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Summary

The thesis deals with the development, modeling, and analysis of demand-based problems containing marketing, operations, and logistics decisions. The problems may be further extended to the concepts of dynamic pricing and marketing that drive the development. Two demand-based problems are presented in the thesis: a) the newsvendor problem, due to its simple structure as a suitable tool for illustrating how facets of marketing may affect decision-making concerning operational problems, and b) the transportation network design problem, where some results and knowledge gained from the newsvendor problem are applied. In the setting presented, the newsvendor is subsequently faced with pricing, advertising, and joint pricing and advertising-sensitive stochastic demand. A demand-related random element comprises the particular marketing decision(s) of a specific form (e.g., multiplicative or additive). It is assumed that a real pricing strategy is captured with a nonlinear decreasing demand function while a suitable advertising strategy results in increased sales. The properties of the obtained optimal decisions for particular models are discussed. The pricing-related results are applied to the stochastic transportation problem, where the stochastic demand is modeled using wait-and-see and here-and-now deterministic (scenario-based) reformulations. A hybrid algorithm composed of a heuristic (genetic) algorithm and an optimization software tool is proposed for solving of a mixed-integer linear as well as a mixed-integer nonlinear problem. Potential applications, especially in waste management, are also discussed at the end of the thesis.

Abstrakt

Tato disertační práce se zabývá vývojem, modelováním a analýzou poptávkově orientovaných úloh, které zahrnují marketingová, operační a logistická rozhodnutí. Úlohy jsou zvoleny tak, aby mohly být dále rozšířeny o koncept tzv. dynamického oceňování a jiných dynamických marketingových rozhodnutí. V práci jsou využity dvě základní poptávkově orientované úlohy: a) úloha kolportéra novin, která je zvolena pro její jednoduchou formu a která tak slouží jako nástroj pro ilustrativní ukázky rozhodovacích procesů v podobných typech úloh, a b) úloha návrhu dopravní sítě, kde jsou využity některé výsledky a znalosti získané při řešení úlohy kolportéra novin. Kolportér (či obecně maloobchodník) čelí náhodné poptávce, která může být postupně ovlivněna oceňováním, marketingovými (tj. reklamními) rozhodnutími a nakonec jejich kombinací. Poptávka obsahuje tedy náhodnou složku, která je pomocí přístupů stochastické optimalizace modelována ve specifickém tvaru (tj. aditivní či multiplikativní tvar). Závislost cena-poptávka je zachycena pomocí nelineární klesající poptávkové funkce, zatímco (vhodná) reklama vede ke zvýšení poptávky (běžně rostoucí s-křivka či konkávní funkce). Výsledky získané při řešení úlohy kolportéra novin s oceňováním jsou následně využity v úloze návrhu dopravní sítě. Tato stochastická úloha je modelována (reformulována) pomocí dvou přístupů stochastické optimalizace: wait-and-see přístup a here-and-now přístup. Jelikož tato implementace vede na lineární či nelineární celočíselnou (navíc scénářovou) úlohu, jsou v práci zmíněny taky výpočetní nástroje. Autor pro řešení používá (původní) tzv. hybridní algoritmus, což je kombinace heuristického (genetického) algoritmu a nástroje optimalizačního softwaru. Potenciální aplikace sestavených modelů, obzvláště v oblasti odpadového hospodářství, jsou diskutovány v závěrečné části disertační práce.

Keywords

Marketing decision-making, pricing, advertising, stochastic demand modeling, newsvendor problem, transportation network design problem, hybrid algorithm

Klíčová slova

Marketingová rozhodování, oceňování, reklama, modelování stochastické poptávky, úloha kolportéra novin, úloha návrhu dopravní sítě, hybridní algoritmus

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I declare that this Ph.D thesis entitled Mathematical Programs for Dynamic Pricing - Demand Based Management is the result of my own autonomous work under the guidance of prof. Kjetil K. Haugen, PhD and I used materials that have been included in the bibliographic references.

Prohlašuji, že jsem dizertační práci *Úlohy matematického programování pro dynamické oceňování* vypracoval samostatně pod vedením prof. Kjetila K. Haugena, PhD, s použitím materiálů uvedených v seznamu literatury.

In Zlín, 15$^{\text{th}}$ of December 2016 Ing. Dušan Hrabec

Preface

This thesis is a result of my PhD studies at the Institute of Mathematics, Faculty of Mechanical Engineering at the Brno University of Technology (BUT), Brno, Czech Republic.

The main objective of the research presented in the dissertation has been to develop new models, in addition to modifying some existing models, within the area of demand-based problems where ordering (or alternatively transporting), amount, and selected marketing decisions (i.e., pricing and advertising) are included as decision variables.

The motivation for developing such demand-based models lies between both practical and theoretical importance. Typically, marketing and operations/logistics decisions are seen independently, although their coordination may lead to better centralized solutions. Therefore, the practical implementation of logistics models involving marketing decisions calls initially for theoretical development. The models developed are formulated in cooperation with colleagues from BUT dealing with waste collection and with colleagues from MUC dealing with dynamic marketing decisions within logistics problems.

Although the thesis is considered to be a monograph, it is mostly a composition of results from selected papers that are unified to form a coherent final text. Note, that for reader's (or reviewer's, respectively) better orientation, the author's publications are cited as, e.g., [A1], [A2], etc.

This work was conducted from September 2011 until December 2016. Throughout the entire period, I have had the honor of working beside Prof. Kjetil K. Haugen from the Molde University College - Specialized University in Logistics (MUC), who served as my main supervisor, and with Pavel Popela (BUT) as co-supervisor.

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During my PhD studies, I received two grants from the Norway/EEA grants programme: one mobility grant in 2014-2015 for my study stay in Molde, and one grant on institutional cooperation in 2016. Therefore, I would like to acknowledge these grants, which aided my work and helped me to complete the research part of the study.

The main parts of the thesis were written during my academic stay at MUC (May 2014 - February 2015). In writing these articles, Kjetil was a constant source of valuable comments and ideas. I must admit, that Kjetil demonstrated (or tried to, at least) "the real research" principles and what research actually consists of. His honest comments (sometimes not very pleasant for me personally) were and perhaps will prove to be very valuable in my further potential academic/research career. I also learned to better supervise of my own work schedule by often struggling with Kjetil's "do it ASAP" advices and Pavel's different approach during our work together.

Therefore, I would like to formally thank to both my supervisors: to Kjetil for his excellent supervision and honest and rapid comments; to Pavel who enabled my entrance in the world of optimization and who gave me the opportunity to work with him during my Master's studies, and later during my PhD studies. Last but not least, he helped me establish the international connections that ultimately led to the beginning as well as the completion of my PhD studies.

During the period of my work, I experienced some beautiful times together with my girlfriend Ilonka during our stay in Molde in 2014-2015. We became engaged during this time and, later on, were married and settled in Zlín (Czech Republic). I will always be grateful to her for this amazing period in our lives and for her steadfast support!

I met many people during all of these years in Brno, Molde, and Zlín. Special mention goes to Jorge Oyola, Uladzimir Rubasheuski, Afonso Sampaio, Honza Novotný, Prof. Asmund Olstad, Honza Roupec, and many others. Some contributed towards my scientific pursuits, while others contributed as friends during these years.

To my wife Ilonka, to my mother, to my brother... and in loving memory of my deceased father. To my family, colleagues, and friends.

Zlín, Czech Republic — Dušan Hrabec
December, 2016

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Chapter 1

Introduction

Modern goods and service production markets are characterized by increased competition. Globalization and the focus on increased global trade drive this development. One obvious consequence is increased demand uncertainty. At the same time, marketing strategies (locally and globally) are a necessity in order to sell almost any product. This combination - increased demand uncertainty and marketing necessity - indicates that formal modeling that spans both dimensions holds managerial importance. The thesis presents a theoretical approach to models that may be used in marketing practices. Therefore, the need for a mathematical modeling of demand-based problems that involve marketing decisions is clear (see, e.g., [A2, 20, 52, 61, 123]).

A simple newsvendor model platform may prove to be a relevant and principal "laboratory" for an increased understanding of how management science may be applied in order to solve these problems. The objective of such a stochastic single-period problem is to determine the ordering quantity for a fixed period of time, maximizing expected total profit. Stochastic programming (SP) is employed as a tool to capture the uncertain demand, and the stochastic models can then be reformulated into its deterministic equivalent. Examples of the newsvendor problem (NP) illustrating the general principles of mathematical and stochastic programming are provided by [24, 99, 104]. The simple structure of NP makes it an ideal tool for examining the interaction of operational and marketing issues and the resultant impact on a decision-making process [86].

Recently, the increased development of pricing as well as dynamic pricing strategies and their further applications in industry may be observed. There are three supporting factors: (a) the increase in the availability of demand data, (b) the ease of changing prices due to new technologies, and (c) the availability of models for both analyzing demand data and for dynamic pricing [29].

Herein, pricing is understood as a problem of a single selling price decision (among others), while the dynamic pricing problem concerns the determination of prices as a decision variable over time for a product under demand and supply constraints.

Another suitable demand-based tool for examining pricing principles is the transportation network design problem (TNDP). The TNDP concerns the optimization of the design of transportation networks. Under our consideration, it contains an embedded network flow problem and consists of transportation and, alternatively, pricing decisions. Such network design problems are also challenging from a computational point of view; therefore, a hybrid algorithm that is composed of a heuristic algorithm and an optimization software tool is developed in the thesis as well.

A number of producers have used innovative marketing strategies to gain an effective control over their inventory. However, coordination between marketing (i.e., pricing and advertising) and production decisions still belongs among the most challenging practical as well as theoretical problems for operations management. The simultaneous determination of ordered quantity, advertising expenditure, and pricing for a product whose demand is random are investigated within the NP in this thesis as well. However, an interesting question arises: will these marketing efforts increase the retailer's order quantities? If the retailer is a price setter, then the answer to the above question is unclear: the retailer may order less, equal, or more. The intuition behind this result can be easily explained. As the market becomes larger, the retailer may set a higher price to earn a higher margin per unit sold while ordering less to reduce the left-over inventory risk [124].

As the title *Mathematical programs for dynamic pricing - Demand based management* indicates, the thesis presents the above-mentioned selected demand-based problems that may be further extended to the concepts of dynamic pricing and marketing that drive their development [83, 101]. The developed two-stage stochastic models contain pricing and/or advertising decisions. Such models are necessary building blocks for more advanced real stochastic dynamic pricing models.

Outline of the thesis

The Introduction Chapter commences with outlining the impetus for the study.

Chapter 2 provides an introduction of the underlying demand-based problems that consist of: (1) the NP in Section 2.1, which is based on paper [A5], and (2) the (stochastic) transportation network design problem in Section 2.2, which is based on papers [A8, A13, A6]. The NP is considered as a single period problem in our case; the applicability as well as the review of various selected NP's is also discussed (Section 2.1.4 and 2.1.5, respectively). In the TNDP, the demand is assumed to be deterministic at first (Section 2.2.1); then, the stochastic demand is modeled using wait-and-see (WS) and here-and-now (HN) deterministic reformulations (Section 2.2.2).

In Chapter 3, general pricing ideas are summarized. Some of them are applied in the newsvendor pricing problem (NPP) in Section 3.1; the section follows results published in paper [A5]. Firstly, a short overview of the NPP is given together with the modeling issues of the NPP (Section 3.1.1). Then, subsequently, the demand function and randomness (Section 3.1.2) and riskless problem (Section 3.1.3) are discussed. The demand is then modeled in the additive (Section 3.1.4) as well as in the multiplicative demand form (Section 3.1.5). Afterward, references to selected up-to-date literature and remarks on their applicability are given (Section 3.1.6). Then, pricing principles are discussed together with references to a decision dependent randomness case in SP (Section 3.1.7). Specific features of the demand function assuming a decision dependent uniform distribution are investigated here. It is assumed that its support size decreases linearly with an increase in price. Under such assumptions, the model has suitable computational features related to the expectation of the objective function. Afterward, the TNDP with pricing is provided in Section 3.2; the section is divided into subsections that follow the results published in papers [A3, A7, A8]. The stochastic price-dependent demand is modeled using a WS deterministic reformulation while the linear pricing function (Section 3.2.1) as well as the nonlinear (isoelastic) pricing function are applied (Section 3.2.2). The second case leads to a mixed-integer nonlinear problem; therefore, a hybrid algorithm is developed and tested for its solution.

Chapter 4 includes advertising-related ideas; the content of its most crucial part was published in paper [A2]. After the literature review is given (Section 4.1), the newsvendor problem with advertising (NPA) is stated, the demand function is formulated, and the advertising response function is defined (Section 4.2). Section 4.3 describes the objective function being rewritten to suit the multiplicative demand model. Then, optimal stocking quantity, which corresponds to a standard NP result, is analyzed and crucial assumptions and theorems are provided herein. A comparison is made between the optimal advertising amount obtained and the result of the riskless problem. Section 4.4 introduces the NPA model in the additive form, and summarizes the general modeling differences and advertising results as related to the multiplicative demand case. Suitable advertising response functions are presented in Section 4.5. Additionally, numerical results are provided for the uniform distribution of the random variable in the multiplicative case for the NPA model as well as numerical figures for the equivalent NP (Section 4.6). Finally, the chapter concludes with Section 4.7, which also features a managerial interpretation. Remarks on the applicability of the results and further research possibilities follow.

Chapter 5 utilizes the experience and knowledge of previous sections dealing with various NPs and combines the pricing and advertising decisions within the NP (Section 5). Problem formulation and the demand function are given in Section 5.1. Then, the marketing-dependent price-multiplicative demand (MDPM) form is applied in the NP and particular decisions are investigated (Section 5.2). The chapter concludes with a literature overview presenting suggestions for further research (Section 5.3).

Finally, Chapter 6 presents a potential pricing application. It deals with the so-called waste processing facility location problem (FLP), which calls for establishing a set of operational waste processing units. The waste management (WM) expenditure of the waste producers is minimized. It is derived from the related waste processing, transportation, and investment costs. An SP approach is used in recognition of the inherent uncertainties in this area. Two relevant models are presented and discussed. Initially, the common transportation network flow model is extended with on-and-off waste-processing capacities in selected nodes, representing the facility location. Subsequently, the randomly-varying production of waste is modeled by a scenario-based two-stage stochastic integer linear program. Finally, selected pricing ideas are employed from demand-based management to model the behavior of the waste producers, who are assumed to be environmentally friendly. The modeling ideas are illustrated by an example of limited size, solved in GAMS. Computations for larger instances were realized with traditional heuristic algorithms, implemented within MATLAB.

The thesis concludes with Chapter 7, where a summary discussion of the obtained results as well as discussions on particular subproblems are provided.

Chapter 2

Underlying Demand Based Problems

2.1. Classical newsvendor problem

This section provides a discussion on the modeling details for the problem of controlling the inventory of a single item with stochastic demand over a single period. This problem is also known as the "newsvendor problem" (NP), the problem faced by a newsvendor trying to decide how many newspapers to stock on a newsstand before observing demand. The objective of this stochastic single-period inventory problem is to determine the order quantity for a fixed time period that will maximize the profit. It is assumed that there is no initial inventory available. Demand is a random variable, represented by a probability distribution. The demand may be either discrete or continuous by the type of stock items. The NP is one of the classical problems in the literature on, e.g., inventory management [2], the stochastic inventory problem [93], and the single-period problem [62].

This problem may be simply explained through the following example by Hill [50]: *"Early each morning, the owner of a corner newspaper stand needs to order newspapers for that day. If the owner orders too many newspapers, some papers will have to be thrown away or sold as scrap paper at the end of the day. If the owner does not order enough newspapers, some customers will be disappointed and sales and profit will be lost. The NP is to find the best (optimal) amount of newspapers to buy that will maximize the expected (average) profit given that the demand distribution and cost parameters are known".*

2.1.1. History

Through references in the literature, the newsvendor (alternatively newsboy) problem has a long and interesting history. The original newsvendor model appears back in 1888, when Edgeworth in [26] developed an idea which deals with a bank cash-flow problem. He applied a series of returns from the Bank of England and used the central limit theorem to determine the optimal cash reserves to satisfy random withdrawals from savers. The optimal solution to this problem is defined as a balance between the expected cost of understocking and the expected cost of overstocking, which was later coined the critical fractile.

The first full-blown model for production planning was investigated by Harris [45] in 1913. The goal was to find the optimal size of production quantity while costs are minimized. In 1934, Wilson [122] extended the work by Harris to establish the optimal size of an order. These models are known as economic order quantity (EOQ) models.

In 1951, Arrow, Harris, and Marschak [2] expressed the famous critical fractile solution for a problem of optimal inventory policy, which is first derived for a simple inventory model in which the future demand flow is constant and the demand and other quantities are known in advance. They also studied uncertain models (both static and dynamic) in [2], where the demand flow is a random variable with a known probability distribution. The optimal stock and the best reordering point are determined as functions of the demand distribution, the cost of making an order, and the penalty of stock depletion.

During the following years, the NP became an important mathematical model in operations management and applied economics used to determine optimal inventory levels. It often serves well for the introduction of the general theoretical as well as practical principles of mathematical-stochastic programming problems (for more details see [104]). The model appears in many forms in the recent literature; it is typically characterized by fixed prices and uncertain demand for a perishable product. For an extensive review see [62, 93, 94]. See also [50, A5] that focused on the simplest classical single-period problem (i.e., without salvage value and shortage penalty cost). [62] furthered the classical single-period problem with the salvage value per unit, i.e., if the order quantity is larger than the realized demand, a single discount is used to sell excess inventory, and with the shortage penalty cost per unit, i.e., if the order quantity is less than the demand, then profit is lost and a penalty occurs. This extension will further be referred to as the underlying NP (subsection 2.1.2).

2.1.2. Underlying newsvendor problem

The following situation is assumed: First, the newsvendor decides on the amount to buy and so he stocks x units of the product for a unit cost c. Then, the selling period begins. If demand ξ is greater than x, all stocked units are sold for revenue px, where p is a unit price, $p > c$. In this case, it is considered a loss given by the unit shortage penalty cost s for all shortages, $\xi - x$, where $s < c$. Otherwise, if demand ξ is less or equal to x, the revenue is only $p\xi$ and the leftovers, $x - \xi$, are salvaged through the unit salvage value v, $v < c$. Then, the objective (profit) function is denoted by $\pi(x, \xi)$ defined as follows:

$$\pi(x,\xi) = \begin{cases} px - cx - s(\xi - x), & \text{for } x < \xi, \\ p\xi - cx + v(x - \xi), & \text{for } x \geq \xi. \end{cases} \tag{2.1}$$

This constitutes the underlying NP. The decision variable is the order quantity denoted by x ($x \geq 0$), while the demand ξ is the random variable, which is not completely known when the decision is made.

Note, that in the "most simple" version of the classical NP, no cost is assumed if the ordered quantity is less than the demand (see, e.g., [A5]). However, in reality failure to meet demand is always associated with a penalty. Here, the newsvendor faces both overage and underage costs - if he orders too much or if he orders too little.

As the objective function involves a random parameter, the optimization model is built as an underlying program; see [104]. Then, the objective is to maximize $\pi(x, \xi)$ and the optimization problem becomes: $\max_x\{\pi(x, \xi) \mid x \geq 0\}$, alternatively:

$$\max_x \; \{p \min\{x, \xi\} - cx - s \max\{\xi - x, 0\} + v \max\{x - \xi, 0\} \mid x \geq 0\}\,. \tag{2.2}$$

Hence, the NP is to find an optimal amount x^* that maximizes the profit. In order to solve this stochastic optimization problem, deterministic reformulations will be further used (see, e.g., [A11]).

Remark 2.1.1. Note that the per-unit holding cost parameter could also be defined: it is usually denoted by h and it is interpreted as $h = -v$ or a value v in (2.1), where $-c \leq v < 0$ (see, e.g., [86]).

Deterministic reformulation for the continuous demand case

Here, a general probability distribution of ξ with a probability density function (pdf) denoted by $f(t)$ is considered, where $f(t) = 0$ for $t \leq 0$ that is caused by the non-negativity of the demand. Then, the expected objective (EO) deterministic reformulation (see, e.g., [90]) is used and the expected profit is expressed by using the Lebesgue-Stieltjes integration, where Π denotes the expected profit:

$$\begin{aligned}\Pi(x) = E_\xi[\pi(x,\xi)] &= \int_{x<t}[px - cx - s(t-x)]f(t)\mathrm{d}t + \int_{x\geq t}[pt - cx + v(x-t)]f(t)\mathrm{d}t \\ &= (p-c)x - p\int_{x\geq t}(x-t)f(t)\mathrm{d}t - s\int_{x<t}(t-x)f(t)\mathrm{d}t + v\int_{x\geq t}(x-t)f(t)\mathrm{d}t.\end{aligned}$$

Firstly, the bounded support of ξ, i.e., $P(\xi \in [A,B]) = 1$ is assumed. Therefore, the expected profit is:

$$\Pi(x) = \begin{cases} (p-c)x - s(\mathrm{E}[\xi] - x), & x < A, \\ (p-c)x - (p-v-s)\int\limits_A^x (x-t)f(t)\mathrm{d}t, & x \in [A,B], \\ p\mathrm{E}[\xi] - cx + v(x - \mathrm{E}[\xi]), & x > B. \end{cases} \tag{2.3}$$

Further, a concrete distribution example - the uniform demand case - is briefly presented. For an extensive literature review of the distributions used see [62]. In particular, calculations with the normal distribution are presented in [90] and [93], the exponential distribution in [56], and the Poisson distribution in [37] and [50].

2.1.3. Optimal order-up-to quantity for specific demand cases

Continuous demand case

Remark 2.1.2. Some authors introduce model (2.2) as the maximization of

$$\pi(x,\xi) = p\min\{x,\xi\} - cx + v(x-\xi)^+ - s(\xi - x)^+,$$

where $(a-b)^+ = \max\{a-b, 0\}$. Then the expected profit can be written as

$$\Pi(x) = p\mathrm{E}_\xi[\min\{x,\xi\}] - cx + v\mathrm{E}_\xi[(x-\xi)^+] - s\mathrm{E}_\xi[(\xi - x)^+]. \tag{2.4}$$

The following calculations of the optimal solution of this problem are mostly from [3]. Further, expression (2.4) is considered. Then, the following particular expected quantities can be expressed.

Expected sales:

$$\mathrm{E}_\xi[\min\{x,\xi\}] = \int_0^\infty \min\{x,t\} f(t)\mathrm{d}t = \int_0^x t f(t)\mathrm{d}t + x\int_x^\infty f(t)\mathrm{d}t,$$

expected leftovers:

$$\mathrm{E}_\xi[(x-\xi)^+] = \int_0^\infty (x-t)f(t)\mathrm{d}t = \int_0^x (x-t)f(t)\mathrm{d}t,$$

and expected shortages:

$$\mathrm{E}_\xi[(\xi-x)^+] = \int_0^\infty (t-x)f(t)\mathrm{d}t = \int_x^\infty (t-x)f(t)\mathrm{d}t.$$

Then, the expected profit is

$$\Pi(x) = p\left(\int_0^x t f(t)\mathrm{d}t + x\int_x^\infty f(t)\mathrm{d}t\right) - cx + v\int_0^x (x-t)f(t)\mathrm{d}t - s\int_x^\infty (t-x)f(t)\mathrm{d}t.$$

Note that $F(t)$ represents the cumulative distribution function (cdf), $F(t) = P(\xi \le t)$ $= \int_0^t f(y)\mathrm{d}y$. It is relatively easy to show that the function $\Pi(x)$ is concave in x:

$$\frac{\mathrm{d}\Pi(x)}{\mathrm{d}x} = p\int_x^\infty f(t)\mathrm{d}t - c + v\int_0^x f(t)\mathrm{d}t + s\int_x^\infty f(t)\mathrm{d}t = p[1-F(x)] - c + vF(x) + s[1-F(x)],$$

$$\frac{\mathrm{d}^2\Pi(x)}{\mathrm{d}x^2} = -pf(x) + vf(x) - sf(x) = f(x)(v-p-s),$$

where $(v-p-s) < 0$. Since, $\int_{-\infty}^\infty f(t)\mathrm{d}t = 1$, then $\frac{\mathrm{d}^2\Pi(x)}{\mathrm{d}x^2} \le 0$ and so Π is concave on $[0,\infty)$ and has a maximum. Then the first order conditions are necessary and sufficient to determine the value of x representing optimum. Let x^* denotes the optimal ordered quantity that satisfies $\frac{\mathrm{d}\Pi(x)}{\mathrm{d}x} = 0$. Then, the optimal order quantity x^* is set such that:

$$F(x^*) = \frac{p+s-c}{p+s-v} \equiv \rho. \tag{2.5}$$

Some authors use the following notation: $c_u = p+s-c$ and $c_o = c-v$, where c_u is the underage cost and c_o the overage cost. Then $F(x^*) = \frac{c_u}{c_o+c_u}$.

Moreover, if F is invertible, then the optimal and unique x^* can be expressed as

$$x^* = F^{-1}\left(\frac{p+s-c}{p+s-v}\right). \tag{2.6}$$

The newsvendor solution can be interpreted as providing the smallest quantity that guarantees that all demand will be satisfied with a probability at least $100\rho\%$. Thus, the profit maximizing solution results in a service level of $100\rho\%$. In practice, managers often specify ρ and then find x accordingly.

Uniform demand: It is assumed that demand ξ has the uniform probability distribution: $\xi \sim U(A,B)$ and so $x \in [A,B]$. Then:

$$\Pi(x) = (p-c)x - (p-v-s)\int_A^x (x-t)\frac{1}{B-A}\mathrm{d}t = (p-c)x - \frac{(x-A)^2}{2(B-A)}(p-s-v)$$

and $\max_x\{\Pi(x)|x \ge 0\} = \max_x\{(p-c)x - \frac{(x-A)^2}{2(B-A)}(p-s-v)|x\ge 0\}$. Therefore

$$x^* = A + \frac{(B-A)(p-c)}{p-s-v}.$$

Example: Let us show the numerical and graphical results for the uniform case for the example presented in [A5] (for the case where $s = 0$ and $v = 0$).

The following values for the parameters of the model are taken into account: $p = 15$, $c = 10$, $s = 0$, $v = 0$, $A = 30$ and $B = 50$.

Then $x_{\max} = \frac{110}{3}$ and $E_{\xi}\pi(x_{\max}, \xi) = 166.\overline{6}$. The objective function is shown in Figure 2.1. Note that in Figure 2.1, the following notation is used: $f_1 = \Pi(x, \xi)$ for $x < A$, $f_2 = \Pi(x, \xi)$ for $x \in [A, B]$ and $f_3 = \Pi(x, \xi)$ for $x > B$, see (2.3).

See also [A5], where a more detailed discussion of the uniform demand distribution, an example, and a graphical solution are provided (for $s = v = 0$).

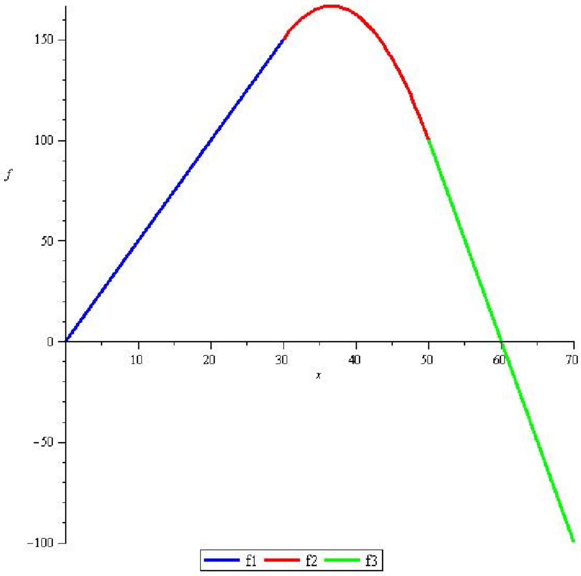


Figure 2.1: The objective function of the example.

Discrete demand case

The optimal order-up-to quantity for the discrete demand case is completely described in [3] as: assume that ξ is a non-negative discrete random variable (represented by a number of a perishable discrete items, e.g., magazines or Christmas trees) with a probability mass function (pmf) given as

$$P\{\xi = \xi_i\} = p_i, \; i = 0, 1, 2, \ldots,$$

where $0 \leq \xi_0 < \xi_1 < \xi_2 < \ldots$ and $0 \leq p_i \leq 1$, that is probability of observing ξ_i. Let $\xi_i \in \Xi$ for $i = 0, 1, 2, \ldots$. Let $F_D(t)$ be the cdf of ξ, i.e., $F_D(t) = \sum_{i:\xi_i \leq t} p_i$. It is shown in [3] that the optimal order-up-to quantity x^* is the smallest x such that

$$F_D(x) \geq \frac{p + s - c}{p + s - v} \equiv \rho_D, \tag{2.7}$$

where ξ_i's need not be integers. The value ρ_D is called the critical ratio or the critical fractile and is always between zero and one. When $c_u = c_o$, the critical ratio is $\rho_D = 0.5$, which is consistent with the intuition that suggests that x^* should be equal to the median demand when the costs are equal. For more details see [50]. Also note that (2.7) includes (2.5) as a special case. Thus, in both the discrete and continuous demand case, the optimal order-up-to quantity is the smallest x that satisfies (2.7):

$$x^* = \min\{\xi \in \Xi : F_D(x) \geq \rho_D\}.$$

Note that [3] focused on a fixed cost objective and a problem of initial inventory. In [50], an example of Poisson distributed demand is provided.

2.1.4. Applicability of the newsvendor problem

The NP is one of the fundamental models of stochastic inventory theory. It is often used to aid decision-making in fashion, the sports industry, and the apparel industry, both at the manufacturing and retailer levels (see [79]), or restaurants with food orders (or the specific order of perishable goods).

According to [120], newsvendor models are also used in capacity management and booking decision in service industries such as hotels and airlines (underbooking wastes a seat, overbooking results in having to make an offer to a passenger to give up a seat). Porteus, in [92], engaged in, among others, its application in flexible medical savings accounts.

Colleagues, in [91], have identified two more original applications: the first one is related to university budget planning and the next one is used to build a model for the design of plant capacity.

2.1.5. Overview of some other newsvendor problem modifications

In this subsection, other modifications of the NP are reviewed. Note that two of them, i.e., the NP with pricing and NP with advertising, as well as their combination, will further be described in upcoming chapters.

Newsvendor problem with pricing: See Section 3.1.

Newsvendor problem with advertising: See Chapter 4.

The multi-period newsvendor problem: The NP presented above is defined for a perishable item. This item is depreciated at the end of each period. In a multi-period problem for this kind of item, it is the same single period problem solved in each period. If there is an item which has a limited lifetime (but longer than one period), it could be modified to resemble the flower-girl problem; see [25].

According to [25], the flower girl problem is defined as: *"The flower-girl sells roses at price p and has to buy them at cost p before she starts selling. Flowers left over at the end of the day can be stored and sold the next day, when she starts selling the old roses. The roses cannot be carried over more than one additional day at the end of which they are thrown away. The demand is random, ξ_t denotes the demand on the t-th day. Whereas x_1 has to be bought without any knowledge of the realization of the random demand, the flower-girl can adapt the subsequent orders x_t, $t > 1$, to the demand observed during the previous days. Her goal is to maximize the total expected profit".*

If the lifetime is not limited, the NP is converted to the multi-period inventory problem with non-perishable goods; see [115]. Then, this problem can be separated into a problem of determining the initial stocking quantity or to a dynamic programming problem, where the stocking quantity can be replenished at any period.

Distribution-free newsvendor problem: In 1958, Scarf [102] formulated the so called distribution-free newsboy problem, where only the mean μ and variance σ^2 are specified, but the demand probability distribution is not known, in general, and pioneered the minimax approach which expresses this problem as involving finding the order quantity that maximizes the expected profit resulting from the worst possible distribution of the demand with the known parameters. Gallego and Moon [36] presented a proof of the optimality of Scarf's ordering rule for the distribution-free newsboy problem. An extension of this problem to a case where a customer may balk if the inventory is low is presented in [79]. Alfares and Elmorra [1] focused on extending the distribution-free newsboy problem to the shortage penalty case. In 1960 and 1961, respectively, Kasugai and Kasegai [59] and [60] applied dynamic programming and the minimax regret ordering policy to the distribution-free multi-period NP, while Vairaktarakis [113] developed several minimax regret models for the distribution-free multi-item NP under a budget constraint and two types of demand uncertainty; for more details see [1].

Loss-averse, risk-averse, and risk-neutrality newsvendor problems: Newsvendor models are usually based upon the assumption of risk neutrality. However, in real situations, there are many reasons to prefer another approach. Loss aversion is based on the premise that losses are considered to be more important than gains. This approach is often preferred in marketing, economics, and finance. This means that a positive parameter can be used (e.g., loss averse coefficient, see [22, 114]), that is a measure of the agent's loss aversion. [22] provides an extensive review of loss and risk aversion in games and decision-making. Wang and Webster [114] have found that if the shortage cost is not insignificant, then a loss-aversion newsvendor may order more than a risk-neutral newsvendor. They have also found a difference between wholesale and retail price: a loss-averse newsvendor's optimal order quantity may increase in wholesale price and decrease in retail price, which can never occur in the risk-neutral newsvendor model. A useful review of the risk-averse NP is provided in [21, 17, 114]. In paper [114], the risk-averse newsvendor is defined as: a newsvendor who turns down a gamble of losing \$100 or gaining \$110, each with a 50% probability, is a risk-averse, i.e., he is risk-averse in the sense that he is unwilling to take a bet that is actuarially fair when facing uncertainty. Loss aversion is distinguished from risk aversion by the presence of a reference point that determines if the payoff is perceived as a loss or a gain, and by a sharp change in the slope of the utility function at the reference point [119]. In recent literature, there are also some papers about the multi-product risk-averse problem [18, 19].

Newsvendor games: Newsvendor games (or the competitive NP) also assume several forms of the model. The simplest way to understand them is to consider the following example by [103]: two newsvendors (player 1 (P_1) and player 2 (P_2)) trying to satisfy the random total market demand with substitutable products. A customer chooses one of them initially. If he can find the item with the newsvendor, he buys it, otherwise he may decide not to buy any product or to go to the second newsvendor and buy it if it is available there. Then, P_i can have both an initial demand (an initial allocation) and an extra demand occurring from the shortage of P_j (a reallocated demand), where $i = 1, 2$, $j = 1, 2$, $i \neq j$. It is quite clear that the decision of one player affects the other. If P_i orders too little, it can cause an increase in the reallocated demand coming to P_2 and the total cost of shortages for P_2 may be higher. If P_1 orders too much, the reallocated demand coming to P_2 increases and the total cost of salvages (or inventory holding cost) for P_2 may be higher. [103] provides a helpful review of this problem. [12] focused on newsvendor games with centralized inventory operations, i.e., they consider a set $U = \{1, \ldots, n\}$ of outlets each with normal distribution and mean μ_i and a standard deviation σ_i, $i \in U$, and they also assume coalition(s) as a subset $S \subseteq U$ of outlets that bands together to centralize inventory, where the game with coalitions is also called the cooperative game (if $S = U$, then S is the so-called grand coalition). Newsvendor games with the possibility of an infinite number of newsvendors are the so-called large newsvendor games [77]. A useful review of the newsvendor games problem is provided in [78], where the authors combine the cooperative and noncooperative games and the simple newsvendor game, the large newsvendor game, and other extensions of the problem. [118] combines the loss-averse NP and the newsvendor game.

2.2. Transportation network design problem

The transportation network design problem (TNDP) deals with the optimization of the design of transportation networks. TNDP remains a challenging research topic in transportation planning, where the objective is to achieve specified objectives (e.g., minimize transportation costs or maximize profits achieved). From constructing new roads, pipelines, power lines, etc., to determining the optimal road toll, TNDP has provided valuable information for capital investment in transportation [67, 4, 112]. In general, a network design problem (NDP) focuses on the building up or modification of a (transportation) network. Discrete NDP deals with the addition of new links to a transportation network. An NDP which deals only with the capacity expansion of existing links in a transportation network is referred to as a continuous NDP. A mixed NDP is a mixture of both; see [39].

The TNDP under our consideration contains an embedded network flow problem. There is a set of source nodes (production facilities) and a set of sink nodes (e.g., customers with demand), and the demand is routed through the network from the sources to the sinks. The aim of the network design is to optimize the network with respect to the overall profit which is clarified in detail later in the chapter. In short, the network operator is paid a certain price for delivering a unit of demand to each customer, but it has to bear the transportation costs and the costs of extending (designing) the network. See Figure 2.2 for an experimental example of the network which will be further used in the thesis for TNDP computations.

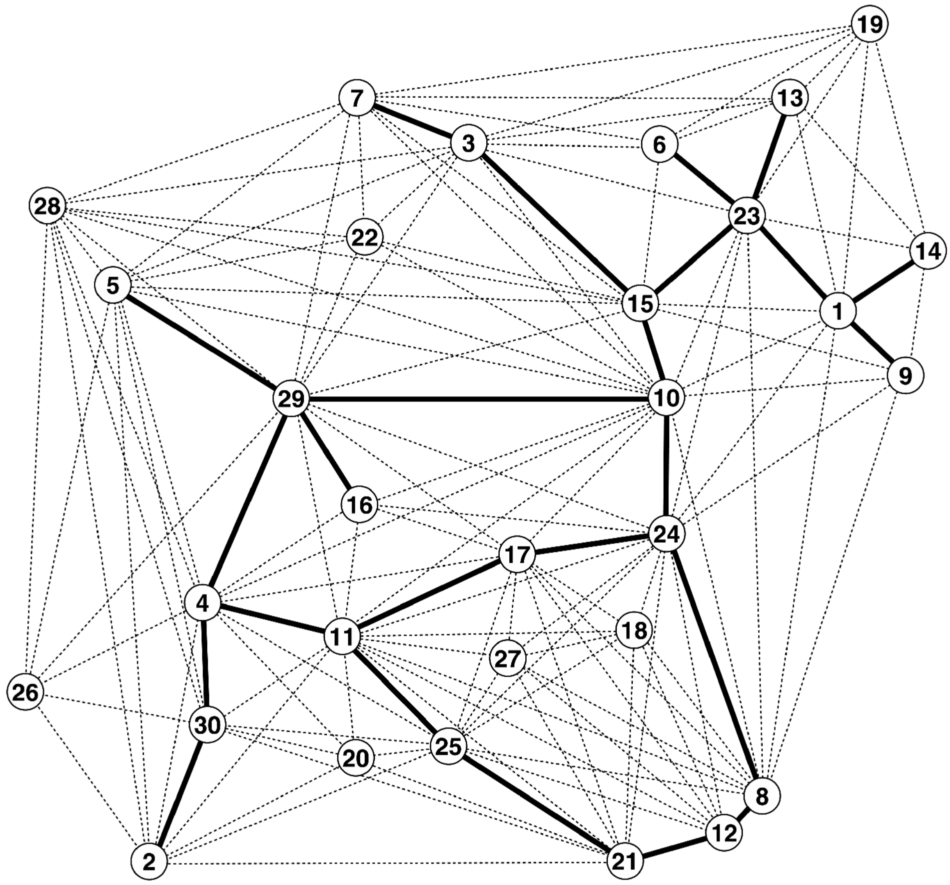


Figure 2.2: Input network structure: bold lines are existing edges and dashed lines are possible edges that can be switched on by 0-1 variables; nodes 1-14 present customers, 15-16 production nodes, 17-30 transition nodes (with no demand allowed) [A1, A8, A13, A6, A9, A7, A3]; see Appendix B.1 for a data set of the network (generated in GAMS software).

Various (algorithmic) approaches have been taken to solve NDP as well as TNDP. Steenbrink [108] and Magnanti and Wong [71] reviewed a range of the NDP's and some earlier algorithms. LeBlanc [67] proposed a branch-and-bound procedure to solve it, but the algorithm did not perform well in large-scale problems. For a detailed review of recent solution techniques, see, e.g., [4, 89].

This thesis deals with a single-commodity stochastic variant of TNDP, where the design of the network is to be conducted under an imperfect knowledge of the future uncertain demand at the sink nodes. The demanded commodity is assumed to be continuously divisible. Herein, the problem is approached using the so-called here-and-now (HN) and the wait-and-see (WS) SP reformulation (see Section 2.2.2); the uncertain demand is captured via available demand information (scenario-based approach; see, e.g., [A11]). See [57, 104] for fundamental ideas on SP.

The next section begins by introducing a deterministic TNDP where the network connects suppliers and customers (see [39] and Figure 2.2); its stochastic forms (i.e., the above-mentioned deterministic reformulations) follow in Section 2.2.2.

2.2.1. Deterministic TNDP

The deterministic transportation network design model, see [39, 10, A13], is the first step towards the development of our model.

The following symbols are used in the model:

• the decision variables:

x_e : the amount of a given product to be transported on edge e,
$\delta_{\mathrm{E}_n} \in \{0,1\}$: 1 if new edge E_n is built, 0 otherwise,

• index sets:

E : set of edges, $e \in E$,
E_n : set of newly built edges, $\mathrm{E}_n \in \mathrm{E}_n,\ \mathrm{E}_n \subset E$,
I : set of customers (or locations with a non-zero demand), $i \in I$,
J : set of production locations (or warehouses), $j \in J$,
K : set of traffic nodes, $k \in K$,
V : set of all nodes (vertices) in the network, $v \in V$, $V = I \cup J \cup K$,

• and parameters:

$A_{v,e}$: incidence matrix, $A_{v,e} = \begin{cases} 1 & \text{if edge } e \text{ leads to node } v, \\ -1 & \text{if edge } e \text{ leads from node } v, \\ 0 & \text{otherwise,} \end{cases}$
b_v : the demand in node v,
c_e : unit transporting cost on edge e,
d_{E_n} : cost of building of a new edge E_n,
p_i : is a unit selling price for customer i.

The profit maximization objective function is

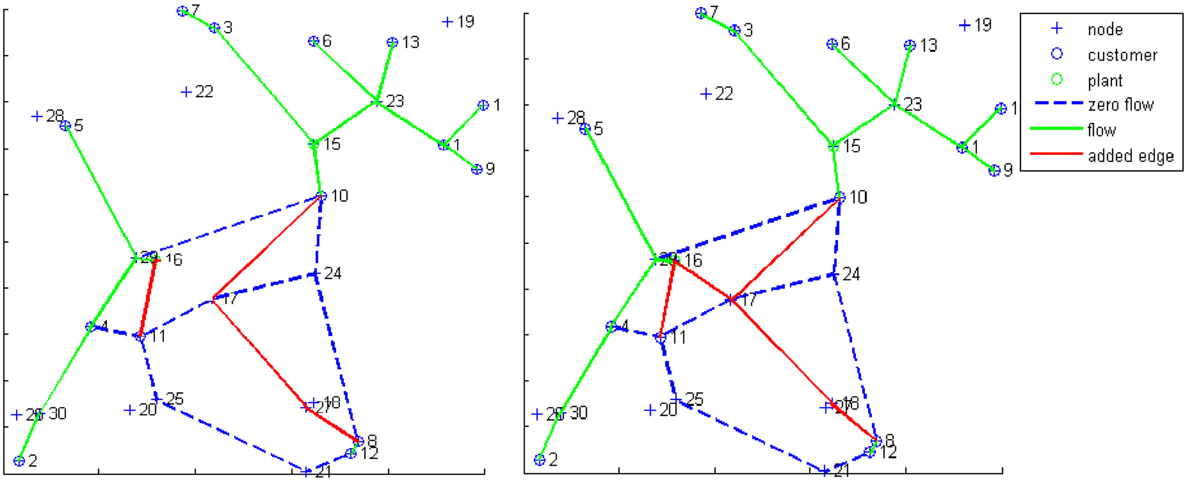
$$\max \sum_{i\in I}\sum_{e\in E} A_{i,e}x_e p_i - \sum_{e} c_e x_e - \sum_{\mathrm{E}_n\in\mathrm{E}_n} d_{\mathrm{E}_n}\delta_{\mathrm{E}_n}; \tag{2.8}$$

the term $\sum_i(\sum_e A_{i,e}x_e)p_i$ defines the income from all customers. The following terms $\sum_e c_e x_e + \sum_{\mathrm{E}_n} d_{\mathrm{E}_n}\delta_{\mathrm{E}_n}$ define the costs of transportation for the produced items and the setting up of new edges in the network. The considered model is balanced, i.e., the sum of demands is equal to the sum of the produced and transported units of goods. This is also represented by transportation balance constraints $\sum_e A_{v,e}x_e = b_v, \forall v \in V$.

So, the mixed-integer linear programming (MILP) model is obtained here and is specified as follows:

$$\begin{aligned} \max \sum_{i\in I}\sum_{e\in E} & A_{i,e}x_e p_i - \sum_{e\in E} c_e x_e - \sum_{\mathrm{E}_n\in\mathrm{E}_n} d_{\mathrm{E}_n}\delta_{\mathrm{E}_n} \\ \sum_{e\in E} A_{v,e}x_e &= b_v, && \forall v \in V, \\ x_{\mathrm{E}_n} &\le \delta_{\mathrm{E}_n}(\sum_{j\in J} -b_j), && \forall \mathrm{E}_n \in \mathrm{E}_n, \\ x_e &\ge 0, && \forall e \in E, \\ \delta_{\mathrm{E}_n} &\in \{0,1\}, && \forall \mathrm{E}_n \in \mathrm{E}_n. \end{aligned} \tag{2.9}$$

See also author's master thesis [A1] for a more detailed model developing, its description and discussion of the related results. The solution, which was obtained by the GAMS software, is visualized in Figure 2.3a; the first scenario from the demand data set, which is provided in Appendix B.2, was used.



(a) Solution of deterministic model (2.9). *(b) Solution of HN stochastic model (2.13).*

Figure 2.3: Two graphical solutions of the problem from Figure 2.2 (by a GAMS solver); see [A1] for their GAMS implementation as well as solution, and see also [A13].

2.2.2. Stochastic TNDP

As the next step, uncertainty is involved in model (2.9); parameter, which is considered to be uncertain, is demand b. This captures such real-world problems, where suppliers have imperfect information about the demands of their (potential) customers. A common approach to deal with the uncertain demand is a scenario-based approach. Therefore, it will be further considered that the supplier (or distributor, or simply decision-maker) knows possible demand values for each potential scenario as well probability of its occurrence.

Therefore, two scenario-based deterministic reformulations will be further used: the HN approach, see [A8, 90] and model (2.13); and the WS approach, see [A1, 90] and model (2.14).

HN reformulation

Firstly, the HN approach (deterministic reformulation) is used; see [A13]. The HN approach means that, based on a knowledge of the potential scenarios (e.g., their potential demand values and probability of each of the scenarios), the decision-maker must make his decisions (i.e., x_e $\forall e \in E$ and δ_{E_n} $\forall \mathrm{E}_n \in \mathrm{E}_n$) before he knows the real demand.

The following notation is different comparing to that from Section 2.2.1:

- the second-stage variables (that relate to the particular scenarios):

  $y^+_{i,s}$: shortages for customer i in scenario s, where $y^+_{i,s} = \max\{b_{i,s} - \sum_e A_{i,e}x_e, 0\}$,
  $y^-_{i,s}$: leftovers for customer i in scenario s, where $y^-_{i,s} = \max\{\sum_e A_{i,e}x_e - b_{i,s}, 0\}$,

- index set:

  S : set of all possible scenarios, $s \in S$, $s = 1, 2, \ldots, m$,

- and parameters:

$b_{i,s}$	:	the demand in node i for scenario s; $b_{i,s} > 0\ \forall i \in I,\ \forall s \in S$,
b_j	:	the production in node j; $b_j < 0\ \forall j \in J$,
b_k	:	the (zero) demand/production in node k; $b_k = 0\ \forall k \in K$,
q_s	:	probability that scenario s occurs, where $0 \leq q_s \leq 1, \forall s \in S$, and $\sum_s q_s = 1$,
r_i^+	:	unit penalty cost for shortages (unsatisfied demand) at customer node i,
r_i^-	:	unit penalty cost for leftovers (redundant units) at customer node i.

So, the objective function is "modified" by a penalty term representing the recourse action for the unsatisfied demand that is specified by the new decision variables $y^+_{i_1,s} \geq 0$ and $y^-_{i,s} \geq 0$. Thus, formula $\sum_s q_s(\sum_i(r_i^- y^-_{i,s} + r_i^+ y^+_{i,s}))$, which represents an additional cost, is subtracted from the objective function (2.8).

The first constraint from the model (2.9) is modified as follows

$$\sum_{e\in E} A_{i,e}x_e + y^+_{i,s} - y^-_{i,s} = b_{i,s},\ \forall i \in I, \forall s \in S, \tag{2.10}$$

$$\sum_{e\in E} A_{j,e}x_e = b_j,\quad \forall j \in J, \tag{2.11}$$

$$\sum_{e\in E} A_{k,e}x_e = b_k,\quad \forall k \in K, \tag{2.12}$$

where equation (2.10) means that the transported units plus missing units minus surplus units must be equal to the customer demand. Equations (2.11) and (2.12) are defined in the same way as (the first constraint) in the model (2.9), but they are defined separately for plants (2.11) and for transition nodes (2.12).

Altogether the updated scenario-based stochastic MILP model is specified as follows:

$$\begin{aligned}
\max\ & \sum_{i\in I}\sum_{e\in E} A_{i,e}x_e p_i - \sum_{e\in E} c_e x_e - \sum_{\mathrm{E}_n\in \mathrm{E}_n} d_{\mathrm{E}_n}\delta_{\mathrm{E}_n} - \sum_{s\in S} q_s \sum_{i\in I}(r_i^- y^-_{i,s} + r_i^+ y^+_{i,s}) \\
\text{s.t.}\ & \textstyle\sum_{e\in E} A_{i,e}x_e = b_{i,s} - y^+_{i,s} + y^-_{i,s}, && \forall i \in I, \forall s \in S, \\
& \textstyle\sum_{e\in E} A_{j,e}x_e = b_j, && \forall j \in J, \\
& \textstyle\sum_{e\in E} A_{k,e}x_e = b_k, && \forall k \in K, \\
& x_{\mathrm{E}_n} \leq \delta_{\mathrm{E}_n} \textstyle\sum_{j\in J}(-b_j), && \forall \mathrm{E}_n \in \mathrm{E}_n, \\
& y^+_{i,s} \leq b_{i,s}, && \forall i \in I, \forall s \in S, \\
& x_e \geq 0, && \forall e \in E, \\
& \delta_{\mathrm{E}_n} \in \{0,1\}, && \forall \mathrm{E}_n \in \mathrm{E}_n, \\
& y^+_{i,s},\ y^-_{i,s} \geq 0, && \forall i \in I, \forall s \in S.
\end{aligned} \tag{2.13}$$

See Figure 2.3b for a graphical solution of the model (2.13).

WS reformulation

Herein, the decision-maker solves each particular scenario independently and so, he can make the decision as a reaction to the obtained results. Nevertheless, the WS approach is employed for our stochastic scenario-based problem in order to examine some attributes of result of each of the scenarios (e.g., "variance" of the network design variables).

In other words, the WS decisions $x_{e,s}$ and $\delta_{\mathrm{E}_n,s}$ are reactions to known $b_{i,s}$ for each particular scenario s. Therefore, in contrast to the HN approach [see model (2.13)], the decision (or set of the decisions, respectively) on transportation as well as on network design is made for each scenario; in other words, both decision variables (i.e., x_e's and δ_{E_n}'s) includes index s. The following notation is different comparing to the model (2.13):

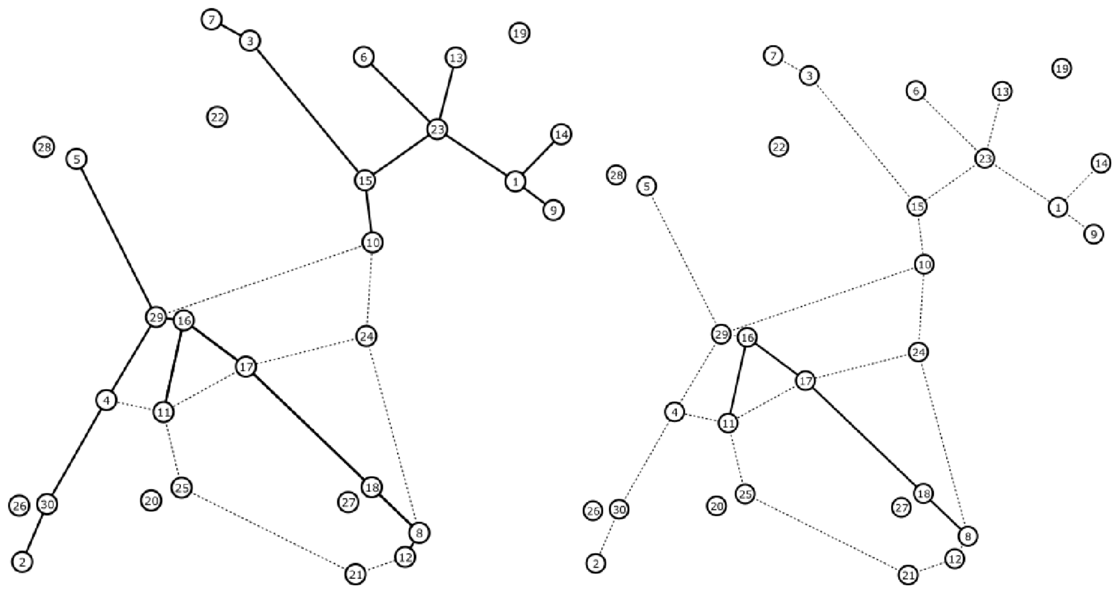
- the decision variables are:

$x_{e,s}$: the amount of a given product to be transported on edge e in scenario s,
$\delta_{\mathrm{E}_n,s}$ $\in$ $\{0,1\}$: 1 if new edge E_n is built in scenario s, 0 otherwise.

So this is a separable model that can be solved by the loop solutions of the separated models involving of objective functions and constraints. In accordance with the above-mentioned notations, the TNDP under the WS approach is formulated as

$$\begin{array}{lrcll}
\forall s \in S: & \max \sum_{i\in I}\sum_{e\in E} A_{i,e}x_{e,s}p_i - \sum_{e\in E} c_e x_{e,s} - \sum_{\mathrm{E}_n\in \mathrm{E}_n} d_{\mathrm{E}_n}\delta_{\mathrm{E}_n,s} - \sum_{i\in I}(r_i^- y_{i,s}^- + r_i^+ y_{i,s}^+) & & & \\
\text{s.t.} & \sum_{e\in E} A_{i,e}x_{e,s} & = & b_{i,s} - y_{i,s}^+ + y_{i,s}^-, & \forall i\in I, \forall s \in S, \\
& \sum_{e\in E} A_{j,e}x_{e,s} & = & b_j, & \forall j \in J, \\
& \sum_{e\in E} A_{k,e}x_{e,s} & = & b_k, & \forall k \in K, \\
& x_{\mathrm{E}_n} & \leq & \delta_{\mathrm{E}_n,s}\sum_{j\in J}(-b_j), & \forall \mathrm{E}_n \in \mathrm{E}_n, \\
& y_{i,s}^+ & \leq & b_{i,s}, & \forall i \in I, \\
& x_{e,s} & \geq & 0, & \forall e \in E, \\
& \delta_{\mathrm{E}_n,s} & \in & \{0,1\}, & \forall \mathrm{E}_n \in \mathrm{E}_n, \\
& y_{i,s}^+,\ y_{i,s}^- & \geq & 0, & \forall i \in I.
\end{array} \tag{2.14}$$

See Figure 2.4 for a graphical solution for one particular scenario (see Appendix B.2).



(a) *Network flow visualization (nonzero $x_{e,s}$).* (b) *Network design visualization ($\delta_{e,s}$).*

Figure 2.4: Graphical results (by GAMS) of one fixed scenario; see [A9].

See also [A6], where the authors solved the model with a hybrid algorithm (see Section 3.2); the (graphical) results (e.g., in Figure 3.2) can be useful to examining some of the quantitative scenario-based network design attributes. ď»ż

Chapter 3

Pricing

Recently, there has been an increasing focus on the development of pricing as well as on marketing strategies and their further applications in industry. The pricing problems concern on the determination of selling prices as a decision variable for a product under the relevant demand and supply constraints.

Although, pricing is applied in the NP (see Section 3.1) and in the TNDP (see Section 3.2) herein, it is not limited for these 2 problems but it can be applied in many other areas. See, e.g., its application in lot-size problems [47], integrated forward logistics network design with pricing for collection of used products [31] or road toll pricing [125]. The author already presented some fundamental ideas for the coordination between production and pricing decisions; see [A2, A5] for the NPP and see [A3, A7, A8] for the TNDP with pricing.

3.1. Newsvendor pricing problem

The first mathematical formulation of price effects in inventory control problems was provided by Whitin [121] in 1955. Until this year, economic theoreticians had disregarded several important aspects, causing Whitin to focus on the hitherto-neglected demand aspects. He adapted the NP, where the unit selling price is a decision variable and where demand linearly depends on the selling price per unit, where the retailer knows a probability distribution of demand. Hence, he knows the amount demanded at any given price. Whitin established a sequential method for firstly determining the optimal ordered quantity as a function of selling price, and then the relevant optimal price. In [75], Mills extended the NPP by a specifying mean demand as a function of the selling price. He refined Whitin's work by modeling the uncertainty in additive form, i.e., the demand is specified as $D(p, \xi) = d(p) + \xi$, where $d(p)$ is a decreasing demand function of price p and ξ is a random variable defined within some range. Specifically, Mills established that the optimal price under uncertain demand is never greater than the optimal price set in the equivalent deterministic monopoly models, called the riskless price. Both Whitin and Mills considered the single period form of the problem, i.e. a static problem, where only a single price and an ordered amount need to be determined. A detailed comparison between the static case and the dynamic case is provided by Karlin and Carr in [58]. For both these cases, additive demand was applied as well as the multiplicative demand case. They defined the multiplicative demand case as $D(p, \xi) = d(p)\xi$. In the dynamic model, i.e., the multi-period problem, they employed the infinite-period approximation to the

$n-$period model. However, they did so under the assumption that a single constant price needed to be determined at the beginning of the planning horizon. They established that the optimal price under multiplicative uncertain demand is never smaller than the riskless price, i.e., an opposite outcome to Mills' outcome for additive uncertainty.

For useful overviews of the pricing case see [14, 42, 43, 86, 127, 129] and see [29, 76] for the dynamic case.

3.1.1. Modeling issues

In the classical newsvendor model (see subsection 2.1), the selling price is considered as exogenous, over which the newsvendor has absolutely no control [20]. Here, the pricing problem is approached as a price-setting newsvendor problem, i.e., two decision variables are defined: the selling price, p, and the ordered quantity, x. Then, the same parameters as in (2.1) are considered: buying cost c, salvage value v, and shortage cost s. The demand function, D, is both stochastic and price dependent, i.e., $D(p,\xi)$ replaces ξ with regards to (2.1), where ξ presents the random term. Thus, for each pair (p,ξ) the retailer knows the resulting demand, but he is unable to predict it in advance because he does not know which value of the demand will assume in reaction to the random term in both cases. The objective (profit) function is:

$$\pi(p,x,\xi) = \begin{cases} px - cx - s[D(p,\xi) - x], & \text{for } x < D, \\ pD(p,\xi) - cx + v[x - D(p,\xi)], & \text{for } x \geq D, \end{cases} \tag{3.1}$$

where x units are stocked at the beginning of the selling period for cost cx. If demand D is greater than x, then the revenue is px and $s[D(p,\xi)-x]$ denotes shortages multiplied by the shortage penalty cost per unit. Otherwise, if demand D is less or equal to an ordered quantity x, the income is only $pD(p,\xi)$ and $v[x - D(p,\xi)]$ denotes leftovers multiplied by salvage value per-unit. Alternatively, (3.1) can be rewritten as:

$$\pi(p,x,\xi) = p\min\{x, D(p,\xi)\} - cx + v[x - D(p,\xi)]^+ - s[D(p,\xi) - x]^+,$$

where $a^+ = \max\{a, 0\}$ and $\min\{a,b\} = a - (a-b)^+$. The goal is to maximize the expected value of the objective function. For this stochastic optimization program the so-called *expected objective reformulation* is defined (see [82, 90]). The expected profit is to be maximized and is defined as:

$$\Pi(p,x) = \mathrm{E}_\xi[\pi(p,x,\xi)] = \mathrm{E}_\xi[p\min\{x, D(p,\xi)\} - cx + v[x - D(p,\xi)]^+ - s[D(p,\xi) - x]^+].$$

Remark 3.1.1. Without the loss of generality, some authors set the parameters s, v to be zero (see, e.g., [129]). In general, the parameters satisfy $s \geq 0$, $c > v \geq 0$, and $p > c$.

3.1.2. Demand function and randomness

This section is focused on the demand function $D(p,\xi)$. Price independent randomness is further assumed in demand. More specifically, the demand function satisfies

$$D(p,\xi) \equiv D(p,\xi_a,\xi_m) = d(p)\xi_m + \xi_a, \tag{3.2}$$

where ξ is a two-dimensional vector, and $\xi = (\xi_m, \xi_a)$ and ξ_m, ξ_a are the random variables. In principle, the stochastic demand curve given by (3.2) should capture a real situation: when the price rises then the demand decreases, i.e., $d(p)$ is assumed as strictly decreasing, i.e., $d'(p) = \frac{\mathrm{d}d(p)}{\mathrm{d}p} < 0$, that presents the dependency between demand and price; the expected value of demand tends toward zero at sufficiently high prices, since demand cannot be negative. The assumption of monotonicity satisfies all common items; only special luxury items are excluded (i.e., Veblen paradox) [129]. Moreover, $d(p)$ is assumed to be a continuous function and twice differentiable. So, $d(p)$ is defined on a closed interval $[c, \overline{p}]$, where $d(\overline{p}) = 0$, see [129].

For the purpose of this thesis, two special cases of the demand function (3.2) are mentioned. Mills in [75] defined the *additive demand* function, where $\xi_m = 1$ (or $P(\xi_m = 1) = 1$) and so $D(p, \xi) = d(p) + \xi_a$. Furthermore, for this case it is assumed that $\mathrm{E}[\xi_a] = 0$. Another special case of the demand function is the *multiplicative demand* case defined by Karlin and Carr [58], where $\xi_a = 0$ (or $P(\xi_a = 0) = 1$) and so $D(p, \xi) = \xi_m d(p)$. Furthermore, here it is assumed that $\mathrm{E}[\xi_m] = 1$. For more details on special cases see [15].

Then, the expectation of D is specified by a function $d(p)$ for both the cases:

$$\mathrm{E}_\xi[D(p, \xi)] = d(p).$$

It is assumed that $F(\cdot)$ represents the cdf of the random variable with pdf $f(\cdot)$. It is also reasonable to consider that F is invertible; see expression (2.6).

Remark 3.1.2. Some authors consider the mean of the random variable to be a general number, e.g., μ, see [86, 134].

Remark 3.1.3. Note that the additive and multiplicative cases, which are applied in most existing literature, are considered under some special assumptions. A different definition is verified by Young [131]. He analyzed a model which combines both the additive and multiplicative effects, defining the demand functions as $D(p, \xi) = d_1(p)\xi + d_2(p)$.

3.1.3. Riskless problem

In the riskless theory demand is considered as strictly a function of the price p, which means that no random factor ξ is considered, see [75]. The related objective function is denoted by $\Psi(p)$ and so the problem is: $\max_p\{\Psi(p) | p \geq c\}$, where

$$\Psi(p) = (p - c)d(p), \tag{3.3}$$

i.e., the profit for a given price in the certainty-equivalent problem. The optimal riskless price p^*_Ψ can be determined by solving the first order condition $\frac{\mathrm{d}\Psi(p)}{\mathrm{d}p} = 0$.

Remark 3.1.4. For the linear function $d(p) = a - bp$, $a, b > 0$, which is often used for the additive demand case, the optimal riskless price is $p^*_\Psi = \frac{a+bc}{2b}$.

Remark 3.1.5. For the isoelastic function $d(p) = ap^{-b}$, $a > 0$, $b > 1$, which is often used for the multiplicative demand case, it is $p^*_\Psi = \frac{bc}{b-1}$.

Yao, Chen and Yan [129] define a class of demand functions with an *increasing price elasticity* (IPE), i.e., functions that satisfy

$$\frac{\mathrm{d}e}{\mathrm{d}p} \geq 0,$$

where $e = -\frac{pd'(p)}{d(p)}$ denotes the *price elasticity* of $d(p)$ that gives the percentage change in demand in response to a one percent change in price (see [132]). The IPE attribute means that for the retailer, it is less desirable to raise the price; in other words, if the price increases by a certain percentage, demand decreases by a larger percentage. The following lemma by [129] guarantees uniqueness of the first order condition solution.

Lemma 3.1.1. If $d(p)$ has increasing price elasticity, then the riskless profit Ψ is quasi-concave (or unimodal) in p in the interval $[c, \overline{p}]$. The optimal riskless price p^*_Ψ can be uniquely determined by solving $\frac{\mathrm{d}\Psi}{\mathrm{d}p} = 0$.

See [129] for proof.

In [129], the authors provide some typical examples with IPE property. Table 3.1 contains the examples and an extension of some other functions used in the literature.

Table 3.1: Typical demand functions with IPE property [129].

Demand/price function $d(p)$	Conditions	Paper
Linear function $d(p) = a(\overline{p} - p)$	$a > 0$	Mills [75]
$d(p) = p^\alpha e^{-\lambda p}$	$\alpha \leq 0, \lambda > 0$	Karlin and Carr [58]
$d(p) = \frac{1}{(1+p)^\alpha}$	$\alpha > 1$	Karlin and Carr [58]
Function $d(p)$ concave in p		Zabel [133], Ha [46], Federgruen and Heching [32]
Isoelastic function $d(p) = ap^{-b}$	$b \geq 2$	Young [131]
Isoelastic function $d(p) = ap^{-b}$	$b > 1$	Petruzzi and Dada [86]
Function $d(p)$ log-concave in p		Rosenberg [95]

3.1.4. Additive demand case

Remember that this case involves the demand function $D(p, \xi)$, which is given by (3.2), modeled such that $\xi_m = 1$ and $\mathrm{E}[\xi_a] = 0$. Then:

$$D(p, \xi) \equiv d(p) + \xi_a. \tag{3.4}$$

Here, the reader is subsequently referred to [16, 75, 86, 124, 128, 129, 134]. The objective function (3.1) can be rewritten by substituting (3.4) and defining the *stocking factor* z, $z = x - d(p)$, as:

$$\pi(p, z, \xi_a) = \begin{cases} p[z + d(p)] - c[z + d(p)] - s(\xi_a - z), & \text{for } z < \xi_a, \\ p[d(p) + \xi_a] - c[z + d(p)] + v(z - \xi_a)], & \text{for } z \geq \xi_a. \end{cases} \tag{3.5}$$

This variable transformation (from x to z) is often used in the newsvendor-related literature (e.g., in [30, 86, 111, 134]); it will be further shown to simplify the computations. It also provides an alternative interpretation of the stocking decision: if the choice of z is greater than the realized value of the random variable ξ_a, then leftovers occur, otherwise shortages occur.

The objective is to maximize the expected profit by choosing p and z. However, the optimal solution is not necessarily an interior solution. In particular, the value of z can be on boundary A or B (see [134]).

The convention put forth by Petruzzi and Dada [86] is further used, with the following quantities subsequently defined as (the expectation operator is further denoted as E[·]):

expected leftovers:

$$\Lambda(z) = \mathrm{E}[(z-\xi_a)^+] = \int_A^z (z-t)f(t)\mathrm{d}t, \tag{3.6}$$

expected shortages:

$$\Theta(z) = \mathrm{E}[(\xi_a - z)^+] = \int_z^B (t-z)f(t)\mathrm{d}t. \tag{3.7}$$

Then, the expected profit is expressed by:

$$\Pi(p,z) = (p-c)[z+d(p)] - (p-v)\int_A^z (z-t)f(t)\mathrm{d}t - s\int_z^B (t-z)f(t)\mathrm{d}t. \tag{3.8}$$

Considering (3.6) and (3.7), the loss function can be expressed as:

$$L(p,z) = (c-v)\Lambda(z) + (p+s-c)\Theta(z),$$

where if z is chosen too high, an overage cost $(c-v)$ appraises each of the $\Lambda(z)$ expected leftovers, and if z is chosen too low, an underage cost $(p+s-c)$ appraises each of the $\Theta(z)$ expected shortages and the expected profit can hence be expressed by:

$$\Pi(p,z) = \Psi(p) - L(p,z), \tag{3.9}$$

the riskless profit, which would occur in the absence of uncertainty (see subsection 3.1.3), less the expected loss that occurs as a result of the presence of uncertainty (see [85, 86]).

Through integration by parts, the expected profit can be expressed from (3.8) as:

$$\Pi(p,z) = (p-c)[z+d(p)] - (p-v)\int_A^z F(t)\mathrm{d}t - s\int_z^B [1-F(t)]\mathrm{d}t. \tag{3.10}$$

Whitin [121] established the sequential method for first determining the optimal value of z as a function of p by using the famous fractile rule for determining z when p is fixed, i.e., the result of the common NP. Let the subscript * denote optimality. By solving the first ordered condition $\frac{\partial \Pi(p,z)}{\partial z} = 0$, the following expression is observed:

$$z^* \equiv z(p) = F^{-1}\left(\frac{p+s-c}{p+s-v}\right). \tag{3.11}$$

The problem of maximizing the expected profit $\Pi(p,z)$ over two variables is now reduced to a maximization problem over the single variable p: $\max_p \Pi(z(p),p)$, and so substituting (3.11) into (3.10) is p^* obtained by solving $\frac{\mathrm{d}\Pi(z(p),p)}{\mathrm{d}p} = 0$. The related derivative is:

$$\frac{\mathrm{d}\Pi(z(p),p)}{\mathrm{d}p} = d(p) + (p-c)\frac{\mathrm{d}d(p)}{\mathrm{d}p} + \frac{c-v}{p+s-v}z^* + \int_A^{z^*} tf(t)\mathrm{d}t. \tag{3.12}$$

The second derivative w.r.t. p is:

$$\frac{\mathrm{d}^2\Pi(z(p),p)}{\mathrm{d}p^2} = (p-c)\frac{\mathrm{d}d^2(p)}{\mathrm{d}p^2} + 2\frac{\mathrm{d}d(p)}{\mathrm{d}p} + \frac{1}{f(z^*)}\frac{(c-v)^2}{(p+s-v)^3} \tag{3.13}$$

See Appendix A.1 for an expression of (3.13) from (3.12).

Linear pricing function

For the linear demand function $d(p) = b - ap$, $a, b > 0$ (see Remark 3.1.4), Zabel [132] developed the sequential method for first determining the optimal value of p as a function of z by using the first partial derivative with respect to p. Further, the method of determining the optimal price is outlined. The optimal selling price can be determined as:

$$p^* \equiv p(z) = p^*_\Psi - \frac{\Theta(z)}{2a} \tag{3.14}$$

where $p^*_\Psi = \frac{b+ac}{2a}$ (see subsection 3.1.3). Equation (3.14) implies that $p^* \leq p^*_\Psi$ (with respect to nonnegativity of $\Theta(z)$, see [75]). Then the boundary condition for p^* is

$$c \leq p^* \leq p^*_\Psi \tag{3.15}$$

(or $c - s \leq p^* \leq p^*_\Psi$, see [134]). The second partial derivative is $\frac{\partial^2 \Pi(p,z)}{\partial p^2} = -2a$ and so Π is concave in p for a given z. Then, z^* can be found by searching through the resulting optimal trajectory to maximize $\Pi(p, z^*)$, see [86, 132]. Zhan and Shen [134] deal with properties of the solution, its unimodality and geometrical interpretation. They also developed an iterative algorithm as well as a simulation based algorithm to solve the system of equations (3.11) and (3.14).

Following, the single period optimal stocking and pricing policy for the additive demand case is to stock $x^* = d(p^*) + z^*$ units to sell at the price p^* per unit.

3.1.5. Multiplicative demand case

For the multiplicative demand case, it is considered that $\xi_a = 0$ and $\mathrm{E}[\xi_m] = 1$ in the demand function $D(p, \xi)$ given by (3.2). See papers by Petruzzi and Dada [86] and Karlin and Carr [58] for seminal work. Therefore, the demand function is defined as:

$$D(p, \xi) \equiv d(p)\xi_m. \tag{3.16}$$

Substituting (3.16) and $z = \frac{x}{d(p)}$ (the stocking factor) to model (3.1), it can be obtained:

$$\pi(p, z, \xi_m) = \begin{cases} pzd(p) - czd(p) - sd(p)(\xi_m - z), & \text{for } z \leq \xi_m, \\ p\xi_m d(p) - czd(p) + vd(p)(z - \xi_m)], & \text{for } z > \xi_m. \end{cases}$$

It can be seen that the effect of z, which is defined in a different way with respect to the additive demand case, is the same as for the additive demand case model (3.5), i.e., if z is larger than ξ_m, leftovers occur; if z is smaller, shortages occur.

For the multiplicative demand case, the following is defined: expected leftovers $d(p)\Lambda(z)$, expected shortages $d(p)\Theta(z)$, riskless profit $\Psi(p)$ and the loss function:

$$L(p, z) = d(p)[(c - v)\Lambda(z) + (p + s - c)\Theta(z)],$$

where $\Lambda(z)$ and $\Theta(z)$ are defined by expressions (3.6) and (3.7); riskless profit is given by (3.3). Expected profit $\Pi(p, z)$ is again expressed by (3.9), where $L(p, z)$ assesses an overage cost $(c-v)$ for each of the $d(p)\Lambda(z)$ expected leftovers when z is chosen too high and an underage cost $(p+s-c)$ for each of the $d(p)\Theta(z)$ expected shortages when z is chosen too low. The expected profit can be expressed from as:

$$\Pi(p,z) = (p-c)zd(p) - (p-v)d(p)\int_A^z F(t)\mathrm{d}t - sd(p)\int_z^B [1-F(t)]\mathrm{d}t. \tag{3.17}$$

The optimal stocking factor z^* can be expressed by similar steps as in the additive case; in addition, z^* is observed identically to that of the additive case, i.e., (3.11). Substituting z^* into (3.17) is p^* obtained by solving $\frac{\mathrm{d}\Pi(p,z(p))}{\mathrm{d}p} = 0$. The related derivative is:

$$\begin{aligned}\frac{\mathrm{d}\Pi(p,z(p))}{\mathrm{d}p} =& z^*d(p) + (p-c)z^*\frac{\mathrm{d}d(p)}{\mathrm{d}p} - (p-v)\frac{\mathrm{d}d(p)}{\mathrm{d}p}\int_A^{z^*} F(t)\mathrm{d}t - d(p)\int_A^{z^*} F(t)\mathrm{d}t \\ &- s\frac{\mathrm{d}d(p)}{\mathrm{d}p}\int_{z^*}^B [1-F(t)]\mathrm{d}t.\end{aligned} \tag{3.18}$$

The second derivative w.r.t. p is:

$$\begin{aligned}\frac{\mathrm{d}^2\Pi(p,z(p))}{\mathrm{d}p^2} =& \frac{\mathrm{d}^2d(p)}{\mathrm{d}p^2}\left[(p+s-c)z^* - (p+s-v)\int_A^{z^*} F(t)\mathrm{d}t + s\int_A^B F(t)\mathrm{d}t - sB\right] \\ &+ 2\frac{\mathrm{d}d(p)}{\mathrm{d}p}\left[z^* - \int_A^{z^*} F(t)\mathrm{d}t\right] + d(p)\frac{1}{f(z^*)}\frac{(c-v)^2}{(p+s-v)^3}.\end{aligned} \tag{3.19}$$

Remark 3.1.6. Considering $\Pi(p,z) = \Psi(p) - L(p,z)$, the necessary optimal condition of the price is as follows:

$$\frac{\mathrm{d}d(p)}{\mathrm{d}p}[p-c-l(p,z)] - d(p)[1-\Theta(z)] = 0. \tag{3.20}$$

This condition is further used to compare the optimal price with the NPPA in Chapter 5.

Isoelastic pricing function

Let the isoelastic demand curve $d(p)$ be defined as

$$d(p) = ap^{-b}, \tag{3.21}$$

where $a > 0, b > 1$.Moreover, it is assumed that $A > 0$ (see [86]). The related partial derivative w.r.t. p is:

$$\frac{\partial\Pi(p,z)}{\partial p} = (b-1)\frac{d(p)}{p}[1-\Theta(z)]\left\{p_\Psi^* + \frac{b}{b-1}\left[\frac{(c-v)\Lambda(z) + s\Theta(z)}{1-\Theta(z)}\right] - p\right\},$$

where p_Ψ^* is the optimal riskless price maximizing the riskless profit $\Psi(p)$: the derivative needed is $\frac{\mathrm{d}\Psi(p)}{\mathrm{d}p} = -(b-1)ap^{-b-1}[p - \frac{bc}{b-1}]$, where $(b-1)ap^{-b-1} > 0$ for $p < \infty$ and so the maximum of function is $\Psi(p)$ is $p_\Psi^* = \frac{bc}{b-1}$. Further, because $1-\Theta(z) \geq A > 0$, for a given z, the unique optimal price p^* as a function of z can be established:

$$p^* \equiv p(z) = p^*_\Psi + \frac{b}{b-1}\left[\frac{(c-v)\Lambda(z) + s\Theta(z)}{1-\Theta(z)}\right]. \tag{3.22}$$

For an exhaustive proof see [86]. From equation (3.22) the boundary condition for p^* can be established:

$$p^* \geq p^*_\Psi,$$

which was found by Karlin and Carr [58] and is opposite the related boundary founded by Mills [75] for the additive case; see expression (3.15).

3.1.6. Some up-to-date results

In order to review some selected up-to-date results, the following definitions are needed (see [124, 129]): the *failure rate function* of the random variable $r(\xi) = \frac{f(\xi)}{(1-F(\xi))}$, the *generalized failure rate function* $g(\xi) = \xi \cdot r(\xi)$ as well as the related property called *generalized strict increasing failure rate (GSIFR)*:

$$g'(\xi) > 0$$

for all ξ. For more details on IPE and GSIFR properties, see [129]. To the author's best knowledge, the GSIFR class of distributions and $d(p)-$functions with IPE property are the major and most up-to-date class in the recent literature.

It can be shown that if the mean demand $d(p)$ has IPE and the distribution F has GSIFR (alternatively IFR for the additive and GIFR for the multiplicative demand case, see [124]), then Π is quasi-concave in p in the range $[c, \overline{p}]$ and thus the first order condition $\frac{\mathrm{d}\Pi(z(p),p)}{\mathrm{d}p} = 0$ has a unique solution (see [124, 129]).

The list of mean demand and random factor distributions used in recent literature references for the additive demand case provided by [129] is also reviewed in Table 3.2. See [129] for a similar table on the multiplicative case.

Table 3.2: Demand functions and distribution function classes for the additive form.

Paper	$d(p)$ function	$F(\xi)$ distribution
Mills [75] (1959)	linear	uniform
Ernst [30] (1970)	linear	PF_2
Zabel [133] (1972)	concave	uniform and exponential
Thowsen [111] (1975)	linear	PF_2
Lau and Lau [66] (1988)	linear	uniform
Polatoglu [88] (1991)	linear	uniform
Ha [46] (2001)	concave	increasing failure rate
Petruzzi and Dada [86] (1999)	linear	increasing failure rate
Zhan and Shen [134] (2005)	linear	nondecreasing failure rate
Yao, Chen and Yan [129] (2006)	IPE	GSIFR

3.1.7. Decision dependent randomness

In this case, the focus is on the fact that ξ depends on p, so it is denoted as $\xi(p)$. As the decision p may influence the probability distribution of ξ, we write about the decision dependent randomness case. Similarly, as in two-stage stochastic programs (see [104]), a separable case is searched for. Therefore, we would like to find a description of demand D by a function separating p, as well as a random variable ξ defined in a new way that does not depend on p (i.e., $D(p,\xi) = g(p,\xi)$ where $g : \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}$). Such models are solved in [76] and [86]. Mostly, the random influence ξ is introduced in a separable additive way. Another previously studied possibility is the multiplicative case discussed in [58]. In [A5], the author introduces the case where the demand-price dependence is linear; however, the coefficients are uniformly distributed and thus the demand is a linear function of price p, i.e., $D(p) = \alpha p + \beta$ where α and β are uniformly distributed dependent random parameters. The uniform distribution is suitable for cases where the bounds of uncertainty are known, otherwise there is a lack of information on the uncertainty. Clearly, this linear dependency does not approximate real situations very well. Some authors use a hyperbolic/isoelastic dependency (see [76]), that can be piece-wise approximated. So, with the regard to the features expanded by Taylor [110], it can be assumed that linear approximation is acceptable.

The expected profit can be expressed as

$$\Pi(p,x) = \mathrm{E}_{\xi}[\pi(p,x,\xi(p))] = (p-c)x - (p-v)\int\limits_{a(p)}^{x}[x-t(p)]\mathrm{d}Ft(p) - s\int\limits_{x}^{b(p)}[t(p)-x]\mathrm{d}Ft(p),$$

or, alternatively, as

$$\Pi(p,x) = \begin{cases} (p-c)x - s(\mathrm{E}[\xi(p)] - x), & x < a(p), \\ (p-c)x - (p-v)\int\limits_{a(p)}^{x}[x-t(p)]\mathrm{d}Ft(p) - s\int\limits_{x}^{b(p)}[t(p)-x]\mathrm{d}Ft(p), & x \in [a(p),b(p)], \\ p\mathrm{E}[\xi(p)] - cx + v(x - \mathrm{E}[\xi(p)]), & x > b(p). \end{cases}$$

Decision dependent uniform distribution

For linearly dependent uniformly distributed random variables α and β (with supports $[\alpha_2,\alpha_1]$ and $[\beta_1,\beta_2]$) such that $\xi(p) \sim U(a(p),b(p))$, it can be obtained:

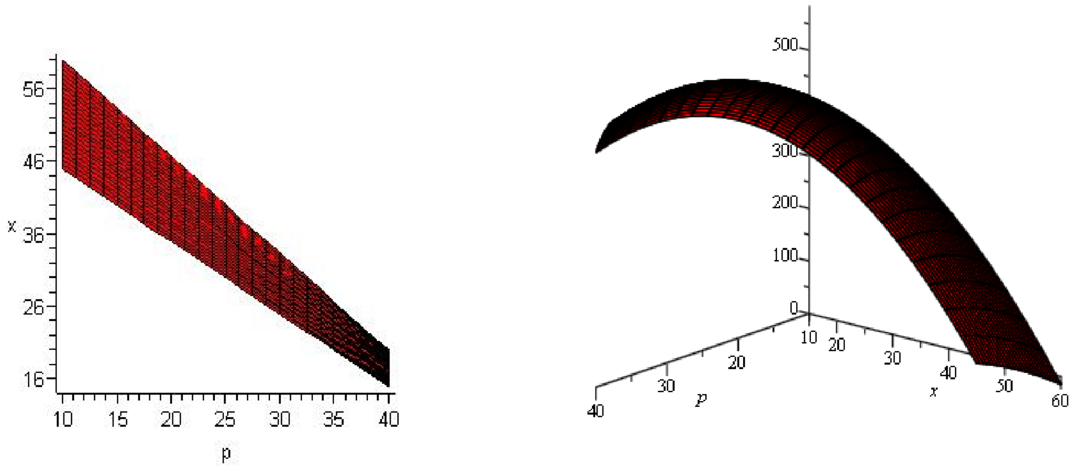
$$\begin{aligned} \Pi(p,x) &= (p-c)x - (p-v)\int_{a(p)}^{x}\frac{x-t(p)}{b(p)-a(p)}dt(p) - s\int_{x}^{b(p)}\frac{t(p)-x}{b(p)-a(p)}dt(p) \\ &= (p-c)x - \frac{(p-v)[x-a(p)]^2}{2[b(p)-a(p)]} - \frac{s[x-b(p)]^2}{2[b(p)-a(p)]}. \end{aligned}$$

Solving the first order condition $\frac{\partial \Pi(p,x)}{\partial x} = 0$, the following expression is obtained for optimal x:

$$x^* = \frac{(c-v)a(p) + (p+s-c)b(p)}{p+s-v}.$$

Substituting x^* into $\Pi(p,x)$ we get $\Pi(x^*,p) = (p-c)x^* - \frac{b(p)-a(p)}{2}[(p-v)\left(\frac{p+s-c}{p+s-v}\right)^2 + s\left(\frac{c-v}{p+s-v}\right)^2]$ which can be expressed as $(p-c)x^* - \frac{b(p)-a(p)}{2}[(p-v)\beta^2 + s\left(\frac{\mathrm{d}\beta}{\mathrm{d}p}\right)^2]$.

Example: Let us take the following values for the model parameters: $p_0 = c = 10$, $p_1 = 40$, $v = 8$, $s = 2$, $a_0 = 45$, $a_1 = 15$, $b_0 = 60$, and $b_1 = 20$; the related $\alpha_1, \alpha_2, \beta_1, \beta_2$ can be derived by common straight line calculations using the facts that $a_0 = a(p_0)$, $a_1 = a(p_1)$, $b_0 = b(p_0)$, and $b_1 = b(p_1)$. Then, $x_{\max} = 27.29$, $p_{\max} = 32.85$, and $\Pi(p_{\max}, x_{\max}) = 564.8$. Therefore, we can see that by considering the price p as a variable, we achieve an improvement in the profit; this is an illustration of the key idea of the introduction of pricing in the model. To illustrate this observation graphically, we further consider the continuous probability distribution analyzed above, $\xi(p) \sim U(a(p), b(p))$, and the aforementioned calculations. See Figures 2 and 3 for a graph of the uniform distribution domain. There is only an "inner part" of the function $\Pi(p, x)$ (that corresponds to f_2 from Figure 2.1) in Figure 3, since the objective function maximum is always achieved here.



(a) Domain (p, x) of the example. *(b) Inner part of the objective function $\Pi(p, x)$.*

Figure 3.1: The domain and the objective function of the example.

3.2. TNDP with pricing

In addition to the network-design decision variables, which represent the inclusion of additional edges (see Section 2.2), pricing variables are included herein. The main idea is as follows: if the network operator has the possibility of decision on price(s) charged to the customers, the demand profile of the customers can adapt and this can lead to overall more sensible network designs. Moreover, since suppliers have imperfect information about the demands of their (potential) customers in real-world problems, a scenario-based approach to the uncertain demand is used (as in subsection 2.2.2).

Thus, the entire Section 3.2 concerns the problem of determining the pricing and production decisions (i.e., transportation and network design) of a single continuously divisible item in TNDP over a single period for a stochastic price-dependent demand. More specifically, in subsection 3.2.1 it is considered that demand is linearly price-dependent, while it is considered that demand is a nonlinear (isoelastic) function of the price in subsection 3.2.2. Remind paper [86] for the linear as well as nonlinear case and [129] for an even more general pricing approach which was mentioned in subsection 3.1.6.

3.2.1. WS reformulation of stochastic TNDP with linear pricing

In this subsection, the demand is a linear function of price. More specifically, the demand is considered to be decreasing, continuous, and defined on a closed interval [86]. Thus, the demand function is defined as (for each scenario s and each customer i):

$$b_{i,s}(p_{i,s}) = \beta_{i,s} - \alpha_{i,s} p_{i,s},$$

where $\alpha_{i,s}$ and $\beta_{i,s}$ capture/reformulate the uncertainty (or uncertain parameters) in the linear demand function $b_{i,s}$ for a concrete customer i and a scenario s. The scenario-based approach assumes to have enough observations of the parameters (for each customer, one observation presents one particular scenario, i.e., one particular market situation). Then, the selling price $p_{i,s}$ is the decision variable (as it is described below). For concrete examples see results and figures at the end of this subsection (e.g., figures 3.3a and 3.3b and Table 3.3, respectively).

Then, the following WS (stochastic) mixed-integer bilinear program is defined. Remind, that the model is solved for each considered scenario:

$$\begin{array}{llrcll}
\forall s \in S: & & & & & \\
\max & \multicolumn{5}{l}{\sum_{i \in I} \sum_{e \in E} A_{i,e} x_{e,s} p_{i,s} - \sum_{e \in E} c_e x_{e,s} - \sum_{E_n \in E_n} d_{\mathrm{E}_n} \delta_{E_n,s} - \sum_{i \in I} (r_i^- y_{i,s}^- + r_i^+ y_{i,s}^+)} \\
\text{s.t.} & & \sum_{e \in E} A_{i,e} x_{e,s} & = & b_{i,s} - y_{i,s}^+ + y_{i,s}^-, & \forall i \in I, \\
& & \sum_{e \in E} A_{j,e} x_{e,s} & = & b_j, & \forall j \in J, \\
& & \sum_{e \in E} A_{k,e} x_{e,s} & = & b_k, & \forall k \in K, \\
& & x_{E_n} & \leq & \delta_{E_n,s} \sum_{j \in J} (-b_j), & \forall E_n \in E_n, \\
& & y_{i,s}^+ & \leq & b_{i,s}, & \forall i \in I, \\
& & x_{e,s} & \geq & 0, & \forall e \in E, \\
& & \delta_{E_n,s} & \in & \{0,1\}, & \forall E_n \in E_n, \\
& & y_{i,s}^+,\ y_{i,s}^- & \geq & 0, & \forall i \in I, \\
& & p_{i,s} & \geq & p_i^{min}, & \forall i \in I, \\
& & p_{i,s} & \leq & p_i^{max}, & \forall i \in I, \\
& & b_{i,s} & = & \beta_{i,s} - \alpha_{i,s} p_{i,s}, & \forall i \in I,
\end{array} \tag{3.23}$$

where the following notation is different comparing to the model (2.14):

• the decision variables are:

$p_{i,s}$: the unit selling price of the product at a node i in scenario s,

• and parameters:

$\alpha_{i,s}$: the slope of the linear demand function $b_{i,s}$ for customer i in scenario s,
$\beta_{i,s}$: the intercept of the linear demand function $b_{i,s}$ for customer i in scenario s,
p_i^{min} : a price lower bound for customer i,
p_i^{max} : a price upper bound for customer i.

Note, that $b_{i,s}$ are the the linear demand functions capturing the decision-dependent stochastic (scenario-based) demand parameter.

Hybrid algorithm for the WS deterministic reformulation

The aforementioned model (3.23) was programmed in GAMS and solved through the use of CPLEX and XA solvers for small test instances obtaining acceptable results. The solution used attempted to solve larger test problems in the same way and has led to increasing computational time needs. It is evident that it will be necessary to use a heuristic-based algorithm for the nonlinear pricing network design problem that follows as a logical step of our model development (see subsection 3.2.2). Thus, the authors of the paper [A7] have decided to utilize previous experience (see, e.g., [A6, A13, 98]) and modified previously-studied and developed hybrid algorithm that combines the GAMS code with a chosen genetic algorithm (GA). The C++ implementation focusing on GAMS-GA interface features is set up for the modified GA that was discussed in [96]. The use of the algorithm was initially considered for TNDP problems in [A13]. The scheme of the algorithm as well as its implementation was already presented in previous papers (e.g., [A6]).

The GA used was presented in the previous work ([A6, A13]). See, e.g., [A6] for a short review of the key ideas of the utilized GA that functions as the main part of the hybrid algorithm or see a detailed description in the next subsection, i.e. subsection 3.2.2. It follows the previous ideas of one of the authors ([96, 97]).

The main idea of the hybrid algorithm is based on the solution of a stochastic program for various sequences of fixed $0-1$ variables repeatedly for each scenario. The initial idea of the algorithm from [A13] was modified in [A6] and is used for this problem, again. The optimal objective function values are obtained together with these sequences of zeros and ones. They serve as the input fitness value plus elements of the populations for GA instances that utilize their own steps (selection, crossover, mutation, and further modifications as limited lifetime and sexual recombinations, see [96]) that are hidden within the GA structure. Updated sequences of zeros and ones are generated by the GA and sent to the GAMS through the updated `$INCLUDE` file, and the computational loop continues until the moment when the satisfactory improvement of the network design is obtained. The algorithmic details follow in subsection 3.2.2; see also Appendix B.1 for selected parts of GAMS code used.

Computations and results

Computational results of the model (3.23) are presented here; for comparison purposes, the test examples from previous work are utilized (see Figure 2.2). The main emphasis is placed on the network design and on the effect of pricing.

Results of the computations: The model (3.23) was solved for 100 scenarios. Results of the computations are described in Figure 3.2, where the line thickness represents frequencies (i.e., for how many scenarios the edge should be built), and hence, probabilities that variables $\delta_{e,s}$ are equal to 1 (or $x_{e,s}$ are non-zero for related edges). The fixed lines are drawn as dashed lines to emphasize the role of the edges generated by the WS computations. It can also be seen that the stochastic demand usually requires new edges to bring about the necessary recourse in the results. In comparison with the HN solutions (cf. [A13]), this can be done in a more flexible and cheaper way. See also Table 3.3, which provides some numeral results of the frequencies for 100 scenarios.

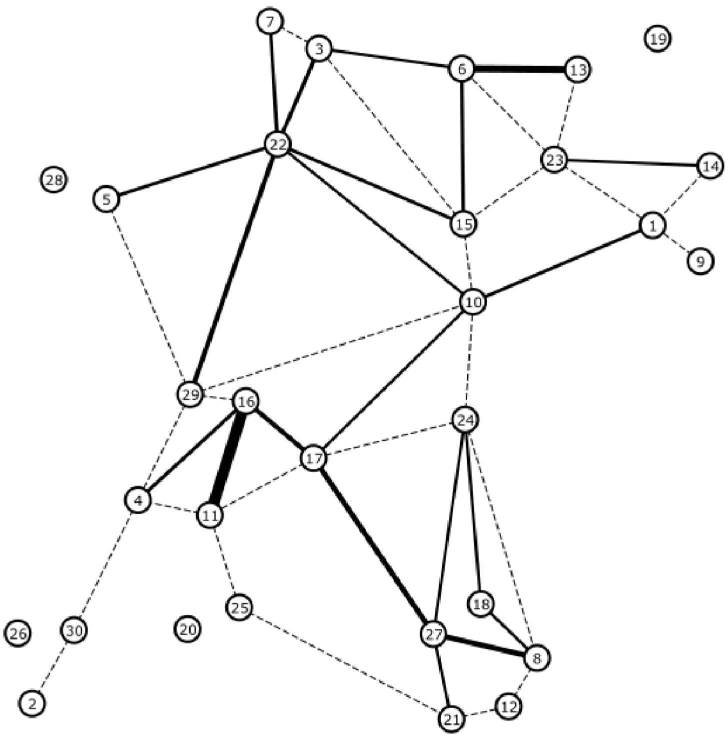
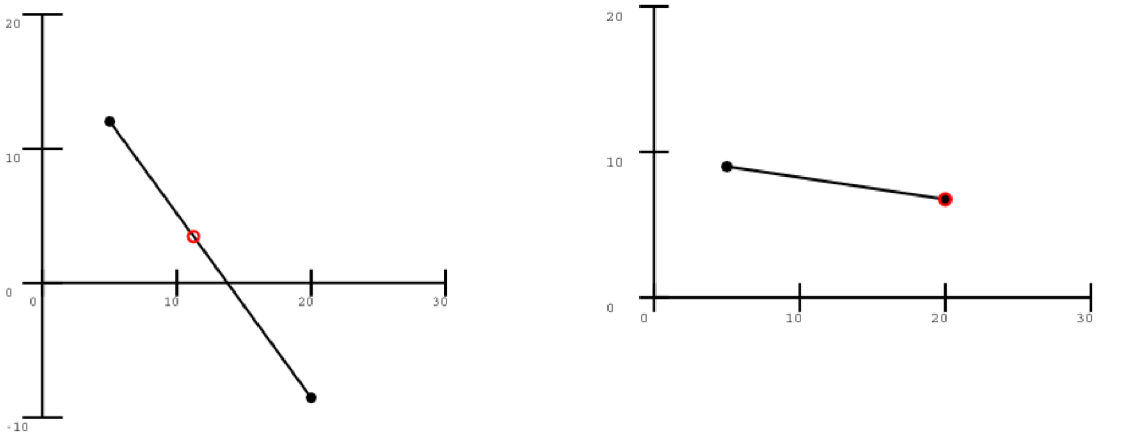


Figure 3.2: Summary results graph of the computations: network design variables for 100 *scenarios; line thickness represents the edge usage frequency.*

Effect of pricing: Some illustrative insights for parameters of the linear function are provided in Table 3.3 and in figures 3.3a and 3.3b. It can be seen that for some of the customers, the price decisions $p_{i,s}$ are determined such that the demand $b_{i,s}$ is equal to 0 (e.g., for $i = 2$). This means that the pricing strategy allows us to reduce the number of supplied customers by the price decision. On the other hand, some of the price decisions are on the bound p_i^{min} or p_i^{max} (e.g., for $i = 6$; see also Figure 3.3b).

Comparing the obtained results with the results published in [A6], it can be seen that the total number of the designed edges (the 0-1 variables) as well as its variability have decreased due to the pricing strategy used. Moreover, pricing usually leads to a significant improvement in the objective function value (it should never obtain the worst solution). Even if the linear pricing does not capture reality very well, it helps us to understand how the pricing technique works, e.g., the obtained results provided in Table 3.3 or Figure 3.3 serve as suitable illustrative tools.



(a) Linear function example ($i = 1$). *(b) Linear function example ($i = 6$).*

Figure 3.3: Linear pricing functions for a fixed scenario ($s = 2$) with highlighted optimal $p^*_{i,s}$; used bounds: $p_i^{min} = 5$, $p_i^{max} = 20$.

Table 3.3: Parameters α and β (for $s = 2$) and computational results (for 100 scenarios); remind that nodes 15 and 16 are production places.

Node	Demand and price				Shortages/Leftovers		Edge	Frequency
i	$\alpha_{i,2}$	$\beta_{i,2}$	$p^*_{i,2}$	$b_{i,2}$	$y^+_{i,2}$	$y^-_{i,2}$	E_n	$\sum_{s=1}^{100} \delta_{E_n,s}$
1	1.370	19.848	11.246	4.441	0	0	16-11	83
2	1.798	19.989	11.119	0	0	0	13-6	42
⋮							17-27	25
6	0.150	10.749	20	7.754	0	0	27-8	25
⋮							⋮	
12	0.295	12.474	20	6.579	6.579	0		
⋮								
15				-63	0	19.090	23-14	1
16				-110	0	96.299	6-3	1

3.2.2. WS reformulation of stochastic TNDP with nonlinear pricing

This subsection presents a scenario-based WS stochastic MINLP, which models the design of a transportation network under (non-linearly) price-sensitive stochastic demand. Due to the growing popularity of the development of pricing strategies and their further applications in industry, the author follows up on previous modeling ideas presented in subsection 3.2.1 (or [A7]), where a MILP with linearly price-dependent stochastic demand was modeled. Thus, this case extends the previous model into a more complex case with a nonlinear (isoelastic) price-demand dependency and, therefore, the authors also modified the previously used hybrid algorithm ([A7, A13]).

Before the stochastic problem and its WS reformulation are presented, the isoelastic pricing function is described.

Isoelastic pricing

Consider a price-setting firm that faces a price-dependent demand function, $b_{i,s}(p_{i,s})$, describing the dependency between price $p_{i,s}$ and demand $b_{i,s}$ for each customer denoted by i and for each possible scenario s. For most goods the elasticity (the responsiveness of quantity demanded to price) is negative, so it can be convenient to write the constant elasticity demand function with a negative sign on the exponent, in order for the coefficient to take on a positive value: $b_{i,s}(p_{i,s}) = \alpha_{i,s} p_{i,s}^{-\beta_{i,s}}$; such isoelastic function should capture (the most common) real-world situations.

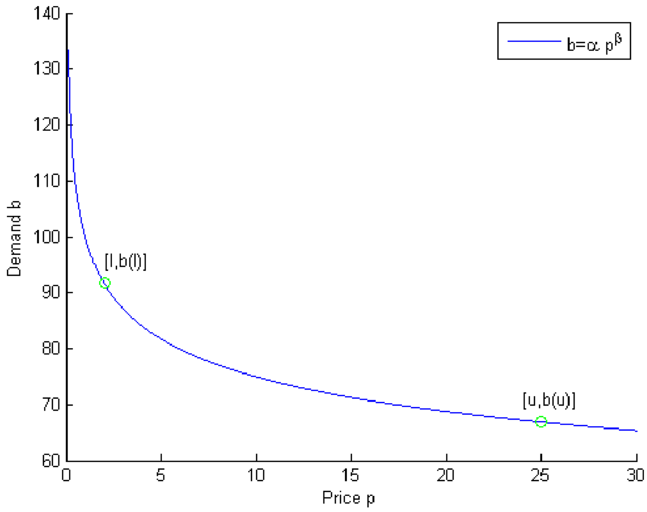


Figure 3.4: Example of an isoelastic demand-price function.

WS stochastic TNDP model with isoelastic pricing function

In order to develop the mathematical model, the following notation must be changed comparing to that used in the model (3.23):

- the (scenario-based) parameters:

$\alpha_{i,s}$: a constant presenting effectiveness of the pricing function $b_{i,s}$ for customer i in scenario s, $\alpha_{i,s} > 0$,

$\beta_{i,s}$: the elasticity (the responsiveness of quantity demanded to price or the magnitude of the responsiveness) of the demand function $b_{i,s}(p_{i,s})$ for customer i in scenario s, $\beta_{i,s} > 1$.

Thus, the stochastic (scenario-based) TNDP with isoelastic pricing is formulated using the WS approach; remind that the model is solved repeatedly, i.e., once for each scenario:

$\forall s \in S:$

$$
\begin{array}{llcll}
\max & \sum\limits_{i\in I}\sum\limits_{e\in E} A_{i,e}x_{e,s}p_{i,s} - \sum\limits_{e\in E} c_e x_{e,s} - \sum\limits_{E_n\in E_n} d_{E_n}\delta_{E_{n,s}} - \sum\limits_{i\in I}(r_i^- y_{i,s}^- + r_i^+ y_{i,s}^+) & & & \\
\text{s.t.} & \sum\limits_{e\in E} A_{i,e}x_{e,s} & = & b_{i,s} - y_{i,s}^+ + y_{i,s}^-, & \forall i \in I, \\
& \sum\limits_{e\in E} A_{j,e}x_{e,s} & = & b_{j,s}, & \forall j \in J, \\
& \sum\limits_{e\in E} A_{k,e}x_{e,s} & = & b_{k,s}, & \forall k \in K, \\
& x_{E_{n,s}} & \leq & \delta_{E_{n,s}} \sum\limits_{j\in J}(-b_j), & \forall E_n \in E_n, \\
& y_{i,s}^+ & \leq & b_{i,s}, & \forall i \in I, \\
& x_{e,s} & \geq & 0, & \forall e \in E, \\
& \delta_{E_{n,s}} & \in & \{0,1\}, & \forall E_n \in E_n, \\
& y_{i,s}^+,\ y_{i,s}^- & \geq & 0, & \forall i \in I, \\
& p_{i,s} & \geq & p_i^{min}, & \forall i \in I, \\
& p_{i,s} & \leq & p_i^{max}, & \forall i \in I, \\
& b_{i,s} & = & \alpha_{i,s}p_{i,s}^{-\beta_{i,s}}, & \forall i \in I.
\end{array}
\tag{3.24}
$$

Obviously, the problem (3.24) is mixed-integer nonlinear, but it seems that the exact solvers deal with a linearized (MILP) version of the problem. Such nonlinear problems often require a heuristic approach, especially large-scale problems. Therefore, a hybrid algorithm is further proposed in the rest of this subsection.

Hybrid algorithm for the WS approach

The above-mentioned model was coded in GAMS and solved by the BARON, MINOS, and CPLEX solvers for suitable test instances. The obtained results are considered acceptable. The next solution attempt targeted large test problems using the same techniques; however, this led to an increase in the computational time required [A3].

Due to the above, the decision to utilize previous experience was made; see [A13, 51]. This resulted in the implementation of a modified hybrid algorithm combining the GAMS code with a selected genetic algorithm (GA). The C++ implementation concentrating on the GAMS-GA interface is developed for the updated GA, as was discussed in [96]. This can also be replaced by other GAs [73]. The principles of the following algorithmic scheme follow the papers [A13] and [51]:

1. Initialize the computer environment for parallel computations.

2. Define the scenario-based GAMS model and load the model and data into *.gms files for each scenario. Specify the control parameters for the GA so that one instance is created for each scenario. The parameters can be defined either by the user (e.g., the population size) or inherited from the GAMS code (e.g., how many edges in the network should be taken into account).

3. Build an initial population for each GA instance. Specifically, the initial values of 0-1 variables must be generated and copied in the `$INCLUDE` files, from which they are read by the GAMS code.

4. The GAMS model is repeatedly solved (in parallel, two loops, one for scenarios and one by population size) by using the MINOS solver. Each run solves the program for the fixed values of 0-1 variables. The profit (or, alternatively, cost) function values are computed (initially in 3. and then in 8.).

5. The best results obtained from GAMS in 4. are saved for comparisons.

6. The termination conditions for the algorithm are tested (in parallel) and the algorithm is terminated if they are met. Otherwise, the algorithm proceeds until the last scenario solution is obtained.

7. Input values for the GA from GAMS results are generated, see step 4. Specifically, the profit function values for each member of the population of the GA are received from the results of the GAMS runs in 4.

8. The GA run leads to an update of the set of 0-1 variables (population), see [96] for details.

Broadly speaking, the GA works with 0-1 variable $\delta_{E_{n,s}}$ for each scenario s, while MINOS solves the remaining nonlinear problem (NLP) for fixed binary variables $\delta_{i,s}$, i.e., MINOS computes an optimal $x_{e,s}$, $p_{i,s}$ as well as value of the objective function. Afterward, the value of the objective/fitness function from model (3.24) is sent back for the solution assessment and then, according to 6., the algorithm continues.

Description of the utilized genetic algorithm

This section shortly reviews the key ideas of the utilized GA that functions as the main part of the hybrid algorithm; see Section 3.2.2. It follows the previous ideas of one of the authors [96]; see also [A13] for its extension.

In general, a set of genetic operators is considered that contains: the crossover operator, the mutation operator, and eventually other problem-dependent or implementation-dependent operators. All of these operators generate descendants from parents. The parent selection operator and the genetic operators have a probabilistic character and the deletion operator is usually deterministic. The fitness value f is a non-negative number, which captures a relative measure of the quality of every individual in the current population. The run of GA used can be described using the following steps: (1) Generation of the initial population (random generation is often used) composed of individuals. (2) Computation of fitness function values related to 1). (3) Parent selection and generation of offspring. (4) Creation of the new population by using deletion operator and addition of offspring generated in the previous step. (5) Mutation. (6) If the stopping rule is not satisfied, go to step 3), otherwise continue to 7). (7) The result is the best individual in the population. It is usually advantageous to use some redundancy in genes, and then the physical length of the genes can be greater than one bit. Such a type of redundancy by shades was introduced by Ryan [100]. To prevent degeneration and deadlock in a local extreme, a limited lifetime of individuals can be used. This limited lifetime is implemented via a death operator [96], which represents something like a continual restart of the GA. Many GAs are implemented on a population consisting of haploid individuals (each individual contains one chromosome). However, in nature, many living organisms have more than one chromosome and there are mechanisms used to determine dominant genes. Sexual recombination generates an endless variety of genotype combinations that increases the evolutionary potential of the population. Since it increases the variation among the offspring produced by an individual, this improves the probability that some of them will be successful in varying and often unpredictable environments. The modeling of sexual reproduction is quite simple. The population is divided into two parts: males and females. One parent from each part is selected for crossover. The sex of the individual is stored in the special gene; this gene is not mutated. The sex of the descendant is determined by a crossover of the sexual genes of the parents, with the descendant placed into the corresponding part of population. The replacement scheme is associated with another problem. To ensure monotonous behavior, incremental replacement (steady-state replacement) was introduced. Least-fit member replacement can be used where one (or more) elements with the worst fitness is replaced, or randomly chosen element(s) can be replaced. Therefore, elitism presents a way to keep monotony while generational replacement is used. One or several best individuals represent the elite. The entire elite is directly taken into the next iteration.

So, the GA used herein for problem-related computations uses ranking selection, haploid chromosomes, shadows, and limited lifetime, as described above. Uniform crossover was used and the probability of mutation of every gene was 5%. Every 0-1 variable was stored in one gene, having a length of 3 bits. This redundant coding uses the shades technique mentioned above. The population size was 20 individuals; such a low value was chosen in relation to the computational complexity of the evaluation of fitness. The maximum number of iterations was limited to 50. The maximum lifetime of an individual was set to 5 iterations.

Computations and results

The main idea of the hybrid algorithm is based on the solution of a stochastic program for various sequences of the fixed $0-1$ variables repeatedly for each scenario. This extends the idea of [A13] with modifications of the hybrid algorithm in Section 3.2.2. Thus, the optimal objective function values are obtained together with these sequences of zeros and ones. They serve as the input fitness value plus elements of the populations for the GA instances that utilize their own above-mentioned steps that are hidden within the GA structure. Updated sequences of zeros and ones are generated by the GA and sent to the GAMS through the updated `$INCLUDE` file and the computational loop continues until a satisfactory improvement of the network design is obtained. For the purpose of future comparison, test examples from [A7] were utilized. The comparison between MINOS and the proposed hybrid solution will be the subject of a future research, but it was already shown through other MINLP problems that the usage of exact solvers is not applicable in real (large) problems due to the huge computational timeframe involved [A7]. Therefore, the use of the hybrid approach has one more reason in the MINLP's.

Figure 2.2 represents an initial visualization of an example. The results are described in Figure 3.5, where the thickness of the lines represents the frequencies of usage in m scenarios, and hence, the probabilities that variables x_e related to the edges are non-zeros. The fixed lines are drawn as dashed lines to emphasize the role of the edges generated by the WS computations. It may also be seen that the stochastic demand usually requires new edges to bring about the necessary adaptation in the results. In comparison with the HN solutions (cf. [A13]), it can be done in a more flexible and cheaper way. Figure 3.5 also shows that only suboptimality has been reached by computations for some scenarios, as extra unnecessary edges are switched on by the GA runs (e.g., 5-28).

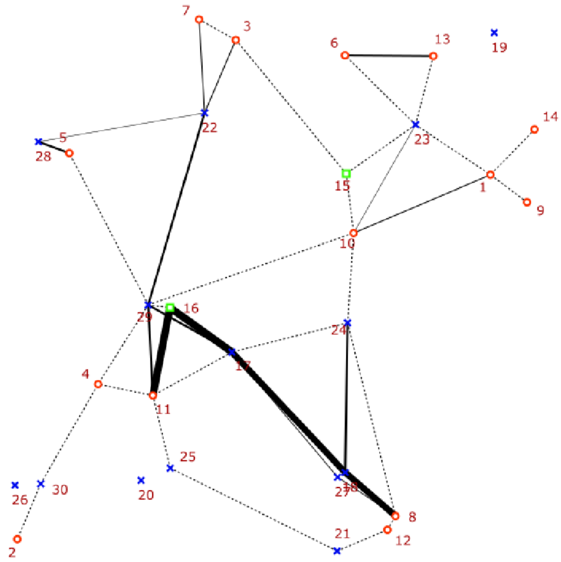


Figure 3.5: Visualization of results for the hybrid algorithm for 100 *scenarios.*

To compare the obtained results, due to the extreme time requirements of finding a traditional GAMS MINLP solution, one scenario case was utilized; visualization of the result is provided in Figure 3.6. However, author leaves further comparison of time requirements as well as the values of objective functions for further research.

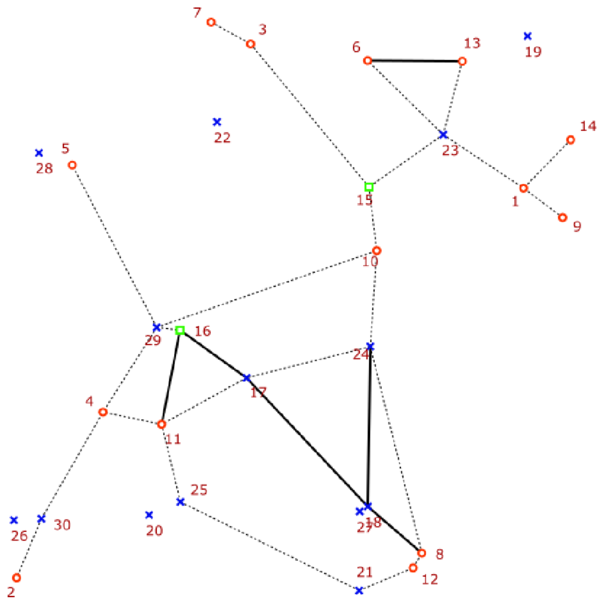


Figure 3.6: Visualization of results from GAMS for 1 *scenario.*

Discussion

The entire Section 3.2 presents a stochastic programming approach to the TNDP with stochastic price-dependent demands while subsection 3.2.2 deals with the isoelastic form of the price-demand function. The proposed mixed-integer nonlinear model is solved with the original hybrid algorithm involving GA for the solution of the WS network design problem. The previously introduced hybrid algorithm (see [A7, A13]) has been modified and successfully tested. This reconfirms authors' conclusions in [A13] about the portability of the approach to other problems.

In author's further research, it is planned to compare (or improve) the proposed hybrid algorithm with similar ideas dealing with differential evolution, specifically with multi-chaotic success-history based parameter adaptation for differential evolution [A4], which is a novel version of the standard GA that, hopefully, may achieve better computational results for the MINLP problems. Moreover, some obvious suboptimalities (see, e.g., Figure 3.5) produced by the GA can easily be eliminated by appending a local search procedure to the GA run.

Similar mixed integer (nonlinear) stochastic programs may appear in many application areas, including NDP [89], traffic networks [51] or waste management problems [A4, 136]. Therefore, the suggested hybrid algorithm can be modified and widely applied.

Chapter 4

Newsvendor Problem with Advertising

Coordinating marketing and production decisions still ranks as one of the most challenging practical and theoretical problems of operations management. Indeed, a number of producers have used innovative marketing strategies to gain effective control of their inventories. This chapter discusses the problem of simultaneously determining the quantity necessary to order and the advertising expenditure required for a product for which demand is random.

Again, the problem is approached by formulating a stochastic single-period problem, which is referred to as the NP.

A typical NP reveals that the quantity ordered maximizes the expected profit. In the setting given herein, the newsvendor is faced with advertising-sensitive stochastic demand where the demand-related random element depends on advertising decisions. It is assumed, that a suitable advertising strategy can lead to increases in sales. Note that the assumption of a fixed price corresponds to an instance of the buyers effectively representing mere price-takers.

Although, NP has been studied for decades, it still serves as a suitable tool to illustrate many new marketing situations ([20, 54, 105, 123]). In this chapter, it is used notation that is frequently utilized for the newsvendor problem with pricing (NPP, see section 3.1) in order to show how marketing aspects interact with production decisions in the newsvendor problem with advertising (NPA). Moreover, inspired by [86], who presented findings on NPP for the linear price-demand function in an additive demand model, as well as the hyperbolic price-demand function for a multiplicative demand case (higher prices cause a decrease in demand), suitable (and more complex) functions related to various advertising situations for additive and multiplicative cases (an increase in advertising expenditure brings about higher sales) are presented. Using suitable notations and procedures, it is the aim to further the understanding of the matter and to present new results.

Note, that this chapter is mostly based on material that was published in [A2].

4.1. Literature review

The various sources of literature pertaining to the subject are detailed below (in addition to the above mentioned topics, i.e. NP and NPP). Prior to introducing the model as one comprising a combination of conceptual notions, a review is given on the primary resources utilized.

4.1.1. Advertising

Advertising policy is a crucial aspect of marketing. The key questions are the following: What is the effect for the retailer? How much should be spent on advertising? How often? What marketing channel should be used? The authors of [11] and [55] published two possible approaches to expenditure frequency - a constant advertising stream (extremely expensive) or pulses of advertising conducted at irregular intervals (the preferred option); see also [63]. A major issue is that concerning the actual effectiveness of advertising. Advertising response functions are typically assumed to adhere to the following: (1) a concave-downward function (usually an advertising response function without a threshold); (2) an S-shaped logistic function (an advertising response function with a threshold); for reference purposes see [5, 7, 55, 68, 107]. In general, the responsiveness of sales to advertising may start to decline beyond a level of spending on advertising ([8]). This phenomenon of diminishing returns is better represented by a downward-sloping curve on a graph in which the advertising levels relate to the sales results. Alternatively, some modelers prefer to assume that the relationship is actually S-shaped. Initially, when advertising budgets are low, sales exhibit no response to advertising at all. It is supposed that it takes some time for advertising to take effect as repeated exposure is required. Afterward, a point of increasing returns is discerned when sales really begin to respond to further advertising once the advertising budget exceeds a minimum, critical-level threshold. Eventually, the curve begins to slope downwards again, when the phase of diminishing returns reappears; see also [44] for further analysis of the S-shaped function. Publications by [64, 80, 87] provide a comprehensive overview on such themes and proceed to detail recent principles of marketing.

4.1.2. The newsvendor problem with advertising

A single period inventory problem with advertising was given by [38]. Furthermore, the interface between decision-making on marketing and manufacturing has been studied by many researchers, examples being [13, 28, 35, 48, 69, 72], to whom reference is made in this paper.

The effect exhibited by advertising on sales represents an important aspect of demand-based problems. When a retailer/newsvendor faces demands of the stochastic advertising-sensitive type, he is forced to make decisions concerning advertising and inventory prior to the demand being met [116]. [63] and [116] assume that the mean demand is strictly increasing and strictly concave in advertising expenditure. [63] provides an extensive review of related literature and the advertising problem, attempting to solve the problem for uniform, exponential and normal demand distributions. [118] investigated circumstances under which advertising leads to increased sales under additive demand. Later,

[117] extended the problem to cover the situation pertaining to a perishable product subject to being ordered under emergency conditions, first for a multiplicative model, then going on to investigate the general demand function in [116]. Finally, [119] applied the results to coordinating a supply chain with advertising, as well as setting a price for the manufacturer.

4.2. Problem formulation and demand function

In the classical NP (see Section 2.1) or in the NPP (Section 3.1), the newsvendor's marketing effort, which can be used to enhance the demand, is not taken into consideration at all [20]. Therefore, the following situation is assumed: First, the retailer has to decide about an amount a to advertise for a product to be sold and simultaneously has to buy and stock x units of the product for a unit cost c. Then, the selling period begins. If demand D is greater than x, all stocked units are sold for revenue px, where p is a unit price, $p > c$. In this case, a loss given by a unit shortage penalty cost s for all shortages, $D - x$, is considered. Otherwise, if demand D is less or equal to x, the revenue is only pD and leftovers, $x - D$, are salvaged through a unit salvage value v, $v < c$. Then, the objective (profit) function is denoted by $\pi(a, x, \xi_m, \xi_a)$ being defined as follows:

$$\pi(a, x, \xi_a, \xi_m) = \begin{cases} px - cx - s[D(a, \xi_a, \xi_m) - x] - a, & x < D, \\ pD(a, \xi_a, \xi_m) - cx + v[x - D(a, \xi_a, \xi_m)] - a, & x \geq D. \end{cases} \tag{4.1}$$

The decision variables are the order quantity denoted by x and the amount spent on the advertising denoted by a, while the demand $D(a, \xi_a, \xi_m)$, which depends on the advertising expenditure a and is affected by the random elements ξ_a, ξ_m, is not completely known when the decisions are made.

One of the keys to understanding the marketing problems lies in the relation between demand and advertising response function. Therefore, the next section focuses on the demand function together with the definition of its related uncertainty. Then, the basic concepts of the advertising response function are presented.

4.2.1. Demand function and randomness

Also this modification of the NP deal with problems where the decision-maker does not know the real demand. Therefore, the demand is further modeled using a (response) function, which can be affected by the advertising expenditure and which somehow depends on a random element (similarly as in Chapter 3.1).

Inspired by many papers on the NPP ([58, 75, 86, 129]) and some on the NPA ([62, 116, 118]), it is further assumed that advertising-related randomness is independent of the demand, which helps to avoid complexities.

Let the demand function be denoted as $D(a, \xi_a, \xi_m)$ now and let it satisfy

$$D(a, \xi_a, \xi_m) = d(a)\xi_m + \xi_a, \tag{4.2}$$

where ξ_a, ξ_m are independent continuous random variables. This chapter further deals with two special cases of demand function $D(a, \xi_a, \xi_m)$: a) the multiplicative demand case (section 4.3) and b) the additive demand case (section 4.4). In order to define the multiplicative demand case, let $\mathrm{P}(\xi_a = 0) = 1$ and let the random variable ξ_m be defined on the domain $[A_m, B_m]$ and satisfy $\mathrm{E}[\xi_m] = 1$. In the additive demand case, let $P(\xi_m = 1) = 1$ and let the random variable ξ_a be defined on the domain $[A_a, B_a]$ and satisfy $\mathrm{E}[\xi_a] = 0$. Then, for both cases, the expectation of D is specified as: $\mathrm{E}[D(a, \xi_a, \xi_m)] = d(a)$.

In comparison with the NPP references where the additive form of the randomness is commonly used for the linear demand function while the multiplicative form for the hyperbolic demand function, this chapter examines the effects of the additive as well as multiplicative form on the optimal advertising strategy for the concave and the S-shaped functions, which, on other hand, corresponds to some ideas in the NPP study [129].

Before the multiplicative demand form is modeled and examined in section 4.3 and the additive demand form in section 4.4, assumptions and properties of the advertising response function $d(a)$ are introduced.

4.2.2. Advertising response function

The response function describes the sales effect of additional amounts of advertising, even though it sometimes illustrates the amount of advertising needed to trigger buying [55]. Two (general) functions that are often used are further assumed: a) the concave response function, which is presented in subsection 4.5 ([55, 63]); b) the S-shaped response function, which is presented in subsection 4.5 [55].

Although the S-shaped function is very important from the marketing literature perspective, it has not yet been considered by researchers dealing with operational research in the discussed context. According to the marketing literature trying to approximate the advertising situations, the S-shaped function is defined as a bounded real function defined for all nonnegative input values with a positive derivative at each point, which is first convex and then concave. It means, in the beginning, when advertising budgets are low, sales do not respond significantly to advertising. It supposedly takes time for advertising wear-in. It can be seen the point of increasing returns, as sales really begin to respond to increased advertising, as the advertising budget exceeds some minimum critical-level threshold. Finally, the curve begins to slope downward again, as once again the diminishing-returns phase appears, see [5, 7].

Let the response function $d(a)$ be continuous, nonnegative, twice-differentiable and increasing on its domain $[0, a_{max}]$ in the advertising expenditure a [62, 68]. Moreover, since $d(0) > 0$ holds, $d(a)$ is positive.

To capture a real situation/dependency between the advertising expenditure and demand, three particular functions are presented in section 4.5, which are provided for their illustrative-suitable behavior.

4.3. Multiplicative demand model

Let the demand function $D(a, \xi_a, \xi_m)$ be defined in the multiplicative form (see subsection 4.2.1) and let $F(\cdot)$ denote a cdf and $f(\cdot)$ be a pdf of ξ_m. In order to assure that demand is positive, it is required that $A_m > 0$. Then, the demand is in the multiplicative form

$$D_M(a, \xi_m) = d(a)\xi_m, \tag{4.3}$$

see [58] for similar ideas in the NPP. The objective function (4.1) can be rewritten by substituting (4.3) and utilizing the 'stocking factor' defined as

$$z = \frac{x}{d(a)}, \tag{4.4}$$

where $z \geq 0$ (note that if $x = 0$ and since $d(a) > 0$, then $z = 0$). A similar transformation of the objective (variable transformation, respectively) has already been used to simplify the calculations in the NPP [86, 134]. It provides an alternative interpretation of the stocking decision: if the choice of z is greater than the realized value of random variable ξ_m, then leftovers occur, otherwise shortages occur [86]. An important managerial interpretation for z demonstrates that, although z is defined differently for each of the two mentioned demand cases (see section 4.4 for the additive case), its meaning is consistent for both: z represents a stocking factor that [86] defined as a surrogate for safety factor by [106].

Then, the NPA is as follows:

$$\pi(a, z, \xi_m) = \begin{cases} pzd(a) - czd(a) - sd(a)[\xi_m - z] - a, & \text{for } z < \xi_m, \\ p\xi_m d(a) - czd(a) + vd(a)[z - \xi_m] - a, & \text{for } z \geq \xi_m. \end{cases} \tag{4.5}$$

For better understanding, notation for both multiplicative and additive cases is unified; the same symbols π are also used for both objective functions involving either x or z variables.

The objective is to maximize the expected profit by choosing a and z. However, the optimal solution is not necessarily an interior solution, in particular, the value of z can be on the boundary, A_m or B_m [134].

The expected profit $\Pi(a, z)$ can be expressed as:

$$\begin{aligned} \Pi(a, z) = \mathrm{E}[\pi(a, z, \xi_m)] = d(a) \int_{A_m}^{z} [pt + v(z - t)]\, F(t)dt \\ + d(a) \int_{z}^{B_m} [pz - s(t - z)]\, F(t)dt - czd(a) - a \end{aligned} \tag{4.6}$$

Defining the *riskless profit* [75, 86], which would occur in the absence of uncertainty, as

$$\Psi(a) = (p - c)d(a) - a, \tag{4.7}$$

and the so-called *expected loss per unit* as

$$l(z) = (c - v)\Lambda(z) + (p + s - c)\Theta(z), \tag{4.8}$$

where $d(a)\Lambda(z)$ denotes *expected leftovers* and $d(a)\Theta(z)$ *expected shortages*, the expected profit given by (4.6) can be rewritten as

$$\Pi(a,z) = \Psi(a) - L(a,z) = d(a)[p - c - l(z)] - a. \tag{4.9}$$

Note that $L(a,z) = d(a)l(z)$ is the *expected loss* that occurs as a result of the presence of uncertainty [86, 106] and $p - c - l(z)$ denotes the so-called *per-unit expected benefit*, i.e., margin minus expected loss. If z is chosen too high, in (4.9) or in (4.8), respectively, an overage cost $(c - v)$ appraises each of the $d(a)\Lambda(z)$ expected leftovers, and, if z is chosen too low, an underage cost $(p + s - c)$ appraises each of the $d(a)\Theta(z)$ expected shortages. The equivalence of expressions (4.6) and (4.9) can be obtained by a sequence of straightforward substitutions; see Appendix A.2.

Remark 4.3.1. The transformation from x to z presents an advantage of determining optimal values a^* and x^* that maximize the expected profit, using the following steps:

1. An optimal stocking factor z^* is first determined using input parameters (subsection 4.3.1). Because of the form of (4.9) and (4.6), z^* can be obtained independently on optimal value a^*.

2. Using a suitable function $d(a)$, an optimal advertising a^* can be expressed and an optimal order quantity x^* is determined such that $x^* = z^* d(a^*)$ (subsection 4.3.2).

4.3.1. Optimal stocking quantity

To maximize $\Pi(a,z)$ over two variables, two steps described in Remark 4.3.1 are followed. Solving the first order condition (with respect to z), an expression for optimal z is expressed as:

$$F(z^*) = \frac{p + s - c}{p + s - v}.$$

It is comparatively easy to show that $\Pi(a,z)$ is concave in z on $[0,\infty)$: $\frac{\partial^2 \Pi(a,z)}{\partial z^2} = (v - p - s)F(z)d(a)$, where $v - p - s < 0$. Moreover, assuming that F is invertible, the optimal and unique z^* can be expressed as

$$z^* = F^{-1}\left(\frac{p + s - c}{p + s - v}\right), \tag{4.10}$$

which corresponds to the standard NP result [93, 121].

4.3.2. Optimal advertising expenditure

Substituting (4.10) into (4.9) leads to the following expected profit expression:

$$\Pi(a, z^*) = d(a)[p - c - l(z^*)] - a, \tag{4.11}$$

where $l(z)$ is given by (4.8). Notice in (4.11) that, if $p - c - l(z^*) < 0$, the expected profit $\Pi(a, z^*)$ is, under our assumptions about $d(a)$ in subsection 4.2.1, negative and strictly decreasing in a, which does not capture any real situation [116], similarly if $p - c - l(z^*) = 0$. This leads to the following assumption.

Assumption 4.3.1. The per-unit expected benefit must be positive, i.e., $p - c - l(z^*) > 0$.

This assumption simply means that the expected profit per unit is greater than zero (price p minus cost c minus expected loss per unit $l(z^*)$ is greater than zero). Otherwise, if a loss is expected, the only good strategy is to "do nothing" ($x = a = 0$ and so $z = 0$), see section 4.6 for illustrative examples. As Assumption 4.3.1 depends on the expected loss function $l(z)$, it also depends on the distribution F. The more shortages or leftovers are expected (caused, for example, by greater variance of the distribution), the greater is the expected loss $l(z)$. Similarly to other parameters from the expected loss function given by (4.8), e.g., as s increases, so does $l(z)$. Since the assumption is crucial for further analysis, more detailed insights for the uniform distribution is provided in subsection 4.6.1.

Further, the expected profit expression given by (4.11) is assumed. Solving the first order condition of $\Pi(a, z^*)$ with respect to a, leads to the following remark.

Remark 4.3.2. The optimal advertising expenditure a^* must satisfy the (necessary) optimality condition, which is given by:

$$\frac{\mathrm{d}d(a)}{\mathrm{d}a} = \frac{1}{p - c - l(z^*)}. \tag{4.12}$$

4.3.3. Monotonocity

Consider the second derivative of the expected profit given by (4.11):

$$\frac{\mathrm{d}^2\Pi(a, z^*)}{\mathrm{d}a^2} = \frac{\mathrm{d}^2 d(a)}{\mathrm{d}a^2}[(p - c - l(z^*)]. \tag{4.13}$$

Then, due to Assumption 4.3.1, the following lemma is obtained from (4.13).

Lemma 4.3.1. The intervals of concavity and convexity of the expected profit $\Pi(a, z^*)$ with respect to a are identical with the intervals of concavity and convexity of the response function $d(a)$.

The following assumption, together with Assumption 4.3.1, will further help us to guarantee solution/optimality uniqueness for selected types of demand functions (i.e. for the concave and the S-shaped function). The assumption results from expression (4.9), or (4.12) respectively.

Assumption 4.3.2. The demand function $d(a)$ satisfies that $\lim_{\Delta a \to 0+} \frac{d(\Delta a) - d(0)}{\Delta a} > \frac{1}{p-c-l(z^*)}$ and $\lim_{\Delta a \to 0+} \frac{d(a_{max}) - d(a_{max} - \Delta a)}{\Delta a} < \frac{1}{p-c-l(z^*)}$.

Remark 4.3.3. In such case, where the function $d(a)$ is defined on a higher range than $[a, a_{max}]$, the conditions can be rewritten to: $\frac{\mathrm{d}d(0)}{\mathrm{d}a} > \frac{1}{p-c-l(z^*)}$ and $\frac{\mathrm{d}d(a_{max})}{\mathrm{d}a} < \frac{1}{p-c-l(z^*)}$.

Concave response function

Suppose that the demand/response function $d(a)$ is strictly concave in domain of a [63], see Figure 4.1. Then, the following theorem can be deduced.

Theorem 4.3.2. If the response function $d(a)$ is strictly concave, then, under assumptions 4.3.1 and 4.3.2, the expected profit $\Pi(a, z^*)$ is strictly concave in a and so the globally optimal advertising expenditure a^* is unique and is given by solution of (4.12) with respect to decision variable a.

Proof. Since the response function $d(a)$ is considered to be strictly concave in its domain then under Assumption 4.3.1 and Lemma 4.3.1, it is obtained that $\Pi(a, z^*)$ is also strictly concave in a, see (4.13). Moreover, under Assumption 4.3.2, the expected profit $\Pi(a, z^*)$ is to be increasing at the initial point and decreasing at the end point. Then, the critical point determined from the optimality condition (4.12) is unique and is the optimal advertising amount a^* (see section 4.6 for illustrative examples). □

S-shaped response function

Theorem 4.3.3. If the response function $d(a)$ is S-shaped, then, under assumptions 4.3.1 and 4.3.2, the expected profit $\Pi(a, z^*)$ is strictly quasi-concave in a and so the globally optimal advertising expenditure is unique and is given by (4.12).

Proof. Since the response function is supposed to be S-shaped, under Assumption 4.3.1, the expected profit function $\Pi(a, z^*)$ is also first convex and then concave in a. Moreover, using Assumption 4.3.2, the expected profit $\Pi(a, z^*)$ increases at the initial point and so it will increase until it reaches its maximum. In other words, $\Pi(a, z^*)$ is strictly quasi-concave in a. Then, from the optimality condition (4.12), one critical point a^* can be expressed that presents the optimal advertising amount, which always lies in the concave range (see section 4.6 one illustrative and one counter example are provided). □

In order to solve the original problem of maximizing the expected value of the objective function given by (4.1) with respect to decision variable x, a final step is to determine an optimal order quantity x^* from (4.4). The pair $[a^*, x^*]$ then presents the optimal solution of the original NPA given by (4.1) for the multiplicative demand case defined by (4.3) for the expected objective function case, see (4.6) and (4.9).

4.3.4. Comparison with riskless problem

Consider the advertising decision without demand uncertainty and note that the profit of such a deterministic problem is called riskless profit, $\Psi(a)$, given by (4.7). Solving the first order condition of $\Psi(a)$ leads to the following necessary optimality condition:

$$\frac{\mathrm{d}d(a)}{\mathrm{d}a} = \frac{1}{p - c}, \tag{4.14}$$

which must be satisfied by the optimal riskless advertising a^*_Ψ.

Remark 4.3.4. If the response function $d(a)$ is either concave or S-shaped, then, under Assumption 4.3.2, the necessary optimal condition (4.14) is also sufficient for the optimal riskless advertising a^*_Ψ as (4.13) and Lemma 4.3.1 can be adequately applied.

Based on the optimality condition (4.14), the following theorem can be proved under assumptions 4.3.1 and 4.3.2 considering concave and S-shaped functions.

Theorem 4.3.4. For the multiplicative demand model, the optimal advertising a^* is always less than or equal to the optimal riskless advertising a^*_Ψ.

Proof. Using expressions (4.12) and (4.14), it can be expressed that $\frac{1}{p-c} \leq \frac{1}{p-c-l(z^*)} \Rightarrow \frac{\mathrm{d}d(a^*_\Psi)}{\mathrm{d}a} \leq \frac{\mathrm{d}d(a^*)}{\mathrm{d}a}$. For both functions, concave and S-shaped, the optimal advertising a^*, if it exists and is greater than zero, belongs to the concave part of $d(a)$. Then, for the concave part of $d(a)$, $\frac{\mathrm{d}d(a)}{\mathrm{d}a}$ is decreasing and so if $\frac{\mathrm{d}d(a^*_\Psi)}{\mathrm{d}a} \leq \frac{\mathrm{d}d(a^*)}{\mathrm{d}a}$ then $a^*_\Psi \geq a^*$. See section 4.6 for illustrative examples. □

Remark 4.3.5. Recall that the optimal price for multiplicative uncertain demand is not less than the riskless price in the NPP [86].

Even though there are similar structures of the the expected profit functions, which is (3.9) in the NPP case, while the NPA equivalent is given by (4.9), the demand function is defined differently: $d(p)$ is decreasing in p in the pricing case, but $d(a)$ increases in a in the advertising case. Therefore, it is not surprising that the observation on the effect of uncertainty given by Theorem 4.3.4 is opposite to its NPP equivalent (see Remark 4.3.5).

4.4. Additive demand model

Let the demand function (4.2) be defined in the additive form (see subsection 4.2.1) and let $F(\cdot)$ denote the cdf and $f(\cdot)$ the pdf of ξ_a. Then, the demand is in the additive form

$$D_A(a, \xi_a) = d(a) + \xi_a. \tag{4.15}$$

Considering model (4.1), demand function (4.15) and defining the stocking factor $z \in \mathbb{R}$ as $z = x - d(a)$ (if $x = 0$ then $a = 0$ but $d(0) > 0$), the following model is obtained:

$$\pi(a, z, \xi_a) = \begin{cases} p[d(a) + z] - c[d(a) + z] - s[\xi_a - z] - a, & \text{for } z < \xi_a, \\ p[d(a) + \xi_a] - c[d(a) + z] + v[z - \xi_a] - a, & \text{for } z \geq \xi_a. \end{cases}$$

The expected profit $\Pi(a, z)$ can be expressed by:

$$\Pi(a, z) = \mathrm{E}[\pi(a, z, \xi_a)] = \Psi(a) - l(z). \tag{4.16}$$

Riskless profit $\Psi(a)$ and expected loss $l(z)$ are given by (4.7) and (4.8) substituting ξ_m in expected quantities expressions, where $\Lambda(z)$ are expected leftovers and $\Theta(z)$ are expected shortages.

From expressions (4.16) it can be seen that the decisions on a and z are made independently, unlike in the multiplicative model (see (4.9) and (4.12)). Therefore, for the additive demand model, the optimal advertising a^* is always equal to the optimal riskless advertising a^*_Ψ (see (4.14)), while the optimal stocking quantity z^* corresponds to that from the multiplicative case, see (4.10).

Indeed in our setting the additive model is questionable: can the optimal advertising be resistant to the demand uncertainty? However, this analysis can help the interested manager to choose a suitable model by using statistical observations and evaluation of the demand variance. Therefore, after two following theorems that are equivalent to that of the multiplicative case are established, numerical results are investigated in Section 4.6.

The difference in observations on optimal advertising between the multiplicative and additive demand cases can be mainly explained by their variances and coefficients of variation, respectively. While in the additive case the variance of the demand is constant (independent of a), i.e., $\sigma^2[D_A(a,\xi_a)] = \sigma_A^2$ that is the *constant variance case*, in the multiplicative case the variance is a function of the response function, i.e., $\sigma^2[D_M(a,\xi_m)] = [d(a)]^2\sigma_M^2$, and the coefficient of variation is constant, i.e., $c_v[D_M(a,\xi_m)] = \sigma_M$ that is the *constant coefficient of variation case*. See [62] for a similar variance analysis.

Theorem 4.4.1. If the response function $d(a)$ is strictly concave, the expected profit $\Pi(a,z^*)$ is strictly concave in a and, under Assumption 4.3.2, the optimal advertising amount is unique and is given by (4.14).

Proof. The proof is much the same as the one of Theorem 4.3.2. □

Theorem 4.4.2. If the response function $d(a)$ is S-shaped, then, under Assumption 4.3.2, the expected profit $\Pi(a,z^*)$ is strictly quasi-concave in a and so the optimal advertising amount is unique and given by (4.14).

Proof. The proof is much the same as the one of Theorem 4.3.3. □

Finally, the optimal order quantity can then be determined as $x^* = z^* + d(a^*)$.

4.5. Suitable examples of the response functions

Advertising concave function without threshold in demand

Here, it is considered an advertising function with diminishing returns, which is given by

$$d(a) = d_0 + \omega a^\alpha, \tag{4.17}$$

where $\alpha \in [0,1]$ and $\omega > 0$ are empirically determined constants indicating the effectiveness of advertising and $d_0 > 0$ represents the initial demand (for $a = 0$). If $\omega = 0$, then the demand is independent of advertising expenditure. The larger the value of α, the more effective advertising is. For more details on the function and its analysis, see [63].

Solving the first order condition for $\Pi(a,z^*)$ with respect to a, we get

$$a^* = \sqrt[1-\alpha]{\omega\alpha\,[p-c-l(z^*)]}. \tag{4.18}$$

Note that the following expressions for optimal advertising expenditure are true for the multiplicative demand case, while, for the additive demand case, it must be substituted $l(z^*) \equiv 0$ to obtain the results, see/compare (4.12) and (4.14).

Figure 4.1 illustrates three particular function examples presented in this section. Section 4.6 provides numerical examples involving these functions.

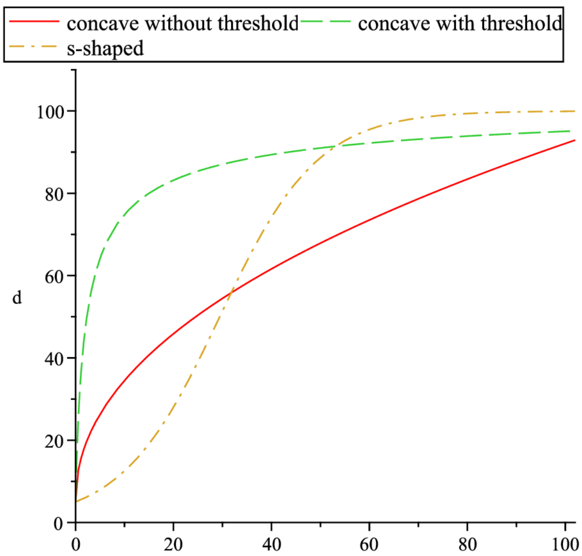


Figure 4.1: Advertising response functions $d(a)$.

Advertising concave function with threshold in demand

Here, the function $d(a)$ has an asymptotic behavior (an upper bound in demand). Based on the empirical study by [23], the following function is suggested as a suitable candidate:

$$d(a) = d_0 + \theta\left[1 - \frac{1}{(a+1)^{\delta}}\right] \tag{4.19}$$

where θ and δ are positive real numbers and d_0 is an initial demand. This function has a horizontal asymptote defined by demand value $\theta + d_0$. The choice of δ decides the speed towards the asymptote. A small value of δ indicates a slow speed while a large value of δ indicates a fast speed.

Similarly, here

$$a^* = \sqrt[\delta+1]{\theta\delta\left[p - c - l(z^*)\right]} - 1.$$

Advertising S-shaped function example

A logistic function/curve represents a typical "S" shape graph. According to theoretical definitions of the S-shaped response function ([5, 7, 55, 68, 107]) and according to the experimental marketing research results (e.g. [23]), it is suggested the following function:

$$d(a) = d_0 + \frac{\theta}{1 + (\frac{\theta-\theta_l}{\theta_l})e^{-\gamma a}}, \tag{4.20}$$

where θ specifies an upper asymptote, γ is a coefficient of growth, θ_l defines a lower asymptote (see, e.g., [34] for a mathematical description/analysis of the function). The interested reader is also referred to other S-shaped functions as the Gompertz function.

Here, the optimal advertising for the S-shaped function is expressed by:

$$a^* = -\frac{\ln\left(\frac{\left\{1-\frac{1}{2}[p-c-l(z^*)]\theta\gamma+\frac{1}{2}\sqrt{-4[p-c-l(z^*)]\theta\gamma+[p-c-l(z^*)]^2\theta^2\gamma^2}\right\}\theta_l}{-\theta+\theta_l}\right)}{\gamma}.$$

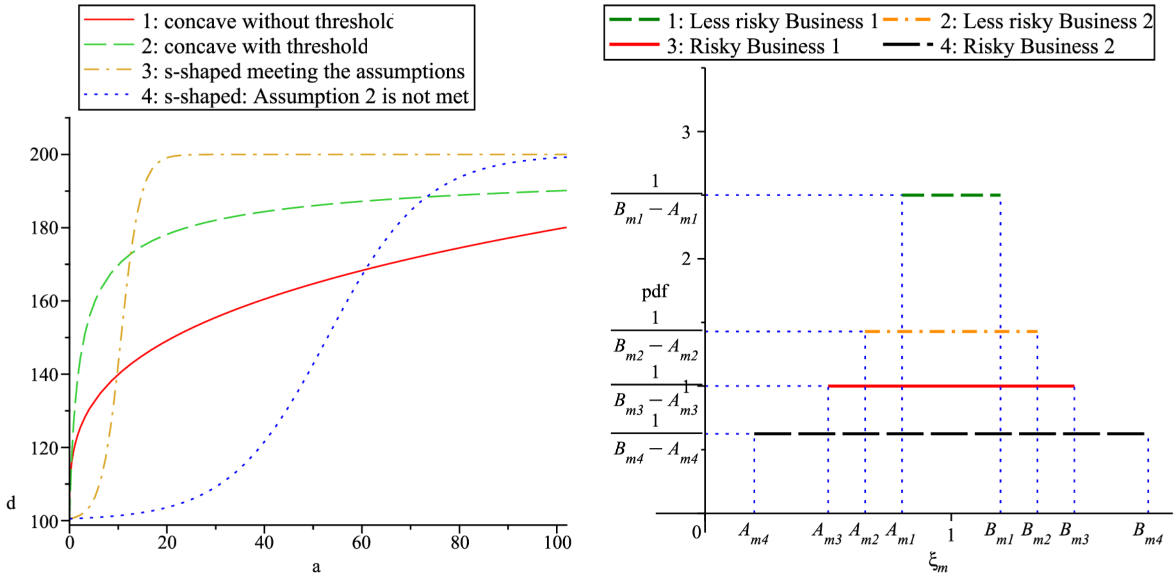
4.6. Numerical examples and the multiplicative case results

So far, conditions for existence of a (non-zero) solution as well as theorems ensuring its uniqueness were discussed; both using the input parameters p, c, s, v, parameters of $d(a)$ and, especially, distribution function F. In order to exemplify managerial information on the optimal decisions, some insights using the uniform distribution of the random variable are further provided; e.g. a situation with minimal knowledge of demand behavior within a given range (a uniform distribution). That is, a situation where it with certainty can be defined an upper and a lower bound for demand; everything between the bounds is equally probable. Alternatively, if only some information is known, it is referred to usage of either the piece-wise uniform (histogram like, i.e. empirical with additional uncertainty) probability distribution or the triangular distribution. Hence, in upcoming examples the NPA is illustrated by using the uniform distribution (subsection 4.6.1). Results are followed by the managerial viewpoint inspired interpretation in subsection 4.7.

In this section, some interesting insights into the decision making described in sections 4.3 and 4.4 are provided. Therefore, in Table 4.1, there are presented parameters leading to illustrative response functions (section 4.2.2), see Figure 4.2a. Note, that one example (i.e. $d_4(a)$) for the S-shaped function, for which Assumption 4.3.2 is not met (and so $\Pi(a, z^*)$ is not quasi-concave), is included; see the left hand derivative, i.e. $\frac{\mathrm{d}d(0)}{\mathrm{d}a}$, for $d_4(a)$ in Table 4.1.

Table 4.1: Numerical examples of response functions given by (4.17)-(4.20).

Type	Indic.	Parameters			$\frac{\mathrm{d}d(0)}{\mathrm{d}a}$	Ass. 4.3.2	$\frac{\mathrm{d}d(150)}{\mathrm{d}a}$	Ass. 4.3.2
(4.17)	$d_1(a)$	$\alpha = \frac{3}{10}$	$\omega = 20$		∞	✓	0.180	✓
(4.19)	$d_2(a)$	$\theta = 100$	$\delta = \frac{1}{2}$		50	✓	0.027	✓
(4.20)	$d_3(a)$	$\theta = 100$	$\theta_l = \frac{1}{2}$	$\gamma = \frac{1}{2}$	0.249	✓	0.165	✓
(4.20)	$d_4(a)$	$\theta = 100$	$\theta_l = \frac{1}{2}$	$\gamma = \frac{1}{10}$	0.050	×	0.001	✓



(a) Response function examples $d_1(a)$ - $d_4(a)$. *(b) Uniform distribution examples U_1 - U_4.*

Figure 4.2: Visualization of numerical examples from tables 4.1 and 4.2.

4.6.1. Uniform distribution

Let the random variable ξ_m be uniformly distributed, i.e., $\xi_m \sim U(A_m, B_m)$. Then, from (4.10), it can be obtained that $z^* = A_m + \frac{(p+s-c)(B_m - A_m)}{p+s-v}$. Substituting z^* into (4.8) leads to $l(z^*) = (z^* - A_m)\frac{c-v}{2} = \frac{B_m - A_m}{2}(c-v)\frac{p+s-c}{p+s-v}$. Using the obtained $l(z^*)$, condition $p - c - l(z^*) > 0$ from Assumption 4.3.1 converts to

$$p - c - \frac{B_m - A_m}{2}(c - v)\frac{p + s - c}{p + s - v} > 0. \tag{4.21}$$

Since the expression (4.21) as well as optimal advertising expenditure a^* crucially depends on price p and cost c as well as on the range $[A_m, B_m]$, which relates to the variance, some insights into impact of the parameters are further provided. Note that variance σ^2 of the uniformly distributed random variable ξ_m is $\frac{1}{12}(B_m - A_m)^2$.

4.6.2. Impact of variance

Considering the demand function $d_1(a)$ and a product with $p = 15$, $c = 10$, $v = 8$, $s = 2$ and $d_0 = 100$, the following four examples of various uniform distributions are introduced in Table 4.2 (see also Figure 4.2b).

Table 4.2: Numerical examples of various uniform distributions U_1 - U_4 for $d_1(a)$.

U_i	A_{mi}	B_{mi}	σ^2	z^*	$l(z^*)$	$p-c-l(z^*)$	a^*	x^*
U_1	0.8	1.2	0.0133	1.111	0.311	4.689	117.6	204.0
U_2	0.65	1.35	0.0408	1.194	0.544	4.456	109.3	217.1
U_3	0.5	1.5	0.0833	1.278	0.778	4.222	101.2	229.9
U_4	0.2	1.8	0.2133	1.444	1.244	3.756	85.6	254.2

Therefore, with increasing variance of the random element, the optimal z^* as well as $l(z^*)$ increases, see (4.21). Since from (4.12), a higher $l(z^*)$ leads to a lower optimal advertising a^*, which corresponds to a lower expected demand $d(a^*)$, the optimal strategy is to buy a higher amount x^* of the product, although a less profit $\Pi(a^*, x^*)$ is expected (see Table 4.3).

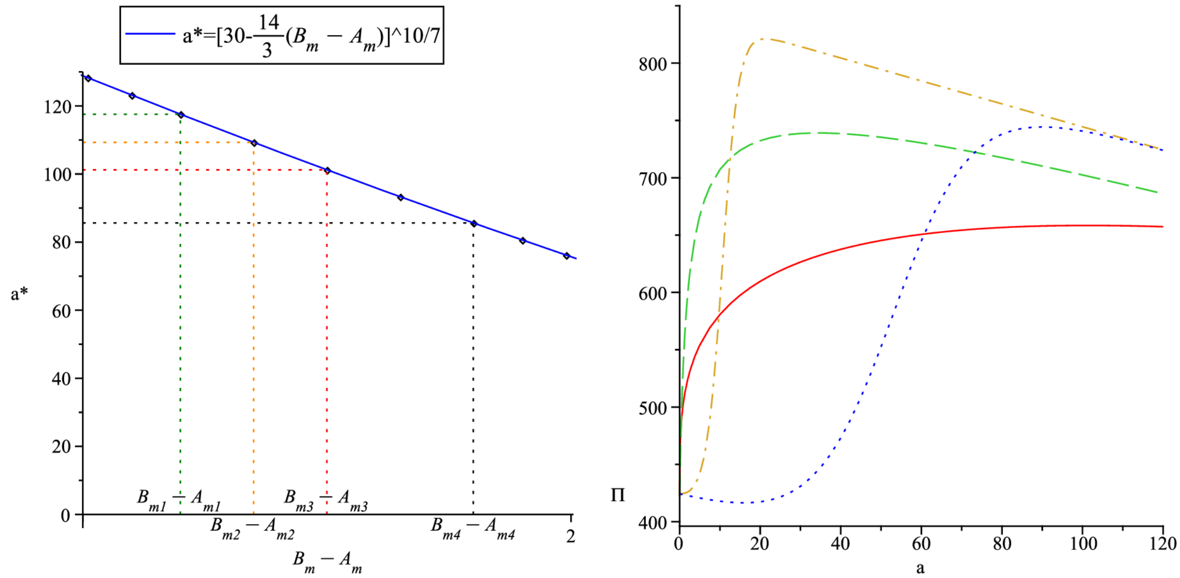
In Figure 4.3a, the decreasing dependency of the optimal advertising a^* on the range $B_m - A_m$ (on the variance, respectively) is illustrated. It can be seen, that for some special cases of response function (e.g. for small α's in function (4.17)), the general dependency between the optimal advertising a^* and variance can be approximated by a linear function, see function graph in Figure 4.3a.

4.6.3. Impact of price and cost changes

Impact on Assumption 4.3.1: Assumption 4.3.1 crucially depends on the margin $(p - c)/p$, see (4.21). The higher the margin the more likely the assumption will be fulfilled. Therefore, an insight into the impact of the $p - c$ change is provided: in Figure 4.4a, there is illustrated the dependency by varying c (by means of (4.21)).

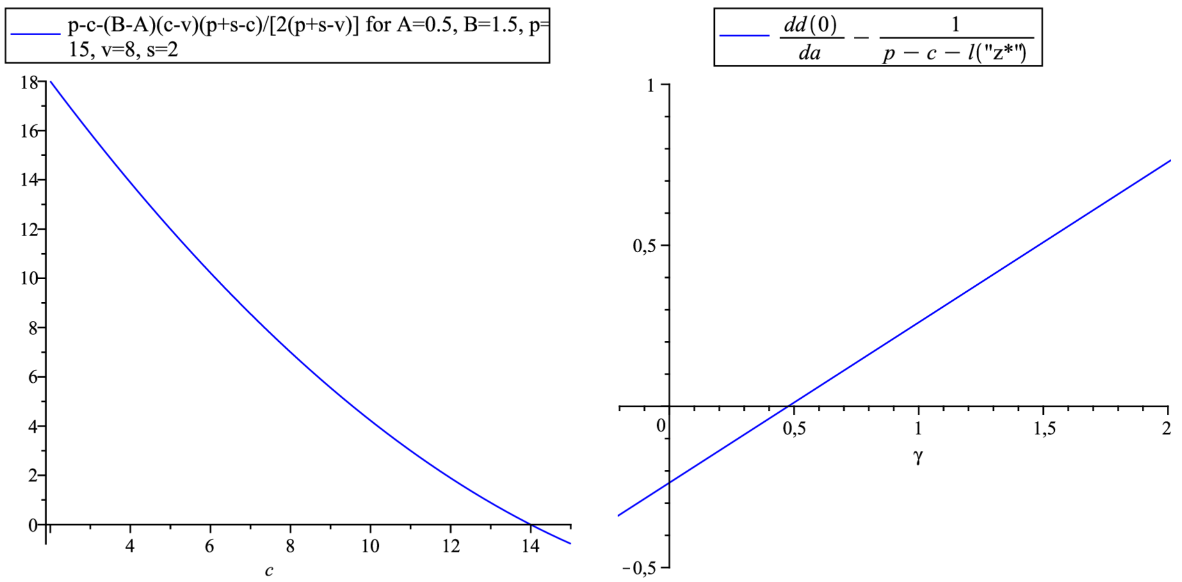
Impact on optimal advertising: Consider the first derivative w.r.t. a of expression given by (4.11) and substitute z^* (and $l(z^*)$ respectively) for the uniform distribution. Then, taking derivative w.r.t. p leads to

$$\frac{\partial(\frac{\partial \Pi(a,z^*)}{\partial a})}{\partial p} = \frac{\mathrm{d}d(a)}{\mathrm{d}a}\left[1 - \frac{B_m - A_m}{2}\frac{(c-v)^2}{(p+s-v)^2}\right] > 0.$$



(a) Dependency between a^* and $B_m - A_m$. (b) Expected profit functions $\Pi(a, z^*)$.

Figure 4.3: Illustration of examples: optimal advertising a^* as function of $B_m - A_m$ and expected profit functions $\Pi(a, z^*)$ related to that from Figure 4.2a (for U_3).



(a) Assumption 4.3.1: expected unit loss $p - c - l(z^*)$, see (4.21), as function of the cost c. (b) Assumption 4.3.2: illustration of dependency on the S-shaped function parameter γ.

Figure 4.4: Visualizations of dependencies between selected parameters and assumptions 4.3.1 and 4.3.2 for the uniform distribution U_3.

Therefore, $\Pi(a, z^*)$ is strictly supermodular in (a, p) (see [118]) and the optimal advertising is strictly increasing in selling price p. With the same procedure but w.r.t. c, it is expressed:

$$\frac{\partial(\frac{\partial \Pi(a,z^*)}{\partial a})}{\partial c} = d(a)\left[\frac{B_m - A_m}{2}\frac{2c - p - s - v}{p + s - v} - 1\right] < 0,$$

which means that $\Pi(a, z^*)$ is strictly submodular in (a, c) and the optimal advertising is strictly decreasing in buying cost c.

Therefore, any increase in the unit profit margin, i.e. range $p - c$ (see [62]), leads to higher optimal advertising expenditure a^*.

4.6.4. Impact of demand function

In Table 4.3, there are provided numerical results of optimal quantities for response function examples $d_1(a)$ - $d_4(a)$ using two uniform distribution examples U_1 and U_3. See also Figure 4.3b for illustration of expected profit functions to the related demand functions from Figure 4.2a.

Table 4.3: Numerical results: optimal quantities for $d_1(a)$ - $d_4(a)$ and U_1, U_3.

	U_1				U_3			
	$d_1(a)$	$d_2(a)$	$d_3(a)$	$d_4(a)$	$d_1(a)$	$d_2(a)$	$d_3(a)$	$d_4(a)$
a^*	117.6	37.0	21.5	91.0	101.2	34.5	21.3	89.9
$d_i(a^*)$	183.6	183.8	200.0	197.8	179.9	183.2	199.5	197.6
x^*	204.0	204.2	221.7	219.8	229.9	234.1	254.9	252.5
$\Pi(a^*, x^*)$	743.2	824.7	914.3	836.6	658.4	739.1	821.2	744.3

In Figure 4.4b, it is illustrated how parameter δ of the S-shaped function affects the Assumption 4.3.2. If the left hand derivative of the response function is less than $1/[p-c-l(z^*)]$, then the Assumption 4.3.2 is violated (left "negative" part of the function in Figure 4.4b). In other words, for small δ's is the advertising not effective in the initial period, i.e. the expected profit is negative (see Figure 4.3b).

4.7. Discussion of the results

The numerical examples presented in Section 4.6 indicate how: a) variance of the random element may affect the optimal advertising as well as order quantity, b) profit margin changes (denoted $p - c$) can impact optimal advertising and expected profit and c) various response functions as well as given various parameters settings representing different markets or market strategies influence the profit.

The illustrative uniform distribution shows how a manager can utilize his often incomplete knowledge on the advertising/demand-related uncertainty. For example, the lack of knowledge can be represented by the lower and upper bounds for the interval specifying the region of uncertainty. Then, no further available information naturally leads to the choice of the uniform distribution. In case of an additional information in the form of a small set of historical data, it may be used the piece-wise uniform probability distribution (with histogram like probability density function) or the triangular distribution that lead to similar computations. Furthermore, it was also shown how the manager can evaluate a particular situation regarding to assumptions 4.3.1 and 4.3.2. Specifically, when Assumption 4.3.1 is violated then the only one optimal strategy is to "do nothing". In addition, when Assumption 4.3.2 is not met then it does not necessarily mean that the advertising strategy cannot lead to increases in sales (especially for the S-shaped function), however, there is no guarantee that a unique solution (advertising expenditure) of the first first order condition will be found or by straightforward application of basic calculus formulas.

Finally, the most interesting ideas for a real-world manager that can be derived from subsection 4.6.1 are related to the choice of a suitable marketing strategy. From our modeling point of view, this marketing strategy is based on the choice of advertising response function. Such a choice can lead to increases in profit, see Table 4.3. However, the situation can become more complex. As it is seen from Figure 4.3b and Table 4.3, the shape of chosen response function can significantly modify the shape of original objective function for the case with no advertising. It may change the optimal solution location from quantitative point of view, however, the shape of the objective changes also qualitatively. Therefore, the manager can design his marketing strategy, and so, the related response function in the way that will influence his operational decision making needs. Thus, although marketing decisions are not easy to perform, even in our fundamental NPA-setting, it was shown that even discussion of concrete explanatory examples can provide valuable insights into decision making, i.e. how to determine (approximate) the optimal decisions and related quantities.

Chapter 5

Newsvendor Problem with Joint Pricing and Advertising

Coordination between advertising, pricing, and production decisions still falls within the very challenging problems associated with operations and computational management; see, e.g., [6, 70]. Typically, marketing decisions (e.g., pricing and advertising) are made independently of knowledge of production and logistics constraints, possibly leading to the sub-optimality of the decisions. Here, marketing decisions are seen together with logistic decisions aiming for improved company performance.

In the aforementioned models the retailer/distributor cannot jointly adjust the price and the advertising amount (marketing effort) to stipulate the market demand. Here, a decision-maker may adjust the current selling price in order to increase or reduce the demand in most cases; moreover, he has the opportunity to influence the final demand by choosing appropriate marketing activities, e.g., providing shelf spaces, promotional displays, advertising, after-sales service support, and other demand-enhancing activities [20]. In a word, price and marketing efforts can be used to affect the final sale of the products ordered, and hence to exert influence on the initial ordering decision [20].

The newsvendor problem with pricing and advertising (NPPA) combines all of the introduced operational and marketing strategies used in sections 2.1, 3.1, and 4. It means that the NPPA is a problem over three decision variables (ordering, pricing, and advertising) and one random variable (influencing the demand).

This thesis follows two papers on the NPPA: [20] and [119]. In the paper by T. Wang and Hu [119], they deal with the multiplicative demand form, where the demand $D(a, p)$ is nonnegative, twice-continuous differentiable, strictly concave, and is defined on $[0, \infty) \times [0, \infty)$; conceivably, $D(a, p)$ is strictly increasing and concave in the advertising premium, while it is strictly decreasing and convex in the sale price. In the paper by Dai and Meng [20], which combines the NPPA with its risk-averse form, two demand cases are investigated: the marketing-dependent price-multiplicative case and the marketing-dependent price-additive case; see Section 5.1.

5.1. Problem formulation and demand function

The NPPA model is similar to the NPA model (4.1); the crucial difference is that p is the decision variable, so the newsvendor faces stochastic demand $D(a, p, \xi_a, \xi_m)$. Thus, the newsvendor simultaneously decides on: the advertising amount a, the selling price p, and the amount x of a product to be stocked and sold. Fsubs Replacing $D(a, \xi_a, \xi_m)$ with $D(a, p, \xi_a, \xi_m)$ in the NPA model (4.1), the NPPA model is expressed as

$$\pi(a, p, x, \xi_a, \xi_m) = \begin{cases} px - cx - s[D(a, p, \xi_a, \xi_m) - x] - a, & \text{for } x < D, \\ pD(a, p, \xi_a, \xi_m) - cx + v[x - D(a, p, \xi_a, \xi_m)] - a, & \text{for } x \geq D. \end{cases} \tag{5.1}$$

5.1.1. Demand function and randomness

Let the demand function be denoted as $D(a, p, \xi_a, \xi_m)$ and let it satisfy

$$D(a, p, \xi_a, \xi_m) = d_1(a)[d_2(p) + \xi_a]\xi_m, \tag{5.2}$$

where ξ_a, ξ_m are independent continuous random variables. [20] refers to two special cases of the demand function $D(a, p, \xi_a, \xi_m)$: a) the marketing-dependent price-multiplicative (MDPM) case and b) the marketing-dependent price-additive (MDPA) case.

In the MDPM case, let $P(\xi_a = 0) = 1$ and let the random variable ξ_m be defined on the domain $[A_m, B_m]$ and satisfy $\mathrm{E}[\xi_m] = 1$. In the MDPA case, let $P(\xi_m = 1) = 1$ and let the random variable ξ_a be defined on the domain $[A_a, B_a]$ and satisfy $\mathrm{E}[\xi_a] = 0$. Then, for both cases, the expectation of D is specified as: $\mathrm{E}[D(a, p, \xi_a, \xi_m)] = d_1(a)d_2(p) \equiv d(a, p)$.

In this thesis, the MDPM case is further investigated.

5.2. MDPM demand model

Let the demand function $D(a, p, \xi_a, \xi_m)$ be defined in the MDPM form and let $F(\cdot)$ and $f(\cdot)$ denote the cdf and pdf of ξ_m, respectively, and let $d(a, p)$ denote a general demand function that will be further specified as $d_1(a)d_2(p)$. Then, the demand is in the MDPM form

$$D_M(a, p, \xi_m) = d(a, p)\xi_m. \tag{5.3}$$

The objective function (5.1) can be rewritten by substituting (5.3) and defining the "stocking factor" as $z = \frac{x}{d(a,p)}$:

$$\pi(a, p, z, \xi_m) = \begin{cases} pzd(a, p) - czd(a, p) - sd(a, p)(\xi_m - z) - a, & \text{for } z < \xi_m, \\ p\xi_m d(a, p) - czd(a, p) + vd(a, p)(z - \xi_m) - a, & \text{for } z \geq \xi_m. \end{cases} \tag{5.4}$$

The objective is to maximize the expected profit $\Pi(a, p, z)$; therefore, the expected profit, where $\Pi(a, p, z) = \mathrm{E}[\pi(a, p, z, \xi_m)]$, is expressed as:

$$\Pi(a, p, z) = \Psi(a, p) - L(a, p, z) = \Psi(a, p) - d(a, p)l(p, z), \tag{5.5}$$

where $\Psi(a,p) = (p-c)d(a,p) - a$ and $l(p,z) = (c-v)\Lambda(z) + (p+s-c)\Theta(z)$. Remember that $d(a,p)\Lambda(z)$ denotes expected leftovers and $d(a,p)\Theta(z)$ expected shortages.

5.2.1. Optimal stocking quantity

To maximize $\Pi(a,p,z)$ over three variables, according to the previous experience from the NPP and NPA, the optimal stocking quantity is expressed as first. Taking $\frac{\partial \Pi(a,p,z)}{\partial z} = 0$, the following expression for the optimal stocking quantity z^* is expressed:

$$z^* = F^{-1}\left(\frac{p+s-c}{p+s-v}\right).$$

Remember, that this quantity corresponds to the standard NP optimal quantity expression (2.6) and to the optimal stocking quantity of the NPP (3.11) as well as of the NPA (4.10).

5.2.2. Optimal price

Taking $\frac{\partial \Pi(a,p,z)}{\partial p} = 0$, it can be observed: $\frac{\partial d(a,p)}{\partial p}[p-c-l(z,p)] - d(a,p)[\Theta(z)-1] = 0$. Similarly, substituting the general demand function $d(a,p)$ with its multiplicative form $d_1(a)d_2(p)$, the following is expressed:

$$\frac{\mathrm{d}d_2(p)}{\mathrm{d}p}[p-c-l(z,p)] - d_2(p)[\Theta(z)-1] = 0. \tag{5.6}$$

Theorem 5.2.1. Under our assumptions (i.e., if $d(p)$ has IPE and cdf has GSIFR), $\Pi(a,p,z)$ is quasi-concave in p, and optimal price of the MDPM model is unique and is always equal to that of the multiplicative form in the NPP.

Proof. The proof is obvious comparing two optimal price conditions, the NPP condition (3.20) and the NPPA condition (5.6); see also [20]. □

Optimal p and z can then be found similarly as in the NPP case; see Section 3.1.

Example: Let $d_2(p) = \tau p^{-\beta}$, which corresponds to the isoelastic pricing function given by (3.21). Substituting the isoelastic function $d_2(p)$ into expression (5.6)

$$\frac{\partial \Pi(a,p,z)}{\partial p} = (\beta-1)\frac{d_2(p)}{p}[1-\Theta(z)]\left\{p_\Psi + \frac{\beta}{\beta-1}\left[\frac{(c-v)\Lambda(z)+s\Theta(z)}{1-\Theta(z)}\right] - p\right\}$$

and so the optimal price expression corresponds to that of the NPP, i.e. (3.22). Note, that p^*_Ψ is the optimal riskless profit; see (5.11) for its example for the isoelastic pricing function. Then, the expression of the optimal price follows:

$$p^* = p^*_\Psi + \frac{\beta}{\beta-1}\frac{(c-v)\Lambda(z)+s\Theta(z)}{1-\Theta(z)}. \tag{5.7}$$

5.2.3. Optimal advertising expenditure

Taking $\frac{\partial \Pi(a,p^*,z^*)}{\partial a} = 0$, it can be observed: $\frac{\partial d(a,p^*)}{\partial a} = \frac{1}{p^*-c-l(z^*,p^*)}$. Substituting the general demand function $d(a,p^*)$ with its multiplicative form $d_1(a)d_2(p^*)$, the following is expressed:

$$\frac{\mathrm{d}d_1(a)}{\mathrm{d}a} = \frac{1}{d_2(p^*)[p^* - c - l(z^*,p^*)]}. \tag{5.8}$$

Optimal advertising depends on the choice of p as well as $d_2(p^*)$; see (5.8). Moreover, with increasing p (or p^*, respectively) the optimal value of $d_2(p^*)$ decreases. Therefore, optimal advertising for the NPPA depends on specification not only $d_1(a)$ but also $d_2(p)$. Remember that p^* is equivalent to that of the NPP (see Theorem 5.2.1).

Example: Consider $d_1(a) = d_0 + \omega a^\alpha$, which corresponds to the advertising response function given by (4.17). Substituting the function into (5.8), the optimal advertising a^* can be expressed as:

$$a^* = \sqrt[1-\alpha]{\omega\alpha d_2(p^*)\,[p^* - c - l(z^*,p^*)]},$$

which is equal to the optimal advertising of the equivalent NPA times $\sqrt[1-\alpha]{d_2(p)}$; see the equivalent NPA result (4.18).

Remember that the optimal ordering quantity is determined as $x^* = z^* d(a^*,p^*)$.

5.2.4. Comparison with riskless problem

Let the riskless objective function be defined as

$$\Psi(a,p) = (p-c)d_1(a)d_2(p) - a.$$

Taking $\frac{\partial \Psi(a,p)}{\partial a} = 0$, an optimality condition for a^*_Ψ is expressed as:

$$\frac{\mathrm{d}d_1(a)}{\mathrm{d}a} = \frac{1}{d_2(p)(p-c)}. \tag{5.9}$$

Then, an equivalent result to that of NPA is observed: if the response function $d_1(a)$ is either concave or S-shaped, then, under Assumption 4.3.2, the necessary optimal condition (5.9) is also sufficient for the optimal riskless advertising a^*_Ψ.

Based on the optimality condition (5.9), the following theorem can be proven under assumptions 4.3.1 and 4.3.2 considering concave and S-shaped functions.

Theorem 5.2.2. For the MDPM, the optimal advertising a^* is always less than or equal to the optimal riskless advertising a^*_Ψ.

Proof. See proof of Theorem 4.3.4. □

Example: Considering $d_1(a) = d_0 + \omega a^\alpha$ and $d_2(p) = \tau p^{-\beta}$, the following expression for a^*_Ψ is observed:

$$a^*_\Psi = \sqrt[1-\delta]{\omega\alpha(p-c)}. \tag{5.10}$$

Taking $\frac{\partial \Psi(a,p)}{\partial p} = 0$, an optimality condition for p^*_Ψ can be expressed as:

$$d_2(p) + (p-c)\frac{\partial d_2(p)}{\partial p} = 0.$$

Example: Considering $d_2(p) = \tau p^{-\beta}$, the following expression for p^*_Ψ is observed:

$$p^*_\Psi = \frac{\beta c}{\beta - 1}. \tag{5.11}$$

Since $\beta - 1 > 0$, $c - v > 0$ and $1 - \Theta(z^*) > 0$, the following theorem can be proved.

Theorem 5.2.3. For the isoelastic pricing function in the MDPM, the optimal pricing p^* is always greater than or equal to the optimal riskless pricing p^*_Ψ.

Proof. See [58, 86] for similar observations and related proofs. □

5.3. Literature review for further research

This section reviews some existing literature references that deal with other cases of the coordination of marketing and production decisions; it may serve as inspiration for further research into the NPPA.

5.3.1. Newsvendor problem with marketing effort

In addition to the literature review provided in Section 4.1 (i.e., literature on the NPA), it is also referred to in the literature dealing with the so-called marketing effort instead of advertising; these are quite similar terms, but such a short note may help a reader to avoid any confusion.

For example, Taaffe et al. [109] deal with the so-called selective newsvendor with marketing effort. Note that they investigate two different variance cases (w.r.t. the marketing-dependency): 1. demand variance independent of marketing effort, 2. demand variance dependent on marketing effort.

5.3.2. Supply chain

In the literature references reviewed herein, the marketing and production decisions are considered within supply chain problems.

He et al. [49] consider a single effort level e to summarize the retailer's activities in promoting sales and $g(e)$ captures the retailer's cost of exerting an effort level e, where: $g(0) = 0$, $g'(e) > 0$, and $g''(e) > 0$. Demand is considered to be stochastically increasing in effort and decreasing in price, i.e., the distribution (of a random variable ξ) satisfies $\frac{\partial F(\xi|(e,p))}{\partial e} < 0$ and $\frac{\partial F(\xi|(e,p))}{\partial p} > 0$. Specifically, the demand function is defined as $D = d(p,e) + \xi$, where $d(p,e)$ decreases in price and is concave and increasing in effort. Later, they provide results for $d(e,p) = a + be - kp$ and $g(e) = \mu e^2/2$, where $a, b, k, \mu > 0$ and ξ is uniformly distributed.

Xu et al. [126] first consider the NPP model; the model includes the following: multiplicative stochastic demand, IPE on mean demand, increasing generalized failure rate on the distribution of the random factor, mean demand increases in price, and the random factor is independent of price; then, under two assumptions, i.e., on IPE and IGFR, $\pi(p,x)$ is quasi-concave. Then, the pricing and marketing-dependent demand function is defined as $D(p,e) = d(p)f(e)\epsilon$. Under some assumptions, they provide a theorem on the quasi-concavity of the profit function. They also provide a dynamic pricing-inventory model.

Taylor [110] deals with the so-called sale timing in a supply chain. The demand is modeled in the additive form and depends on both effort and pricing. They provide a result for the linear demand case: $D(e,p,\xi) = a + e + \xi - bp$, where the cost of effort is quadratic.

Hong et al. [52] consider a problem of joint advertising, pricing and collection decisions in the (remanufacturing) supply chain. The demand function is considered as $D(a,p) = \phi - \beta p + \theta\sqrt{a}$, s.t. $\phi, \beta, \theta > 0$,where ϕ is the market scale, β is the price elasticity coefficient, and θ is the demand sensitivity to the advertising level of the retailer. They assume that the retailer's demand function is downward sloping with regard to the self-price effect and upward sloping with regard to the self-advertising effect, with cost of marketing effort "only" being a (which corresponds to the advertising case).

5.3.3. Risk averse newsvendor

Dai and Meng [20] present risk-averse under marketing and pricing dependency. The dependency is defined in the following two forms (which are also mentioned above):

a) the MDPM model, where $d(e,p,\xi) = \alpha(e)\beta(p)\xi$, $\xi \in [L,U]$, $0 \leq L < U$, and $var(\xi) < \infty$. $\beta(p)$ is defined on a closed interval $[c, \bar{p}(e)]$, and continuous, strictly decreasing, and twice-differentiable in p, where $\bar{p}(e)$ is the maximum admissible price for the given effort level e;

b) the MDPA demand model, where $d(e,p,\xi) = \alpha(e)[\beta(p) + \xi]$, $\xi \in [L,U]$, $L < 0 < U$, and $E\xi = 0$, $Var\xi < \infty$.

For both cases, they assume that $\alpha(0) = 1$. If $e = 0$, then it is reduced to the price-additive, and price-multiplicative model, respectively. Moreover, $\bar{p}(e)$ is non-decreasing in e since $\alpha(e)$ is an increasing function of e. In particular, $\beta(\bar{p}(e)) = 0$ for the MDPM demand model and $\beta(\bar{p}(e)) + L = 0$ for the MDPA demand model.

Chapter 6

Waste Processing Facility Location Problem with Stochastic Programming: Models and Solutions

The growing concern for the environment leads to the integration of new solutions into traditional waste management (WM) in practice. About 3 billion tons of wastes are generated in the European Union countries yearly; see [9]. Moreover, due to population increase, the migration of non-EU inhabitants, and economic development in the EU countries, the amount of waste generated is rapidly increasing [27, 41]. Therefore, municipal solid waste producers often face problems of insufficiency in available facility capacities to meet future waste disposal demands [53].

Municipal WM consists of various activities that can be clustered into four processing steps: waste generation, collection, transformation, and disposal [40]. This chapter (and paper [A12] respectively) deals with the second stage: collection that also involves waste transportation to waste processing units. Hence, the chapter concerns mathematical modeling and related decision-support computations of the optimal WM, including facility location planning, in this step; see, e.g., [41] for an extensive review of WM modeling, and see also [136] for facility location in the context of the so called waste-to-energy plant planning. So, WM decision-making problems belong to the class of optimization problems, whose importance has recently significantly increased in practice. Therefore, the mathematical modeling of particular situations and the related computational support can help decision-makers to control WM as well as to achieve cost savings [39].

Existing modeling and solution challenges are related to the fact that the studied problems often combine deterministic and stochastic parameters together with nonlinear terms and both continuous and discrete decision variables. Since many parameters in such a WM system can be uncertain, the straightforward applicability of deterministic mathematical programming methods can be doubtful [53]. Thus, to model real world requirements in a suitable way, the SP approach has been selected and applied in the model building process.

Among the above-mentioned problems, it is focused on a so-called waste processing facility location problem (FLP) that defines the task of choosing the set of open and running waste processing units in the best way from the point of view of total expected costs; see also [A4]. Thus, the facility location decisions must be made when a logistics system is started from scratch, i.e., when new products or services are launched or when existing product distribution or services are expanded [39]. Specifically, in this paper, it is dealt only with waste producer preferences, and so the related processing, transportation, and investment costs are minimized.

In this chapter, the FLP is considered within the transportation network. In general, the network design of transportation problems still belongs to interesting research topics in transportation planning [61, 135]. Various approaches have been taken to solve network design problems; see [71] and [108] for a review of network design problems and see [4] for a detailed review of solution techniques. See also [A6] for author's previous ideas and further references on a hybrid computational approach to network design problems dealing mostly with switching on and off the edges and arcs of the transportation network.

The next sections of the chapter are organized as follows. Section 6.1 describes the developed FLP within waste transportation network design models. Two considered models are subsequently presented, described, and discussed. Firstly, a common transportation network flow is enriched with the on-off waste processing capacities in the chosen nodes to represent the facility location. Then, the randomly-varying waste production is modeled by scenarios and a two-stage stochastic integer linear program is obtained. As the second step, the environmentally-friendly behavior of waste producers through ideas inspired by the utilization of pricing mechanisms in operations research problems is suggested and modeled. The discussed modeling ideas are explained by means of an explanatory example in Section 6.1 The results of computations that were realized for various larger instances with the utilization of both traditional and heuristic algorithms by using model and algorithm implementations in GAMS and MATLAB are commented on in Section 6.2. Finally, Section 6.3 concludes the chapter and outlines some directions for future research and suggests some new computational and modeling ideas for future development.

6.1. Models and explanatory examples

In this section, the cost-minimizing stochastic mixed integer nonlinear program for the above-mentioned problem is developed in two steps. The introduced models use the following sets of indices, parameters, and decision variables. The sets of indices are as follows:

I : set of transportation network related nodes representing places, $i \in I$,
E : set of transportation network related edges representing routes, $e \in E$,
S : set of included scenarios representing uncertainty, $s \in S$.

In this case, nodes with waste producers, transition places, and waste processing units can be identified. In addition, it is distinguished between existing processing units and those units that can be newly established. The edges of model routes may serve for the transportation of waste. The structural information describing the network is completed with the following input parameters:

$a_{i,e}$	:	network description by node-edge incidence matrix,
$b_{i,s}^-$	:	available amount of produced waste in node i for scenario s,
b_i^+	:	available waste processing capacity in node i,
c_e	:	cost per transported unit of waste by edge e,
f_i	:	cost per processed unit of waste in node i,
g_i^-	:	cost per unprocessed waste left in node i,
g_i^+	:	cost per unit of unused capacity in node i,
h_i	:	cost per switched on processing unit in node i,
q_s	:	probability of achieving scenario s.

It is further assumed that the waste producers considered in the model coordinate their decision steps and behave as one decision maker. So, among the model elements, the following decision variables are included:

$x_{e,s}$	:	waste transported by edge e for scenario s, bounded by $x_{U,e}$,
$y_{i,s}$	:	amount of waste processed in node i by scenario s,
$u_{i,s}^-$	:	amount of untransported waste from node i for scenario s,
$u_{i,s}^+$	:	amount of unused processing capacity in node i for scenario s,
$v_{i,s}^-$	:	amount of waste transported from node i for scenario s (negative),
$v_{i,s}^+$	:	amount of waste transported to node i for scenario s,
δ_i	:	indicator of switching on-off extra waste processing capacity in i.

The first model is a scenario-based two-stage MILP that is described as follows:

$$\min \sum_{s\in S} q_s \left(\sum_{e\in E} c_e x_{e,s} + \sum_{i\in I} (f_i y_{i,s} + g_i^- u_{i,s}^- + g_i^+ u_{i,s}^+) \right) + \sum_{i\in I} h_i \delta_i \tag{6.1}$$

$$\text{s.t.} \quad \sum_{e\in E: a(i,e)>0} a_{i,e} x_{e,s} = v_{i,s}^+, \quad \forall i \in I, s \in S, \tag{6.2}$$

$$\sum_{e\in E: a(i,e)<0} a_{i,e} x_{e,s} = -v_{i,s}^-, \quad \forall i \in I, s \in S, \tag{6.3}$$

$$y_{i,s} + u_{i,s}^+ = b_i^+ \delta_i, \quad \forall i \in I, s \in S, \tag{6.4}$$

$$-b_{i,s}^- + v_{i,s}^+ = v_{i,s}^- + y_{i,s} + u_{i,s}^-, \quad \forall i \in I, s \in S, \tag{6.5}$$

$$x_{e,s}, y_{i,s}, u_{i,s}^-, u_{i,s}^+, v_{i,s}^-, v_{i,s}^+ \geq 0, \quad \forall i \in I,\ e \in E, s \in S, \tag{6.6}$$

$$x_{e,s} \leq x_{U,e}, \quad \forall e \in E, s \in S, \tag{6.7}$$

$$\delta_i \in \{0, 1\}, \quad \forall i \in I. \tag{6.8}$$

The objective function (6.1) minimizes the total cost that is a sum of the scenario-related costs involving transportation costs, processing costs, penalizing costs for remaining waste, penalizing costs for unused capacity, and investment costs following investment decisions that must be the same for all scenarios. Eq. (6.2) means that all flows entering node i are summarized as $v_{i,s}^+$. Similarly eq. (6.3) says that all flows leaving node i are summarized as $v_{i,s}^-$. Eq. (6.4) represents a constraint on the processed amount of waste that is bounded by processing unit capacity. This equation also allows processing units to switch on for new waste. To make differentiate between already-built processing units and newly established ones, the value of the first stage decision variables δ_i can be fixed. So, the value 0 is used for transition nodes and value 1 is utilized for the existing processing units. Eq. (6.5) provides the balance constraint of inputs and outputs in node i. Finally, (6.6)–(6.8) specify the domains of the decision variables. For the initial explanation, the transportation network were utilized (in Figure 6.1). Such a simple example can be solved

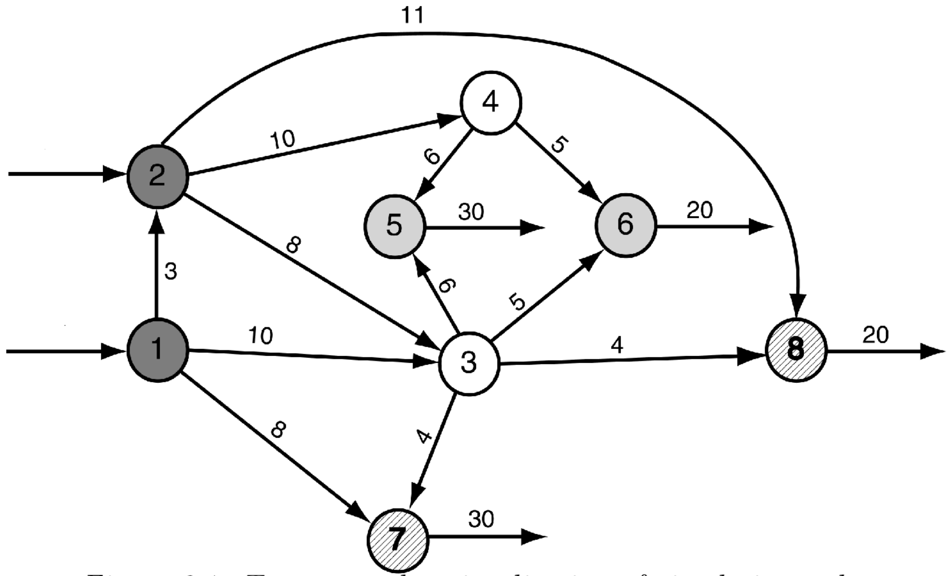
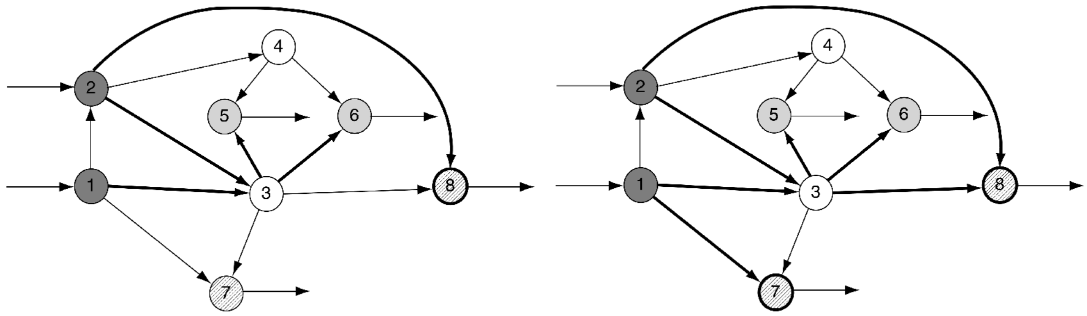


Figure 6.1: Test network - visualization of simple input data

almost intuitively for one scenario case. Therefore, the output for the single scenario is listed in the form of a GAMS results file that also contains all of the input data:

```
Input data and results for data case: 01 - mid waste production
================================================================================
Total optimal cost        zmin =          2650.00
Partial optimal cost       h*d =           600.00 investment of new units
Partial optimal cost p*(gM*uM) =             0.00 average for unprocessed waste
Partial optimal cost p*  (c*x) =           900.00 average transportation costs
Partial optimal cost p*  (f*y) =          1100.00 average for processing waste
Partial optimal cost p*(gP*uP) =            50.00 average for unused capacity
================================================================================
S1 scenario optimal cost gM*uM =             0.00      p(S1 ) = 1.000000
   scenario optimal cost   c*x =           900.00
   scenario optimal cost   f*y =          1100.00
   scenario optimal cost gP*uP =            50.00
================================================================================
nodes                 i |     N1     N2     N3     N4     N5     N6     N7     N8
investment costs      h |    0.0    0.0    0.0    0.0    0.0    0.0 1000.0  600.0
building unit         d |    0.0    0.0    0.0    0.0    1.0    1.0    0.0    1.0
product             h*d |    0.0    0.0    0.0    0.0    0.0    0.0    0.0  600.0
================================================================================
Scenario S1 with probability p(S1 ) = 1.000000
--------------------------------------------------------------------------------
nodes                 i |     N1     N2     N3     N4     N5     N6     N7     N8
produced waste    bM>= |   35.0   30.0    0.0    0.0    0.0    0.0    0.0    0.0
--------------------------------------------------------------------------------
left waste           uM |    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0
--------------------------------------------------------------------------------
related cost         gM |  100.0  100.0  100.0  100.0  100.0  100.0  100.0  100.0
product            gMuM |    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0
--------------------------------------------------------------------------------
output flow          vM |   35.0   30.0   45.0    0.0    0.0    0.0    0.0    0.0
                    -vM |  -35.0  -30.0  -45.0    0.0    0.0    0.0    0.0    0.0
--------------------------------------------------------------------------------
   j   c*x    c      x | ax(N1) ax(N2) ax(N3) ax(N4) ax(N5) ax(N6) ax(N7) ax(N8)
E1a2   0.0  3.0    0.0 |    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0
E1a3 350.0 10.0   35.0 |  -35.0    0.0   35.0    0.0    0.0    0.0    0.0    0.0
E1a7   0.0  8.0    0.0 |    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0
E2a3  80.0  8.0   10.0 |    0.0  -10.0   10.0    0.0    0.0    0.0    0.0    0.0
E2a4   0.0 10.0    0.0 |    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0
E2a8 220.0 11.0   20.0 |    0.0  -20.0    0.0    0.0    0.0    0.0    0.0   20.0
E3a5 150.0  6.0   25.0 |    0.0    0.0  -25.0    0.0   25.0    0.0    0.0    0.0
E3a6 100.0  5.0   20.0 |    0.0    0.0  -20.0    0.0    0.0   20.0    0.0    0.0
E3a7   0.0  4.0    0.0 |    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0
```

```
E3a8   0.0  4.0   0.0 |    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0
E4a5   0.0  6.0   0.0 |    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0
E4a6   0.0  5.0   0.0 |    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0
-------------------------------------------------------------------------------
c*x= 900.0  out:   vP |    0.0    0.0   45.0    0.0   25.0   20.0    0.0   20.0
       in:  -vM |  -35.0  -30.0  -45.0    0.0    0.0    0.0    0.0    0.0
-------------------------------------------------------------------------------
capacity         bP>= |    0.0    0.0    0.0    0.0   30.0   20.0   30.0   20.0
unused             uP |    0.0    0.0    0.0    0.0    5.0    0.0    0.0    0.0
processed           y |    0.0    0.0    0.0    0.0   25.0   20.0    0.0   20.0
-------------------------------------------------------------------------------
related cost        f |    0.0    0.0    0.0    0.0   20.0   20.0   10.0   10.0
product            fy |    0.0    0.0    0.0    0.0  500.0  400.0    0.0  200.0
-------------------------------------------------------------------------------
cost unused        gP |   10.0   10.0   10.0   10.0   10.0   10.0   10.0   10.0
product          gPuP |    0.0    0.0    0.0    0.0   50.0    0.0    0.0    0.0
===============================================================================
```



(a) Results for 1 scenario. *(b) Results for 5 scenarios.*

Figure 6.2: Test network - visualization of results.

Additionally, Figure 6.2a shows the effect of one scenario that leads to the additional switching on of available capacity in node 7 (see boldface circle) and extra routes (see boldface edges) used for waste transport. More scenarios taken into account obviously lead to an increase in newly-used processing units (see both nodes 7 and 8) and additional routes used for transportation; see Figure 6.2b.

To generalize the model, the pricing related ideas mentioned above are introduced. Therefore, it is assumed that waste producers, who are trying to minimize their total costs, can improve their behavior and influence the amount of waste as the decision variable. Consequently, the prices may change. It is reasonable to assume the monopolistic type of behavior is derived from the set of waste processors and from the government, who decide upon the related prices. So, the second considered and generalized model is the following scenario-based two-stage stochastic MINLP:

$$\min \sum_{s \in S} q_s \left(\sum_{e \in E} c_e(x_{e,s}) x_{e,s} + \sum_{i \in I} (f_i(y_{i,s}) y_{i,s} + g_i^-(\bar{b}_i) u_{i,s}^- + g_i^+ u_{i,s}^+) \right) + \sum_{i \in I} h_i \delta_i \tag{6.9}$$

$$\begin{aligned}
\text{s.t.} \quad \sum_{e \in E: a(i,e) > 0} a_{i,e} x_{e,s} &= v_{i,s}^+, && \forall i \in I, s \in S, && (6.10)\\
\sum_{e \in E: a(i,e) < 0} a_{i,e} x_{e,s} &= -v_{i,s}^-, && \forall i \in I, s \in S, && (6.11)\\
y_{i,s} + u_{i,s}^+ &= b_i^+ \delta_i, && \forall i \in I, s \in S, && (6.12)\\
-b_{i,s}^- + v_{i,s}^+ &= v_{i,s}^- + y_{i,s} + u_{i,s}^-, && \forall i \in I, s \in S, && (6.13)\\
\bar{b}_i + \varepsilon_{i,s} &= b_{i,s}^-, && \forall i \in I, s \in S, && (6.14)\\
x_{e,s}, y_{i,s}, u_{i,s}^-, u_{i,s}^+, v_{i,s}^-, v_{i,s}^+ &\geq 0, && \forall i \in I, e \in E, s \in S, && (6.15)\\
x_{e,s} &\leq x_{U,e}, && \forall e \in E, s \in S, && (6.16)\\
\delta_i &\in \{0,1\}, && \forall i \in I, && (6.17)\\
b_{L,i} \leq \bar{b}_i \leq b_{U,i},\ b_{i,s}^- &\geq 0, && \forall i \in I, s \in S. && (6.18)
\end{aligned}$$

In the second model (6.9)–(6.18), most of the constraints [see (6.10)–(6.13), (6.15)–(6.17) and compare with (6.2)–(6.8)] remain the same; however, several important modifications have been included. The cost coefficients newly depend on the decision variables [see the objective function (6.9)], and the functions $c_e(x_{e,s})$, $f_i(y_{i,s})$, and $g_i^-(\bar{b}_i)$ were introduced instead of the coefficients c_e, f_i, and g_i^- respectively. It is also assumed that a decrease of the amount transported or processed will lead to an increase of the related unit cost specified by the price coordinating processing units. Similarly, it is assumed that the unit governmental penalty for unprocessed waste will increase with decreasing production of the waste. See Figure 6.3 for an example of the $f_i(y_{i,s})$ function.

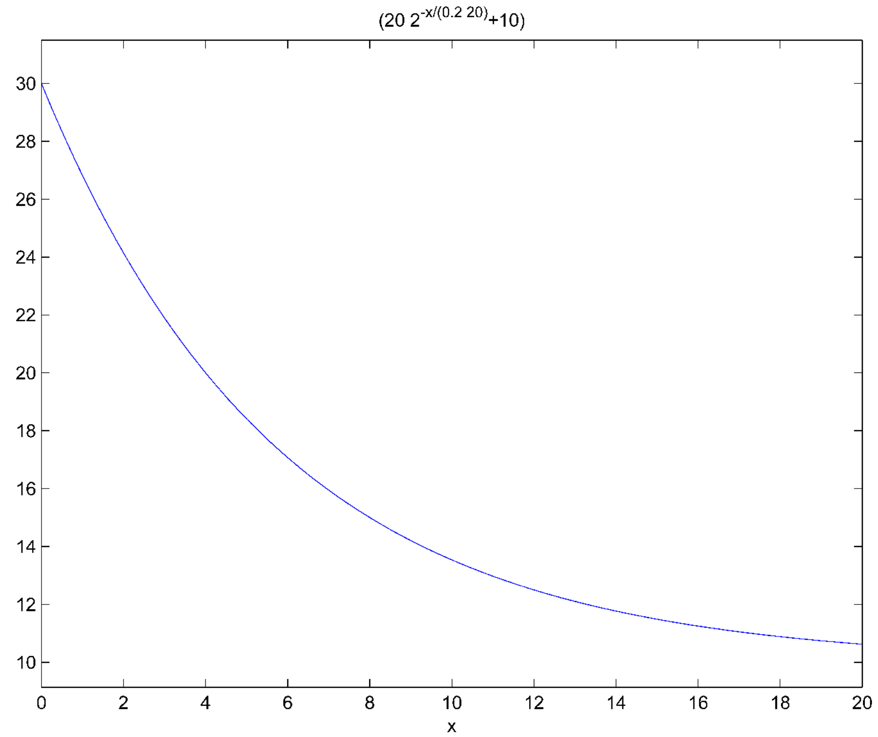
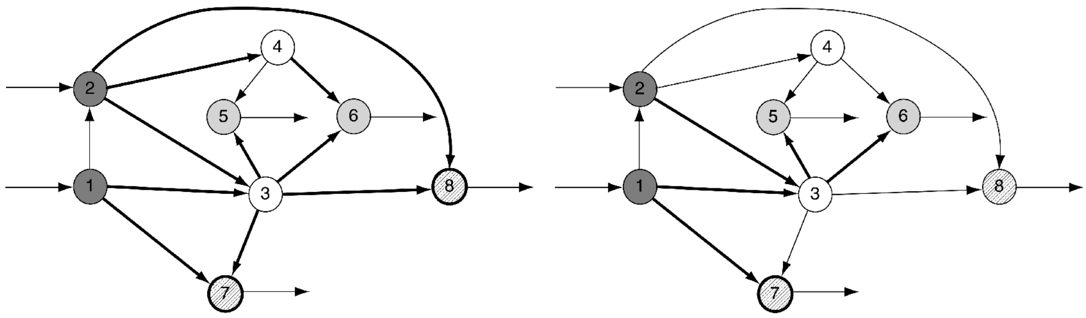


Figure 6.3: modeling pricing ideas.

It is highlighted the fact that under the assumption of the strict monotonicity of these functions, traditional pricing related formulas can appear in the case that it is decided to deal with inverse functions. However, the related interpretation derived from the viewpoint of the producers seems to be unrealiztic for such a case. Therefore, the original pricing ideas were converted into the final ones that are included in the model. The decision of the waste producers wit regard to the amount of the waste delivered for processing is denoted by $\bar{b}_i$ and changes are only allowed within the bounds $b_{L,i}$ and $\leq b_{U,i}$, see (6.18). Random disturbances following this decision modeled by $\varepsilon_{i,s}$ are expected. Then, the $b^-_{i,s}$ is a dependent variable defined by (6.14).



(a) Results for 50 scenarios *(b) Results for pricing-like mechanism*

Figure 6.4: Test network - visualization of results.

The last figures in this section illustrate the effect of pricing ideas included; see model (6.9)–(6.18). Allowing price changes will motivate waste producers to increase, e.g., recycling attempts and it may also reduce their total costs, waste produced, and waste processed. Specifically, fewer processing units must be opened and fewer routes are used cf. Figures 6.4a involving a solution for 50 scenarios for the first model (6.1)–(6.8) and 6.4b describing the results for the second model (6.9)–(6.18).

6.2. Computations and results

Two above-mentioned models were programmed in GAMS and solved through the use of BARON, MINOS, and CPLEX solvers for small test instances obtaining acceptable results. The next computations were realized for larger instances of the model (6.9)–(6.18). However, solution difficulties have appeared when the original GAMS code was applied, as computations have led to increasing computational time requirements. Therefore, heuristics have been discussed and the previous authors' ideas related to the suitable hybrid algorithm have been detailed; see [A6]. Instead of previous implementations based on a combination of the GAMS and C++ codes, it was preferred the complete implementation in MATLAB. This implementation combines the `fmincon` function with genetic algorithm implementation to follow the algorithmic scheme:

1. Set up an instance of a scenario-based two-stage mixed integer nonlinear program in MATLAB. Set up control parameters for the genetic algorithm implemented in MATLAB.

2. Create an initial population for the GA instance. So, the initial values of $0-1$ variables are generated and fixed to obtain a scenario-based (separable) nonlinear program.

3. Several runs of random generators are needed for a specified population size and number of considered scenarios. Repeatedly run the `fmincon` procedure in MATLAB to obtain the set of scenario-related solutions. Each run solves the program for the fixed values of $0-1$ variables.

4. The objective function values are also computed for new individuals created by means of the genetic operators, initially in 2. and then in 3. Store the best results obtained from MATLAB (the optimal objective function values and optimal values of all variables for all scenarios) for comparisons.

5. Test the algorithm termination rules and stop in case of their satisfaction. Otherwise continue until the moment when the last scenario solution is obtained.

6. Generate input values for the GA from `fmincon` results; see step 4. Specifically, the objective function values for each member of the population of the GA are obtained from results of the runs in 3.

7. Run GA to update the set of $0-1$ variables (population); see, e.g., [74] for details. Return to step 3.

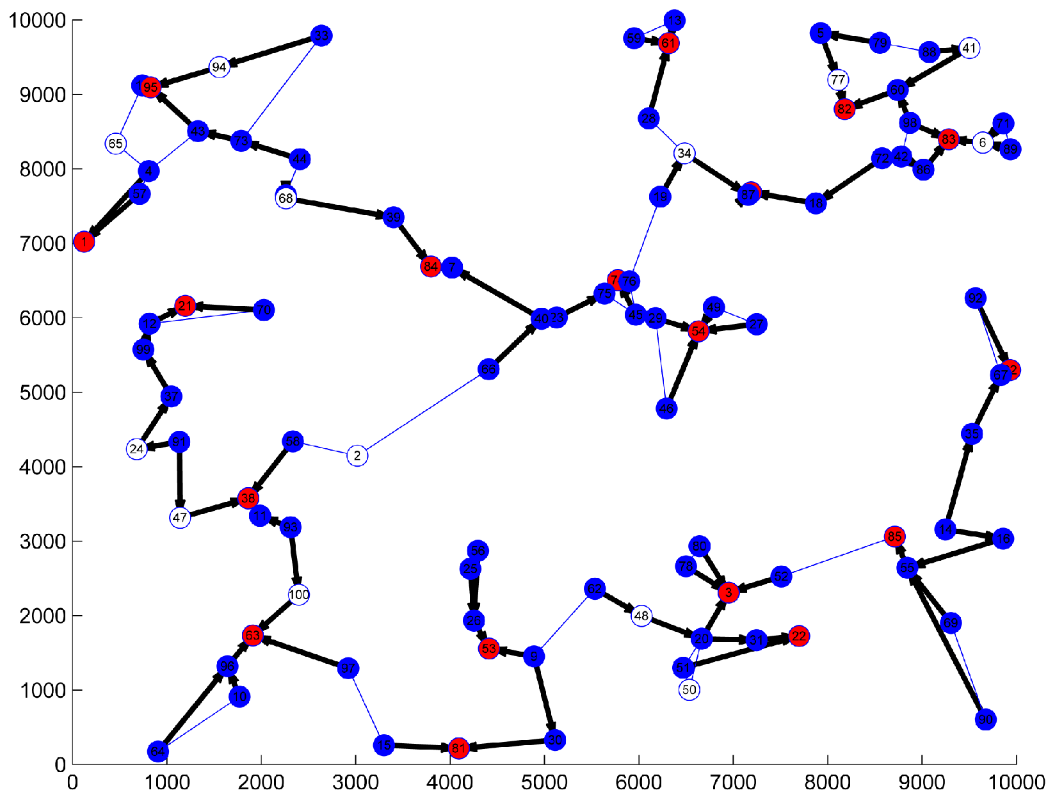


Figure 6.5: Visualization of hybrid algorithm results.

The results obtained by the hybrid algorithm implementation in MATLAB are illustrated for one instance of data and model (6.9)–(6.18) in Figure 6.5. A special post-processing procedure, dynamically supporting the visualization of the obtained results, has been implemented in MATLAB as well. The red nodes represent constructed waste processing units (e.g., incinerators), the blue nodes identify waste producers, and the white nodes are transition nodes. Let us emphasize that for these test computations to simplify the data instance coding, it is not assumed any already-built waste processing units. Then, the edges are differentiated by the flow. The black edge denotes a non-zero flow while the blue edge identifies a zero flow. Similarly, instances of various sizes have been tested and the collected experience is contained in Table 6.1. The average computational times show the expected increasing trends with an increase in the number of nodes and number of scenarios.

Table 6.1: Test results.

Number of nodes	10			20		40		50
Number of scenarios	1	5	10	1	5	1	3	1
Computational time [s]	27	137	184	46	1070	288	3027	427
Number of nodes	12			24		42		55
Number of scenarios	1	5	10	1	5	1	3	1
Computational time [s]	32	151	193	62	1122	309	3227	493
Number of nodes	14			28		44		60
Number of scenarios	1	5	10	1	5	1	3	1
Computational time [s]	39	163	205	71	1197	327	3302	564
Number of nodes	16			32		46		65
Number of scenarios	1	5	10	1	5	1	3	1
Computational time [s]	45	182	226	83	1251	339	3411	617
Number of nodes	18			28		48		70
Number of scenarios	1	5	10	1	5	1	3	1
Computational time [s]	53	199	245	96	1307	378	3571	691

6.3. Discussion

In the paper [A12], authors generalized a well-known FLP to the specific problems of waste processing; see [130] and [136]. They adopted the standpoint of the waste producers and minimize the WM costs which they face, which is derived from the related processing, transportation, and investment costs. They built two stochastic programs starting from the transportation network flow model with on-and-off waste-processing capacities in selected nodes and randomly-varying waste production modeled by scenarios. Then, pricing ideas from revenue management have been utilized to allow for the environmentally-friendly behavior of waste producers. For computational purposes, a modified hybrid algorithm is implemented in MATLAB and the obtained results are visualized. It is also assumed, as new waste management technologies are developed and various vehicles are needed during waste collecting, fleet size and vehicle routing problems (see, e.g., [84]) will necessarily be solved as part of the so-called smart cities projects.

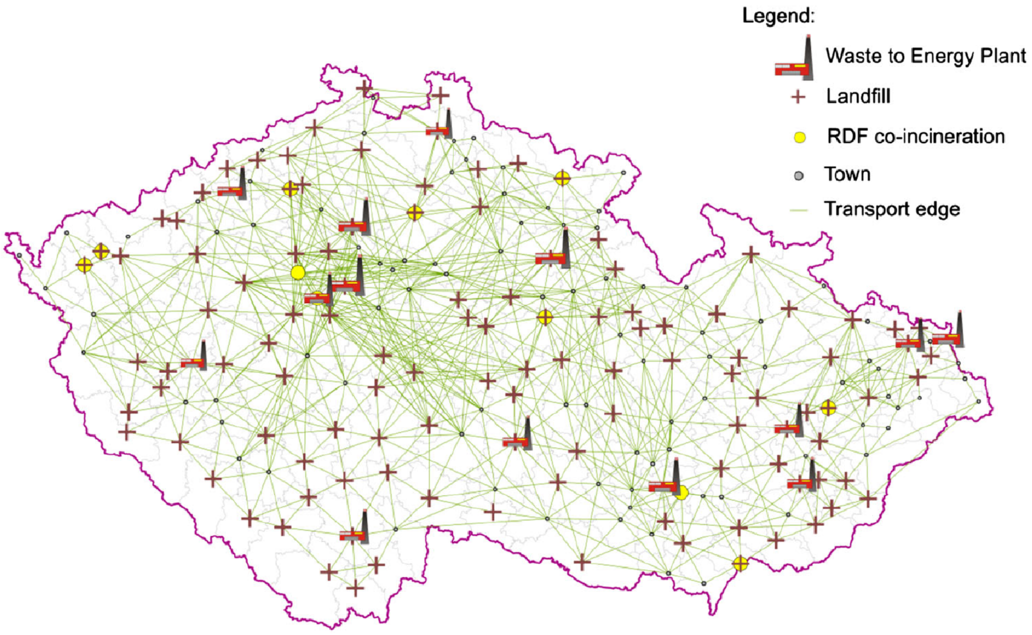


Figure 6.6: Real-world transportation network for the Czech Republic.

Chapter 7

Conclusions and Further Research

The thesis is based on two underlying demand-based problems: the newsvendor problem and the transportation network design problem. Both problems are used throughout the work as tools for examining how marketing decisions may affect the particular decisions of each of the underlying problems. Therefore, this chapter is further divided into particular conclusions on the newsvendor problem and, later, on the transportation network design problem.

7.1. Marketing decisions within NP's

NP and NPP

The NP (section 2.1) as well as the NPP (section 3.1) review some selected existing results that are further utilized in follow-up sections on the NPA and on the NPPA; in these sections, that are the core of the newsvendor related branch of the thesis, original results are provided.

However, in addition to the review contributions of the NP and NPP sections, there are also some original contributions in the work. Especially, the section on the NPP provides original insights into decision dependent randomness as a currently challenging research topic.

NPA

This study demonstrates the possibility of solving an extension of the NP model in which stocking quantity and advertising expenses are set simultaneously and in which the so-called multiplicative and additive demand cases were utilized; in order to deal with the level of uncertainty. The major point of the research was to obtain an appropriate model for demand with an appealing and intuitive economical interpretation - but one that still unified the existing concepts. Notation was adopted as used in the scientific literature available (on the NPP) so as to provide a simple structure for the model and its solution.

Employing this notation, an algorithm was devised to solve the problem that comprised the following steps: 1. Using input parameters, first an optimal stocking factor

z^* is determined; 2. By applying z^* and a concrete (suitable) function $d(a)$, an optimal advertising a^* can be expressed while z^* and a^* determine the optimal order quantity x^*.

Two appropriate-general demand functions are identified that stem from the literary sources on economics: the concave and the S-shaped response (demand) functions. For both of these, we not only solved the relevant problem, but also provided some necessary conditions to guarantee a solution that is also unique.

An important finding concerns the difference between Theorem 4.3.4 and the related observation for the additive case. It was revealed that optimal advertising in the multiplicative demand case never exceeds the optimal advertising in the equivalent deterministic model while, in the additive case, it is always equal; i.e., advertising is independent of the uncertainty involved. This difference can be explained by expressions (4.9) and (4.16), which show that for the additive case the loss function is independent of the advertising amount a. The reference sources mentioned ([58, 75, 86]) give rise to an appropriate discussion on a similar situation for NPP where the optimal price under conditions of multiplicative uncertain demand is never less than the riskless price while the optimal price under additive uncertain demand is never greater than the optimal riskless price.

The difference in observations on optimal advertising between the multiplicative and additive demand cases can also be explained by their variances and coefficients of variation, respectively. While in the additive case the variance of the demand is constant (independent of a), in the multiplicative case the variance is a function of the response function and the coefficient of variation is constant.

Several original, definite functions were identified that brought about potentially suitable advertising response functions, which subsequently produced several results and illustrations of numerical examples for the uniform distribution of the random variable.

NPPA: Comparison with NPP and NPA

In the section on the NPPA, the demand was modeled on the multiplicative-demand price-multiplicative demand form that corresponds to a combination of multiplicative demand forms in the NPP and the NPA. The observation for this case (i.e., NPPA in MDPM form) is equivalent to that of NPA and NPP: the optimal price for the multiplicative uncertain demand (in the isoelastic form) is always greater than or equal to the optimal riskless price while the optimal advertising (in the concave or S-shaped form) is always less than or equal to the optimal riskless advertising.

An interesting question arises: will marketing efforts increase the retailer's order quantities? Then, the following intuitive explanation follows: as the market becomes larger, the retailer may set a higher price to earn a higher margin per unit sold while ordering less to reduce the left-over inventory risk [124]. However, the intuitive explanation is disproved by our efforts: the optimal price is equivalent in the NPPA as well as in the NPP case (for the introduced multiplicative models), while the optimal advertising depends on the optimal price as well as on the price-demand (pricing) function and its optimal value.

From a managerial perspective, most of the findings given herein might seem somewhat theoretical. However, a problem that simultaneously comprises advertising and pricing decisions should be of greater practical importance. Obviously, the same holds for the indication that uncertainty does not necessarily lead to greater advertising expenses. It is the intention to continue to pursue such a direction in future research, with the expectation of achieving results that are both theoretically and practically relevant. The results add valuable managerial insight into these problems, which may be of higher importance in future volatile and globalized markets. The newsvendor platform is simple and does not cover the most complex practical situations. Still, the need for more research in this direction is evident.

Further research

The main challenging topics that were identified through the work consist of: an extension of the advertising response functions to other S-shaped functions (e.g., [65]), its generalization, and a focus on the variability of the random factor [62] as decision-dependent randomness [A5]. Moreover, the advertising decisions can be combined with other operational decisions such as pricing [86]; other aspects such as risk analysis [54] or any combination of the topics [20] are also challenging. Cost variability ideas, e.g., [81], may also lead to interesting new results.

7.2. Stochastic TNDP's with (and without) pricing

Section 2.2 subsequently describes models on: deterministic TNDP and stochastic TNDP with wait-and-see as well as here-and-now deterministic reformulations (approaches) that are further used in Section 3.2, where pricing is applied in the linear as well as in the nonlinear price-demand dependency.

Section 3.2.1 presents a specific network design problem with uncertain demands leading to the large-scale separable mixed integer bilinear program. The previously introduced hybrid algorithm (see [A6, A13]) has been modified and successfully tested for a more general bilinear case in comparison with the previous network flow case. It has proven our assumption that a suitable pricing strategy can decrease the variability of the network design solution. The linear function used in this section is not fully in accordance with real situations. However, the linear function used in our model may be replaced with a nonlinear one (e.g., isoelastic or hyperbolic); nevertheless, such a nonlinear function can be approximated by a piecewise linear function and the approach of the section can prove to be useful.

Such cases (i.e., the isoelastic or hyperbolic pricing functions) lead to stochastic mixed integer nonlinear models that provide new challenges and difficulties for our hybrid algorithmic approach. Therefore, the hybrid algorithm, which is suggested in Section 3.2.2, was modified for needs of such stochastic mixed-integer nonlinear problems. Apparently, the algorithm is portable and can be widely applied, especially, in large-scale problems where the exact solutions are not available or prove to be time-consuming.

Further research

A particularly interesting motivating application problem has recently appeared in the case of waste-to-energy generation. The problem of finding the optimal waste-to-energy plant capacity with respect to uncertain future demand for heat and electricity was discussed in [33]. Such a problem in its reduced form can be initially modeled by the NP. This simplification can be very useful for initial managerial strategic decisions about the principal investment level. This decision can further be made more precise by the utilization of a more complex model. In addition, the usual interplay between the strategic decision regarding capacity and returns under uncertain demands can be originally enriched by the impact of advertising. The advertising process is then related to the information campaign directed towards the waste producers while the pricing mechanism is linked to a so-called gate fee price for the processed waste. Hence, we plan to use the present research results in this recent real-world managerial problem, combining strategic investment policy and operational decisions with respect to uncertain demands.

Some of the ideas have already been utilized and published by the author in the following different but similar topics (papers): the vehicle routing problem with profits [A14] and the facility location problem for waste management [A4, A12].

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Appendix A

Supplementary calculations to the NP's

A.1. NPP supplementary calculations

(3.12) → (3.13): Consider the first derivative of the expected profit w.r.t. p given by expression (3.12). Moreover the following is considered: $\frac{\mathrm{d}z^*}{\mathrm{d}p} = \frac{1}{f(F^{-1}(\rho))}\frac{\mathrm{d}\rho}{\mathrm{d}p}$, where $\rho = \frac{p+s-c}{p+s-v}$ and where F^{-1} denotes the inverse function to function F. Then, the second derivative is as follows:

$$\begin{aligned}\frac{\mathrm{d}^2\Pi(z(p),p)}{\mathrm{d}p^2} =&(p-c)\frac{\mathrm{d}^2 d(p)}{\mathrm{d}p^2} + 2\frac{\mathrm{d}d(p)}{\mathrm{d}p} - \frac{\mathrm{d}\rho}{\mathrm{d}p}z^* + \frac{c-v}{p+s-v}\frac{1}{f(F^{-1}(\rho))}\frac{\mathrm{d}\rho}{\mathrm{d}p} + \frac{\mathrm{d}\rho}{\mathrm{d}p}z^* \\ &- F(F^{-1}(\rho))\frac{1}{f(F^{-1}(\rho))}\frac{\mathrm{d}\rho}{\mathrm{d}p} + \rho\frac{1}{f(F^{-1}(\rho))}\frac{\mathrm{d}\rho}{\mathrm{d}p}\end{aligned}$$

Then, substituting back the stocking factor $z^* = F^{-1}(\rho)$, the following is obtained:

$$\frac{\mathrm{d}^2\Pi(z(p),p)}{\mathrm{d}p^2} =(p-c)\frac{\mathrm{d}d^2(p)}{\mathrm{d}p^2} + 2\frac{\mathrm{d}d(p)}{\mathrm{d}p} + \frac{1}{f(z^*)}\frac{(c-v)^2}{(p+s-v)^3},$$

which proves the expression (3.13).

(3.17) → (3.18) → (3.19): Calculations in the multiplicative demand form are similar to that in the additive form (see above). Alternatively, see [129, 124]

A.2. NPA supplementary calculations

Herein, it is shown that the expected profit expressions (4.6) and (4.9) are equivalent. First, the NPA is considered in the form (4.5). Then, it is clear that

$$\Pi(a,z) = E_\xi[-czd(a) + pd(a)\min(z,\xi) - sd(a)\max(\xi - z, 0) + vd(a)\max(z-\xi,0) - a].$$

Substituting expected quantities given by (3.6) and (3.7) it is obtained that:

$$\begin{aligned}\Pi(a,z) =& -czd(a) + pd(a)\left[\int_A^z tf(t)dt + \int_z^B zf(t)dt\right] - sd(a)\int_z^B (t-z)f(t)dt \\ & + vd(a)\int_A^z (z-t)f(t)dt - a.\end{aligned}$$

This expression is clearly equivalent to (4.6). Considering that $z = \int_A^B zf(t)dt$ it is further observed that:

$$\begin{aligned}\Pi(a,z) =& -cd(a)\int_A^B zf(t)dt + \underbrace{pd(a)\left[\int_A^z tf(t)dt + \int_z^B tf(t)dt\right]}_{pd(a)\mathrm{E}[\xi]} \\ & - pd(a)\int_z^B tf(t)dt + cd(a)\int_A^B tf(t)dt + - \underbrace{cd(a)\int_A^B tf(t)dt}_{cd(a)\mathrm{E}[\xi]} \\ & + pd(a)\int_z^B zf(t)dt - sd(a)\int_z^B (t-z)f(t)dt + vd(a)\int_A^z (z-t)f(t)dt - a.\end{aligned}$$

Substituting back the expected quantities given by (3.6) and (3.7) and considering that $\mathrm{E}[\xi] = 1$ (see Section 4.3), we obtain:

$$\begin{aligned}\Pi(a,z) &= \underbrace{(p-c)d(a) - a}_{\Psi(a)} - \underbrace{d(a)\left[(c-v)\Lambda(z) + (p+s-c)\Theta(z)\right]}_{L(a,z)} \\ &= \Psi(a) - L(a,z).\end{aligned}$$

Then, it is proved that the expected profit expressions given by (4.6) and (4.9) are equivalent.

Appendix B

GAMS Data Set for the TNDP's

This file is a supplement material to papers [A8, A9, A6, A7, A3, A10, A13]. It is also published for all researchers dealing with TNDP that want to use our network setting and compare the results, alternatively. Therefore, in order to make our research more accessible, we provide (experimental) data of our network that we use through our papers dealing with modeling and solving techniques for stochastic TNDP, see, e.g., [A9].

B.1. Gams code: Data set

Further text/code parts are GAMS implementations of our network.

```
Option Seed=6;
Option OPTCR=0;
Option OPTCA=0;

Scalars n number of nodes /30/
        c1 number of customers /14/
        c2 number of plants   /2/
        g price /50/;
```

This means that we further work with a network of 30 points, where indices $1, 2, ..., 14$ (defined as Icus in GAMS) represents customers, $15, 16$ (Ipla) plants and $17, .., 30$ (Inod) "general" nodes. Coordinations of the nodes are randomly generated below (uniform distribution); using "Option seed = 6" guarantees that the "random numbers" are always the same.

```
Sets i points /1*30/
     s  scenarios /1*100/
     j coordinates / Xcoor, Ycoor /
 k edges
/1-8, 1-9, 1-10, 1-13, 1-14, 1-15, 1-19, 1-23, 1-24, 2-4, 2-5, 2-11, 2-20, 2-21, 2-25, 2-26, 2-28, 2-30,
3-5, 3-6, 3-7, 3-10, 3-13, 3-15, 3-19, 3-22, 3-23 ,3-28, 3-29, 4-2, 4-5, 4-10, 4-11, 4-16, 4-17, 4-20,
4-25, 4-26, 4-28, 4-29, 4-30, 5-2, 5-3, 5-4, 5-7, 5-10, 5-15, 5-22, 5-26, 5-28, 5-29, 5-30, 6-3, 6-7,
6-13, 6-15, 6-19, 6-23, 7-3, 7-5, 7-6, 7-10, 7-13, 7-15, 7-19, 7-22, 7-28, 7-29, 8-1, 8-9, 8-10, 8-11,
```

```
8-12, 8-17, 8-18, 8-23, 8-24, 8-25, 8-27, 9-1, 9-8, 9-10, 9-14, 9-15, 9-24,10-1, 10-3, 10-4, 10-5, 10-7,
10-8, 10-9, 10-11, 10-15, 10-16, 10-17, 10-22, 10-23, 10-24, 10-28, 10-29, 11-2, 11-4, 11-8, 11-10, 11-12,
11-16, 11-17, 11-18, 11-20, 11-21, 11-24, 11-25, 11-27, 11-29, 11-30, 12-8, 12-11, 12-17, 12-18, 12-21,
12-24, 12-25, 12-27, 13-1, 13-3, 13-6, 13-7, 13-14, 13-19, 13-23, 14-1, 14-9, 14-13, 14-19, 14-23, 15-1,
15-3, 15-5, 15-6, 15-7, 15-9, 15-10, 15-22, 15-23, 15-28, 15-29, 16-4, 16-10, 16-11, 16-17, 16-24,
16-29, 17-4, 17-8, 17-10, 17-11, 17-12, 17-16, 17-18, 17-21, 17-24, 17-25, 17-27, 17-29, 18-8, 18-11,
18-12, 18-17, 18-21, 18-24, 18-25, 18-27, 19-1, 19-3, 19-6, 19-7, 19-13, 19-14, 19-23, 20-2, 20-4, 20-11,
20-21, 20-25, 20-30, 21-2, 21-11, 21-12, 21-17, 21-18, 21-20, 21-24, 21-25, 21-27, 21-30, 22-3, 22-5, 22-7,
22-10, 22-15, 22-28, 22-29, 23-1, 23-3, 23-6, 23-8, 23-10, 23-13, 23-14, 23-15, 23-19, 23-24, 24-1,
24-8, 24-9, 24-10, 24-11, 24-12, 24-16, 24-17, 24-18, 24-21, 24-23, 24-25, 24-27, 24-29, 25-2, 25-4, 25-8,
25-11, 25-12, 25-17, 25-18, 25-20, 25-21, 25-24, 25-27, 25-30, 26-2, 26-4, 26-5, 26-28, 26-29, 26-30,
27-8, 27-11, 27-12, 27-17, 27-18, 27-21, 27-24, 27-25, 28-2, 28-3, 28-4, 28-5, 28-7, 28-10, 28-15,
28-22, 28-26, 28-29, 28-30, 29-3, 29-4, 29-5, 29-7, 29-10, 29-11, 29-15, 29-16, 29-17, 29-22, 29-24,
29-26, 29-28, 30-2, 30-4, 30-5, 30-11, 30-20, 30-21, 30-25, 30-26, 30-28/

k1(k) edges /1-8, 1-10, 1-13, 1-15, 1-19, 1-24, 2-4, 2-5, 2-11, 2-20, 2-21, 2-25, 2-26, 2-28, 3-5, 3-6,
3-10, 3-13, 3-19, 3-22, 3-23 ,3-28, 3-29, 4-2, 4-5, 4-10, 4-16, 4-17, 4-20, 4-25, 4-26, 4-28, 5-2, 5-3,
5-4, 5-7, 5-10, 5-15, 5-22, 5-26, 5-28, 5-30, 6-3, 6-7, 6-13, 6-15, 6-19, 7-5, 7-6, 7-10, 7-13, 7-15, 7-19,
7-22, 7-28, 7-29, 8-1, 8-9, 8-10, 8-11, 8-17, 8-18, 8-23, 8-25, 8-27, 9-8, 9-10, 9-14, 9-15, 9-24, 10-1,
10-3, 10-4, 10-5, 10-7, 10-8, 10-9, 10-11, 10-16, 10-17, 10-22, 10-23, 10-28, 11-2, 11-8, 11-10, 11-12,
11-16, 11-18, 11-20, 11-21, 11-24, 11-27, 11-29, 11-30, 12-11, 12-17, 12-18, 12-24, 12-25, 12-27, 13-1,
13-3, 13-6, 13-7, 13-14, 13-19, 14-9, 14-13, 14-19, 14-23, 15-1, 15-5, 15-6, 15-7, 15-9, 15-22, 15-28,
15-29, 16-4, 16-10, 16-11, 16-17, 16-24, 17-4, 17-8, 17-10, 17-12, 17-16, 17-18, 17-21, 17-25, 17-27,
17-29, 18-8, 18-11, 18-12, 18-17, 18-21, 18-24, 18-25, 18-27, 19-1, 19-3, 19-6, 19-7, 19-13, 19-14,
19-23, 20-2, 20-4, 20-11, 20-21, 20-25, 20-30, 21-2, 21-11, 21-17, 21-18, 21-20, 21-24, 21-27, 21-30,
22-3, 22-5, 22-7, 22-10, 22-15, 22-28, 22-29, 23-3, 23-8, 23-10, 23-14, 23-19, 23-24, 24-1, 24-9, 24-11,
24-12, 24-16, 24-18, 24-21, 24-23, 24-25, 24-27, 24-29, 25-2, 25-4, 25-8, 25-12, 25-17, 25-18, 25-20,
25-24, 25-27, 25-30, 26-2, 26-4, 26-5, 26-28, 26-29, 26-30, 27-8, 27-11, 27-12, 27-17, 27-18, 27-21,
27-24, 27-25, 28-2, 28-3, 28-4, 28-5, 28-7, 28-10, 28-15, 28-22, 28-26, 28-29, 28-30, 29-3, 29-7,
29-11, 29-15, 29-17, 29-22, 29-24, 29-26, 29-28, 30-5, 30-11, 30-20, 30-21, 30-25, 30-26, 30-28/

Parameter Node(i,j);
          Node(i,j) = uniform(0,100);

Parameter p(s) probability of a scenario s /1 0.2, 2 0.2, 3 0.2, 4 0.2, 5 0.2/;

Parameters
c(k) transporting cost (k)
/1-8 102, 1-9 1, 1-10 17, 1-13 13, 1-14 2, 1-15 18, 1-19 18, 1-23 5, 1-24 26, 2-4 22, 2-5 131, 2-11 24,
2-20 14, 2-21 89, 2-25 21, 2-26 3, 2-28 138, 2-30 3, 3-5 27, 3-6 10, 3-7 1, 3-10 35, 3-13 34, 3-15 19,
3-19 58, 3-22 5, 3-23 29, 3-28 35, 3-29 62, 4-2 22, 4-5 46, 4-10 60, 4-11 3, 4-16 7, 4-17 16, 4-20 8,
4-25 8, 4-26 11, 4-28 51, 4-29 6, 4-30 9, 5-2 131, 5-3 27, 5-4 46, 5-7 21, 5-10 71, 5-15 67, 5-22 16,
5-26 96, 5-28 1, 5-29 21, 5-30 95, 6-3 10, 6-7 19, 6-13 7, 6-15 12, 6-19 19,6-23 6, 7-3 1, 7-5 21, 7-6
19, 7-10 45, 7-13 48, 7-15 28, 7-19 76, 7-22 8, 7-28 26, 7-29 71, 8-1 102, 8-9 87, 8-10 70, 8-11 53,
8-12 1, 8-17 33, 8-18 3, 8-23 134, 8-24 33, 8-25 44, 8-27 3, 9-1 1, 9-8 87, 9-10 26, 9-14 5, 9-15 29,
9-24 31, 10-1 17, 10-3 35, 10-4 60, 10-5 71, 10-7 45, 10-8 70, 10-9 26, 10-11 42, 10-15 3, 10-16 30,
10-17 18, 10-22 23, 10-23 11, 10-24 7, 10-28 87, 10-29 37, 11-2 24, 11-4 3, 11-8 53, 11-10 42, 11-12 51,
11-16 7, 11-17 6, 11-18 33, 11-20 6, 11-21 36, 11-24 33, 11-25 5, 11-27 30, 11-29 7, 11-30 13, 12-8 1,
12-11 51, 12-17 35, 12-18 3, 12-21 2, 12-24 37, 12-25 41, 12-27 3, 13-1 13, 13-3 34, 13-6 7, 13-7 48,
13-14 10, 13-19 3, 13-23 4, 14-1 2, 14-9 5, 14-13 10, 14-19 8, 14-23 12, 15-1 18, 15-3 19, 15-5 67,
15-6 12, 15-7 28, 15-9 29, 15-10 3, 15-22 18, 15-23 5, 15-28 83, 15-29 38, 16-4 7, 16-10 30, 16-11 7,
16-17 4, 16-24 28, 16-29 1, 17-4 16, 17-8 33, 17-10 18, 17-11 6, 17-12 35, 17-16 4, 17-18 17, 17-21 36,
17-24 12, 17-25 12, 17-27 17, 17-29 6, 18-8 3, 18-11 33, 18-12 3, 18-17 17, 18-21 5, 18-24 20, 18-25 27,
18-27 1, 19-1 18, 19-3 58, 19-6 19, 19-7 76, 19-13 3, 19-14 8, 19-23 9, 20-2 14, 20-4 8, 20-11 6,
20-21 34, 20-25 1, 20-30 9, 21-2 89, 21-11 36, 21-12 2, 21-17 36, 21-18 5, 21-20 34, 21-24 45, 21-25 25,
21-27 5, 21-30 77, 22-3 5, 22-5 16, 22-7 8, 22-10 23, 22-15 18, 22-28 24, 22-29 32, 23-1 5, 23-3 29,
23-6 6,  23-8 134, 23-10 11, 23-13 4, 23-14 12, 23-15 5, 23-19 9, 23-24 35, 24-1 26, 24-8 33, 24-9 31,
24-10 7, 24-11 33, 24-12 37, 24-16 28, 24-17 12, 24-18 20, 24-21 45, 24-23 35, 24-25 33, 24-27 21,
24-29 35, 25-2 21, 25-4 8, 25-8 44, 25-11 5,  25-12 41, 25-17 12, 25-18 27, 25-20 1, 25-21 25, 25-24 33,
25-27 24, 25-30 15, 26-2 3, 26-4 11, 26-5 96, 26-28 103, 26-29 32, 26-30 1, 27-8 3, 27-11 30, 27-12 3,
27-17 17, 27-18 1, 27-21 5, 27-24 21, 27-25 24, 28-2 138, 28-3 35, 28-4 51, 28-5 1, 28-7 26, 28-10 87,
28-15 83, 28-22 24, 28-26 103, 28-29 26, 28-30 102, 29-3 62, 29-4 6, 29-5 21, 29-7 71, 29-10 37, 29-11 7,
29-15 37, 29-16 1, 29-17 6, 29-22 32, 29-24 35, 29-26 32, 29-28 26, 30-2 3, 30-4 9, 30-5 95, 30-11 13,
30-20 9, 30-21 77, 30-25 15, 30-26 1, 30-28 102/

d(k1)  cost of building a new edge (j) - default value 0
/ 1-8 326, 1-10 40, 1-13 40, 1-15 51, 1-19 64, 1-24 73, 2-4 61, 2-5 364, 2-11 96, 2-20 33, 2-21 285,
2-25 83, 2-26 5, 2-28 551, 3-5 64, 3-6 21, 3-10 139, 3-13 82, 3-19 140, 3-22 11, 3-23 92, 3-28 99,
3-29 123, 4-2 61, 4-5 112, 4-10 193, 4-16 19, 4-17 56, 4-20 23, 4-25 24, 4-26 30, 4-28 103, 5-2 364,
5-3 64, 5-4 112, 5-7 68, 5-10 170, 5-15 268, 5-22 38, 5-26 231, 5-28 2, 5-30 344, 6-3 21, 6-7 60,
6-13 22, 6-15 44, 6-19 47, 7-5 68, 7-6 60, 7-10 126, 7-13 115, 7-15 67, 7-19 274, 7-22 15, 7-28 73,
```

```
7-29 199, 8-1 326, 8-9 314, 8-10 196, 8-11 148, 8-17 119, 8-18 10, 8-23 428, 8-25 142, 8-27 8, 9-8 314,
9-10 84, 9-14 17, 9-15 103, 9-24 111, 10-1 40, 10-3 139, 10-4 193, 10-5 170, 10-7 126, 10-8 196, 10-9 84,
10-11 134, 10-16 107, 10-17 64, 10-22 46, 10-23 43, 10-28 244, 11-2 96, 11-8 148, 11-10 134, 11-12 121,
11-16 27, 11-18 131, 11-20 25, 11-21 145, 11-24 107, 11-27 121, 11-29 23, 11-30 36, 12-11 121, 12-17 69,
12-18 638, 12-24 120, 12-25 165, 12-27 306, 13-1 40, 13-3 82, 13-6 22, 13-7 115, 13-14 36, 13-19 9,
14-9 17, 14-13 36, 14-19 30, 14-23 49, 15-1 51, 15-5 268, 15-6 44, 15-7 67, 15-9 103, 15-22 43, 15-28 232,
15-29 150, 16-4 19, 16-10 107, 16-11 27, 16-17 14, 16-24 110, 17-4 56, 17-8 119, 17-10 64, 17-12 69,
17-16 14, 17-18 40, 17-21 72, 17-25 29, 17-27 47, 17-29 13, 18-8 10, 18-11 131, 18-12 338, 18-17 40,
18-21 19, 18-24 62, 18-25 53, 18-27 1, 19-1 64, 19-3 140, 19-6 67, 19-7 274, 19-13 9, 19-14 30, 19-23 29,
20-2 33, 20-4 23, 20-11 25, 20-21 95, 20-25 2, 20-30 21, 21-2 251, 21-11 145, 21-17 72, 21-18 19,
21-20 95, 21-24 145, 21-27 11, 21-30 276, 22-3 11, 22-5 38, 22-7 15, 22-10 46, 22-15 43, 22-28 58,
22-29 116, 23-3 92, 23-8 427, 23-10 43, 23-14 49, 23-19 29, 23-24 97, 24-1 73, 24-9 111, 24-11 107,
24-12 120, 24-16 110, 24-18 62, 24-21 145, 24-23 97, 24-25 132, 24-27 58, 24-29 139, 25-2 83, 25-4 24,
25-8 142, 25-12 165, 25-17 29, 25-18 53, 25-20 2, 25-24 132, 25-27 88, 25-30 58, 26-2 5, 26-4 30,
26-5 231, 26-28 371, 26-29 103, 26-30 2, 27-8 8, 27-11 121, 27-12 306, 27-17 47, 27-18 1, 27-21 11,
27-24 58, 27-25 88, 28-2 551, 28-3 99, 28-4 103, 28-5 2, 28-7 73, 28-10 244, 28-15 232, 28-22 58,
28-26 371, 28-29 71, 28-30 245, 29-3 123, 29-7 199, 29-11 23, 29-15 150, 29-17 13, 29-22 116, 29-24 139,
29-26 103, 29-28 71, 30-5 344, 30-11 36, 30-20 21, 30-21 276, 30-25 53, 30-26 2, 30-28 245/;

Parameters  qplus(i) compensation
            qminus(i) compensation;

Loop(Icus, qplus(Icus)=100);
Loop(Icus, qminus(Icus)= 100);
```

The incidence matrix $A(i,k)$ can be implemented via "include" statement in GAMS as follows:

```
\$INCLUDE D:\....\incidence_matrix.txt
```

The same can be done with the demand file. It can either be randomly generated as

```
Parameters  pro(i) capacity of i(rpoduction)
            dem(s,i) capacity of i in scenario s(demands)

Loop(s, Loop(Icus, dem(s,Icus)=uniformint(10,20)));
Loop(Ipla, pro(Ipla)=-(sum(Icus, sum(s,dem(s,Icus))))/(c3*card(s)));
```

or you can use our file:

```
\$INCLUDE D:\....\demand.txt
```

See next section - it is shown how can the file look like.

The network is presented in, beside others, Figure 2.2 (alternatively, Figure 2 in [A9]); see the paper for our optimization model, hybrid/heuristic solution technique as well as graphical solution of the two-stage stochastic (scenario-based) TNDP.

B.2. Demand.txt

```
Table dem(s,i) demand
    1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
1  17 16 13 17 16 19 11 10 14 10 17 13 11 17 138 63 0  0  0  0  0  0  0  0  0  0  0  0  0  0
2  13 11 17 10 17 10 10 13 13 16 12 11 10 10 63 110 0  0  0  0  0  0  0  0  0  0  0  0  0  0
3  13 15 15 19 17 11 14 13 10 19 18 18 16 18 152 64 0  0  0  0  0  0  0  0  0  0  0  0  0  0
```

4	13	19	12	15	15	13	17	16	13	16	18	14	15	19	138	77	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	12	13	12	16	10	13	15	14	10	18	12	18	19	16	84	114	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	17	10	16	12	13	13	12	10	15	17	18	12	12	10	109	78	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	19	15	11	13	16	19	19	16	14	13	15	11	13	14	65	143	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	11	18	10	11	13	11	18	13	15	18	19	13	12	18	121	79	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	11	13	12	14	18	13	12	18	15	15	14	11	19	11	110	86	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	16	10	18	11	17	15	16	18	16	13	14	13	17	12	135	71	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	13	17	14	11	15	14	17	11	18	14	17	19	10	15	67	138	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	15	12	14	12	14	15	13	13	19	10	17	10	18	14	93	103	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	10	13	19	12	12	19	16	13	13	15	18	14	15	19	90	118	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	12	11	13	17	18	18	19	14	13	11	17	19	17	10	92	117	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	19	12	18	17	14	19	11	19	10	10	15	14	16	15	104	105	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	12	16	12	17	13	15	17	18	12	12	16	12	15	15	111	91	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	16	16	17	11	16	15	15	11	17	13	15	14	11	19	117	89	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	18	16	13	12	14	10	11	15	15	13	17	12	12	14	79	113	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	16	13	17	15	15	11	10	14	14	15	15	18	19	11	116	87	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	18	17	16	16	11	14	11	15	15	17	13	16	12	17	75	133	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	18	12	18	19	17	15	18	15	12	12	18	19	13	11	123	94	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	17	13	14	11	13	13	10	13	12	14	12	11	14	15	116	66	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	17	17	11	14	14	17	15	18	12	17	18	14	18	18	146	74	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	12	13	17	13	19	13	16	17	17	12	18	15	11	18	64	147	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	16	14	14	13	11	16	10	18	15	17	13	17	13	18	149	56	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	19	13	14	10	19	10	12	12	15	14	14	10	12	19	107	86	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	13	15	19	15	14	19	16	13	14	16	14	19	14	15	133	83	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	16	16	19	17	11	11	18	15	11	10	14	10	12	14	125	69	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	18	10	15	16	18	11	14	18	15	13	11	14	11	10	72	122	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	10	11	15	13	13	19	18	19	16	13	18	19	11	16	67	144	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	12	11	10	17	17	15	19	12	16	17	13	12	12	16	71	128	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	15	10	18	15	19	10	14	19	14	14	12	18	13	17	93	115	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	19	18	13	16	15	14	14	12	10	14	18	15	18	14	153	57	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	12	11	10	10	19	10	16	19	12	10	18	16	17	16	73	123	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	18	15	16	13	13	13	11	10	18	15	19	18	18	14	114	97	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	12	17	13	15	13	15	14	10	17	11	10	13	10	18	95	93	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	11	14	12	14	17	15	15	13	15	14	12	15	16	14	50	147	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	14	14	10	15	13	13	13	11	17	12	13	12	14	14	136	49	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	11	19	10	18	19	19	11	17	17	14	13	16	12	15	132	79	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	10	10	10	15	10	15	13	15	17	14	16	11	18	14	50	138	0	0	0	0	0	0	0	0	0	0	0	0	0	0
41	17	11	16	13	18	13	11	12	18	18	12	19	16	15	152	57	0	0	0	0	0	0	0	0	0	0	0	0	0	0
42	16	18	17	18	15	19	16	13	14	15	14	15	18	11	75	144	0	0	0	0	0	0	0	0	0	0	0	0	0	0
43	10	15	10	12	18	14	17	17	14	17	18	13	14	12	66	135	0	0	0	0	0	0	0	0	0	0	0	0	0	0
44	15	14	17	17	15	13	13	19	14	16	11	11	19	18	84	128	0	0	0	0	0	0	0	0	0	0	0	0	0	0
45	16	11	19	10	10	10	14	12	13	15	15	10	12	18	56	129	0	0	0	0	0	0	0	0	0	0	0	0	0	0
46	16	17	13	14	12	17	13	13	12	12	13	17	18	19	92	114	0	0	0	0	0	0	0	0	0	0	0	0	0	0
47	11	19	16	16	15	12	16	14	15	14	18	14	15	13	92	116	0	0	0	0	0	0	0	0	0	0	0	0	0	0
48	16	12	14	18	15	13	14	15	12	15	18	17	11	16	123	83	0	0	0	0	0	0	0	0	0	0	0	0	0	0
49	14	19	11	12	14	11	10	18	12	12	16	15	11	13	137	51	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	17	15	17	16	18	12	14	14	10	12	19	12	11	11	87	111	0	0	0	0	0	0	0	0	0	0	0	0	0	0
51	12	11	14	10	11	11	10	18	18	12	16	17	12	12	100	84	0	0	0	0	0	0	0	0	0	0	0	0	0	0
52	10	11	13	17	15	15	10	13	18	15	12	14	19	19	126	75	0	0	0	0	0	0	0	0	0	0	0	0	0	0
53	18	15	14	10	18	19	10	19	12	17	19	13	15	17	98	118	0	0	0	0	0	0	0	0	0	0	0	0	0	0
54	14	12	18	14	14	18	16	16	17	11	15	11	18	14	109	99	0	0	0	0	0	0	0	0	0	0	0	0	0	0
55	19	12	16	10	16	15	10	11	15	16	14	11	15	16	62	134	0	0	0	0	0	0	0	0	0	0	0	0	0	0
56	15	15	17	15	13	16	17	19	13	15	13	16	19	11	126	88	0	0	0	0	0	0	0	0	0	0	0	0	0	0
57	19	15	18	17	19	13	17	18	15	19	17	13	10	11	105	116	0	0	0	0	0	0	0	0	0	0	0	0	0	0
58	12	19	12	11	10	13	10	10	19	16	11	19	18	14	114	80	0	0	0	0	0	0	0	0	0	0	0	0	0	0
59	17	17	17	14	11	12	19	18	13	18	13	16	10	14	73	136	0	0	0	0	0	0	0	0	0	0	0	0	0	0
60	12	11	14	15	16	14	15	17	17	12	12	14	14	15	91	107	0	0	0	0	0	0	0	0	0	0	0	0	0	0
61	10	13	15	19	13	18	13	13	14	10	15	16	10	13	108	84	0	0	0	0	0	0	0	0	0	0	0	0	0	0
62	10	13	14	13	14	13	19	18	16	14	12	17	13	13	88	111	0	0	0	0	0	0	0	0	0	0	0	0	0	0
63	15	10	16	12	14	19	13	18	17	19	12	18	16	19	132	86	0	0	0	0	0	0	0	0	0	0	0	0	0	0
64	13	10	16	11	12	16	11	17	18	10	10	17	16	14	62	129	0	0	0	0	0	0	0	0	0	0	0	0	0	0
65	19	13	19	14	16	12	18	18	13	16	17	10	18	13	124	92	0	0	0	0	0	0	0	0	0	0	0	0	0	0
66	19	19	16	18	14	14	11	14	10	10	17	10	10	16	100	98	0	0	0	0	0	0	0	0	0	0	0	0	0	0
67	11	18	13	14	13	18	13	16	15	15	12	14	10	15	61	136	0	0	0	0	0	0	0	0	0	0	0	0	0	0
68	14	15	16	15	13	11	16	15	15	11	12	19	15	11	65	133	0	0	0	0	0	0	0	0	0	0	0	0	0	0
69	14	16	16	12	17	16	17	15	11	12	19	15	14	11	124	81	0	0	0	0	0	0	0	0	0	0	0	0	0	0
70	14	15	16	11	17	11	14	18	10	19	14	18	10	18	146	59	0	0	0	0	0	0	0	0	0	0	0	0	0	0
71	11	19	13	14	16	10	19	13	14	13	13	17	18	17	75	132	0	0	0	0	0	0	0	0	0	0	0	0	0	0
72	14	13	19	15	15	15	13	17	19	19	12	14	12	11	54	154	0	0	0	0	0	0	0	0	0	0	0	0	0	0
73	11	14	15	10	19	11	16	11	11	10	10	12	13	17	103	77	0	0	0	0	0	0	0	0	0	0	0	0	0	0
74	19	16	16	12	19	10	16	15	10	18	10	19	19	14	142	71	0	0	0	0	0	0	0	0	0	0	0	0	0	0

75	16	13	10	12	14	16	12	17	15	14	16	18	14	19	70	136	0	0	0	0	0	0	0	0	0	0	0	0	0	0
76	13	11	11	17	19	13	13	16	15	13	16	13	15	12	73	124	0	0	0	0	0	0	0	0	0	0	0	0	0	0
77	18	19	12	11	15	14	14	11	13	17	19	10	10	14	136	61	0	0	0	0	0	0	0	0	0	0	0	0	0	0
78	10	15	18	10	15	12	17	15	18	13	19	14	11	18	60	145	0	0	0	0	0	0	0	0	0	0	0	0	0	0
79	16	15	17	15	12	13	15	15	15	11	18	13	16	17	66	142	0	0	0	0	0	0	0	0	0	0	0	0	0	0
80	15	14	10	12	13	14	14	13	18	15	15	10	16	19	85	113	0	0	0	0	0	0	0	0	0	0	0	0	0	0
81	12	13	19	12	16	12	18	17	16	18	16	17	12	17	76	139	0	0	0	0	0	0	0	0	0	0	0	0	0	0
82	14	15	15	14	15	16	18	19	14	14	15	19	11	16	94	121	0	0	0	0	0	0	0	0	0	0	0	0	0	0
83	19	17	13	17	13	10	15	13	13	18	13	13	14	17	149	56	0	0	0	0	0	0	0	0	0	0	0	0	0	0
84	16	11	14	17	19	18	11	18	15	14	18	16	15	18	141	79	0	0	0	0	0	0	0	0	0	0	0	0	0	0
85	16	18	13	11	13	11	18	14	19	18	11	15	14	10	132	69	0	0	0	0	0	0	0	0	0	0	0	0	0	0
86	19	11	16	14	19	13	17	12	19	10	17	14	10	16	51	156	0	0	0	0	0	0	0	0	0	0	0	0	0	0
87	14	10	11	19	11	15	17	11	14	18	12	14	15	18	77	122	0	0	0	0	0	0	0	0	0	0	0	0	0	0
88	11	17	16	11	16	12	18	13	12	14	18	17	10	11	123	73	0	0	0	0	0	0	0	0	0	0	0	0	0	0
89	11	17	12	16	16	15	13	10	17	18	18	11	14	17	104	101	0	0	0	0	0	0	0	0	0	0	0	0	0	0
90	16	16	11	13	13	12	19	19	14	18	16	19	11	13	73	137	0	0	0	0	0	0	0	0	0	0	0	0	0	0
91	18	11	10	17	10	19	10	19	14	15	16	11	17	16	61	142	0	0	0	0	0	0	0	0	0	0	0	0	0	0
92	15	13	19	18	14	11	12	16	19	10	11	11	16	17	135	67	0	0	0	0	0	0	0	0	0	0	0	0	0	0
93	13	10	17	10	18	13	16	17	12	11	17	13	10	10	71	116	0	0	0	0	0	0	0	0	0	0	0	0	0	0
94	17	12	16	13	16	17	13	13	10	17	18	12	16	18	132	76	0	0	0	0	0	0	0	0	0	0	0	0	0	0
95	15	13	16	11	15	13	11	10	12	13	12	11	19	16	67	120	0	0	0	0	0	0	0	0	0	0	0	0	0	0
96	12	18	18	10	14	11	15	10	12	18	18	14	19	11	81	119	0	0	0	0	0	0	0	0	0	0	0	0	0	0
97	12	13	18	12	10	18	15	17	18	16	12	17	17	15	156	54	0	0	0	0	0	0	0	0	0	0	0	0	0	0
98	11	10	17	10	14	16	12	15	16	16	19	10	15	15	103	93	0	0	0	0	0	0	0	0	0	0	0	0	0	0
99	16	13	19	19	16	19	10	12	11	14	17	15	16	19	116	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0
100	11	15	16	13	18	13	16	17	12	14	17	14	13	12	149	52	0	0	0	0	0	0	0	0	0	0	0	0	0	0

B.3. Incidence_matrix.txt

Table A(i,k) incidence matrix of the graph/network

	1-9	1-14	1-23	2-30	3-7	3-15	4-11	4-29	4-30	5-29	6-23	7-3	8-12	8-24	9-1	10-15	10-24	10-29	11-4	11-17	11-25	12-8	12-21	13-23	14-1	15-3	15-10
1	-1	-1	-1												1										1		
2				-1																							
3					-1	-1						1														1	
4							-1	-1	-1										1								
5										-1																	
6											-1																
7					1							-1															
8													-1	-1								1					
9	1														-1												
10																-1	-1	-1									1
11							1												-1	-1	-1						
12													1									-1	-1				
13																								-1			
14		1																							-1		
15						1										1										-1	-1
16																											
17																				1							
18																											
19																											
20																											
21																							1				
22																											
23			1								1													1			
24														1			1										
25																					1						
26																											
27																											
28																											
29								1		1								1									
30				1					1																		

\+

	15-23	16-29	17-11	17-24	21-12	21-25	23-1	23-6	23-13	23-15	24-8	24-10	24-17	25-11	25-21	29-4	29-5	29-10	29-16	30-2	30-4
1							1														
2																				1	
3																					
4																1					1
5																	1				
6								1													
7																					
8											1										
9																					
10												1						1			
11			1											1							
12					1																
13									1												
14																					
15	-1									1											
16		-1																	1		
17			-1	-1									1								

18																				
19																				
20																				
21				−1	−1									1						
22																				
23	1					−1	−1	−1	−1											
24			1							−1	−1	−1								
25					1								−1	−1						
26																				
27																				
28																				
29		1													−1	−1	−1	−1		
30																			−1	−1

\+

	1-8	1-10	1-13	1-15	1-19	1-24	2-4	2-5	2-11	2-20	2-21	2-25	2-26	2-28	3-5	3-6	3-10	3-13	3-19	3-22	3-23	3-28	3-29	4-2	4-5	4-10	4-16	4-17	4-20	4-25
1	−1	−1	−1	−1	−1	−1																								
2							−1	−1	−1	−1	−1	−1	−1	−1										1						
3															−1	−1	−1	−1	−1	−1	−1	−1	−1							
4							1																	−1	−1	−1	−1	−1	−1	−1
5								1							1										1					
6																1														
7																														
8	1																													
9																														
10		1															1									1				
11									1																					
12																														
13			1															1												
14																														
15				1																										
16																											1			
17																												1		
18																														
19					1														1											
20										1																			1	
21											1																			
22																				1										
23																					1									
24						1																								
25												1																		1
26													1																	
27																														
28														1								1								
29																							1							
30																														

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	4-26	4-28	5-2	5-3	5-4	5-7	5-10	5-15	5-22	5-26	5-28	5-30	6-3	6-7	6-13	6-15	6-19	7-5	7-6	7-10	7-13	7-15	7-19	7-22	7-28	7-29	8-1	8-9	8-10	8-11	8-17	8-18
1																											1					
2			1																													
3				1									1																			
4	−1	−1			1																											
5			−1	−1	−1	−1	−1	−1	−1	−1	−1	−1						1														
6													−1	−1	−1	−1	−1		1													
7						1								1				−1	−1	−1	−1	−1	−1	−1	−1	−1						
8																											−1	−1	−1	−1	−1	−1
9																												1				
10							1													1									1			
11																														1		
12																																
13															1						1											
14																																
15								1								1						1										
16																																
17																															1	
18																																1
19																	1						1									
20																																
21																																
22									1															1								
23																																
24																																
25																																
26	1									1																						
27																																
28		1									1														1							
29																										1						
30												1																				

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	8-23	8-25	8-27	9-8	9-10	9-14	9-15	9-24	10-1	10-3	10-4	10-5	10-7	10-8	10-9	10-11	10-16	10-17	10-22	10-23	10-28	11-2	11-8	11-10	11-12	11-16	11-18
1									1																		
2																						1					
3										1																	
4											1																
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List of symbols and abbreviations used

Abbreviations

EO	Expected objective
FLP	Facility location problem
GA	Genetic algorithm
HN	Here-and-now
IP	Integer programming
LP	Linear programming
MDPA	Marketing-dependent price-additive (demand)
MDPM	Marketing-dependent price-multiplicative (demand)
MILP	Mixed-integer linear programming/programme
MINLP	Mixed-integer nonlinear programming/programme
NDP	Network design problem
NLP	Nonlinear programming
NP	Newsvendor problem
NPA	Newsvendor problem with advertising
NPP	Newsvendor problem with pricing (or Newsvendor pricing problem)
NPPA	Newsvendor problem with (joint) pricing and advertising
SP	Stochastic programming
s. t.	subject to
TNDP	Transportation network design problem
WM	Waste management
WS	Wait-and-see

Symbols: Unified

$\mathbb{R}$	the set of real numbers
$\mathbb{R}^n$	the set of real n-tuples

Symbols: Newsvendor problems (NP, NPP, NPA and NPPA)

a	advertising amount, alternatively function parameter in the NPP
A, B	lower and upper bounds of the random variable
b	function parameter in the NPP
c	buying cost per unit
d	demand function (i.e. $d(p), d(a)$ and $d(a, p)$)
D	general demand random factor-dependent function
e	price elasticity function; alternatively, marketing effort

List of symbols

$E(\cdot)$	expectation operator
$f(\cdot)$	probability distribution/density function (pdf)
$F(\cdot)$	cumulative distribution function (cdf)
g	generalized failure rate function
L	(expected) loss function
p	price per unit (selling)
r	failure rate function
s	shortage cost
t	variable of integration (instead of random variable ξ)
v	salvage value
x	first-stage decision variable - an ordering quantity
z	stocking factor
α	a function parameter
β	function parameter
δ	a response function parameter
γ	a response function parameter
Λ	expected leftovers
π	profit (objective) function
Π	expected profit function
Ψ	riskless profit function
ω	a response function parameter
ρ	critical ratio in the NPs
θ	a response function parameter
Θ	expected shortages
ξ	random (stochastic) variable
Ξ	set of random variables, $\xi \in \Xi$

Symbols: Unified for transportation problems (TNDPs and FLP)

E	set of edges, $e \in E$
I	set of nodes in the network, $i \in I$
S	set of scenarios, $s \in S$
c	transporting cost per unit (by an edge)
q	probability of achieving a scenario
x	first-stage decision variable - a transporting quantity

List of symbols

Symbols: Transportation network design problems (TNDPs)

A	network description by node-edge incidence matrix
b	demand, alternatively production or amount at a node
d	designing edge cost
p	price per unit (selling)
p^{max}	a price upper bound
p^{min}	a price lower bound
r^-	unit penalty cost for leftovers
r^+	unit penalty cost for shortages
y^-	second-stage decision variable - leftovers for a customer in a scenario
y^+	second-stage decision variable - shortages for a customer in a scenario
δ	0-1 (integer) network design variable

Symbols: Facility location problem (FLP)

a	network description by node-edge incidence matrix
b^-	available amount of produced waste in node
b^+	available waste processing capacity in node
c	transporting cost per unit
f	cost per processed unit of waste in node
g^-	cost per unprocessed unit of waste in a node
g^+	cost per unused unit of capacity in a node
h	switched on processing unit cost in a node
u^-	amount of untransported waste
u^+	amount of unused processing capacity
v^-	amount of waste transported from a node
v^+	amount of waste transported to a node
x	first-stage decision variable - a transporting quantity
y	amount of waste processed in a node
δ	0-1 (integer) variable - indicator of switching on-off extra waste processing capacity