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CONCEPT AND COMPARISON OF CLASSIC AND FUZZY REGULATOR FOR AUTOMATIC FLIGHT LEVEL CONTROL

NÁVRH A POROVNÁNÍ KLASICKÉHO A FUZZY REGULÁTORU PRO AUTOMATICKÉ UDRENÍ VÝšKY LETU

DIPLOMOVÁ PRÁCE
MASTER'S THESIS

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Abstrakt
Účelem této práce je navrhnout zjednodušené modely klasického a fuzzy regulátoru pro automatické udržení výšky letu a porovnat jejich vlastnosti. Cílem je vyšetřit, zda fuzzy regulátor neprojeví lepší chování než klasický. Prostředkem pro návrh a srovnání vlastností obou regulátorů je posouzení odezev modelu systému letadlo-regulátor na požadavek změny výšky a modelu turbulence. Simulace jsou realizovány s pomocí prostředí MATLAB SIMULINK.

Summary
The purpose of this treatise is to design simplified models of classic and fuzzy controllers for automatic flight level control and compare their qualities. The goal is to investigate whether fuzzy regulator shows better behaviour then the classic one. Instrument for design and comparison of qualities of both regulators is examination of responses of the model aircraft-regulator on requested change of height and on model of turbulence. Simulations are realized with the aid MATLAB SIMULINK enviroment.

Klíčová slova
automatické systémy řízení letu, regulace, fuzzy řízení

Keywords
automatic flight control systems, regulation, fuzzy control

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I declare that this thesis is my authentic work.

Zdeněk Uhlíř

Contents

1. Introduction

This treatise deal with automatic flight control systems especially with comparison of classic design methods based on linear control theory and new approach of fuzzy linguistic models. It was chosen the application to height hold system. Design of automatic flight control systems is very complicated task because aircraft is complex system which we aren't able to describe accurately. As it will be seen in the subsequent text there's needed to make many limiting assumptions to obtain reasonable description of such such systems. Therefore it seems natural to expect that the approach of fuzzy logic by which we design controllers on basis of models of experienced human operator will be quite handy. In the following chapters will be derived simplified model of aircraft longitudinal dynamics for the concrete machine. For that model will be designed classic and fuzzy controller. We put emphasis on simulations of the control process by using the MATLAB SIMULINK environment. There will investigated behaviour of both regulators in chosen conditions (atmospheric turbulence, request on change of height) and then compared their qualities.

2. Basic Terms of Aviation

The aim of this chapter is to describe the flight of an airplane with consideration on just those data we need to control the flight level. However at the beginning we need to define basic terms of aviation. Then in chapter 3 we will put together the equations of motion of an aircraft for concrete machine: Charlie – a very large, four-engined, passenger jet aircraft.

2.1. Parts of an Airplane

Fuselage – The true body of an airplane.

Wings – This is the part of the airplane which generates most of the lift (i.e. force which holds the plane in the air).

Horizontal Stabilizers – Two small wings on the plane's tail which prevent up and down motion.

Vertical Stabilizer – Small wing on the tail which keeps the nose of the plane from swinging from side to side.

There are also parts used for manoeuvering the aircraft called ***control surfaces***. The purpose of those hinged parts is controlling of the aircraft's motion via their ability to change the amount of generated force by the wings. Forces and moments generated by the movement of the plane through the air are determined by geometrical characteristics of it's parts (It will be discussed in Chapter 2.2 Geometrical Characteristics of an Airplane.):

Elevator – The hinged part of horizontal stabilizer. By changing the amount of generated force is able to control up and down motion.

Rudder – Part of vertical stabilizer used for deflecting of the airplane's tail.

Control surfaces situated on the wings:

Aileron – Roll the wings from side to side.

Spoiler – Can also roll the plane from side to side. Spoilers change the amount of force generated by the wings because they disrupt the flow over the wing when deployed and that's why the lift is decreasing.

Flaps – Situated on the rear of the wing near the fuselage. Used during takeoff and landing for increasing the amount of produced force.

Slats – On the front and also used during takeoff and landing to produce additional force.

2.2. Geometrical Characteristics of an Airplane

As it has been said in Chapter 2.1 there are generated forces due to the movement of the plane through the air on its parts. One of the most important factors affecting the amount and orientation of generated forces is geometrical characterization of the aircraft.

2.2.1. Wings

Wing area viewed from above is bounded by leading edge in front, trailing edge in the back and the wing tips on both sides, finally the shape of the wing area is called a planform. Furthermore cut of the wing viewed from side is called an airfoil.

Top View:

Span, s – Distance from tip to tip.

Chord, $\bar{c}$ – Distance between leading and trailing edge.

Centerline – Bisector of two symmetric parts. Situated on axis x_B (body-fixed axis system, it will be discussed in Chapter 2.3 Axis Systems).

The Wing Area, A – Projected area of the planform bounded by leading and trailing edge.

Aspect Ratio, AR – This is a measurement of how long and slender is a wing from tip to tip. For rectangular planform is denoted by: $AR = \frac{s}{c}$

Generally (for various planforms):

$$AR = \frac{s^2}{c}$$

Side View: A cut through the wing perpendicular to y_b axis (body-fixed axis system, see Chapter 2.3 Axis Systems) gives side view called an airfoil. This is very important characteristic because air flow around any object causes generating of aerodynamic forces (lift and drag). It means that the shape of the airfoil directly affects the amount and orientation of those forces. Principal of the inception included other influence will be discussed in Chapter 2.4 Forces and Moments on an Airplane.

Chord Line – Straight line from the leading to trailing edges.

Mean Camber Line – All its points lie halfway between upper and lower surfaces.

Camber – The maximum distance between chord and mean camber lines. This is the measure of the airfoil curvature.

Thickness – The maximum distance between upper and lower surfaces.

Front View: For better roll stability there is an angle between both the right and left wing and local horizontal called the dihedral angle.

2.2.2. Fuselage

This part of the plane doesn't have bigger significance for generating of lift, but of course increases drag and that's why optimal shape is needed. The weight of an airplane is distributed all along the aircraft, but the fuselage with passengers and cargo contribute a significant portion of the weight of an aircraft. The most important reason why the fuselage is discussed here is that center of gravity (c.g. in the following text) is located inside the fuselage.

2.3. Axis Systems

They are divided in particular to observational (tightly connected with any observation post, for example the Earth or galaxy) and dynamic (always dependent in some way on an airplane and its flight).

The Earth Axis System – It's used as a reference system, whose origin is regarded as being fixed at the center of the Earth, and to express gravitational effects, altitude, horizontal distance and the orientation of the aircraft. The fact that the Earth axis system is used as a reference system reflects problems with dynamic situation. It rarely lasts for more than a few minutes so a more convenient inertial reference frame is needed.

The x-axis points north, the y-axis points east and the orthogonal triad is completed when the z-axis points down. It's the observational system and by disregarding of the Earth rotation we obtain inertial system.

The Body-fixed Axis System – It's origin is located identically at an aircraft's c.g. The x-axis points forward out of nose, the y-axis points out through the starboard (right) wing and the z-axis points down. This is dynamic system.

The aerodynamic forces and moments depend only upon the angles α and β, which orient the total velocity vector, $\vec{V_T}$, in relation to the axis X_B (the body axis system). The angular orientation of the body axis system (X_B, Y_B, Z_B) with respect to the Earth axis system (X_E, Y_E, Z_E) depends strictly upon the orientation sequence:

- Rotate X_E, Y_E, Z_E through an azimuthal angle, Ψ, about X_E to reach intermediate axes X_1, Y_1, Z_1.
- Rotate these axes X_1, Y_1, Z_1 through an angle of elevation, Θ, about Y_1 to reach a second, intermediate set of axes X_2, Y_2, Z_2.
- Rotate axes X_2, Y_2, Z_2 through an angle of bank, Φ, about X_2 to reach the body axes X_B, Y_B, Z_B.

The Stability Axis System – The X_S axis is chosen to coincide with the velocity vector, $\vec{V_T}$, at the start of the motion. Between X_S and X_B there is a trimmed angle of attack, α_0. The equations of motion derived by using the stability axis system are the special subset of the set derived by using the body axis system because it's a special version of the body-fixed axis system and is used for characterization of an airplane movement in the range of small perturbations from the original settled statement (It's often considered as symmetrical.).

The Wind Axis System – This is dynamic system oriented with respect to the aircraft's flight path and that's why timevarying terms which correspond to the moments and cross-products of inertia appear in the equations of motion. This fact complicates the analysis of motion. Dependent in particular on the way of mass flowing around an airplane. It's frequently used in American literature.

The Experimental Axis System – For measuring in the wind tunel. The y-axis, $y_{ex.}$, is orthogonal to the scales' and aircraft's plane of symmetry, the z-axis, $z_{ex.}$, is scale swivel's axis and finally the x-axis, $x_{ex.}$, is dependent only on the system of scales. Another kind of dynamic system.

2.3.1. Movements of an Airplane

The movement of an airplane is described by using the body axis system. It's an expresion of resultant force and moment influence on an airplane directed along X_B, Y_B, Z_B axes.

U, V, R – These are the forward, side and yawing velocities.

L, M, N – Roll, pitch and yaw moments.

Φ, Θ, Ψ – Roll, pitch and yaw angles.

2.4. Forces and Moments on an Airplane

Weight : This force is generated by the gravitational attraction of the earth on the airplane. It can described by the Newton's second law of motion $\vec{F_g} = m.\vec{g}$. It is always directed toward the center of the earth (along the z_E axis).

Thrust : This force is generated by the propulsion system to move the airplane through the air (directed along the x_B axis).

2.4.1. Aerodynamic Forces and Moments

Lift : This force is generated by the motion of the airplane through the air to overcome the weight force. Generating of the lift explains Bernoulli's equation. When fluid flow around any solid the velocity is increasing because of longer distance that molecules have to travel around. Bernoulli's equation says that the static pressure is decreasing in this case. So the issue is that the velocity of fluid on upper surface is not the same as on lower surface, identically the pressure. Difference between upper and lower pressure is overpressure and it gives us the lift. Every part of the airplane generates lift, but most of it is generated by the wings.

Factors affecting lift:

The Object – Shape and size (geometrical characteristics).

The Motion – Velocity and inclination (expressed by the angle of attack, a) to flow.

The Air – Mass, viscosity and compressibility.

The Lift Coefficient, c_l: Experimentally determined number which express all the complex dependencies of shape, size, inclination and flow conditions.

Free stream lift coefficient:

$$c_{l0} = \frac{2L}{\rho V^2 A}$$

Drag : It's closely associated with the lift. Same thing is that the drag is generated by the motion of the airplane through the air. The drag is induced by the friction between the surface of the airplane and molecules of gas.

note: Both the lift and the drag are components of the same force affecting.

3. The Equations of Motion of an Aircraft

This chapter deals with derivation of the equations of motion of an aircraft for purposes of automatic flight control system design. It's a general theory which I used here just to be obvious how the equations of motion later used was obtained. I followed theory given in Mc.Lean [1] for derivation of equations.

3.1. The Equations of Motion of a Rigid Body Aircraft

- The distance between any points on the aircraft do not change in flight.
- The aircraft's motion has six degrees of freedom which means that the movement of an airplane can be described as the translation of c.g. and turning around the c.g.
- Generally an airplane's movement can be divided into *longitudinal* and *lateral* motion.
- Longitudinal motion is projection of general motion to the plane of an airplane symmetry.
- Lateral motion is projection of general motion to the plane which is orthogonal to the plane of symmetry.

Deflections of control surfaces:

Longitudinal motion: δ_E, δ_{Th}

Lateral motion: δ_R, δ_A

3.1.1. Translational Motion

- Newton's Second Law of Motion

$$\vec{F} = \frac{d}{dt}(m\vec{V_T}) \tag{3.1}$$

$$\vec{M} = \frac{d}{dt}(\vec{H}) \tag{3.2}$$

Where $\vec{F}$ represents the sum of all externally applied forces, $\vec{M}$ represents the sum of all applied torques and $\vec{H}$ is the angular momentum. There are three components in the sum of external forces: aerodynamic (lift and drag), gravitational (weight) and propulsive (thrust). The last one is produced by expending some of the vehicle mass but the mass, m, can be assumed as a constant and the thrust which is equal to the relative velocity between the exhausted mass and the aircraft and the change of the aircraft's mass/unit time can be treated as an external force without impairing the accuracy of the equations of motion. It's assumed that there will be no change in the propulsive force and then changes in the aircraft's state of motion can occur if and only if there are changes in either the aerodynamic or gravitational forces (or both).

The sums of applied forces and torques consist of an equilibrium and perturbational components:

$$\vec{F} = \vec{F_0} + \Delta\vec{F} = m\frac{d}{dt}(\vec{V_T}) \tag{3.3}$$

$$\vec{M} = \vec{M_0} + \Delta\vec{M} = \frac{d}{dt}(\vec{H}) \tag{3.4}$$

The subscript 0 means equilibrium component and Δ the component of perturbation. By using the Earth axis system as an inertial reference system the components of perturbation can be expressed as follows:

$$\Delta\vec{F} = m\frac{d}{dt}(\vec{V_T})_E \tag{3.5}$$

$$\Delta\vec{M} = \frac{d}{dt}(\vec{H})_E \tag{3.6}$$

The equilibrium flight is unaccelerated along a straight path so the linear velocity vector relative to fixed space is invariant and the angular velocity is zero that's why both $\vec{F_0}$ and $\vec{M_0}$ are zero.

The rate of change of $\vec{V_T}$ relative to the Earth axis system:

$$\frac{d}{dt}(\vec{V_T})_E = \frac{d}{dt}\vec{V_T}\bigg|_B + \vec{\omega} \times \vec{V_T} \tag{3.7}$$

Where $\vec{\omega}$ is the angular velocity of the aircraft with respect to the body fixed axis system.

Both velocities can be written as the sum of their corresponding components with respect to X_B, Y_B, Z_B:

$$\vec{V_T} = \vec{i}U + \vec{j}V + \vec{k}W \tag{3.8}$$

$$\vec{\omega} = \vec{i}P + \vec{j}Q + \vec{k}R \tag{3.9}$$

$$\frac{d}{dt}\vec{V_T}\bigg|_B = \vec{i}\dot{U} + \vec{j}\dot{V} + \vec{k}\dot{W} \tag{3.10}$$

and the cross-product, $\vec{\omega} \times \vec{V_T}$, is given by:

$$\vec{\omega} \times \vec{V_T} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ P & Q & R \\ U & V & W \end{bmatrix} \tag{3.11}$$

$$\vec{\omega} \times \vec{V_T} = \vec{i}(QW - VR) + \vec{j}(UR - PW) + \vec{k}(PV - UQ) \tag{3.12}$$

So the components of the perturbation force can be expressed:

$$\Delta\vec{F} = \vec{i}\Delta F_x + \vec{j}\Delta F_y + \Delta\vec{k}F_z \tag{3.13}$$

Now we obtain:

$$\Delta F_x = m(\dot{U} + QW - VR) = \Delta X \tag{3.14}$$

$$\Delta F_y = m(\dot{V} + UR + PW) = \Delta Y \tag{3.15}$$

$$\Delta F_z = m(\dot{W} + VP - UQ) = \Delta Z \tag{3.16}$$

The notation $\Delta X, \Delta Y, \Delta Z$ follows the American custom.

3.1.2. Rotational Motion

At first let's define the angular momentum:

$$\vec{H} = \mathbf{I}\vec{\omega} \tag{3.17}$$

Where $\mathbf{I}$ is the inertia matrix defined as:

$$\mathbf{I} = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix} \tag{3.18}$$

I_{ii} denotes a moment of inertia and I_{ij} a product of inertia $j \neq i$.

$$\vec{M} = \frac{d}{dt}\vec{H} + \vec{\omega} \times \vec{H} \tag{3.19}$$

By using transformation from body axes to the Earth axis system the last equation can be re-expressed as:

$$\vec{M} = \mathbf{I}(\frac{d}{dt}\vec{\omega} + \vec{\omega} \times \vec{\omega}) + \vec{\omega} \times \vec{H} \tag{3.20}$$

However,

$$\vec{\omega} \times \vec{\omega} = \vec{0} \tag{3.21}$$
$$\frac{d}{dt}\vec{\omega} = \vec{i}\dot{P} + \vec{j}\dot{Q} + \vec{k}\dot{R} \tag{3.22}$$
$$\tag{3.23}$$

and

$$\vec{\omega} \times \vec{H} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ P & Q & R \\ h_x & h_y & h_z \end{bmatrix} \tag{3.24}$$

where h_x, h_y and h_z are the components of $\vec{H}$ obtained from expanding equation $\vec{H} = \mathbf{I}\vec{\omega}$ thus:

$$h_x = I_{xx}P - I_{xy}Q - I_{xz}R \tag{3.25}$$
$$h_y = -I_{yx}P + I_{yy}Q - I_{yz}R \tag{3.26}$$
$$h_z = -I_{zx}P - I_{zy}Q + I_{zz}R \tag{3.27}$$

The aircrafts are symmetrical about the plane XZ and consequently it is generally the case that:

$$I_{xy} = I_{yz} = 0 \tag{3.28}$$

Therefore:

$$h_x = I_{xx}P - I_{xz}R \tag{3.29}$$
$$h_y = I_{yy}Q \tag{3.30}$$
$$h_z = -I_{zx}P + I_{zz}R \tag{3.31}$$

and

$$\Delta M_x = I_{xx}\dot{P} - I_{xz}(\dot{R} + PQ) + QR(I_{zz} - I_{yy}) = \Delta L \tag{3.32}$$
$$\Delta M_y = I_{yy}\dot{Q} + I_{xz}(p^2 - R^2) + PR(I_{xx} - I_{zz}) = \Delta M \tag{3.33}$$
$$\Delta M_z = I_{zz}\dot{R} - I_{xz}\dot{P} + PQ(I_{yy} - I_{xx}) + I_{zz}QR = \Delta N \tag{3.34}$$

The notation $\Delta L, \Delta M, \Delta N$ follows the American custom.

3.1.3. Notes

- **For flight simulation work is not entirely convenient the derivation of the equations by using a body axis system.**
- **In model of large aircraft's flight (transporters for example) the terms which characterize the angular motion are frequently neglected because those aircrafts can't generate large angular rates (this is the case of this treatise).**
- **Equations invoking other assumptions:**
  1. **The body axes coincide with the principal axes.** $\Longleftrightarrow I_{xz}$ **(the product of inertia) is sufficiently small to allow of its being neglected.**
  2. **Low maximum values of angular velocity.** $\Longleftrightarrow$ **The terms PQ, QR and** $\mathbf{P^2 - R^2}$ **can be neglected.**
  3. $\mathbf{R^2 \ll P^2} \Longleftrightarrow \mathbf{R^2}$ **is often neglected.**
- **The neglecting of such terms can be practised only after very carefully consideration of both the aircraft's characteristics and the AFCS (Automatic Flight Control Systems) problem.**

3.1.4. Axis Transformations

Mutual orientation of the Earth axis system to body-fixed axis system is denoted by a sequence of three rotations, for each rotation a transformation matrix is applied to the variables. The total transformation matrix, is obtained by taking the product of the three matrices, multiplied in the order of the rotations.

So the Earth axis system incorporates the gravity vector, $\vec{g}$, and there's the way how it can be expressed in the body-fixed axis system:

1st Rotation – Azimuth Ψ

$$T_\Psi = \begin{bmatrix} \cos\Psi & \sin\Psi & 0 \\ -\sin\Psi & \cos\Psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.35}$$

2nd Rotation – Pitch Θ

$$T_\Theta = \begin{bmatrix} \cos\Theta & 0 & -\sin\Theta \\ 0 & 1 & 0 \\ \sin\Theta & 0 & \cos\Theta \end{bmatrix} \tag{3.36}$$

3rd Rotation – Roll Φ

$$T_\Phi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Phi & \sin\Phi \\ 0 & -\sin\Phi & \cos\Phi \end{bmatrix} \tag{3.37}$$

The total transformation matrix T, called ***the direction cosine array***, is defined as:

$$T = [T_\Psi][T_\Theta][T_\Phi] \tag{3.38}$$

So it means that:

$$T = \begin{bmatrix} \cos\Psi\cos\Theta & \sin\Psi\sin\Theta & -\sin\Theta \\ (\cos\Psi\sin\Theta\sin\Phi - \sin\Psi\cos\Phi) & (\sin\Psi\sin\Theta\sin\Phi + \cos\Psi\cos\Phi) & \cos\Theta\sin\Phi \\ (\cos\Psi\sin\Theta\cos\Phi + \sin\Psi\sin\Phi) & (\sin\Psi\sin\Theta\cos\Phi - \cos\Psi\sin\Phi) & \cos\Theta\cos\Phi \end{bmatrix} \tag{3.39}$$

Now the vector $\vec{g}$ will be expressed in the body-fixed axis system as follows:

$$\vec{g} = g(-\sin\Theta\vec{i} + \cos\Theta\sin\Phi\vec{j} + \cos\Theta\cos\Phi\vec{k}) \tag{3.40}$$

3.1.5. The Gravity Contributions to the Equations of Motion

For nonextra-atmospheric flight it's assumed that gravity acts at the centre of gravity (c.g.). When the centres of mass and gravity coincide in an aircraft, then there is no external moment produced by gravity about c.g. and that's why gravity contributes only to the external force vector, $\vec{F}$, for the body axis system. The gravity vector, $m\vec{g}$, is directed along Z_E axis so there's needed the projection to body-fixed axes:

$$\begin{aligned} \partial X &= m\vec{g} \quad \sin[-\Theta] &&= \quad -m\vec{g} \quad \sin\Theta \\ \partial Y &= m\vec{g} \quad \cos[-\Theta] \quad \sin\Phi &&= \quad m\vec{g} \quad \cos\Theta \quad \sin\Phi \\ \partial Z &= m\vec{g} \quad \cos[-\Theta] \quad \cos\Phi &&= \quad m\vec{g} \quad \cos\Theta \quad \cos\Phi \end{aligned} \tag{3.41}$$

Where Θ represents the angle of elevation between $m\vec{g}$ and the $Y_B Z_B$ plane and Φ represents the bank angle between the Z_B axis and projection of $m\vec{g}$ on the $Y_B Z_B$ plane.

Now it's needed to relate two new motion variables, Θ and Φ, and their derivatives to the angular velocities, P, Q and R, because in general those two angles are not simply the integrals of the angular velocity, P and Q. In very high speed flight the gravitational vertical is seen as rotating, but aircraft speeds being very low compared to orbital velocities, so the vertical may be regarded as fixed. The expression depends upon the angular velocity of the body axes about the vector $m\vec{g}$ called azimuth rate, $\dot{\Psi}$. The projection of $\dot{\Psi}$ in the $Y_B Z_B$ plane is normal to both $\dot{\Phi}$ and $\dot{\Theta}$.

$$\begin{aligned} P &= \dot{\Phi} - \dot{\Psi}\sin\Theta \\ Q &= \dot{\Theta}\cos\Phi + \dot{\Psi}\cos\Theta\sin\Phi \\ R &= -\dot{\Theta}\sin\Phi + \dot{\Psi}\cos\Theta\cos\Phi \end{aligned} \tag{3.42}$$

also:

$$\begin{aligned}\dot{\Phi} &= P + \dot{\Psi}\sin\Theta \\ \dot{\Theta} &= Q\cos\Phi - R\sin\Phi \\ \dot{\Psi} &= \frac{R\cos\Phi}{\cos\Theta} + \frac{Q\sin\Phi}{\cos\Theta}\end{aligned} \tag{3.43}$$

Finally by using substitution, it can be easily shown that:

$$\dot{\Phi} = P + R\tan\Theta\cos\Phi + Q\tan\Theta\sin\Phi \tag{3.44}$$

Where Φ, Θ and Ψ are called Euler angles.

3.1.6. Linearization of the Inertial and Gravitational Terms

Now it's possible to express the resultant force affecting on the aircraft, where equations (3.14)-(3.16) and (3.32)-(3.34) represent the inertial forces acting on the aircraft. Equations (3.41) represents the contribution of the forces due to gravity. This resultant affecting represents the accelerations which would be measured by sensors located on the aircraft. Input axes of the sensors would be coincident with the body axes X_B, Y_B and Z_B.

So, the external forces affecting is:

$$\begin{aligned}X &= \Delta X + \partial X \\ Y &= \Delta Y + \partial Y \\ Z &= \Delta Z + \partial Z\end{aligned} \tag{3.45}$$

Which means

$$\begin{aligned}X \triangleq ma_{x_{cg}} &= m(\dot{U} + QW - RV + g\sin\Theta) \\ Y \triangleq ma_{y_{cg}} &= m(\dot{V} + RU - PW - g\cos\Theta\sin\Phi) \\ Z \triangleq ma_{z_{cg}} &= m(\dot{W} + PV - QU - g\cos\Theta\cos\Phi)\end{aligned} \tag{3.46}$$

for forces and

$$\begin{aligned}L &= \dot{P}I_{xx} - I_{xz}(\dot{R} + PQ) + (I_{zz} - I_{yy})QR \\ M &= \dot{Q}I_{yy} + I_{xz}(P^2 - R^2) + (I_{xx} - I_{zz})PR \\ N &= \dot{R}I_{zz} - I_{xz}\dot{P} + PQ(I_{yy} - I_{xx}) + I_{xz}QR\end{aligned} \tag{3.47}$$

for moments.

There are also needed the auxiliary equations (3.42) since they relate the Euler angles, Ψ, Θ and Φ, to the angular velocities, R, Q and P.

Equations (3.46)-(3.47) cannot be solved analytically and would require the use of a computer. By considering the aircraft to comprise two components (a mean motion and a dynamic motion which accounts for perturbations about the mean motion) some simplification is possible. In this form of analysis *the assumption of small perturbations* is needed. Those small perturbations are inhibited when the flight is stable and uninhibited when the flight is unstable(according to the theory of small oscillations).

So, every motion variable is considered to have two components like:

$$U \triangleq U_0 + u$$

for example.

The trim, or equilibrium, values are denoted by a subscript 0 and the small perturbation values by the lower case letter.

There can't be translational or rotational acceleration in trim. Hence:

$$\begin{aligned}
X_0 &= m(Q_0W_0 - R_0V_0 + g\sin\Theta_0)\\
Y_0 &= m(R_0U_0 - P_0W_0 - g\cos\Theta_0\sin\Phi_0)\\
Z_0 &= m(P_0V_0 - Q_0U_0 - g\cos\Theta_0\cos\Phi_0)\\
L_0 &= (I_{zz} - I_{yy})Q_0R_0 - I_{xz}P_0Q_0\\
M_0 &= I_{xz}(P_0^2 - R_0^2) + (I_{xx} - I_{zz})P_0R_0\\
N_0 &= P_0Q_0(I_{yy} - I_{xx}) + I_{xz}Q_0R_0
\end{aligned} \tag{3.48}$$

Steady rolling, pitching and yawing motion can occur in the trim condition.

The perturbed equations of motion (sines and cosines are approximated to the angles themselves, the products and squares of the perturbed quantities are negligible):

$$\begin{aligned}
dX &= m(\dot{u} + W_0q + Q_0w - R_0v - V_0r + g\cos\Theta_0\theta)\\
dY &= m[\dot{v} + R_0u + U_0r - P_0w - W_0p - (g\cos\Theta_0\sin\Phi_0)\phi + (g\sin\Theta_0\sin\Phi_0)\theta]\\
dZ &= m[\dot{w} + P_0v + V_0p - Q_0u - U_0q + (g\cos\Theta_0\sin\Phi_0)\phi + (g\sin\Theta_0\cos\Phi_0)\theta]\\
dL &= I_{xx}\dot{p} - I_{xz}\dot{r} + (I_{zz} - I_{yy})(Q_0r + R_0q) - I_{xz}(P_0q + Q_0p)\\
dM &= I_{yy}\dot{q} - I_{xz}(2R_0r - 2P_0p) + (I_{xx} - I_{zz})(P_0r + R_0p)\\
dN &= I_{zz}\dot{r} - I_{xz}\dot{p} + (P_0q + Q_0p)(I_{yy} - I_{xx}) + I_{xz}(Q_0r + R_0q)
\end{aligned} \tag{3.49}$$

Where Ψ_0, Θ_0 and Φ_0 represent steady orientations and Ψ, θ and ϕ the perturbations in the Euler angles. Sometimes there's required the components of angular velocity representing the rotation of the body-fixed axis system relative to the Earth axis system (half of the set of auxiliary perturbation equations):

$$
\begin{aligned}
p &= \dot{\phi} - \dot{\Psi}\sin\Theta_0 - \theta(\Psi_0\cos\Theta_0) \\
q &= \dot{\theta}\cos\Phi_0 - \theta(\dot{\Psi}_0\sin\Phi\sin\Theta_0) + \dot{\Psi}\cos\Theta\sin\Psi_0 \\
r &= \dot{\Psi}\cos\Theta_0\cos\Phi_0 - \phi(\dot{\Psi}_0\cos\Theta_0\sin\Phi_0 + \dot{\Psi}_0\cos\Phi_0) - \dot{\theta}\sin\Phi_0 - \theta(\dot{\Psi}_0\sin\Theta_0\cos\Phi_0)
\end{aligned}
\tag{3.50}
$$

Now the equations (3.49)-(3.50) are linear, but still too cumbersome for general use, and that's why considering of flight cases with simpler trim conditions is commonly used in AFCS studies.

So a case of great interest is the straight steady, symmetric flight, with its wings level. Steady flight means, that the rates of the components of linear and angular velocity are zero. All the trimmed conditions can be expressed as follows:

1. The straight flight is motion with the components of angular velocity being zero and it implies
$$\dot{\Psi}_0 = \dot{\Theta}_0 = 0.$$
2. The symmetric flight is motion with the plane of symmetry fixed in space during the manoeuvre taking place and it implies
$$\Psi_0 = V_0 = 0.$$
3. Flying with wings level implies
$$\Phi_0 = 0.$$

When those concrete trim conditions are assumed, the aircraft will have particular values of U_0, W_0 and Θ_0. Moreover it may be assumed that:

$$Q_0 = P_0 = R_0 = 0$$

It follows that equations (3.49)-(3.50) can be written in the simplified form. The whole set is given below in two distinct groups:

1. The Longitudinal Motion:
$$
\begin{aligned}
x &= m(\dot{u} + W_0 q + g\cos\Theta_0\theta) \\
z &= m(\dot{w} - U_0 q + g\sin\Theta_0\theta) \\
m_1 &= I_{yy}\dot{q}
\end{aligned}
\tag{3.51}
$$
Where m_1 is subscripted by 1 not to confuse with the aircraft's mass, m.
2. The Lateral Motion:
$$
\begin{aligned}
y &= m(\dot{v} + U_0 r - W_0 p - g\cos\Theta_0\phi) \\
l &= I_{xx}\dot{p} - I_{xz}\dot{r} \\
n &= I_{zz}\dot{r} - I_{xz}\dot{p}
\end{aligned}
\tag{3.52}
$$

And the auxiliary equations:

$$
\begin{aligned}
p &= \dot{\phi} - \dot{\Psi}\sin\Theta_0 \\
q &= \dot{\theta} \\
r &= \dot{\Psi}\cos\Theta_0
\end{aligned}
\tag{3.53}
$$

3.2. Complete Linearized Equations of Motion

In the subsequent text will be discussed only equations of longitudinal motion because just this set is needed for modelling of a height hold system.

3.2.1. Expansion of Aerodynamic Force and Moment Terms

A Taylor series is used to expand the left-hand side of the equations of motion about the trimmed flight condition. Besides a contribution from the components of perturbed forces and moments there is a contribution from control surfaces (for example: elevator, E; rudder, R; thrust or throttle, T; flaps, F; spoilers, sp, ailerons, A), and that's why there's needed to introduce some additional terms into the Taylor series. Some terms depending on other motion variables, like θ, are omitted because they are generally insignificant.

As example, how to expand the left-hand side of the equations of motion, simple case of straight steady, symmetric flight with wings level is used. Furthermore they are simplified for longitudinal motion and it is assumed that only elevator deflection is involved in the control of this motion:

$$\begin{aligned}
\frac{\partial X}{\partial u}u &+ \frac{\partial X}{\partial \dot{u}}\dot{u} + \frac{\partial X}{\partial w}w + \frac{\partial X}{\partial \dot{w}}\dot{w} + \frac{\partial X}{\partial q}q + \frac{\partial X}{\partial \dot{q}}\dot{q} + \frac{\partial X}{\partial \delta_E}\delta_E + \frac{\partial X}{\partial \dot{\delta}_E}\dot{\delta}_E \\
&= m(\dot{u} + W_0 q + g\cos\Theta_0\theta) \\
\frac{\partial Z}{\partial u}u &+ \frac{\partial Z}{\partial \dot{u}}\dot{u} + \frac{\partial Z}{\partial w}w + \frac{\partial Z}{\partial \dot{w}}\dot{w} + \frac{\partial Z}{\partial q}q + \frac{\partial Z}{\partial \dot{q}}\dot{q} + \frac{\partial Z}{\partial \delta_E}\delta_E + \frac{\partial Z}{\partial \dot{\delta}_E}\dot{\delta}_E \\
&= m(\dot{w} - U_0 q + g\sin\Theta_0\theta) \\
\frac{\partial M}{\partial u}u &+ \frac{\partial M}{\partial \dot{u}}\dot{u} + \frac{\partial M}{\partial w}w + \frac{\partial M}{\partial \dot{w}}\dot{w} + \frac{\partial M}{\partial q}q + \frac{\partial M}{\partial \dot{q}}\dot{q} + \frac{\partial M}{\partial \delta_E}\delta_E + \frac{\partial M}{\partial \dot{\delta}_E}\dot{\delta}_E = I_{yy}\dot{q}
\end{aligned} \tag{3.54}$$

Simplified notation:

$$\begin{aligned}
X_x &= \frac{1}{m}\frac{\partial X}{\partial x} \\
Z_x &= \frac{1}{m}\frac{\partial Z}{\partial x} \\
M_x &= \frac{1}{I_{yy}}\frac{\partial M}{\partial x}
\end{aligned} \tag{3.55}$$

Where X_x, Z_x and M_x are the stability derivatives.

3.2.2. Equations of Longitudinal Motion

Following simplified notation equations (3.54) can be rewritten:

$$\begin{aligned}
\dot{u} &= X_u u + X_{\dot{u}}\dot{u} + X_w w + X_{\dot{w}}\dot{w} + X_q q + X_{\dot{q}}\dot{q} - W_0 q - g\cos\Theta_0\theta + X_{\delta_E}\delta_E + X_{\dot{\delta}_E}\dot{\delta}_E \\
\dot{w} &= Z_u u + Z_{\dot{u}}\dot{u} + Z_w w + Z_{\dot{w}}\dot{w} + Z_q q + Z_{\dot{q}}\dot{q} + U_0 q - g\sin\Theta_0\theta + Z_{\delta_E}\delta_E + Z_{\dot{\delta}_E}\dot{\delta}_E \\
\dot{q} &= M_u u + M_{\dot{u}}\dot{u} + M_w w + M_{\dot{w}}\dot{w} + M_q q + M_{\dot{q}}\dot{q} + M_{\delta_E}\delta_E + M_{\dot{\delta}_E}\dot{\delta}_E
\end{aligned} \tag{3.56}$$

and

$$\dot{\theta} = q \tag{3.57}$$

It follows from studying the aerodynamic data of a large number of aircraft that not every stability derivative is significant and a number can be neglected. Before ignoring stability derivatives, it is important to check the appropriate aerodynamic data because stability derivatives depend both upon the aircraft being considered and the flight condition which applies. According to McLean [1] those stability derivatives are insignificant and can be ignored without loss of generality:

$$X_{\dot{u}},\ X_q,\ X_{\dot{q}},\ X_{\dot{w}},\ X_{\delta_E},\ X_{\dot{\delta}_E},\ Z_{\dot{u}},\ Z_{\dot{w}},\ Z_{\dot{q}},\ Z_{\dot{\delta}_E},\ M_{\dot{u}},\ M_{\dot{q}},\ M_{\dot{\delta}_E}.$$

The stability derivative Z_q is often large but ignored if the trimmed forward speed, U_0, is large. This is just the case of the aircraft modeled in this treatise.

Hence, simplified equations of longitudinal motion are:

$$\begin{aligned}
\dot{u} &= X_u u + X_w w - W_0 q - g\cos\Theta_0\theta \\
\dot{w} &= Z_u u + Z_w w + U_0 q - g\sin\Theta_0\theta + Z_{\delta_E}\delta_E \\
\dot{q} &= M_u u + M_w w + M_{\dot{w}}\dot{w} + M_q q + M_{\delta_E}\delta_E \\
\dot{\theta} &= q
\end{aligned} \tag{3.58}$$

The motion and control variables, u, w, q, θ and δ_E, have units $m.s^{-1}$ and $rad.s^{-1}$. Hence the stability derivatives are dimensional.

3.3. Equations of Motion in Stability Axis System

As it has been discussed in Chapter 2 Basic Terms of Aviation the aerodynamic forces depend upon geometrical characteristics, speed of the airflow and the angle of attack. Those facts implies that orientation of the airflow is needed. So there's used stability axis system to express the forces of lift and drag relative to body-fixed axis system. The velocity and its components are relative in the sense of airframe to air mass.

The stability axis system relative to the body-fixed axis system orient the angle of attack, α, and the angle of sideslip, β.

- The velocity components along the body axes:

$$\begin{aligned} U_\alpha &= V_{T_\alpha} \cos\beta \cos\alpha \\ V_\alpha &= V_{T_\alpha} \sin\beta \\ W_\alpha &= V_{T_\alpha} \cos\beta \sin\alpha \end{aligned} \tag{3.59}$$

- Symmetric flight in the steady state pointing into the relative wind. V_0 and W_0 are both zero, so α_0 and β_0 are zero.
  Then:

$$U_0 = V_T \tag{3.60}$$

- Initial inclination to the horizon at some flight path angle, γ_0:

$$\Theta_0 \triangleq \gamma_0 + \alpha_0 \tag{3.61}$$

  Where α_0 is zero of course.

  The initial alignment has no effect on the body-fixed character of the axis system. There's held the body-fixed frame of reference while measuring the motion due to perturbations. The alignment of the stability axis system with respect to the body-fixed axis system changes as a function of the trim conditions.
  In chosen frame of reference, where $W_0 = 0$ and $\Theta_0 = \gamma_0$, equations (3.58) may be expressed as:

$$\begin{aligned} \dot{u} &= X_u u + X_w w - g\cos\gamma_0\theta \\ \dot{w} &= Z_u u + Z_w w + U_0 q - g\sin\gamma_0\theta + Z_{\delta_E}\delta_E \\ \dot{q} &= M_u u + M_w w + M_{\dot{w}}\dot{w} + M_q q + M_{\delta_E}\delta_E \\ \dot{\theta} &= q \end{aligned} \tag{3.62}$$

3.4. Equations of Motion for Steady Manoeuvring Flight Conditions

It follows from the theory described in Chapter 3.1.6 that trim conditions of steady flight are used to eliminate initial forces and moments from the equations of motion and Chapter 3.2 shows how to expand them to contribution from linear and angular velocities and control surfaces by using the Taylor series. Together the equations of motion for chosen flight case are obtained.

Commonly used steady flight conditions:

Steady, Straight Flight • All time derivatives are zero and there is no angular velocity about c.g.

- The assumption of symmetric flight implies that the bank angle, Φ, is zero.

Steady Turns • All time derivatives are all zero again.

- The Euler angles, Φ and Θ, are also zero.
- The rate of turn, $\dot{\Psi}$, is constant.

Steady Pitching Flight • V, P, R, Φ and Ψ are all zero.

- The pitching velocity, Q, is constant.
- The linear velocities, U and W, do vary with time.

Steady Rolling (Spinning) Flight • Cannot be easily simplified. So, there's needed special methods. See, for example, Thelander (1965) for such methods.

3.4.1. Steady, Straight, Symmetric Flight

There's not considered lateral motion for the needs of automatic flight level control in this treatise. So, this is the flight case being hold and the only perturbation involved is pitching movement.

Hence, the initial conditions are:

$$\begin{aligned} X_0 &= mg\sin\Theta \\ Z_0 &= -mg\cos\Theta \\ Y_0 &= L_0 = M_0 = N_0 \end{aligned} \tag{3.63}$$

and the equations of perturbed motion:

$$\begin{aligned} x &= m(\dot{u} + W_0 q + g\cos\Theta_0\theta) \\ z &= m(\dot{w} - U_0 q + g\sin\Theta_0\theta) \\ m_1 &= I_{yy}\dot{q} \end{aligned} \tag{3.64}$$

According to Chapter 3.2 the Taylor series is used to expand the left-hand side of the equations (3.64). The perturbed forces, x, z and m_1 have a contribution from only one control surface, the elevator. By using simplified notation, ignoring insignificant stability derivatives and adding of needed auxiliary equation, the following set of equations is obtained:

$$\begin{aligned} \dot{u} &= X_u u + X_w w + X_q q - W_0 q - g\cos\Theta_0\theta \\ \dot{w} &= Z_u u + Z_w w + U_0 q - g\sin\Theta_0\theta + Z_{\delta_E}\delta_E \\ \dot{q} &= M_u u + M_w w + M_{\dot{w}}\dot{w} + M_q q + M_{\delta_E}\delta_E \\ \dot{\theta} &= q \end{aligned} \tag{3.65}$$

Using the stability axis system, in which $W_0 = 0$ and $\Theta_0 = \gamma_0$, another set of equations is obtained:

$$\begin{aligned} \dot{u} &= X_u u + X_w w - g\cos\gamma_0\theta \\ \dot{w} &= Z_u u + Z_w w + U_0 q - g\sin\gamma_0\theta + Z_{\delta_E}\delta E \\ \dot{q} &= M_u u + M_w w + M_{\dot{w}}\dot{w} + M_q q + M_{\delta_E}\delta_E \\ \dot{\theta} &= q \end{aligned} \tag{3.66}$$

3.5. Additional Motion Variables

Another motion variables than the primary ones may be interesting for designing of AFCSs. They are usually those which can be measured by the sensor commonly available on aircraft.

3.5.1. Longitudinal Motion

Normal acceleration for perturbed motion – Measured at the c.g. of the aircraft and defined as:

$$a_{z_{cg}} = (\dot{w} - U_0 q) \tag{3.67}$$

– For small angles of attack, α, $w \simeq U_0\alpha$. Thus:

$$a_{z_{cg}} = U_0(\dot{\alpha} - q) \tag{3.68}$$

– Measured in units of g:

$$n_{z_{cg}} = \frac{a_{z_{cg}}}{g} \tag{3.69}$$

– Normal acceleration due to gravity when an aircraft changes its altitude:

$$a_{z_{cg}} = \dot{w} - U_0 q - g \tag{3.70}$$

– At some point on the fuselage centre line distant from the c.g. by l_x:

$$a_{z_x} = \dot{w} - U_0 - l_x\dot{q} \tag{3.71}$$

Height of the aircraft c.g. above the ground – By definition:

$$\ddot{h}_{cg} = -a_{z_{cg}} \tag{3.72}$$

therefore

$$\ddot{h} = -\dot{w} + U_0 q \tag{3.73}$$

and

$$\dot{h}_{cg} = -w + U_0\theta \tag{3.74}$$

so, the height of the aircraft c.g. above the ground is:

$$h_{cg} = U_0 \int \theta dt - \int w dt = U_0 \int \gamma dt \tag{3.75}$$

from that another variation of normal acceleration is obtained:

$$n_{z_{cg}} = \frac{-U_0\dot{\gamma}}{g} \tag{3.76}$$

Acceleration sensitivity – This variation of load factor with the angle of attack is an important aircraft parameter defined as(following McLean [1]):

$$\begin{aligned} n_{z_\alpha} &= \frac{U_0}{g}\frac{(Z_{\delta_E}M_w - M_{\delta_E}Z_w)}{(M_{\delta_E} - Z_{\delta_E}\frac{M_q}{U_0})} \\ &\simeq \frac{U_0}{gM_{\delta_E}}(Z_{\delta_E}M_w - M_{\delta_E}Z_w) \end{aligned} \tag{3.77}$$

since, for conventional aircraft $M_{\delta_E}Z_w \gg Z_{\delta_E}M_w$ this relation become:

$$n_{z_\alpha} = \frac{-Z_w U_0}{g} \tag{3.78}$$

and for straight and level flight at 1g:

$$n_{z_\alpha} = -Z_w U_0 = \frac{C_{L_\alpha}}{C_L} \tag{3.79}$$

where C_{L_α} is the lift curve slope and C_L is the coefficient of lift.

3.6. State-Space Representation of Aircraft

3.6.1. The State Equation

It's a natural form in which is possible to represent the equation of motion of an aircraft and a first order, vector differential equation. Its most general expression is:

$$\dot{\vec{x}} = A\vec{x} + B\vec{u} + E\vec{d} \tag{3.80}$$

where $\vec{x} \in R^n$ is the state vector of the state variables, $\vec{u} \in R^m$ is the control vector of the control input variables, $\vec{d} \in R^l$ is the disturbance vector, A is the state coefficient matrix, B is the driving matrix and E is the disturbance coefficient matrix. The term $E\vec{d}$ introduces to the state equation the influence of various disturbances like atmospheric turbulence. Special methods used for its introduction to the state equation will be discussed in Chapter 5. In this chapter $\vec{d}$ will be regarded as a null vector.

3.6.2. The Output Equation

It's an algebraic equation which depends solely upon the state vector and, occasionally, upon the control vector also. The output equation is needed when the concern is with motion variables. Its form containing noise effects of the sensors:

$$\vec{y} = C\vec{x} + D\vec{u} \tag{3.81}$$

where $\vec{y} \in R^p$ is the output vector of the output variables, C is the output matrix, D is the direct matrix.

3.6.3. Equations of Motion of Steady, Straight, Symmetric Flight

There's used the stability axis system because, in accordance with McLean [1], this is most convenient for AFCS work.

Only Deflection of Elevator Involved

According to Chapter 3.4.1 the state vector is defined as:

$$\vec{x} = \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} \tag{3.82}$$

and it is assumed that an aircraft is controlled only by means of elevator deflection, δ_E. So, its control vector is defined as:

$$\vec{u} \triangleq \delta_E \tag{3.83}$$

then, there's obtained from equations (3.66):

$$A \triangleq \begin{bmatrix} X_u & X_w & 0 & -g\cos\gamma_0 \\ Z_u & Z_w & U_0 & -g\sin\gamma_0 \\ \tilde{M}_u & \tilde{M}_w & \tilde{M}_q & \tilde{M}_\theta \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{3.84}$$

$$B \triangleq \begin{bmatrix} X_{\delta_E} \\ Z_{\delta_E} \\ \tilde{M}_{\delta_E} \\ 0 \end{bmatrix} \tag{3.85}$$

where stability derivative, X_{δ_E}, is not ignored.

Then the whole state equation is given by:

$$\begin{aligned}
\dot{\vec{x}} &= A\vec{x} + B\vec{u} \\
\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} &= \begin{bmatrix} X_u & X_w & 0 & -g\cos\gamma_0 \\ Z_u & Z_w & U_0 & -g\sin\gamma_0 \\ \tilde{M}_u & \tilde{M}_w & \tilde{M}_q & \tilde{M}_\theta \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} X_{\delta_E} \\ Z_{\delta_E} \\ \tilde{M}_{\delta_E} \\ 0 \end{bmatrix} \delta_E
\end{aligned} \tag{3.86}$$

There's not possible to be terms involving the first (or even higher) derivatives of any of the state or control variables on the right hand side of the state equation. Thus, $\dot{w}$, which depends only upon $\vec{x}$ and $\vec{u}$, has to be substituted. This substitution is meaning of tilde in the matrix. Corresponding dependency is:

$$\dot{w} = Z_u u + Z_w w + U_0 q - g\sin\gamma_0\theta + Z_{\delta_E}\delta E \tag{3.87}$$

and substituting for $\dot{w}$ in the equation for $\dot{q}$ gives:

$$\dot{q} = \tilde{M}_u w + \tilde{M}_w w + \tilde{M}_q q + \tilde{M}_\theta\theta + \tilde{M}_{\delta_E}\delta_E \tag{3.88}$$

where

$$\begin{aligned}
\tilde{M}_u &= (M_u + M_{\dot{w}} Z_u) \\
\tilde{M}_w &= (M_w + M_{\dot{w}} Z_w) \\
\tilde{M}_q &= (M_q + U_0 M_{\dot{w}}) \\
\tilde{M}_\theta &= (-g M_{\dot{w}} \sin\gamma_0) \\
\tilde{M}_{\delta_E} &= (M_{\delta_E} + M_{\dot{w}} Z_{\delta_E})
\end{aligned} \tag{3.89}$$

The purpose of this treatise is the flight level control and that's why corresponding output variable is the height of an aircraft c.g., h_{cg}.

Using the equation (3.74):

$$\dot{h}_{cg} = -w + U_0\theta \tag{3.90}$$

we introduce new state variable:

$$x_5 = h \tag{3.91}$$

and that's why:

$$\vec{x} \triangleq \begin{bmatrix} u \\ w \\ q \\ \theta \\ h \end{bmatrix} \tag{3.92}$$

therefore matrices A and B become:

$$A \triangleq \begin{bmatrix} X_u & X_w & 0 & -g\cos\gamma_0 & 0 \\ Z_u & Z_w & U_0 & -g\sin\gamma_0 & 0 \\ \tilde{M}_u & \tilde{M}_w & \tilde{M}_q & \tilde{M}_\theta & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & U_0 & 0 \end{bmatrix} \tag{3.93}$$

$$B \triangleq \begin{bmatrix} X_{\delta_E} \\ Z_{\delta_E} \\ \tilde{M}_{\delta_E} \\ 0 \\ 0 \end{bmatrix} \tag{3.94}$$

Then the whole state equation is given by:

$$\begin{aligned} \dot{\vec{x}} &= A\vec{x} + B\vec{u} \\ \begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \\ \dot{h} \end{bmatrix} &= \begin{bmatrix} X_u & X_w & 0 & -g\cos\gamma_0 & 0 \\ Z_u & Z_w & U_0 & -g\sin\gamma_0 & 0 \\ \tilde{M}_u & \tilde{M}_w & \tilde{M}_q & \tilde{M}_\theta & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & U_0 & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \\ h \end{bmatrix} + \begin{bmatrix} X_{\delta_E} \\ Z_{\delta_E} \\ \tilde{M}_{\delta_E} \\ 0 \\ 0 \end{bmatrix} \delta_E \end{aligned} \tag{3.95}$$

Changes of Elevator and Thrust Involved

Corrections of perturbed motion occur changes in the angle of attack, α, which affects in cooperation with thrust the amount of lift force. So, it's useful the thrust not to be constant but control it as well.

In this case the driving matrix B and the control vector $\vec{u}$ will become:

$$B \triangleq \begin{bmatrix} X_{\delta_E} & X_{\delta_T} \\ Z_{\delta_E} & Z_{\delta_T} \\ \tilde{M}_{\delta_E} & \tilde{M}_{\delta_T} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{3.96}$$

$$\vec{u} \triangleq \begin{bmatrix} \delta_E \\ \delta_T \end{bmatrix} \tag{3.97}$$

And now the state equation with putting emphasis on the height of an aircraft c.g. will be:

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \\ \dot{h} \end{bmatrix} = \begin{bmatrix} X_u & X_w & 0 & -g\cos\gamma_0 & 0 \\ Z_u & Z_w & U_0 & -g\sin\gamma_0 & 0 \\ \tilde{M}_u & \tilde{M}_w & \tilde{M}_q & \tilde{M}_\theta & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & U_0 & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \\ h \end{bmatrix} + \begin{bmatrix} X_{\delta_E} & X_{\delta_T} \\ Z_{\delta_E} & Z_{\delta_T} \\ \tilde{M}_{\delta_E} & \tilde{M}_{\delta_T} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_E \\ \delta_T \end{bmatrix} \tag{3.98}$$

Where $\tilde{M}_{\delta_T} = (M_{\delta_T} + M_{\dot{w}} Z_{\delta_T})$.

Final Form of the Equations of Motion for a Particular Aircraft and Flight Conditions

There will be discussed a model of the aircraft referred to as CHARLIE in this treatise, which is a very large, four-engined, passenger jet aircraft. Name and experimentally measured data for this airplane were given in Mc.Lean [1]. There is considered flight condition where the aircraft is flying straight, level and has the value of $\gamma_0 = 0$, $\alpha_0 = 0$ and stability derivative, $X_{\delta_E} = 0$. It came out during experiments with the model that forward speed need to be controlled too. So the equations of motion given by (3.98) take the following form:

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \\ \dot{h} \end{bmatrix} = \begin{bmatrix} X_u & X_w & 0 & -g & 0 \\ Z_u & Z_w & U_0 & 0 & 0 \\ \tilde{M}_u & \tilde{M}_w & \tilde{M}_q & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & U_0 & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \\ h \end{bmatrix} + \begin{bmatrix} 0 & X_{\delta_T} \\ Z_{\delta_E} & Z_{\delta_T} \\ \tilde{M}_{\delta_E} & \tilde{M}_{\delta_T} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_E \\ \delta_T \end{bmatrix} \tag{3.99}$$

The block-diagram representation used for modelling in MATLAB SIMULINK is shown on following figure.

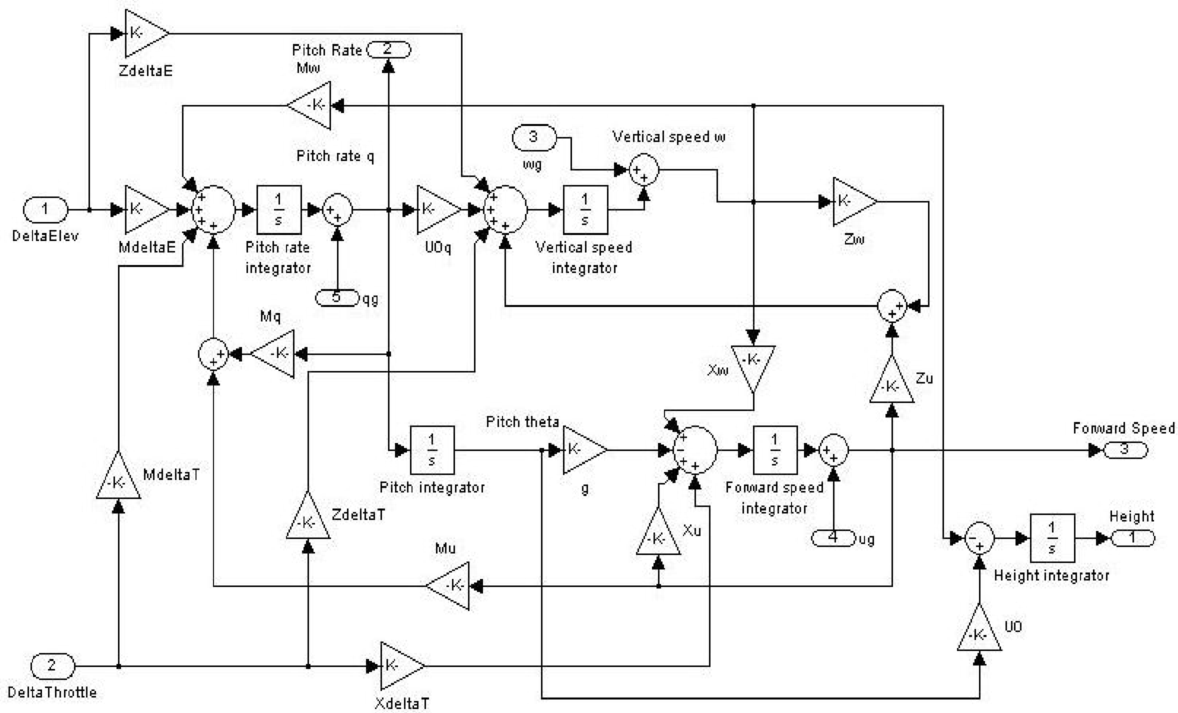

Figure 3.1: Block-diagram representation of the longitudinal dynamics.

4. Aircraft Stability and Dynamics

In this chapter will be discussed stability of an aircraft by using the equations of motion which have been derived in chapter 3. Concern of this treatise lies only with the longitudinal motion and that's why the corresponding set of equations will be used. It is considered that all the assumptions used for deriving of the equations of motion holds.

4.1. Longitudinal Stability

4.1.1. Short Period and Phugoid Modes

The dynamic stability characterize the eigenvalues of the state coefficient matrix, A. Those eigenvalues are roots of the equation

$$|\lambda I - A| = 0 \tag{4.1}$$

from which by expanding the determinant, the longitudinal stability quartic is obtained:

$$\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0 \tag{4.2}$$

Dynamically Stable Aircraft:

- All its eigenvalues, being real, have negative values.
- All its complex eigenvalues have negative real parts.

Dynamically Unstable Aircraft:

- Any complex eigenvalue has zero, or positive, value.

Short period and phugoid modes arise from invariable factorizing of the longitudinal stability quartic into two quadratic factors which can be done according to Mc Lean [1] in the following manner:

$$(\lambda^2 + 2\zeta_{ph}\omega_{ph}\lambda + \omega_{ph}^2)(\lambda^2 + 2\zeta_{sp}\omega_{sp}\lambda + \omega_{sp}^2) \tag{4.3}$$

Phugoid Mode

It's characterized by an oscillation of long period. The first factor of the equation (4.3) corresponds to this mode of an aircraft motion, where ω_{ph} is the natural frequency and ζ_{ph} is the damping ratio.

Short Period Mode

It's a rapid, relatively well-damped motion to which corresponds the second factor of the equation (4.3). The short period mode has frequency ω_{sp} and damping ratio ζ_{sp}.

4.1.2. Longitudinal Stability of the Modeled Aircraft

Data were given in [1], appendix B.2.3, page 559. It's very large, four-engined, passenger jet aircraft. There's considered steady, straight flight, at Mach 0.8 ($U_0 = 250\frac{m}{s}$) and at a height of 6 100 m. According to equation (3.93) and data corresponding to chosen flight condition we obtain:

$$A = \begin{bmatrix} -0.0002 & 0.026 & 0 & -9.81 & 0 \\ -0.09 & -0.624 & 250 & 0 & 0 \\ -0.000007 & -0.0045632 & -0.843 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 250 & 0 \end{bmatrix} \tag{4.4}$$

This state coefficient matrix has the eigenvalues:

$$\lambda_{1,2} = -0.734 \pm 1.0628i \tag{4.5}$$

$$\lambda_{3,4} = 0.0004 \pm 0.0489i \tag{4.6}$$

$$\lambda_5 = 0 \tag{4.7}$$

So the aircraft is dynamically unstable. Note that the eigenvalue, λ_5, is stated here only for completeness. As it can be seen from the equations, the additional equation for height is linearly dependent. Hence the original 4×4 matrix A describes the same dynamics and the eigenvalues corresponding to previous theory of aircraft stability are $\lambda_{1,2}$ and $\lambda_{3,4}$.

5. Disturbances Affecting Aircraft Motion

In this chapter will be shortly described derivation of simplified model of turbulence used for simulations. The analysis of turbulence uses statistical methods. Following theory given in Mc.Lean [1] will be obtained simplified model of atmospheric turbulence.

5.1. A Discrete Gust Function

According to Mc.Lean [1] the mathematical model with the most general acceptance for fixed-wing aircraft is given by:

$$x_g(t) = \frac{k}{T}(1 - \cos(\frac{2\pi}{T})t) \tag{5.1}$$

T is the duration of the gust.

$$T = \frac{L}{U_0} \tag{5.2}$$

The scale length L is the wavelength of the gust in meters and k is a scaling factor selected to achieve the required gust intensity. The gust wavelength is traditionally taken to be equal to twenty-five times the mean aerodynamic chord, $\bar{c}$, of the wing of the aircraft.

$$L = 25\bar{c} \tag{5.3}$$

It means $L = 207.5\text{m}$ for CHARLIE with $\bar{c} = 8.3\text{m}$.

5.2. Power Spectral Density Functions

It's a statistical theory allows us to represent the atmospheric turbulence as a stationary, random process. The power spectral density (PSD) of any function, $x(t)$, is a real function which provides information of how the mean squared value of $x(t)$ is distributed with frequency, ω. It's defined by the following equation.

$$\Phi(\omega) = \lim_{\substack{\triangle\omega \to 0 \\ T \to \infty}} \frac{1}{T\triangle\omega} \int_0^T x^2(t, \omega, \triangle\omega)\mathrm{d}t \tag{5.4}$$

This PSD function has units of either ms^{-1} or rads^{-2}.

Dryden PSD Function in Terms of Spatial Frequency

This type is commonly used in AFCS work for its simplicity.

$$\Phi(\Omega) = \frac{\sigma^2 L}{\pi} \frac{(1 + 3L^2\Omega^2)}{(1 + L^2\Omega^2)^2} \tag{5.5}$$

It can be transformed to the spectral domain corresponding to chosen aircraft in particular flight condition by following equation.

$$\Phi(\omega) = \frac{\Phi(\Omega)}{U_0} \tag{5.6}$$

5.3. Obtaining the Linear Filter for Continuous Gust Representation

To generate gust signals with the required intensity, scale lengths and PSD functions, white noise with a PSD function $\Phi_N(\omega)$, is used to provide an input signal to a linear filter given by:

$$\Phi_i(\omega) = |G_i(s)|^2_{s=j\omega} \Phi_N(\omega) \tag{5.7}$$

The formulas needed to obtain the linear filters for gust velocities are derived under following assumptions:

1. According to Mc.Lean [1] the dependence of scale length is defined for heights greater than $L = 207.5\text{m}$ as:

$$L_u = L_w = 207.5\text{m} \tag{5.8}$$

2. Atmospheric turbulence is a stationary random process.
3. The turbulence field is frozen with respect to time.
4. The statistical characteristics of turbulence are defined for the stability axis system of the aircraft.
5. The intensity of the three translational components of the turbulence are isotropic, i.e.:

$$\frac{\sigma_u^2}{L_u} = \frac{\sigma_w^2}{L_w} \tag{5.9}$$

Under those assumptions we obtain following formulas:

$$G_u(s) = \frac{\sqrt{\frac{2U_0\sigma_u^2}{L_u\pi}}}{(s + \frac{U_0}{L_u})} \tag{5.10}$$

$$G_w(s) = \sqrt{\frac{3\sigma_w^2}{L_w\pi U_0}} \frac{s + \frac{U_0}{\sqrt{3}L_w}}{(s + \frac{U_0}{L_w})^2} \tag{5.11}$$

For thunderstorms, at any height:

$$\sigma_u = \sigma_w = 7\text{ms}^{-1} \tag{5.12}$$

This results in the linear filters given by:

$$G_u(s) = \frac{7.51}{s + 1.205} \tag{5.13}$$

$$G_w(s) = \frac{0.03s + 0.021}{s^2 + 2.41s + 1.452} \tag{5.14}$$

The gust velocities are now obtained according to Mc.Lean [1] by:

$$u_g(s) = G_u(s)\eta(s) \tag{5.15}$$

$$w_g(s) = G_w(s)\eta(s) \tag{5.16}$$

$$q_g(s) = -\dot{\alpha}_g \tag{5.17}$$

Where $\eta(s)$ is the signal from the white noise source and $\alpha_g = -\frac{w_g}{U_0}$ because the translational velocity of turbulence is defined as positive stability axes. The block-diagram representing this model is shown on following figure. Pulse generator and switches are added to ensure 50 seconds duration of turbulence.

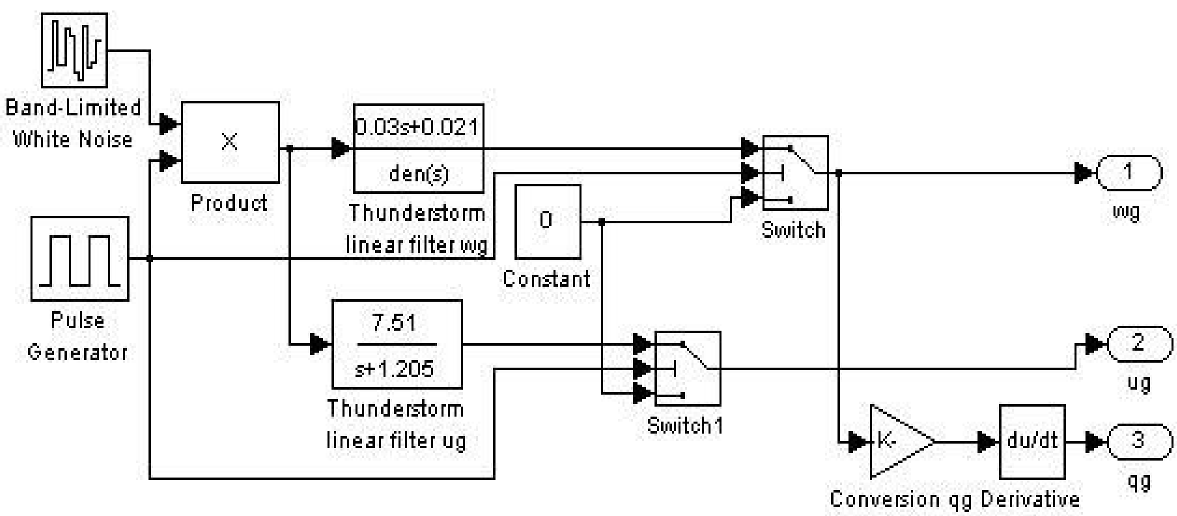

Figure 5.1: Model of turbulence.

6. General Theory of Control

6.1. Introduction

Nowadays automatons, computers and AI components stand in for human activities. This complicated process, where human activities are replaced by various machines, is called ***automation***. The automation is not related only to manual work but various machines. Computers with corresponding programs etc. are able to substitute human control function. Right this issue is the case of interest of this treatise because the task of automatic flight level control is to stand in for pilot's effort to ensure settled flight.

With the theory of regulation deals the science discipline called ***cybernetics***. As its founder is regarded the American mathematician Norbert Wiener who as the first worked on theory of feedback systems of control in his famous book ***Cybernetic or Control and Communication in the Animal and the Machines*** from the year 1948. Cybernetics is science which investigate general characteristics and natural relations of control in biological, technical and social systems. Concern of this treatise lies from all parts of the cybernetics especially with the ***theory of automatic control***. Now lets define the control term:

Control is aimed affecting on the controlled object in such way to reach specific prescribed goal.

Control is divided into manual and automatic after the way it's realized. Nice example is piloting by human representing manual control and by autopilot on the other side.

Other and very important dividing is in the sense of the feedback:

Operating – Control without the feedback.

Regulation – Control with the feedback. Regulation is upkeeping of certain physical quantity on the constant value or on in accordance with any rule changing value. Actual values of this quantity ascertained during the regulation are compared with the value it should have got. Based on recognized deviations there are interventions to the regulated process to shift off those deviations.

Now lets notice dividing by principle of affecting on the controlled system:

Logical Control – This type of control use two-valued quantities. It means that there are always only two possibilities formally expressed by 0 and 1. Relations between quantities are called logical functions and control circuits working based on this principle are logical control circuits.

Continuous Control – The action intervence is set continuously and consequently data about the controlled system are measured as quantities continuous in time. Continuous control system creates uninterrupted binding between input and output.

Discrete Control – This kind of control is a consequence of using of computers as regulators, although the beginnings are wedded with control of continuous systems discretely measured. The control computers aren't able to process continuous signal and that's why it's needed to transfer it to the discrete signal. Therefore discrete control system creates relation between the inputs and outputs as relations between progressions of impulses sampled in sequence which is given by so called sample period. The regulated quantity is not between the moments of sampling measured and the action quantity is not even adjusted in this time. The discrete control with very short sampling period is approximately similar to continuous.

Fuzzy Control – It's not based on controlled system and its mathematical model. The system is controlled according to rules like ***"if ... then"*** which correspond to the orders of an expert who is able to stand in for automatons to control the system. This type of control is suitable for the systems which we can't describe but we can control them and just the difference between the control based on the mathematical model of the system and on the model of human control is the issue of this treatise. There's possible to determine the output without the knowledge of formulas describing the relations between the input and output.

So that's enough for basic terms. In the following text will be dealt with continuous and fuzzy control with the feedback.

6.2. Continuous control

6.2.1. Basic terms

Regulation circuit – The system in which the regulation is realized.

Regulator – Control system which realize regulation by any device.

Regulated system – Controlled system which is the object of regulation.

Regulated quantity, $y(t)$ – The output from regulated system which is kept on required value by regulator.

Control quantity, $w(t)$ – Used for setting of the regulated quantity value.

Regulatory deviation, $e(t)$ – It's a difference between regulated and control quantities.

$$e(t) = w(t) - y(t) \tag{6.1}$$

Action quantity, $u(t)$ – This is the output quantity of the regulator which is the input quantity of the regulated system. Used to interfere to the regulated proces to reach minimal or even zero value of $e(t)$.

Disturbance quantities, $v_1(t), \ldots, v_n(t)$ – Unpredictable influence upon the regulated quantity. The subject of regulation is balancing of the disturbances effects.

6.2.2. Laplace Transform of the State Equations to the Transfer Matrix

Generally Laplace transform is defined by

$$G(s) = \mathcal{L}[G(t)] = \int_0^\infty e^{-st} G(t) dt \tag{6.2}$$

where $G(s)$ is the transfer matrix of the system.

Solution of the system of ordinary differential equations describing dynamics of the aircraft is obtained by the inverse Laplace transform given by following equation.

$$\mathcal{L}^{-1}[G(s)] = g(t) = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} e^{st} G(s) ds \tag{6.3}$$

Where γ is a real number so that the contour path of integration is in the region of convergence of $G(s)$. Function $g(t)$ describes behaviour of the system and it's defined for all $t \in \langle 0, \infty)$.

Definition of the transfer matrix

$$\vec{Y}(s) = G(s)\vec{U}(s), \tag{6.4}$$

where $\vec{Y}(s)$ is the Laplace image of the output vector $\vec{y}(t)$, $\vec{U}(s)$ is the Laplace image of the input vector $\vec{u}(t)$ and $G(s)$ is the transfer matrix, whose elements are the transfers between individual inputs and outputs.

State equations of the system

$$\dot{\vec{x}}(t) = A\vec{x}(t) + B\vec{u}(t) \tag{6.5}$$
$$\vec{y}(t) = C\vec{x}(t) + D\vec{u}(t) \tag{6.6}$$

Laplace transformation of those equations

$$s\vec{X}(s) - \vec{x}(0) = A\vec{X}(s) + B\vec{U}(s) \tag{6.7}$$
$$\vec{Y}(s) = C\vec{X}(s) + D\vec{U}(s), \tag{6.8}$$

where $\vec{x}(0) = \vec{0}$ are the initial conditions and by using them we obtain

$$(sI - A)\vec{X}(s) = B\vec{U}(s), \tag{6.9}$$

where I is the unit matrix. By using the equation (6.9) it's possible to determine the image of the state

$$\vec{X}(s) = (sI - A)^{-1}B\vec{U}(s). \tag{6.10}$$

Therefore the image of the output equation is

$$\vec{Y}(s) = [C(sI - A)^{-1}B + D]\vec{U}(s) \tag{6.11}$$

and now by using the equation (6.4) we obtain **the transfer matrix of the system**

$$G(s) = C(sI - A)^{-1}B + D. \tag{6.12}$$

7. Classic Controller

7.1. Mathematical Model of Classic Controller

7.1.1. Transfer Matrix of the System

It has been derived linearized state equations of motion in chapter 3. They will be used now for obtaining the transfer matrix of the system. Consider state equation given by (3.98). First we need to calculate the matrix $(sI-A)^{-1}$:

$$(sI-A)^{-1} = \frac{adj(sI-A)}{\det(sI-A)} \tag{7.1}$$

Where $(sI-A)^{-1}$ is $n \times n$ matrix and $\Delta_{long}(s)$ is simplified notation for determinant of the system given by:

$$\Delta_{long}(s) = (s^n + a_1 s^{n-1} + \ldots + a_{n-1}s + a_n) \tag{7.2}$$

Where the coefficients $a_1, \ldots, a_n$ are given by evaluating the determinant of the state coefficient matrix A. Roots of this equation are called **poles of the system** and they are those eigenvalues of A that specify longitudinal stability of an aircraft as was discussed in section 4. So the poles determine phugoid and short period modes and that's why resulting aircraft dynamic behaviour too.

7.1.2. Feedback Controller

Following equations describe the generalized linear feedback controller.

$$\dot{\vec{x}}_c = A_c\vec{x}_c + B_c\vec{y} \tag{7.3}$$

$$\vec{y}_c = C_c\vec{x}_c + D_c\vec{y} \tag{7.4}$$

Closed-loop control is achieved when the control law

$$\vec{u} = \vec{y}_c \tag{7.5}$$

is satisfied. Similarly to previous theory we are able to obtain transfer matrix of the controller and by using block-diagram algebra transfer matrix of the closed-loop system. The goal is to find such parameters (coefficients) of the controller which ensure required behaviour of the closed-loop system. It can be done through the linear control theory. The first step in this process is to define suitable pole locations in complex domain for the closed-loop system to achieve our requirements (primarily the poles must have negative real parts for the system to be stable). It was chosen to find parameters experimentally because we had no expert information which was proper pole locations for CHARLIE.

7.1.3. Concept of Classic Controller

As it has been said in previous section, the parameters of the classic controller were found through experiments with mathematical model. MATLAB SIMULINK was used for that purpose. This design environment allows to represent dynamical system in block diagram for analysis and simulation. Classic controller were designed and adjusted through observations of responses on various disturbances (turbulence and request for change of height) to ensure the best possible behaviour. Factors affecting the design process were stability, overshooting, overload (for comfort), oscillations during the control process and control rate.

Choice of Controlled Variables

There's not needed to control all the state variables which is called the full-state feedback. Besides, often it's not possible to measure all variables. For the purpose of automatic flight level control is needed to measure height, h, of course. The second controlled variable is pitch rate, q, because aircraft changes its pitch rate during the change of height and it's needed to bring deviation in pitch rate to zero when approaching required height. Those two variables were used for the control of elevator (action variable is δ_E) where signal from pitch rate was used as damping signal for pitching moment to be stabilized on required height. The third and the last variable to control is forward speed, u, because it came out during experiments that the aircraft changed u significantly during the change of height. Signal from the forward speed determine change in thrust, δ_T.

Final Form of Control Law

The best response on change of height during turbulence was obtained with proportional-derivational (PD) regulator for height and proportional (P) regulators for pitch rate and forward speed. Proportional regulator is simple amplifier of measured regulatory deviation, derivational regulator is amplifier of derivative of regulatory deviation and proportional-derivational is their sum. Hence, the final form of control law for CHARLIE in flight conditions given in previous text is expressed by following equations.

$$\delta_E = K_{PhE}h + K_{DhE}\dot{h} + K_{PqE}q = -0.00005h - 0.0004\dot{h} + q \tag{7.6}$$

$$\delta_T = K_{PuT}u = -5000u \tag{7.7}$$

Where $K_{PhE} = -0.00005$, $K_{DhE} = -0.0004$, $K_{PqE} = 1$ and $K_{PuT} = -5000$ are parameters of the controller. Corresponding block-diagram is shown on figure 7.1.

Block-Diagram of the Whole System

The real altimeter has a delay. So there was added according to Mc.Lean[1] simplified model to reflect that fact. The very same thing is valid for elevator and engine

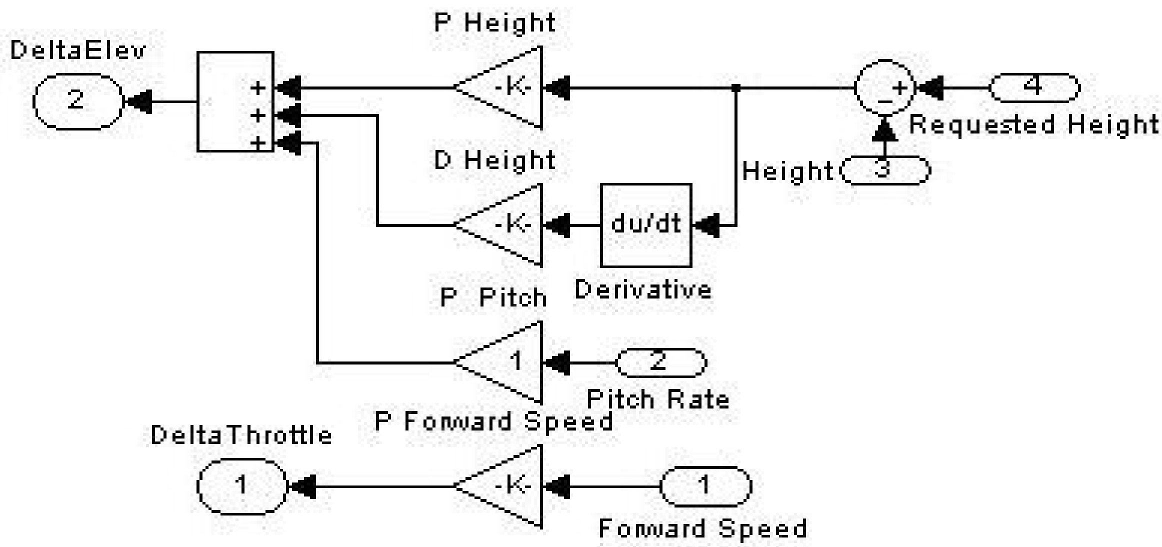

Figure 7.1: Block-diagram of the classic controller.

and furthermore they have physical limits how fast and how big control actions may be. Engine and actuator delays were added according to Mc.Lean[1] and rate limiters were added to ensure realistic rate of control action. Both are shown on figures. Maximum magnitude is given by amplification. It means that parameters were chosen to hold control actions in realistic ranges. This will be discussed together with results in section 7.2.

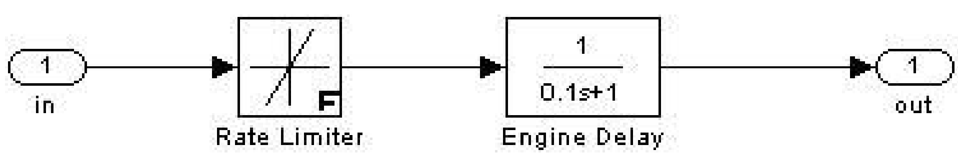

Figure 7.2: Block-diagram of the engine.

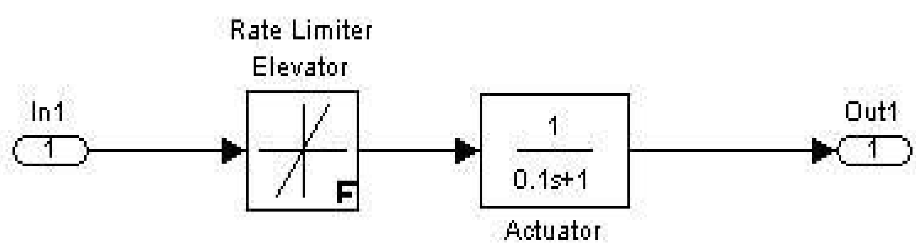

Figure 7.3: Block-diagram of the elevator.

7.2. Results of Simulation

It was chosen for simulation to request the change of height 10m in thunderstorm turbulence with duration of 150s. As it can be seen from figure 7.5 maximal overshoot from requested value of height is around 10.6m and control rate is 300s. In this time is regulatory deviation lesser than 5%. According to Mc.Lean[1] the available

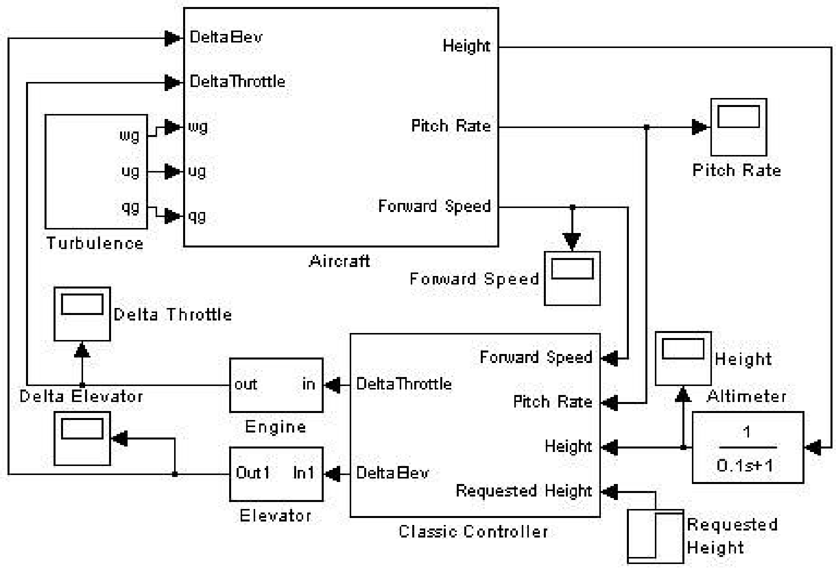

Figure 7.4: Block-diagram of the whole system.

excess thrust for CHARLIE is 525KN and only 10% can be used by the actuator. We have maximum thrust deflection 15KN which is perfectly in the range.

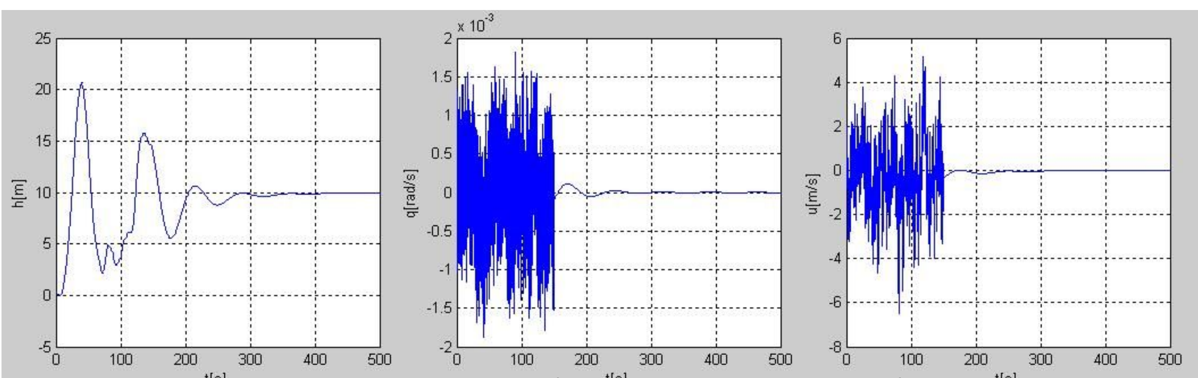

Figure 7.5: Height, pitch rate and forward speed response of classic controller

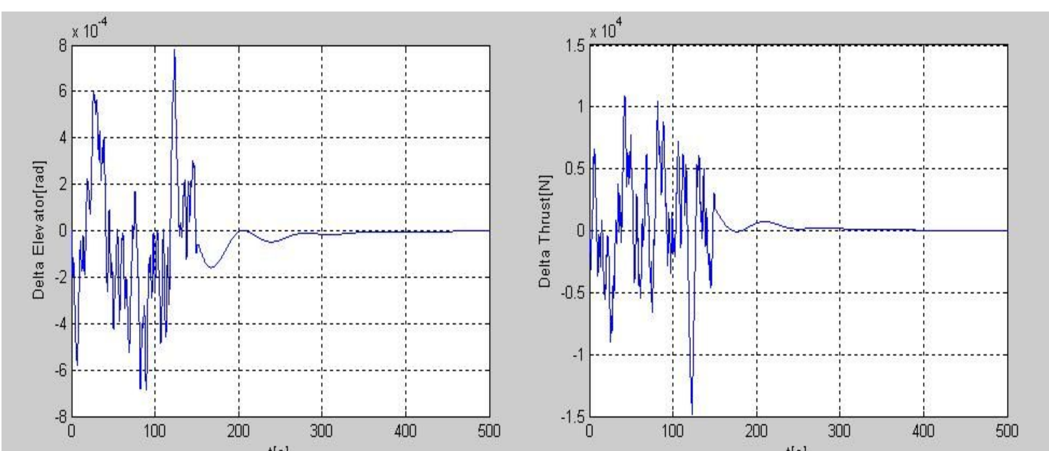

Figure 7.6: Control actions of classic controller

8. Fuzzy Controller

The construction of fuzzy controller is based on methods of fuzzy logic. This new specialization of mathematic is able to process vague information. When there is putting emphasis on accuracy of system's description it always brings many problems with correct description. It's coming into view that accuracy is principally unapproachable so the effort to reach absolute accuracy always leads to the unsolveable contradiction between relevance and accuracy of any information. This principal called L.A.Zadeh

The Principal of Incompatibility:

When there is increasing complexity of any system our ability to describe it accurately is decreasing. It means that after exceeding certain limit of complexity relevance and accuracy become mutually contrary characterizations.

Using the methods of fuzzy logic it's possible to apply vague information from natural language to control.

The aim of following three section is to shortly describe the basic terms needed to derive linguistic model of controller. It's only an extract from theory available in a number of publications. I've tried to hint in this theory what was used for building the model of fuzzy controller.

8.1. Theory of Fuzzy Sets

8.1.1. Motivation

According to the Russell's paradox there's a contradiction in the naive set theory. For a set S containing sets X that are not members of themselves is not possible to judge, whether S is member of itself. This problem can be solved when we concede not only statements like an element pertains or doesn't to a set but everything between them. It means that characteristic function of a set can hold not only values 0 and 1 but continuously all between them, so the borders between sets are ***fuzzy***. It leads to formulation of the *crisp set* C for classic set where are only two possibilities of membership and *fuzzy set* F where we have more possibilities of membership.

8.1.2. Membership function

Definition: The membership function μ_F of a fuzzy set F is given by

$$\mu_F : U \to \langle 0, 1 \rangle \tag{8.1}$$

where U is *universe*, the set containing all elements, from which we select.

Shapes of membership functions: Membership function used in this treatise are triangular and trapezoidal. Let $u, \alpha, \beta, \gamma, \delta \in U$.

Λ - function (triangular)

$$\Lambda(u, \alpha, \beta, \gamma) = \begin{cases} 0 & u < \alpha \\ (u-\alpha)/(\beta-\alpha) & \alpha \leq u \leq \beta \\ (\gamma-u)/(\gamma-\beta) & \beta \leq u \leq \gamma \\ 0 & u > \gamma \end{cases} \tag{8.2}$$

Π - function (trapeoidal)

$$\Pi(u, \alpha, \beta, \gamma, \delta) = \begin{cases} 0 & u < \alpha \\ (u-\alpha)/(\beta-\alpha) & \alpha \leq u \leq \beta \\ 1 & \beta \leq u \leq \gamma \\ (\delta-u)/(\gamma-\delta) & \gamma \leq u \leq \delta \\ 0 & u > \delta \end{cases} \tag{8.3}$$

Membership functions as areas: Vague terms like cold, pleasantly and hot for example is possible to express by shifting a membership function through universe,U. We use following terms to define areas of universe.

PB – Positive Big

PBS – Positive Big Small

PM – Positive Medium

PSB – Positive Small Big

PS – Positive Small

ZO – Zero

NS – Negative Small

NSB – Negative Small Big

NM – Negative Medium

NBS – Negative Big Small

NB – Negative Big

8.1.3. Basic Properties and Operations

Fuzzy set:

$$A = \{(u, \mu_A(u))/u \in U\} \tag{8.4}$$

Support:

$$S(A) = \{x/\mu_A(u) > 0\} \tag{8.5}$$

Convex fuzzy set:

$\forall x, y \in U$ and $\forall \lambda \in \langle 0, 1 \rangle$ holds

$$\mu_A(\lambda x + (1-\lambda)y) \geq \min(\mu_A(x), \mu_A(y)). \tag{8.6}$$

Intersection and union:

There are more possibilities how to define intersection and union of fuzzy sets. General approach uses *triangular norms (t-norms)* and *triangular t-conorms (s-norms)*. In this treatise is used Zadeh's definition.

$$\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x)) \tag{8.7}$$
$$\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x)) \tag{8.8}$$

Fuzzy number:

is fuzzy set, A, defined on the real axis with following properties: A is normal, convex fuzzy set with the limited support.

8.2. Theory of Fuzzy Logic - Approximate Reasoning

Approximate reasoning works with *fuzzy propositions* which have the truth value in the interval $[0, 1]$. Its rule with the following form of *fuzzy implication* has vague input information.

IF *fuzzy proposition* **THEN** *fuzzy proposition*

Those fuzzy propositions are *the linguistic variables* and they may consist of many fuzzy propositions connected by logical conjunctions *and, or.* The first proposition is *the antecedent* and the second is *the consequent.*

8.2.1. Linguistic Variable

Linguistic variable is the fundamental element representing a knowledge. The one often used in *fuzzy regulation* is *deviation*, e, taking values from the set of terms Ł$_e$\{ NB, NBS, NM, NSB, NS, ZO, PS, PSB, PM, PBS, PB\} for the case of this treatise. Linguistic variable can be described by ordered quartet:

$$\langle e, \text{Ł}e, U, M_e \rangle \tag{8.9}$$

Where e is symbolic name of the linguistic variable, $Łe$ is ordered set of word values variable can take, U is the universe (numerical range physical variable can take) and $M_e : Łe \longrightarrow \mu_{Łe}$ is function mapping the word values into the values of universe (its argument is the word value and returns the meaning of this value in terms of fuzzy sets).

8.2.2. Fuzzy Propositions

Atomic fuzzy proposition can be obtained from natural language in three steps:

1. Physical variable "deviation" indicate by e.
2. Its value "negative big" indicate by NB.
3. Then we obtain an atomic fuzzy proposition e is NB.

The membership function, μ_{NB}, defines its meaning and the value of μ_{NB} determines the degree with which the crisp value of physical variable deviation belong to the fuzzy set NB. Atomic fuzzy propositions can be connected by logical operations.

Fuzzy Conjunction

When there are two atomic fuzzy propositions $p : X$ is A and $q : Y$ is B, where the fuzzy sets A, B are defined on the same universe, U, then the meaning of the compound proposition $p \wedge q$ is given by intersection of fuzzy sets $\mu_{A \cap B}$.

Fuzzy Disjunction

When there are two atomic fuzzy propositions $p : X$ is A and $q : Y$ is B, where the fuzzy sets A, B are defined on the same universe, U, then the meaning of the compound proposition $p \vee q$ is given by union of fuzzy sets $\mu_{A \cup B}$.

Fuzzy Implication

There are many types of implication in fuzzy logic. The most popular type in fuzzy regulation and used in this treatise is Mamdani implication. It's defined by $(p \Rightarrow q) = (p \wedge q)$ where operation $\wedge$ uses t-norm min.

8.3. Fuzzy Regulator

Basic terms of continuous control given in section 6.2.1 remain the same. Difference is that fuzzy regulator is based on another principle where mathematical characterization is not needed. It's sufficient to known way how to control called *regulation strategy* which is described by the set of human-like commands of the **IF-THEN** type. So the only requirement is knowledge of the regulation strategy and there's not needed to look for mathematical description of the regulated process.

8.3.1. Structure of Fuzzy Regulator

Fuzzyfication component – This modul conducts normalization and fuzzyfication. Normalization is conversion of physical variables to the normalized universe based on scale. Fuzzyfication converts sharp values ofnormalized input variables into the fuzzy sets.

Knowledge basis – It's the ground of the fuzzy regulator constituted by a language description(a set of the **IF-THEN** rules).

Inference mechanism – This part processes the inputs in accordance with the knowledge basis which means that the rough deduction is conducted. The result is the output fuzzy set which represents an action intervention.

Defuzzyfication component – It conducts defuzzyfication of the output fuzzy set which means that this set is transformed to a concrete number. The result is specific action intervention. The smallest of maximum method (SoM) was used to avoid to big control actions.

So the simplified procedure of fuzzy regulation is:

1. Definition of the input and output variables.
2. Choice of the regulator type and the way of the rough deduction.
3. Construction of the knowledge basis.
4. Definition of the language context for all variables.

Types of fuzzy regulators

There's the same dividing like in classic regulation. We have P, PD, PI and PID regulators but in the means of the fuzzy logic. Fuzzy regulator is nonlinear function defined by the **IF-THEN** rules.

Setting the Regulator Parameters

It's done through normalization in fuzzyfication component by choice of scale.

8.4. Concept of Fuzzy Controller

Model was built by using MATLAB fuzzy toolbox. Input and output variables remain the same as for classic controller. Linguistic context was chosen as discussed in previous text. Fuzzy inference system is of Mamdani type. Membership functions are triangular near zero value of linguistic variable and trapezoidal otherwise to ensure fine regulation near requested value. Mapping of the set of terms into universe is not the same for all variables. Every variable matches different mapping as came out during experiments with model. See for example figure 8.1. Universe for every used variables were derived from classic controller then adapted for fuzzy controller experimentally and they are given as follows: $U_h = \langle -50, 50 \rangle$, $U_{\dot{h}} = \langle -6, 6 \rangle$, $U_q = \langle -10^{-4}, 10^{-4} \rangle$, $U_{\delta_E} = \langle -0.002, 0.002 \rangle$, $U_u = \langle -10, 10 \rangle$, $U_{\delta_T} = \langle -10^4, 10^4 \rangle$.

8.4.1. Regulation strategy

Type of fuzzy regulator is similar to classic controller but there are some differences. There was defined the set of 120 coarse rules as common fuzzy PD regulator (see figure 8.2). Damping signal from pitch rate were used to define the set of 11 fine rules when h and $\dot{h}$ are both Z. That's the reason why the universe of pitch rate is so small. Control of pitch rate is needed only when approaching requested value to stabilize aircraft movement. Response of regulator without the set of fine rules settled in oscillations with constant amplitude and frequency and had permanent regulatory deviation. The set of fine rules is defined as follows:

For both h and $\dot{h}$ are Z,

IF q is NB THEN δ_E is NB,

else IF q is NBS THEN δ_E is NBS,

else IF q is NM THEN δ_E is NM,

else IF q is NSB THEN δ_E is NSB,

else IF q is NS THEN δ_E is NS,

else IF q is Z THEN δ_E is Z,

else IF q is PS THEN δ_E is PS,

else IF q is PSB THEN δ_E is PSB,

else IF q is PM THEN δ_E is PM,

else IF q is PBS THEN δ_E is PBS,

else IF q is PB THEN δ_E is PB.

Forward speed controller is simple fuzzy P regulator:

IF u is NB THEN δ_T is PB,

else IF u is NBS THEN δ_T is PBS,

else IF u is NM THEN δ_T is PM,

else IF u is NSB THEN δ_T is PSB,

else IF u is NS THEN δ_T is PS,

else IF u is Z THEN δ_T is Z,

else IF u is PS THEN δ_T is NS,

else IF u is PSB THEN δ_T is NSB,

else IF u is PM THEN δ_T is NM,

else IF u is PBS THEN δ_T is NBS,

else IF u is PB THEN δ_T is NB.

The block-diagram of fuzzy controller is shown on figure 8.3. All the other parts of the system remain the same as for classic controller.

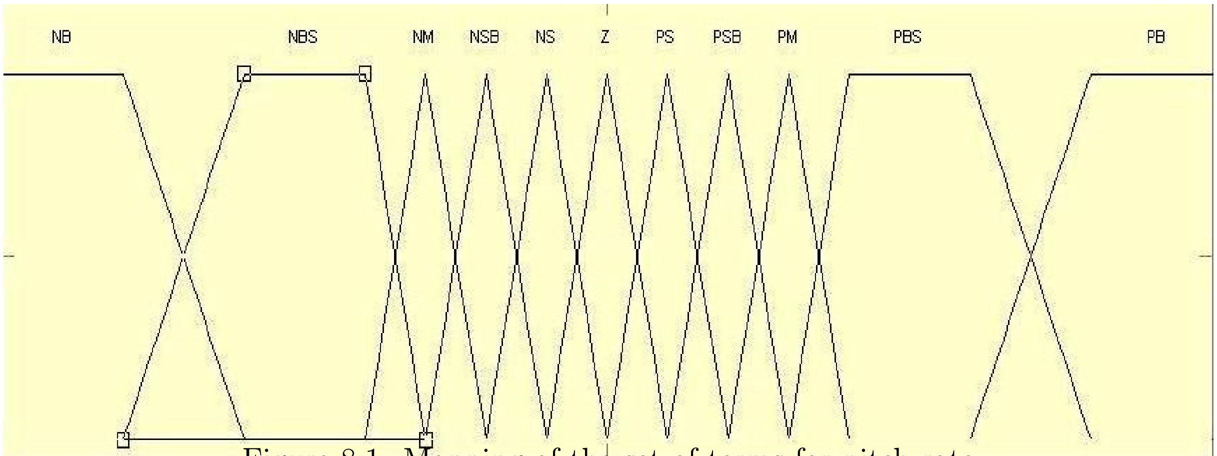

Figure 8.1: Mapping of the set of terms for pitch rate

		derivation of height deviation										
		NB	NBS	NM	NSB	NS	Z	PS	PSB	PM	PBS	PB
	NB	PB	PB	PB	PB	PB	PB	PBS	PM	PSB	PS	Z
	NBS	PB	PB	PB	PB	PB	PBS	PM	PSB	PS	Z	NS
	NM	PB	PB	PB	PB	PBS	PM	PSB	PS	Z	NS	NSB
	NSB	PB	PB	PB	PBS	PM	PSB	PS	Z	NS	NSB	NM
	NS	PB	PB	PBS	PM	PSB	PS	Z	NS	NSB	NM	NBS
h	Z	PB	PBS	PM	PSB	PS		NS	NSB	NM	NBS	NB
	PS	PBS	PM	PSB	PS	Z	NS	NSB	NM	NBS	NB	NB
	PSB	PM	PSB	PS	Z	NS	NSB	NM	NBS	NB	NB	NB
	PM	PSB	PS	Z	NS	NSB	NM	NBS	NB	NB	NB	NB
	PBS	PS	Z	NS	NSB	NM	NBS	NB	NB	NB	NB	NB
	PB	Z	NS	NSB	NM	NBS	NB	NB	NB	NB	NB	NB

Figure 8.2: Coarse rules for fuzzy height controller

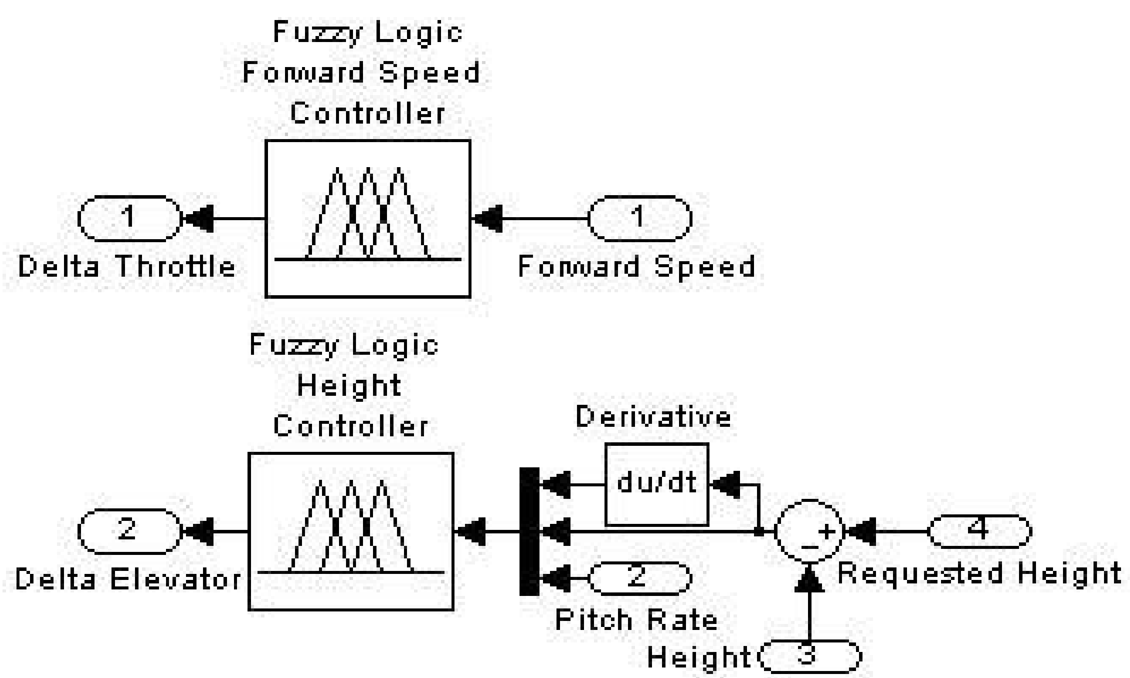

Figure 8.3: Block-diagram of fuzzy controller

8.5. Results of Simulation

Simulation was performed under the same conditions as for classic controller. As it can be seen from figures 8.4 and 8.5 maximum overshoot is around 4.2m and control rate is around 175s. Maximum thrust deflection is 8KN. Unfortunately we have still small oscillations in pitch rate that we can't remove. It's a consequence of inaccurate regulation. We don't have precise value of 0 but certain area given by the membership function Z. So we can't never reach the accurate requested value of controlled variable because there will be always area (smaller than Z) where controller doesn't regulate. It should be the task of subsequent research how to inhibit such oscillations. As it can be seen from figure 8.5 oscillations in pitch rate causes oscillations of the elevator which can cause detrition.

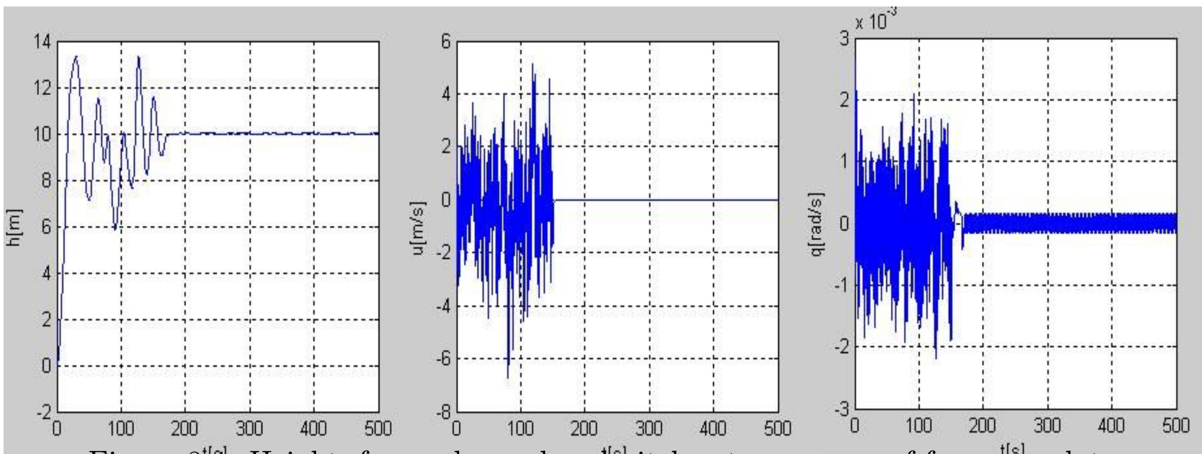

Figure 8.4: Height, forward speed and pitch rate response of fuzzy regulator

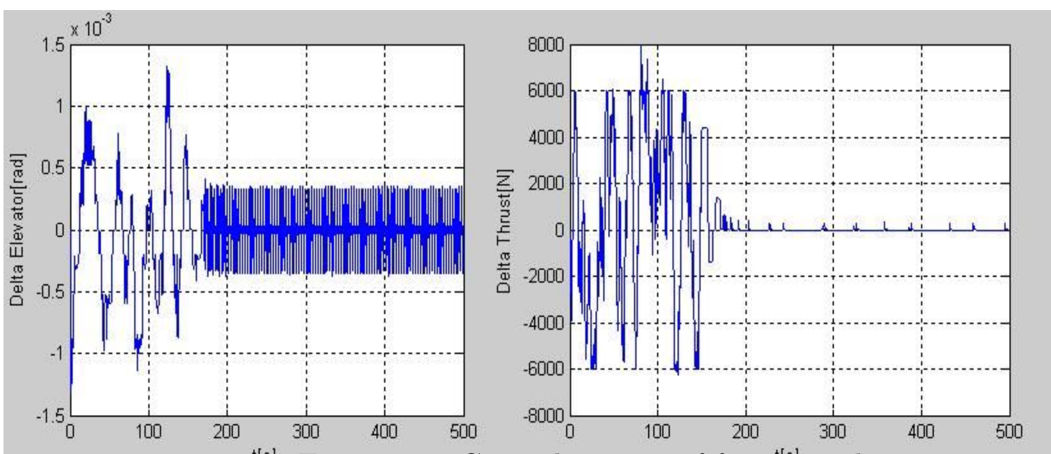

Figure 8.5: Control actions of fuzzy regulator

9. Conclusion

As it can be found out from the results of simulations fuzzy controller shows much better behaviour than the classic one. Maximum overshoot of fuzzy regulator is approximately 4.2m against 10m for classic controller. The resulting control rate of fuzzy regulator 175s is very good too especially when we realize that the duration of simulated thunderstorm turbulence was 150s! On the other side the classic controller has control rate for that simulation 300s. When we notice maximum throttle deflections of the both regulators fuzzy controller is again markedly better with 8KN which is nearly half of the result of the classic regulator which has 15KN. It shows possible good economical acceptability of fuzzy controllers. Better stability of the fuzzy regulator was perceived already during the design process. Universe for control and action quantities of fuzzy controller was set in the first step to reflect the structure of the classic regulator. It's worth to mention that especially possible deflection of elevator as a result of signal from regulatory deviation was significantly increased by enlargement of U_{δ_E} and the system remained perfectly stable. On the other side it came out that the fuzzy controller had problems when approaching the requested value. There remained oscillations in the pitch rate which caused subsequent oscillations of the elevator. This should be the task for further research how inhibit those oscillations not to cause detrition of the elevator. Another open question is what happens when any defect of the aircraft occurs. To simulate such situation it's needed to have much more data about the modeled aircraft.

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