9. ATTACHMENTS

ATTACHMENT I

Mathematical formulation

The objective functions and endogenous restrictions have the following expressions:

$$\max Z = \sum_{i=1}^{I} \sum_{p=1}^{P} v_{ip} \cdot x_{ip}$$

Subject to:

$$\sum_{p=1}^{P} x_{ip} \le 1 \quad \forall i = 1, 2, \dots, n$$

Where

 \boldsymbol{x}_{ip} is a binary variable so that:

 $x_{ip} = \left\{$

0 in other cases

 ${f v}_{ip}$ is the coefficient of the objective function, such as HF or NPV

The exogenous constraints have the following expressions:

• Flow constraints:

$$(1+\alpha)\sum_{\substack{i=1\\I}}^{I} v_{ip} x_{ip} \le \sum_{\substack{i=1\\I}}^{I} v_{i(p+1)} x_{i(p+1)}, \forall p = 1, \dots, P$$
$$(1-\alpha)\sum_{\substack{i=1\\I}}^{I} v_{i(p-1)} x_{i(p-1)} \ge \sum_{\substack{i=1\\I}}^{I} v_{ip} x_{ip}, \forall p = 1, \dots, P$$

Where:

 $\pmb{\alpha}$ is the fractional difference permitted in the flow level between two consequential period

• Adjacency constrains:

$$M \cdot x \le A$$
$$M = A + B$$

Where:

A = Adjacency matrix

M = Modified adjacency matrix

x = Control vector

1 is an $(n \times 1)$ unit vector

ATTACHMENT II



































