BRNO UNIVERSITY OF TECHNOLOGY

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INSTITUTE OF MATHEMATICS

ÚSTAV MATEMATIKY

GAME THEORY IN WASTE MANAGEMENT

TEORIE HER V ODPADOVÉM HOSPODÁŘSTVÍ

DOCTORAL THESIS

DIZERTAČNÍ PRÁCE

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BRNO 2023

Abstract

Game theory handles tasks such as cooperation, competition, and self-regulation in the environment, where numerous agents with conflicting goals are involved. These conflicts of interest are extremely common, when dealing with environmental sustainability and circular economy. This Ph.D. thesis is devoted to applications of game theory in waste management, with an emphasis on Waste-to-Energy treatment of non-recyclable waste. After an introduction, the fundamental background of game theory is summarized, providing an overview of the current state of knowledge. Then, recent applications of game-theoretic techniques in sustainability research are reviewed to emphasize the novelty of the work. In the end, the author's own contribution in the application of non-cooperative and cooperative games to problems of waste management is presented. In particular, this Ph.D. thesis is focused on the Waste-to-Energy plants' price-setting game and the waste producers' cost minimization game. Theoretical properties of these games are studied in detail. The original algorithms for bilevel optimization problems and dynamic coalition formation are proposed to solve the considered games. The case studies' results demonstrate rational outcomes of the conflicts and prove that the proposed approaches to the considered waste management problems are reasonable.

Abstrakt

Teorie her se zabývá tématy, jako je spolupráce, konkurence a seberegulace v prostředí, kde je zapojeno mnoho entit s protichůdnými cíly. Rozdílné zájmy jsou běžné při řešení environmentální udržitelnosti a oběhového hospodářství. Tato Ph.D. práce je věnována aplikacím teorie her v odpadovém hospodářství s důrazem na energetické zpracování nerecyklovatelného odpadu. Po úvodu je shrnuto základní pozadí teorie her, které poskytuje přehled o současném stávu poznání. Poté jsou přezkoumány novodobé aplikace metod teorie her v problematice udržitelnosti, aby se zdůraznila aktuálnost práce. V závěru je uveden vlastní přínos autora v aplikaci nekooperativních a kooperativních her v oblasti odpadového hospodářství. Konkretně je Ph.D. práce zaměřena na hru o stanovení cen zařízeními pro energetické využití odpadů a hru o minimalizaci nákladů producentů odpadů. Jsou podrobně studovány teoretické vlastnosti těchto her. Pro řešení uvažovaných her jsou navrženy originální algoritmy pro problémy dvouúrovňové optimalizace a vytváření dynamických koalic. Výsledky případových studií ukazují racionální vyústění konfliktů a dokazují, že navrhované přístupy k uvažovaným problémům odpadového hospodářství jsou rozumné.

Keywords

game theory, optimization, waste management, decision-making, sustainability

Klíčová slova

teorie her, optimalizace, odpadové hospodářství, rozhodování, udržitelnost

Reference

ERYGANOV, Ivan. *Game Theory in Waste Management.* Brno, 2023. Doctoral thesis. Brno University of Technology, Faculty of Mechanical Engineering. Supervisor doc. Mgr. Jaroslav Hrdina, Ph.D.

Game Theory in Waste Management

Declaration

I hereby declare that this Ph.D. thesis was prepared as an original work by the author under the supervision of doc. Mgr. Jaroslav Hrdina, Ph.D. Furthermore, I declare, that I cited all literature, publications, and other sources used for this document.

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Ivan Eryganov
September 27, 2023

Acknowledgements

I would like to express my gratitude to doc. Mgr. Jaroslav Hrdina, Ph.D. for his hard and time-consuming work, which made this thesis as complete, as it is presented. I also would like to thank Ing. Radovan Šomplák, Ph. D. and doc. Mgr. Petr Vašík, Ph.D. for their support throughout my Ph.D. study. Special gratitude is addressed to my wife. Thank you for always being there for me.

Contents

Chapter 1

Introduction

According to [91], game theory (GT) focuses on mathematical models of complex interactions among rational participants of the formalized conflict. GT enables the description of the natural and logical development of such conflicts. It anticipates possible outcomes of situations in which decision-makers with different goals are involved and can affect each other [85]. Among other applications, it can imitate rationality and optimize arbitrary complex engineering systems, where different system parts are considered to be players performing various, often conflicting, tasks. GT has become an essential framework in the past years, since the number of applications involves multiple users, where disagreements between them are incredibly likely or even unavoidable [81]. These disagreements are common to a wide range of disciplines such as economics, computer science, social sciences, or engineering. Among all these disciplines, this doctoral thesis is focused on sustainability research, circular economy (CE), and efficient green waste management (WM).

The structure of this doctoral thesis is the following. Initially, a general description of the sustainability concept and a discussion about the role of decision-making (DM) in sustainability research are given to promote GT as a suitable DM method within the area. Chapter 2 consists of an overview of the main GT concepts, their mathematical descriptions and properties. This section provides mathematical background, which is considered necessary in order to understand and assess current research trends. To highlight the contribution of the work, the general review of the recent articles focused on the applications of GT in particular fields, requiring sustainable development, is presented in Chapter 3. Thus, the theoretical part of this work is followed by the results of the performed review. These results are thoroughly discussed in order to identify existing research gaps. After that, author's original research within GT in WM is demonstrated in Chapter 4. This last section demonstrates current applications of cooperative GT (CGT) and non-cooperative GT (NGT) approaches to WM problems. In particular, this application section is focused on the problems of a gate fee-setting in competitive environment and of a fair waste treatment costs distribution and union formation between municipalities in WM networks. To solve the former problem, the original algorithm based on bilevel programming techniques is proposed. Moreover, an existence of Nash equilibrium (NE) in this setting is studied from a theoretical point of view. The properties of the latter problem are extensively studied and the solution is proposed using the coalition formation approach and the Shapley value estimate. The definitions and analyses of these practical problems are complemented by the necessary theoretical insights and reviews on the related topics. Functionality of both approaches is demonstrated on a realistic case study inspired by the WM situation in the Czech Republic.

1.1 Potential of game theory in sustainability research

The rapid growth of the world population, urbanization, and industrialization are current trends, that lead to a substantial and continuous increase in consumption of goods, energy, and primary resources [82]. One of the most negative consequences of such trends is environmental degradation represented by water, air, and land pollution, non-speaking of overwhelming greenhouse gases emissions having undeniable climate change impact [86]. Scarcity of available resources and irresponsible consumption, contributing to the above-mentioned consequences, emphasize the importance of sustainability [79]. Sustainability is the ability of an economy to retain or improve the level of economic, environmental, and social resources over generations [73]. Thus, it is common to think of this concept as of something that considers and contributes to the three main aspects: economic, environmental, and social [6]. Among all these dimensions, this work is mainly focused on the economic consequences of environmental sustainability principles implemented into waste treatment policy.

Environmental sustainability is a conservation concept that entails the provision of current and future generations with services and resources without endangering the health of ecosystems [57]. When embedding principles of environmental sustainability into economic processes, conflicts are expected to arise, since different stakeholder groups have their own interests and priorities. An integrative DM process should enable to erase such problems and help to achieve cooperation between stakeholders [86].

1.1.1 Challenges of decision-making in environmental sustainability

Environmental sustainability problems are often characterized by the need for a practical DM solution [24]. However, DM within this context represents an eminent challenge, since numerous social, financial, and political consequences of possible decisions have to be considered. Different tools have been developed and applied to face the above-described complications and many others arising during the DM processes. For example, DM methods, such as sensitivity analysis, stochastic analysis, and mathematical programming, can serve as a helpful basis for finding sustainable solutions. Now, implications of recent articles dealing with DM in environmental sustainability within different fields of research will be summarized.

The sustainability of hydropower development has been studied in [67] in detail. Whereas hydropower is one of the most spread renewable sources of energy, it might bring negative environmental (and consequently social) impacts. The authors concluded, that commonly applied cost-benefit analysis does not fully take into account the sustainability of the designed system. Thus, as it was already mentioned, the main challenge is to consider all possible dimensions impacted by the prepared project. As a possible solution, authors propose their own evaluation criteria system and multi-criteria DM method. In [56], authors reviewed DM tools and methods for solid WM systems. It has been emphasized, that the main challenge is to capture cooperative incentives of stakeholders, as well as their possibly conflicting distinct objectives. The same problem has been pointed out in [79], where DM in the context of sustainable energy, water, and food nexus systems has been reviewed. The authors highlighted the difficulty of taking into account different sectors, agents, and uncertainties. The importance and challenges of DM in the management of the closed-loop supply chains (CLSC) have been discussed in [98]. It has been pointed out, that the main challenge of DM in the general framework of supply-chain management is an interdependence of agents' decisions. According to the above-mentioned implications, it can be concluded, that the most frequent difficulties occurring during the DM process, related to the environmental sustainability research, are the necessity to consider conflicting objectives, multiple assessment criteria, and interdependence between the involved agents.

1.1.2 Game theory as a decision-making method

Among all possible DM methods, this doctoral thesis proposal is focused on GT. The reasoning beneath such a choice is rather plain: GT is a relatively young (in the context of mathematics) framework and possibly can complement (or fully replace) some DM methods. In the above-mentioned work [79], the authors concluded, that currently applied tools cannot properly represent strategic interactions and trade-offs between stakeholders. Exactly GT proves itself as an efficient and practical DM tool in a multi-stakeholder interdependent environment. The authors of [56] highlighted, that the GT approach can contribute to the sustainability of a solid WM system. Also, in [98], the authors concluded, that GT provides a quantitative insight into the allocation of costs and benefits within the CLSC. Thus, it can be concluded, that all the above-mentioned works agreed on GT potential in environmental sustainability research. In particular, GT has a potential to serve as a powerful tool for researchers to overcome the occurring challenges and to explore topic of CE [17], being a substantial tool of sustainable development.

1.2 Motivation and goals

Well-planned WM is an essential part of CE, and behavioral modeling, describing the ever-changing decisions of the involved agents, is its key aspect [1]. This doctoral thesis is devoted to applications of CGT and NGT to WM problems, which are of critical importance for the modern society. As it was already indicated in the beginning of this chapter, the considered problems are the non-cooperative gate fee setting game between waste treatment facilities and the municipalities' cooperative waste treatment cost game.

The main goals of this doctoral thesis are:

- to present an overview of the GT theoretical concepts, with an emphasis on branches, solutions, and specific game types, which will be used later in the application section;
- to review recent applications of GT in environmental sustainability research within different fields in order to identify currently existing research gaps;
- to formulate and analyze WM problems using CGT and NGT;
- to design algorithms for solving these problems;
- to implement bilevel programming techniques into the price-setting problem.

Chapter 2

Theoretical aspects of game theory

By the nature of a studied conflict and given set of rules from which possible interactions are derived, GT has been traditionally divided into two branches: non-cooperative and cooperative. NGT deals with strategic choices in settings, in which there are no binding agreements between players, and they are independently trying to improve their own welfare [85]. Compared to NGT, CGT studies behavior of players in a situation, when they can improve their payoffs by merging into coalitions [88]. Evolutionary GT (EGT), pioneered in [36], is another GT branch distinguished by many authors. Original biology-inspired EGT assumed that players could not choose a strategy, but only inherit it from ancestors or obtain it due to mutation [102]. Among these widely accepted branches, other, yet insufficiently developed, branches such as quantum GT [65] can be found. Due to nature of the thesis, we have mainly focused on deterministic games with fully rational players. The basic classification of GT, major game types, fundamental solution concepts, and related algorithms are depicted in Figure 2.1. Clearly, this classification cannot be viewed as complete, but might provide the reader with some introductory image of GT. Since the applied part of this doctoral thesis devotes itself to the application of the NGT and the CGT framework to WM, the theoretical concepts related to these branches of GT will be discussed in the following sections. Concepts, which are not related to the case studies and underlying WM games, will be briefly discussed for the purpose of simple acquaintance or completely omitted in some cases. For example, in this chapter, we have omitted concept of fuzzy games, random games, or games with bounded rationality.

2.1 Non-cooperative game theory

Due to the historical development of GT [107], its non-cooperative branch is diverse and well-studied [91]. Early applications of non-cooperative games can be found in classical studies of oligopoly competition by Cournot [21], Bertrand [9], or Stackelberg [108]. In the following subsection, possible approaches, solution algorithms, games types, and representations of NGT are summarized according to [85].

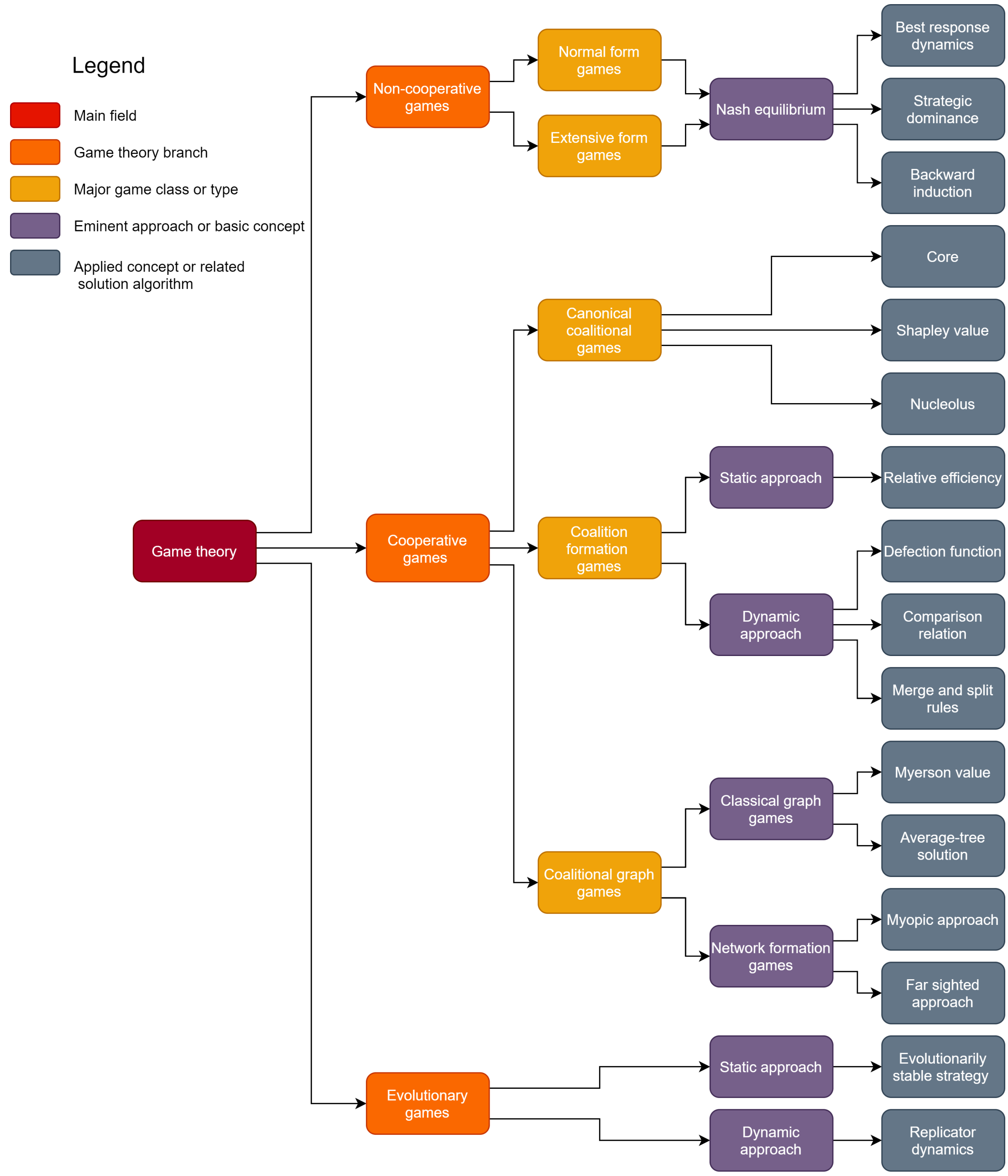

Figure 2.1: Game theory basic structure

2.1.1 Representation, types, properties

Non-cooperative games can be represented in extensive form or normal form. Extensive form employs a game tree as a representation tool, where nodes represent states of the game, whereas arcs represent possible moves. Alternatively, non-cooperative games in normal form consist of a list of strategies for the players together with their payoff functions defined for each strategy profile. Formally, normal form game can be defined as follows.

Definition (Normal form game)**.** A non-cooperative game in normal form is defined as the structure $G = (N, (X_i, \pi_i)_{i \in N})$, where $N = \{1, 2, 3, \dots, n\}$ is the set of players, X_i, $i \in N$, is the individual strategy set of i-th player, with element $x_i \in X_i$ called strategy, and $\pi_i : \Pi_{i \in N} X_i \to \mathbb{R}, i \in N$, is the payoff function of the i-th player. The joint strategy set will be further denoted as $X_N := \Pi_{i \in N} X_i$, and its element $x = (x_1, \dots, x_n) \in X_N$ is called a strategy profile.

A particular case of normal form games are finite games, where the strategy sets of players have finite cardinality $|X_i| < \infty, \forall i \in N$ [39]. In some cases, finite games can be represented via matrix of payoffs. Non-cooperative games can be either static or dynamic. Static games takes place only once, and all players make decisions simultaneously [85]. The latter group can be divided into two classes: sequential and repeated games. In sequential games, agents play a game in turns, dividing it into smaller subgames [84]. A static game, occurring over many periods (possibly infinitely many), is called a repeated game [8]. Regarding the amount of the available information, non-cooperative games can be divided into games with perfect or imperfect information and complete or incomplete information [85]. Perfect information means, that each player possesses all available information about every event, that has occurred during previous stages of the game. Complete information implies, that players' utility functions, payoffs, and strategy sets are assumed to be common knowledge [72].

2.1.2 Nash equilibrium

The NE is a cornerstone of non-cooperative solutions [80]. The NE is a stable outcome, in which no player has the intention to change his strategy, while other players keep theirs unchanged [85]. Its precise definition is following.

Definition (Nash equilibrium)**.** The NE is a strategy profile $x^* \in X_N$, such that

$$\pi_j(x_j^*, x_{-j}^*) \geq \pi_j(x_j, x_{-j}^*), \forall x \in X_N, \forall j \in N,$$

where (x_j, x_{-j}^*) stands for $(x_1^*, ..., x_{j-1}^*, x_j, x_{j+1}^*, ..., x_n^*)$.

Existence of NE is one of fundamental questions of GT. It is closely related to existence of a fixed point of a correspondence [39].

Definition (Correspondence and its properties)**.** Let $X \subseteq \mathbb{R}^n$ and $Y \subseteq \mathbb{R}^m$, where $n, m \in \mathbb{N}$. Then, a correspondence F from X to Y is a map $F : X \to 2^Y$. We say, that a correspondence F is

- non-empty-valued if $\forall x \in X$, $F(x)$ is non-empty subset of Y;
- closed-valued if $\forall x \in X$, $F(x)$ is closed subset of Y;
- convex-valued if $\forall x \in X$, $F(x)$ is convex subset of Y;

Now, we generalize notion of continuity for correspondences.

Definition (Upper and lower hemicontinuity)**.** A correspondence F

- is upper hemicontinuous if, for each sequence $\{x_k\} \subseteq X$ converging to $\hat{x} \in X$ and each open set $Y^* \subseteq Y$, such that $F(\hat{x}) \subseteq Y^*$, there is $k_0 \in N$ such that, for each $k \geq k_0$, $F(x_k) \subseteq Y^*$.

- is lower hemicontinuous if, for each sequence $\{x_k\} \subseteq X$ converging to $\hat{x} \in X$ and each open set $Y^* \subseteq Y$, such that $F(\hat{x}) \cap Y^* = \emptyset$, there is $k_0 \in N$ such that, for each $k \geq k_0$, $F(x_k) \cap Y^* = \emptyset$.

In particular, we focus on the best-response correspondences of players.

Definition (Best-response correspondence)**.** Let $G = (N, (X_i, \pi_i)_{i \in N})$ be a normal-form game. Then, we define best-response correspondence $B_j : X_{-j} \to 2^{X_j}$, where $X_{-j} := \Pi_{i \in N, i \neq j} X_i$ with $x_{-j} \in X_{-j}$, as

$$B_j(x_{-j}) = \{\tilde{x}_j \in X_j : \; \pi_j(\tilde{x}_j, x_{-j}) \geq \pi_j(x_j, x_{-j}), \forall x_j \in X_j\}.$$

Clearly, best-response correspondences do not have to be well-defined. We also define $B : X_N \to 2^{X_N}$ for each $x \in X_N$, as $B(x) := \Pi_{i \in N} B_i(x_{-i})$.

Then, to establish the so-called Nash theorem, which states sufficient conditions for NE existence, we introduce Kakutani fixed-point theorem. The formulations of the two following theorems and the quasi-concavity definition are taken form [39].

Theorem 2.1.1 (Kakutani fixed-point theorem)**.** *Let $X \subseteq R^n$ be a nonempty, convex, and compact set. Let $F : X \to X$ be an upper hemicontinuous, non-empty-valued, closed-valued, and convex-valued correspondence. Then, F has a fixed-point.*

Proof. Proof can be found in [39]. □

At last, the property of quasi-concavity has to be introduced.

Definition (Quasi-concavity)**.** Let $m \in \mathbb{N}$ and $X \subseteq \mathbb{R}^m$ be a convex set. A function $f : X \to \mathbb{R}$ is quasi-concave if, for each $r \in \mathbb{R}$, the set $\{x \in X : f(x) \geq r\}$ is convex.

The main point of the Nash theorem is that in certain situations underlying best-response correspondences fulfill assumptions of Kakutani fixed-point theorem.

Theorem 2.1.2 (Nash theorem)**.** *Let $G = (N, (X_i, \pi_i)_{i \in N})$ be a normal-form game such that, for each $i \in N$,*

- *X_i is a nonempty, convex, and compact subset of $\mathbb{R}^{m_i}$ for some $m_i \in \mathbb{N}$;*

- *π_i is continuous;*

- *For each x_{-i}, $\pi_i(x_i, \cdot)$ is quasi-concave on X_i.*

Then, the game G has NE.

Proof. Proof can be found in [39]. □

It can be seen, that this theorem cannot be applied to games with discontinuous payoff functions or non-convex strategy sets, which are quite common when dealing with practical problems. Another substantial problem is that this theorem does not discuss uniqueness of the NE. Indeed, it can be pointed out, that possible existence of multiple NEs, as well as non-existence of the NE, might dramatically complicate prediction of the outcome. In such situations, utilization of the refined or generalized solution concepts might be useful. One of the main such concepts is mixed NE.

Until now, we were discussing only so-called pure strategies, where players choose only one particular strategy. However, the concept of mixed strategy assumes that player might plays strategies randomly. When working with mixed strategies, player $i \in N$ does not choose strategy x_i, but some probability distribution s_i over the strategy set X_i. Now, a mixed extension of the normal form game will be formally introduced according to [39].

Definition (Mixed extension)**.** Let $G = (N, (X_i, \pi_i)_{i\in N})$ be a finite game. The mixed extension $E(G) := (N, (S_i, \hat{\pi}_i)_{i\in N})$ of G, is the strategic game with:

- $S_i := \{s_i \in [0,1]^{X_i} : |\{x_i \in X_i : s_i(x_i) > 0\}| < \infty$ and $\sum_{x_i\in X_i} s_i(x_i) = 1\}, \forall i \in N$,
- $S_N := \Pi_{i\in N} S_i$;
- $s(x) := s_1(x_1) \cdot ... \cdot s_n(x_n)$, for each $s \in S_N$;
- $\hat{\pi}_i(s) := \sum_{x\in X_n} \pi_i(x)s(x)$.

Then, for a finite game G, the following theorem holds.

Theorem 2.1.3 (Existence of equilibrium in mixed games)**.** *Let $G = (N, (X_i, \pi_i)_{i\in N})$ be a finite game. Then, the mixed extension $E(G) := (N, (S_i, \hat{\pi}_i)_{i\in N})$ of G has NE.*

Proof. Existence of NE for $E(G)$ follows directly from the Nash theorem [39]. □

One of the most popular algorithms for finding the NE in normal form games (except for direct extensive search in finite games) are best-response dynamics (BRD) [71] and implementation of strategy domination [76]. The former algorithm represents a process, during which each player iteratively plays his/her best response to actual rivals' strategy profile. The main idea of the algorithm is natural: a process starts at a given point and at each iteration player chooses a strategy from his or her best-response correspondence. The new starting strategy profile for the next player is obtained from a chosen strategy of the previous player. If the algorithm converges to some strategy profile, then this strategy profile is the NE. The main disadvantage of this algorithm is the fact that it can get stuck in a cycle. Moreover, in some cases, finding a $B_j(x_{-j})$ might be a challenging task. Figure 2.2 explains the main principles of the algorithm. Before we proceed to the strategy domination approach, some important aspects of the BRD process should be discussed, since this algorithm will play a major role with respect to subject of the Ph.D. thesis.

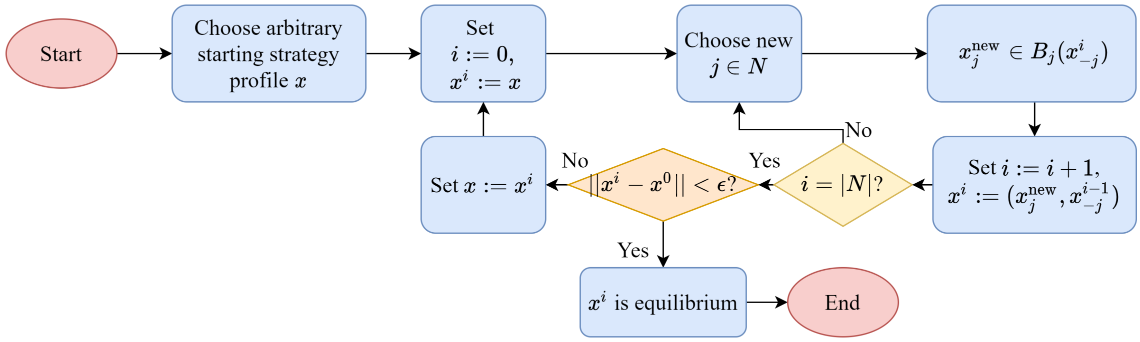

Figure 2.2: Sketch of best-response dynamics algorithm

Properties of best-response dynamics. Unfortunately, there is no general results on convergence of BRD. It has been proven, that for certain types of games, such as potential [75] or aggregative games [25], convergence is assured. However, these games assume that players' utility functions have some common denominator or their best-response correspondences can be jointly described by one general function. Therefore, to cover «complementary» class of games, literature also focuses on the study of the so-called uncoupled BRD processes, where players payoff/responses do not depend on payoffs of the other players (or they cannot be directly incorporated into π_i due to lack of knowledge). Thus, player i can choose best response only based on available joint strategy profile x and properties of π_i. In case BRD is uncoupled, its convergence cannot be guaranteed. In fact, even if we modify original uncoupled BRD, general uncoupled dynamics which can guarantee convergence to NE ceases to exist [45]. From Figure 2.2, it is also not clear in which order players make their decisions. The order in which players update their actions is called playing sequence. In [49], it has been demonstrated, that, compared to fixed cyclic order, random playing sequence (using uniform distribution over N) converges to a pure NE (if it exists) in almost all games. On the contrary, the probability of finding pure NE using fixed playing sequence goes to 0 with increasing $|N|$ or/and $|X_i|, i \in N$. Thus, Figure 2.2 does not demonstrate the particular approach, but rather presents a possible mix of both playing sequences. During every inner cycle a new player has to take turn, but there is no fixed strict ordering. Convergence speed also dramatically depends on properties of the underlying game. For example, [27] have shown that in potential games fixed cyclic ordering is the fastest possible option. Simulation results presented in [49] demonstrate that the speed of convergence in generic games is slower for random playing sequences. In general, if we focus on uncoupled BRD processes, the speed of convergence is at least exponential function of $|N|$ [46].

The idea of the strategy domination algorithm dwells in the iterative removal of dominated strategies. The strategy dominance relation can be defined as follows [76].

Definition (Strategy dominance)**.** A strategy $x_j^1 \in X_j$ is dominated by a strategy $x_j^2 \in X_j$ iff

$$\pi_j(x_j^2, x_{-j}) \geq \pi_j(x_j^1, x_{-j}),\ \forall x_{-j} \in X_{-j},$$

and

$$\exists x_{-j} \in X_{-j} : \pi_j(x_j^2, x_{-j}) > \pi_j(x_j^1, x_{-j}).$$

In the first step, all the dominated strategies of the first player are removed, and his or her strategy set is reduced to a set of undominated strategies. In the second step, the same approach is applied for the second player using the new reduced strategy set of the first player and so forth. If such an algorithm leads to a singleton, then this strategy profile is NE.

2.2 Cooperative game theory

In general, a coalitional or cooperative game is uniquely defined by pair (N, v), where N is a set of players and v is a coalition value function, that assigns each coalition $S \subseteq N,\ S \neq \emptyset$, (a binding agreement of players to act as a single entity) its worth (the total utility that can be obtained by S) in the game [88]. The definition of v determines the so-called form and properties of the game. In this work, we focus only on games with transferable utility (TU), where coalition's worth has a monetary equivalent, that can be divided between participants of the coalition [88]. Thus, we can formally define a value function v of TU-game as a function $v : 2^N \times A_1 \times ... \times A_k \to \mathbb{R}$, where $A_1, ..., A_k$ are possible additional spaces than can be considered. The conventionally added assumption $v(\emptyset) = 0$ implies, that in absence of cooperation, no value is produced. The main principle of a TU-game is that, if we denote player's $i \in N$ payoff as x_i, then any utility allocation $(x_i)_{i \in S}$, such that

$$\sum_{i \in S} x_i \leq v(S),$$

can be achieved by players in $S, \forall S \subseteq N$ [88]. The set of all so-called feasible payoff vectors of the game (N, v) [88] will be denoted as

$$X^*(N, v) = \{(x_i)_{i \in N} | \sum_{i \in N} x_i \leq v(N)\}.$$

Further, we will also use equivalent notation of payoff vectors $x := (x_i)_{i \in N}$. According to the classification of coalitional games proposed in [91], there are three major distinct classes:

- canonical coalitional games;
- coalition formation games;
- coalitional graph games.

The first two classes will be introduced in detail in the following subsections. Coalitional graph games are out of scope of this doctoral thesis.

2.2.1 Canonical coalitional games

Canonical coalitional games are the most common type of the cooperative games. All the concepts presented in this subsection can be found in [88], [85] and [38]. There are three main properties, that coalitional game should possess in order to be classified as a canonical [91]:

Characteristic function form. The first one is that coalitional game has to be in characteristic form [88]. For many readers, this requirement could seem redundant, since numerous publications do not make difference between concept of a value function and concept of characteristic function. However, the main property of a game in characteristic form is that the value of coalition S depends solely on coalition members, without dependence on how $N \setminus S$ is structured. Thus, $v : 2^N \to \mathbb{R}$ and additional sets from the definition of the value function are all singletones. In some literature, authors refer to this property as to absence of externalities.

Advantages of cooperation. The second property of canonical games is that cooperation is always prosperous (or at least guarantees the same utility). This property corresponds to superadditivity of the value function [88], i.e. it should fulfill:

$$v(S \cup T) \geq v(S) + v(T), \forall S, T \subseteq N, S \cap T = \emptyset.$$

Thus, no player can do worse by cooperating, than acting non-cooperatively. Sometimes, it is sufficient to consider only weakly superadditive games fullfilling

$$v(S \cup \{i\}) \geq v(S) + v(\{i\}), \forall i \in N, \forall S \subseteq N \setminus \{i\}.$$

If $\forall S, T \subseteq N, S \cap T = \emptyset$, the equality holds instead of inequality in the superadditivity definition, then we say that the characteristic function is additive. The additivity assumption can be equivalently rewritten as:

$$\sum_{i \in S} v(\{i\}) = v(S), \forall S \subseteq N.$$

Games with additive characteristic function are called inessential. Inessentiality derives from absence of a space to negotiate: payoff of a player i is uniquely determined by a value $v(\{i\})$. Additive games are subclass of the so-called constant sum games for which

$$v(S) + v(N \setminus S) = v(N), \forall S \subseteq N,$$

holds. Games with constant sum are often employed to describe political negotiations. It is peculiar, that the fact that a game is not essential does not imply it to be inessential. Gilles [38] defines essential game as a one, which fulfills

$$\sum_{i \in N} v(\{i\}) < v(N).$$

Thus, essentiality means, that if all players cooperate they are able to produce nontrivial amount of wealth, which can be distributed. The requirement of the superadditivity can be restrictive, and many authors require cohesiveness

$$v(N) \geq \sum_{S \in \mathcal{P}} v(S), \ \forall \mathcal{P} \in \mathcal{P}_N,$$

where $\mathcal{P}_N$ is a set of all partitions of N, or the weaker property

$$v(N) \geq \sum_{i \in N} v(\{i\}).$$

If the particular game (N, v) fails to satisfy superadditivity, it can be studied as (N, v_{SA}) using superadditive cover of a characteristic function

$$v_{SA}(S) = \max_{\mathcal{P} \in \mathcal{P}_S} \sum_{T \in \mathcal{P}} v(T).$$

It is important to note, that superadditivity is necessary only when working with the value function that has «positive connotation»: for example, if it describes profits. However, when $v(S)$ represents costs inflicted by S, the game is called a cost game and, in order to motivate players to cooperate, it should posses property of subadditivity:

$$v(S \cup T) \leq v(S) + v(T), \forall S, T \subseteq N, S \cap T = \emptyset.$$

Such a game can alternatively be studied by its savings formulation. Assume a cost game (N, v_{cost}), then a corresponding savings game $(N, v_{savings})$ can be defined as

$$v_{savings}(S) = \sum_{i \in S} v_{cost}(\{i\}) - v_{cost}(S).$$

The last note regarding superadditive games is their connection to class of monotonic games, where

$$v(S) \leq v(T), \forall S \subseteq T \subseteq N.$$

The two properties are not equivalent, since there is no general relationship between these classes of games. However, if, for a superadditive game (N, v), $v(S) \geq 0, \forall S \subseteq N$, it is always monotonic.

Distribution of wealth. Third and the last main property of canonical coalitional games is that their main objectives are to study possibility of forming grand coalition N and its properties, and to design allocations of the value produced by grand coalition $v(N)$ between players. The fact that the main interest is only focused on the study of the grand coalition can be explained by superaddtivity of canonical games. Due to this property, formation of the grand coalition is almost inevitable, since it is the most profitable possibility to all of the players.

Thus, the two fundamental questions of canonical coalitional games are: which payoffs can guarantee stability of the grand coalition and which payoffs distribute $v(N)$ between players in a “fair” way? These questions are addressed by a so-called solution concept.

Solutions of coopertaive games

In terms of CGT, a solution is defined as a function $\sigma : \Gamma = \{(N, v)\} \to 2^{X^*(N,v)}$, that assigns each game (N, v) a subset $\sigma(N, v) \subseteq X^*(N, v)$ [88]. In the following paragraphs, the most well-known solutions of canonical coalitional games will be defined and discussed. The less requiring solution is a set of preimputations, which can be defined as follows.

Definition (Preimputation)**.** A payoff vector $x \in X^*(N, v)$ is a preimputation, i.e. $x \in PI(N, v)$, if it satisfies the condition of group rationality also known as Pareto optimality or efficiency

$$\sum_{i \in N} x_i = v(N).$$

Thus, preimputations distribute wealth $v(N)$ completely. However, such a distribution does not reflect any negotiation power of players. The set of imputations embeds the potential claims of the players in the most simple fashion: it does not distribute to a player an amount of wealth, which is less than he/she is able to ensure on his/her own.

Definition (Imputation)**.** A payoff vector $x \in PI(N, v)$ is an imputation, i.e. $x \in I(N, v)$, if it satisfies condition of individual rationality

$$x_i \geq v(\{i\}), \forall i \in N.$$

For example, in inessential games there is always only one imputation:

$$I(N, v) = (v(\{1\}), ..., v(\{n\}).$$

Imputations can be compared by a relation of domination [38]: imputation x dominates imputation y through coalition $S, S \neq \emptyset$, iff

$$x_i > y_i, \forall i \in S,$$

$$\sum_{i \in S} x_i \leq v(S).$$

Such relation is denoted as $(x >_S y)$. We say, that imputation x dominates imputation y, i.e. $x > y$, if there exists a coalition S such that $x >_S y$. If for some $x \in I(N, v), \nexists y$, such that $y > x$, then we say that x is undominated. The domination relation leads us directly to the one of the most fundamental solution concepts in CGT: the core. The core $C(N, v)$ can be described via the set of undominated imputations in two ways:

1. $x \in C(N, v) \Rightarrow x \in I(N, v)$ and x is undominated;

2. For superadditive game (N, v), $x \in C(N, v) \Leftrightarrow x \in I(N, v)$ and x is undominated.

Alternatively, the Core can be defined as a set of imputations fulfilling coalitional rationality.

Definition (Core)**.** Core is a set

$$C = \{x | x \in I(N, v), \sum_{i \in S} x_i \geq v(S), \forall S \subseteq N\}.$$

Thus, core is a stable set of imputations, in which no group of players has an incentive to deviate and form a smaller coalition instead of N. The core is always well-defined, but in general does not satisfy non-emptiness [88]: for example, constant-sum games have non-empty core iff they are inessential. In general, verification of a core non-emptiness is an NP-complete problem, since increase in number of subsets with respect to number of players $|N|$ cannot be bounded by a polynomial function. This fact naturally brings the question: for which games the core is non-empty and how do we efficiently verify its non-emptiness?

To prove core non-emptiness, the linear programming techniques and duality are employed [11]. In particular, the idea of the proof lies in showing that solution of the following optimization problem

$$\min_{x_i, i \in N} \sum_{i \in S} x_i \tag{2.1}$$

$$\text{s.t.} \sum_{i \in S} x_i \geq v(S), \forall S \subseteq N, \tag{2.2}$$

is $v(N)$. This fact will directly imply existence of Pareto optimal coaltitionally rational imputation and, as a result, $C(N, v) \neq \emptyset$. To proceed further, notion of the characteristic vector $\lambda_S \in \mathbb{R}^n$ for $S \subseteq N$ is necessary:

$$\lambda_S^i = \begin{cases} 1, & \text{if } i \in S, \\ 0, & \text{otherwise.} \end{cases}$$

These characteristic vectors will help us to formulate the so-called dual problem (2.1,2.2). In the following paragraph, the duality principles of linear programming are explained in accordance with [85].

Duality principle of linear programming. Assume $m, n \in \mathbb{N}$ and the primal linear programming problem

$$\max_{x_j, j=1,\dots,n} \sum_{j=1}^{n} c_j x_j,$$

$$\text{s.t.} \sum_{j=1}^{n} a_{ij} x_j \leq b_i, \ i = 1, \dots, m,$$

$$x_j \geq 0, \ j = 1, \dots, n.$$

Then, its dual problem can be defined as

$$\min_{y_i, i=1,\dots,m} \sum_{i=1}^{m} b_i y_i$$

$$\text{s.t.} \sum_{i=1}^{m} a_{ij} y_i \geq c_j, \ j = 1, \dots, n.$$

$$y_i \geq 0, \ i = 1, \dots, m.$$

Then, for these problems the principles of weak and strong duality hold:

- **Weak:** For feasible x of primal problem and feasible y of dual problem $\sum_{j=1}^{n} c_j x_j \le \sum_{i=1}^{m} b_i y_i$ holds.
- **Strong:** If the primal and dual problems are both feasible, then there exist an optimum $\tilde{x}$ of the primal problem and optimum $\tilde{y}$ of its dual such that $\sum_{j=1}^{n} c_j \tilde{x}_j = \sum_{i=1}^{m} b_i \tilde{y}_i$.

Thus, according to the strong duality principle, the value of the previously introduced optimization problem (2.1,2.2) equals to $v(N)$ only if for all feasible vectors δ_S of its dual problem

$$\max_{\delta_S, S \subseteq N} \sum_{S \subseteq N} \delta_S v(S)$$

$$\text{s.t.} \sum_{S \subseteq N} \delta_S \lambda_S = \lambda_N,$$

$$\delta_S \ge 0, \forall S \subseteq N,$$

following relation

$$v(N) \ge \sum_{S \subseteq N} \delta_S v(S)$$

holds. However, [11] and [97], were able to efficiently reduce the set of δ_S for which this condition should be checked. They (independently) established the sufficient and necessary condition for a non-empty core using the concept of balanced collection.

Definition (Balanced collection)**.** A collection $\mathcal{B} \subseteq 2^N, \emptyset \notin \mathcal{B}$, is called balanced if positive numbers $\delta_S, S \in \mathcal{B}$, exist such that

$$\sum_{S \in \mathcal{B}} \delta_S \lambda_S = \lambda_N.$$

The weights $\delta_S, S \in \mathcal{B}$, from the previous definition are called a system of balanced weights. Then, the weak form of the Bondareva-Shapley theorem states that a game has non-empty core iff

$$v(N) \ge \sum_{S \in \mathcal{B}} \delta_S v(S)$$

for each balanced collection $\mathcal{B}$ and each system of balanced weights $\delta_S, S \in \mathcal{B}$. Games for which previous expression holds are called balanced. The strong form of the same theorem states that the same inequality has to be valid only for *minimal* balanced collections. These are balanced collections that do not contain a proper balanced subcollection or, equivalently, have a unique system of balancing weights.

From the theoretical point of view, there exist another type of games, that always have a non-empty core: these are convex games [88]. Convex games are games for which the following inequality holds

$$v(S \cup T) + v(S \cap T) \ge v(S) + v(T), \forall S, T \subseteq N,$$

or, alternatively, convexity can be represented as

$$v(T \cup \{i\}) - v(T) \ge v(S \cup \{i\}) - v(S), \forall i \in N, \forall S \subseteq T \subseteq N \setminus \{i\}.$$

Previous inequality states that the game is convex if and only if marginal contribution of each player to coalition is non-decreasing with respect to set-theoretic inclusion. In other words, each player has an incentive to join larger coalition because his possible payoff will increase. Convexity can be also described as a "snow-ball effect". Convexity of the game implies its balancedness and as a result non-emptiness of the core.

Still, a game might possibly have an empty core or, on the contrary, relatively large core, so it can lose its informativeness. Then, alternative solution concepts such as ϵ-core can be employed [96].

Definition (ϵ-core)**.** For $\epsilon \in \mathbb{R}$, ϵ-core is a set

$$C_\epsilon \ (N, v) = \{x | x \in PI(N, v) \text{ and } \sum_{i \in S} x_i \geq v(S) - \epsilon, \forall S \subset N\}.$$

Clearly, depending on the choice of the parameter ϵ, ϵ-core can be seen as a relaxation of the group rationality ($\epsilon > 0$) or as a more restrictive solution concept ($\epsilon < 0$). From the concept of ϵ-core, the least-core can be deduced [96].

Definition (Least-core)**.** Least-core of a game (N, v) is an intersection of all non-empty ϵ-cores

$$LC(N, v) = \cap_{\epsilon \in \mathbb{R}: C_\epsilon \neq \emptyset} C_\epsilon.$$

The least-core is a reasonable alternative to the core in case of an empty core. In the end of this subsection, we introduce a solution concept known as the Weber set, which is strongly related to the core since it is one of so-called core covers [38].

Definition (Core cover)**.** Solution $\sigma(N, v)$ is a core cover if

$$C(N, v) \subseteq \sigma(N, v)$$

holds for every game (N, v).

In this section, we focus solely on the definition of Weber set on 2^N. In general, Weber set can be defined on $\Omega \subseteq 2^N$ fulfilling some regularity properties described in [38].

Definition (Weber set on 2^N)**.** Weber set on 2^N is defined as

$$W(N, v) = conv\{x^\alpha \in \mathbb{R}^n \mid \alpha \text{ is a permutation on N}\},$$

where *conv* denotes convex hull of a set,

$$x_i^\alpha = v(R_i) - v(R_i \setminus \{i\}),$$

and

$$R_i = \{\alpha(1), ..., \alpha(j)\}, \text{ where } j \in N \text{ such that } \alpha(j) = i.$$

With respect to $W(N, v)$, convex games demonstrate noteworthy properties since

$$C(N, v) = W(N, v) \Leftrightarrow (N, v) \text{ is convex.}$$

All the above-mentioned solution concepts are defined as subsets of imputations with particular requirements on their elements. However, a proper mathematical definition of solution states that it is a function which assigns to every game a subset of feasible allocations of the game. Thus, a solution is not always a set, but can be just one allocation.

Solution as a single allocation

The solution concepts, mentioned in the previous subsection, mostly suffer from the same disadvantages: they can be empty or, on the contrary, too large. These complications serve as a motivation for application of the solution concepts result of which yields a unique allocation vector. We have already pointed out some inconsistencies in literature regarding essentiality and inessentiality. There are also inconsistencies regarding the value concept. Gilles [38] defines value directly as a function

$$\phi : \Gamma = \{(N, v)\} \to \mathbb{R}^n$$

without any further assumptions. However, in [88], it is stated that if solution σ fulfills $|\sigma(N, v)| = 1$ for every (N, v) it only possesses the property of being single-valued. The authors establish further non-trivial properties for a single-valued solution to be called value. In fact, this problem does not have any major consequences, since these properties are fulfilled by all the single-valued solution concepts discussed in this subsection.

Shapley value. One of the most popular single-valued solution concepts is the Shapley Value (Shapley defined it as a value of the game) [95]. The Shapley Value ϕ (ϕ_i will denote payoff obtained by player i via the Shapley value) has been defined as the unique solution that satisfies the four axioms:

- Efficiency: $\sum_{i \in N} \phi_i(N, v) = v(N)$;
- Symmetry: $v(S \cup \{i\}) = v(S \cup \{j\}), \forall S \in 2^N, i, j \notin S, \Rightarrow \phi_i(N, v) = \phi_j(N, v)$;
- Dummy: $v(S \cup \{i\}) = v(S), \forall S \in 2^N, i \notin S, \Rightarrow \phi_i(N, v) = 0$;
- Additivity: u, v are characteristic functions $\Rightarrow \phi(N, u+v) = \phi(N, u) + \phi(N, v)$, where $u + v(S) := u(S) + v(S),\ \forall S \subseteq N$.

Efficiency makes Shapley value a preimputation. Symmetry says that if two players have the same contributions to each coalition, then their payoffs have to be the same. Dummy axiom states, that if player does not contribute to any coalition, his payoff should be zero. Additivity connects values of different games and contribute to uniqueness of the Shapley value. Proof of uniqueness can be found in [88].

At first, we demonstrate a way of computation of the Shapley value based on the unanimity games [38] and Harsanyi dividends [44].

Definition (Unanimity game)**.** For a players set N and any nonempty $S \subseteq N$, the value function of the corresponding unanimity game (N, v_S) is defined as

$$v_S(T) = \begin{cases} 1, & \text{if } S \subseteq T, \\ 0, & \text{otherwise.} \end{cases}$$

The set of unanimity games $\{(N, v_S) | S \subseteq N\}$ forms a basis of all cooperative games (N, v) with the set of players N [88]. Exactly Harsanyi dividends λ_S [44] are used to represent games in this basis:

$$v(T) = \sum_{S \subseteq N,\ S \neq \emptyset} \lambda_S v_S(T),$$

where

$$\lambda_S = v(S) - \sum_{M \subset S} \lambda_M.$$

Then, in terms of the Harsanyi dividends, the Shapley value can be expressed as

$$\phi_i\ (N, v) = \sum_{S \subseteq N:\ i \in S} \frac{\lambda_S}{|S|}.$$

If we interpret a Harsanyi dividend as a real «added value» of coalition S with respect to its subsets, then the Shapley value equally distributes the generated wealth in accordance with these contributions. An alternative and a more classical formulation of the Shapley value is

$$\phi_i(N, v) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(|N| - |S| - 1)!}{|N|!} (v(S \cup \{i\}) - v(S)).$$

This formulation makes one great disadvantage of the Shapley value obvious: summation proceeds through all possible subsets of grand coalition and might lead to combinatorial explosion in case of N with a large cardinality. The expression above provides another possible interpretation of the Shapley value. It assigns to each player his expected payoff in the following situation: players arrive randomly and each order has the equal probability, when player arrives he/she obtains his/her marginal contribution to the coalition of the already arrived players. From such an interpretation, we can directly establish the probabilistic formulation of the Shapley value, which is directly connected to the Weber set [38]

$$\phi_i(N, v) = \frac{1}{n!} \sum_{\alpha \in \mathfrak{G}_N} x_i^{\alpha},$$

where $\mathfrak{G}_N$ is the collection of all permutations on N with $n! = |\mathfrak{G}_N|$. In this doctoral thesis, this formulation will prove itself considerably useful, since it will allow for estimation of the Shapley value in polynomial time eliminating potential problems with its computational complexity. Now, we will discuss the relation between the Core, the Weber set and the Shapley value. It is important to note that the last formulation directly implies that the Shapley value is a center of gravity of the Weber set, which coincide with the core in the case of convex games. However, in general, the Shapley value does not have to be contained in the core. Moreover, the Shapley value might not be even an imputation, if the considered game is not superadditive.

Nucleolus. The second most popular single-valued solution is the nucleolus [100]. The idea behind the nucleolus is finding the imputation, which makes the most unsatisfied coalition as satisfied as possible. More specifically it is such an imputation, that lexicographically minimizes dissatisfactions of the coalitions measured by the excesses

$$e(x, S) = v(S) - \sum_{i \in S} x_i.$$

To provide a better image about excesses, it can be emphasized that

$$x \in I(N,v) \Rightarrow e(x,N) = 0,$$

$$x \in C(N,v) \Rightarrow e(x,S) \leq 0, \forall S \subseteq N.$$

Assume $\theta(x) \in \mathbb{R}^{2^n}$ will denote the vector with its elements being excesses of all coalitions arranged in non-increasing order, i.e. with elements $\theta_k(x) = e(S_k, x), k = 1, .., 2^n$, such that $e(S_k, x) \geq e(S_{k+1}, x)$. Then, an imputation $x \in I(N,v)$ is lexicographically smaller than $y \in I(N,v)$, i.e. $\theta(x) <_l \theta(y)$, iff

$$\exists l \in \mathbb{N}, 1 \leq l \leq 2^n : \theta_j(x) = \theta_j(y), \forall j < l \text{ and } \theta_l(x) < \theta_l(y).$$

We write $\theta(x) \leq_l \theta(y)$, in case $\theta(x) <_l \theta(y)$ or $\theta(x) = \theta(y)$. Using these relation, the nucleolus can be defined as follows [100].

Definition (Nucleolus)**.** Consider a game (N,v), such that $I(N,v) \neq \emptyset$. Then, the nucleolus η of the game (N,v) is

$$\eta(N,v) = x, \text{ s.t. } x \in I(N,v) \text{ and } \theta(x) \leq_l \theta(y), \forall y \in I(N,v).$$

In fact, nucleolus can be defined on arbitrary subset of $\mathbb{R}^n$, not only on $I(N,v)$ [100]. Later, we will use this generalized definition of nucleolus on arbitrary set X:

$$\eta(N,v,X) = \{x | x \in X \text{ and } \theta(x) \leq_l \theta(y), \forall y \in X\},$$

with $\eta(N,v) = \eta\left(N,v,I(N,v)\right)$. If X is compact, then nucleolus is non-empty, and if it is convex, then nucleolus is unique. Thus, solution $\eta(N,v)$ is unique, as an imputation it satisfies individual and group rationality, and additionally it satisfies symmetry and dummy axioms of the Shapley value [38]. Moreover, nucleolus always exists and if the core is not empty, it contains nucleolus. Nucleolus also belongs to every non-empty ϵ-core, and, as a result, to the least core [100]. The computation of the nucleolus dwells in the sequential minimization of the excesses using linear programming techniques until unique solution is obtained. Unfortunately, this algorithm requires at most $2^n - 1$ steps. Thus, analogically to the Shapley value, there is a problem with the computational complexity. Precise description of the algorithm can be found in [39].

The original definition of the nucleolus has been introduced using the so-called 0-normalized games, for which $v(\{i\}) = 0, \forall i \in N$ [100]. The notion of a payoff vector $(x_i)_{i \in N}$ was then automatically assuming that $x_i \geq 0, \forall i \in N$, and $\sum_{i \in N} x_i = v(N)$ [100]. However, this approach can easily be generalized using concept of strategic equivalence presented in [88].

Definition. Games (N,v) and (N,w) are strategically equivalent if there $\exists \alpha > 0$ and $(\beta_i)_{i \in N}$, such that

$$w(S) = \alpha v(S) + \sum_{i \in S} \beta_i, \forall S \subseteq N.$$

This relation is equivalence on the space of games with players set N. Thus, strategically equivalent games have the same properties and a solution of one game is a simple linear transformation of the analogical solution of the other game (if these properties and solutions preserve linear transformations). Clearly, every game (N, v) is strategically equivalent to a 0-normalized game through the choice

$$\alpha = 1,\ \beta_i = -v(\{i\}), \forall i \in N.$$

Thus, the main assumption of $I(N, v) \neq \emptyset$ remains natural.

2.2.2 Coalition formation games

Coalition formation games can be either in characteristic or partition form [91]. In games in partition form, the value of the coalition $v(S, \mathcal{P})$ depends not only on participants of coalition S, but in addition on the actual coalition structure $\mathcal{P} \in \mathcal{P}_N$, where $\mathcal{P}_N$ is set of all partitions of players set N [88]. Thus, the value of S depends on the cooperation between external players outside of S given by $\mathcal{P}$. However, games with $v : 2^N \times \mathcal{P}_N \to \mathbb{R}$ are significantly computationally complex and are out of scope of these Ph.D. thesis. Coalition formation games are generally not superadditive, because cooperation can often bring additional costs implying that formation of the grand coalition is not always desirable and optimal. Their main objective is to study the final state of the game and describe the properties of the resulting structure. In addition, impact of possible changes in game environment on the game outcome and wealth distribution can be studied. In general, coalition formation games can be divided into two basic types: static and dynamic [91].

Static coalition formation games

Static coalition formation games dwell in study of already imposed coalition structure, that can be predefined by some external factor. Such games can be uniquely defined by triple $(N, v, \mathcal{P})$ [5]. This subclass is not dramatically distinct from canonical cooperative games and can be viewed as their superstructure. The concepts from previous section, can be preserved in static coalition formation games with suitable modifications. The major change is that we have to replace Pareto optimality with the so-called relative efficiency/rationality. All the concepts presented in this section can be found in the original work [5].

Definition (Relative efficiency)**.** The payoff vector x is relative efficient for the static coalition formation game $(N, v, \mathcal{P})$ iff

$$\sum_{i \in S} x_i = v(S), \forall S \in \mathcal{P}.$$

Then, we introduce the generalization of $X^*(N, v)$ of a canonical game for the static coalition formation game $(N, v, \mathcal{P})$ [5]:

$$\tilde{X}(N, v, \mathcal{P}) = \{x | \sum_{i \in S} x_i = v(S), \forall S \in \mathcal{P}\}.$$

Now, we can proceed to the study of the previously introduced solution concepts with respect to static coalition formation games.

Solutions of games with coalition structure. The main point of this discussion is to establish, which solution concepts possess so-called restriction property. This property dwells in fact, that in order to compute the solution of static coalition formation game, it is sufficient to compute its classical games analogy separately with respect to each coalition $S \in \mathcal{P}$ [5]. Thus, this property can be formally defined as follows.

Definition (Restriction property)**.** Assume the solution of the coalition formation game

$$\sigma^{CF} : \Gamma^{CF} = \{(N, v, \mathcal{P})\} \to 2^{\tilde{X}(N,v,\mathcal{P})}.$$

We say that σ^{CF} has a restriction property if

$$\sigma^{CF}(N, v, \mathcal{P}) = \Pi_{S\in\mathcal{P}}\sigma^{CF}(S, v|_S, \{S\}) = \Pi_{S\in\mathcal{P}}\sigma(S, v|_S), \forall(N, v, \mathcal{P}) \in \Gamma^{CF},$$

where $v|_S$ denotes restriction of v onto set $S \in \mathcal{P}$.

At first, we focus on the coalition formation analogy of the Shapley value.

Definition (Coalition formation Shapley value)**.** There exist, unique single-valued solution $\phi^{CF}(N, v, \mathcal{P})$ that satisfies:

- **Relative efficiency:** For all $S \in \mathcal{P}$, $\sum_{i\in S} \phi_i^{CF}(N, v, \mathcal{P}) = v(S)$;
- **Symmetry:** For all permutations $\alpha \in \mathfrak{G}_N$ such that $\alpha(S) = S, \forall S \in \mathcal{P}$, the following expression holds

  $$\sum_{i\in S} \phi_i^{CF}(N, \alpha v, \mathcal{P}) = \sum_{i\in\alpha(S)} \phi_i^{CF}(N, v, \mathcal{P}), \forall S \subseteq N,$$

  where $\alpha v(S) = v(\alpha(S))$ and $\alpha(S) = \{\alpha(i)|i \in S\}$.
- **Additivity:** $\phi^{CF}(N, v + w, \mathcal{P}) = \phi^{CF}(N, v, \mathcal{P}) + \phi^{CF}(N, w, \mathcal{P})$;
- **Dummy:** If $v(S \cup \{i\}) = v(S), \forall S \subseteq N, i \notin S$, then $\phi_i^{CF}(N, v, \mathcal{P}) = 0$.

It is important to note, that we have introduced the novel definition of symmetry, which is equivalent to the one already presented in the definition of the Shapley value, which describes the so-called equal treatment property [38]. An expression for computation of $\phi^{CF}(N, v, \mathcal{P})$ is omitted, due to the following theorem.

Theorem 2.2.1. *Coalition formation Shapley value ϕ^{CF} has a restriction property.*

Proof. Proof can be found in [5]. □

Thus, $\phi^{CF}(N, v, \mathcal{P})$ can be computed for each player $i \in S$ from set $S \in \mathcal{P}$, using the relation:

$$\phi_i^{CF}(N, v, \mathcal{P}) = \phi_i(S, v|_S).$$

Now, our attention will be focused on nucleolus and core for static coalition formation games. It has been already emphasized, that nucleolus can be defined on an arbitrary set, therefore generalization of nucleolus for coalition formation games is straightforward [5].

Definition (Nucleolus of coalition formation game)**.** For static coalition formation game (N, v, P), nucleolus is defined as

$$\eta^{CF}(N, v, \mathcal{P}) = \eta\left(N, v, \tilde{I}(N, v, \mathcal{P})\right),$$

where $\tilde{I}(N, v, \mathcal{P}) = \{x \in \tilde{X}(N, v, \mathcal{P}) | x_i \geq v(\{i\}), \forall i \in N\}$.

Analogically, we can generalize the core as follows [5].

Definition (Core of coalition formation game)**.** For static coalition formation game $(N, v, \mathcal{P})$, core is defined as

$$C^{CF}(N, v, \mathcal{P}) = \{x \in \tilde{X}(N, v, \mathcal{P}) | \sum_{i \in S} x_i \geq v(S), \forall S \subseteq N\}.$$

Unfortunately, following theorem [5] complicates situation with computation and study of these concepts within class of static coalition formation games.

Theorem 2.2.2. *Core C^{CF} and nucleolus η^{CF} of coalition formation game do **not** have a restriction property.*

Proof. Counterexamples can be found in [5]. □

Nevertheless, [5] were able to find some connection between game $(N, v, \mathcal{P})$ and games played between players in $S \in \mathcal{P}$. For example, for static coalition formation game $(N, v, \mathcal{P})$, the following relation holds

$$x \in C^{CF}(N, v, \mathcal{P}) \Rightarrow (x_i)_{i \in S} \in C^{CF}(S, v|_S, \{S\}), \forall S \in \mathcal{P}.$$

The practical implications of these theoretical findings can be summarized as follows. Among the Shapley value, the nucleolus and the core, only the former concept has some consistency with respect to coalition formation games: the Shapley value of sub-games on coalitions defined by partition is the Shapley value of the whole coalition formation game. Thus, for the Shapley value, there is no question about which solution $\left(\phi_i^{CF}(N, v, P)\right)_{i \in N}$ or $\Pi_{S \in \mathcal{P}, i \in S} \phi_i(S, v|_S)$ has to be interpreted as correct and there is no need to modify computational process, which can be performed on a smaller coalitions instead of the grand one. Unfortunately, this does not hold for the nucleolus or the core. Whereas each point of the core (including the nucleolus) of the $(N, v, \mathcal{P})$ is in the core of $(S, v|_S, \{S\})$, this is not possible to directly use the core or the nucleolus of these sub-games to obtain these solutions for the whole game without modification of the value function. Therefore, to obtain the complete image of the game, all solutions has to be calculated separately, but their results might be contradictory. This ambiguity is one of the main drawbacks of static coalition formation approach employing the core or the nucleolus concepts.

Dynamic coalition formation games of a distributed type

Whereas static games study properties of the already given structure, dynamic games analyze the process of the formation of suitable coalition structure for the given game and its stability and evolution in the face of the internal and external factors. In dynamic games, the main point is finding coalition formation that maximizes welfare of the players in a pre-defined sense. However, a framework of coalition formation games of dynamic type, is not so formalized, and, as a result, is a more diverse and application-specific branch [91]. There are two major distinct approaches: centralized and distributed. The centralized approach dwells in finding partition of a set of players, which maximizes welfare, by iterating over all partitions of N. Such extensive search is generally NP-complete due to the fact, that number of partitions grows exponentially depending on the cardinality of N [93]. Thus, in many application-oriented problems, study of coalition formation process in a distributed manner is preferred. Distributed coalition formation is based on the idea of the autonomy of players decision about joining a coalition. Considerations, established in [4], provide necessary concepts for the description of the general process of distributed coalition formation by the means of merges and splits. This approach is based on the three following concepts.

Comparison relation. At first, relation, which enables to compare collections of coalitions, has to be defined. For the sake of clarity, we define the collection of the coalitions, as a family $C = \{C_1, \ldots, C_l\}$ of mutually disjoint subsets of N. Recall, that partition is a particular case of collection, for which $\cup_{i=1}^{l} C_i = N$ holds. Further, for $C = \{C_1, \ldots, C_l\}$, the notation $\cup C$ will stand for $\cup_{i=1}^{l} C_i$. Collections A and B can be compared, iff $\cup A = \cup B$ holds. Then, a comparison relation $\rhd$ can be defined as follows.

Definition (Comparison relation)**.** Comparison relation $\rhd$ is an irreflexive, transitive relation, which satisfies $\forall A, B, C, D:\ \cup A = \cup B,\ \cup C = \cup D,\ (\cup A) \cap (\cup C) = \emptyset$, two following conditions of the monotonicity:

$$A \rhd B,\ C \rhd D \Rightarrow A \cup C \rhd B \cup D,$$

$$A \rhd B \Rightarrow A \cup C \rhd B \cup C.$$

Intuitive meaning of notation $A \rhd B$ is that A partitions some set in a way, that is preferred by elements of this set over B. A comparison relation $\rhd$ is semi-linear, if for all collections $A, B : \cup A = \cup B$, either $A \rhd B$ or $B \rhd A$. In a framework of CGT, collections of coalitions can be compared by the mean of the value function of the game: for the collections A and B

$$A \rhd B \Leftrightarrow v(A) \rhd v(B),$$

where for the collection $A = \{A_1, \ldots, A_m\},\ v(A) = \{v(A_1), \ldots, v(A_m)\}$. Then, we could define semi-linear comparison relations between sets $v(A) = \{v(A_1), \ldots, v(A_m)\}$ and $v(B) = \{v(B_1), \ldots, v(B_n)\}$ of reals such as

- utilitarian order: $A \rhd_{ut} B \Leftrightarrow v(A) \rhd_{ut} v(B) \Leftrightarrow \sum_{i=1}^{m} v(A_i) > \sum_{i=1}^{n} v(B_i)$;
- Nash order: $A \rhd_{Nash} B \Leftrightarrow v(A) \rhd_{Nash} v(B) \Leftrightarrow \Pi_{i=1}^{m} v(A_i) > \Pi_{i=1}^{n} v(B_i)$.

The utilitarian order states, that collections of coalitions, that guarantee greater social welfare (total utility), are preferred by the players, whereas the Nash order states, that collections, which provide equal distributions, are preferred (note, that Nash order is reasonable only for non-negative values of $v(S)$).

Merge and split rules. Once a comparison order is defined, dynamic process of distributed coalition formation can be started through two possible rules on the set of partitions of the players set:

- merge: $\{T_1, \ldots, T_k\} \cup R \to_M \{\cup_{j=1}^k T_j\} \cup R$, if $\{\cup_{j=1}^k T_j\} \triangleright \{T_1, \ldots, T_k\}$;
- split: $\{\cup_{j=1}^k T_j\} \cup R \to_S \{T_1, \ldots, T_k\} \cup R$, if $\{T_1, \ldots, T_k\} \triangleright \{\cup_{j=1}^k T_j\}$.

In the above-described rules, $\{T_1, \ldots, T_k\} \cup R$ is some partition of the players' set N, where R represents partition of other players, who are not involved in any $T_i, i = 1, \ldots, k$. For the comparison relation $\triangleright$, every iterative application of the merge and split rules terminates [4].

These theoretical rules can be explained as follows. Assume utilitarian order and coalition structure $\mathcal{C}_0 = \{S_1, S_2, S_3\} \cup R$, where $S_i,\ i = 1, 2, 3$, are disjoint coalitions and R represents partition of other players, who are not involved in any $S_i,\ i = 1, 2, 3$. If $v\left(\cup_{i=1}^3 S_i\right) > \sum_{i=1}^3 v\left(S_i\right)$, then application of merge rule will lead to a new coalition structure $\mathcal{C}_1 = \{\cup_{i=1}^3 S_i\} \cup R$. Analogically, if coalition structure $\mathcal{C}_0 = \{\cup_{i=1}^3 S_i\} \cup R$ is assumed and $v\left(\cup_{i=1}^3 S_i\right) < \sum_{i=1}^3 v\left(S_i\right)$, then the result of the split rule will be $\mathcal{C}_1 = \{S_1, S_2, S_3\} \cup R$. Now, our attention can be focused on the two following questions:" under what conditions does different sequences of merge and split rules lead to the same outcome?" and "is this outcome unique?".

Defection function. Important concept, used for checking stability and uniqueness of the coalition formation outcome, is defection function $\mathcal{D}$, that assigns to each partition $\mathcal{P} \in \mathcal{P}_N$, some collections $\mathcal{D}(\mathcal{P})$ of the grand coalition. In other words, $\mathcal{D}(\mathcal{P})$ consists of collections, that can be formed by players in N by leaving partition $\mathcal{P}$. The two most obvious ways of defining $\mathcal{D}$ are $\mathcal{D}_c$, which assigns to every partition $\mathcal{P}$ the family of all collections in N, and $\mathcal{D}_p$, which assigns to every partition $\mathcal{P}$ the family $\mathcal{P}_N$. Then, for the given defection function $\mathcal{D}$ and comparison relation $\triangleright$ stability of a partition $\mathcal{P} = \{S_1, ..., S_k\}$ can be defined as follows.

Definition. Partition $\mathcal{P}$ is $\mathcal{D}$-stable iff

$$C[\mathcal{P}] \triangleright C,\ \forall C \in \mathcal{D}(\mathcal{P}),\ C[\mathcal{P}] \neq C,$$

where

$$C[\mathcal{P}] = \{S_1 \cap (\cup C), \ldots, S_k \cap (\cup C)\} \setminus \{\emptyset\}.$$

Thus, the $\mathcal{D}$-stability means, that each collection C from $\mathcal{D}(\mathcal{P})$ in the frame of partition $\mathcal{P}$ is «better» compared to C. However, checking both $\mathcal{D}_c$ and $\mathcal{D}_p$ stability types for large games through extensive search is computationally challenging. Whereas for the case of semi-linear comparison a $\mathcal{D}_p$-stable partition always exists, the $\mathcal{D}_c$-partition does not need to exist even under this assumption. Moreover, they are not directly connected to process of merging and splitting: terminal partition obtained by merge and split rules is not guaranteed to be $\mathcal{D}_c$- or $\mathcal{D}_p$-stable. Due to these complications, another type of stability is applied. This is $\mathcal{D}_{hp}$ type stability, where $\mathcal{D}_{hp}$ is a defection function, which assigns to each partition $\mathcal{P}$ the collection $\mathcal{D}_{hp}(\mathcal{P})$ consisting of all partitions, that can be obtained from $\mathcal{P}$ by performing exactly one merge or split operation. $\mathcal{D}_{hp}$-stable partition can be found as an outcome of iterating merge and split rules. Thus, it always exists. In [3], it is proven, that $\mathcal{D}_{hp}$-stable partition is unique $\mathcal{D}_{hp}$-stable and unique $\mathcal{D}_p$-stable partition for the case of the utilitarian order. To summarize, whereas the mechanism of comparison relation built-in into merge and split rules enables to obtain some predictable outcome of the coalition formation process, defection function-based stability describes properties of such an outcome.

Chapter 3

Review

The main goal of this section is to provide a thorough overview of the recent publications about general applications of GT in sustainability research. Search for the articles has been performed via the Web of Science Core collection database using the keyword «game theory application». Only articles in English published between 2017 and 2020 (when this research has started) have been considered. This 3 years range has helped us to rationally limit the amount of the found articles. Articles with solely theoretically oriented research, or not related to sustainability research have been excluded from the review. As a result, a complete review of 33 articles has been performed. The reviewed articles have been characterized according to GT branches, applied theoretical concepts, and other important factors in «General review.xlsx» of Appendix.

3.1 Review summary

The main findings and implications of the performed review will be summarized and discussed in this section. In particular, the following areas, where GT is applied to embed sustainability principles, have been studied: WM (related to 21.2% of the reviewed articles), supply chain management (33.3%), policy design (24.2%), water resource management (18.2%) and energetics (18.2%).

Supply chain management and waste management. Fields of WM and of supply chain management are very similar, since practical WM is mainly based on optimal management of the supply chain and employs many techniques from operations research [7]. When managing waste or general supply chain, it is common, that there exist some pre-defined roles, that are assigned to players. These roles are frequently placed on the different level of hierarchy, such as in [112]. This is why Stackelberg games are of such popularity in these fields of research. The simplest Stackelberg model includes two entities, the leader and follower, and both entities are trying to anticipate decision of the opponent. In fact, the Stackelberg game can be viewed as a particular case of a bilevel optimization problem [60]. Therefore, along with the backward induction, the Karush-Kuhn-Tucker (KKT) conditions are used to establish the NE in the Stackelberg games. Some of the reviewed articles are focused on comparison of a centralized cooperation model and a non-cooperative model within the supply chain or the WM network [37]. It is mainly considered, that the non-cooperative case is more insightful and realistic, than the mandatory cooperation. However, it is not realistic to completely neglect possibility of cooperation. Application of differential games, which bear continuous game process, has been demonstrated in [41]. It is a promising approach, that combines optimal control theory and GT.

Policy design. In this area, it is important to study reaction [31] and process of adaptation [16] to newly implemented policies, with a focus on a stable outcome of such dynamic processes. This is a reason, why evolutionary games are most common in a field of policy design, since EGT provides an explicitly dynamical point of view missing from the traditional theory [94]. Moreover, it enables to assume limited rationality of involved parties. Consequently, the most popular solution concepts in policy design games are evolutionarily stable strategy (ESS) and replicator dynamics [94]. CGT can be potentially applied to design a fair distribution of emission reductions using its solution concepts [64].

Water resource management. Since irresponsible water consumption represents externalities for other users, cooperation in management of water resources is vital. Due to this fact, CGT is expectedly the most popular GT branch within this area of research. Moreover, it enables to define solutions of water management games, which reflect legislative specifics, as well as the nature of a water resource, and locations of users upstream and downstream of a river [105]. CGT also enables to distribute water cleaning costs in a fair way, which is a common problem in this field [14]. As in the area of policy design, when some political or biological issues are involved, non-cooperative games in matrix form are applied [42]. The fuzzy coalitional games [87], where players take part in a coalition only to a certain extent, are also considered in this area [53]. The water resource management is the only field, where coalition formation games and graph games have been considered. Graph games [77] are applied to management of water [105], since the graph structure enables to incorporate geographical connections between subjects. The coalitional graph games also possess solution concepts, which can take into account role of the player in the network structure [50].

Energetics. Due to the generality of this field, it demonstrates a rather uniform application of the majority of GT concepts. In energetics, CGT enables to design a fair distribution of energy between users [68], optimal capacities of system parts [43] or reasonable allocations of energy costs [69]. Compared to other fields, it even employs solution concept of the Core [43]. When dealing with the energetical supply, Stackelberg games are applied [92], since again, the supplier and consumer are clearly defined roles for system participants. Energetics also demonstrates the application of classical normal form games, which are solved through NE using the derivatives of payoff functions [70].

3.2 Findings and suggestions

From the reviewed articles, it can be deduced, that GT can be used as a fully-right standalone DM method, as well as a complement to other DM methods. GT can be applied to design government policies, supply chains, resource allocations, and real, tangible engineering projects. Moreover, it can take into account uncertainties and probabilities of different types, including fuzzy sets, and serve as a basis for performing sensitivity analysis. The review confirms, that GT is frequently applied to environmentally oriented research, but also reveals, that game-theoretic approaches require further improvement. While future applications of NGT should embed models considering more complex leader-follower games and uncertainties, in CGT-related research, more attention should be paid to game classes describing cooperation restrictions. Moreover, computational complexity makes applications of GT to instances with many players inconvenient. This problem represents a substantial research gap and a potential direction for further studies. Overcoming the computational issues, might considerably promote application of GT across different fields of research. The main findings of the review and the resulting implications can be summarized as follows.

- Almost all reviewed articles, considering cooperative games, deal only with canonical coalitional games, ignoring other classes. There is an obvious absence of application-oriented research sufficiently employing coalition formation and coalitional graph games.

- Applications of NGT dominate over other studied branches. This fact can be easily explained by diversity of its approaches, intuitive solution concepts, and relatively simple formalization of the game process. Thus, NE is expectedly the most eminent concept of GT. Due to supply chain management importance, backward induction (often accompanied by the KKT and bilevel programming) also plays a significant role in contemporary research. While some instances of the Stackelberg games are well studied, instances with multiple leaders achieving mutual equilibrium represent a research gap. Naturally, replicator dynamics and the ESS are mentioned in all articles, where EGT is applied. The core and the Shapley value are also of great significance, but some of the cooperative instances are studied without particular wealth distribution suggestions.

- Only slightly more than half (57%) of articles have considered real data in the performed case studies. Due to the applied nature of the performed review, it can be viewed as insufficient. Evidently, there are not enough applications of GT to real instances.

- 75% of the articles consider either 2 or 3 players, while only 3 articles consider more than ten players. Therefore, there is insufficient application of GT to instances with a large number of agents.

3.3 Scope of the research

The author has decided to focus on the application of GT to the problems of sustainable WM, with respect to the found research gaps. Since problems of environmental degradation and depletion of natural resources have become eminent challenges for the modern society, WM is a currently actively developing area of research, which aims to embed sustainability principles into the majority of the underlying processes. WM is closely connected to the concept of CE because one of their common goals is an effective treatment of solid waste. In particular, the European Union (EU) countries have recently adopted the CE Package (CEP) [18] to legislatively embed principles of CE and effective WM. CEP sets up a series of goals which dwell in a decrease of the amount of solid waste that is being landfilled and in an increase of its material and energy recovery. This package includes the requirement, that all EU member countries must recycle at least 60% of produced mixed solid waste (MSW) by 2030 and 65% by the year 2035 [26]. According to the update of the above-described package, only 10% of the total MSW amount can be landfilled by the year 2035 [19]. These milestones gradually become part of member states' legislation, including prioritized energy recovery of MSW in appropriate facilities. Such incentives require reliable strategic planning from all involved stakeholders: government, waste treatment facilities managers and investors, WM companies, and municipalities. This fact makes study of sustainable WM networks crucial.

Currently employed game-theoretic models in this area lack more sophisticated approaches, real data-based case studies, and are often limited to comparison of fully cooperative and non-cooperative cases, or to solution of simple matrix-form games. Thanks to cooperation on research projects with the Institute of Process Engineering, Faculty of Mechanical Engineering, Brno University of Technology, real data and operation conditions of WM networks, in the form of waste production, price levels, capacities and infrastructure, are available to experiment with designed approaches under conditions, that are maximally close to real ones. In the next chapter, it will be also demonstrated, that the considered area of research possesses previously mentioned complex instances of Stackelberg games with multiple leaders, that have not been sufficiently studied. These games deal with setting of gate fee for waste treatment facilities and their optimal capacity design. Moreover, WM has a great potential in application of classes of cooperative games with cooperation restrictions. In particular, dynamic coalition formation games between municipalities are of great interest, since individual municipalities generally are not able to efficiently and economically dispose of the produced waste. The considered task also brings further challenges, such as development and employment of the approaches, that are able to handle large number of players. The proposed game theoretic approaches to the considered problems will be validated through realistic case studies[1].

[1]Data and results of the case study from section 4.1, resp. 4.2, can be found in «Data 1.xlsx», resp. «Data 2.xlsx», of Appendix.

Chapter 4

Games in waste management

The main issues of WM are monitoring and regulation of the collection, transportation, treatment, and disposal of waste [1]. Whereas the recyclable waste fits perfectly into the design of CE closed production cycles, the non-recyclable fraction of MSW cannot be utilized in the same way. However, the energy potential of non-recyclable waste can be restored through Waste-to-Energy (WtE) technology [59]. It is expected that WtE plants will play an important role in waste treatment under CEP legislative changes [74]. Whereas in the past, incineration of MSW has been a source of substantial pollution, nowadays, due to the continuous development of WtE technology, WtE plants can serve as an environmentally friendly source of energy [110]. In [89], the WtE environmental impact has been thoroughly studied. The research concluded that WtE, as a combination of WM practice and electricity sources, can provide climate change benefits. However, if it is considered a renewable energy source solely, it cannot compete with other sources regarding greenhouse gas emissions. On the other side, it is more stable than wind power or solar energy [111]. Thus, the embedment of the WtE plants into cities' smart-energy grids might help to increase the sustainable production of energy and solve the problem of overwhelming energy demand expected in the near future [104].

4.1 Waste-to-energy plants price-setting

Expectedly, the actual capacities of already existing waste treatment facilities can be insufficient for efficient waste energy recovery in the future. Therefore, new waste treatment facilities will be needed [52]. The placement of a new WtE facility is strongly impacted by the existing infrastructure of the considered region and therefore does not suggest vast space for possible decisions. On the other side, the optimal capacity design brings numerous variants that should be assessed correctly. Such strategical decisions should be made with the help of suitable DM methods. Moreover, it should be supported by a reliable analysis of the current WM situation, since the accurate estimate of potential occupancy of capacity, and a realistic gate fee will enable to correctly anticipate return on investment and the financial feasibility of the whole project. However, in most operational research models employed in WM [7], gate fees are assumed to be external fixed parameters that have been set or optimized centrally. Such assumption neglects individual behaviors of WtE plants management and cannot describe a real conflict of interests in a waste treatment market. Therefore, there is an open problem of how to efficiently anticipate the gate fees, which will realistically reflect the WM network setting.

This part of the doctoral thesis deals with the problem of the WtE plans' optimal gate-fee setting in a competitive environment under limited capacities. It focuses on an identification of the stable gate fee state in an already built WM network (locations, productions and capacities are already given) that can be used in strategical planning and should ensure efficient and financially sustainable waste energy recovery. The problem of WtE plants' price-setting (setting of gate fee) will be studied as a normal formal game of WtE plants with gate fee being their strategies. The primary purpose of the research is to find a gate fees NE such that none of WtE plants would like to change its gate fee. After a formal introduction of the problem and extensive investigation of the bilevel optimization methods, the novel heuristic approach is proposed to solve an optimal price-setting problem for each Waste-to-Energy (WtE) plant separately. Combined with the BRD algorithm, it enables the search for NE of the considered game. The functionality of the proposed approach is validated by an exemplary problem of the DM process on the optimal capacity design of the newly planned WtE facility in the Czech Republic. At the end, the existence of the specific NE generalization for the newly introduced class of price-setting games is studied.

4.1.1 Problem statement

The detailed formulation of the considered problem can be described as follows. Consider the already built WM network. WtE plants with different capacities and waste producers (mainly cities or agglomerations) with different waste productions are presented in an area. Each WtE plant is interested in maximizing its income by setting the optimal gate fee, which will be sufficiently high or/and will attract waste producers. WtE plant income is presented as a product of its gate fee and the total amount of waste sent to this WtE plant by waste producers. The main assumption is that landfilling of utilizable waste is substantially limited, according to [19]. This fact forces waste producers to treat all produced non-recyclable waste using the services of WtE plants. Each waste producer's main interest is to reduce costs for waste treatment. These costs are represented as a product of the amount of waste sent to a particular WtE plant and the sum of gate fee and transportation costs. Another important assumption is that, whereas WtE plants located in an area are individually maximizing their income, waste producers are cooperatively minimizing their total waste treatment costs. The cooperating waste producers reflect the current trend when municipalities tend to create unions to lower their waste treatment costs [29]. The schematic explanation of the revenue maximization by a WtE plant is depicted in Figure 4.1, where the entities' objectives are highlighted in bold, and their constraints are highlighted in italics. The exchange of decision variables is depicted using arrows.

From Figure 4.1, it can be seen that setting the optimal gate fee for a particular WtE plant corresponds to solving the bilevel optimization problem, with the WtE plant on the upper level of the hierarchy and waste producers as one entity

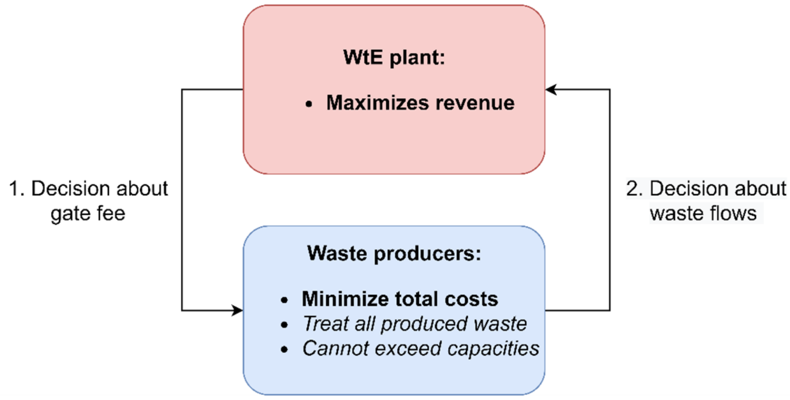

Figure 4.1: Revenue maximization problem

on the lower level. Formally, bilevel optimization can be defined as a mathematical programming problem with constraints determined by another optimization problem.

The conflict of WtE plants' interests will certainly occur since each plant will operate with its gate fee to obtain a greater part of the fixed total demand (total waste production of the whole region). Plants' capacities and relative locations of WtE plants and waste producers define the market power of WtE plants, i.e., how great a gate fee WtE plant can set and still not a substantial loose part of demand. The considered problem can be seen as a classical normal form game, which is played on the upper hierarchy level between WtE plants, where optimizing the payoff function of a player leads to a bilevel programmming problem. It was decided to apply a non-cooperative approach to the price-setting problem; cooperation between WtE plants would mean the existence of illegal collusion about the gate fees level. The NE is assumed to be the searched stable outcome. One of the main complications is that setting the optimal price for one WtE plant is already an NP-hard bilevel programming problem [13]. Therefore, the established task comprehends two distinct challenging steps:

- a solution of the price-setting bilevel programming problem with one WtE plant, maximizing its revenue on the upper level and cooperating waste producers, minimizing their total costs on the lower level;
- a determination of the NE of the price-setting normal form game between WtE plants.

Now, the mathematical formalization of the considered problem will be given.

4.1.2 Model and game

Let $N = \{1, \dots, n\}$ be a set of WtE plants; $w_1^c, \dots, w_n^c$ denotes their capacities and $C_1^g, \dots, C_n^g$ denotes their strategy sets (sets of possible gate fees) with an element $c_j^g \in C_j^g, j \in N$. The set of producers is $M = \{1, \dots, m\}$. Their waste productions are $w_1^p, \dots, w_m^p$. Transportation costs are given by the matrix $\left[c_{i,j}^t\right]$, where $c_{i,j}^t$ represents the cost of waste transportation from the producer $i \in M$ to the plant $j \in N$. In the following expressions, $x_{i,j}$ denotes the amount of waste sent by the producer $i \in M$

to the WtE plant $j \in N$ in tonnes. For each WtE plant $j \in N$, the payoff function π_j is defined as

$$\pi_j\left(c_1^g, \ldots, c_n^g\right) = \sum_{i \in M} c_j^g x_{i,j}^*, \tag{4.1}$$

where $(x_{i,j}^*)_{i \in M, j \in N} \in \{(x_{i,j}^*)_{i \in M, j \in N}\}$, such that

$$\{(x_{i,j}^*)_{i \in M, j \in N}\} = \arg \min_{x_{i,j}: i \in M, j \in N} \sum_{j \in N} \sum_{i \in M} \left(c_{i,j}^t + c_j^g\right) x_{i,j}, \tag{4.2}$$

$$s.t. \sum_{i \in M} x_{i,j} \leq w_j^c, \ \forall j \in N, \tag{4.3}$$

$$\sum_{j \in N} x_{i,j} = w_i^p, \ \forall i \in M, \tag{4.4}$$

$$x_{i,j} \geq 0, \quad \forall i \in M, \quad \forall j \in N. \tag{4.5}$$

The $(x_{i,j}^*)_{i \in M, j \in N}$ describe resulting non-negative (4.5) waste flows after cooperative minimization of total costs by cities (4.2) and the fact, that they have to dispose of all waste they produce (4.4) and cannot exceed capacities of WtE plants (4.3). The set $\{(x_{i,j}^*)_{i \in M, j \in N}\}$ is not necessarily a singleton. To prevent ambiguity, in this work, a risk-averse leader, who wants to create a financial cushion, is considered. Thus, the worst possible waste distribution scenario $(x_{i,j}^*)_{i \in M, j \in N}$ for the WtE plant will be taken among all possible arguments of optima of the above-presented mathematical programming problem. To make the problem of waste producers feasible, it is necessary to assume $\sum_{i \in N} w_i^c \geq \sum_{j \in M} w_j^p$. By now, two of three necessary elements of the normal form game of WtE plants have been established: the set of players $N = \{1, \ldots, n\}$ and their payoff functions $\pi_j(c_1^g, \ldots, c_n^g), \ j \in N$, have been defined. To thoroughly study the properties of the problem, the whole set of positive reals will be considered as a strategy space of possible gate fees. Thus, the considered game can be represented as a triple $G = (N, (\pi_j, C_j^g)_{j \in N})$, where $C_j^g = (0, \infty), \forall j \in N$.

The above-defined payoff functions are not differentiable or continuous. As a result, their derivatives cannot be described in order to analytically find the NE. Author's first paper on this topic [28] has considered applying BRD to discrete sets of possible gate fees. Compared to the original work on this topic [83], the cardinality of the sets of possible gate fees for which equilibrium can be found was substantially enlarged. In [83], the NP-hard problem of setting the optimal price between one WtE plant and all waste producers has been solved by a simple combinatorial approach through simple iteration over all possible strategies. However, such an approach does not reflect reality, where WtE plants can choose from the continuous sets of gate fees. Then, an achieved equilibrium might seem artificial because players were not allowed to play optimal strategy and arbitrarily change it. This is the reason why we will focus on bilevel programming methods in the next section: it will enable us to consider continuous strategy spaces, find optima faster and better reflect reality.

4.1.3 Bilevel programming

Firstly, we will analyze the $\max_{c_{j'}^g \in_{j'}^g} \pi_{j'}$ for an arbitrary $j' \in N$ and for the given gate fees of rivals. This can be seen as an instance of bilevel ***bilinear*** programming, where the WtE plant on the upper level maximizes its income by setting the optimal gate fee, whereas waste producers on the lower level minimize the sum of their waste treatment costs. This problem will be further referred to as $MR_{j'}$. This section is devoted to the proper inroduction of the bilevel programming and to the review of the bilevel programming research related to our problem. Also we will discuss the common elements of particular instances of so-called general taxation problem (GTP) and of $MR_{j'}$.

Theoretical background

The framework of bilevel optimization involves convex, non-convex, and mixed-integer programming (MIP) and enables the model of hierarchical situations when the response of lower level entities impacts the decisions of the upper level authority. Bilevel programming is NP-hard in general. Obviously, $MR_{j'}$ belongs to this class of mathematical programming problems. Let $F : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$, $G : \mathbb{R}^n \to \mathbb{R}^q$, $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$, $g : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^p$, and $X \subseteq \mathbb{R}^n$, $T \subseteq \mathbb{R}^m$, are closed sets. Then, according to [23], general bilevel programming problem can be mathematically expressed as

$$\min_{x \in X} F(x, y), \tag{4.6}$$

$$s.t.\ \ G(x) \leq 0, \tag{4.7}$$

$$(x, y) \in \mathbf{gph}\ \psi, \tag{4.8}$$

where $\mathbf{gph}\ \psi := \{(x, y) \in \mathbb{R}^n \times \mathbb{R}^m \mid y \in \psi(x)\}$ is a graph of solution set mapping ψ,

$$\psi(x) := \{y \in Y(x) \cap T \mid f(x, y) \leq \phi(x)\},$$

with an optimal value function ϕ

$$\phi(x) := \min_{y \in T} f(x, y),$$

$$s.t.\ \ g(x, y) \leq 0,$$

and a feasible set mapping Y

$$Y(x) := \{y \mid g(x, y) \leq 0\}.$$

It can be percived as a hierarchical problem, where leader pursues the objective of (4.6)-(4.8), trying to anticipate reaction of the follower, who tries to solve

$$\min_{y \in T} f(x, y),$$

$$s.t.\ \ g(x, y) \leq 0.$$

However, this general definition of the bilevel programming problem is not precise, since $\psi(x)$ is not necessary a singleton, implying that $x \to F(x, y)$ is not a function, but rather a point-to-set mapping. This fact causes the ambiguity in choice of a solution from $\{F(x, y) \mid y \in \psi(x)\}$ for a particular x. To overcome this problem, three conventional approaches may be applied:

1. **Optimistic**: Leader has a belief, that follower will behave «friendly» and, in case of numerous lower level solutions, will choose the solution, that is the best for the leader. This leads to the problem

$$\min_{x \in X} \phi_o(x),$$
$$s.t.\ \ G(x) \leq 0,$$

where $\phi_o(x) := \min_{y \in \psi(x)} F(x, y)$.

2. **Pessimistic**: Leader works with the worst-case scenario, solving

$$\min_{x \in X} \phi_p(x),$$
$$s.t.\ \ G(x) \leq 0,$$

where $\phi_p(x) := \max_{y \in \psi(x)} F(x, y)$.

3. **Selection function approach**: The leader is able to perfectly anticipate the reaction of the follower to each decision x, i.e. $y(x) \in \psi(x)$ for all x. Then, this unique reaction can directly be transferred to the upper level problem (4.6)-(4.8):

$$\min_{x \in X} F\left(x, y(x)\right),$$
$$s.t.\ \ G(x) \leq 0.$$

Both previously introduced cases are particular instances of more generic selection function approach.

As we have already identeified, the pessimistic approach in the choice of $(x^*_{i,j})_{i \in M, j \in N}$ from $\{(x^*_{i,j})_{i \in M, j \in N}\}$ will be employed. Thus, our pessimistic bilevel bilinear programming problem $MR_{j'}$, can be rewritten as

$$\max_{c^g_{j'} > 0} \min_{(x^*_{i,j})_{i \in M, j \in N} \in \psi(c^g_{j'})} \sum_{i \in M} c^g_j x^*_{i,j},$$

where

$$\psi(c^g_{j'}) := \{(x_{i,j})_{i \in M, j \in N} \in Y \cap \mathbb{R}_0^{mn,+} \mid f\left(c^g_{j'}, (x_{i,j})_{i \in M, j \in N}\right) \leq \phi(c^g_{j'})\},$$

$$\phi(c^g_{j'}) := \min_{y \in \mathbb{R}_0^{mn,+}} f\left(c^g_{j'}, (x_{i,j})_{i \in M, j \in N}\right)$$

$$s.t.\ \sum_{j \in N} x_{i,j} = w^p_i,\ \forall i \in M,$$

$$\sum_{i \in M} x_{i,j} \leq w^c_j,\ \forall j \in N,$$

$$Y := \{(x_{i,j})_{i \in M, j \in N} \mid \sum_{j \in N} x_{i,j} = w^p_i,\ \forall i \in M,\ \text{and}\ \sum_{i \in M} x_{i,j} \leq w^c_j,\ \forall j \in N\},$$

and

$$f\left(c^g_{j'}, (x_{i,j})_{i \in M, j \in N}\right) := \sum_{j \in N} \sum_{i \in M} \left(c^t_{i,j} + c^g_j\right) x_{i,j}.$$

At the end of this theoretical section, some basic image of classical bilevel programming solution methods will be provided.

How to solve bilevel programming problems? The most classical way of solving the bilevel programming problems is via KKT conditions. In this paragraph, we introduce only the so-called classical KKT transformation [23]. Let us assume that $T = \mathbb{R}^m$ and $Y(x) = \{y \mid g(x, y) \leq 0\}$. We additionaly assume, that $g(x, y)$ is convex for a fixed x and

$$\exists \hat{y} : \; g(x, \hat{y}) < 0 \text{ (\textbf{Slater's regularity condition})}.$$

Then, classical KKT conditions state, that $y \in \psi(x)$ if and only if

$$0 \in \partial_y f(x, y) + \lambda^T \partial_y g(x, y),$$

$$\lambda \geq 0,$$
$$\lambda^T g(x, y) = 0,$$

where $\partial_y f(x, y)$ denotes subdifferential (set of all subgradients) of function $f(x, y)$ for a fixed x. Thus, the originally introduced bilevel programming problem can be rewritten using lower-level KKT conditions as following single-level problem

$$\min_{x,y} F(x, y)$$

$$G(x) \leq 0$$
$$0 \in \partial_y f(x, y) + \lambda^T \partial_y g(x, y),$$
$$\lambda \geq 0, g(x, y) \leq 0, \lambda^T g(x, y) = 0,$$
$$x \in X.$$

Clearly, newly introduced dual variables λ bring complications, that the original formulation of the bilevel problem and its classical KKT transformation are not completely equivalent. Therefore, it is important to establish relationship between solution of original bilevel problem and its KKT transformation. If we denote

$$\Lambda(x, y) := \{\lambda \geq 0 \mid 0 \in \partial_y f(x, y) + \lambda^T \partial_y g(x, y), \lambda^T g(x, y) = 0\},$$

then we have the two following theorems [23], specifying the relationship between solutions of both problems.

Theorem 4.1.1. *Let the lower-level problem be a convex optimization problem and assume that Slater's condition is satisfied for all $x \in X$ with $\psi(x) \neq \emptyset$. A feasible point $(\hat{x}, \hat{y})$ of the original bilevel probelm is a local optimal solution of this problem iff $(\hat{x}, \hat{y}, \hat{\lambda})$ is a local optimal solution of KKT transformed problem for each $\hat{\lambda} \in \Lambda(\hat{x}, \hat{y})$.*

Proof. Proof can be found in [22]. □

Theorem 4.1.2. *Let $(\hat{x}, \hat{y}, \hat{\lambda})$ be a global optimal solution of KKT transformed problem and assume $f(x, y), g_i(x, y)$, are convex for every fixed $x \in X$ and that Slater's constraint qualification is satisfied for the lower-level problem for each $x \in X$. Then, $(\hat{x}, \hat{y})$ is a global optimal solution of the bilevel optimization problem.*

Proof. Proof can be found in [22]. □

Thus, under some additional assumptions, we can globally solve bilevel problem by solving its classical KKT transformation (what might also be a challenging task in practice). Particular methods of solving the above-presented problem are designed with respect to specific properties of the involved functions. Now, it is suitable to analyze already established approaches used in contemporary research dealing with the problems of pricing and bilevel optimization.

Literature review on price-setting

The product's pricing has always been and is still the key question in economics, as it is one of the main aspects affecting a firm's revenue [32]. The problem of a firm that maximizes its revenue, under the assumption that customers are maximizing their utility from the product, has been vastly studied in the literature. The work of Van Hoesel [106] confirmed the direct connection between the general Stackelberg pricing game and bilevel programming. This connection holds due to the hierarchical structure of the pricing problems. In fact, [106] has focused his study of pricing games on the network pricing problem (NPP), being an instance of the GTP proposed in [62] (further «toll-setting problem» will be used as an equivalent for NPP). In GTP, the leader imposes taxes on commodities transported through the abstract network by a follower to maximize profit, whereas the follower minimizes transporting costs. Indeed, numerous pricing problems correspond to the GTP. This is why it was decided to split the review of the current state-of-art into two parts: the first one is focused on the price-setting problems presented in the literature, whereas the second part is devoted solely to the GTP and its instances.

To ensure high relevance of the performed review, the main interest has been focused on the recent review papers on the bilevel optimization, from which articles focused on pricing and toll-setting have been extracted. In particular, the survey of mixed-integer bilevel approaches [58], a general review on classical bilevel optimization with an emphasis on evolutionary approaches [101], article on bilevel intermodal pricing [103] and extensive review of pessimistic bilevel optimization approaches [66] have been considered. To complement the found papers, the search in the Scopus database using pairs (and triplets in case of numerous results) of the following keywords has been performed:

- general taxation problem;
- highway network problem;
- price setting;
- bilevel optimization;
- bilevel bilinear problem;
- Stackelberg game.

Then, relevant papers have been divided into two groups mentioned above and detailly reviewed. The results of the review of general pricing problems can be found in Table 1 of the author's paper [30].

The main feature of our problem are limited capacities of WtE plants, which substantially complicate the solution. Only a few papers consider some analogy of these capacities. Anjost et al. [2] studied the model where only part of the lower level decision variables have an upper bound. Moreover, the integer nature of some variables has simplified single-level reformulation. The work [35] also assumes analogical constraints. Still, the problem formulation again contains integer variables, and the application specifics enable convenient linearization of bilinear terms during reformulation into a single-level problem. Feng et al. [33] also consider the analogy of capacitated arcs, but, compared to cooperating waste producers considered in this paper, the authors have assumed equilibrium on the lower level, which enabled reformulation into a mixed-integer quadratically constrained optimization problem. Zheng et al. [113] considered capacitated depots, but the capacity is given for each product separately, implying their mutual independence.

Thus, the analogical problem has not been studied in the considered papers. Another peculiar finding is that the pessimistic approach considered in this paper is enforced using a simple numerical trick, which has been also applied in [10]. It dwells in the addition of an artificial small constant, which makes the leader's services more expensive than services of other suppliers. One of the most interesting papers is [99], where the closely related problem of product line pricing is studied. Whereas it has an analogical structure (though formulated as a single-level problem), it differs in the following important assumptions:

- the leader does not assume the limited production capacities of the competitors (analogy of capacity of other WtE plants), which leads to maximally risk-averse behavior;
- the customers are not forced to buy products, whereas waste producers (in fact, customers of the WtE sector) have to treat all produced waste;
- integer nature of the customer-product relationship (each customer buys at most one product) simplifies the potential embedment of capacity constraints.

Moreover, under the assumptions of this work, the heuristics proposed in [99] degenerates into an enumeration procedure. Regarding the search for equilibrium between leaders, Myklebust et al. [78] assumed the stationary prices of the competitors' products since changing competitors' prices would substantially complicate the problem. The same is valid for the work [99]. The problem of establishing the equilibrium between leaders has been considered only in one paper: Reisi et al. [90] studied the version of the equilibrium problem with equilibrium constraints. However, this version has been simplified by an assumption that enabled a direct search for equilibria via the backward induction. Thus, from the perspective of the upper level normal form game, the lack of related research can be aslo confirmed.

The first part of the review has confirmed the necessity to focus on the GTP: the majority of the papers mention NPP or GTP. For example, the envy-free pricing studied in [34] is solved with the help of the NPP. Now, the GTP will be shortly introduced.

General taxation problem literature review

Labbé et. al [62] have thoroughly studied theoretical properties of GTP. In this problem, the leader imposes taxes on a commodities transported through the abstract network by follower, maximizing his profit, while follower minimizes his costs. Now, assume that x and y are vectors of reals describing quantitative levels of taxed and untaxed types of activities, respectively. Vector T will denote taxes imposed on the x. Let F and f denote the leader's and follower's objective functions, respectively. Then, the corresponding bilevel programming problem can be expressed as follows

$$\max_{T \in \Theta, x, y} F(x, y, T),$$

$$\min_{x,y} f(x, y, T),$$

$$s.t. (x, y) \in \Pi,$$

where Θ is set of feasible taxes and Π is set of feasible activities.

Such a model can describe multiple possible situations, when T can represent not only taxes, but also subsidies, while x and y can represent consumption or production levels. After describing this model, Labbé et. al [62] focus on the simplified bilevel bilinear model, which clearly has the same structure as $MR_{j'}$:

$$\max_{T,x,y} Tx,$$

$$\text{s.t. } TC \geq e,$$

$$\min_{x,y} (c + T)x + dy,$$

$$s.t. \ Ax + By \geq b,$$

where C, d, and e are vectors of reals and A and B are real matrices of suitable dimensions (the original notation from [62] has been preserved). It is important to notice that c corresponds to the costs of the activities x before the tax were imposed. The description, provided in [62], illustrates that the leader's objective function is neither continuous nor convex, but it is piecewise linear and left continuous in the optimistic case.

Under assumptions that $\Pi = \{(x, y) | \ Ax + By \geq b\}$ is bounded and $\{y | y \geq b\}$ is non-empty, the leader's objective function is bounded from above and the whole problem can be reformulated using KKT conitions as a single-level bilinear problem. However, such reformulation might bring extensive amount of the additional variables for the large instances complicating computation of the global optima for the off-the-shelf solvers (their can stuck in numerous local optima). Alternatively, it can be reformulated as a linear bilevel programming problem, which, however, does not necessarily simplifies the task.

Then, Labbé et. al [62] proceed to NPP (originally called a road pricing model), being one of the most common instances of the GTP. In NPP, an authority (leader) tolls a specified arc of a multicommodity transportation network, while the remaining arcs bear only fixed costs, and the users (followers) of the network travel on the shortest path (minimum sum of initial costs and tolls) between their relative origins and destinations [47]. Now, we will formally introduce NPP.

Assume a multicommodity network represented by a set of nodes $\mathcal{N}$, a set of arcs $\mathcal{A} \cup \mathcal{B}$, and a set of pairs $\{(o^k, d^k) : k \in \mathcal{K}\}$ for the commodities $k \in \mathcal{K}$, associated with a demand η^k. A subset $\mathcal{A}$ represent arcs, which are owned by authority and can be tolled by a $t_a, a \in \mathcal{A}$, whereas $\mathcal{B}$ represent toll-free arcs. For a commodity $k \in \mathcal{K}$ and arc $a \in \mathcal{A} \cup \mathcal{B}$, c_a^k denotes the cost of travelling via a. The flow variables are denoted by x_a^k, where $a \in \mathcal{A} \cup \mathcal{B}$, and $k \in \mathcal{K}$. Then, NPP can be formulated as,

$$\max_{t_a, a \in \mathcal{A}, x_a^k, k \in \mathcal{K}, a \in \mathcal{A} \cup \mathcal{B}} \sum_{k \in \mathcal{K}} \sum_{a \in \mathcal{A}} \eta^k t_a \tilde{x}_a^k \tag{4.9}$$

$$s.t.\ t_a \geq 0,\ \forall a \in \mathcal{A}, \tag{4.10}$$

$$(\tilde{x}_a^k)_{k \in \mathcal{K}, a \in \mathcal{A} \cup \mathcal{B}} \in \arg \min_{x_a^k, k \in \mathcal{K}, a \in \mathcal{A} \cup \mathcal{B}} \sum_{k \in \mathcal{K}} \left(\sum_{a \in \mathcal{A}} \left(c_a^k + t_a \right) x_a^k + \sum_{a \in \mathcal{B}} c_a^k x_a^k \right), \tag{4.11}$$

$$s.t. \sum_{a \in i^- \cap \mathcal{A}} x_a^k + \sum_{a \in i^- \cap \mathcal{B}} x_a^k - \sum_{a \in i^+ \cap \mathcal{A}} x_a^k - \sum_{a \in i^+ \cap \mathcal{B}} x_a^k = \begin{cases} -1, \text{ if } i = o^k \\ 1, \text{ if } i = d^k \\ 0, \text{ otherwise} \end{cases},\ \forall k \in \mathcal{K},\ \forall i \in \mathcal{N}, \tag{4.12}$$

$$x_a^k \in \{0, 1\},\ \forall k \in \mathcal{K}, \forall a \in \mathcal{A}. \tag{4.13}$$

where i^+, resp. i^-, denotes arcs with i as its head, resp. tail. This problem has been proven to be generally NP-hard, even for instances without congestion (capacitated arcs). Under assumptions, that no negative cost cycle can occur and there always exist toll-free path for each commodity, the NPP formulation can be reformuilated as an integer programming problem, since each origin-destination path will carry either total demand or zero. Unfortunately, this assumption doesn't hold in case of the congested arcs, making linearization of terms problematic. Moreover, numerate Lagrange multipliers will complicate the situation.

For the case of the single toll arc a, the solution can be find in polynomial time. Let $\gamma_k(t_a)$ denotes the shortest path cost for the pair (o^k, d^k), $k \in \mathcal{K}$, for a toll t_a. If we set $\pi_k = \gamma_k(\infty) - \gamma_k(0)$. Then, assuming ordering

$$\pi_{k_1} \geq \pi_{k_2} \geq ... \geq \pi_{k_{|\mathcal{K}|}},$$

the optimal toll t_a can be computed as

$$t_a = \pi_{k_{i^*}},$$

$$i^* \in \arg\max_i \{\pi_{k_i} \sum_{j \leq i} \eta^{k_j} :\ \pi_{k_i} \geq 0\}.$$

The review of papers focused on GTP and its instances can be found in Table 2 of the author's paper [30]. The work of Bouhtou et al. [12] is similar to the studied problem, but does not consider the main complication of our model: capacity constraints. Due to omitted capacities, the authors were able to find the optimal solution in polynomial time using the enumeration procedure. However, in the problem considered in this paper, the assumption of cooperating followers and capacitated arcs makes it hard to anticipate the behavior of followers and changes in waste flows. There are only two works with the same research subject: [54] and [55]. Evolutionary approaches presented in [109] and [40] are out of scope of this thesis. Kalashnikov et al. [54] considered four different heuristic approaches for toll-setting problems with congestion (capacitated arcs). In particular, the penalization function approach, quasi-Newton method, sharpest ascent method, and direct search via Nelder-Mead algorithm. These algorithms can handle the capacitated toll-setting problem: for example, for medium-sized problems, it takes from 7 up to 15 minutes for these algorithms to find a solution. Compared to the papers mentioned above, $MR_{j'}$ has a much simpler structure, that should be exploited when computing optimum: it has only one tolled arc controlled by j'. Moreover, there is no available data about the efficiency of computation process of the above-mentioned algorithms in the case of single tolled arc and numerous commodities.

Heilporn et al. [47] focus on instances reflecting the structure of an actual toll highway: the network is composed of a toll path (the highway) and toll-free arcs linking the origins, highway entrances, exits, and destinations. This problem is called the Highway NPP (HNPP). It is assumed that all arcs controlled by an authority present a complete bipartite subgraph and for every commodity exists the toll-free path from its origin to its destination. The main distinction of HNPP from NPP, which makes it not a particular case of the NPP, but its variant, is the assumption that followers do not re-enter the highway. This is ensured via Triangle and Monotonicity inequalities. Clearly, the existence of one tolled arc (one-arc highway) axiomatically fulfills these assumptions. These properties enabled Heilporn et al. [47] to suggest a simple and efficient reformulation of the HNPP into MIP (solvable in polynomial time for a single tolled arc or a single commodity). This reformulation also enabled solving other pricing problems: it has been demonstrated that the envy-free pricing problem can be reduced to basic HNPP [34]. Moreover, the equivalence between HNPP and the product line pricing problem [99] has been shown in [48]. However, the main drawback of the work of Heilporn et al. are unconstrained arcs in a network.

One of the main ideas implied by Kalashnikov et al. [54] is that approximation of derivatives enables capturing the followers' behavior. Kalashnikov et al. [55] have exploited the related idea of finding the maximum of the leader function via iterated sensitivity analysis performed on the lower level linear programming problem to find a suitable increase in the leader's function. This approach has been applied to indirectly model followers' behavior in the non-constrained arc and in the constrained case [55] , where equilibrium on the lower level has been considered to fairly solve the congestion problem.

The solution idea. Exactly the combination of the MIP reformulation proposed by Heilporn et al. [47] and of the idea analogical to [55] has inspired the development of a new heuristic approach providing the near-optimal solution for $MR_{j'}$. Whereas, in the latter work, the follower's behavior has been anticipated via small perturbations in flows, in this work, a completely new iterative solution approach is presented. It is suggested to neglect the idea of approximation of objective function derivatives. The proposed approach captures the followers' behavior via iterative update of their optimal flows after the solution of the risk-averse revenue maximization problem of the leader: the iterative adjustment of the lower level solution enables to estimate the optimal price of the upper level. The whole leader problem is formulated based on MIP reformulation proposed by [47] with novel additions, enabling the embedding of leader capacities constraints and new inequalities reflecting his ability to raise gate fees by neglecting some of the flows.

4.1.4 Finding the optimal gate fee

In this section, the previously introduced idea of finding the solution will be further formalized. In particular, the different formulations of HNPP, the establishment of the relation between HNPP and $MR_{j'}$, and precise description of the proposed algorithm and commentary on it will be introduced.

Highway network pricing problem

In this subsection, we focus on the particular instance of HNPP called Constrained Complete Toll NPP. In the original work [47], Heilporn et al. have introduced three main versions of the problem: Basic NPP (additive tolls and forbidden re-entry), General Complete Toll NPP (arbitrary non-additive tolls and complete toll subgraph) and Constrained Complete Toll NPP (analogical to General Complete Toll NPP with additional real-life constraints). In order to introduce Constrained Complete Toll NPP, new notation is necessary. For a commodity $k \in \mathcal{K}$ and a toll arc $a \in \mathcal{A}$, c_a^k denotes the cost of travel through the path $o^k \to t(a) \to h(a) \to d^k$ before imposing tolls, where $t(a), h(a) \in \mathcal{N}$, are the entry (tail node of a) and exit (head node of a) of the highway, respectively [47]. The corresponding flow variable is denoted by x_a^k. The travel cost on the toll-free path $o^k \to d^k$ is denoted by c_{od}^k corresponding flow variable x_{od}^k. Using this notation, Triangle and Monotonicity constraints on network can be introduced.

- **Triangle constraints:**

$$t_a \leq t_b + t_c \ \forall a, b, c \in \mathcal{A}:$$

$$t(a) = t(b), \ h(b) = t(c), \ h(c) = h(a).$$

- **Monotonicity constraints:**

$$t_a \geq t_b, \ \forall a, b \in \mathcal{A} : t(a) = t(b) < h(a) = h(b) + 1$$

$$\text{or } t(a) = t(b) - 1 < h(a) = h(b)$$

$$\text{or } t(a) = t(b) > h(a) = h(b) - 1$$

$$\text{or } t(a) = t(b) + 1 > h(a) = h(b),$$

where nodes are represented by indices, which are ordered increasingly with respect to direction (see [47]).

Then, the following bilevel formulation of the Constrained Complete Toll NPP can be obtained

$$\max_{t_a, x_a^k, x_{od}^k, a \in \mathcal{A}, k \in \mathcal{K}} \sum_{k \in \mathcal{K}} \sum_{a \in \mathcal{A}} \eta^k t_a \tilde{x}_a^k \tag{4.14}$$

$$s.t.\ t_a \geq 0,\ \forall a \in \mathcal{A}, \tag{4.15}$$

$$\left(\tilde{x}_a^k, \tilde{x}_{od}^k\right)_{a \in \mathcal{A}, k \in \mathcal{K}} \in \arg \min_{x_a^k, x_{od}^k, a \in \mathcal{A}, k \in \mathcal{K}} \sum_{k \in \mathcal{K}} \sum_{a \in \mathcal{A}} \left(c_a^k + t_a\right) x_a^k + c_{od}^k x_{od}^k, \tag{4.16}$$

$$s.t. \sum_{a \in \mathcal{A}} x_a^k + x_{od}^k \ = 1,\ \forall k \in \mathcal{K}, \tag{4.17}$$

$$x_a^k \in \{0, 1\},\ \forall k \in \mathcal{K}, \forall a \in \mathcal{A}. \tag{4.18}$$

The constraint (4.16) is the so-called shortest-path constraint. The constraint (4.17) on the lower level ensures that the commodity cannot be assigned to both tolled and toll-free paths simultaneously. Under the assumptions of the problem, the requirement of x_a^k to be binary is redundant and it can be taken from the closed interval between zero and one. Introducing linearizng variables

$$p_a^k = \begin{cases} t_a, & \text{if commodity } k \text{ travels through arc } a, \\ 0, & \text{otherwise.} \end{cases} \tag{4.19}$$

and replacing the lower-level problem with its KKT conditions, alternative formulation can be obtained [47]. However, shortly after, it is demonstrated, that dual variables are redundant, when expressing lower level optimality in term of path flows. This fact enables us to obtain the following compact mixed-integer problem CCTNPP

$$\max_{p_a^k, t_a, x_a^k, a \in \mathcal{A}, k \in \mathcal{K}} \sum_{k \in \mathcal{K}} \sum_{a \in \mathcal{A}} \eta^k p_a^k, \tag{4.20}$$

$$s.t. \sum_{a \in \mathcal{A}} \left(c_a^k x_a^k + p_a^k\right) + c_{od}^k \left(1 - \sum_{a \in \mathcal{A}} x_a^k\right) \leq c_b^k + t_b,\ \forall k \in \mathcal{K}, \forall b \in \mathcal{A}, \tag{4.21}$$

$$p_a^k \leq M_a^k x_a^k,\ \forall k \in \mathcal{K}, \forall a \in \mathcal{A}, \tag{4.22}$$

$$t_a - p_a^k \leq N_a \left(1 - x_a^k\right),\ \forall k \in \mathcal{K}, \forall a \in \mathcal{A}, \tag{4.23}$$

$$p_a^k \leq t_a,\ \forall k \in \mathcal{K}, \forall a \in \mathcal{A}, \tag{4.24}$$

$$p_a^k \geq 0,\ \forall k \in \mathcal{K}, \forall a \in \mathcal{A}, \tag{4.25}$$

$$x_a^k \in \{0, 1\},\ \forall k \in \mathcal{K}, \forall a \in \mathcal{A}, \tag{4.26}$$

$$\sum_{a \in \mathcal{A}} x_a^k \leq 1,\ \forall k \in \mathcal{K}, \tag{4.27}$$

where $M_a^k = \max\{0, c_{od}^k - c_a^k\}$ and $N_a = \max_{k\in\mathcal{K}} M_a^k$. Constraints (4.21) ensure the optimality of the chosen path for each commodity $k \in \mathcal{K}$, whereas constraints (4.22)-(4.24) ensure that revenue variable p_a^k fulfills the linearization assumption (4.19).

This formulation coincides with the reformulation given in [99] to the problem of product line pricing. As already mentioned, Heilporn et al. [48] have indicated a close relation between a generic NPP, CCTNPP, and the product line pricing problem. Labbe and Violin [63] also highlighted the parallel between a product's pricing and a highway. Certainly, a similarity between the $MR_{j'}$ and the CCTNPP with the single tolled arc can be observed. The schematic representation of CCTNPP with the single tolled arc and three commodities is given in Figure 4.2.

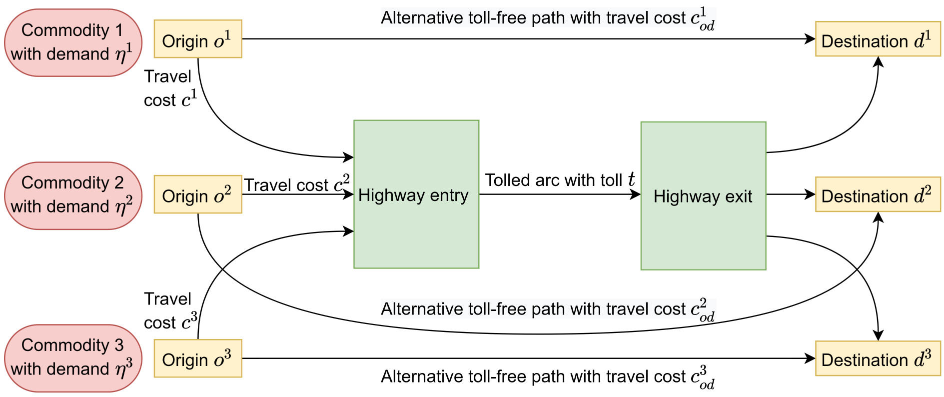

Figure 4.2: Structure of CCTNPP

The «aim» of a commodity is to be transported with minimal costs. Analogically, a waste producer aims to treat waste with minimal costs. Whereas the owner of the arc sets the toll, the WtE plant sets the gate fee. Let toll t be identified with the gate fee $c_{j'}^g$ of j', $\mathcal{K}$ be identified with a set of waste producers M, price of untolled highway travel c^k be identified with transportation costs $c_{i,j'}^t$, origins of commodities o^k be identified with locations of waste producers, and alternative optimal route costs c_{od}^k be identified with alternative optimal waste treatment option costs $c_{i,j}^t + c_j^g$, and destinations d^k be identified with successful treatment of waste. Further, for the sake of convenience, we will use simplified notation $N \setminus \{i\} := N \setminus i$. Then, $MR_{j'}$ can be represented analogically to CCTNPP as it is depicted in Figure 4.3 for the case $j' = \{2\}$.

However, the most challenging difference between these problems is that CCTNPP does not involve capacity constraints on an arc (analogy of WtE plants capacities constraints). This fact brings many complications, since, due to limited capacities, a waste producer can choose a non-optimal waste treatment possibility to reduce the costs of another waste producer and achieve a minimal sum of total costs. As a result, the behavior of waste producers will not correspond to the behavior of commodities. In order to proceed to heuristical solution of the problem, the exact solution of the modified problem should be established at first.

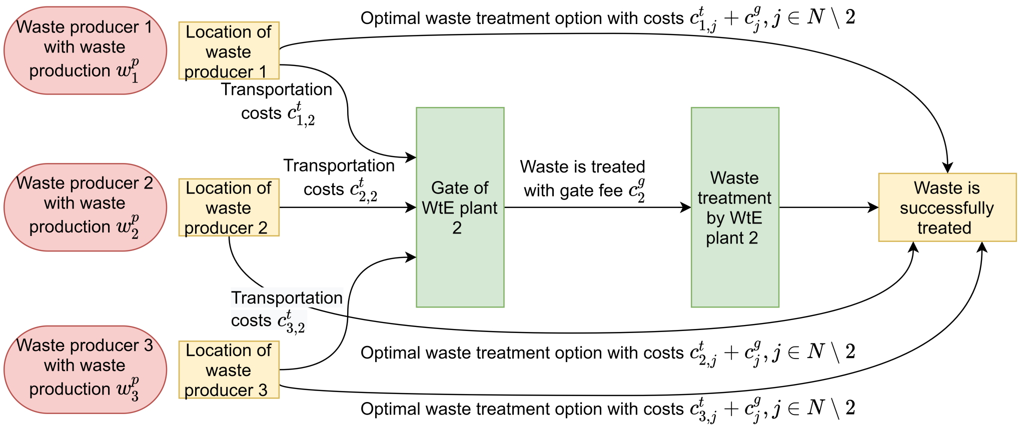

Figure 4.3: Sturcture of $MR_{j'}$

Risk-averse price-setting

Consider the point of view of one of WtE plants j' and setting, in which only the following information is available to j': gate fees of other WtE plants, waste production for each waste producer in the region, and, obviously, the capacity of its own waste treatment facility. Whereas such a situation is improbable, exactly this assumption will enable to model $MR_{j'}$ as CCTNPP and to embed capacity constraints into the problem afterward. Since capacities of other WtE plants are unknown, j' has to make a decision about its attitude to possible risks in this uncertain situation. If j' accepts the risk-averse behavior, it has to work with the worst possible scenario. Therefore, j' will try to solve the $MR_{j'}$, where the capacity constraint holds only for the WtE plant managed by itself. Further, this problem will be denoted as $MR_{j'}RA$. The following way of finding the solution to $MR_{j'}RA$, which can be viewed as a three-step algorithm, is proposed.

At first, a linear programming problem, corresponding to minimization of the total costs by waste producers, assuming infinite capacities of WtE plants from $N \setminus j'$ and absence of j' in the network, has to be solved. It can be formulated as $LP_{j'}RA$:

$$\min_{x_{i,j}, i\in M, j \in N\setminus j'} \sum_{j\in N\setminus j'} \sum_{i \in M} \left(c_{i,j}^t + c_j^g\right) x_{i,j}, \tag{4.28}$$

$$s.t. \sum_{j \in N\setminus j'} x_{i,j} = w_i^p, \ \forall i \in M, \tag{4.29}$$

$$x_{i,j} \geq 0, \quad \forall i \ \in M, \quad \forall j \in N \setminus j'. \tag{4.30}$$

Once the solution of the $LP_{j'}RA$ is obtained, take $(x_{i,j}^{*,j'})_{i\in M, j\in N\setminus j'} \in \arg LP_{j'}RA$. Non-uniquness of $LP_{j'}RA$ solution does not have an impact on the following considerations. Now, when the optimal waste flows from $LP_{j'}RA$ are known, the $MR_{j'}RA$ can be solved as CCTNPP with a single toll arc in two steps. A relation between the role of variables and parameters in the CCTNPP and new formulation $HNP_{j'}RA$ is given by the following Table 4.1.

Table 4.1: Roles of variables in CCTNPP and $HNP_{j'}RA$

CCTNPP	$\mathcal{A}$	$k \in \mathcal{K}$	η^k	p_a^k	c_a^k	c_{od}^k	x_a^k	t_a
$HNP_{j'}RA$	Single arc	(i,j), $i \in M, j \in N \setminus j'$	$x_{i,j}^{*,j'}$	$p^{i,j}$	$c_{i,j'}^t$	$c_{i,j}^t + c_j^g$	$q^{i,j}$	$c_{j'}^g$

Thus, $MR_j'RA$ can be reformulated as a problem $HNP_{j'}RA$

$$\max_{c_{j'}^g, p^{i,j}, q^{i,j}, i \in M, j \in N \setminus j'} \sum_{i \in M} \sum_{j \in N \setminus j'} x_{i,j}^{*,j'} p^{i,j}, \tag{4.31}$$

$$\text{s.t.} \ \left(c_{i,j'}^t q^{i,j} + p^{i,j}\right) + \left(c_{i,j}^t + c_j^g\right)\left(1 - q^{i,j}\right) \leq c_{i,j'}^t + c_{j'}^g, \forall i \in M, \forall j \in N \setminus j', \tag{4.32}$$

$$p^{i,j} \leq M^{i,j} q^{i,j}, \ \forall i \in M, \forall j \in N \setminus j', \tag{4.33}$$

$$c_{j'}^g - p^{i,j} \leq N\left(1 - q^{i,j}\right), \ \forall i \in M, \forall j \in N \setminus j', \tag{4.34}$$

$$p^{i,j} \leq c_{j'}^g, \ \forall i \in M, \forall j \in N \setminus j', \tag{4.35}$$

$$p^{i,j} \geq 0, \ \forall i \in M, \forall j \in N \setminus j', \tag{4.36}$$

$$q^{i,j} \in \{0,1\}, \ \forall i \in M, \forall j \in N \setminus j', \tag{4.37}$$

$$\sum_{i \in M} \sum_{j \in N \setminus j'} q^{i,j} x_{i,j}^{*,j'} \leq w_{j'}^c, \tag{4.38}$$

where $M^{i,j} = \max\{0, c_{i,j}^t + c_j^g - c_{i,j'}^t\}$, $\forall i \in M, \forall j \in N \setminus j'$, and $N = \max M^{i,j}$. Newly imposed inequality (4.38) will prevent the exceeding of the capacity of the WtE plant j'. However, due to the integer nature of variables $q^{i,j}$, the WtE plant j' can not completely engage its capacity, what is clearly possible in the original setting. To take into account this complication and solve the occurred problem, the following modification $HNP_{j'}RA\ FULL$ of $HNP_{j'}RA$, which is based on its optimal solution $\left((p^{*,i,j}, q^{*,i,j})_{i \in M, j \in N \setminus j'}, c_{j'}^{*,g}\right) \in \arg HNP_{j'}RA$, has to be solved.

$$\max_{c_{j'}^g, p^{i,j}, q^{i,j}, i \in M, j \in N \setminus j'} \sum_{i \in M} \sum_{j \in N \setminus j'} x_{i,j,new}^{*,j'} p^{i,j}, \tag{4.39}$$

$$s.t. \ \left(c_{i,j'}^t q^{i,j} + p^{i,j}\right) + \left(c_{i,j}^t + c_j^g\right)\left(1 - q^{i,j}\right) \leq c_{i,j'}^t + c_{j'}^g, \ \forall i \in M, \forall j \in N \setminus j', \tag{4.40}$$

$$p^{i,j} \leq M^{i,j} q^{i,j}, \ \forall i \in M, \forall j \in N \setminus j', \tag{4.41}$$

$$c_{j'}^g - p^{i,j} \leq N\left(1 - q^{i,j}\right), \ \forall i \in M, \forall j \in N \setminus j', \tag{4.42}$$

$$p^{i,j} \leq c_{j'}^g, \ \forall i \in M, \forall j \in N \setminus j', \tag{4.43}$$

$$p^{i,j} \geq 0, \ \forall i \in M, \forall j \in N \setminus j', \tag{4.44}$$

$$q^{i,j} \in \{0,1\}, \ \forall i \in M, \forall j \in N \setminus j', \tag{4.45}$$

$$\sum_{i \in M} \sum_{j \in N \setminus j'} q^{i,j} x_{i,j,new}^{*,j'} \leq w_{j'}^c, \tag{4.46}$$

$$c_{j'}^{g} \leq c_{j'}^{*,g}, \tag{4.47}$$

where

$$x_{i,j,new}^{*,j'} = \begin{cases} x_{i,j}^{*,j'}, \text{ if } q^{*,i,j} = 1, \\ \min\{x_{i,j}^{*,j'}, w_{j'}^{c} - \sum_{i \in M} \sum_{j \in N \setminus j'} q^{*,i,j} x_{i,j}^{*,j'}\}, \text{ if } q^{*,i,j} = 0. \end{cases}$$

Inequality (4.46) will enable utilization of the whole capacity, whereas (4.47) prevents the repetition of calculations already performed during the solution of $HNP_{j'}RA$. Then, the optimal solution of $HNP_{j'}RA\ FULL$ is also assumed to be a solution to $MR_{j'}RA$. It is important to note, that in case $HNP_{j'}RA$ is infeasible, it is sufficient to directly solve $HNP_{j'}RA\ FULL$ without constraint (4.47) and assume, that all $q^{*,i,j}$ are zeros.

Suggested approach

The setting described in the previous subsection enables to fully embed the considered problem of gate fee setting into the framework of HNPP. However, the previously mentioned risk-averse approach might impose too strong and unrealistic restrictions. For example, such an approach can accept the idea that all waste produced in the region can be sent to only one WtE plant, which is rather improbable for large-scale cases. Thus, in this subsection, a heuristic algorithm for solving the original problem $MR_{j'}$, which is based on the approach presented in the previous subsection, is proposed. This suggested algorithm embeds the capacities of other WtE plants into a DM process and can be described as follows.

First step. Solve the problem $LP_{j'}WITHOUT$ and obtain information about the current state of the network without WtE plant j'.

$$\min_{x_{i,j}, i \in M, j \in N \setminus j'} \sum_{j \in N \setminus j'} \sum_{i \in M} \left(c_{i,j}^{t} + c_{j}^{g}\right) x_{i,j}, \tag{4.48}$$

$$\text{s.t.} \sum_{i \in M} x_{i,j} \leq w_{j}^{c}, \forall j \in N \setminus j', \tag{4.49}$$

$$\sum_{j \in N \setminus j'} x_{i,j} = w_{i}^{p}, \ \forall i \in M, \tag{4.50}$$

$$x_{i,j} \geq 0, \ \ \forall i \ \in M, \ \ \forall j \in N \setminus j'. \tag{4.51}$$

Second step. Set $(x_{i,j}^{*,j'})_{i \in M, j \in N \setminus j'} \in \arg LP_{j'}WITHOUT$. Solve the problem $HNP_{j'}RA$ and consequently $HNP_{j'}RA\ FULL$. The first two steps provide the main body of the algorithm with the relevant estimate of the network starting state and the gate fee $c_{j'}^{start,g} \in \arg HNP_{j'}RA\ FULL$ is the starting price in the iterative solution process. Currently, the capacity constraints hold for every WtE plant in the network.

Third step. Solve the $LP_{j'}$, corresponding to the lower-level problem in the original bilevel formulation $MR_{j'}$ with $c^g_{j'} = c^{start,g}_{j'}$, to obtain the current state of the network:

$$\min_{x_{i,j}, i\in M, j\in N} \sum_{j\in N}\sum_{i\in M} \left(c^t_{i,j} + c^g_j\right) x_{i,j}, \tag{4.52}$$

$$s.t. \sum_{i\in M} x_{i,j} \ \leq w^c_j, \forall j \in N, \tag{4.53}$$

$$\sum_{j\in N} x_{i,j} \ = w^p_i, \ \forall i \in M, \tag{4.54}$$

$$x_{i,j} \geq 0, \quad \forall i \ \in M, \quad \forall j \in N. \tag{4.55}$$

In each iteration, this step corrects the reactions of the follower to the newly chosen $c^{start,g}_{j'}$, so that leader has actual information about current flows for the given gate fee.

Fourth step. Set $(x^{*,j'}_{i,j})_{i\in M, j\in N\setminus j'} \in \arg LP_{j'}$. Solve the problem $HNP_{j'}CAP$

$$\max_{c^g_{j'}, p^{i,j}, q^{i,j}, i\in M, j\in N} \sum_{i\in M}\sum_{j\in N} x^{*,j'}_{i,j} p^{i,j}, \tag{4.56}$$

$$s.t. \left(c^t_{i,j'} q^{i,j} + p^{i,j}\right) + \left(c^t_{i,j} + c^g_j\right)\left(1 - q^{i,j}\right) \leq c^t_{i,j'} + c^g_{j'}, \ \forall i \in M, \forall j \in N \setminus j', \tag{4.57}$$

$$c^t_{i,j'} q^{i,j'} + p^{i,j'} \ + L^i(1 - q^{i,j'}) \leq c^t_{i,j'} + c^g_{j'}, \forall i \in M, \tag{4.58}$$

$$c^g_{j'} - p^{i,j} \leq N\left(1 - q^{i,j}\right), \ \forall i \in M, \forall j \in N, \tag{4.59}$$

$$p^{i,j} \leq M^{i,j} q^{i,j}, \ \forall i \in M, \forall j \in N, \tag{4.60}$$

$$p^{i,j} \leq c^g_{j'}, \ \forall i \in M, \forall j \in N, \tag{4.61}$$

$$p^{i,j} \geq 0, \ \forall i \in M, \forall j \in N, \tag{4.62}$$

$$q^{i,j} \in \{0, 1\}, \ \forall i \in M, \forall j \in N, \tag{4.63}$$

$$\sum_{i\in M}\sum_{j\in N} q^{i,j} x^{*,j'}_{i,j} \leq w^c_{j'}, \tag{4.64}$$

where

$$M^{i,j} = \max\{0, c^t_{i,j} + c^g_j - c^t_{i,j'}\}, \ \forall i \in M, \forall j \in N \setminus j',$$

$$M^{i,j'} = \max\{0, L^i - c^t_{i,j'}\},$$

$$L^i = \min_{j\in N\setminus j'} \{c^t_{i,j} + c^g_j \mid c^t_{i,j} + c^g_j > c^t_{i,j'} + c^{start,g}_{j'}\},$$

and $N = \max M^{i,j}$. In case L^i is not defined due to emptiness of the underlying set, L^i can be set as sufficiently large number. Consequently, solve modification $HNP_{j'}CAP\ FULL$: modify flows analogous to the previous subsection and add a constraint (4.47) describing that the gate fee can only be lowered compared to the optimum found via $HNP_{j'}CAP$. These two problems describe the adaptation of the leader to the current flows that have been changed in the previous step. Novel, newly introduced constraint (4.58) reflects the possible choice of abandoning some of the current non-zero waste flows to j' in order to increase the price and potentially obtain higher revenue. Set $c^{opt,g}_{j'} \in \arg\ HNP_{j'}CAP\ FULL$.

Fifth step. Raise $c_{j'}^{opt,g}$ and solve $LP_{j'}$ with $c_{j'}^{g} = c_{j'}^{opt,g}$, until the first decrease in $\sum_{i\in M} x_{i,j'}^{*,j'}$, where $(x_{i,j}^{*,j'})_{i\in M, j\in N\setminus j'} \in \arg\ LP_{j'}$. This is a simple computational check in case the WtE plant j' might still be the best waste treatment option due to the filled capacities of the other plants.

Sixth step. If $c_{j'}^{opt,g}$ from the previous step guarantees greater revenue than $c_{j'}^{start,g}$, then set $c_{j'}^{start,g} = c_{j'}^{opt,g}$ and go back to the third step. Otherwise, the solution $c_{j'}^{start,g}$ is found, END. This is a classical search stop condition, where the main body of a cycle runs as long as it can find a better solution.

Commentary. The algorithm is meant to produce the optimal or near-optimal solution. To create an artificial upper bound for gate fees and to ensure the requirement that for every commodity exists the toll-free path from its origin to its destination, a «virtual» WtE plant with a fixed gate fee and a capacity that can meet waste production of the whole region has to be considered. It was stated that the pessimistic approach would be applied in the case of multiple solutions on the lower level. However, all presented MIPs are defined for the optimistic approach. Embeddment of the pessimistic approach into them can be done by adding a sufficiently small number ϵ to all $c_{i,j'}^{t}$. It will help to choose a solution that is smaller than limit of nearly optimal solutions by the ϵ and to avoid numerous evaluations, which will not substantially improve the objective function value. To not distort optima by this numerical adjustment, it is recommended to set an ϵ to a decimal number, which has order of magnitude equal to min(order of magnitude that is lower than the order of magnitude of any transportation costs, order of magnitude of the fixed gate fees). Thus, if integer costs and gate fees are considered, it is advised to set ϵ=0.1. Moreover, in the fifth step of the algorithm, it is advised to raise $c_{j'}^{opt,g}$ by ϵ to cover all possible waste distributions on the lower level.

The linear programming problems solved during third and fifth step of the presented algorithm are solved using the pessimistic approach for the leader with the original $c_{i,j'}^{t}$ without adjustments. This can be ensured by finding arbitrary solution $(x_{i,j}^{*,j'})_{i\in M, j\in N} \in \arg LP_{j'}$ with $c_{i,j'}^{t}$. Then, to obtain pessimistic argument it is sufficient to solve $LP_{j'}PES$:

$$\min_{x_{i,j}, i\in M, j\in N} \sum_{i\in M} \left(c_{i,j'}^{t} + c_{j'}^{g}\right) x_{i,j'}, \tag{4.65}$$

$$s.t. \sum_{i\in M} x_{i,j} \ \leq w_j^c, \forall j \in N, \tag{4.66}$$

$$\sum_{j\in N} x_{i,j} \ = w_i^p,\ \forall i \in M, \tag{4.67}$$

$$\sum_{j\in N}\sum_{i\in M} \left(c_{i,j}^{t} + c_{j}^{g}\right) x_{i,j} = \sum_{j\in N}\sum_{i\in M} \left(c_{i,j}^{t} + c_{j}^{g}\right) x_{i,j}^{*,j'}, \tag{4.68}$$

$$x_{i,j} \geq 0, \quad \forall i \ \in M, \quad \forall j \in N. \tag{4.69}$$

Generation of the shortest paths in the preprocessing step [106] may help to minimize the number of the waste treatment options represented by arcs (waste producer will be connected to his best option and to tolled arc). However, it is redundant in the considered case, since such preprocessing is almost equivalent to the solution of the problem. Also, the costs of each arc will iteratively change during BRD: previously redundant information should be considered in the next step. Basic single tolled arc problems without congestion solved during the algorithm are simple and are solvable in polynomial time. As it was already described in bilevel programming section, it is sufficient to order differences $c^t_{i,j} + c^g_j - c^t_{i,j'}, \forall i \in M, \forall j \in N$, and perform a simple sequential evaluation of the leader's objective function with a gate fee equal to these differences in the decreasing order. However, this representation does not consider the leader's capacity constraint and the inequality enabling the renouncing of some waste flows sent to the leader. Therefore, CCTNPP has seemed like a more suitable formulation, which better represents the structure of the problem, and might enable convenient generalization and future work with the inequalities, which will reduce the feasible region, so the solution can be found faster.

The heuristic's testing

In this section, the attention will be solely focused on testing the proposed method's ability to solve the general bilevel price-setting problem without searching for the NE (BRD functionality will be demonstrated in the case study section). An application to artificial WM network instances has been considered to validate the proposed bilevel programming algorithm. Now, the instance generation rules will be described in detail.

- A random number n of local WtE plants between 10 and 20 is generated. Capacities of WtE plants are generated randomly within a range of 25 kt to 350 kt. Their gate fees are chosen randomly between 40 €/t and 100 €/t.

- A number $m = kn$ of municipalities is generated, where k is a random number between 5 and 15. For k municipalities, waste production is generated within a range of 100 kt to 300 kt (representing large cities). For the remaining $k(n-1)$ municipalities, it is generated within a range of 5 kt to 50 kt (small and medium-sized municipalities).

- Then, these municipalities are randomly placed on a map. The map is considered to have a size of 450×300 square units of length (in particular, square kilometers are considered). However, only a range of $(50, 400) \times (50, 250)$ is considered for the municipalities. The WtE plants are randomly assigned to the municipalities.

- Additionally, 1 to 5 foreign WtE plants are randomly generated on the map within a range $(0, 50) \cup (400, 450) \times (0, 50) \cup (250, 300)$. Each plant's capacity equals the total waste production of all municipalities. All foreign WtE plants have the same gate fee of 1.5 times the maximum of local WtE plants' gate fees.

- Transportation costs are generated using the euclidean distance between the municipality and WtE plant. The distance is multiplied by a randomly generated coefficient within a range of 0.1 to 0.4 €/km.
- Locations of WtE plants and municipalities, transportation costs coefficients, gate fees, and waste productions are generated using a continuous uniform distribution. All other values are generated using a discrete uniform distribution over integers within the defined ranges.
- The generated waste productions are then rounded to two decimal places, transportation costs are rounded to an integer, and gate fees are rounded to one decimal place (thus, $\epsilon = 0.1$ can be set). This is done to computationally simplify the algorithm and to enhance the speed of checking the heuristic's correctness.

Since the heuristic will be later applied to an exemplary case study, the ranges were chosen to generate WM networks comparable to the Czech Republic's WM situation. Each map generated in the above-described way is considered an artificial scenario, for which an optimal gate fee has been subsequently established for each local WtE plant. The total of 20 scenarios have served as an input: 10 scenarios where $\sum_{i \in M} w_i^p$ is greater than total capacity of local WtE plants and 10 scenarios where $\sum_{i \in M} w_i^p$ is less than total capacity of local WtE plants have been taken into consideration. Such diversification of scenarios makes it possible to test situations when the main competitors are foreign WtE plants, as well as instances when competition takes place within a local WM network. The results are then compared to the one obtained via the complete enumeration procedure of the precision $\epsilon = 0.1$. It dwells in a successive increase of a gate fee from zero with step 0.1 and a calculation of the revenue for each linear problem solution under this gate fee. All computations were performed using the CPLEX solver within GAMS. The results and basic scenarios information are presented in Table 4.2.

One iteration of the follower's problem during enumeration lasts for approximately 0.25 seconds with $1,574$, resp. $2,236$, solutions performed in case of sufficient, resp. insufficient, capacities of local WtE plants on average. On the other side, to solve one iteration of the MIP formulation approximately 10 times more time is needed with only 4.5 iterations performed on average. Whereas ten scenarios with insufficient capacities require averagely 1.3 iterations and lose averagely 3.34% compared to optimal objective function value, the remaining scenarios are more computationally challenging (7.5 iterations are required), which do not substantially affect average loss of 3.67%. In 87% procents of the cases, loss was less than 10% and, in the worst case, loss was 45%. The maximal number of iterations that has been performed during one run of the algorithm is 46. The more detailed analysis of errors did not demonstrate some obvious pattern in the behavior of the heuristic and its performance with respect to the setting of the scenarios. Potentially, greater loss can be implied by an unrealistic input or it can be the result of complex interactions of the parameters with the shape of the generated network. It can be seen that the proposed algorithm is able to handle the randomly generated scenarios time-efficiently without substantial loss in an objective function value in most of the cases.

Table 4.2: Results of the algorithm validation

Scenario	**Number of local WtE plants**	**Total local WtE capacity [kt]**	**Average gate fee of local WtE plants [EUR/t]**	**Number of municipalities**	**Total waste production [kt]**	**Gate fee of foreign WtE plants [EUR/t]**	**Average transportation costs [EUR/t]**	**Average loss in objective function value [%]**
1	11	2,107	77.58	88	3,854	144,1	21.83	2.37%
2	12	2,35	79.85	72	3,209	146,7	44.20	5.08%
3	14	3,659	74.81	196	8,051	147,9	43.60	0.24%
4	12	2,036	69.88	168	6,884	139	77.47	1.18%
5	15	2,031	67.44	75	2,822	134,3	30.65	4.76%
6	18	2,985	76.34	198	7,639	143,4	44.80	0.05%
7	18	3,335	63.32	144	5,337	143,7	64.30	9.13%
8	16	2,844	71.34	224	8,518	145,5	72.17	0.94%
9	12	2,939	76.66	72	3,046	148,5	37.73	6.89%
10	20	4,677	68.17	200	7,252	149,9	34.87	2.76%
11	20	4,229	74.91	120	4,216	148,7	56.77	12.17%
12	17	4,065	72.15	85	2,954	137,6	61.81	1.02%
13	20	4,807	69.36	140	4,519	147,8	61.55	2.70%
14	17	3,715	71.78	85	3,127	145,4	62.08	5.92%
15	15	3,209	64.24	75	2,897	130,1	60.01	1.34%
16	20	4,101	72.60	100	3,613	146,9	22.26	0.91%
17	11	2,629	77.46	55	2,335	146,1	26.55	1.16%
18	17	4,756	71.47	85	3,183	148,4	57.33	2.92%
19	19	4,006	75.72	95	3,558	146,8	50.00	4.63%
20	19	4,141	64.33	95	3,664	149,2	27.71	3.96%

4.1.5 Exemplary case study

In this section, the Czech Republic exemplary case study will demonstrate how the proposed approach could be applied to design the optimal capacity of the future WtE plant. Moreover, the numerical results of the proposed bilevel programming heuristics algorithm will be presented. It is assumed that in the Czech Republic, there are 16 WtE plants (the founding of 12 of them is currently planned). However, some waste producers from the Czech Republic might use the services of facilities in the nearby countries (Germany and Austria). To create an upper boundary on the possible gate fee and ensure the existence of the «toll-free» path, these facilities are represented as three WtE plants with a fixed gate fee of 100 €/t and the capacity corresponding to the total waste production of the whole Czech Republic.

To compete with these foreign facilities, it is planned to build one more WtE plant in the Czech Republic (WtE plant «Otrokovice»), and the question of optimal capacity design arises. To optimally estimate the capacity, it is advised to «place» this facility in the currently existing network and find the NE of the considered WtE plants price-setting game using the suggested approach: BRD based on the proposed bilevel programming heuristics. The resulting price state will enable the establishment of the waste flows and revenues of all WtE plants in the network. This process, iteratively repeated for each capacity design, will provide an image of the expected revenue of the planned facility, which can be compared to required investments. The starting point of the whole process for each WtE plant (except the foreign plants) is assumed to be the gate fee of 50 €/t, and the first capacity design is 25 kt/y. To computationally simplify the algorithm, the transportation costs are assumed to be integers (thus, ϵ=0.1). Productions, as well as capacities, are assumed to be annual.

Unfortunately, the BRD failed to find an NE during the first attempt. When the σ, defining stopping condition of the algorithm in Figure 2.2, is considered to be too small, the algorithm gets stuck in the cycle. This fact can be explained, by the hypothesis, that when continuous strategy sets are assumed, the change of the gate fee is expected to be always profitable. This would lead to non-existence of the fixed-point in best-response correspondences, and, as a result, the NE would cease to exist in a general game. This possible explanation will be studied in detail at the end of the section devoted to WtE price-setting. To overcome this complication, it is assumed that, when the norm of the difference vector is less than 1, no substantial change in the gate fees vector has occurred, and the algorithm will be stopped. This assumption will enable to prevent the cyclic nature of the price-setting game, when players successively lower their prices to obtain greater demand. Under assumption σ=1, the gate fee stable outcomes were computed for the suggested capacities from 25 kt to 350 kt with the step of 25 kt. The capacity usage and the estimated revenue of the planned WtE plant «Otrokovice» are presented in Table 4.3. The table confirms that the proposed model is reasonable: capacity increase causes a gradual decrease in gate fees for all of the considered WtE plants. Thus, in accordance with basic economy rules, the greater «supply» (capacity) leads to a lower price (gate fee). Clearly, to improve the reliability of the found solutions, the impact of the input parameters and

Table 4.3: Results for «Otrokovice»

Capacity [kt]	**Gate fee [€/t]**	**Obtained amount of waste [kt]**	**Employed capacity**	**Revenue [T€]**
25	68.8	6.54	26.17%	450.21
50	55.9	36.93	73.85%	2,064.21
75	54.6	67.47	89.97%	3,684.07
100	53.2	84.60	84.60%	4,500.81
125	52.9	103.14	82.51%	5,456.18
150	50.8	146.09	97.40%	7,421.55
175	50.5	152.88	87.36%	7,720.50
200	51.5	163.94	81.97%	8,442.81
225	49.3	163.94	72.86%	8,082.15
250	48.9	239.66	95.87%	11,719.57
275	47.6	265.91	96.69%	12,657.26
300	46.8	252.75	84.25%	11,828.56
325	48	265.91	81.82%	12,763.62
350	48.6	260.06	74.30%	12,638.91

initial point choice on the algorithm precision and speed of convergence should be studied in the future.

To choose an appropriate capacity design for a particular WtE project, the revenues from waste treatment have to be compared with the initial investments. For the sake of simplicity, the solved task does not consider operational costs and revenues related to heat and electricity selling. In the case of investment costs, it is important to reflect decreasing unit costs when increasing capacity. The costs for particular capacity variants are estimated by adopting the following formula from [20]:

$$I = I_R \frac{C}{C_R}^{0.75},$$

where I represents investments and C represents the capacity of the facility. Subscript R denotes the reference number. For the case presented herein, the reference numbers were set to $I_R = 4$ M€/y and $C_R = 100$ kt/y. Figure 4.4 illustrates the results for the considered capacity variants. The profitability of investment can be easily compared via ratios illustrated by a line. Figure 4.4 demonstrates that the greater capacity does not always guarantee a better ratio between revenue and investments. Thus, the market power induced by a greater capacity does not automatically ensure a greater return on investment but has phase-shifting properties. For example, only after trespassing the capacity of 225 kt/y the WtE plant again obtains an advantageous position on the WM market and can pursue a greater return on investment. The decision about the optimal capacity directly depends on the available capital for the investment. For example, if the maximal possible investment is around 7 M€/y, it is reasonable to invest less and build a WtE plant with a capacity of 150 kt/y. Now, suppose the management of the WtE plant can ensure greater resources for the investment. Then, it is more profitable to invest approximately 8 M€/y and build a facility with a capacity of 250 − 275 kt/y (higher precision can be achieved by choice of the smaller step).

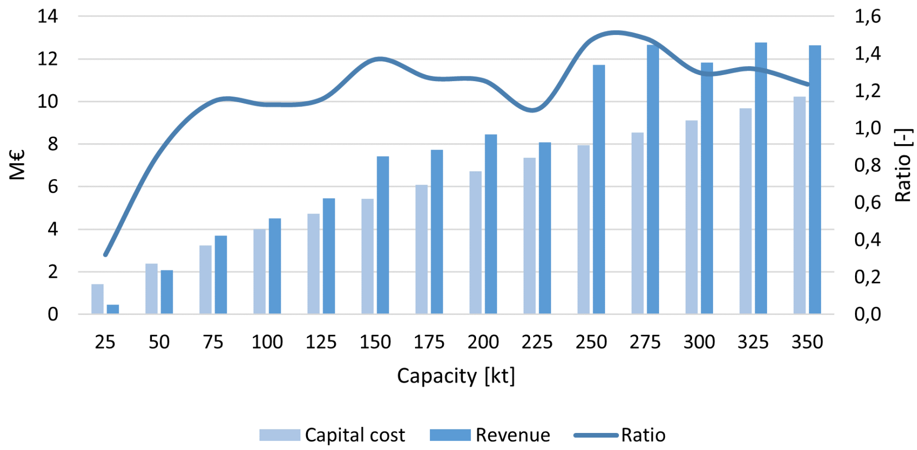

Figure 4.4: Ratio of revenue vs. capital cost

Numerical results of the heuristic algorithm

To verify that the algorithm is also able to provide the optimal or near-optimal solution in the realistic scenario, its performance has been compared to the classical enumeration of the precision ϵ. In particular, gate fee vectors from the last iteration of BRD have been used as an input describing fixed gate fees of competitors. Thus, 17 different cases (each for one of 17 competing WtE plants) have been calculated for 14 capacity designs. Table 4.4 represents information about non-optimal solutions found by the proposed heuristics.

The heuristics failed to find an optimum solution only in 44 cases out of the considered 238, only 10 of which have led to a loss greater than 1%. Moreover, the largest difference between found optimum and the optimum established by the algorithm is 1.1. Thus, Table 4.4 confirms the potential of the proposed algorithm on the realistic data: it produces an optimal solution in most cases. Due to comparability of the artificial scenarios to the exemplary case study input data, the computational time of one iteration remains approximately the same. Thus, the case study motivated by the realistic data also proves that the algorithm solves underlying NP-hard problems cardinally faster with an average objective function value optimality loss of 0.18%. Since the underlying motivation was to provide fast input into the BRD evaluation cycle, the proposed heuristics can be considered suitable. The presented apparatus can provide a realistic estimate of the optimal gate fee for a particular WtE plant, which enables finding the NE of the WtE plant's price-setting game.

Now, we proceed to a more theoretical study of the presented WtE plants' game (and of the analogical games) in order to establish some general conclusions about NE existence.

Table 4.4: Numerical results for the heuristic

Capacity [kt]	WtE plant	Iterations	Found optimum	Real optimum	Loss in revenue
25	Praha	2	100.7	99.7	2.64%
25	Brno	2	78.8	78.7	0.76%
25	Liberec	1	95.6	95.5	1.79%
25	Ústí nad Labem	1	94.5	93.8	6.70%
50	Tábor	2	74.8	74.9	0.13%
50	Hradec Králové	2	73.8	73	0.11%
75	Liberec	1	79.5	79.6	0.13%
75	Vsetín	1	50.5	50.4	0.37%
100	Liberec	1	80.8	79.8	0.66%
100	Ústí nad Labem	1	77.8	77.9	0.13%
125	Liberec	1	75.9	76	0.13%
125	Most	1	76.9	77	0.13%
125	Ústí nad Labem	2	72.8	73	0.27%
150	Hradec Králové	3	70.5	69.8	0.20%
150	Ústí nad Labem	1	75.5	75.6	0.13%
175	Praha	3	79.6	79.5	2.86%
175	Liberec	1	75.4	75.5	0.13%
175	Ústí nad Labem	3	72.3	72.4	0.14%
175	Vsetín	1	47.4	47.5	0.21%
225	Brno	2	60.4	60.2	0.56%
225	Ústí nad Labem	2	73.9	74	0.14%
250	Tábor	2	68	67.9	0.95%
250	Liberec	1	72.9	73	0.14%
250	Most	1	74.9	74.8	3.90%
250	Otrokovice	2	48.9	49.9	2.00%
250	Vsetín	1	45.9	46.9	0.35%
275	Liberec	1	73.2	73.3	0.14%
275	Melník	2	71.5	71.2	5.25%
275	Jihlava	1	62.4	62.5	0.16%
275	Otrokovice	2	47.6	48.7	2.26%
300	Brno	2	56.5	56.6	0.02%
300	Hradec Králové	4	65	65.5	0.76%
300	Liberec	1	72.9	72.5	1.35%
300	Otrokovice	3	46.8	47.8	2.09%
325	Brno	2	57.5	57.4	0.71%
325	Hradec Králové	1	66.3	65.9	0.53%
325	Ústí nad Labem	1	70.7	70.8	0.14%
325	Jihlava	1	62.4	62.2	0.33%
325	Otrokovice	1	48	48.2	0.41%
325	Zlín	1	47.9	48	0.21%
350	Tábor	4	67.3	67.2	0.95%
350	Brno	1	58	57.8	0.54%
350	Liberec	1	73.1	73.2	0.14%
350	Ústí nad Labem	3	70	70.1	0.14%

4.1.6 Price-setting game and its properties

Our empirical results have pointed out possible non-existence of NE in problems of price-setting. This is why this section is devoted to analysis of the newly introduced class of price-setting games and to research on existence of NE in games of this type. In particular, we are interested in proving the fact that, under some real-world constraints and limitations, there might be no stable price state for sufficiently small artificial parameter ϵ from previous section. Before we define a price-setting game, a concept of market situation should be discussed.

Definition (Market situation)**.** The market situation

$$MS = (M, N, R, (t_{i,j})_{i \in N \cup R, j \in M}, (c_i)_{i \in N \cup R}, (d_j)_{j \in M}, x_{ref})$$

is defined by the set of customers $M = \{1, ..., m\}$, $|M| \geq 1$, the set of domestic producers $N = \{1, ...n\}$, $|N| \geq 2$, the set of foreign producers $R = \{n+1, ..., r\}$, $|R| \geq 1$, transportation costs per unit of goods $t_{i,j} \geq 0$, $\forall j \in M$, $\forall i \in N \cup R$, needed to transport unit of product from producer $i \in N \cup R$ to consumer $j \in M$, production capacities $c_i > 0$, $\forall i \in N \cup R$, of producers, and demands $d_j > 0$, $\forall j \in M$, of consumers. Foreign producers are participants of the market creating the reference price $x_{ref} > 0$.

Further, to simplify some expressions, we will use notation $\tilde{N} = N \cup R$. We also would like to describe role of x_{ref} in more details. In our study, the reference price x_{ref} is a price of a product on a foreign market, so, when the price on the domestic market exceeds the reference price (and potential transportation costs), it is more economic to import the product. Thus, it indeed creates «reference» for domestic producers and establishes price ceiling after trespassing which, domestic market begin to lose customers. Now, we can proceed to the definition of the price-setting game associated with a market situation.

Definition (Price-setting game)**.** Let us assume the market situation MS. Then, we define the price-setting game $G = (N, (X_i, \pi_i)_{i \in N})$ associated with MS as a game between players from a set N, where strategy of each player is represented as a price $x_i \in X_i = (0, \infty), \forall i \in N$. Elements of R are not part of the game itself, and they prices are fixed as $x_i = x_{ref}, \forall i \in R$. Then, each player's payoff function $\pi_i(x), i \in N$, is defined as

$$\pi_i(x) = \sum_{j \in M} x_i q^*_{i,j}, \text{ where } (q^*_{l,j})_{l \in \tilde{N}, j \in M,} \in Q,$$

$$s.t. \sum_{j \in M} x_i q^*_{i,j} \leq \sum_{j \in M} x_i q_{i,j}, \ \forall (q_{l,j})_{l \in \tilde{N}, j \in M} \in Q,$$

where set Q is defined as

$$Q = \arg \min_{q_{l,j}, l \in \tilde{N}, j \in M} \sum_{j \in M} \sum_{l \in \tilde{N}} (x_l + t_{l,j}) q_{l,j},$$

$$s.t. \sum_{j \in M} q_{l,j} \leq c_l, \ \forall l \in \tilde{N},$$

$$\sum_{l \in \tilde{N}} q_{l,j} = d_j, \ \forall j \in M,$$

$$q_{l,j} \geq 0, \ \forall j \in M, \ \forall l \in \tilde{N}.$$

Thus, domestic producers are independently maximizing their profits, whereas customers are minimizing their total costs, while aiming at completely satisfying their demands without capacity overruns. The above-defined game is designed to model markets with a high level of government interference, where costs, that occur during operation, are negligible compared to initial capital investments: this is why the payoff function does not involve fixed or variable costs. In order to ensure the correct definition of the payoff function, we have employed the already introduced pessimistic approach, i.e., that in the case of the existence of multiple solutions to the lower level customers' cost minimization problem, the solution, which is the most unfavorable to the producer i is chosen. This choice is crucial since the following problem can occur at NEs of games solved using an optimistic approach.

Example 4.1.3. Assume market situation MS with following parameters: $M = \{1\}$, $N = \{1, 2\}$, $R = \{3\}$, $d_1 = 5$, $x_{ref} = 4$, $c_1 = c_2 = c_3 = 10$, and $t_{l,j} = 0, \forall l \in \tilde{N}, j \in M$. Then, the optimistic NE of the associated game G is price state $x^* = (4, 4)$ which guarantees to both players payoff $\pi_1(x^*) = \pi_2(x^*) = 20$.

One can see, that optimistic NE might create overoptimistic expectations and some kind of «vacuum»: these expectations can not be fulfilled for all of players since the consumer will spend only 20. In other words, the optimistic approach has doubled expected profit of producers compared to amount of money spent by consumer. Thus, the suggested pessimistic approach is more reasonable than the optimistic one. It is also important to note, that it is required, that consumers' demand is completely satisfied, leading to the necessary feasibility assumption that total market production capacity is greater or equal to the total demand, i.e.

$$\sum_{i \in \tilde{N}} c_i \geq \sum_{j \in M} d_j \ (\textbf{feasibility}).$$

However, the stronger assumption should be imposed on game in order to make its study reasonable:

$$c_i \geq \sum_{j \in M} d_j, \forall i \in R, \ (\textbf{boundness}).$$

Now, we will demonstrate a possible problem, which occurs when boundness assumption is not fulfilled.

Example 4.1.4. Assume market situation MS with following parameters: $M = \{1\}$, $N = \{1, 2\}$, $R = \{3\}$, $d_1 = 5$, $x_{ref} = 4$, $c_1 = c_2 = 3$, $c_3 = 1$, and $t_{l,j} = 0, \forall l \in \tilde{N}, j \in M$. In this setting, $q^*_{1,1} \geq 1$ and $q^*_{2,1} \geq 1$ hold for any $x \in X_N$. Thus, $\forall x \in X_N$ and for each $i \in N$

$$\pi_i(x_1 + \delta_1, x + \delta_2) \geq \pi_i(x_1, x_2)$$

always holds for arbitrary $\delta_i > 0, i \in N$. Such situation will lead to the state where consumer will have to pay infinite amount of money for goods.

As we can see, insufficient capacities of foreign producers might make optimization problems of domestic producers unbounded. Clearly, boundness imply the feasibility assumption, making it redundant. Before a discussion on existence of NE is given, the description of payoff function π_i and of $\sum_{j\in M} q^*_{i,j}$ for $i \in N$, fixed strategy profile (x_{-i}), and given x_{ref} should be provided.

Properties of the payoff function and the lower-level optimal solution. Assume some $i \in N$, fixed strategy profile (x_{-i}), and given x_{ref}. Then, let us describe $\sum_{j\in M} q^*_{i,j}$ as a function of x_i. Due to the nature of linear programming problems, their solutions are convex combinations of extreme points or directly extreme points (in case problems are bounded). This implies that $\sum_{j\in M} q^*_{i,j}$ as a function of x_i is non-increasing piece-wise constant and right continuous [62]. This properties should hold, since otherwise it will be a contradiction with optimality of $(q^*_{l,j})_{l\in\tilde{N},j\in M}$ and its pessimistic property with respect to i. If this function will be multiplied by a variable $x_i > 0$, we will obtain a piece-wise linear (where each segment is increasing) and a right continuous payoff function $\pi_i(x_i)$ [62]. Now, the concept of NE in the considered class of games can be discussed.

Concept of δ-equilibrium

Unfortunately, the definition of the problem violates the existence of NE. For the above-defined payoff function, a more profitable strategy can always be found: it is sufficient to choose the price, which will shift the payoff closer to the peak of the «optimal» linear segment. The peak istelf is «absent»: in pessimistic approach it is only a limit of the payoff function from the left, which corresponds to an optimistic approach optimal solution (which does not have to be unique). Thus, player is always able to choose some sufficiently small $\delta > 0$, such that, for a fixed (x_{-i}), given x_{ref}, and arbitraty x_i

$$\pi_i(x_i^{opt} - \delta) \geq \pi_i(x_i)$$

where x_i^{opt} denotes the optimistic approach optimal price. However, if we assume, that players can be satisfied with the «nearly» optimal solution, then it is possible to define the following alternative to the pure NE concept.

Definition (δ-NE)**.** Let us assume a normal form game $G = (N, (X_i, \pi_i)_{i\in N})$ with $X_i = (0, \infty)$, $\forall i \in N$. Then, we define δ-NE, $\delta > 0$, as a strategy profile $\tilde{x} \in X_N$, such that $\tilde{x}_i = x^{\delta}_{\lim,i} - \delta$, where $x^{\delta}_{\lim,i}$ fulfills

$$\lim_{x_i \to x^{\delta-}_{\lim,i}} \pi_i(x_i, \tilde{x}_{-i}) \geq \pi_i(x_i, \tilde{x}_{-i}), \forall x_i \in (\delta, \infty).$$

This way we avoid the concept of the classical NE, replacing it with the strategy profile that might be arbitrarily close to a profile that is NE in a sense of limit. Such a definition makes perfect sense with respect to the application-oriented nature of this work: the price is always set in some currency, which has the lowest possible order of magnitude. For example, when we work with euros, the price can be changed only by cents, meaning the closest possible price to $x^{\delta}_{\lim,i}$ is $x^{\delta}_{\lim,i} - \delta$ with $\delta = 0.01$. We would like to additionally emphasize that the concept of δ-NE is not necessarily unique.

Existence of δ-NE

We begin this section with the study of δ-NE existence in market situations without transportation costs. It will be demonstrated, that capacities of producers significantly impact existence of δ-NE. In particular, we will prove that, under the certain assumptions, there exists a $\gamma > 0$ such that, for every δ smaller than γ, δ-NE for non-zero transportation costs game will cease to exist. However, before starting our theoretical considerations we would like to demonstrate, that concept of δ-NE exists at least for some games. Thus, we introduce the following simple example.

Example 4.1.5. Assume market situation MS with following parameters: $M = \{1\}$, $N = \{1, 2\}$, $R = \{3\}$, $d_1 = 6$, $x_{ref} = 4$, $c_1 = 4, c_2 = 3$, and $c_3 = 10$, and $t_{l,j} = 0, \forall l \in \tilde{N}, j \in M$. Let us set $\delta = 1$. Then, δ-NE is a state $\tilde{x} = (3, 2)$ which guarantees payoffs $\pi_1(\tilde{x}) = 9$ and $\pi_2(\tilde{x}) = 6$.

Zero transportation costs

In this part, we consider only price-setting games G associated with MS, where $t_{i,j} = 0, \forall i \in \tilde{N}, j \in M$. Further, we will use notation $x_{\lim,i}(x_{-i})$ describing all $x_{\lim,i}$ such that

$$\lim_{x_i \to x_{\lim,i}^-} \pi_i(x_i, x_{-i}) \geq \pi_i(x_i, x_{-i}), \forall x_i \in X_i.$$

Notation $x^{\delta}_{\lim,i}(x_{-i})$ will be used analogically. Then, we begin with the following lemma.

Lemma 4.1.6. Assume the price-setting game G associated with zero transportation costs MS fulfilling boundness and arbitrary strategy profile $\hat{x} \in X_N$. Then, $\forall i \in N$, it holds that

$$x_{\lim,i} \in x_{\lim,i}(\hat{x}_{-i}) \Rightarrow x_{\lim,i} = \hat{x}_l \text{ for some } l \in \tilde{N} \setminus \{i\}.$$

Proof. Further, we will denote $\sum_{j \in M} q^*_{i,j}$ for a fixed $\hat{x}_{-i}$, $i \in N$, as $\sum_{j \in M} q^*_{i,j}(x_i)$. Due to zero transportation costs,

$$\sum_{j \in M} q^*_{i,j}(x_i) > 0, \ 0 < x_i < \min_{l \in \tilde{N} \setminus \{i\}} \hat{x}_l,$$

holds. Then, due to boundness, there is also such $x_i \in X_i$ that $\sum_{j \in M} q^*_{i,j}(x_i) = 0$. Thus, there will always occur at least one decrease in $\sum_{j \in M} q^*_{i,j}(x_i)$ for some $x_i \in X_i$. Now, let us assume, that we have some point of decrease $x^*_i \in X_i$, i.e.

$$\sum_{j \in M} q^*_{i,j}(x^*_i) < \sum_{j \in M} q^*_{i,j}(x^*_i - \delta), \ \delta \in (0, x^*_i),$$

since $\sum_{j \in M} q^*_{i,j}(x_i)$ is non-increasing and only right continuous. However, decrease can occur only if there $\exists l \in \tilde{N}$ such that $x^*_i - \delta < \hat{x}_l \leq x^*_i$, since otherwise it will imply a contradiction due to absence of decrease direction for the objective function. Then, we can obtain

$$\lim_{\delta \to 0^+} x^*_i - \delta < \hat{x}_l \leq x^*_i \Rightarrow x^*_i = \hat{x}_l.$$

Thus, every possible decrease x_i^* should be given by some $\hat{x}_l, l \in \tilde{N} \setminus \{i\}$. The previously discussed properties of $\pi_i(x_i, \hat{x}_{-i})$, imply that $x_i^* = \hat{x}_l, l \in \tilde{N} \setminus \{i\}$, are only possible points fulfilling

$$\lim_{x_i \to x_i^{*,-}} \pi_i(x_i, \hat{x}_{-i}) \geq \pi_i(x_i, \hat{x}_{-i}), \forall x_i \in (x_i^* - \epsilon, x_i^* + \epsilon),$$

for some $\epsilon > 0$. Therefore, one or more of such $x_i^* = \hat{x}_l, l \in \tilde{N} \setminus \{i\}$, should fulfill

$$\lim_{x_i \to x^{*-}} \pi_i(x_i, \hat{x}_{-i}) \geq \pi_i(x_i, \hat{x}_{-i}), \forall x_i \in X_i.$$

□

Then, we can proceed to the following theorem on δ-NE existence for price-setting games associated with a particular group of MS with zero transportation costs.

Theorem 4.1.7. *For any zero transportation costs price-setting game G fulfilling boundness and*

$$\sum_{l \in N \setminus \{i\}} c_l > \sum_{j \in M} d_j, \forall i \in N, \ (\textbf{\textit{absence of dictator}}),$$

δ-NE exists for every δ.

Proof. Assume arbitrary value $\delta > 0$. If we construct a price state $\tilde{x}$ with

$$0 < \tilde{x}_i \leq \delta, \forall i \in N,$$

then this state is always a δ-NE. Due to absence of dictator, every x_i with $x_i \geq \delta$ will always lead to $\pi_i(x_i, \tilde{x}_{-i}) = 0$. Thus, $x^{\delta}_{\text{lim},i}(\tilde{x}_{-i}) = (\delta, \infty)$, since every $x^{\delta-}_{\text{lim},i}$ from such $x^{\delta}_{\text{lim},i}(\tilde{x}_{-i})$ trivially satisfies the expression

$$\lim_{x_i \to x^{\delta-}_{\text{lim},i}} \pi_i(x_i, \tilde{x}_{-i}) \geq \pi_i(x_i, \tilde{x}_{-i}), \forall x_i \in (\delta, \infty),$$

where both sides equal 0. Therefore, if we choose $x^{\delta}_{\text{lim},i} = \tilde{x}_i + \delta, \forall i \in N$, then our $\tilde{x}$ will completely meet the definition of δ-NE. □

Absence of dictator ensure that there is some amount of demand over which players might possibly compete. However, the theorem points out an interesting drawback of δ-NE for the MS with this property: some strategy profiles are δ-NE only due to the fact, that players cannot play their optimal prices with respect to the given price state. This problem does not occur when capacity dictator exists, as we will demonstrate in the following theorem.

Theorem 4.1.8. *Assume zero transportation costs MS fulfilling boundness and that $\exists i^* \in N$ such that*

$$\sum_{j \in M} d_j > c_{i^*}, \ \sum_{j \in M} d_j > \sum_{k \in N \setminus \{i^*\}} c_k \ \textit{and} \ \sum_{j \in M} d_j < \sum_{k \in N} c_k. \ (\textbf{\textit{existence of dictator}})$$

Then, for the associated price-setting game G, there $\exists \delta$ such that δ-NE does not exist.

Proof. The theorem can be proven via contradiction. Assume there exist δ-NE profile $\tilde{x}$ for each $\delta > 0$. From Lemma 4.1.6, it follows that $x_{\lim,i^*} = \tilde{x}_l, l \in \tilde{N} \setminus \{i\}$, for all $x_{\lim,i^*} \in x_{\lim,i^*}(\tilde{x}_{-i^*})$. If we consider $\delta \in (0, x_{ref})$, then, due to existence of dictator,

$$x^{\delta}_{\lim,i^*} = \tilde{x}_l, l \in \tilde{N} \setminus \{i\}, \text{ for all } x^{\delta}_{\lim,i^*} \in x^{\delta}_{\lim,i^*}(\tilde{x}_{-i^*}),$$

with

$$\lim_{x_{i^*} \to x^{\delta-}_{\lim,i^*}} \pi_{i^*}(x_{i^*}, \tilde{x}_{-i^*}) > 0.$$

At first, we focus ourselves on the case $x^{\delta}_{\lim,i^*} = x_{ref}$. Clearly, we can order elements of $\tilde{x}$ as

$$\tilde{x}_{i_1} \leq ... \leq \tilde{x}_{i_{n-1}} \leq \tilde{x}_{i^*} < x_{ref}.$$

If we consider $\delta \in (0, \frac{x_{ref}}{3})$, it should hold that $\tilde{x}_{i_{n-1}} = x_{ref} - \delta$ or $\tilde{x}_{i_{n-1}} = \tilde{x}_{i^*} - \delta$, since otherwise it will be in contradiction with the $\tilde{x}$ being a δ-NE. It also important to note, that existence of dictator implies

$$\lim_{x_{i^*} \to x^-_{ref}} \sum_{j \in M} q^*_{i^*,j}(x_{i^*}, \tilde{x}_{-i^*}) < c_{i^*}.$$

Now, we introduce notation

$$\gamma_i := \frac{x_{ref} \Delta^{\min}_i}{2c_i},$$

where

$$\Delta^{\min}_i := \min_{x_{-i} \in X_{-i}, 0 < \epsilon < x_{ref}} \left(\lim_{x_i \to (x_{ref} - \epsilon)^-} \sum_{j \in M} q^*_{i,j}(x_i, x_{-i}) - \lim_{x_i \to x^-_{ref}} \sum_{j \in M} q^*_{i,j}(x_i, x_{-i}) \right),$$

$$s.t. \lim_{x_i \to (x_{ref} - \epsilon)^-} \sum_{j \in M} q^*_{i,j}(x_i, x_{-i}) > \lim_{x_i \to x^-_{ref}} \sum_{j \in M} q^*_{i,j}(x_i, x_{-i}).$$

Then, for sufficiently small δ, such that $\delta \in (0, \frac{x_{ref}}{3}) \cap (0, \gamma_{i^*})$, choice $x_{\lim,i^*}(\tilde{x}_{-i}) = x_{ref}$ does not guarantee the greatest possible payoff. Player i^* is able to ensure greater payoff by choosing $x_{ref} - 2\delta$ as a limit. Indeed, if we consider such δ, it will imply

$$\delta < \frac{x_{ref} \Delta_{i^*}(\tilde{x}_{-i^*})}{2 \lim_{x_{i^*} \to (x_{ref} - 2\delta)^-} \sum_{j \in M} q^*_{i^*,j}(x_{i^*}, \tilde{x}_{-i^*})},$$

where

$$\Delta_{i^*}(\tilde{x}_{-i^*}) = \lim_{x_{i^*} \to (x_{ref} - 2\delta)^-} \sum_{j \in M} q^*_{i^*,j}(x_{i^*}, \tilde{x}_{-i^*}) - \lim_{x_{i^*} \to x^-_{ref}} \sum_{j \in M} q^*_{i^*,j}(x_{i^*}, \tilde{x}_{-i^*}).$$

However, using this expression, we can obtain

$$\lim_{x_{i^*} \to x^-_{ref}} x_{i^*} \sum_{j \in M} q^*_{i^*,j}(x_{i^*}, \tilde{x}_{-i^*}) < \lim_{x_{i^*} \to (x_{ref} - 2\delta)^-} x_{i^*} \sum_{j \in M} q^*_{i^*,j}(x_{i^*}, \tilde{x}_{-i^*}).$$

This fact is in direct contradiction with definition of δ-NE, proving that in case $x^{\delta}_{\lim,i^*} = x_{ref}$ our concept of δ-NE does not exist for $\delta \in (0, \frac{x_{ref}}{3}) \cap (0, \gamma_{i^*})$.

Now, we will focus our attention on the second possible case, where $x^{\delta}_{\lim,i^*} = \tilde{x}_l, l \in N \setminus \{i^*\}$. Consider δ such that

$$\delta \in \left(0, \frac{x_{ref}(\sum_{j\in M} d_j - \sum_{k\in N\setminus\{i^*\}} c_k)}{2c_{i^*}}\right).$$

Such choice implies

$$2\delta c_{i^*} < x_{ref}(\sum_{j\in M} d_j - \sum_{k\in N\setminus\{i^*\}} c_k),$$

leading to the fact, that

$$\tilde{x}_l > 2\delta$$

should hold, since otherwise $x^{\delta}_{\lim,i^*} = \tilde{x}_l, l \in N \setminus \{i^*\}$, will be contradicted. Then, there are two possibilities:

$$\lim_{x_{i^*}\to\tilde{x}_l^-} \sum_{j\in M} q^*_{i^*,j}(x_{i^*}, \tilde{x}_{-i^*}) < c_{i^*} \text{ or } \lim_{x_{i^*}\to\tilde{x}_l^-} \sum_{j\in M} q^*_{i^*,j}(x_{i^*}, \tilde{x}_{-i^*}) = c_{i^*}.$$

In case strict inequality holds, it implies that

$$\lim_{x_l\to(\tilde{x}_l+\delta)^-} \pi_l(x_l, \tilde{x}_{-l}) = 0.$$

However,

$$\lim_{x_l\to(\tilde{x}_l-\delta)^-} \pi_l(x_l, \tilde{x}_{-l}) > 0$$

is an obvious contradiction with assumption that $\tilde{x}$ is a δ-NE.

Otherwise, consider that $\lim_{x_{i^*}\to\tilde{x}_l^-} \sum_{j\in M} q^*_{i^*,j}(x_{i^*}, \tilde{x}_{-i^*}) = c_{i^*}$ holds. Let us assume following ordering of elements from $\tilde{x}$:

$$... \leq \tilde{x}_{i^*} < \tilde{x}_l \leq ... < x_{ref}.$$

Thus, there should exist $l^* \in N \setminus \{i^*\}$ with

$$\tilde{x}_{l^*} + \delta = x_{ref}$$

such that

$$\lim_{x_{l^*}\to x_{ref}^-} \sum_{j\in M} q^*_{l^*,j}(x_{l^*}, \tilde{x}_{-l^*}) < c_{l^*},$$

since otherwise $\tilde{x}$ cannot be δ-NE or it will directly imply that

$$\lim_{x_{l^*}\to(\tilde{x}_{l^*}+\delta)^-} \pi_{l^*}(x_{l^*}, \tilde{x}_{-l^*}) = 0,$$

what can be easily prevented by choosing $x^{\delta}_{\lim,l^*} = \tilde{x}_{i^*}$. Then, the rest of the proof completely corresponds to the case $x^{\delta}_{\lim,i^*} = x_{ref}$ with respect to l^* and $\delta \in (0, \gamma_{l^*})$.

Thus, for every

$$\delta \in (\cap_{i\in N}(0,\gamma_i)) \cap \left(0, \frac{x_{ref}(\sum_{j\in M} d_j - \sum_{k\in N\setminus i^*} c_k)}{2c_{i^*}}\right) \cap (0, \frac{x_{ref}}{3}) \neq \emptyset,$$

the concept of δ-NE does not exist. □

The previous proof has led us to the following corollary.

Corollary 4.1.9. Assume market situation MS fulfilling boundness and existence of dictator. Then, for the associated price-setting game G, there $\exists\gamma$, s.t. for all $\delta \in (0, \gamma)$, δ-NE ceases to exist.

Indeed, if we set

$$\gamma = \min\left\{ (\min_{i\in N} \gamma_i), \frac{x_{ref}(\sum_{j\in M} d_j - \sum_{k\in N\setminus i^*} c_k)}{2c_{i^*}}, \frac{x_{ref}}{3} \right\},$$

then for each $\delta, 0 < \delta < \gamma$, the proposed δ-NE will cease to exist. Unfortunately, we were not able to prove an existence of the analogical threshold in the case of general transportation costs.

General transportation costs

Non-zero transportation costs complicate study of δ-NE existence representing important competitive advantage for some of the players. The main problem is that, for the price-setting game G associated with general MS fulfilling boundness and arbitrary strategy profile $\hat{x} \in X_N$, we have

$$x_{\lim,i} \in x_{\lim,i}(\hat{x}_{-i}) \Rightarrow x_{\lim,i} = \hat{x}_l + t_{l,j} - t_{i,j} \text{ for some } l \in \tilde{N} \setminus \{i\},\ j \in M,$$

for every $i \in N$. Thus, transportation costs bring asymmetry into the game and it is not possible to generalize the considerations established in Theorem 4.1.8 and prove problem with optimality of playing x_{ref}. Indeed, each player may play one of many possible «versions»

$$x_{ref} + t_{r,j} - t_{i,j} \text{ for some } r \in R,\ j \in M.$$

At least, we were able to deduce the assumption, that will prevent the situation described in Theorem 4.1.7:

- For each $i \in N$, $\exists! j^i \in M$, such that

$$\min_{l\in\tilde{N}\setminus\{i\},} t_{l,j^i} - t_{i,j^i} > 0, \text{ (\textbf{internal competitiveness})}.$$

Internal competitiveness ensures that each i can always achieve non-zero profit with

$$\lim_{x_i \to (\min_{l\in\tilde{N}\setminus\{i\},} t_{l,j^i} - t_{i,j^i})^-} \pi_i(x_i, x_{-i}) > 0$$

under any given x_{-i}. Thus, $\delta < \min_{i\in N} \min_{l\in\tilde{N}\setminus\{i\},} t_{l,j^i} - t_{i,j^i}$ will make implications of Theorem 4.1.7 impossible. Now, we will briefly discuss implications of our theoretical findings.

Discussion

Clearly, it is rather hard to directly establish value of γ from Corollary 4.1.9, due to necessity of finding all $\Delta_i^{min}, i \in N$. In practice, it is possible to establish lower boundary for $\Delta_i^{min}, i \in N$. In the case study, the smallest order of magnitude for the capacities and demands is 10^{-2}. Thus, $\Delta_i^{min}, i \in N$, will never be lower than 0.01. This estimate will enable us to directly calculate lower boundary for γ. Clearly, the resulting value will be less than the considered $\epsilon = 0.01$. However, it is important to note that the resulting γ does not need to be a «tight» boundary. Thus, it is not guaranteed, that the established γ will be the greatest possible. We believe that the considerations established in subsection 4.1.6 will enable us to generalize Theorem 4.1.7 and to estimate «tighter» upper boundary for γ in the future.

4.2 Waste producers' costs minimization

The upcoming CEP legal changes will substantially affect municipalities due to more complex and expensive waste treatment in the future. Thus, it is also essential to model and study the implementation of WtE technology from the municipalities point of view, considering their objectives of WM cost minimization. The way how municipalities financially handle new legal requirements will substantially impact sustainability of WtE plants and, as a result, of the energy produced there. To react to the up-coming legal changes, it is beneficial to create municipal unions, focused on the cooperation in WM. Such municipal unions help to lower waste treatment costs and to optimize waste collection. Whereas full cooperation axiomatically assumed in [51] can be considered as the most desirable outcome, it may not correspond to the realistic one due to circumstances/settings. In fact, such a centralized approach cannot properly model individual incentives of municipalities and interactions between them. This behavioral aspect becomes crucial during planning of municipal budgets and negotiations about the legal form of municipal units' cooperation. Therefore, it is necessary to study formation of municipal unions in a dynamic manner. Moreover, the distribution of resulting costs across municipal units should be assessed with respect to their locations and waste productions. Such cost analysis will enable to estimate future realistic WM tariffs, providing important information for municipal councils.

In this section, the problem of municipal unions formation will be represented as a distributed dynamic coalition formation game, which is able to capture noncooperative incentives of municipalities, while they are pursuing cost minimization, as well as their cooperation and trade-offs. At first, we deduce the characteristic function, introduce the canonical coalitional game and study its properties. Then, the discussion on the implementation of coalition formation and the proposed costs distribution is given. After that, in the case study, the outcome, achieved through iterating merges and splits with respect to utilitarian order, is presented. Since the Shapley value is commonly applied in different problems involving optimal allocation and configuration of profits, costs, resources, or capacities, it is suggested as a particular proposal, of how to distribute costs (including appropriate financial compensations) among resulting coalitions' participants in a fair way. The point of the core will be also calculated to provide an alternative point of view on possible stable distribution of costs in case of full cooperation.

The originality and contribution of this part consists of application of the distributed dynamic coalition formation framework to waste treatment costs minimization game. It must be emphasized that, according to the performed review, the author has not found out any cases of coalition formation applied to the analogical problematics. The suggested approach is justified by theoretical properties of the game, that have been studied in detail. To overcome a problem with large players set, innovative coalition formation algorithm for games with numerous players has been developed. In this algorithm, penalization for cooperation serves as an instrument, which impacts the behavior of the waste producers and the size of the resulting coalitions. Also, none of the reviewed articles has considered application of the Shapley value to such a large set of players (up to 47), where computation of a solution becomes a challenging task. Due to this reason, the sampling Shapley value estimation [15] has been applied (deterministic approach is not suitable for such large games).

4.2.1 Problem definition

The general case of the problem considers a nonspecific area in which WtE plants with different capacities are situated. Waste producers (municipalities) with different locations and waste productions treat their waste using services of the available WtE plants. The model works with the already existing WM network. Assuming limited or banned landfilling, waste producers are forced to treat produced waste using services of WtE plants. Gate fees of WtE plants are assumed to be external fixed parameters (which can be obtained using approach from the previous section). Waste producers minimize their total waste treatment costs, consisting of transportation and waste processing costs. Cooperation occurs when instead of competing over the free capacities, some producers create union and reserve capacities of nearby WtE plants to waste producers with unfavorable locations. This enables them to reduce their waste treatment costs in exchange for the financial compensation, from which some of the cooperating waste producers, that have renounced these capacities, might substantially benefit. Now, the deduction of the appropriate value function v will be discussed in detail.

Deduction of the value function

- **TU vs non-TU:** Since v should reflect costs for waste treatment, which are commonly represented by their monetary value, the considered conflict of waste producers should be modeled as a TU-game.

- **Form of the game:** From the theoretical chapter, it can be seen, that games in partition function form are much more computationally complex. Facing this complication, only value functions in the characteristic form will be considered.

- **Underlying normal-form game:** A characteristic function of a cooperative game can be derived from utility functions of the non-cooperative conflict. The two main approaches are α- and β-functions. The former function is derived under condition that outsiders cooperate to maximize costs of coalition after it has minimized them. The latter approach swaps order of decisions. However, these approaches are based on the underlying non-cooperative game in normal form, which cannot be appropriately defined for the case of waste producers' conflict. Due to limited capacities, WtE plants, which locations and gate fees are the most favorable for a player, can have no free capacity depending on the established order, in which waste producers make their decisions about waste treatment. The importance of the DM order implies non-relevance of the normal form approach. Moreover, it is not realistic to assume that $N \setminus S$ will be aimed at damaging coalition S at all cost.

- **Objective of outsiders:** Another possibility is γ-function approach [38], which reasonably admits that players outside of coalition individually pursue their own interest instead of damaging their cooperating opponents. However, it should be modified to avoid the above-mentioned ordering problem. For this sake, full cooperation between players in $N \setminus S$ will be assumed and these players will cooperatively minimize the sum of their own total costs. Unfortunately, even in a such setting order is crucial.

- **Optimistic vs pessimistic approach:** The setting, in which coalition S makes decision as first, can be viewed as the most optimistic approach for total waste treatment costs estimation. However, it is not always appropriate to estimate costs using an optimistic variant of situation development. This is the reason why the pessimistic setting, in which the coalition S makes decision after $N \setminus S$, has been preferred as more representative way of defining the $v(S)$ for the waste producers cost reduction game. Moreover, this approach can be viewed as completely risk-averse attitude when coalitions do not possess information about their mutual composition, making exclusion of partition function approach reasonable.

Thus, the main idea is to propose the value function, which will reflect a realistic worst-case scenario of the WtE treatment costs minimization by an arbitrary municipal union. In the following mathematical programming problem, notation is given as follows: M is set of WtE plants, N is set of waste producers, S is coalition of municipalities (subset of N), $v(S)$ is value function of S (total annual waste treatment costs of S), remaining notation coincides with the model from the previous section. Then, waste producers' cost reduction game can be defined as a pair (N, v), where N is a set of waste producers and v is the value function defined as

$$v(S) = \min_{x_{i,j}, i \in S, j \in M} \sum_{j \in M} \sum_{i \in S} (c^t_{i,j} + c^g_j) x_{i,j}, \tag{4.70}$$

$$\text{s.t.} \sum_{i \in S} x_{i,j} \leq w^c_j - \sum_{i \in N \setminus S} x'^{,N \setminus S}_{i,j}, \ \forall j \in M, \tag{4.71}$$

$$\sum_{j \in M} x_{i,j} = w^p_i, \ \forall i \in S, \tag{4.72}$$

$$x_{i,j} \geq 0,\ \forall i\ \in S,\ \forall j \in M, \tag{4.73}$$

$$(x_{i,j}^{\prime,N\setminus S})_{i\in N\setminus S,\ j\in M} \in \arg costs_{N\setminus S} \tag{4.74}$$

$$costs_{N\setminus S} = \min_{x_{i,j}, i\in N\setminus S,\ j\in M} \sum_{j\in M} \sum_{i\in N\setminus S} \left(c_{i,j}^t + c_j^g\right) x_{i,j}, \tag{4.75}$$

$$s.t. \sum_{i\in N\setminus S} x_{i,j} \leq w_j^c,\ \forall j \in M, \tag{4.76}$$

$$\sum_{j\in M} x_{i,j} = w_i^p,\ \forall i \in N \setminus S, \tag{4.77}$$

$$x_{i,j} \geq 0,\ \forall i\ \in N \setminus S,\ \forall j \in M. \tag{4.78}$$

Each waste treatment costs component is represented as linear variable costs, where the amount of waste is multiplied by transportation cost and gate fee per ton of waste. Most of the constraints are the same as in lower-level problem of WtE plants' price-setting. For the sake of clarity, we describe their role once more time. Expression (4.70) represents the minimal amount of total costs, that can be achieved by coalition S. Constraints (4.72), (4.73), and (4.77), (4.78), ensure that all waste is treated, and forbid negative waste flows. Constraint (4.76) ensures, that the capacity of WtE plants cannot be exceeded, when computing optimal waste flows of coalition $N \setminus S$ in expression (4.75). Constraint (4.71) guarantees, that coalition S optimizes its waste flows on the capacities remaining after $N \setminus S$. This value function describes the pessimistic setting, in which the coalition S makes decision after the coalition $N \setminus S$, and is assumed to describe upper bound of WM costs of coalition S. The considered v has been originally presented in [83]. It is crucial to assume, that the total capacities of regional WtE plants should be greater than (or equal to) total waste production in a region. Thus, once more, the main assumption of the whole model is

$$\sum_{i\in N} w_i^p \leq \sum_{j\in M} w_j^c.$$

It is important to explain, how particular solution $(x_{i,j}^{\prime,N\setminus S})_{i\in N\setminus S,\ j\in M}$ can be chosen, since in case of ambiguity it can drastically affect $v(S)$.

1. At first, the problem $costs_{N\setminus S}$ is solved.

2. After that, we obtain the particular solution $(x_{i,j}^{\prime,N\setminus S})_{i\in N\setminus S,\ j\in M}$ as

$$(x_{i,j}^{\prime,N\setminus S})_{i\in N\setminus S,\ j\in M} \in \arg \max_{x_{i,j}, i\in N,\ j\in M} \sum_{j\in M} \sum_{i\in N} \left(c_{i,j}^t + c_j^g\right) x_{i,j},$$

$$s.t. \sum_{i\in N} x_{i,j} \leq w_j^c,\ \forall j \in M,$$

$$\sum_{j\in M} x_{i,j} = w_i^p,\ \forall i \in N,$$

$$\sum_{j \in M} \sum_{i \in N \setminus S} \left(c^t_{i,j} + c^g_j\right) x_{i,j} = costs_{N \setminus S},$$

$$x_{i,j} \geq 0, \ \forall i \ \in N, \ \forall j \in M.$$

However, when performing simulations in the case study, the arbitrary solution of $costst_{N \setminus S}$ is taken, to better reflect randomness of real world decisions and limited rationality of outsiders. Now, the effect that can be achieved through cooperation will be explained and demonstrated in detail.

4.2.2 Motivational examples

In this section, two exemplary problems will be presented. Each problem has the same data for the sake of better demonstration of the cooperation restriction impact on a game outcome. Resulting costs will be compared by the means of the Shapley value computed for each problem. The first exemplary game (N, v) is represented by Figure 4.5, where used notation fully corresponds to the previously given description. From the practical point of view, such a setting can be explained in the following way. In the case N is formed, waste producer 2 will willingly choose the more expensive services of the WtE plant 1 in order to reduce total costs by leaving free capacity of WtE plant 2 to waste producers 1 and 3.

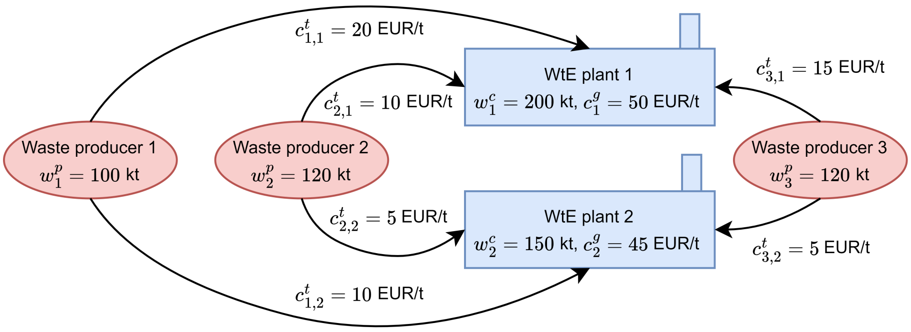

Figure 4.5: Exemplary problem without cooperation restrictions

Increased expenses of waste producer 2 will be then compensated by waste producers 1 and 3 from the money they spared, because even with such compensation their costs will be less, than in a case with absence of cooperation. The second exemplary problem is represented in Figure 4.6. The yellow shape presents a natural or legal barrier, which in a certain way, divides waste producers in the considered area. This setting can be represented by a static coalition formation game $(N, v, \mathcal{P})$ with the pre-defined coalition structure $\mathcal{P} = \{\{1, 2\}, \{3\}\}$.

In Table 4.5, the values of the characteristic function for both considered games is presented.

Table 4.5: The characteristic function values in MEUR

Game/Coalition	{1}	{2}	{3}	{1,2}	{1,3}	{2,3}	{1,2,3}
(N, v)	7	7.2	7.8	13.75	14.35	14.25	19.75

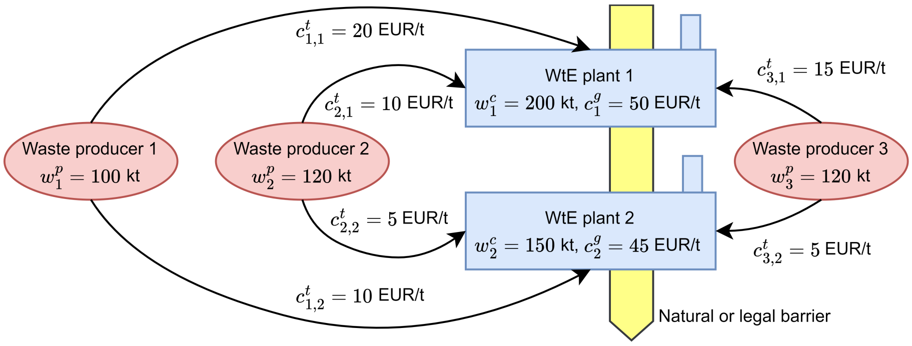

Figure 4.6: Exemplary problem with cooperation restrictions imposing pre-defined coalition structure

Then, the Shapley values for each game can be obtained in accordance with Table 4.6. It can be highlighted, that costs distributed on a basis of the Shapley value are at

Table 4.6: The Shapley values in MEUR

Game/Waste producer	1	2	3
(N, v)	6.35	6.4	7
$(N, v, \mathcal{P})$	6.775	6.975	7.8

least the same as they were in a case of absence of cooperation for each player. Thus, cooperation in each setting has proven itself as profitable. It enables municipality, which does not have WtE infrastructure and is distant from other WtE plants, to lower its waste treatment costs through negotiation with the closest municipality, that is situated near some WtE plant. The latter municipality can choose to treat its waste at another WtE facility to let the former municipality minimize its transportation costs (in real life, it is enough to subsidize transportation of former municipality). The part of occurred financial surplus, i.e., difference between the potential non-cooperative scenario costs and the real costs achieved through cooperation, can be then transferred to the latter municipality as a compensation. Now, we will study the theoretical properties of the considered game.

4.2.3 Properties of the game

Throughout the whole section, we make the following assumption:

- Each considered waste producers' cost reduction game (N, v) has unique solutions $(x'^{,S}_{i,j})_{i \in S, j \in M} = \arg costs_S$, $\forall S \subseteq N$.

Though, this assumption might seem quite strong, it is necessary in order to be able to study properties of the considered game and compare the underlying linear programming problems. When solving practical problems, addition of sufficiently small random $\epsilon \in \mathbb{R}$ (positive as well as negative) to each considered transportation cost might help to create unique decrease directions to meet this assumption. We begin with the properties, that might have practical consequences with respect to costs distribution and coalition formation process during our case study.

Cohesivity and balancedness

The game (N, v) considered in the motivational example is cohesive. When studying a cohesive game using merge and split rules in terms of utilitarian order and $\mathcal{D}_{hp}$ or $\mathcal{D}_p$ stability, this property implies that if a merge and split process will start from N, then it will never split. We begin with the following lemma.

Lemma 4.2.1. Assume a waste producers' cost reduction game (N, v). Then,

$$\sum_{i \in S} x'^{,S}_{i,j} \leq \sum_{i \in T} x'^{,T}_{i,j}, \forall S \subseteq T \subseteq N, \forall j \in M.$$

Proof. This relationship can be proven by a contradiction. Assume there exist $\tilde{j} \in M$, $T \subseteq N$ and $S \subseteq T$, such that

$$w^c_{\tilde{j}} \geq \sum_{i \in S} x'^{,S}_{i,\tilde{j}} > \sum_{i \in T} x'^{,T}_{i,\tilde{j}}.$$

Since $(x'^{,S}_{i,j})_{i \in S, j \in M}$ is the unique optimal solution for $costs_S$, there should exist $\delta > 0$ and $\hat{i} \in S, \hat{j} \in M \setminus \{\tilde{j}\}$, such that a solution constructed as

$$\left(x'^{,T}_{\hat{i},\hat{j}} - \delta, x'^{,T}_{\hat{i},\tilde{j}} + \delta, (x'^{,T}_{i,j})_{i \in T, j \in M \setminus \{\hat{j}, \tilde{j}\}}\right)$$

will lower value of $\sum_{j \in M} \sum_{i \in S} (c^t_{i,j} + c^g_j) x_{i,j}$ compared to $(x'^{,T}_{i,j})_{i \in T, j \in M}$, while the value of $\sum_{j \in M} \sum_{i \in T \setminus S} (c^t_{i,j} + c^g_j) x_{i,j}$ will remain the same. Therefore, the following relation will hold

$$\sum_{j \in M \setminus \{\hat{j}, \tilde{j}\}} \sum_{i \in T} (c^t_{i,j} + c^g_j) x'^{,T}_{i,j} + (c^t_{\hat{i},\hat{j}} + c^g_{\hat{j}})(x'^{,T}_{\hat{i},\hat{j}} - \delta) + (c^t_{\hat{i},\tilde{j}} + c^g_{\tilde{j}})(x'^{,T}_{\hat{i},\tilde{j}} + \delta) < costs_T.$$

Thus, we were able to construct solution that produces value which is less than $costs_T$ under the same constraints. However, this is a contradiction with optimality of $(x'^{,T}_{i,j})_{i \in T, j \in M}$ for $costs_T$. □

Then, the following relationship holds.

Lemma 4.2.2. Assume a waste producers' cost reduction game (N, v). Then,

$$costs_T \leq costs_{T \setminus S} + v(S), \forall S \subseteq T \subseteq N.$$

Proof. It is sufficient to prove that optimal solution $\left((x^{*,S}_{i,j})_{i \in S, j \in M}, (x'^{,T \setminus S}_{i,j})_{i \in T \setminus S, j \in M}\right)$ that produces value $costs_{T \setminus S} + v(S)$ for $\sum_{i \in T} (c^t_{i,j} + c^g_j) x_{i,j}$ is always feasible for problem $costs_T$. The main interest dwells in the problem with capacity constraints, since other constraints are trivially satisfied. It is clear that by the definition of $v(S)$

$$\sum_{i \in S} x^{*,S}_{i,j} + \sum_{i \in N \setminus S} x'^{,N \setminus S}_{i,j} \leq w^c_j, \forall j \in M,$$

holds. However, from Lemma 4.2.1,

$$\sum_{i \in S} x^{*,S}_{i,j} + \sum_{i \in T \setminus S} x'^{,T \setminus S}_{i,j} \leq \sum_{i \in S} x^{*,S}_{i,j} + \sum_{i \in N \setminus S} x'^{,N \setminus S}_{i,j} \leq w^c_j, \forall j \in M,$$

which completes the proof. □

With respect to the statement of the previous lemma it is important to note, that $costs_N = v(N)$ holds. Lemma 4.2.2 also enables necessary to establish quite obvious relationship between $costs_S$ and $v(S)$.

Corollary 4.2.3. Assume a waste producers' cost reduction game (N, v). Then,

$$costs_S \leq v(S), \forall S \subseteq N.$$

Proof. It is a direct consequence of Lemma 4.2.2 for $S \subseteq S \subseteq N$:

$$costs_S \leq costs_{S \setminus S} + v(S) = v(S).$$

□

Now, we can proceed to the main theorem on cohesivity of waste producers' cost reduction games.

Theorem 4.2.4. *General waste producers' cost reduction game (N, v) is cohesive.*

Proof. Assume arbitrary partition $\mathcal{P} = \{S_1, ..., S_k\} \in \mathcal{P}_N$ of N with $k = 2, ..., n$ (case $k = 1$ is trivial). Then, by Lemma 4.2.2, we have

$$v(N) \leq v(S_1) + costs_{N \setminus S_1}.$$

By subsequent application of Lemma 4.2.2,

$$v(N) \leq v(S_1) + v(S_2) + costs_{N \setminus (S_1 \cup S_2)} \leq ... \leq \sum_{i=1}^{k-1} v(S_i) + costs_{S_k}.$$

Then, with the help of Corollary 4.2.3, we obtain

$$v(N) \leq \sum_{i=1}^{k-1} v(S_i) + costs_{S_k} \leq \sum_{i=1}^{k} v(S_i),$$

which completes the proof. □

Thus, when playing as one large entity, total costs of the waste treatment in a region are as minimal as possible. The game from the motivational example also has a non-empty core (the Shapley value of the game belongs to the core). However, Shapley value, that has been chosen as a suitable distribution of waste treatment costs, does not necessarily belong to the core of the non-convex game. Therefore, it might be beneficial to consider the core distribution to compare this stable solution to the Shapley value. We have focused ourselves on finding point $(x_i)_{i \in N}$ of the core for every cost minimization game (N, v). By finding such a distribution, the balancedness of the general waste producers costs minimization game will be automatically proven. The cohesivity of the general game (N, v) has motivated us to study costs of each $i \in N$ when $v(N)$ is calculated, since it is the optimal partition with respect to social welfare. At first, the following important lemma is introduced.

Lemma 4.2.5. Let us assume waste producers' cost reduction game (N, v). Further, we will use notation $costs_N^{N\setminus S} := \sum_{j\in M}\sum_{i\in N\setminus S}(c_{i,j}^t + c_j^g)x_{i,j}^{\prime,N}$. Then, the following relation holds

$$costs_{N\setminus S} \leq costs_N^{N\setminus S}, \forall S \subseteq N.$$

Proof. The vector $(x_{i,j}^{\prime,N})_{i\in N\setminus S, j\in M}$ from the solution of $costs_N = v(N)$ is always feasible for $costs_{N\setminus S}$:

$$\sum_{i\in N}(x_{i,j}^{\prime,N}) \leq c_j, \forall j \in M \Rightarrow \sum_{i\in N\setminus S}(x_{i,j}^{\prime,N}) \leq c_j, \forall j \in M.$$

Since other constraints and objective functions of both problems coincide, minimization of $costs_{N\setminus S}$ is always less or equal to $costs_N^{N\setminus S}$. □

Then, we proceed to the next lemma about the relationship of $costs_N^S$ and $v(S)$.

Lemma 4.2.6. Let us assume waste producers' cost reduction game (N, v). Then, the following relation

$$costs_N^S \leq v(S), \forall S \subseteq N,$$

holds.

Proof. Clearly,

$$v(N) = costs_N^S + costs_N^{N\setminus S}$$

holds. Additionally, from Lemma 4.2.2,

$$v(N) \leq v(S) + costs_{N\setminus S}$$

holds. Thus, we have

$$costs_N^S + costs_N^{N\setminus S} \leq v(S) + costs_{N\setminus S}$$

or, equivalently,

$$costs_N^S + costs_N^{N\setminus S} - costs_{N\setminus S} \leq v(S).$$

However, from Lemma 4.2.5 it follows that

$$costs_N^S + \epsilon \leq v(S),$$

for some $\epsilon \geq 0$, implying

$$costs_N^S \leq v(S).$$

□

Now, the main theorem on core of the general waste producers cost reduction game can be established.

Theorem 4.2.7. *Let us assume waste producers cost reduction game (N, v). Further assume the costs distribution $(\hat{x}_i)_{i\in N}$ such that*

$$\hat{x}_i = \sum_{j\in M}(c_{i,j}^t + c_j^g)x_{i,j}^{\prime,N}.$$

Then, $(\hat{x}_i)_{i\in N} \in C(N, v)$.

Proof. Two following properties of $(\hat{x}_i)_{i\in N}$ should be proven:

$$\sum_{i\in N} \hat{x}_i = v(N)$$

and

$$\sum_{i\in S} \hat{x}_i \leq v(S), \forall S \subseteq N.$$

Clearly,

$$\sum_{i\in N} \hat{x}_i = \sum_{i\in N}\sum_{j\in M}(c^t_{i,j} + c^g_j)x'^{,N}_{i,j} = v(N).$$

Then, using the Lemma 4.2.6,

$$\sum_{i\in S} \hat{x}_i = \sum_{i\in N}\sum_{j\in M}(c^t_{i,j} + c^g_j)x'^{,N}_{i,j} = costs^S_N \leq v(S), \forall S \subseteq N.$$

□

The result of the previous theorem and equivalence between balancedness and core non-emptiness imply the following corollary.

Corollary 4.2.8. Every waste producers' cost reduction game (N, v) is balanced.

Now, we proceed to another important property that might substantially impact the distributed dynamic coalition formation process.

Subadditivity

It is important to note, that the game (N, v) from the motivational example is subadditive. Unfortunately, this important property is not satisfied for all games of the considered type.

Lemma 4.2.9. Waste producers cost reduction games are not subadditive in general.

Proof. We prove the lemma via constructing a counter-example. Assume a waste producer game with the following setting: $N = \{1, 2, 3, 4\}$, $M = \{1, 2, 3\}$, $c^g = (50, 50, 50)$, $w^p = (100, 100, 100, 50)$, $w^c = (100, 100, 150)$, and the following transportation costs matrix

$$c^t = \begin{pmatrix} 10 & 4 & 20 \\ 8 & 2 & 20 \\ 20 & 2 & 4 \\ 20 & 20 & 2 \end{pmatrix}.$$

It can be easily verified that

- $costs_{\{2,3,4\}} = 13200$ with non-zero $x'_{2,2} = 100, x'_{3,3} = 100, x'_{4,3} = 50$,
- $costs_{\{1,3,4\}} = 13400$ with non-zero $x'_{1,2} = 100, x'_{3,3} = 100, x'_{4,3} = 50$,
- $costs_{\{3,4\}} = 7800$ with non-zero $x'_{3,2} = 100, x'_{4,3} = 50$,

implying

- $v(\{1\}) = 6000$ with non-zero $x^*_{1,1} = 100$ and non-trivial constraint $x_{1,1} \leq\ 100$,
- $v(\{2\}) = 5800$ with non-zero $x^*_{2,1} = 100$ and non-trivial constraint $x_{2,1} \leq\ 100$,
- $v(\{1,2\}) = 12800$ with non-zero $x^*_{1,3} = 100, x^*_{2,1} = 100$ due to non-trivial constraints $x_{1,1} + x_{2,1} \leq 100,\ x_{1,3} + x_{2,3} \leq 150 - 50$.

It can be seen that

$$v(\{1\}) + v(\{2\}) < v(\{1,2\}).$$

Thus, we can conclude that waste producers' cost reduction games do not satisfy subadditivity in general. □

The proof of Lemma 4.2.9 also makes it possible to establish the following corollary about properties related to subadditivity.

Corollary 4.2.10. Waste producers' cost reduction games are not weakly subadditive or convex in general.

Unfortunately, it is rather challenging to establish some easily verifiable condition for subadditivity or convexity, since the relationship between $\sum_{i\in N\setminus(S\cup T)} x'^{,N\setminus(S\cup T)}_{i,j}$ and $\sum_{i\in N\setminus S} x'^{,N\setminus S}_{i,j} + \sum_{i\in N\setminus T} x'^{,N\setminus T}_{i,j}$ for some $j \in M$ can be hardly predicted.

Additivity

Since some games are not subadditive, it was decided to focus on studying a condition (put on input parameters of the game) that makes cooperation during the game non-trivial for at least one coalition. Thus, our aim is to establish easily verifiable condition, that will demonstrate if the game is or is not additive. At first, let us focus on the relationship between $\sum_{T\in\mathcal{P}} costs_T$ and $costs_S$ for arbitrary partition $\mathcal{P} \in \mathcal{P}_S$ of $S \subseteq N$. We begin with the following lemma.

Lemma 4.2.11. Assume a waste producers' cost reduction game (N, v). Then,

$$\sum_{T\in\mathcal{P}} costs_T \leq costs_S, \forall S \subseteq N, \forall \mathcal{P} \in \mathcal{P}_S, S \subseteq N.$$

Proof. Assume arbitrary partition $\mathcal{P}$ of S. Since feasible regions of each $costs_T$, $T \in \mathcal{P}$, are not interrelated, $\sum_{T\in\mathcal{P}} costs_T$ can be equivalently reformulated as a problem with the same objective function as $costs_S$. Thus, we only have to demonstrate, that feasible region of $costs_S$ is a subset of a feasible region of $\sum_{T\in\mathcal{P}} costs_T$. An arbitrary vector $(x_{i,j})_{i\in S, j\in M}$ from feasible region of $costs_S$ trivially satisfies 4.78 and 4.77 of $\sum_{T\in\mathcal{P}} costs_T$. Moreover,

$$\sum_{i\in S} x_{i,j} \leq w^c_j,\ \forall j \in M \Rightarrow \sum_{i\in T} x_{i,j} \leq w^c_j,\ \forall T \in \mathcal{P}, \forall j \in M,$$

which completes the proof. □

The following lemma will demonstrate under which condition equality holds in the relationship established above.

Lemma 4.2.12. Assume a waste producers' cost reduction game (N, v). Then,

$$\sum_{i \in N} x'^{,i}_{i,j} \leq w^c_j, \forall j \in M \Rightarrow x'^{,S}_{i,j} = x'^{,i}_{i,j}, \ \forall S \subseteq N, \forall i \ \in S, \ \forall j \in M.$$

Proof. The unique $(x'^{,i}_{i,j})_{i \in S, j \in M}$ composed of solutions $(x'^{,i}_{i,j})_{j \in M}$ of $costs_i$ fulfills 4.78 and 4.77 of $costs_S$. Then,

$$\sum_{i \in N} x'^{,i}_{i,j} \leq w^c_j, \forall j \in M \Rightarrow \sum_{i \in S} x'^{,i}_{i,j} \leq w^c_j, \forall j \in M,$$

making it feasible and, as a consequence of Lemma 4.2.11 for $\mathcal{P} = \{i | i \in S\} \in \mathcal{P}_S$, the unique optimal solution for the $costs_S$. □

Corollary 4.2.13. Assume a waste producers' cost reduction game (N, v). Then,

$$\sum_{i \in N} x'^{,i}_{i,j} \leq w^c_j \Rightarrow \sum_{T \in \mathcal{P}} costs_T = costs_S, \forall S \subseteq N, \forall \mathcal{P} \in \mathcal{P}_S.$$

Then, the following implication of $\sum_{i \in N} x'^{,i}_{i,j} \leq w^c_j, \forall j \in M$, can be established.

Lemma 4.2.14. Assume a waste producers' cost reduction game (N, v). Then,

$$\sum_{i \in N} x'^{,i}_{i,j} \leq w^c_j, \forall j \in M \Rightarrow x^{*,S}_{i,j} = x'^{,i}_{i,j}, \forall S \subseteq N, i \in N, j \in M.$$

Proof. An assumption $\sum_{i \in N} x'^{,i}_{i,j} \leq w^c_j, \forall j \in M$, and Lemma 4.2.12 imply that $(x'^{,i}_{i,j})_{i \in S, j \in M}$ is always feasible for $v(S)$, since

$$\sum_{i \in S} x'^{,i}_{i,j} \leq w^c_j - \sum_{i \in N \setminus S} x'^{,i}_{i,j} = w^c_j - \sum_{i \in N \setminus S} x'^{,N \setminus S}_{i,j}, \ \forall j \in M.$$

Then, from Corollary 4.2.3 and uniqueness of $(x'^{,i}_{i,j})_{i \in S, j \in M}$, we have

$$x^{*,S}_{i,j} = x'^{,i}_{i,j}, \forall S \subseteq N, i \in N, j \in M.$$

□

Now, we can proceed to the first theorem on the addititivty of the studied game.

Theorem 4.2.15. *Assume a waste producers' cost reduction game* (N, v). *Then,*

$$\sum_{i \in N} x'^{,i}_{i,j} \leq w^c_j, \forall j \in M \Rightarrow (N, v) \ \textit{is additive.}$$

Proof. From Lemma 4.2.14, we have,

$$v(S) = \sum_{i \in S} v(\{i\}) = \sum_{i \in S} costs_{\{i\}}, \forall S \subseteq N.$$

□

As a result, non-existence of a conflict over WtE capacities between waste producers, when they are individually optimizing costs on the «empty» network, is a sufficient condition for the game to be additive. Now, the necessity of the established condition will be proven. At first, we demonstrate that additivity of the game brings following implications.

Lemma 4.2.16. Assume a waste producers' cost reduction game (N, v). Then,

$$(N, v) \text{ is additive} \Rightarrow x^{*,S}_{i,j} = x^{\prime,S}_{i,j} = x^{\prime,i}_{i,j}, \;\; \forall S \subseteq N, \forall i \; \in S, \;\; \forall j \in M.$$

Proof. Lemma 4.2.2 and Corollary 4.2.3 lead us to the following expression:

$$v(N) \leq costs_{N \setminus S} + v(S) \leq v(N \setminus S) + v(S), \forall S \subseteq N.$$

However, additivity of (N, v) implies that

$$v(N) = v(N \setminus S) + v(S), \forall S \subseteq N,$$

and

$$costs_{N \setminus S} + v(S) = v(N \setminus S) + v(S), \forall S \subseteq N.$$

As a result, we obtain

$$costs_{N \setminus S} = v(N \setminus S), \forall S \subseteq N. \tag{4.79}$$

Then, (4.79) and uniqueness of solution of $costs_{N \setminus S}$ imply, that

$$x^{*,S}_{i,j} = x^{\prime,S}_{i,j}, \;\; \forall S \subseteq N, \forall i \; \in S, \;\; \forall j \in M.$$

At the same time, additivity of the game and (4.79) also imply, that

$$\sum_{T \in \mathcal{P}} costs_T = costs_S, \forall \mathcal{P} \in \mathcal{P}_S, \forall S \subseteq N,$$

and, from Lemma 4.2.11 and uniqueness assumption, we obtain

$$x^{*,S}_{i,j} = x^{\prime,S}_{i,j} = x^{\prime,i}_{i,j}, \;\; \forall S \subseteq N, \forall i \; \in S, \;\; \forall j \in M.$$

□

After that, the necessity of the proposed condition can be established in the following theorem.

Theorem 4.2.17. *Assume a waste producers' cost reduction game* (N, v). *Then,*

$$(N, v) \text{ is additive} \Rightarrow \sum_{i \in N} x^{\prime,i}_{i,j} \leq w^c_j, \forall j \in M.$$

Proof. The proof can be performed by a contradiction. Assume, that there exists some $j \in M$, such that $\sum_{i \in N} x^{\prime,i}_{i,j} > w^c_j$. Then, $(x^{\prime,i}_{i,j})_{i \in N, j \in M}$ is infeasible for $v(N)$, being a contradiction with the additivity of the game due to Lemma 4.2.16. □

Corollary 4.2.18. Assume a waste producers' cost reduction game (N, v). Then,

$$\sum_{i \in N} x^{\prime,i}_{i,j} \leq w^c_j, \forall j \in M \Leftrightarrow (N, v) \text{ is additive.}$$

Thus, in waste producers' costs minimization game, cooperation might bring benefits, when for at least two waste producers the most economical optimistic option of the individual waste treatment becomes infeasible due to limited capacities.

Summary of theoretical findings and their implications with respect to case study

Since necessary and sufficient condition of additivity has been obtained, it is possible to verify if the study of the particular WM setting is reasonable. At the same time, if the game is not additive, it will always have a core element

$$(\hat{x}_i)_{i\in N} \in C(N,v),\ \hat{x}_i = \sum_{j\in M}(c^t_{i,j}+c^g_j)x'^{,N}_{i,j}, \forall i \in N,\ \text{such that } \exists S \subset N : \sum_{i\in S}\hat{x}_i > v(S),$$

due to general balancedness and cohesivity of waste producers' cost reduction games. Thus, after verifying $\sum_{i\in N} x'^{,i}_{i,j} \leq w^c_j, \forall j \in M$, we are able to calculate a stable allocation of the total waste treatment costs $v(N)$. The whole process requires only $|N|+1$ linear programming problems to be solved.

However, in practice, when large number of players is considered (case of the national WM network), it is rather improbable, that the grand coalition N will form. Indeed, to sustain such a large network of micro-regions (consisting of smaller municipalities), additional investments might be needed. This consideration can be supported by our finding that the waste producer's game might not be subadditive. Thus, it is not completely reasonable to study this game using the canonical cooperative games. Therefore, it was decided to employ the coalition formation approach. Since there is no reason to assume existence of some pre-defined coalition structure, we have chosen to apply dynamic approach. The centralized dynamic approach is impractical for the large games. This is why its dynamic alternative will be employed through merges and splits. In the next section, the proposed implementation of the coalition formation process will be discussed.

Distributed dynamic coalition formation

Whereas the concepts from theoretical section provide necessary elements to formalize dynamic coalition formation, they do not explain, how outcome of such process should be computed. Moreover, a particular implementation of the merge and split process might directly affect a found stable outcome. In this work, the following implementation is suggested (the implementation has been programmed in MATLAB).

The initial coalition structure is assumed to correspond to the state with no cooperation among players. The merge rule is always applied as first and operates exclusively on pairs of coalitions. Coalitions to be merged are subsequently taken from a set of all available pairs of coalitions in coalition structure. If the merge operation is performed, coalition structure is updated, and merge rule application starts again. When no merge operation can be performed, the algorithm proceeds to the application of a split rule. It iterates over all coalitions in the coalitional structure and checks the split operation assumption for every partition of the currently processed coalition. Partitions are taken from a set of all possible partitions. If the split operation is performed, the coalition structure is updated, and the split rule continues to run. When no split operation can be performed, the process proceeds to merge rule application. If in one full cycle (one application of merge rule and one application of split rule) no merge or split operation has been performed, the merge and split algorithm ends. The ordering in combinatorial sets (set of all pairs of coalitions and set of all partitions) is obtained via the MATLAB function „nchoosek". The assumptions about a starting coalition structure and the application of merge rule on pairs of coalitions are aimed at sustaining computational complexity on the desired level. Since every game has been proven to be cohesive, if the merge and split process with respect to utilitarian order will start from N, it will not be splitted any more. In the case of strict cohesivity, any starting profile will lead us to N, if we consider not only pairs but all possible merges. Thus, when no additional costs are assumed, the first merge operation might result into complete cooperation and the formation of the grand coalition. In such case, the large set of players will potentially lead to the combinatorial explosion during the split operation, since all possible partitions must be checked. To overcome this potential problem, it has been decided to embed additional cooperation costs into the considered approach. Such penalization might reflect increasing financial costs for retaining efficient communication between coalition participants and coordination of mutual actions.

Additional costs algorithm. In order to capture the impact of additional cooperation costs, the definition of value function has been modified to:

$$v^*(S) = v(S) + \sum_{i \in S} \sqrt{|S| - 1} \frac{p}{100} v(\{i\}).$$

The value function now represents the sum of the original value function and additional cooperation costs, which are represented as a sum of value function values corresponding to the individual micro-regions contained in $S \subseteq N$. The latter term is multiplied by a square root of coalition S size minus one to embed nonlinear penalization of greater coalitions (with $v^*(i) = v(i)$). To obtain uniform coalition, the latter term is also multiplied by a penalization term $p \in [0, 100]$, which will be further used as an instrument to manipulate with the coalition formation process. In practice, it is almost impossible to find a general cost function describing the costs of cooperation. It is intuitively clear, that it will have positive correlation with the cardinality of the coalition, therefore the proposed function is in line with the basic premise. The exact idea of the manipulation with penalization dwells in an algorithm, which is aimed at obtaining the collation structure with the maximal average coalition size, through iterative alternation of penalization decreases and increases. The design of the proposed algorithm is sketched in Figure 4.7. In Figure 4.7, p with the lower subscript represents particular value of penalization, k is step with which penalization changes in each iteration, $\mathcal{C}_j = \{S_1, \dots, S_m\}$ is a particular coalition structure, and $a_{\mathcal{C}_j} = \frac{|N|}{m}$ is an average coalition size under structure $\mathcal{C}_j$. The structure $\mathcal{C}_{start}$ represents starting coalition structure for application of merge and split algorithm (it corresponds to fully non-cooperative case only during the first penalization decrease).

Distributing the costs

Unfortunately, it may not be possible to generalize the proposed core solution

$$\hat{x}_i = \sum_{j \in M} (c^t_{i,j} + c^g_j) x'^{,N}_{i,j}, \forall i \in N,$$

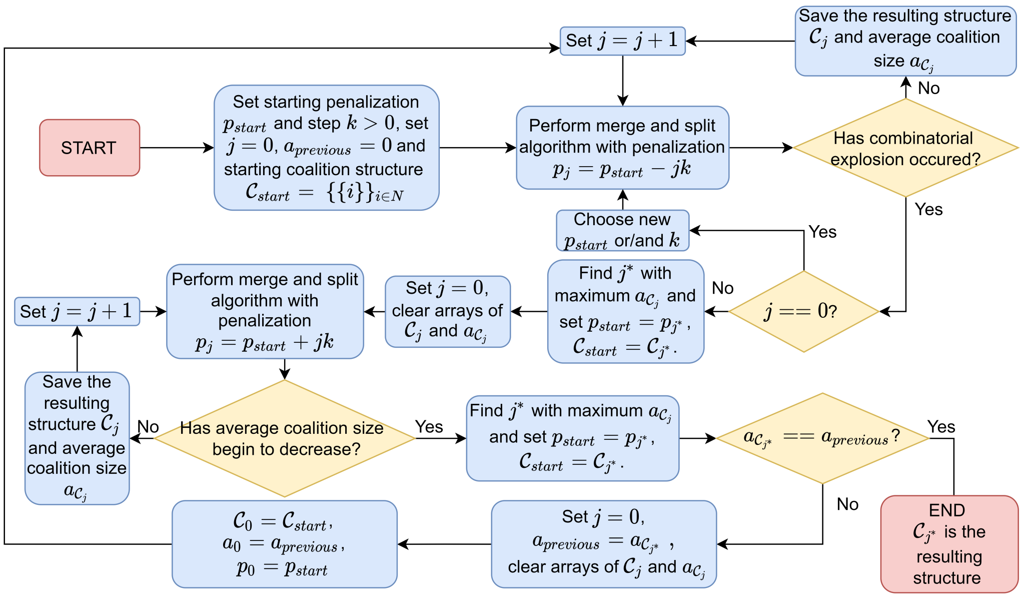

Figure 4.7: Penalization-based coalition formation algorithm

into domain of coalition formation games. Indeed, $(\hat{x}_i)_{i \in N}$, resp. $(\hat{x}_i)_{i \in S}, S \subset N$, may not belong to $C^{CF}(N, v, \mathcal{P})$, resp. $C^{CF}(S, v|_S, \{S\})$. Moreover, these cores may be empty. Therefore, the proposed core distribution will be used only for the sake of comparison to demonstrate how «far» the Shapley value will be from the stable distribution of $v(N)$.

The Shapley value has a reduction property and this solution concept will be utilized to suggest fair distribution of costs among resulting coalitions of municipal units. Due to the size of the players' set in the considered case study (47), a Shapley value estimation, based on sampling theory [15], will be employed. In [15], the Shapley value is estimated as the mean value of the marginal contribution of the player to coalition of player's predecessors in permutation (players assigned to a smaller natural numbers than the considered player), taken from a sample, in which each permutation on N is included with the probability equal to $1/|N|!$. Now, the sampling algorithm will be formalized with respect to the definition of the Shapley value utilizing Weber strings/permutations. Assume sample of m permutations. In step k of the algorithm, permutation $\alpha_k \in \mathfrak{S}_N$ is taken from the sample and

$$x_i^{\alpha_k} = v(R_i) - v(R_i \setminus \{i\}),$$

where

$$R_i = \{\alpha_k(1), ..., \alpha_k(j)\},\ j \in N \text{ such that } \alpha_k(j) = i,$$

is calculated for each $i \in N$. Then, after the m steps, the Shapley value estimate $\hat{\phi}(N, v)$ can be obtained as

$$\hat{\phi}_i(N, v) = \frac{\sum_{k=1}^{m} x_i^{\alpha_k}}{m}.$$

Such an estimate can be calculated in polynomial time, it is also unbiased and consistent [15].

4.2.4 Case Study

The case study dwells in the application of the modified merge and split algorithm to the waste producers' cost game, where the set of players consists of 47 micro-regions (municipalities with extended authority), which are presented within three regions of the Czech Republic: the Zlín Region, the Olomouc Region, and the South Moravian Region. In order to meet the requirement, that all waste meant for energy recovery can be handled by the Czech Republic's WM network, WtE plants, that do not exist, but are currently being planned, have also been assumed. This makes a total of seven WtE plants. The data on waste generation of the micro-regions has been provided by the Ministry of the Environment; financially sustainable gate fees, capacities, and transportation costs have been obtained from [61]. The additivity condition has been checked and the game in the considered setting is not inessential. As it was already mentioned, the initial coalition structure corresponds to the state with no cooperation among the micro-regions, i.e. the process starts with 47 disjoint coalitions, each represented by only one municipality. For the case study, starting penalization value has been set to 2 and the step has been set to 0.1. This relatively low penalization might be explained by a pessimistic setting of the problem, where only large coalitions might substantially reduce their total costs through cooperation. A schematic merge and split process for the penalization of 1.2 during first penalization decrease is depicted in Figure 4.8. The algorithm run information is presented in Table 4.7.

Table 4.7: Average coalition size changes

Penalization	1st decrease	1st increase	2nd decrease	2nd increase	3rd decrease
2	1.044				
1.9	1.119				
1.8	1.119				
1.7	1.119				
1.6	1.093				
1.5	1.119	1.306			
1.4	1.237	1.343		1.382	
1.3	1.424	1.382		1.382	
1.2	**1.469**	**1.469**	1.469	1.469	
1.1	Err		1.469	1.469	
1			**1.567**	**1.567**	**1.567**
0.9			Err		Err

The 3rd increase column has been omitted, since it fully copies the 2nd increase column. In each penalization increase step, few more iterations have been computed to ensure, that average coalition size is consistently decreasing. All resulting coalitions with the cardinality greater than one, can be considered as a steady and stable outcome. The map of the resulting structure is depicted in Figure 4.9.

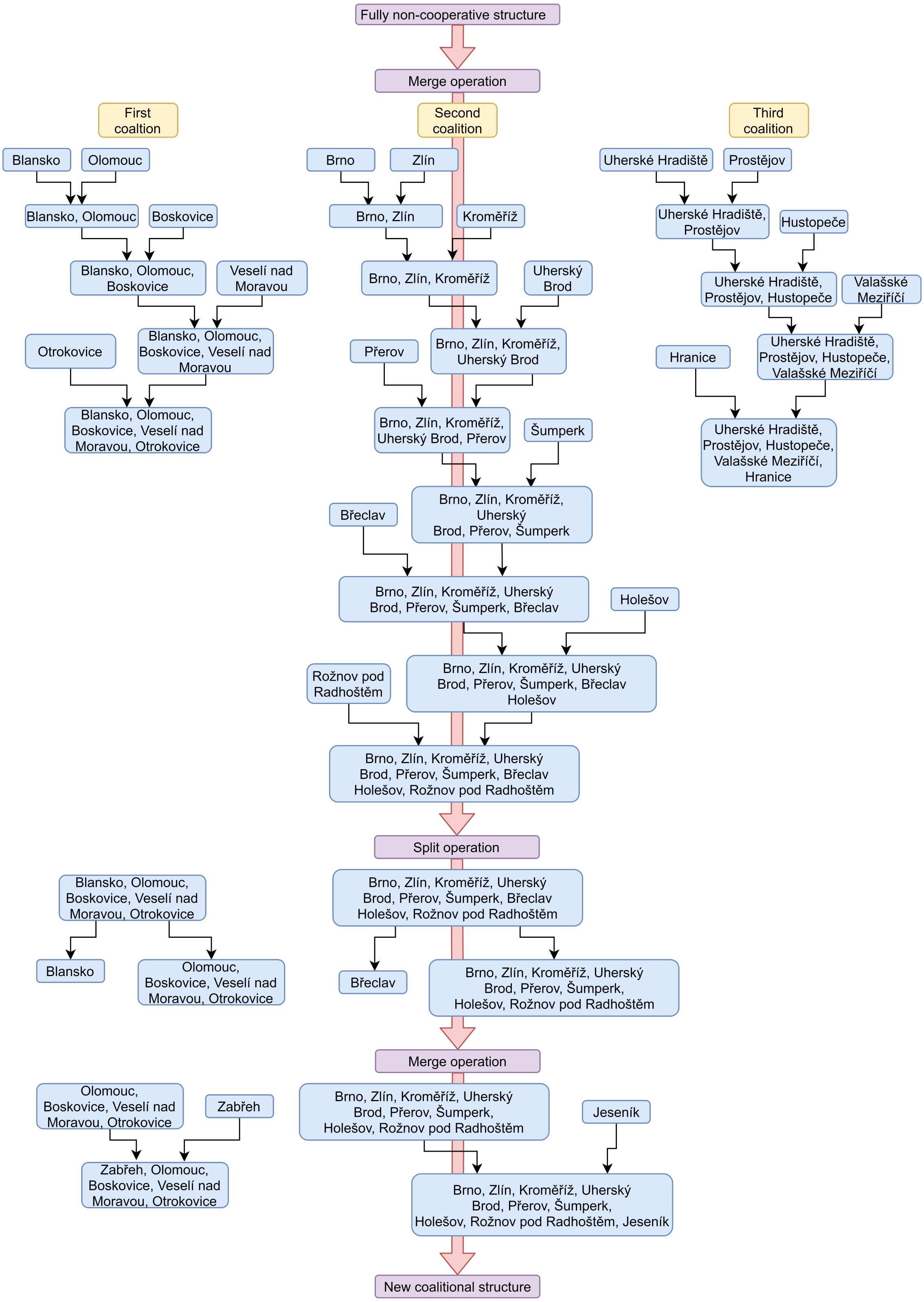

Figure 4.8: Merge and split full run for the penalization of 1.2

Discussion

In this case study, the algorithm has enabled to create three “clusters”, which attracted a certain number of micro-regions, due to substantial total costs decrease

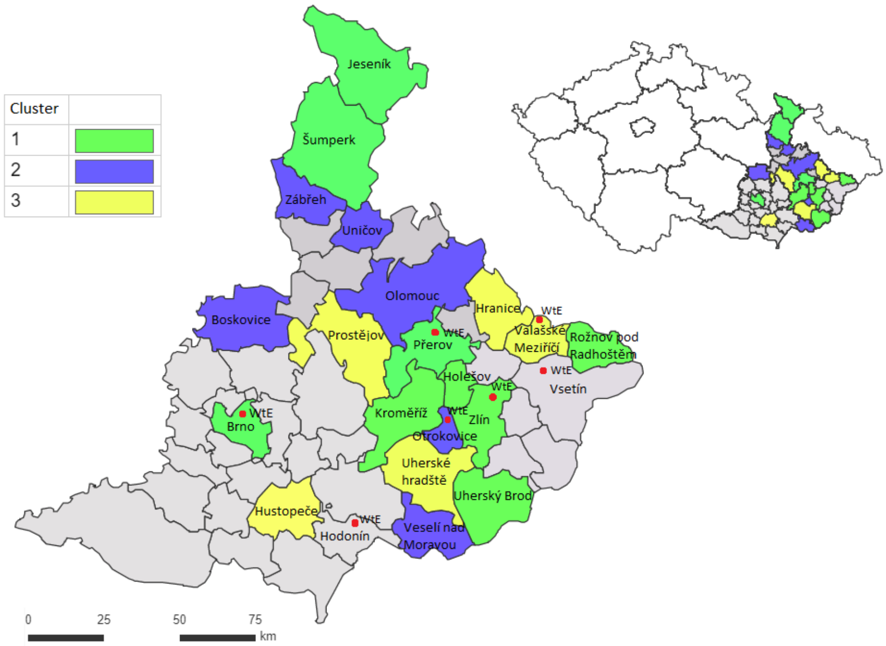

Figure 4.9: Map of the resulting municipal unions

regardless of the applied penalization. These coalitions can be referred to as the most profitable, while other micro-regions are not interested in cooperation under additional cooperation costs. This implies, that cooperation cannot enable them sufficient compensation due to their waste productions and locations with respect to the WtE plants, which are indicated by red dots in Figure 4.9. These dots do not correspond to the real or planned location of WtE and only indicate their existence in a micro-region. Evident geographical inconsistencies in coalitions can be explained by the fact, that the considered micro-regions already represent aggregated smaller cities. Moreover, the planning of the waste collection is not taken into account in the model, which might promote cooperation between distant micro-regions.

Clearly, the proposed algorithm must be further improved to provide precise instructions in case of possible irregularities. A more comprehensive study of the development of average coalition size depending on penalization is also needed. Moreover, other "uniformity" metrics such as geometric mean might be worth considering. The merging of pairs of coalitions remains the main disadvantage of the current implementation, but it is necessary to mitigate the risk of combinatorial explosion. When working with smaller player sets, merging three or more coalitions into one could also be considered.

The stable outcome. From Figure 4.9, it can be seen, that the resulting coalitions are not spatially consistent: cooperation of distant micro-regions can be profitable. Thus, formation of municipal unions cannot be solved solely intuitively based on geographical vicinity between subjects, as it is usually done in practice. The resulting coalitions have showed that micro-regions, where WtE facilities are situated, tend to be major players of their coalitions, around which other players are gathering.

Due to assumed zero transportation costs, these “centers” tend to reserve capacities of their WtE plants to other participants of corresponding coalitions. While the greatest coalition consists of three such “centers” (“Brno”, “Přerov”, and “Zlín”), which explains its greater size, another coalition has occurred around “Otrokovice” with a WtE plant of large capacity and competitive gate fee for its region. The last coalition has been created around “Valašské Meziříčí” with a WtE plant, which, though of a relatively small capacity and higher gate fee, still provides possibility to achieve smaller waste treatment costs for local micro-regions. However, the existence of a WtE plant within a micro-region does not always guarantee that such micro-regions will attract others. For example, “Hodonín” micro-region, which has its own WtE facility, does not serve as a gathering “center” for any coalition. This fact can be explained by the fact that “Hodonín” is situated close to “Brno”, but its WtE plant is uncompetitive compared to “Brno” WtE plant. It can be concluded that obtained results lack irrationalities and the presented approach has potential in research on this topic. The case study results validate the proposed method and indicate, that the developed approach can be applied to locations with analogous demographical conditions.

The proposed distributions of waste treatment costs. The Shapley value has been chosen as a fair method of a total waste treatment cost distribution between micro-regions. Three possible scenarios have been considered to provide a better image about the role of cooperation in the presented problem. These scenarios are the following: I. fully non-cooperative case, II. fully cooperative case, III. stable outcome with non-cooperating outsiders (three proposed coalitions are considered and remaining micro-regions do not cooperate). For the sake of better comparison, all scenarios have been computed using the original function v. The suggested point of the core $C(N, v)$ has been calculated only for the fully cooperative case. The proposed costs distributions are presented in «Shapley values.docx» of Appendix. The results for the I. scenario are represented by total waste treatment costs per ton of waste. The results of scenarios II. and III. and the proposed core distribution are represented by percentual savings compared to the I. scenario. The sampling method has been employed to estimate the Shapley value of coalitions with cardinality greater than 7, where the sample size has been set to 10,000.

At first, it is necessary to emphasize that estimates of Shapley value in II. and III. scenarios are smaller than $v(\{i\})$ values of I. scenario. Thus, under both scenarios players were able to prosper from cooperation. Expectedly, micro-regions in which WtE plants are situated play a major role in their coalitions. This fact has also manifested itself through the suggested costs distributions. Mainly, micro-regions with production, which is smaller than capacity of their local WtE plant, can achieve substantial savings through cooperation. Other micro-regions in these coalitions, can also save considerable amount of money, especially if their waste production is high with respect to their geographical area. Therefore, micro-regions in which waste treatment facilities are situated and micro-regions with locally above-average waste generation should be maximally interested in cooperation and initiate the creation of municipal unions in order to substantially lower their waste treatment costs. While pursuing their own wealth, they can also reduce the financial impact of legal changes on the other micro-regions. As expected, global cooperation, corresponding to the II. scenario, is the most profitable outcome for everyone. According to the performed estimate, in case of global cooperation, all players can lower their total waste treatment costs. While the III. scenario represents an opportunity to lower waste treatment costs for members of the previously described coalitions, it should be noted, that it cannot offer such substantial savings that can be achieved through the II. scenario. It should be concluded that decision of the micro-regions to cooperate is based on all considered factors. Waste productions and locations play an equally eminent role in the process of coalition formation. The «attractiveness» of a micro-region in coalition formation is not guaranteed exclusively by existence of a nearby WtE or large waste production, rather it is a combination of both factors. There is obviously no intention to cooperate with micro-regions with small waste productions, since they cannot offer any benefits to their partners. Then, after passing a certain threshold, where waste production becomes sufficient with respect to location of a micro-region, attractiveness of the micro-region begins to grow. Due to the clear implication, that some micro-regions might play fundamental role in their coalitions, currently widely applied policy of equal waste treatment tariffs in municipal unions should be revised.

The proposed core distribution demonstrates that, in case of full cooperation, some waste producers are able to achieve enormous savings. They can save twice more than under the distribution proposed by Shapley value for fully cooperative scenario II. The large differences between the Shapley value and core point indicate that Shapley might not be stable distribution. However, it distributes costs in a more fair, uniform way. Indeed, in some cases stable core distribution provide savings comparable to III. scenario or does not provide any savings at all.

Sensitivity analysis of the proposed method. To verify the stability of the model and assess the impact of the input variables on the outcome, a sensitivity analysis has been conducted. Whereas some significant biasedness of the data on WtE plants and transportation can hardly be assumed, the main source of the variability of the whole model is considered to dwell in the waste production data. After a brief analysis of the available waste production time series data, it has been concluded, that in the last ten years average fluctuation in waste production for the considered micro-regions was around 5%. Therefore, it has been suggested to generate 10 new scenarios, using 10 randomly generated samples from the continuous uniform distribution on the interval $[-0.05; 0.05]$. These sample data have been multiplied by the originally considered waste amounts and then added to the original waste production data. Thus, the effect of imprecision in the waste production data on the outcome of the model has been studied using these 10 scenarios. For each of these scenarios, the resulting coalition structure and resulting waste costs distribution have been computed using the above-proposed coalition formation method. Summarization of the sensitivity analysis results can be described as follows:

- The micro-regions, that participate in a union in the original scenario, also participate in unions generated in 85% of the new scenarios on average.

- On the contrary, micro-regions, that are not interested in cooperation in the original scenario, cooperate only in 26.7% of the newly generated scenarios on average.
- 11 out of 20 micro-regions, that cooperate in the original scenario, cooperate in all 10 scenarios.
- The number of unions in the generated scenarios varies from 2 to 5, with the modus equal to 3 (4 out of 10 scenarios), being the number of clusters in the original scenario.
- From 1081 of all possible pairs of micro-regions, only 16 pairs cooperate in more than half of scenarios. 15 of these 16 pairs also participate in the same union in the original scenario.

From the above-presented points, it can be concluded, that the sensitivity analysis has demonstrated the relative stability of the method. Evidently, there are micro-regions that have a strong incentive to cooperate, as well as there are micro-regions, that are not interested in joining coalitions in the most of scenarios. Moreover, there is an obvious trend in cooperation between particular micro-regions. Clearly, the algorithm is quite sensitive with respect to the resulting number of clusters, which can be partially explained, that the 5% change in waste production of the large micro-region represents a substantial change of the original setting.

Chapter 5

Conclusion

In this Ph.D. thesis, application of GT approaches to problems of WtE treament of non-recyclable waste in WM networks has been demonstrated. The work has provided theoretical insight into domain of NGT and CGT. The latter branch has been discussed with respect to class of canonical coalitional games and coalition formation games. The performed review has enabled us to establish existing research gaps. These gaps have highlighted the contribution of this thesis. In particular, the autor's original research has been aimed at two types of games.

The WtE plants' price-setting problem has been thoroughly studied from two perspectives: setting the optimal prices for one WtE plant and the search for NE between WtE plants. The problem has been defined as a normal-form game of WtE plants, with gate fee as their strategies. Such a game has peculiar properties, wherein maximizing a player's payoff leads to a bilevel programming problem between one WtE plant and waste producers. However, these instances of bilevel optimization cannot be solved in polynomial time. After the extensive investigation of the bilevel optimization methods, the novel heuristic approach to solve the considered bilevel problem has been proposed. The approach considers that a simple iterative update of the lower-level linear problem solution provides sufficiently reliable estimates of waste flows, concerning which the optimization on the upper level is performed. Algorithm performance has been validated via testing and exemplary case study: it has been shown that it provides fast solutions to the considered problem and produces optimal solutions in approximately 60% of artificial scenarios and in nearly 85% of realistic cases. The research has also filled the gap in the current game-theoretic literature since the solution of the NP-hard optimization problem is only an instrument to find the NE in the WtE plants' network. Combined with the BRD algorithm, the heuristic enabled the search for NE under the assumption of continuous strategy sets. This approach should provide more realistic insight into the reaction of other WtE plants to changes in gate fees. Thus, the estimate of optimal waste flows and gate fees in the WM network provides more reliable input to decision-makers. The proposed method can be potentially applied to assess the feasibility of the investments in new WtE plants. In particular, the exemplary problem motivated by the Czech Republic data demonstrated how the approach could be applied in practice to design the capacity of the WtE plant. The optimal capacity of the facility, which is being planned in one of the regions, was proposed with respect to the analogous projects and actual waste production in the Czech Republic. The found stable gate fee outcomes exhibit economically reasonable behavior of waste treatment market participants, verifying that the developed tool can be used to simulate the market environment for the WtE facility. While solving the exemplary problem, the hypothesis about the non-existence of the NE in the considered game has been proposed. The existence of the NE has been studied for the whole class of the originally introduced price-setting games. Since the classical NE concept does not exist for the pessimistic setting, the author has proposed the modified concept of δ-NE. Existence of the δ-NE under different assumptions put on capacities and transportation costs has been studied.

The waste producer's cost reduction game has been defined to suggested the most suitable municipal unions for adaptation to new waste treatment legislative. The strong connection between the studied theoretical concepts and the real-world waste treatment problem has been showed. The cohesivity and balancedness of the studied class of games has been proven. Moreover, the easily verifiable necessary and sufficient condition of additivity has been established. The practical implications of the game properties has been discussed. The related research has provided concepts and instruments to study the formation of coalitions and distribution of costs for general TU-game with numerous players. The proposed method handles distributed coalition formation via merge and split rules under utilitarian order relation. In order to reasonably implement merge and split rules into the considered game, a cooperation costs model has been introduced. It has helped to achieve a more realistic outcome, which considers the possible suboptimality of the grand coalition and nonlinearly growing costs for creating a sustainable coalition of large number of players. The penalization percent has been used as the main instrument through which uniform coalition structure can be obtained and computational complexity can be retained at the desired level. The distribution of costs for the resulting coalition structure has been suggested on the basis of sampling Shapley value and the point of the core. Real WM data for the Czech Republic and distributed coalition formation between 47 micro-regions have been analyzed. After the application of the presented method, slightly less than half of micro-regions were engaged into some coalition under resulting coalition structure and their saves were varying from around 2% up to 8% compared to non-cooperative case. The estimated costs have provided an insight into how cooperation might affect the municipal budgets under transition from landfilling to WtE technology. The resulting coalitions can be viewed as a potential suggestion of which municipal unions should be formed. The case study data revealed that micro-regions possessing their own WtE infrastructure can substantially lower their total waste treatment costs via renouncing the capacities to other participants of the coalition. Brief sensitivity analysis has been performed, to assess impact of changes in waste production of the micro-regions (being the main source of the model variability) on the resulting costs of municipalities. The results demonstrated, that, when it is profitable for a municipality to cooperate, it tends to do so in majority of scenarios. Regarding the future research, we establish four possible directions:

- there is an opportunity to embed reconsideration of the waste flows with respect to capacities constraint into the heuristics from section 4.1. to improve the performance of the method;

- the detailed study of the possible generalization of Theorem 4.1.8 for arbitrary price-setting game;
- the estimation of the nucleolus for the waste producer's cost reduction game
- an embedment of waste collection within the established municipal unions into the waste producer's cost reduction game.

To summarize the whole work:

- the new price-setting approach, combining bilevel optimization techniques and GT, should help to ensure efficient and financially sustainable waste energy recovery;
- the presented coalition formation approach has a potential to serve as a basis for design of tariffs for different public services or for design of unions in arbitrary cost minimization problem, where cooperation between subjects is possible.

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Abbreviations

BRD	Best-Response Dynamics
CE	Circular Economy
CEP	Circular Economy Package
CGT	Cooperative Game Theory
CLSC	Closed-Loop Supply Chain
DM	Decision-Making
EGT	Evolutionary Game Theory
ESS	Evolutionarily Stable Strategy
EU	European Union
GT	Game Theory
GTP	General Taxation Problem
HNPP	Highway Network Pricing Problem
KKT	Karush-Kuhn-Tucker
MIP	Mixed-Integer Programming
MSW	Mixed Solid Waste
NE	Nash Equilibrium
NGT	Non-cooperative Game Theory
NPP	Network Pricing Problem
TU	Transferable Utility
WM	Waste Management
WtE	Waste-to-Energy

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