tzeng, G., Wang, H. F. And Wen, P. Multiple criteria decision making. New York: Springer, 1995. ISBN 3-540-94297-1.

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Declaration

I declare that I have worked on my bachelor thesis titled " Multi-criteria decision making for personal investments" by myself and I have used only the sources mentioned at the end of the thesis. As the author of the bachelor thesis, I declare that the thesis does not break any copyrights.

In Prague on 15.03.2023　　　　　＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿

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Multi-criteria decision making for personal investments

Abstract

This bachelor thesis devotes to the application of multi-criteria decision making for personal investment in residential real estate.

The theoretical part presents the framework for multi-criteria decision making (MCDM), including a description of several methods and all the necessary elements of the multi-criteria analysis model.

The practical part enables the determination of the characteristics of the desired demand and the selection of the best alternative from the options presented. The chosen method for identifying the most optimal solution is TOPSIS. Weight and rating analysis of the individual criteria are done using AHP comparison matrix. After performing all the calculations, the alternative with the greatest relative closeness value is selected as the most desirable solution.

Keywords: Multiple criteria decision-making, TOPSIS, weighted sum approach, Saatys method

Vícekriteriální rozhodování pro osobní investice

Abstrakt

Tato bakalářská práce se věnuje aplikaci vícekriteriálního rozhodování pro osobní investice do rezidenčních nemovitostí.

Teoretická část představuje rámec pro vícekriteriální rozhodování (MCDM), včetně popisu několika metod a všech potřebných prvků modelu vícekriteriální analýzy.

Praktická část umožňuje určení charakteristik požadované poptávky a výběr nejlepší alternativy z nabízených možností. Zvolenou metodou pro identifikaci nejoptimálnějšího řešení je TOPSIS. Váhová a hodnotící analýza jednotlivých kritérií se provádí pomocí AHP srovnávací matice. Po provedení všech výpočtů je jako nejžádoucnější řešení vybrána alternativa s největší relativní blízkostí.

Klíčová slova: Vícekriteriální rozhodování, TOPSIS, metoda váženého součtu, Saatyho metoda

Table of content

1 Introduction

The history of decision science reports significant advances in operations research, behavioral scientists, and mathematicians in structuring and solving complex problems. The decision maker is expected to examine a set of potential and possible alternatives and determine the optimal or most preferred one. Decision makers should take an intuitive approach when they are dealing with problems with a single criterion. This is quite simple, since you need to choose an alternative with a very high preference. The decision-making process becomes more complex when it requires the evaluation of many criteria that are conflicting in nature and have different weights. This requires the development of methods that can take into account the trade-offs between criteria and alternative and real problems and concerns of decision makers. The problems associated with evaluating multiple criteria and alternatives are usually considered as part of the multi-criteria decision making (MCDM) domain.

The domain of decision making involves three kinds of analysis:

1. Descriptive analysis: it actively involves the decision maker to draw conclusions from a graphical or tabular presentation of data/information.

2. Prescriptive Analysis: It uses techniques to enable decision makers to improve analysis.

3. Normative Analysis: It deals with problems that broadly require the participation of the decision maker in problem solving through the evaluation of a set of arguments trying to present a right and wrong position regarding the situation. It requires cross-examination of opinions and reaching consensus in making a specific decision. (Thakkar, 2021).

In recent decades, multi-criteria decision making (MCDM) method has become an important subject in many areas for decision-making from social life to scientific research, from engineering to economics. In the context of personal investments, MCDM can be used to help an individual weigh and balance different factors that may influence their investment decisions, such as risk vs. return, short-term vs. long-term goals, and diversification.

2 Objectives and Methodology

2.1 Objectives

The main objective of the thesis is to determine the best real estate for personal investment based on the criteria provided, using the methods of multi-criteria analysis.

2.2 Methodology

The research focuses on identifying the various criteria that investors typically consider when evaluating real estate investment opportunities, such as property type and size, price and rental income. It also investigates the different multi-criteria decision-making methods that investors can employ to analyze and prioritize these criteria, including the Analytic Hierarchy Process (AHP), the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) and Weighted Sum Method

A mixed-methods research design is employed, combining both quantitative and qualitative approaches. The quantitative research involves TOPSIS method to compare and evaluate different investment opportunities based on their criteria and preferences. The qualitative research includes a review of the multi-criteria decision-making literature.
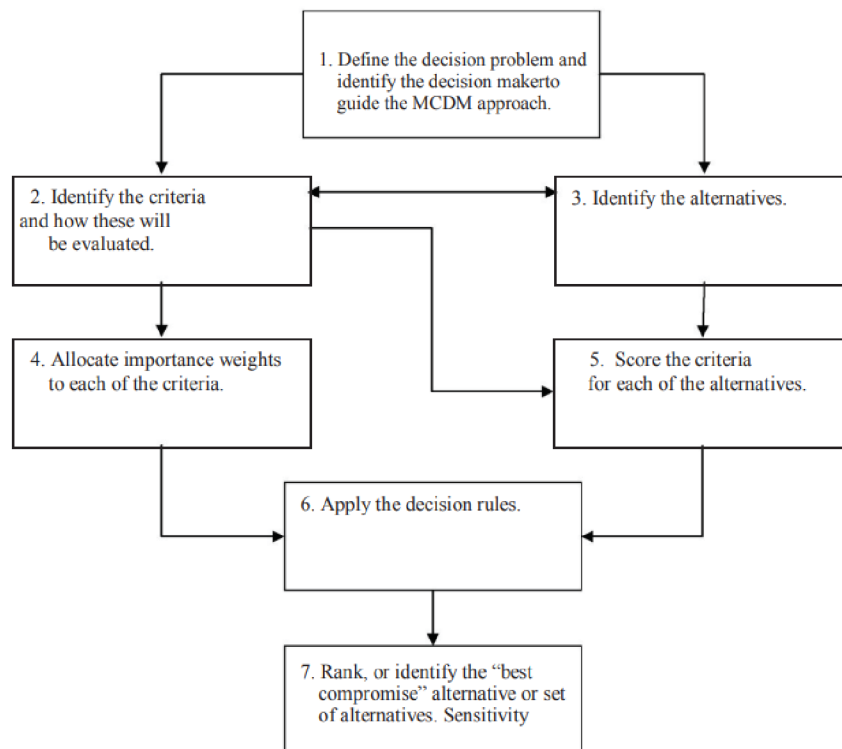
3 Literature review

3.1 Decision making process

According to Robert Harris, decision making is: the process of identifying and choosing alternatives based on values and preferences of decision maker. Making a decision implies that there are alternative choices to be considered, and in such a case we want not only to identify as many of these alternatives as possible but to choose the one that best fits with our goals, objectives, desires, values, and so on." (Harris, 1998).

Decision making is a process of selecting a single alternative from a set of alternatives in a systematic and logical way. The basic step by step process (which are shown in the figure 1) involved in decision making is called a decision-making process (Fülöp, 2000).

Figure 1. Decision Making Process


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┌──────────────────────────────┐
│ 1. Define the decision problem and │
│    identify the decision maker to  │
│    guide the MCDM approach.        │
└──────────────────────────────┘

┌──────────────────────┐        ┌──────────────────────┐
│ 2. Identify the criteria │      │ 3. Identify the alternatives. │
│    and how these will    │      └──────────────────────┘
│    be evaluated.         │
└──────────────────────┘

┌──────────────────────┐        ┌──────────────────────┐
│ 4. Allocate importance weights │  │ 5. Score the criteria    │
│    to each of the criteria.    │  │    for each of the alternatives. │
└──────────────────────┘        └──────────────────────┘

           ┌──────────────────────┐
           │ 6. Apply the decision rules. │
           └──────────────────────┘

           ┌──────────────────────┐
           │ 7. Rank, or identify the "best │
           │    compromise" alternative or set │
           │    of alternatives. Sensitivity │
           └──────────────────────┘
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Source: (Fülöp, 2000)

Below described step-by-step decision making process.

1. Problem definition: Identifying and clearly defining the problem to be solved or decision to be made.

2. Identify the criteria: Identifying and defining of the criteria by which alternatives will be distinguished should be based on objectives. A decision problem containing many criteria is especially useful for deriving better alternatives.

3. Identify alternatives: A large part of the decision-making process involves the analysis of a finite set of alternatives. All available alternatives are compared to the selected aspects, and then any alternatives that do not match the aspects are excluded until there is no one alternative.

4. Allocate importance weights to each criterion: The weights to the criteria are assigned accordingly and implement pairwise comparison.

5. Score the criteria for each of the alternative: The matrix is formed by evaluating the criteria for each alternative, and this matrix is applied to the decision rules.

6. Apply the decision rules: Based on the criterion weights and criterion scores of each alternative, decision rules must be applied to determine the potential and suitable alternative.

7. Identify the best alternative: A suitable alternative is determined in the last step of the decision model through evaluation, and thus the goal is achieved.

Based on type of decision problem, decision making (DM) is divided into two types of single criterion and multi criteria decision making. Single criterion decision making models - the type when choosing an alternative, the overall goal is completely dependent on single criteria.

3.2   Multiple criteria decision making

Multicriteria analysis is generally defined as "a decisionaid and a mathematical tool allowing the comparison of different alternatives or scenarios according to many criteria, often conflicting, in order to guide the decision maker toward a judicious choice" (Roy, 1996).

According to the book (Köksalan, 2011) the field of Multiple Criteria Decision Making (MCDM) can be viewed as both old and new, depending on perspective. It is old because throughout history, individuals have always had to make trade-offs between objectives when making decisions. One of the earliest recorded discussions on this topic can be attributed to Benjamin Franklin an American statesman, who used a method of weighing trade-offs when making important decisions. Today, MCDM continues to be an active area of research for many scholars. It can be framed as an evaluation problem, where the decision maker must

select from a finite set of discrete alternatives, or as a design problem, where mathematical models are employed to describe the decision alternatives.

A typical example of a decision-making situation assisted by MCDA methods: Prioritizing local or central government spending, ranking researchers or students for research grants or scholarships, choosing a new home, car or smartphone, etc.

MCDA includes the following four key components (Anon., n.d.):

1) the alternatives to be assessed.

2) the criteria by which the alternatives are evaluated.

3) the weight criteria which measure the relative importance of the criteria.

4) the decision makers whose preferences should be represented

All multicriteria decision making problems share the following common characteristics (Witold Pedrycz, 2010) :

- o multiple criteria: each issue has multiple criteria, which can be targets or attributes
- o conflicting criteria: several criteria contradict each other
- o incommensurable units: criteria can have different units of measurement
- o design: solutions to multicriteria decision-making problems are either to develop the best alternatives or to choose the best of pre-specified finite alternatives.

Criteria - form the standard of judgment or the rules of eligibility testing.

Objectives are a reflection of the desire to make decisions and indicate the direction on which DMs want to focus.

Goals create objectives. Goals are objects desired by the DMs and expressed in terms of a particular state in space and time. So while the goals provide direction, objectives measure how you should follow that direction.

Attributes are the qualities or characteristics ascribed to alternatives.

According to (Witold Pedrycz, 2010) multiple criteria decision-making problems can be divided into two large classes: multi-objective decision-making and multi-attribute decision-making. Multi-attribute decision-making problems involve selecting the "best" alternative from a set of preselected alternatives. Multi-attribute decision making is concerned with

determining preferences among the available alternatives, which are characterized by multiple and conflicting attributes. Multi-objective decision-making problems is a continuous type of multi-criteria decision-making, the main characteristics of which are that decision makers must solve several problems, while these goals are incommensurable and contradict each other.

The main difference between these two classes is that multiobjective decision-making concentrates on continuous solution spaces, while multiattribute decision-making focuses on problems with discrete solution spaces.

There are many methods that can be used to solve problems. The choice of the best MCDA method varies depending on the source and nature of the information used to inform decision making, as well as the model that decision makers believe is appropriate for their abilities. Each MCDA method has its own calculation method by which alternatives are queued, and using specific methods with the same input will not necessarily lead to the same end result. Methods can be selected according to the type of result. Methods such as AHP, MULTIMOORA, MAUT, Weighted Sum Method, Weighted Product Method can be used if the result is required to compare values. AHP, TOPSIS, and VIKOR, COPRAS, STEP and other methods can be used to reach the defined goal and to find the best alternative from the provided options (Alessio Ishizaka, 2013).

There are two main types of situations that require a multi-criteria approach (Witold Pedrycz, 2010):

- Problems, the consequences of the solution of which cannot be assessed by one criterion. These problems are related to the analysis of models that include both economic and physical indicators, as well as the need to take into account indicators that are difficult to assess.
- Problems that can be solved based on one criterion. However, if the uncertainty of information does not allow obtaining an unambiguous solution, then these problems can be reduced to multi-criteria decision-making by applying additional criteria, including those of a qualitative nature.

Bernard Roy (Roy, 1996) identifies four different categories of problems for which MCDA can be useful:

- The choice problematique: to make a simple choice from a variety of alternatives.

- The sorting problematique: to sort activities into classes or categories such as "definitely acceptable", "possibly acceptable but needs more information", and "definitely unacceptable".
- The ranking problematique: to place actions in some form of preference ordering.
- The description problematique: to describe actions and their consequences in a formalized and systematic way so that decision makers can evaluate these actions.

3.3 Saaty's Analytic Hierarchy Process as a decision-making method

Statistics show that the most used method worldwide is AHP. Probably the reason is that it is easy to understand and learn, and it is not mathematically complex (Nolberto Munier, 2021). The Analytic Hierarchy Process (AHP) was proposed by Saaty. AHP is a multi-criteria decision-making method that allows decision-makers to prioritize and evaluate different options based on a set of criteria. AHP involves breaking down a complex decision problem into a hierarchical structure of criteria and sub-criteria, and then determining the relative importance of each criterion through pairwise comparisons (Tzeng, 1995).

Saaty developed the following steps for applying the AHP (Saaty, 2008):

1. Define the problem and determine its goal. The problem for analysis is selected from all those that are considered important or difficult enough for analysis.

2. Structure the decision hierarchy. This structure is built "from the top with the goal of the decision, then the objectives from a broad perspective, through the intermediate levels (criteria on which subsequent elements depend) to the lowest level (which usually contains a list of alternatives)" (Saaty, 2008). It is necessary to build the model in such a way that it is possible to identify criteria and alternatives that are really relevant. At this stage, decision makers should rule out alternatives that are considered unfeasible (R. Russo, 2015).

3. Construct a set of pairwise comparison matrices (size n×n, where n denotes the number of criteria being compared to a specific goal). "Each element at the top level is used to compare the elements at the level directly below it" (Saaty, 2008).

In order to make comparisons, we need a scale of numbers that shows how many times one element is more important or dominates another element in terms of the criterion or property by which they are compared. Table 1 shows the scale (Saaty, 2008).

Table 1. The fundamental scale of absolute numbers

Intensity of importance	Definition	Explanation
1	Equal Importance	Two activities contribute equally to the objective
3	Moderate importance	Experience and judgements slightly favour one activity over another
5	Strong importance	Experience and judgements strongly favour one activity over another
7	Very strong	An activity is favoured very strongly over another
9	Extreme importance	The evidence favouring one activity over another is of the highest possible order of affirmation

Source: (Saaty, 2008)

After comparison expert and writes the magnitude of the preference of the i-th criterion relative to the j-th criterion into the Saaty matrix S = (s$_{ij}$) (Šubrt, 2015):

$$S = \begin{pmatrix} 1 & S_{12} & \cdots & S_{1n} \\ 1/S_{12} & 1 & \cdots & S_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ 1/S_{1n} & 1/S_{2n} & \cdots & 1 \end{pmatrix} \qquad (1)$$

The elements of this matrix are usually not perfectly consistent, e.i that $S_{hj} = S_{hi} \times S_{ij}$ does not apply for all h, i, j = 1, 2, ... n.

4. Calculate the relative weight of the elements for each level using the following steps: add the value of the columns to normalize the matrix; sum the rows in the normalized matrix to obtain the relative priority of the criteria; evaluate the consistency of the matrix; for each criterion, preliminary steps must be performed; calculate the values of each alternative for each criterion included in one matrix; add the values of each alternative to get the final value. The best alternative is the one with the highest value (R. Russo, 2015)

The matrix S is normalized to matrix N in order to perform a consistency check. The N matrix would look like the following:

$$N = \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{bmatrix} \qquad (2)$$

where $w_{ij} = s_{ij}$ (Sweta Bhattacharya, 2017)

Afterwards,

$$w_{ij} = \frac{s_{ij}}{\sum_{i=1}^{n} s_{ij}}$$ (3)

Where denominator is equal to sum of all the columns.

The relative weight of each row is computed by dividing the sum of the values of each row by n. Therefore, the weight of "i" can be calculated as (Sweta Bhattacharya, 2017):

$$i = w_{ij} = \frac{\sum_{i=1}^{n} w_{ij}}{n}$$ (4)

The matrix S is considered as consistent if S× w = n × w and the equation is considered as an Eigen value problem wherein the largest Eigen value is $\lambda_{max} \geq n$. The consistency ratio can is given by (Sweta Bhattacharya, 2017):

$$CR = \frac{CI}{RI} = \frac{Consistency\ index}{Relative\ index}$$ (5)

$$CI = \frac{\lambda_{max} - n}{n-1}$$ (6)

$$RI = \frac{1,98\ (n-2)}{n}$$ (7)

The value of CR determines the consistency of the judgement. If the CR < 0.10 then the matrix is of an acceptable consistency, otherwise revision of the a$_{ij}$ values is recommended (Sweta Bhattacharya, 2017). Cases where Saaty's matrix is inconsistent are very frequent, especially for larger tasks. The inconsistency may be due to an error in entering the weight ratio estimates, when the expert did not perform any check on his estimates (Šubrt, 2015).

Saaty proposed several numerical ways by which it is possible to estimate the weights. The procedure for calculating weights is most often used as standardized geometric mean of the rows of the Saaty matrix (least squares logarithmic method). We can calculate standardized geometric mean using following equation (Šubrt, 2015):

$$b_i = \sqrt[n]{\prod_{j=1}^{n} S_{ij}}$$ (8)

The weights are then calculated by normalizing the b$_i$ values (Šubrt, 2015):

$$v_i = \frac{b_i}{\sum_{i=1}^{n} b_i} \tag{9}$$

5. Review the final decision. This step is necessary to verify that the results of the AHP application are consistent with expectations, and if deficiencies are identified, a review of the previous process is necessary. Model if needed needs to be supplemented to include elements or criteria not previously considered (R. Russo, 2015).

6. Decision documentation. The final stage includes full documentation of all of the above stages in order to capture and analyze the process (Sweta Bhattacharya, 2017).

3.4 TOPSIS

Technique for Order Performance by Similarity to Ideal Solution (TOPSIS) was proposed by Hwang and Yoon to determine the best alternative based on the concepts of the compromise solution and it is the most known technique for solving MCDM problems (Gwo-Hshiung Tzeng, 2011). The fundamental idea of TOPSIS is that the chosen alternative should have the shortest distance to a positive ideal solution (PIS) and the furthest distance to the negative ideal solution – NIS. (Alessio Ishizaka, 2013).

The procedure of TOPSIS method is described by the following steps (Alessio Ishizaka, 2013):

The performances of n alternatives a with respect to m criteria i are collectes in decision matrix $X = (x_{ia})$, where i = 1, ..., m and a = 1, ..., n.

Step 1. The performance of different criteria are normalized in order to be able to compare the measure on different units. The calculation of normalized ratings can be done by 2 forms.

Form 1 (distributive normalization)

The performances are divided by the square root of the sum of each squared element in a column

$$r_{ia} = \frac{x_{ia}}{\sqrt{\sum_{a=1}^{n} x_{ia}^2}} \tag{10}$$

From 2 (ideal normalization)

- Maximizing criterion: the performances are divided by the highest value in each column and $u_a^+ = \max\ (x_{ai})$

$$r_{ai} = \frac{x_{ai}}{u_a^+} \tag{11}$$

- Minimizing criterion: each performance is divided by the lowest value in each column and $u_a^- = \min\ (x_{ai})$

$$r_{ai} = \frac{x_{ai}}{u_a^-} \tag{12}$$

Step 2. Computing the weighted normalized preference values. In this step decision-maker represents each of the feasible alternalives, as a weighted normalized vector in a following way

$$v_{ai} = w_i * r_{ai} \tag{13}$$

Step 3. Calculation distances to an ideal and anti-ideal virtual action using weighted scores. The three ways in which these virtual actions can be defined are described below. The TOPSIS methods equate the ideal alternative to the positive ideal solution (PIS) and the anti-ideal – to the negative ideal solution (NIS) (Babak Zolghadr-Asli, 2021).

a) Collecting the best and worts scores on each criterion of the normalized decision matrix

For the ideal action:

$$A^+ = (v_1^+, \dots, v_m^+) \tag{14}$$

$$v_i^+ = max_a(v_{ai}) \tag{15}$$

For the anti-ideal action:

$$A^- = (v_1^-, \dots, v_m^-) \tag{16}$$

$$v_i^- = min_a(v_{ai}) \tag{17}$$

b) Assuming an absolute ideal and anti-ideal point, $A^+ = (1, \dots, 1)$ and $A^- = (0, \dots, 0)$

c) The ideal and anti-ideal points are determined by the decision maker, but these points must lie between the ideal and anti-ideal points calculated with the two other methods described above.

Step 4. Calculation the distance for each action to the ideal action,

$$d_a^+ = \sqrt{\sum_i(v_i^* - v_{ai})^2}, \text{ a} = 1, ..., \text{m} \tag{18}$$

And the anti-idel action

$$d_a^- = \sqrt{\sum_i(v_i^- - v_{ai})^2}, \text{ a} = 1, ..., \text{m} \tag{19}$$

Step 5. Computing the relative closeness to the ideal solution.

$$C_a = \frac{d_a^-}{d_a^+ + d_a^-} \tag{20}$$

$C_a \in [0,1]$, $C_a = 1$ alternative is the ideal solution, while alternative is the anti-ideal solution if $C_a = 0$ (Alessio Ishizaka, 2013).

Step 6. Ranking the alternatives. The alternative with the greatest relative closeness value is the most desirable solution and the alternative with the lowest value represents the least desirable solution (Babak Zolghadr-Asli, 2021).

3.5 The Weighted Sum Method

"The weighted sum method (WSM), also called the simple additive weighting (SAW) method, is the best known and simplest MADM method for evaluating a set of alternatives against a set of decision criteria. The main logic of WSM is to obtain a weighted sum of the performance values of the common attributes of each alternative" (Babak Zolghadr-Asli, 2021).

It is based on the principle of utility maximization, but assumes only a linear utility function.

To apply weighted sum approach to MCDM problems, a fundamental requirement is to aggregate criteria such that an overall decision function is formed (represented as a scalar value). However, this process highlights the need to overcome incommensurability of criteria, due to which different criteria cannot be combined into a single decision function. Therefore, it is necessary to convert all criteria to a unit-less, uniform scale. This is done by normalizing each criterion such that the value of the criterion lies in a $0 - 1$ range. Then,

weights are assigned to each normalized criterion according to the desire of the decision maker. Finally, all weighted values of criteria are added. (Shafiqur Rehman, 2017).

The procedure for applying the weighted sum method is below:

Step 1. Defining the decision-making problem. In this step the decision-maker determine the elements ideal of the decision-matrix: H = $(H_1, H_2, \ldots, H_n)$ and negative-ideal alternatives D = $(D_1, D_2, \ldots, D_n)$.

Step 2. Creation of the normalized matrix R = (r_{ij}), the elements of which are obtained from the criterion matrix Y using the transformation formula:

$$r_{ij} = \frac{Y_{ij} - D_j}{H_j - D_j} \tag{21}$$

Where,

Y_{ij} – the actual value of the original matrix

D_j – the negative-ideal alternative for j-th criteria

H_j – the ideal alternative for j-th criteria

Step 3. Aggregating the preference of alternatives in following way

$$u\,(a_i) = \sum_{j=1}^{k} v_j r_{ij} \tag{22}$$

Where,

$v = (v_1, \ldots, v_k)$ – is a vector of criteria weights

$u\,(a_i)$ – are partial utility functions of individual criteria

Step 4. The alternatives are arranged according to the decreasing values of the utility function. An alternative which has reached the maximum utility value is selected as „best".

The weighted sum method is a special case of the utility function method. The utility function value lie in the interval ⟨0,1⟩ and the more advantageous the variant, the higher the value of the utility function (Jan Získal, 2010).

4 Practical part

This chapter will demonstrate in practice the theory of problem solving, in particular, the choice of residential real estate for personal investment for the person making this decision – for my father. Filters that were used for the selection of apartments:

1) Type: two-bedroom apartments (2+kk)
2) The state of the object: only buildings under construction
3) Location: Prague 5 and Prague 6 (according to personal preference)
4) Price: up to 8,000,000 CZK

4.1 Alternatives

1. Two-bedroom apartment in Hábova street, Praha – Stodůlky (a_1)

Figure 2. Apartment in Habova street (a_1)

Source: sreality.cz

2. Two-bedroom apartment in Drnovská street, Praha 6 – Ruzyně (a_2)

Figure 3. Apartment in Drnovska street (a$_2$)

Source: sreality.cz

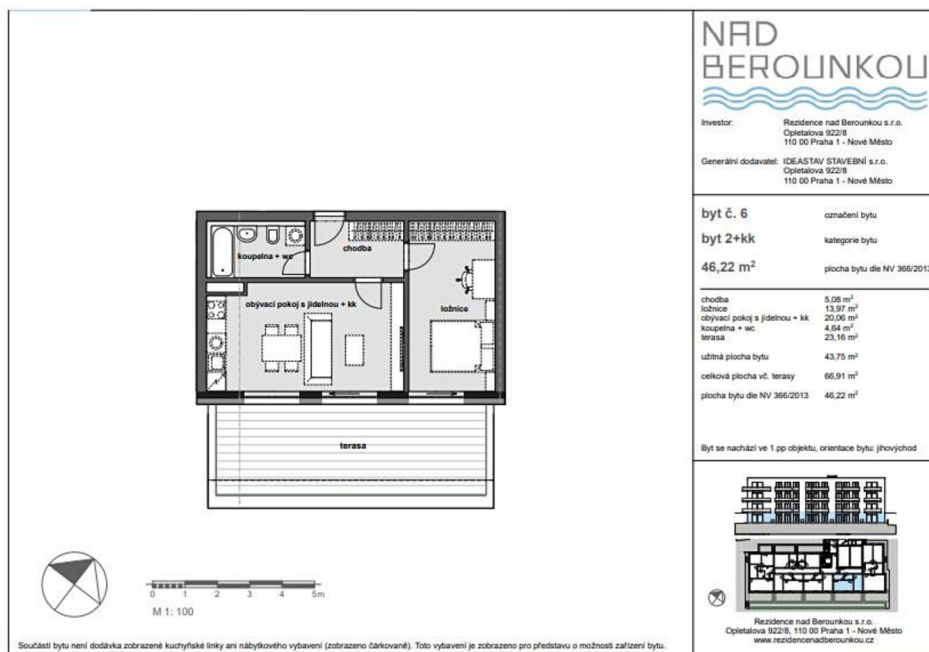3. Two-bedroom apartment in Hofbauerova street, Praha 6 – Řepy (a$_3$)

Figure 4. Apartment in Hofbauerova street, (a$_3$)

Source: sreality.cz

4. Two-bedroom apartment in Karlická street, Praha 5 – Radotín (a$_4$)
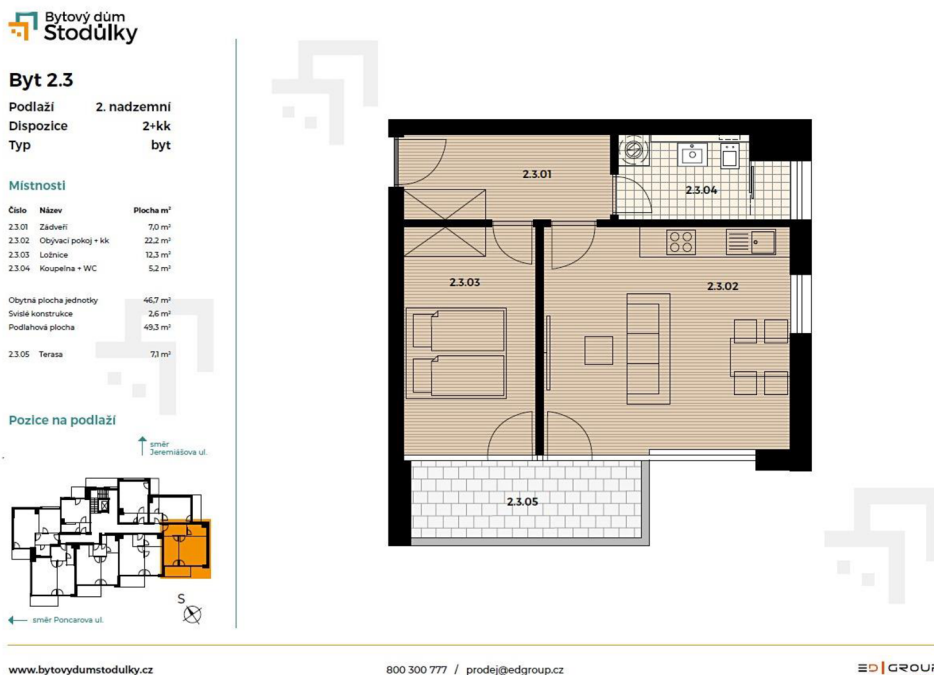
Figure 5. Apartment in Karlicka street (a₄)

Source: sreality.cz

5. Two-bedroom apartment in Jeremiášova street, Praha – Stodůlky (a₅)
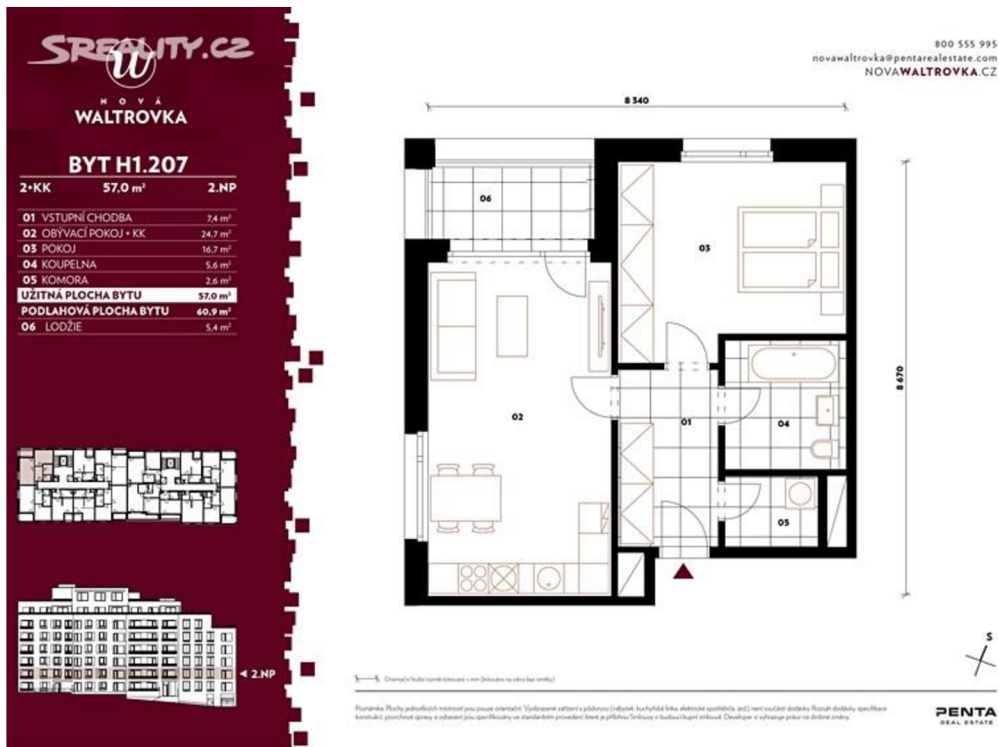
Figure 6. Apartment in Jeremiasova street (a₅)

Source: sreality.cz

6. Two-bedroom apartment in Radlická street, Praha 5 (a_6)jm

Figure 7. Apartment in Radlicka street (a_6)

Source: sreality.cz

4.2 Criteria

All criteria will be reffered as $c_1 - c_7$ and will be used in the following multi-criteria analysis tables. Description of the criteria is provided below:

1. Price (c_1)

Purchase price is one of the most important criteria. The price will be in CZK. This is minimization criterion.

2. Square meters (c_2)

Refers to the total usable floor area. Maximization criterion.

3. Floor (c_3)

The best floor in an apartment can depend on a variety of factors, including personal preferences, building amenities, and location. As apartments on higher floors offer more privacy and pleasant views so will take this criterion as a maximization

4. Energy label (c_4)

In Czech language - průkaz energetické náročnosti budovy (PENB).With the Building Energy Label, you can better assess the energy efficiency of the property you buy and get an idea of what your future energy costs will be. This is minimization criterion.

From January 1, 2022 PENB only allows the construction of buildings, which must meet at least category B - a very economical building. To make comparison as easy as possible, 7 energy efficiency classes have been created (Anon., 2023):

A - Extremely economical
B - Very economical
C - Economical
D - Less economical
E - Inefficient
F - Very wasteful
G - Extremely wasteful

5. Balcony (c_5)

Since the presence of a balcony can increase the value of an apartment and increase its selling price or potential rental income, this will be a maximization criterion. Terraces will also be included in this criterion.

6. Subway (c_6)

The proximity of the subway means a more comfortable life, as it allows you to quickly get to work or study. With the help of the metro, a person minimizes the time spent in traffic jams and can easily get to any part of the city. Also within walking distance from the subway is always a developed infrastructure. Housing near the metro station is highly liquid - such apartment can be sold or rented out without any problems. Therefore, this is maximization criterion.

7. Rental price (c_7)

The rental price is selected according to a similar apartment. Since we are interested in returning the invested money as quickly as possible, rental price is the maximization criterion. This criterion expressed in Czech crown.

The alternatives and their values are shown in the table below (table 2):

Table 2. Initial data

	Price	m^2	Floor	Energy Label	Balcony	Subway	Rental price
a1	7617950	44	6	B	Balcony	Yes	18500
a2	7412000	53	3	B	Balcony	No	20000
a3	5990000	49	1	B	No	No	16000
a4	6258638	46	1	A	Terrace	No	18000
a5	6969000	47	2	A	Terrace	Yes	19000
a6	7916500	57	2	B	Balcony	No	22000

Source: own processing

To further carry out pairwise comparison and use the TOPSIS method for calculation, it is necessary to convert the qualitative assessment to quantitative values. The word-described criteria c_6 will be transformed into binary form: yes – 1, no – 0.

Transformation of criteria c_4 and c_5 is as follows:

- Energy label: A =1 and B=2;

- Balcony: terrace = 2, balcony =1, nothing = 0;

Table 3. Quantitative evaluation of initial data

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
a_1	7617950	44	6	2	1	1	18500
a_2	7412000	53	3	2	1	0	20000
a_3	5990000	49	1	2	0	0	16000
a_4	6258638	46	1	1	2	0	18000
a_5	6969000	47	2	1	2	1	19000
a_6	7916500	57	2	2	1	0	22000

Source: own processing

4.3   The Pairwise Comparison

The weights of the criteria were obtained through Saaty's method. Each element in the pairwise comparison matrix is selected based on an implicit scale used by the decision maker to show relative preference values for each given alternative.

The comparison matrix in table 4 gathers the pairwise comparisons between the criteria. All comparisons are positive. The comparisons on the main diagonal are 1 because a criterion is compared with itself. The matrix is recirpocal because the upper triangle is the reverse of the lower triangle.

Table 4. Comparison matrix

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
c_1	1,00	5,00	7,00	5,00	7,00	5,00	1,00
c_2	0,20	1,00	5,00	3,00	5,00	0,33	0,20
c_3	0,14	0,20	1,00	0,20	0,33	0,20	0,11
c_4	0,20	0,20	5,00	1,00	5,00	0,33	0,20
c_5	0,14	0,20	3,00	0,20	1,00	0,14	0,11
c_6	0,20	3,00	5,00	3,00	7,00	1,00	0,33
c_7	1,00	5,00	9,00	5,00	9,00	3,00	1,00

Source: own processing

Next, it is important to determine the Consistency Ratio. The CR using formulas 5,6 and 7 is equal to 0,095 which means that the Saaty's matrix is consistent and we can proceed to the calculations of geometric means (b_i). Then the found values will be used to calculate the criteria weights (v_i), which in turn are needed for the TOPSIS procedure. Geometric means and criteria weights are shown in table 5.
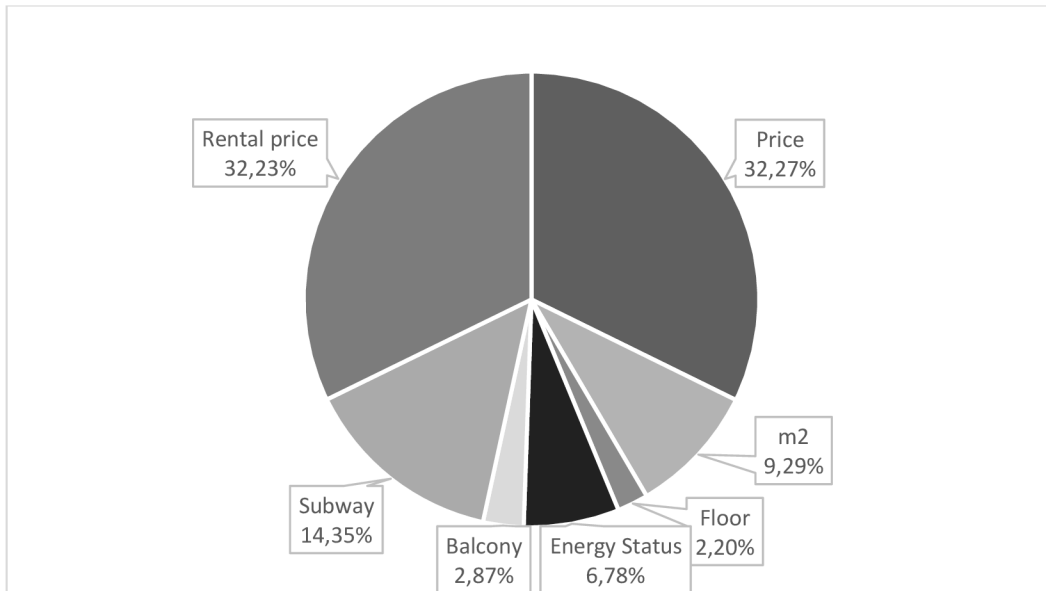
Table 5. Criteria weights

	b_i	v_i
c_1	3,47548	0,32273
c_2	1,00000	0,09286
c_3	0,23726	0,02203
c_4	0,73060	0,06784
c_5	0,30951	0,02874
c_6	1,54486	0,14345
c_7	3,47142	0,32235
c_8	3,47548	0,32273
c_9	1,00000	0,09286
c_{10}	0,23726	0,02203

Source: own calculation

The biggest value among all criteria is price of apartment (32.27%), the next is rental price– 32,23 % and subway - 14,35%. All criteria weights are shown in the pie chart (Figure 8).

Figure 8. Pie chart of criteria weights

Source: own processing

4.4 TOPSIS

Now, after determining the criteria weights, we will find the best alternative by applying the TOPSIS method. For subsequent calculations, we use the criteria weights obtained by the method of pairwise comparisons.

Table 6. Initial matrix for the TOPSIS method

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
a_1	7617950	44	6	2	1	1	18500
a_2	7412000	53	3	2	1	0	20000
a_3	5990000	49	1	2	0	0	16000
a_4	6258638	44	1	1	2	0	18000
a_5	6969000	47	2	1	2	1	19000
a_6	7916500	57	2	2	1	0	22000
weights	0,32273	0,09286	0,02203	0,06784	0,02874	0,14345	0,32235
criterion	min	max	max	min	max	max	max

Source: own calculation

Firstly, we should calculate normalized ratings using the formula (10). A normalized criterial matrix $R = (r_{ij})$ is provided below.

Table 7. Normalized criterial matrix R

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
a_1	0,4404	0,3649	0,8090	0,4714	0,3015	0,7071	0,3974
a_2	0,4285	0,4395	0,4045	0,4714	0,3015	0,0000	0,4296
a_3	0,3463	0,4064	0,1348	0,4714	0,0000	0,0000	0,3437
a_4	0,3618	0,3649	0,1348	0,2357	0,6030	0,0000	0,3866
a_5	0,4029	0,3898	0,2697	0,2357	0,6030	0,7071	0,4081
a_6	0,4576	0,4727	0,2697	0,4714	0,3015	0,0000	0,4726

Source: own calculation

In second step, the weighted normalized preference values will be calculated according to the formula (13). A normalized weighted matrix is represented below.

Table 8. Normalized weighted matrix

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
a_1	0,14212	0,03388	0,01782	0,03198	0,00867	0,10144	0,12810
a_2	0,13828	0,04081	0,00891	0,03198	0,00867	0,00000	0,13848
a_3	0,11175	0,03773	0,00297	0,03198	0,00000	0,00000	0,11079
a_4	0,11676	0,03388	0,00297	0,01599	0,01733	0,00000	0,12464
a_5	0,13001	0,03619	0,00594	0,01599	0,01733	0,10144	0,13156
a_6	0,14769	0,04389	0,00594	0,03198	0,00867	0,00000	0,15233

Source: own calculation

In the next steps we will calculate distances of all variants to an ideal (h) and anti-ideal (d) alternatives using weighted scores. It will be done by selecting the best and worts scores on each criterion of the normalized weighted matrix (which is shown in table 9)

Table 9. Ideal and anti-ideal alternatives

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
h	0,11175	0,04389	0,01782	0,01599	0,01733	0,10144	0,15233
d	0,14769	0,03388	0,00297	0,03198	0,00000	0,00000	0,11079

Source: own calculation

Now it is important to calculate the distance for each alternative to the ideal alternative (d_a^+) and the anti-idel one (d_a^-). Results are provided in table 10.

Table 10. Distances for individual alternatives to the ideal and anti-ideal alternatives

	d^+	d^-
a_1	1,2458	1,308098
a_2	0,8841	0,888977
a_3	0,6743	0,665505
a_4	0,7973	0,808726
a_5	1,0447	1,120917
a_6	0,8763	0,880143

Source: own calculation

Then we compute indicator c_i - the relative closeness to the ideal solution and provide results in table 11.

Table 11. Relative indicator c_i

	c_i
a_1	0,512189718
a_2	0,501363629
a_3	0,496717883
a_4	0,503543415
a_5	0,517598543
a_6	0,501086831

Source: own calculation

Finally, in order to select the efficient alternative, we rank all results from table 11 in descending order, the alternative with the greatest relative closeness value is the most desirable solution.

Table 12. Ranking the alternatives

	c_i
a_5	0,517598543
a_1	0,512189718
a_4	0,503543415
a_2	0,501363629
a_6	0,501086831
a_3	0,496717883

Source: own calculation

5 Discussion of Results

According to the table 12, in which the alternatives were ranked the most efficient alternative is apartment in Jeremiášova street, Praha – Stodůlky (a_5) with the c_i value - 0.5176. While the alternative a_5 has a lower price and a larger area, a_1 can also be considered as a solution with close contender index c_i = 0.5122. The third best alternative is apartment in Karlická street, Praha 5 – Radotín (a_4) with a general utility score of 0,5035. Alternatives a_5 and a_4 have a terrace, more space and better energy efficiency than alternative a_1. Nevertheless, the alternative a5 will be recommended as an solution, since it has almost the best parameters among the other proposed options.

5.1 Sensitivity analysis

To improve the solution, a sensitivity analysis of criterion weights has been included to assess the stability of the resulting ranks. Sensitivity analysis will show the change in the efficient alternative when the weight of the purchase price (criterion c_1) changes. Table 13 lists the original weight of c_1 (0,3227) and artificial weights, starting from 0,1 and incrementally increasing by 0,1 until reaching a final weight of 0,9. Changing the weight of one criterion will change the others, while ratio between them will remain the same.
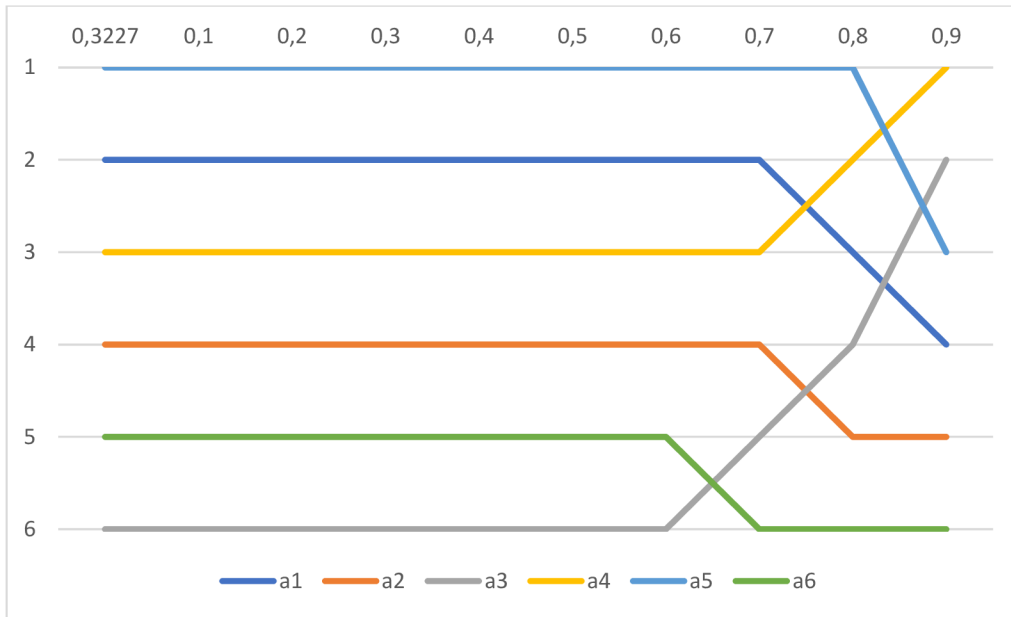
Table 13. Change in alternatives ranking

weight	0,3227	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9
rank										
a_1	2	2	2	2	2	2	2	2	3	4
a_2	4	4	4	4	4	4	4	4	5	5
a_3	6	6	6	6	6	6	6	5	4	2
a_4	3	3	3	3	3	3	3	3	2	1
a_5	1	1	1	1	1	1	1	1	1	3
a_6	5	5	5	5	5	5	5	6	6	6

Source: own calculation

The figure (figure 9) provided below illustrate the obtained.results from table 13.

Figure 9. Sensitivity analysis

Source: own processing

We can see from the figure that up to a weight of 0,6, the alternatives are in the same positions, and only from the original value of 0,3227, the alternative a_6 moves from 5th place to 6th, and after reaching a weight of 0,6, alternative a_3 begins to take a leading position, and this is quite understandable, because this alternative has the lowest purchase price. Also, alternatives a_2 and a_1, after reaching a weight of 0,7 lose their positions, at the same time, alternative a_4 moves from 3_{rd} to 2_{nd} place, and then even to 1_{st} place, which can also be explained - alternative a_4 is second in price after alternative a_3. So, TOPSIS analysis is more reliable for weights up to 0,6, when the contribution of the c_1 criterion weight increases by almost 30%, which indicates that we can be confident in the recommended solution.

6 Conclusion

In accordance with the tasks set, the presented work considers a multi-criteria analysis of alternatives, including a step-by-step description of one of the multi-criteria analysis methods - TOPSIS. The practical section focused on a specific example, taking into account the relevant information from the theoretical part.

The goal of this thesis was to identify the best real estate for a decision maker for investing money. To achieve this, the decision maker established 7 criteria and 6 proposals were selected according to 4 filters. Criteria weights were determined using the Saaty method. And after that, the calculation of the definition of an effective alternative was conducted using the TOPSIS method.

After calculation, apartment in Jeremiášova street, Praha – Stodůlky (alternative a_5) was selected as the recommended alternative based on the highest value of the indicator representing the distance of proposal a_5 from the negative ideal alternative.

Sensitivity analysis was performed to demonstrate how the price criterion affected the results of the MCDA model. The change in this particular criterion was chosen due to the fact that the c1- price criterion has one of the largest weights according to the results provided in Table 5.

The result showed that alternative a_5 is an effective alternative in terms of price weighting scales (0,3227 – 0,8). In the demonstrated graph (Figure 9), up to a weight of 0,7, the alternatives remain in the same ranked positions, which tells us that we can recommend alternative a_5 to the decision maker.

The findings suggest that multi-criteria decision-making methods can help investors make investment decisions. However, the effectiveness of these methods depends on the specific investment scenario and criteria that are most important to the investor.

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8 List of tables and figures

8.1 List of tables

8.2 List of figures