Czech University of Life Sciences Prague

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Bachelor Thesis

Application of mathematical model in decision support

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Faculty of Economics and Management

BACHELOR THESIS ASSIGNMENT

Bc. Daneliya Tleuf

Business Administration

Thesis title

Application of mathematical model in decision support

Objectives of thesis

Main goal of bachelor thesis is to select and apply method of operational research for real managerial decision making problem. The partial goals are systematic description of the decision situation and collection of the real data. The theoretical studies will describe chosen methods and will serve as a basis for the mathematical model construction.

Methodology

The first part of the metodology will be based on the study and analysis of literature. In the second part, we will choose the most suitable method based on the studied materials for the real problem and consider selected method in practice. According to the detail description of the decision situation the mathematical model will be constructed. The input data will be collected and the results calculated according to the selected methodology.The results will be interpreted and solution of the managerial problem will be proposed.

The proposed extent of the thesis

30-40 pages

Keywords

managerial decision making, mathematical model, operation research

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Recommended information sources

COLSON, Gerard, DE BRUYN, Christian. Models and Methods in Multiple Criteria Decision Making. Oxford: Paperback,1985. 254 p. ISBN 978-0-08-037938-8.

FUENTE, Angel. Mathematical Methods and Models for Economists. Cambridge: Cambridge University Press, 2000. 835 p. ISBN 0521585295.

Expected date of thesis defence

2017/18 WS – FEM (February 2018)

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Supervising department

Department of Systems Engineering

Electronic approval: 30. 10. 2017	Electronic approval: 1. 11. 2017
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Prague on 25. 02. 2018

Declaration

I declare that I have worked on my bachelor thesis titled "Application of mathematical model in decision support" by myself and I have used only the sources mentioned at the end of the thesis. As the author of the bachelor thesis, I declare that the thesis does not break copyrights of any their person.

In Prague on 30.11.2018 ______________________________

Acknowledgement

I would like to thank doc. Ing. Ludmila Dömeová, CSc, for valuable advice and support during my work on this bachelor thesis.

Application of mathematical model in decision support

Abstract

The bachelor thesis deals with a topic of the application of mathematical model in decision support. Main goal is to select and apply method of operational research for real company.

The theoretical part will describe the main concepts of multiple criteria decision analysis, scoring methods and compromising methods. In the practical part, calculations will be made, based on the methods, which were previously described, in the theoretical part. After receiving the results, they will be interpreted and the company will be offered beneficial solutions of the management problem.

Keywords: managerial decision-making, mathematical model, operation research

Aplikace matematického modelu v rozhodování

Abstrakt

Bakalářská práce se zabývá aplikací matematického modelu při rozhodování. Hlavním cílem je vybrat a aplikovat metodu operačního výzkumu pro vybranou firmu.

V teoretické části jsou popsány hlavní pojmy analýzy rozhodování s více kritérii, metody hodnocení a kompromisní metody. V praktické části jsou provedeny výpočty založené na metodách, které byly dříve popsány v teoretické části. Výsledky jsou nabídnuty společností jako prospěšné řešení problému řízení.

Klíčová slova: manažerské rozhodování, matematický model, operační výzkum

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1 Introduction

Decision making process is the most frequent process human is facing with in daily basis. It is well known that the human being is a decision maker by nature. Decision making is something people do without thought, without worry each and every day.

Making decisions is the process of distinguishing and selecting a choice among possible alternatives and then evaluating the consequences of that choice.

This process is a part of everyday life. Sometimes people are able to make decisions in short term period of time with possibility to correct it in future, such decisions might be less important or probably are simple enough to explain with basic language, and that will not ultimately require hours of reflection. Meanwhile, more complex decisions takes more time or might never happen.

To be sufficient on the daily basis it is necessary to identify the decision that an individual is facing. The problem might be simple enough or oppositly extremly hard. After analysis with a clear ideas in mind next step is research of all possible options, listing all of them and solutions that are available, no matter how extreme they may first appear. Now is the right time to select the best possible solution.

Mathematical models help solve the decision-making problems. There are many methods that could help you make the most appropriate solution.

2 Objectives and Methodology

2.1 Objectives

Main goal of bachelor thesis is to select and apply method of operational research for real managerial decision-making problem. The partial goals are systematic description of the decision situation and collection of the real data. The theoretical studies will describe chosen methods and will serve as a basis for the mathematical model construction.

2.2 Methodology

The bachelor thesis deals with multi-criteria decision making problems, which is divided into two main parts.

The first part of the methodology is based on the study and analysis of literature.

In this part describes methods of weighting criteria:

- Sequence Method
- Score method
- Fuller triangle
- Saatys method

and methods of choosing compromise variants:

- Conjunctive and disjunctive methods
- Simple additive weighting method
- TOPSIS

In the second part are applied the most suitable method based on the studied materials for the real problem and consider selected method in practice.

According to the detail description of the decision situation, the mathematical model is constructed. The input data are collected and the results calculated according to the selected methodology. The results are interpreted and solution of the managerial problem proposed.

3 Literature Review

3.1 Multiple Criteria Decision Making

We make many decisions every day. Decision is a choice of alternative from many existing alternatives. Sometimes it is a short-term decision, but sometimes you need to do long-term decisions, which may affect your life. It is necessary to take a more careful approach to the issue of choice. One quantitative indicator can characterize decision. If there is only one criterion, it is easily find the best solution. However, usually we have situation when we have more than one criterion. Using more than one criterion may create a conflict, because most of the time criteria are controversial. Multiple Criteria Decision Making (MCDM) deals with defining and solving decision making and planning problems involving multiple criteria. There is no unique optimal solution of problems and it is necessary to use the decision makers' subjective preferences to make decision. The purpose of multiple criteria decision making models is to eliminate ineffective alternatives, choose the most appropriate one or sort them out. [1]

Multiple criteria decision-making is divided into two categories:

- Multi Objective Decision Making (MODM) is a sub-discipline of MCDM that regards decision making problem with infinite and not countable number of alternatives, which are defined by restrictive conditions.
- Multi Attribute Decision Making (MADM) considers decision making problem with a limited number of predetermined alternatives, which represented according to individual criteria. Through this method, we can find a better alternative or set of alternatives.

3.2 Multiple criteria decision analysis

Multi-criteria analysis of variants belongs to a set of techniques for MCDM. As was mentioned, this model contains limited number of predetermined alternatives, which represented according to individual criteria, therefore the final number of *m* variants rated in the criteria. Objective of this model is to find the most appropriate solution that ranked as the best according to the given criteria. Evaluation of all variants allows to sort them. After evaluation of all alternatives according to the given criteria, we are able to construct decision matrix Y. [1]

$$Y = \begin{pmatrix} y_{11} & y_{12} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & y_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ y_{m1} & y_{m2} & \cdots & y_{mn} \end{pmatrix} \tag{3.1}$$

3.2.1 Types of alternatives

There are several types of alternatives:

- **Dominated alternative** is an alternative that has at least one best criterion among other alternatives and at the same time, none of the criteria is the worse among other variants.
- **Non-dominated alternative** is an alternative that usually do not dominate each other. This alternative may be better in some criteria and in some be worst among all the alternatives.
- **Ideal alternative** is the best alternative among all the criteria. Usually there is no such ideal alternative, because it would automatically be the most effective alternative. Other alternatives will be dominated and the ideal alternative will be the optimal.
- **Basal alternative** is the opposite of the ideal alternative. Non-ideal alternative is the worst among all the criteria. As well as the ideal alternative usually does not exist.
- **Optimal alternative** exists in situation when only one solution is acceptable.
- **Compromise alternative** is non-dominated alternative which is recommended as solution of the decision making problem. The choice of the best compromise alternative depends on how it is determined, and its distance from the basal alternative. In situation where decision making process has more non-dominated alternatives, compromise alternative is the best solution.

3.2.2 Types of criteria

Each alternative we consider beside the criterion. The value of the criteria can be qualitative or quantitative. In the decision-making model, it is very important to denote the criteria correctly. The criteria should be independent, consider all aspects of the complex selection and the number of criteria must not be very high, otherwise the model would be unclear. [3]

Depending on the quantifiable of the criterion, criteria divided into:

- **Quantitative criteria** are numerical values, which are objectively measurable.
- **Qualitative criteria** are values, which cannot be objectively measured. Usually values are estimated by decision makers and called subjective criteria. Criteria can be measured, if measured alternative is better, equal or worse than other alternatives.

Depending on the nature of the criterion, we distinguish:

- **Maximization criteria –** higher value is taken to be a better value. Decision maker prefers higher values to lower values.
- **Minimization criteria** – opposite situation, lower value is taken to be a better value. Decision maker prefers lower values to higher ones.

A further assessment of the criteria is the preference or the importance of the criterion compared to the other criteria. These preferences express:

There are several types of preference criteria information:

- **No preference information** – information about preferences are not exists. This type is acceptable only for criteria.
- **Nominal information** – also for the criteria, expressed only aspiration level which divides the variants into acceptable and unacceptable ones.
- **Ordinal information** – represents the ordering of the criteria according to the importance of the criteria or to the order of variants according to criteria evaluation.
- **Cardinal information** – is information, which includes both quantitative and qualitative data. For the criterion preference, this information is expressed by its weight. The variation evaluation is the numerical expression. It says how much more than one is better than the other is, and is most often a numerical expression.

3.2.2.1 Preference criteria

The importance of the criteria over the other criteria defines the preference criteria. Types of preference criteria according different kinds of information: [1]

Aspiration level is applicable under the condition that nominal information of criteria is known and all information of alternatives is in a cardinal form. Methods are based on comparing the values of all alternatives with the aspiration levels of all criteria. Typically, the set of variants is divided into two groups. Variants that have worse critical values than set limits, these variants called unacceptable variants. In addition, variants that have better critical values than set limits, these variants called acceptable variants. Aspiration level method is suitable for reducing number of variants. [4]

Order of criteria is applicable under the condition that criteria information is ordinal. The criteria are ranked from important to less important ones, but these criteria do not carry information about how much one criterion is more important than another. [2]

Weights of the criteria is a value from the interval {0; 1}, which is represent the importance of this criterion compared to other criteria. They represent the relative importance of the provided criteria. More important criterion has a greater weight. The sum of the weights of all criteria is equal to one. [4]

3.3 Methods of weighting criteria

The initial step of the analysis of the model is to identify criteria weights. Sometimes it is very difficult to determine the preference of a criterion, especially if the decision maker is not able to distinguish the importance of this criterion. Criteria Weights depend on their importance. Higher value of the criteria has the higher weight; lower value of criteria has the lower weight. [4]
The criteria weight expresses the relative importance of the criterion for the decison maker. Methods for critria weights assesment:

- Sequence method
- Score method
- Fuller triangle
- Saaty method

Methods with ordinal information are Lexicographic Method, Sequence Method, Fuller´s Method, Saaty Method.

3.3.1 Sequence method

Sequence method is based on ordering the criterion from the most important to the least important. The most important criterion is assigned the value k (where k is the number of all criteria) each subsequent criterion will be assigned the value k–1. The least important criterion among all the criteria will be equal to 1. If the criteria have an equal importance, then each criterion will be assigned a value according to the average order. The sum of all weights should be equal to 1. [3]

$$v_j = \frac{b_j}{\sum_{j=1}^{n} b_j}, j = 1, 2, \dots, n \qquad (3.2)$$

The weights of each criterion are determined by the sum of the allocated numbers divided by the total sum of the allocated numbers.

3.3.2 Score method

It is necessary for the decision maker to be able to quantitatively evaluate the preference of the criterion using scoring scale. More important criterion has more points, for example from 0 to 10 points it depends on scoring scale. Score method uses a process similar to the sequence method for calculating criteria weights. The sum of the points that are assigned to the criteria must be equal to the maximum of the scale. The decision maker or makers will evaluate individual criteria and then the number of points will divide the sum of points for the individual criteria. [9]

3.3.3 The method of Fuller triangle

This method is considered ordinal information in the form of a relationship between two criteria. In each pair, we choose criterion that is more important. Decision maker may define criterion *j* like a more important than criterion *l*, while criterion *l* is considered as a less important as criterion *j*. Calculation of weights is similar to calculation of the weights of the sequence method. We measure the weight of a given criterion as a proportion of the number of labels of this criterion and the number of comparisons of all criteria. [6] [7]

Table 1 - Scheme of Fuller triangle

1	1	1	...	1
2	3	4	...	k
	2	2	...	2
	3	4	...	k
			...	...
			...	...
			k-2	k-2
			k-1	k
				k-1
				k

Source: (FIALA, 2013)

3.3.4 Saaty method

This method is applied when only one decision maker evaluates the problem. Saaty method is one of the most used methods of calculation of criterion weights. This method determines the inconsistency of the pairwise comparison matrix. Saaty method can be divided into two steps. The first step is determination of preferences between each pair of criteria and the second step is determination of criterion weights. The advantage of Saaty method is that decision maker can express their preferences verbally rather than numerically. [10]

Usually for evaluation are used a nine-point scale scheme with values 1, 3, 5, 7, 9, but it is also possible to use intermediate values 2, 4, 6, 8. Even values are used to more accurately determine preferences.

1 – equal importance

3 – moderate importance

5 – strong importance

7 – very strong importance

9 – extreme importance

2, 4, 6, 8 – intermediate values

Decision maker compares all the pairs of criteria and writes preferences into the Saaty matrix $S = (s_{ij})$.

$$S = \begin{pmatrix} 1 & s_{12} & \dots & s_{1n} \\ 1/_{s_{12}} & 1 & \dots & s_{2n} \\ \dots & \dots & \dots & \dots \\ 1/_{s_{1k}} & 1/_{s_{12}} & \dots & 1 \end{pmatrix} \tag{3.3}$$

Elements of matrix s_{ij} are represented by preference value of i-th criterion against j-th criterion. The Saaty matrix is always a square matrix *n*x*n*. If the value of i-row and j-column is equal, then this preference is written as $s_{ij} = 1$. Otherwise, if j-th criterion is more preferable than i-th criterion, then the preference value is equal to the inverted value.

The degree of consistency can be calculated using the consistency index defined as follows:

$$I_s = \frac{\lambda_{max} - n}{n - 1} \tag{3.4}$$

Where λ_{max} is the largest value of the Saaty matrix S and n is the number of criteria. Matrix is considered to be enough consistent if the I_s is <0.1.

There are several methods of determination of the weights; most frequently used weighted geometric average of the Saaty matrix.

$$b_i = \sqrt[n]{\prod_{j=1}^{n} s_{ij}} \tag{3.5}$$

After normalizing averages, weights calculated by normalizing the b_i value.

$$w_i = \frac{b_i}{\sum_i^n b_i} \tag{3.6}$$

3.4 Compromising models

Next step after weighting criteria is to find the most appropriate alternative among the all alternatives. In this part of bachelor thesis describes compromising methods. These methods help to find the most approximate to the ideal solution alternative. All methods are divided into major classis according to the type of information from the decision maker. Methods that do not require information about preference criteria include Sequence method and Score method. Methods requiring aspiration level are represented by the Conjunctive method and the Disjunctive method. Ordinary information methods are the

Lexicographic method and the Permutation method. Cardinal information methods are the Simple Additive Weighting method, AHP method or TOPSIS method. [8]

3.4.1 Conjunctive and Disjunctive methods

Using aspiration levels of the criteria, we determine acceptable alternatives that must meet all aspiration levels, otherwise alternatives are excluded. In the case of disjunctive method, we accept alternatives that meet at least one requirement. [8]

3.4.2 Simple Additive Weighting

Main goal of the Simple Additive Weighting (SAW) method is to find the maximal trade-off alternative. This method is based on the weighted average. The advantage of the SAW method is that it is a proportional linear transformation of the cardinal information which means that the order of values of the standardized scores remains equal. Liner function takes values from 0 to 1. The basal alternative according to the given criterion will be equal to 0. The ideal alternative will be 1. [7] [8]

$$u(a_i) = \sum_{i=1}^{m} v_j u_j(y_{ij}) \tag{3.7}$$

The SAW process includes the following steps:

1. Convert minimization criteria to maximization criteria.
2. Determination of the ideal *H* and basal *D* solutions.
3. Creating a standardized matrix A = a_{ij}, using a following formula:

$$r_{ij} = \frac{y_{ij} - d_j}{h_j - d_j} \tag{3.8}$$

4. Calculation of aggregate function of trade-off using the formula:

$$u(a_i) = \sum_{j=1}^{n} v_j r_{ij} \tag{3.9}$$

5. Last step is to sort all alternatives according to the trade-off values, from the highest value to the lowest one. Alternative with the highest values are considered as a solution to the problem.

3.4.3 TOPSIS

The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) is based on measuring distance from the ideal and basal solutions. A feature of the ideal alternative is the vector of the best criteria. The main concept is that the chosen alternative is the closest to the ideal solution and in the longest distance from non-ideal solution. To use this method, all the criteria must be the maximization type. Criteria of the type of minimization must be simply transferred to the maximization type. [5]

Process of TOPSIS consists of these steps:

1. Creating an evaluation matrix (y_{ij}).
2. Calculation of normalized ratings by using normalisation method:

$$r_{ij} = \frac{y_{ij}}{\sqrt{\sum_{i=1}^{p} y_{ij}^2}} \tag{3.10}$$

3. Determination of the relative importance matrix by multiplying values of the matrix by normalized weights of criterion.

$$w_{ij} = v_j \cdot r_{ij} \tag{3.11}$$

4. Build the ideal solution H_1, H_2, …, H_k and basal solutions D_1, D_2, …, D_k, where H_j=max(w_{ij}) and D_j=min(w_{ij}) for j=1, 2, …, k.
5. Measuring distance from the ideal solution (d_i^+)) and basal solutions (d_i^-). The values belong to the interval from 0 to 1. The ideal variant represents 1, the basal variant represents 0.

$$d_i^+ = \sqrt{\sum_{j=1}^{k} (w_{ij} - h_j)^2} \tag{3.12}$$

$$d_i^- = \sqrt{\sum_{j=1}^{k} (w_{ij} - d_j)^2} \tag{3.13}$$

6. Calculation of relative closeness to the Dj solution (basal alternative).

$$c_i = \frac{d_i^-}{d_i^+ + d_i^-} \tag{3.14}$$

Finally, the alternatives are sorted according to the c_i. The highest rated variant can be considered as a solution to the problem.

4 Practical Part

This part is devoted to the practical application of theory. The practical part was written by using knowledge from the theoretical part of the bachelor thesis. This part will be based on the application of mathematical models in decision support that can help to choose the most appropriate solution to the management problem.

4.1 Interview

An interview is a kind of conversation whose goal is to get the needed information. To obtain the necessary information about the needs of the optical center, interview was conducted with the ophthalmologist of this center in order to get the most complete information about the problem. This method helps to avoid errors in making decision and to have a better understanding of the problem. The purpose of this interview was to get information about how to solve this problem, for example, existence of the requested conditions.

4.2 Characteristics of optical center

Optical center Glaz is a private medical facility that provides its services to clients who needs an ophthalmology consultation. This center provides such services as personal consultation with an ophthalmologist, the sale of optical eyeglasses, sunglasses and eyeglass frames, the sale of contact lenses and related accessories.

4.3 Requirements for choosing an ophthalmoscope

The center of optics, it was decided to buy an ophthalmoscope for everyday use. After an interview with an ophthalmologist from this organization, the following requirements were identified:

- Optical power of a lens should be at least 20
- Aperture/filter dial should be at least 1
- The cost of equipment should not exceed 80 000 RUB

4.4 Determination of the criteria

Before proceeding to the analysis of options it is necessary to set appropriate criteria. Based on the criteria, a set of the alternatives will be created. The researcher will determine which criteria will be maximized and minimized types. After the interview, it was determined that five criteria would be considered.

4.4.1 Optical power of a lens

This criterion is the most important than others. Optical power is measured in diopters (D). The greater the number of diopters has an ophthalmoscope; the more convenient it will be to use it, the scope of use of the device increases. This criterion is maximization type.

4.4.2 Aperture/filter dial

Power supply is also one of the most important factors. It is necessary to specify not only the basic parameters of the device. There are three types of charging device. The first type uses batteries for charging; the second uses adapter and the third can be charged, as from the battery, as from the AC/DC adapter. For a user, the best alternative is to have an ophthalmoscope, which is charged from both, the batteries and the AC/DC adapter. This criterion will convert to maximizing.

4.4.3 Price

The price is the amount of money we can spend on the purchase of an ophthalmoscope. Criterion is maximizing. An organization may purchase a machine for the amount of 80,000 RUB.

4.4.4 Usability

Also, the device should be convenient to use. In total there are two possibilities: non-ergonomic and ergonomic. To measure this criterion, the ergonomic value will be equal to 1 and non-ergonomic value will be equal to 0.

4.5 Variants

For processing the analysis, ophthalmoscope were selected according to requirements. This corresponds to the offer of thirteen models from six manufacturers. All values of the criteria were found on the website.

List of individual models:

- Piccolight E50 – helps to determine the healt of the optic disc
- Beta 200 – modern optical device which determine the condition of the optic nerve disc
- Mini 3000 - modern optical device which determine the condition of the eye retina and vessels
- Eurolight E10 ; Eurolight E30 ; Eurolight E36, 3.5B -helps to determine the healt of the optic disc
- Piccolight E56 – powerful xenon lamp ensures excellent fundus illumination
- Coaxial – Significantly improves the quality of visualization and allows medical checks without pupil

Table 2 - List of suggested variants

	Optical power of a lens (D)	Aperture/ Filter dial	Power supply	Price (RUB)	Usability
Uni III	20	1	battery	11630	non-ergonomic
Piccolight E50	20	1	battery	6800	ergonomic
Beta 200	30	6	battery	31000	non-ergonomic
Mini 3000	20	5	AC/DC adapter	15946	ergonomic
Eurolight E10	20	1	battery	8000	non-ergonomic
Eurolight E30	20	1	both	9900	non-ergonomic
Eurolight E36, 3.5B	20	6	both	23400	non-ergonomic
Prestige Coaxial-Plus	30	6	AC/DC adapter	34103	non-ergonomic
Piccolight E56	20	6	battery	8550	non-ergonomic
MedCenter5000 Eurolight E25	35	5	both	106710	non-ergonomic
MedCenter5000 Piccolight E55	20	5	both	26240	non-ergonomic
MedCenter5000 Piccolight E56	20	6	both	30610	non-ergonomic
Coaxial	30	7	battery	45895	ergonomic
Eurolight E36, 2.5B	20	6	both	12300	non-ergonomic

(Source: own processing)

4.6 Determination of the criteria weights using Saaty method

To determine the weights of the criteria, we use Saaty method, which was described in chapter (3.4.4). This method was chosen because only one evaluator conducts evaluation. Evaluator decides how much one criterion exceeds another one. The preference should be defined for each pair of criteria for each alternative. The disadvantage of this method can be that the decision maker evaluates the criteria subjectively. This part of the decision-making process is the most important one, because based on criteria evaluation will be prepared final solution.

The most important criterion for selection is an optical power of a lens and availability of aperture/filter dials. Other parameters, such as power supply is important too, but this criterion is not priority one.

After evaluating, matrix was compiled in table 3 and the weights were calculated.

Table 3 - Saaty matrix

	C1	C2	C3	C4	C5	R_i	V_i
C1	1	7	5	5	9	4,3597	0,5299
C2	1/7	1	1/6	1/3	7	0,5610	0,0682
C3	1/5	6	1	5	8	2,1689	0,2636
C4	1/5	3	1/5	1	6	0,9364	0,1138
C5	1/9	1/7	1/8	1/6	1	0,2013	0,0245
					SUM	8,2273	1

(Source: own processing)

Each criterion was evaluated according to the nine-point scale scheme; the numbers of this scale represents preferences. After the distribution of the preference, the weight of each criterion was calculated. The higher the weight, the more important the criterion.

4.7 Selecting a compromise variant

4.7.1 Simple Additive Weighting

The Simple Additive Weighting method is used to select the most appropriate alternative. The first step for using this method is to prepare a matrix in which the individual variants, the evaluation criteria, criteria characters and weights. Criterion of price (C4) is minimizing, remaining criteria is maximizing.

Table 4 - Matrix for applying a weighted sum method

	C1	C2	C3	C4	C5
V1	20	1	2	11630	0
V2	20	1	2	6800	1
V3	30	6	2	31000	0
V4	20	5	1	15946	1
V5	20	1	2	8000	0
V6	20	1	3	9900	0
V7	20	6	3	23400	0
V8	30	6	1	34103	0
V9	20	6	2	8550	0
V10	20	5	3	26240	0
V11	20	6	3	30610	0
V12	30	7	2	45895	1
V13	20	6	3	12300	0
Criteria weights	MAX	MAX	MAX	MIN	MAX
Criteria character	0,5299	0,0682	0,2636	0,1138	0,0245

(Source: own processing)

The table consists of an actual data. For each criterion was defined a characteristics, minimizing or maximizing. In this case there is only one criterion having a minimalizing character, this is the price. Also there is a row with the criterion weights. Based on data from the table were found the ideal (*H*) and basal (*D*) variants.

Table 5 - Ideal and basal alternatives

	C1	C2	C3	C4	C5
H	30	7	3	6800	1
D	20	1	1	45895	0

(Source: own processing)

After setting the criteria weights and determining the ideal and basal variant, necessary to calculate the values of the standardized matrix.

Based on the data from the tables, we obtained a standardized matrix R using the formula in Chapter 3.5.1. The standardized matrix represents a matrix of utility value, values ranging from 0 to 1. The basal variant corresponds to the value 0 and the ideal value to 1.

Table 6 - Standardized decision matrix

	C1	C2	C3	C4	C5
V1	0	0	0,5000	0,8765	0
V2	0	0	0,5000	1	1
V3	1	0,8333	0,5000	0,3810	0
V4	0	0,6667	0	0,7661	1
V5	0	0	0,5000	0,9693	0
V6	0	0	1	0,9207	0
V7	0	0,8333	1	0,5754	0
V8	1	0,8333	0	0,3016	0
V9	0	0,8333	0,5000	0,9552	0
V10	0	0,6667	1	0,5027	0
V11	0	0,8333	1	0,3910	0
V12	1	1	0,5000	0	1
V13	0	0,8333	1	0,8593	0
Criteria character	0,5299	0,0682	0,2636	0,1138	0,0245

(Source: own processing)

Final step is to determine the value of the aggregate trade-off function for each variant. Depending on the values, we can easily calculate the rank order for each criterion. The ranks will be also presented into the table:

Table 7 - Order of alternatives (SAW)

	Trade-off	rank
V1	0,2315	12
V2	0,2701	10
V3	0,7619	1
V4	0,1571	13
V5	0,2421	11
V6	0,3684	6
V7	0,3859	5
V8	0,6211	3
V9	0,2973	9
V10	0,3663	7
V11	0,3649	8
V12	0,7544	2
V13	0,4182	4

(Source: own processing)

The results of ranking are recommend for purchase a Heine Beta 200 ophthalmoscope. Nevertheless, it is worth noting that the second in terms of ranking KaWe

Eurolight E36, 2.5B is not so significantly different from the recommended alternative, they are almost equivalent.

The best alternative according simple additive weighting method is Heine Beta 200 ophthalmoscope (V3). This version has high optical power, it also has six filters and use batteries for charging. The price is adoptable according to the characteristics of the device. The disadvantage is that the design of the device is not ergonomic.

4.7.2 TOPSIS

TOPSIS is one of the compromising methods for solving decision making problem, which allows evaluating alternatives in terms of their distance from the ideal and basic variants.

For further calculations, the decision matrix and criteria weights obtained from Saaty method (table 3) will used for calculations.

Table 8 - Decision matrix for applying a weighted sum method

	C1	C2	C3	C4	C5
V1	20	1	2	11630	0
V2	20	1	2	6800	1
V3	30	6	2	31000	0
V4	20	5	1	15946	1
V5	20	1	2	8000	0
V6	20	1	3	9900	0
V7	20	6	3	23400	0
V8	30	6	1	34103	0
V9	20	6	2	8550	0
V10	20	5	3	26240	0
V11	20	6	3	30610	0
V12	30	7	2	45895	1
V13	20	6	3	12300	0
Criteria weights	MAX	MAX	MAX	MIN	MAX
Criteria character	0,5299	0,0682	0,2636	0,1138	0,0245

(Source: own processing)

First, we need to transfer minimization criteria into the maximization one using formulas (3.10), which mentioned in chapter (3.4.3). Next step is to construct a standardized matrix R using formula (3.11). After this transformation, we have a normalized matrix *R*.

Table 9 - Normalized decision matrix R

	C1	C2	C3	C4	C5
V1	0,2443	0,0560	0,2374	0,3417	0,0000
V2	0,2443	0,0560	0,2374	0,3985	0,5774
V3	0,3665	0,3359	0,2374	0,1138	0,0000
V4	0,2443	0,2799	0,1187	0,2909	0,5774
V5	0,2443	0,0560	0,2374	0,3844	0,0000
V6	0,2443	0,0560	0,3560	0,3620	0,0000
V7	0,2443	0,3359	0,3560	0,2032	0,0000
V8	0,3665	0,3359	0,1187	0,0773	0,0000
V9	0,2443	0,3359	0,2374	0,3779	0,0000
V10	0,2443	0,2799	0,3560	0,1698	0,0000
V11	0,2443	0,3359	0,3560	0,1184	0,0000
V12	0,3665	0,3919	0,2374	-0,0614	0,5774
V13	0,2443	0,3359	0,3560	0,3338	0,0000

(Source: own processing)

After normalization of the matrix R, we may to construct a weighted normalized matrix W.

Table 10 - Normalized decision matrix W

	C1	C2	C3	C4	C5
V1	0,1295	0,0038	0,0626	0,0389	0,0000
V2	0,1295	0,0038	0,0626	0,0453	0,0141
V3	0,1942	0,0229	0,0626	0,0129	0,0000
V4	0,1295	0,0191	0,0313	0,0331	0,0141
V5	0,1295	0,0038	0,0626	0,0437	0,0000
V6	0,1295	0,0038	0,0939	0,0412	0,0000
V7	0,1295	0,0229	0,0939	0,0231	0,0000
V8	0,1942	0,0229	0,0313	0,0088	0,0000
V9	0,1295	0,0229	0,0626	0,0430	0,0000
V10	0,1295	0,0191	0,0939	0,0193	0,0000
V11	0,1295	0,0229	0,0939	0,0135	0,0000
V12	0,1942	0,0267	0,0626	-0,0070	0,0141
V13	0,1295	0,0229	0,0939	0,0380	0,0000

(Source: own processing)

Next step will be determination the ideal and basal alternatives.

Table 11 - Ideal and basal alternatives

	C1	C2	C3	C4	C5
D	0,1942	0,0267	0,0939	0,0453	0,0141
H	0,1295	0,0038	0,0313	-0,0070	0,0000

(Source: own processing)

After determining the basal and ideal variants, the last step is to calculate the distance of the individual alternatives from the ideal and basic ones. The value of c_i is obtained from the formula, and these values are written in the table as the most important values. These variables are in the interval from 0 to 1, the basal alternative is equivalent to 0, and the ideal is 1.

All calculation steps are described in the capitol (3.4.3).

Table 12 - Order of alternatives (TOPSIS)

	d^+	d^-	c_i	rank
V1	0,0770	0,0555	0,4188	12
V2	0,0755	0,0626	0,4534	10
V3	0,0474	0,0770	0,6192	1
V4	0,0912	0,0452	0,3313	13
V5	0,0768	0,0596	0,4370	11
V6	0,0702	0,0790	0,5293	4
V7	0,0700	0,0720	0,5071	5
V8	0,0739	0,0693	0,4839	7
V9	0,0734	0,0620	0,4578	9
V10	0,0716	0,0696	0,4928	6
V11	0,0736	0,0685	0,4821	8
V12	0,0610	0,0768	0,5573	2
V13	0,0668	0,0794	0,5431	3

(Source: own processing)

The final step is to determine the rank of alternatives with relative distance of variants from basal variant. It is clear from the table (12), that the best alternative is the Heine Beta 200 ophthalmoscope. On the second and third place are WelchAllyn Coaxial and KaWe Eurolight E36, 2.5B.

Results and Discussion

In order to determine the most appropriate solution to the problem, two multi-criteria decision analysis were used. The weights of each criterion were determined by using Saaty method. Method of simple additive weighting and TOPSIS were used for estimation of individual variants.

Using simple additive weighting method, Heine Beta 200 was chosen as the most preferred ophthalmoscope. This device has a high power of lenses; this variant has one of the highest value of performance characteristics. There is large number of filters. The price for this product is quite high, but it is commensurate with its characteristics.

The second applied method was TOPSIS, which also suggested Heine Beta 200 to us as the most peered, compromise variant. Based on the received data, this variant has a big gap from the second and third alternatives in the rating. At the same time, the second and third variants have a relatively small difference.

Comparing the results of these two methods, we can confidently say that the Heine Beta 200 is considered as the most acceptable for buying according to the results of simple additive weighting method and TOPSIS. At the same time, simple additive weighting method represents WelchAllyn Coaxial as an alternative to a better choice, since the differences of these alternatives are very small. The third place occupies Prestige Coaxial-Plus. According to the evaluation of TOPSIS, WelchAllyn Coaxial is the most similar to KaWe Eurolight E36, 2.5B, and those alternatives occupy 2 and 3 places in rating.

Table 13 - Final order of given alternatives

	SAW		TOPSIS	
	Trade-off	rank	c_i	rank
V1	0,2315	12	0,4188	12
V2	0,2701	10	0,4534	10
V3	0,7619	1	0,6192	1
V4	0,1571	13	0,3313	13
V5	0,2421	11	0,4370	11
V6	0,3684	6	0,5293	4
V7	0,3859	5	0,5071	5
V8	0,6211	3	0,4839	7
V9	0,2973	9	0,4578	9
V10	0,3663	7	0,4928	6
V11	0,3649	8	0,4821	8
V12	0,7544	2	0,5573	2
V13	0,4182	4	0,5431	3

(Source: own processing)

5 Conclusion

The main goal of the bachelor thesis is to determine the most appropriate ophthalmoscope for optical center Glaz using mathematical models. The decision was resolved by method of the multiple criteria decision analysis. To determine a compromise alternative were used: Saaty method, Simple Additive Weighting method and TOPSIS.

The literary part was describes basic concepts and models of multi-criteria analysis. Also, based on theoretical knowledge, appropriate models were chosen to extend based on the situation.

In the practical part, the medical device (ophyholmaskopes) was introduced. Selection criteria have been set according to user requirements and wishes. Using Saaty method, weights were calculated to determine the most important criterion for the optical lens and price.

For determination the most appropriate solution was used Simple Additive Weighting method and TOPSIS. The first method, Simple Additive Weighting, determined that the Heine Beta 200 is the most acceptable for buying according to the results. The same TOPSIS confirmed that the Heine Beta 200 is the best option for the acquisition. Comparing the solutions of two mathematical methods, we obtained only one compromise solution.

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