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EXOTIC OPTIONS AND THEIR FEASIBLE USAGE AS INVESTMENT INSTRUMENT

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Abstract:

The main goal of the diploma thesis is to determine if the exotic options are suitable for hedging risks related to currency exchange rates and to propose the best application of them. The thesis is aimed on specific family of exotic options: path-dependent exotic options. Three types of the exotic options are analyzed and then tested within them and also with the vanilla option. The thesis main outcome is recommendation of feasible usage of tested exotic options.

Keywords:

Exotic Options, Path-Dependent Exotic Options, Asian Options, Barrier Options, Binary Options, Black-Scholes Pricing Model.

Abstrakt:

Diplomová práce primárně řeší zda jsou exotické opce vhodné pro zajištění kurzových rizik a přináší návrh vhodné aplikace exotických opcí. Práce je zaměřena na úzkou skupinu exotických opcí, tzv. Path-Dependent opce. Tři často používané typy těchto opcí jsou analyzovány a testovány jak mezi sebou tak pro lepší porovnání i s klasickou vanilla opcí. Hlavním výstupem diplomové práce je návrh vhodného využití testovaných exotických opcí.

Klíčová slova:

Exotické opce, Path-Dependent exotické opce, Asijské opce, Bariérové opce, Binární opce, Black-Scholesův oceňovací model.

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Čestné prohlášení

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Podpis

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1 Introduction

The dictionary defines a derivative in the chemistry field as “a substance that can be made from another substance”. The financial derivatives are very similar to that. The values of these financial instruments are derived from a value of some kind of underlying. The underlying could be almost anything; it is most commonly financial asset or rate. In addition there are some derivatives that are linked to stock indexes, weather, or some circumstance. It is for example number of bankruptcies among a group of selected companies. The Bank for International Settlements conducted statistic research and claims that size of the market for derivatives exceeded 500 trillion dollars in 2007 [4] [17]. Therefore, derivatives market generates huge amount of different financial instruments. One kind of these financial derivatives is financial options.

These options have long tradition. The earliest usage of the options could be tracked to the ancient Greece, where old philosopher Thales negotiated for the use of olive presses for the upcoming spring while it was a still winter. He paid small amount in advance that gave him a right to buy as first as harvest came for the negotiated price. The options were also used during the seventeenth century in Holland during the tulip bulb craze. In the USA, options first appeared in the 1790 during the beginning of the NY Stock Exchange. The put-call parity concept was originally identified, as conversion was understood in the end of the nineteenth century by Russell Sage. He was a railroad speculator and is recognised as a father of modern option trading [16].

Exotic options have been on the market also for a long time. It's almost 40 years up to date. The barrier options have appeared in the U.S. OTC market in 1967. It was Down-and-Out option first, than in 80's Up-and-Out option took place as significant OTC product. Despite its long history the term of Exotic options is relatively new. First terms used for these special options were “boutique” options or “designer” options. The first use of the term “Exotic Option” is tracked to 1990 when Mark Rubinstein used it in the series of articles. He was trying to find out simple pricing models based on Black-Scholes pricing model that couldn't fit in customised OTC market precisely [14].

The evolution of exotic option was necessary. Exotic options are more flexible than their vanilla counterparts. As the market matured over the time and option pricing became more transparent, specialised structures started using customised instruments for hedging and portfolio insurance strategies. Since vanilla options were too expensive for some goals, new options needed to be engineered. Many of those new products have never existed before [14].

It is clear therefore that options went through long evolution and market has developed exotic options for several reasons. Those reasons could be concluded as:

1. Exotic options are usually cheaper than vanilla options.
2. Exotic options are more flexible and could be designed for special cases.
3. Increased understanding and transparency of exotic options contributed to widespread of usage of them.
4. Moreover, Exotic options have attracted speculators who are seeking bigger yields if the user's view of the market is correct. Exotic options usually require users to make special bet on the direction of the market.

1.1 The goals of the diploma thesis

The main goal of the diploma thesis is to test practical use of the chosen exotic options as an instrument for hedging currency exchange rate risks. The main outcome is the recommendation whether the tested options are suitable for hedging or not. They are compared within themselves and with the plain vanilla option.

The partial goal of the thesis is an analysis of the selected exotic options (asian options, barrier options and binary options). This analysis helps to understand all the major features of selected exotic options; therefore it will significantly contribute in achieving of the main goal of the diploma thesis. Partial goals are:

- Asian options analysis
- Barrier options analysis
- Binary options analysis

1.2 Methodology

The main goal is reached by comparative method and synthesis supported by mathematical models (binomial pricing model, Black-Scholes pricing model and its modifications). The selected options are tested for hedging of currency exchange rate purposes.

The partial goal is reached by the method of analysis also supported by previously mentioned mathematical models.

The methodology of research is quantitative; type of research is deductive with applied outcome.

All the mathematical models are executed by computer software dedicated to option pricing calculations based on Black-Scholes equation. The software is accessible at: <http://formulapages.com/>.

2 Options

Financial options are term derivative instruments that give the holder a right to buy or sell certain asset (underlying) for negotiated price at negotiated moment or during specified period. The counterparty of the deal known as the writer does have a potential obligation. He must sell or buy underlying asset if holder chooses so. The holder pays option premium for this right to writer of the option at the time of opening contract.

Financial options are contingent claims; therefore they are different from other derivatives by being potential asset for one party and potential liability for another. These circumstantialities in the values are subjects of probability theory. Thus, all options pricing have to take probability into an account [21].

2.1 Option value

The value of the option is developing during the time and is dependent on two components. Those components are time value and intrinsic value. The time value is also known as instrumental value or extrinsic value.

The intrinsic value is equal to the price of exercising the option now. If the value is higher than 0, that the option is in-the-money, otherwise is the option out-of-the-money. Because the owner would never exercise option to loose money but let the option expire without exercising it, therefore option doesn't have negative value. Intrinsic value is dependent on the price of underlying asset.

The time value of the option is on the other hand speculative value that changes during the lifetime of the option. The time value is decreasing as option matures and is equal to zero at the expiration. The time value reflects the probability of the option to become profitable for the owner at the time of maturity. In addition, the time value is derived of the underlying asset's volatility.

At expiration the value of the option is equal to the intrinsic value because the time value is equal to zero [21].

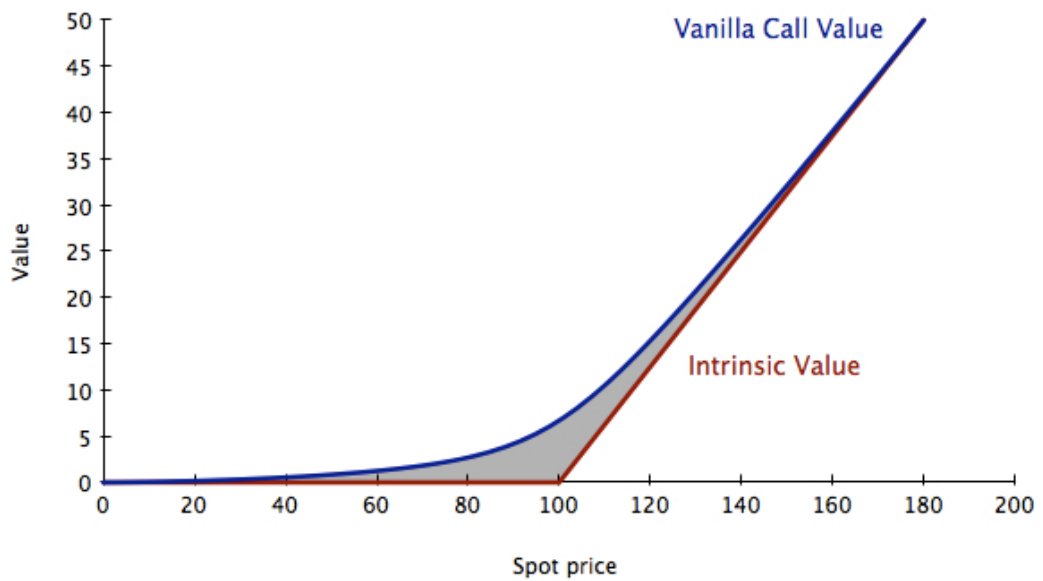


Chart 2.1: Vanilla Call Option Value

2.2 Basic distinction of financial options

There are many types of financial options and they distinguish from each other in many aspects. Regular kinds of options are called plain vanilla options or simply vanilla options. These options are pure options with no additional features added. All other options belong to exotic options' family. In addition, both vanilla and exotic options are either call or put and European or American kind.

2.2.1 Call Option

The call option gives holder right to buy an underlying asset at the time of expiration at the strike price. Holder is going to exercise the option only in case if the spot price of the underlying asset exceeds strike price at the moment of expiration. In other case the holder won't execute the option and let it expire. On the other hand issuer (writer) is obligated to sell this underlying asset to the holder of the option if he ask for it.

The holder of the option has therefore theoretical possibility to generate unlimited surplus while his loss is limited to the option premium. On the other hand, the issuer is in exactly opposite position. His surplus is limited to the option premium and loss can be limitless.

An investor who expects price of the underlying asset to rise would buy the call option.

2.2.2 Put Option

An option to sell an underlying asset is called put option. It gives holder of the option right to sell the underlying asset to the issuer at the time of expiration at the strike price. The holder will exercise the option in case of price drop of the underlying asset.

In this case, the holder of the option doesn't have opportunity to gain limitless profit. His surplus is limited by the strike price on top and by zero on bottom because the price of the underlying can't be negative. Maximum profit can be therefore calculated as strike price minus option premium.

An investor who expects price of the underlying asset to decline would buy the put option.

2.2.3 European and American options

The financial options can be exercised in two different ways.

European option: This style of the option can be exercised only at the expiry date.

American option: If the option can be exercised anytime during its lifetime then it is American type of the option.

If an American and a European option are constructed otherwise same way and having same strike price etc., the American option is going to be worth as much as European option or more. The higher value of the American option is a consequence of opportunity to exercise the option before its maturity. In addition an investor holding American option won't probably execute the option before the expiration because the value of the option also contains time value. Therefore, the investor will rather sell option before expiration then exercising it immediately, sacrificing the time value.

2.3 Option transactions

Before 1973 all options were over-the-counter (OTC) type of contract. That means a broker on behalf of the buyer and the seller individually negotiated those contracts. After 1973 the trading on an official Chicago Board Option Exchange (CBOE) began, with trading initially only in call options on some of the most heavily traded stocks. The listing of the options on an exchange increased competition in this field, therefore the cost of the option contract significantly decreased.

Nowadays, options are traded on the most major exchanges worldwide. They are no longer restricted to equity options but options are also written on futures, government bonds, commodities, currencies etc. On the other hand, the OTC market is still important for the option markets and its main task is to meet special client's needs.

This is where exotic option contracts are created. Many options are registered and settled via a clearing house. This institution is also responsible for collection of margin from the option's issuers. It is a guarantee that writer will be able to meet his obligations if the asset price moves against him [21].

2.3.1 Opening and closing of the position

If the seller of the option didn't previously buy the option from another investor but he's is creating brand new option, than it is writing (issuing) the option. Seller is in the short position and on the other hand buyer of the option is in the long position.

With regard to closing position have writer and holder several alternatives.

- **Selling the option**

A holder of the option can always sell this derivative to the 3rd party anytime before expiry date. Selling is often the best way how to close out a position if there is still time remaining before expiration. More time before expiration more time value option usually has. Selling of the option also means that holder will not need to take a position in the underlying asset.

- **Exercising the option**

An American style option can be exercised anytime on or before the expiry date. If investor exercises the option only the intrinsic value is going to be exercised and potential time value will be lost. Exercising of the option actually means buying or selling the underlying asset. This would be appropriate if there is almost no time value of the option and/or the investor wants to buy the underlying asset in case of a call or sell the underlying asset in case of the put option.

- **Getting assigned**

The other side in long position can assign the issuer of the option who is in short position. The party in short position has no control over this and must simply fulfil the obligation of the option contract. That means selling or buying underlying asset from the holder at the strike price.

- **Letting the option expire**

An option will expire worthless if the option is either at-the-money or out-of-the-money on the date of expiry. Letting option expire worthless is only good alternative when is the option out-of-the-money. The investor loses his premium if option expires worthless.

- **Offsetting the option**

Offsetting is used very often and it's simply a method of reversing the original transaction to close the contract. Holder can always sell an existing option he previously bought back to the writer. Offsetting transaction is usually the best way how to close option position if there is still some time left to expiration [12] [21].

2.3.2 Use of the options

Options can be used for two primary uses: speculation and hedging.

Speculating

An investor who believes that a particular stock is going to rise can simply open a position and buy stock of that company. If his prediction is correct, he can make money. If he's wrong, he will lose some. If he invests to the share that rises from \$250 USD to \$270 USD he makes a profit of \$20 USD that is 8%. If the same stock falls to \$230 USD he loses \$20 USD per share, which is also 8%. The investor can also speculate that share will rise within next three months; hence he decides to buy a call option with strike price \$250 USD and expiry in three months. If he invests his money to the option and not directly to the asset itself, he's going to be exposed to much greater risk of losing all his money. On the other hand his potential surplus is much greater as well.

The investor can also speculate on the fall of the shares. If he thinks so he can sell shares or buy put option. If he's speculating by selling shares he doesn't own (short-sale) the situation would be very similar to the call option situation described

previously. If the investor wants to speculate that share will drop within next three months down to \$230 USD, he will buy a put option with strike price \$250 USD and expiry date in three months [21].

Options can be a cheap way of exposing a portfolio to a large amount of risk.

Hedging

Simply said, the hedging means reducing of the risk. Or moving the risk to another subject. Constructing the portfolio of the assets and derivatives that are in the proper correlations could do this. For example investor can purchase both asset and put option written for this asset. What will happen is when asset price falls, price of the put option rises and vice versa. The portfolio that contains only assets falls when the asset price drops. If the portfolio has only the put options it rises when the asset price falls. The main goal is to find ratio at which a unpredictable movement in the asset does not result in any unpredictable movement in the value of the portfolio. This ratio would be risk free [21]. There are many applicable hedging strategies.

2.4 Pricing of the options

Pricing of the options is the most complicated discipline in financial options theory and there are many academics focused on this challenge. The biggest problem was to determine proper price for the option premium.

Payoff of the call option is priced at expiration as the

$$C_T = \max (S_T - K, 0)$$

where the S_T stands for asset price at expiration, K stands for strike price and C_T stands for value of Call option at expiration. It is read as the greater of the difference between the asset price at expiration and the strike or zero. That means the holder retains intrinsic value of the option or nothing whatever is greater. $S - K$ is the difference between spot and strike. If negative than the holder would prefer to let the option expire and receive nothing [5].

Payoff of the put option is

$$P_T = \max (K - S_T, 0)$$

with additional P_T that stands for put option value at expiration.

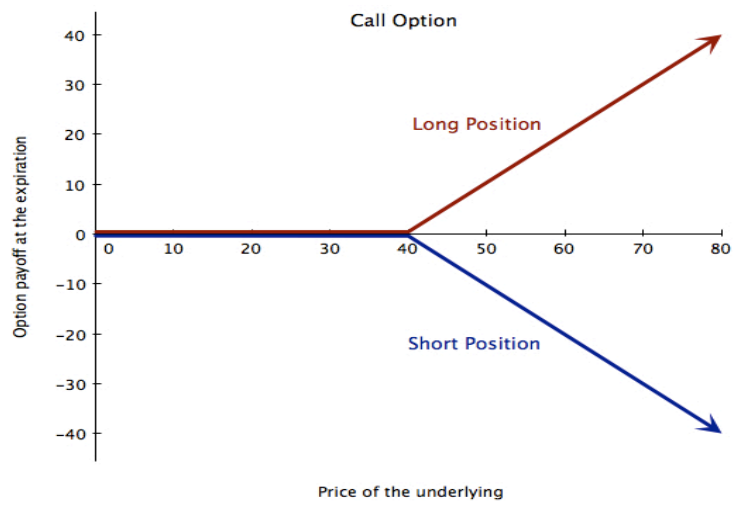


Chart 2.2: Vanilla Call Option Payoff

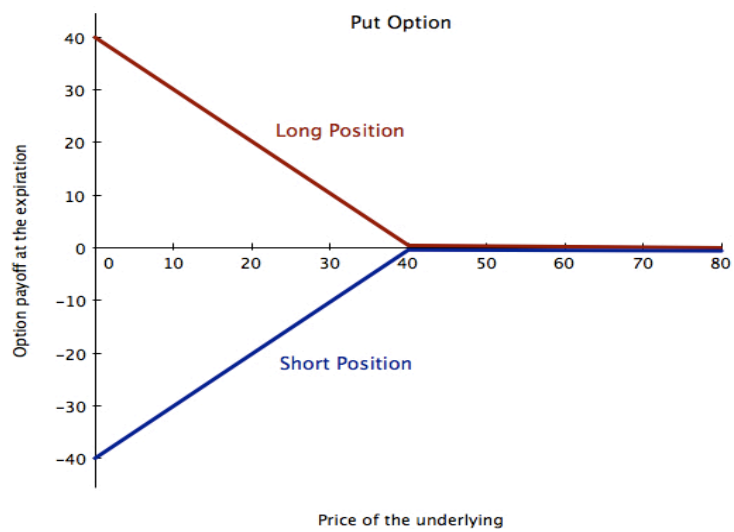


Chart 2.3: Vanilla Put Option Payoff

If there is and option premium taken into the consideration the call option payoff would be modified to

$$C_T = \max (S_T - K, 0) - P$$

where the additional P stands for premium paid by the holder to the writer.

In addition put option payoff can be described as

$$P_T = \max (K - S_T, 0) - P$$

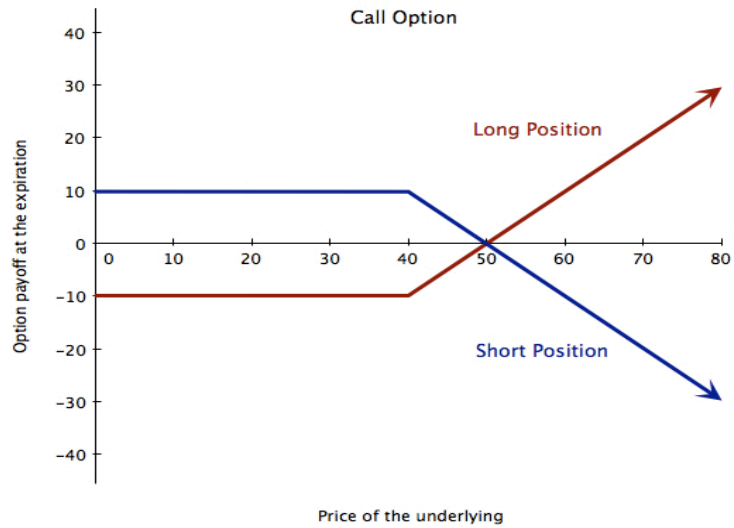


Chart 2.4: Vanilla Call Option profit/loss

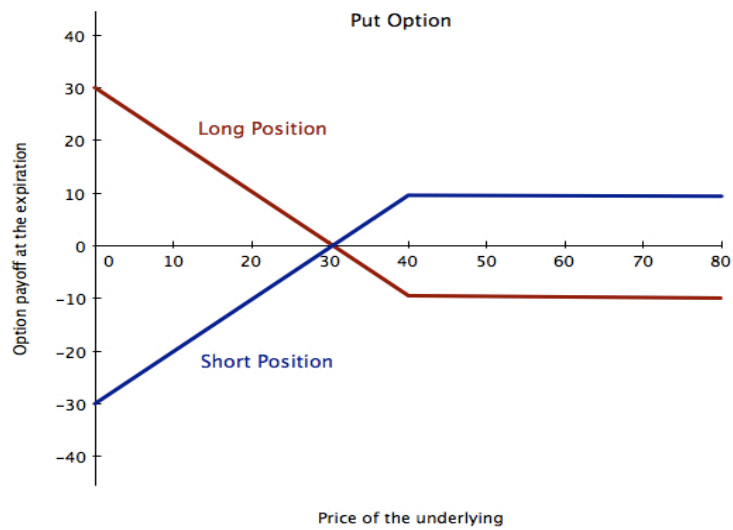


Chart 2.5: Vanilla Put profit

2.4.1 Binomial pricing model

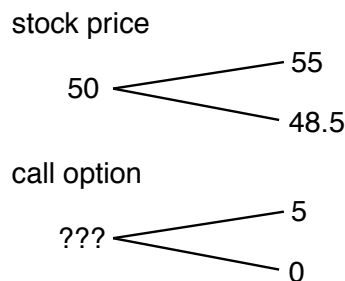
The price of the option is usually non-linear function dependent on the underlying asset and other variables like interest rates etc. Any pricing model is basically trying to describe the stochastic process taken by the underlying asset on which the option is written.

The binomial model is assuming that price of the underlying asset occurs only in certain time intervals. The model involves constructing of a “tree” that represents different possible paths that the price of the underlying asset can follow. Hence it is called the Binomial tree.

The binomial pricing model simply assumes that the stock price follows simple stationary binomial process. Therefore, the price can go only up or down by given probability. If the price of the underlying follows such a process and when there is a risk-free asset than the options written on the underlying asset can be priced [3] [12].

Example:

Consider a stock with actual price of \$50 USD per share. Suppose that the stock price can go either up by 10% or down by -3%. Therefore, the stock price at the end of the period could be \$55 or \$48,50. If there also exists a call option written on this stock with strike price $K = \$50$, then these three assets will have the following payoff patterns:



The payoff of the option can be replicated by a linear combination of the stock and the bond. Number of shares is denoted by A and number of government bonds by B. This will exactly replicate the option's payoff. The system of equations:

$$55A + 1,06B = 5$$

$$48,5A + 1,06B = 0$$

After solving the equations, we get:

$$A = 0,769231 \text{ and } B = -35,1959$$

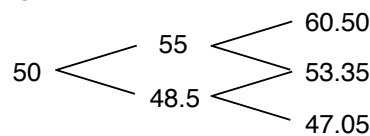
Therefore, the purchasing 0,77 shares and borrowing \$35,20 at 6% of interest for the one period will generate payoffs of \$5 USD if the stock price goes up and \$0 if the stock price goes down. The price of the option must be equal to the cost of replicating its payoffs, thus *call option price* = $0,769231 * 50 - 35,1959 = \$3,2656$

This is also called “pricing by arbitrage”, that means if two assets or sets of assets (in this case option versus the portfolio of shares and bonds) have the same payoffs, they must have the same market price.

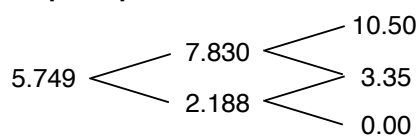
A two-date example:

What happens if the time to maturity is longer is described on following example. There is a two-date example with the strike price of \$50 USD as previous example. The binomial tree will have following pattern [3].

Stock price



Call option price



2.4.2 Black-Scholes pricing model

In 1973 Fisher Black and Myron Scholes developed and published their option pricing formula and started the modern option-pricing era. This elegant formula has forever changed the way in which practitioners and theoreticians view the pricing of financial derivatives.

The formula itself is very important and very widely used nowadays. Authors of the formula were awarded with Nobel price in economics. The formula contains advanced probabilistic techniques from martingale theory and stochastic calculus, which are accessible only to a small group of experts with a high degree of mathematical sophistication, like Black and Scholes were [7].

This formula is based on independent development of underlying asset. The price of the asset is following Brownian motion also known as random walk.

Black-Scholes equation:

$$\text{Call option: } C(S, T) = SN(d_1) - Ke^{-rt} N(d_2)$$

$$\text{Put option: } P(S, T) = Ke^{-rt} N(d_2) - SN(-d_1)$$

where:

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T - t)}{\sigma\sqrt{T - t}}$$

$$d_2 = d_1 - \sigma\sqrt{T - t}$$

where:

N is the normal distribution function, σ is the implied volatility level, S = Stock price,

K = Strike Price, r = the interest rate, T = time to expiration

The formula represents and shows the correlation within things we already know.

- **N** is the normal distribution function. The people in financial world have admitted that only predictable thing about stock prices is that they are randomly distributed. This can be best described by standard normal distribution. Hence, the authors of the formula choose this pattern.
- **σ (sigma)** stands for the volatility level of the option. The volatility of the option has big effect on the time value of the option. Higher the volatility is higher the time value is also. Therefore, the premium rises as with the volatility.
- **S** is the stock price. The stock price itself affects directly the price of the premium as the price of the underlying is connected with the range or move.
- **K** refers to the strike price. Strike price is very important because it is determining the payoff of the option ($S - K$ for the call options or $K - S$ for the put options). Therefore, the **K** is essential to option pricing and has to be involved in the formula.
- **r** stands for the interest rate involved in the model. In the original Black-Sholes model it is used for the risk free rate of return, thus government bonds return. In regular application this rate usually refers to the rate the trader pays or receive if he's lending or borrowing money. Higher interest rates make call options more expensive but on the other hand they make put options cheaper.
- **T** stands for the time to expiration. The greater the time to the expiration is, the greater impact of the volatility and interest rate on the option will be. Therefore, as the time to expiration grows, makes the option premium more expensive [7].

The Black-Scholes pricing model has also several flaws. It is assuming constant volatilities over the time of the life of the option. The reality is that volatilities may change during this time. It also counts with one interest rate but they may differ among lenders and borrowers. The equation is also not taking dividends into the consideration. Despite all the flaws it is widely used in the modern financial world.

3 Exotic Options

There are many different types and subtypes of exotic options. Because of the size limitation of this thesis only the path-dependent exotic options are analyzed.

3.1 Path-Dependent Options

“The Path-Dependent options” is family of options where the payoff is dependent on historical values of the underlying asset over a certain period of time. This payoff can be also dependent on some special behaviour of the asset. This kind of options debuted in 1982 and after that they have made their way in many places. They have been used, as instruments in risk management and also for investment needs. First Path-Dependent options that came into an action were Lookback options. Owner of this kind of option can look back over the life of the option and choose to buy or sell underlying asset at the best price that occurred during the period, hence the name Lookback option [1] [13].

This path dependency attribute gained interest of the investors for their design match with some financial contracts. Investors tend to use them for securing potential losses. For example, they can be used for hedging average exchange rate of foreign currency. In addition, they allow investors to better use their information of volatility, asset price etc.

The most common representatives of Path-Dependent exotic options are:

- Asian Options
- Lookback Options
- Barrier Options
- Binary Options
- etc...

3.1.1 Asian options

Asian options are one of the most popular exotic options and are also known as average options. Asian options have origin in Tokyo office of Banker's trust in 1987, hence the term "Asian Options". The special feature of this kind of option is related with the payoff, which depends on average price of underlying asset. In other words, the path taken by the underlying asset over a fixed period of time determines the payoff. Therefore, the Asian options belong to the class of path dependent exotic options.

On this basis it might be inferred that Asian options are suitable for hedging cash flow where the hedger is concerned with the average of the cash flow over the time. In addition, Asian options would be also suitable for hedging average price of a stock where the hedger is purchasing larger amount of stock on the financial market over a certain period of time. Another benefit of using Asian options could be that Asian options can lower the effect of high price volatility of certain asset related for example with a low liquidity. Therefore, many of commodity contracts are secured by Asian options and they are settled with regard to average price of the commodity over a period of time [23] [22] [15].

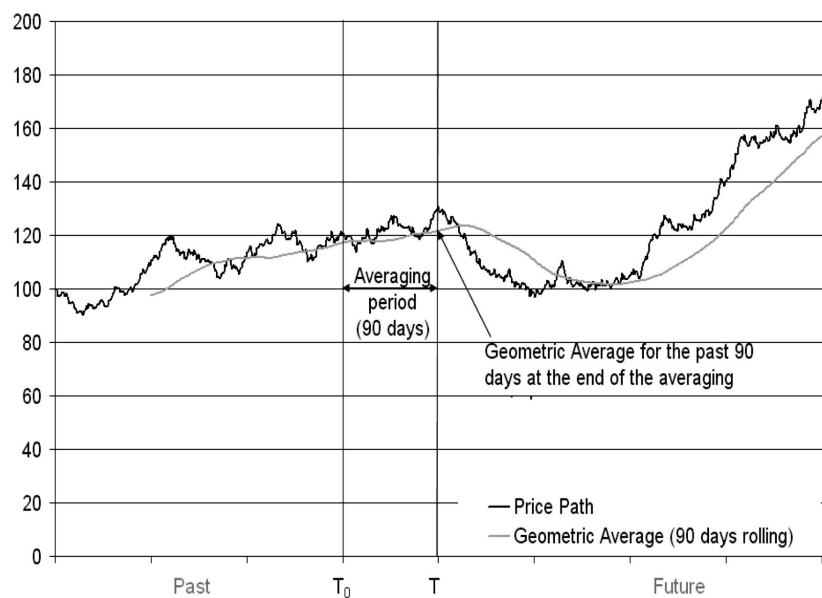


Chart 3.1: Asian option [22]

Previous figure shows how the geometric average path is less volatile than price path. As a consequence of lower volatility of the average price (in this example: geometric) is the risk also lower. Therefore, the Asian option is usually cheaper as shown on following chart.

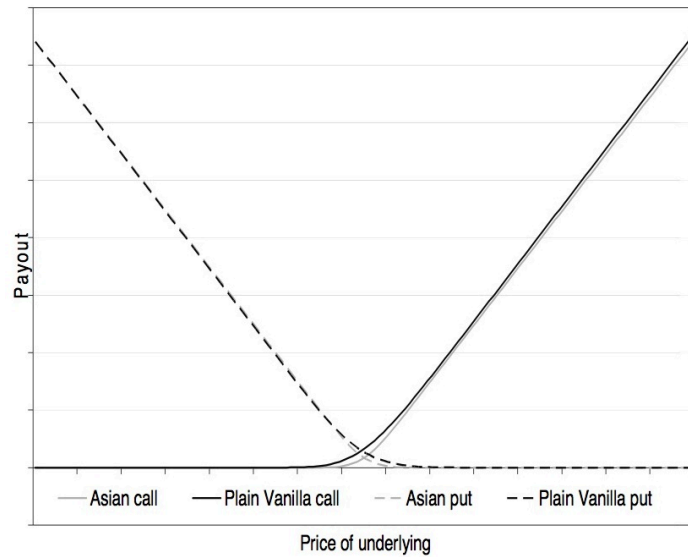


Chart 3.2: Asian option underlying price [22]

Asian options are distinguished in two major categories, thus four cases are stated in the following table.

Product name	Payoff
Average Rate Call Option	$C_T = (A_T - K, 0)$
Average Rate Put Option	$P_T = (K - A_T, 0)$
Average Strike Call Option	$C_T = (S_T - A_T, 0)$
Average Strike Put Option	$P_T = (A_T - S_T, 0)$

Table 3.1: Average rate asian option example

K denotes the strike, S_T the spot time at expiration time, A_T the average, C_T call option payoff, P_T put option payoff.

Average Rate Options

In this case, the value of the Average Rate Asian Option is derived from the average spot price of the underlying asset over a certain period and the strike price. The call option is in the money if the average rate is higher than strike value. Thus, the owner can exercise the option and his profit will be the difference between the average and the strike.

Example:

The European investment fund is planning to increase share in NWR Company whereas the current price of the stock is 280 CZK. Stocks of this company can be purchased at Prague Stock Exchange market. The fund is expecting growing potential of this stock and they need to purchase large amount of NWR stock. The company has decided to divide this purchase in smaller portions, which will be executed every day during next 15 business days, because of the insufficient supply of the stock. The fund is aware of the possibility of growing price and wants to secure this average price at 280 CZK. Hence, the acquirement of an Average Rate Call Option is going to take a place. The option will be settled for the 15 days period, amount of the stock that fund wants to acquire and the strike price will be 280 CZK.

It means that if the prediction of the fund is correct and the average price of the stock will be higher than strike price, than the option issuer will pay the difference between average price and the strike price. On the other hand, if the average price drops below the strike price (280 CZK), the option will be out of money, therefore it won't be executed.

Average Strike Option

The specialty of this option is the fact that average price of the underlying asset is not used to determine value of the option at the time of its expiration. On contrary, it's the strike price that is derived from the average price of underlying asset. The call option is "in the money" if the spot price is higher than strike at the moment of the expiration. Therefore, the owner is going to acquire the value of the difference between spot price and strike.

Example:

An European subsidiary of American company has to translate all profits in the end of the year at average exchange rate. The averaging dates are the last days of every month. To transfer the profits in the end of the year, the subsidiary will need to sell EUR. The subsidiary is unsure of the future development of the exchange rate of the EUR. They want to protect themselves against negative exchange rate movement, but would like to benefit from the appreciation of the EUR. Thus, the subsidiary will purchase Average Strike Put Option with averaging dates at the end of every month.

It means that if the average rate at year end is above the spot price at the time of expiration the subsidiary will exercise the option and issuer will pay the difference between the average rate (the rate that need to be used for profit translation) and the spot price (the current rate at the time of transaction).

On the other hand, if the average rate drops below the spot price at the expiry the option will be “out of money”, but the subsidiary can use the advantage of lower exchange rate at the moment of the transfer.

The European subsidiary has decided to secure 1 000 000 EUR. Spot price is 1,3847 USD / EUR in the beginning of fiscal year. Number of averaging is 12.

Spot reference	1,3847 EUR-USD
Maturity	1 year
Volatility	13%
Number of a averaging	12
Notional	1 000 000 EUR
Company sells	EUR put USD call
Premium	0,02332 EUR
Premium total	23 320 EUR

Table 3.2: Average strike asian put option calculation

Following chart describes movement of the currencies exchange rate during the fiscal year. Blue line represents spot price and red line states for average price of the EUR/USD

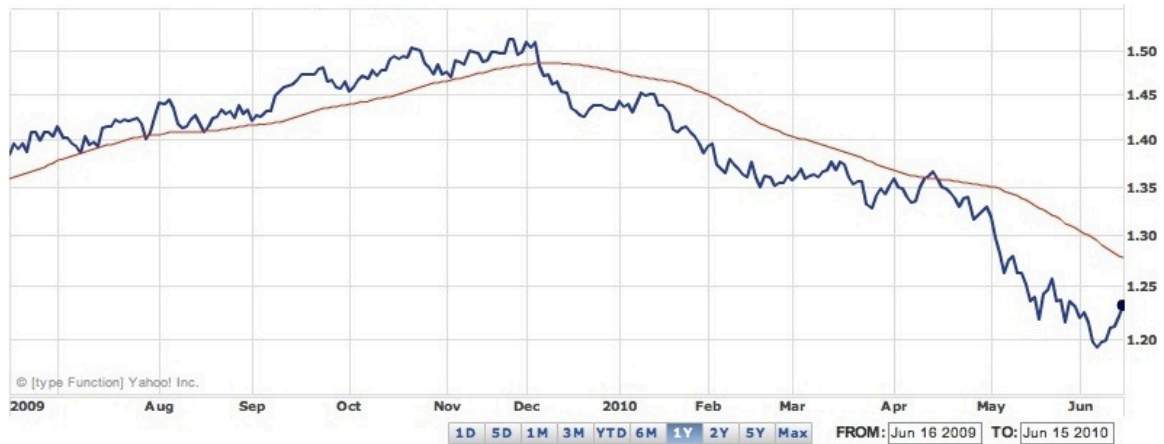


Chart 3.3: EUR/USD exchange rate 16/6/2009 – 15/6/2010

Since the spot price dropped below average price, option will be executed. Details are stated in the following table.

Spot price at the expiration [S_T]	1,2322 EUR-USD
Average price at the expiration [A_T]	1,27
Payoff: $P_T = (A_T - S_T, 0)$	0,0378 EUR per unit
Payoff total	37 800 EUR

Table 3.3: Payoff details

The company executed Average Rate Asian Option and received 37 800 EUR from the issuer.

It could be therefore recognised, that Asian options perfectly suits needs for hedging average prices for certain time period as described in the examples.

3.1.2 Barrier options

Barrier options are similar to vanilla options in a way of determining payoff at the time of maturity and we can also distinguish between American, European, call or put barrier options. In addition, they are bringing one special feature into action. This feature is called barrier and it's determining whether the option can be exercised or not. The options become activated or void if the underlying asset reaches predetermined value. This value is called the barrier.

Barrier options are one of the oldest exotic options and they have become increasingly popular due to additional flexibility brought by the barrier. They are generally cheaper than their vanilla counterparts due to the barrier [22].

The value of the option is dependent on development of price of underlying asset during the option lifetime. Therefore it must be recognised that barrier option is path-dependent.

Knock-out and Knock-in barrier options

A Knock-out call option gives buyer a right to purchase underlying asset on a specified expiration date at the strike price (K) under condition that price of underlying asset never hits or crosses predetermined barrier (B). The holder will exercise the option only if at the expiration date the spot price is above the strike and if the spot has never reached the barrier during the lifetime of the option (case of American style barrier that is the most common) or at the time of expiration (case of European style barrier). Moreover, a down-out and up-out barrier options could be recognised.

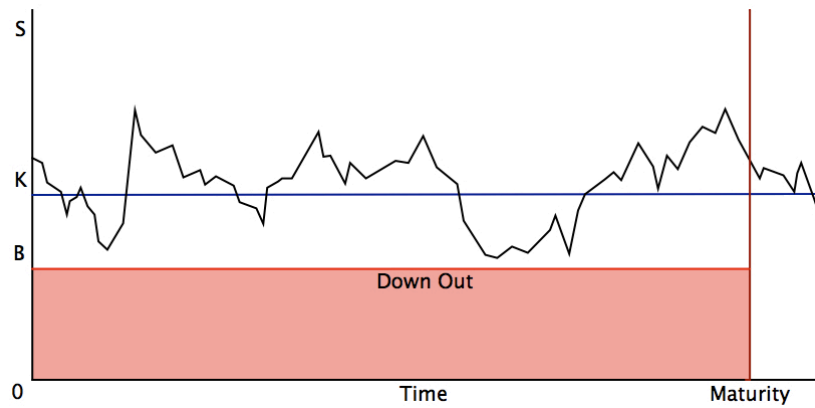


Chart 3.4: Down and Out barrier example

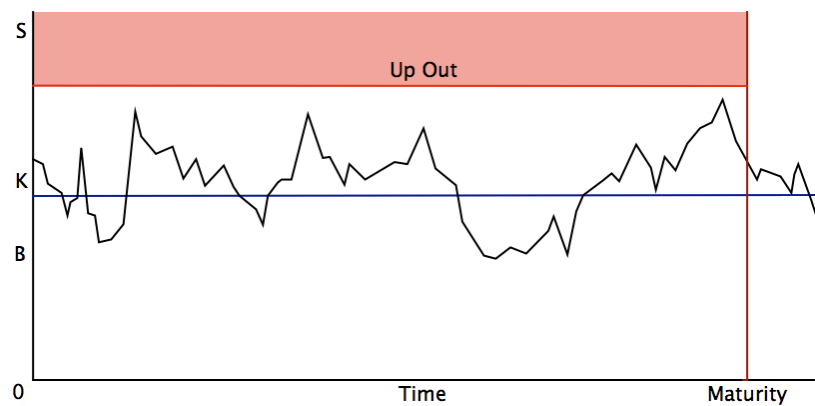


Chart 3.5: Up and Out barrier example

On the other hand, a Knock-in call option gives buyer a right to purchase underlying asset on a specified expiration date at the strike price (K) only if the price of underlying asset crosses predetermined barrier (B), hence the option becomes activated. The holder will exercise the call option only if at the expiration date the spot price is above the strike. Additionally, a down-in and up-in barrier options could be recognised.

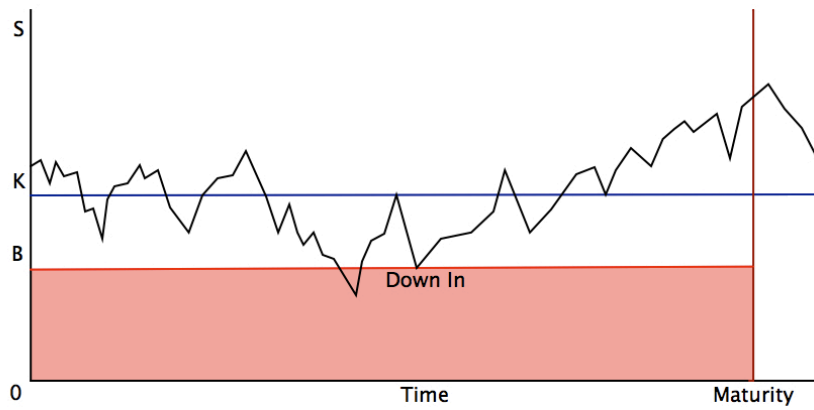


Chart 3.6: Down and In barrier example

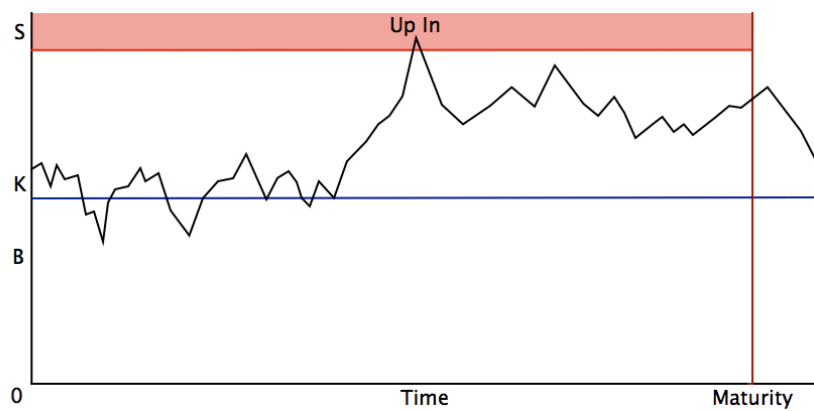


Chart 3.7: Up and In barrier example

An overview of barrier options and the effect of crossing the barrier is expressed in following tables.

Knock-Out options		Value	
Type	Barrier location	Crossed	Not Crossed
Down-and-Out Call	Below spot	0	Vanilla Call
Up-and-Out Call	Above spot	0	Vanilla Call
Down-and-Out Put	Below spot	0	Vanilla Put
Up-and-Out Put	Above spot	0	Vanilla Put

Table 3.4: Knock-Out options value

Knock-In options		Value	
Type	Barrier location	Crossed	Not Crossed
Down-and-In Call	Below spot	Vanilla Call	0
Up-and-In Call	Above spot	Vanilla Call	0
Down-and-In Put	Below spot	Vanilla Put	0
Up-and-In Put	Above spot	Vanilla Put	0

Table 3.5: Knock-In options value

Barrier Option Features' Analysis

- Barrier

There are two types of barriers. The most common barriers are of American style that means the barrier is active during entire life of the option. Therefore, anytime the spot price reaches the barrier between activation and maturity, the options turn worthless. On the other hand, if the barrier is active only at the time of maturity it is European style of barrier option.

If the investor believes that spot will remain within certain range during the life of the option, it is possible to obtain barrier option with both down and up barriers.

In addition, the barrier could be distinguished by the time of which is barrier active. In addition to standard barriers there are also “Partial barriers” and “Forward starting barriers”. The partial barriers are active during initial period while the forward starting barriers are active only over the latter period until the maturity [6] [8].

- Rebate

The rebate is an amount paid by the issuer of the option to the holder of the option if the option fails to knock-in during it's lifetime or if the knock-out option cross the barrier and becomes worthless. The payment of the rebate takes place either at the maturity or the first time the spot price reaches barrier. Nevertheless, the rebate is not the necessary part of the option contract and can be completely separated and traded separately or it can be dismissed from the contract entirely [22].

- Knock-out event

Breaching the barrier could be determined by several events. Option can be knocked in or knocked out by simple hitting of the barrier by the spot price. In addition, there are kinds of the barrier options that depends on whether the underlying asset reaches and goes beyond a certain price level within a certain time interval for pre-determined period of time. Those kinds of options are called Parisian options. Since they are harder to be knocked out, the premium for this option will be higher than similar barrier option but lower than its vanilla counterpart [19].

It is clear therefore that barrier options have many different variables thus can be highly customized for the holders' needs.

The work of Derman and Kani [8] is giving examples of barrier options without rebate and compares their theoretical values with European vanilla options. The down-and-out call option is very close with its value to the vanilla option, because it gets knocked out when the price of the stock is down where the vanilla option has low value. On the other hand down-and-out put option has much lower premium than vanilla put, because it gets knocked out in the levels of underlying asset where the vanilla put generates high profit to the holder. This rule can be observed in all variations of barrier options.

The following tables compare up-and-out and up-and-in barrier options with European vanilla option.

Up-and-Out Options	
Spot reference	100
Strike	100
Barrier	120
Rebate	0
Maturity	1 year
Dividend Yield	5%
Volatility	25% per year
Risk-Free Rate	10%
European Vanilla Call Value	11,434
Up-and-Out Call Value	0,657
European Vanilla Put Value	7,344
Up-and-Out Put Value	6,705

Table 3.6 [8]

Up-and-In Options	
Spot reference	100
Strike	100
Barrier	120
Rebate	0
Maturity	1 year
Dividend Yield	5%
Volatility	25% per year
Risk-Free Rate	10%
European Vanilla Call Value	11,434
Up-and-Out Call Value	10,779
European Vanilla Put Value	7,344
Up-and-Out Put Value	0,639

Table 3.7 [8]

According to first table, the Up-and-In Call option is worth 0,657 that is significantly less, then 11,434 of European Vanilla Call option. On the other hand price of Up-and-Out Call option with its price of 10,779 is very close to the European Vanilla Call worth of 11,434.

The sum of the values of the barrier options is equal to value of European Vanilla Call option.

Up-and-Out + Up-and-In = Vanilla option

Down-and-Out + Down-and-In = Vanilla option

Barrier options proved to be cheaper than Vanilla options. In addition they are more flexible in meeting investor's hedging needs. The amount of the risk that is going to be transferred to the option issuer is going to be set by the barrier. Barrier setting has direct influence on option's premium price.

Therefore it is valuable alternative for plain vanilla options.

3.1.3 Binary options

Binary options also known as digital options or “all or nothing” options have long history at the over-the-counter market. The special feature of this option is that a binary option’s payoff does not increase the more it is in the money at expiration. The issuer of the option will pay fixed amount of money or certain asset to the holder of call binary option if the underlying reaches predetermined price. In addition, put binary option is in the money if underlying won’t reach predetermined level [11]. The binary options are similar to European type of barrier options in a way of penetrating a barrier at the option expiration, however the payoff is calculated different way (see previous chapter).

Cash-or-nothing option

The issuer of this option will pay the holder fixed amount if the option ends up in the money.

Cash-or-nothing call is in the money if the spot is above the strike at maturity.

Cash-or-nothing put is in the money if the spot is below the strike at maturity.

Asset-or-nothing option

In this case, issuer will pay price equal to the underlying asset if the option ends up in the money.

Asset-or-nothing call is in the money if the spot is above the strike at maturity.

Asset-or-nothing put is in the money if the spot is below the strike at maturity.

As with many exotic options, binary options have several customized subtypes to meet specific financial goals of investors. Those accessorized binary options are for example *one-touch binary options*, where the pay off is executed if the price of underlying hits the strike price anytime before expiration. This could be also called American type of binary option. Another special binary option is *Ladder binary option*, which contains a series of payoffs at different levels. In addition, the *Range binary option* is in the money if the price of the underlying asset stays within a certain range. [22] [11]. This feature is also very similar to barrier option with two barriers.

Payoff

The binary options' payoff is simple. If the price of underlying asset is above the strike, a call pays a fixed amount (or price of the underlying asset) while put would be out of the money. If the price of underlying asset is below the strike a put pays fixed amount (or price of the underlying asset) while call would be worthless.

Binary Call Option	$C_T = \begin{cases} 0 & \text{if } S_T \leq K \\ 1 & \text{if } S_T > K \end{cases}$
Binary Put Option	$P_T = \begin{cases} 1 & \text{if } S_T \leq K \\ 0 & \text{if } S_T > K \end{cases}$

Table 3.8: Binary Options payoff

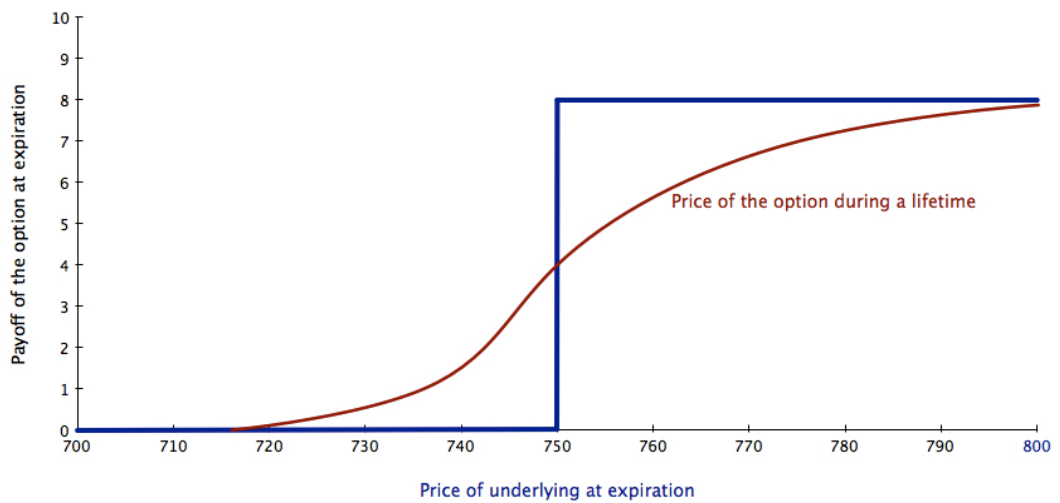


Chart 3.8: Cash-or-nothing call option

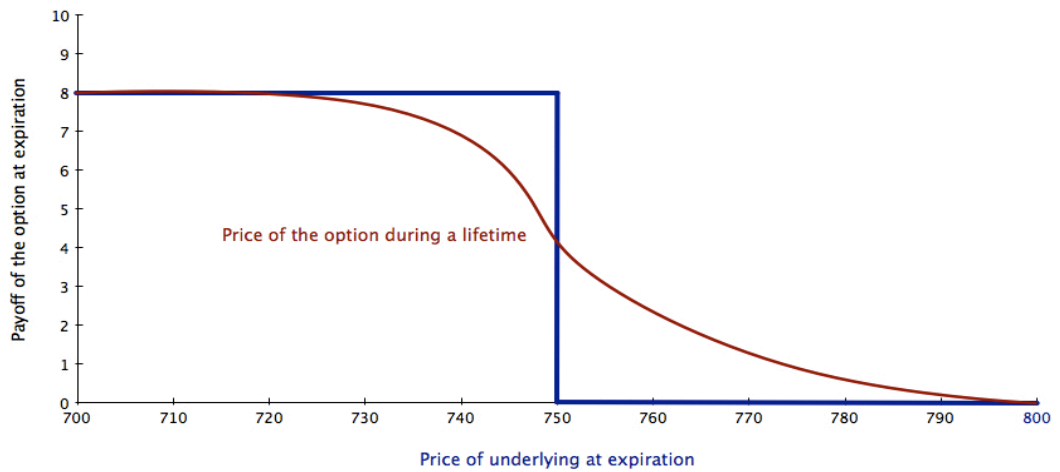


Chart 3.9: Cash-or-nothing put option

The Cash-or-nothing options' price before maturity reflects a probability forecast of whether the option ends up in the money or not. Therefore, they help to reduce uncertainty of investors.

For example, if the Cash-or-nothing call binary option has a \$100 USD payoff, the value can be considered as the probability in percents that the contract will expire in the money. In other words, if the price of the option is \$95 USD one day prior expiration, there is 95% chance that price of the underlying asset will settle above the strike price at the time of expiration.

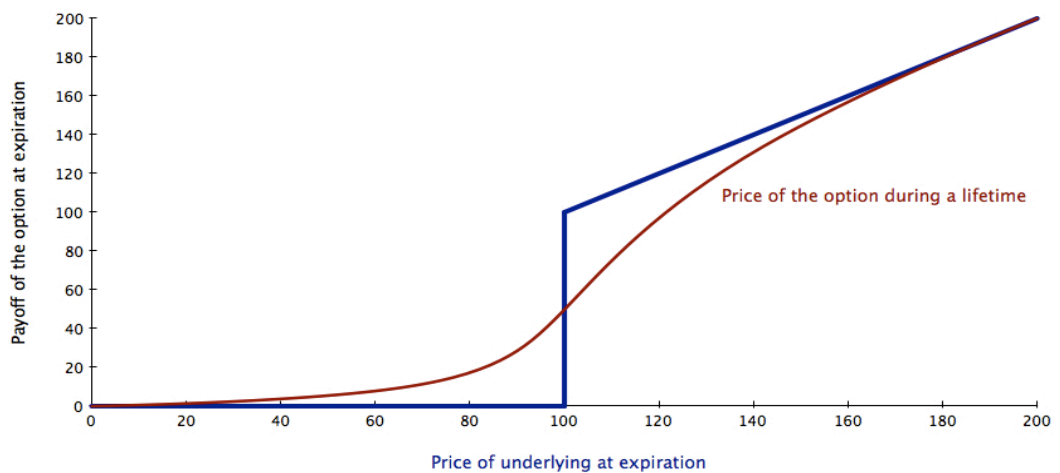


Chart 3.10: Asset-or-nothing call option

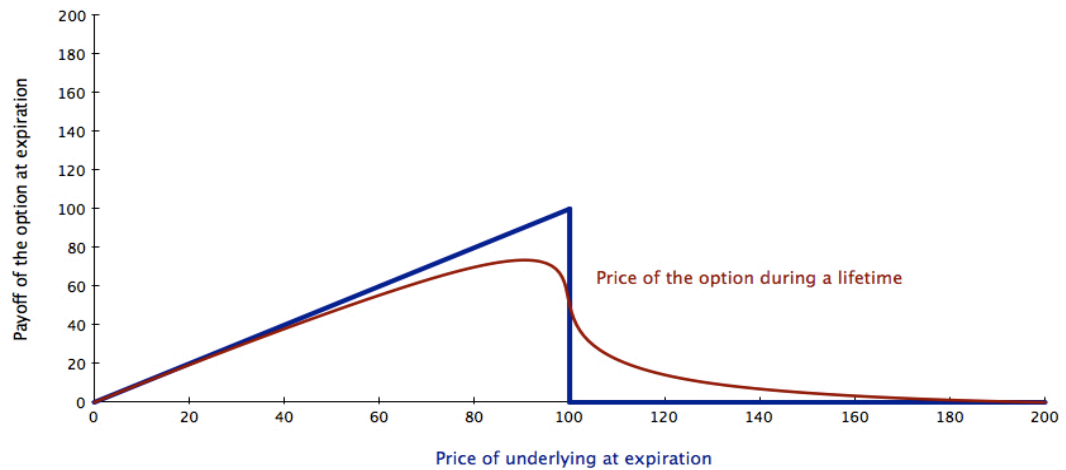


Chart 3.11: Asset or nothing put option

Wystup [22] describes binary options as an instrument highly used by speculators who bet on a rising or falling rates of underlying. Hedging oriented clients often purchase binary options as a rebate and place their binary option against the strategy they believe to work out. Therefore, they are able to receive some compensation if their strategy fails.

The author sees hedging with this instrument as very limited due to the fixed payoff. Nevertheless, following example investigates how could be binary option used for hedging foreign currency exchange rate risk.

Example:

An American company is purchasing goods from European mechanical engineering company. The invoice for €500 000 EURO is payable in 6 months. Company won't purchase Euros now because they don't want to lock their free capital in foreign currency. In addition, they are aware of possibility that Euro will evaluate, which would make this transaction more expensive.

Current USD/EUR exchange rate is 1,26 USD/EUR and at current exchange of €500 000 EUR costs \$630 000 USD. The company wants to protect itself against appreciation of EUR but is willing to spent up to \$661 010 USD. On the other hand they want to profit on possible Euro depreciation. \$661 010 equals to €500 000 with exchange rate of 1,322 USD/EUR.

Scenario 1 – Cash-Or-Nothing Call Option – Cash amount \$31 010 USD

First option is to buy a Cash-Or-Nothing Call Option with \$31 010 USD payoff if the USD/EUR exchange rates crosses 1,3 at the time of the expiration. This amount would compensate losses generated by stronger Euro.

Spot	1,26
Strike	1,3
Volatility	15%
Maturity	0,5
Cash amount	31010
Premium	11010

Table 3.9: Cash-Or-Nothing Call Option #1,

The following table 3.10 shows how does exchange rate influence balance and hedged balance. Balance shows the results if the company wouldn't purchase any instrument, therefore would be fully exploited to the risk. In addition, hedge stands for how much would company loose or profit if the instrument is purchased with regard to actual exchange rate.

Exchange rate	1,18	1,22	1,26	1,29999	1,3	1,34
Balance	40000	20000	0	-19995	-20000	-40000
Hedge	28990	8990	-11010	-31005	0	-20000

Table 3.10: Cash-Or-Nothing Call Option #1 results

Break even: $(630\ 000 - 11\ 010) / 500\ 000 = 1,238$

Max loss 31 010: $(630\ 000 + 31\ 010 + 20\ 000) / 500\ 000 = 1,362$

By executing this scenario, company would break even at USD/EUR exchange rate of 1,238 USD/EUR. If the exchange rate drops more, they will start gain profit. On the other hand, the loss is approaching 31 010 at the 1,3 exchange rate that is limit. If exchange rate hits 1,3 the option will be executed and loss will be reduced to the 0. The big flaw of this scenario is the fact that with every additional 0,04 points to the

exchange rate generates 20000 USD loss. The loss will reach the maximum possible amount of \$31 010 USD again at 1,362 USD/EUR.

Scenario 2 - Cash-Or-Nothing Call Option – Cash amount \$20 000 USD

Second option is to buy a cheaper Cash-Or-Nothing Call option with lower payoff of \$20000 USD. This payoff is executable if exchange rate hits 1,3 at the time of the expiration.

Spot	1,26
Strike	1,3
Volatility	15%
Maturity	0,5
Cash amount	20000
Premium	7101

Table 3.11: Cash-Or-Nothing Call Option #2

Table 3.12 is similar to the table 3.10 in scenario 1 only with different values.

Exchange rate	1,18	1,22	1,26	1,29999	1,3	1,34
Balance	40000	20000	0	-19995	-20000	-40000
Hedge	32899	12899	-7101	-27096	-7101	-27101

Table 3.12: Cash-Or-Nothing Call Option #2 results

Break even: $(630\ 000 - 7\ 101) / 500\ 000 = 1,246$

Max loss 31 010: $(630\ 000 + 31\ 010 + 12\ 899) = 1,3478$

If company executes the scenario company would break even at USD/EUR exchange rate of 1,246 USD/EUR that is little bit higher and better then in scenario 1. In addition, the loss is approaching 27 101 at the 1,3 exchange rate, which is limit. After hitting the 1,3 rate, loss is reduced with payoff to -7 101. As well as with previous scenario, every additional 0,04 pint added to the exchange rate generates 20 000 USD loss. In this case \$31 010 USD loss will be reached again at 1,3478 USD/EUR that is lower than in first scenario. Hence this solution is more risky.

Scenario 3 – Plain Vanilla Call Option

The last option that is going to be examined is purchasing of Vanilla Call Option. The details are stated in following table 3.13.

Underlying	€500 000
Spot	1,26
Strike	1,26
Volatility	15%
Maturity	0,5
Premium	0,052
Premium total	26 000

Table 3.13: Plain Vanilla Option

The following table is similar to the table in scenario 1 only with different values.

Exchange Rate	1,18	1,22	1,26	1,3	1,34
Balance	40000	20000	0	-20000	-40000
Hedge	14000	-6000	-26000	-26000	-26000

Table 3.14: Plain Vanilla Option results

By executing this scenario, company wouldn't need to be afraid of paying more than \$656000 USD. But on the other hand since the premium is so big, company will break even if the exchange rate drops down to 1,208 USD/EUR. The company would still generate loss at 1,22 USD/EUR exchange rate.

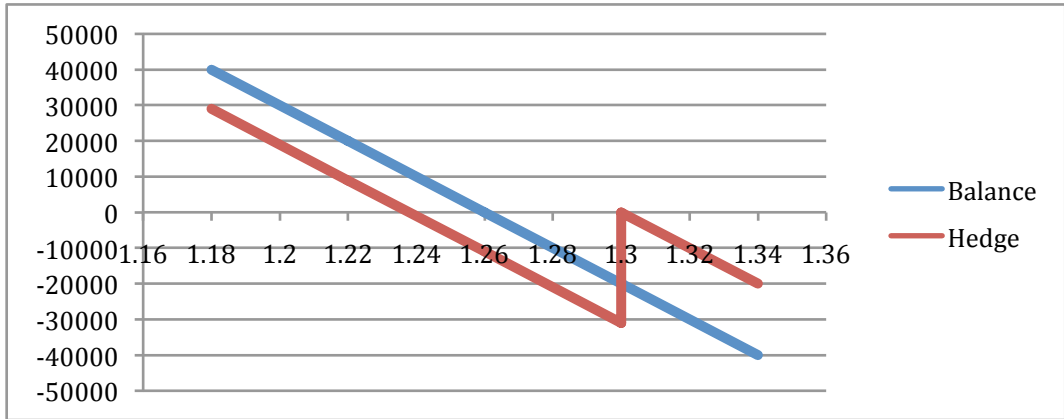


Chart 3.12: Scenario 1 chart

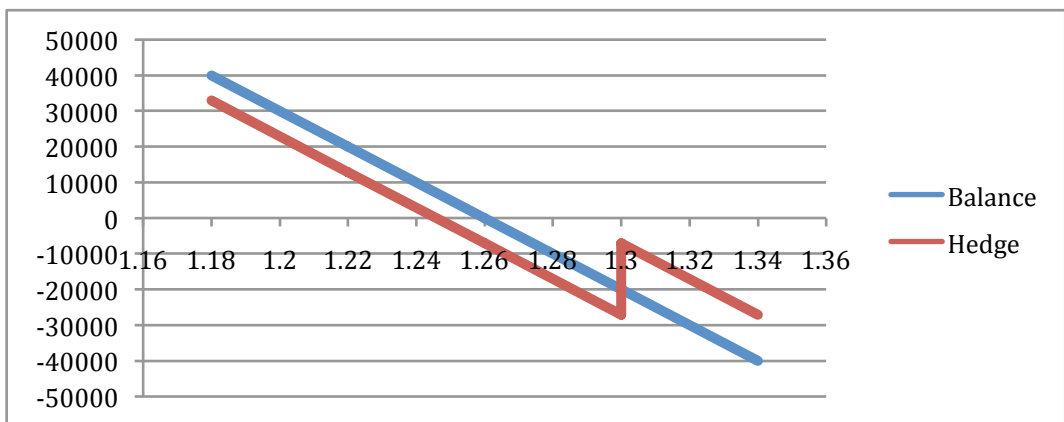


Chart 3.13: Scenario 2 chart

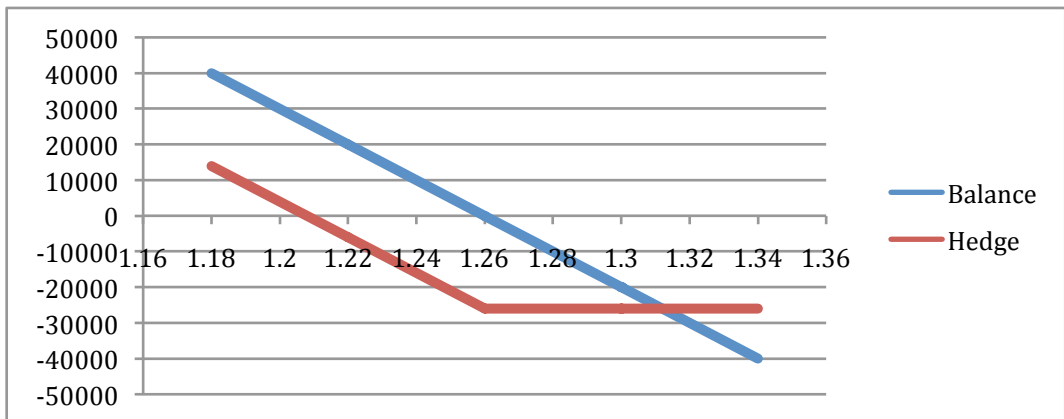


Chart 3.14: Scenario 3 chart

The binary option is much cheaper than vanilla counterpart that can make it more attractive. Investor can receive surplus faster but on the other hand is not fully covered against the risk connected with greater depreciation of USD against EUR. If the company is willing to take bigger risk than they will execute scenario 2. More risk

averse investor will choose scenario 1, and if the company wants to be sure that exchange rate will be secured, than they will choose plain vanilla option.

With regard to binary options analysis, it could be therefore concluded, that binary options are more suitable for speculations and less for hedging.

3.2 Conclusions of the analysis of chosen exotic options

Asian options proved to be perfect for hedging average values during certain time period. Average Rate Asian option is going to be executed if the investor wants to lock specific rate at his will (for example exchange rate 1,3 USD/EUR). On the other hand, the average strike is suitable to hedge average market rate as it develops. Therefore, the investor will be able buy underlying asset for average market price during hedged period of time or lower price. Other use than hedging of average values is speculative.

Barrier options are less expensive and more flexible than vanilla options. They can be used as vanilla options but they also bring more flexibility into an action. The investor can decide better how is he going to be hedged. The variability of the barrier options is high as the investor can choose the nature of the barrier, number of barriers and rate of the barrier.

Binary options as the barrier options have the barrier that in this case needs to be crossed in order for the option to end up in-the-money. In addition, Binary options have been recognized unsuitable for hedging, as there is no payoff development. Hence, the Binary options are highly speculative.

4 Proposal of feasible usage of selected exotic options

Exotic options can be used for both speculative and hedging purposes. In this chapter the author evaluates feasible usage of selected and previously analyzed types of the exotic options as an instrument for hedging risks and proposes the suitable use. The model of a Czech mechanical engineering company manufacturing machinery for German company who gets paid in Euros is created. Exotic options are used for the hedging of interest rate risk exposure. In addition hedging with the vanilla option is also taken into the consideration and evaluated in contrast with its exotic counterparts.

Every proposed scenario of hedging with certain exotic option is evaluated with regard to level of security, cost of the option and contains recommendation whether this solution is feasible or not.

The model:

A Czech engineering company “Machine a.s.” exporting to the European market is selling machinery in Euros. The company is going to accept €700 000 EUR for the shipment of mechanical devices. Both sides of the business agreed that payment would be divided in two payments. First one €300 000 EUR is due in 6 months right after the delivery. The rest (€400 000) of the payment will be paid in following 6 months. The Czech company has to deal with exchange rate risk and wants to hedge this future cash flow. Nevertheless, the Machine a.s. is unsure about further movement of the exchange rate. The company wants to benefit from eventual depreciation of CZK, thus the futures or forwards cannot be used.

One of the solutions is to hedge the risk with Plain Vanilla European Put option or with one of the exotic modifications of financial options.

Current exchange rate 25,25 CZK/EUR

The company wants sell Euro for 25,25 CZK/EUR.

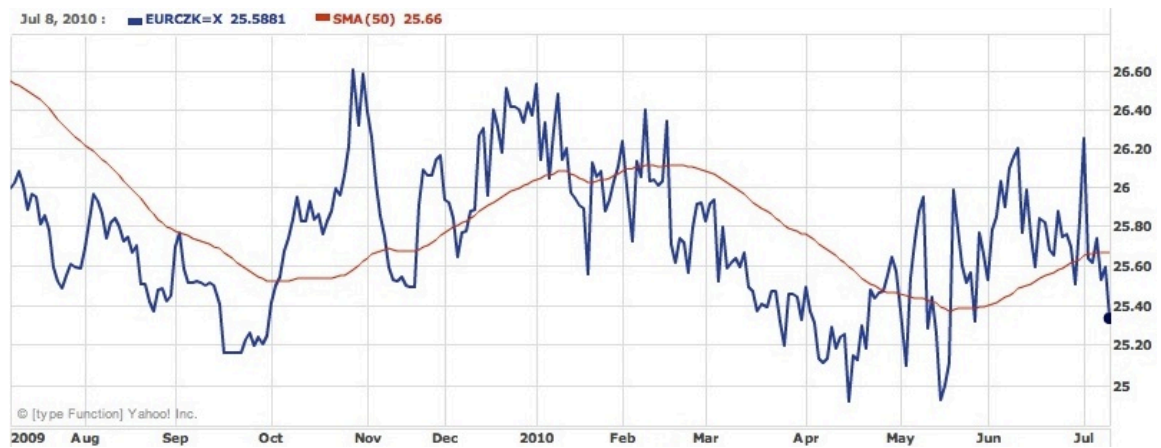


Chart 4.1: CZK / EUR exchange rate

With regard to previous analysis of exotic options four hedging possibilities with use of exotic options are examined and compared with Plain Vanilla option solution.

Scenarios with vanilla and exotic options:

- Plain Vanilla Put Options
K = 25,25 CZK/EUR
- Asian Average Rate Put Option
K = 25,25 CZK/EUR, 12 averaging in the beginning of every month
- Barrier Down-and-Out Put Option
K = 25,25 CZK/EUR, lower barrier = 24,25 CZK/EUR
- Barrier Down-and-In Put Option
K = 25,25 CZK/EUR, lower barrier = 24,75 CZK/EUR
- Binary Cash-or-Nothing Put Option
K = 24,25 CZK/EUR, cash amount = 400 000 CZK

4.1 Proposal #1: Plain Vanilla Put Option

Hedging with the plain vanilla option will reduce the risk fully with no additional conditions and is the most conservative from proposed solutions. The company is going to use European Call option to hedge the risk of negative exchange rate movement.

The company needs to secure two separate cash flows. First one of €300 000 EUR is due in six months followed by €400 000 EUR in additional six months. Therefore, two European puts are going to be used. First one will reach maturity in 6 months (0,5 years), second one in 12 months (1 year). Both are going to be purchased at the same moment.

Black-Scholes pricing model has been used to price the premium of the option. Details are stated in following tables 4.1 and 4.2.

Plain Vanilla Put Option #1	
Underlying in €	300 000
Spot	25,25
Strike	25,25
Volatility	3,55%
Maturity	0,5
Premium CZK	0,2466
Premium tot CZK:	73 980

Table 4.1: Plain Vanilla Put Option #1

Plain Vanilla Put Option #2	
Underlying in €	400 000
Spot	25,25
Strike	25,25
Volatility	3,55%
Maturity	1
Premium CZK	0,3401
Premium tot CZK:	136 040

Table 4.2: Plain Vanilla Put Option #2

The following table 4.3 shows possible development of the exchange rate and contains calculations of a balance, payoff and a hedged outcome. Balance is showing the development of additional costs in case of non-hedged exchange rate. Payoff shows simply the amount of money paid by the issuer to the holder if the option is exercised. Hedge stands for the result of hedged balance.

Exchange rate	24,50	24,75	25,00	25,25	25,50	25,75
Balance	-525 000	-350 000	-175 000	0	175 000	350 000
Payoff	525 000	350 000	175 000	0	0	0
Payoff - premium	314 980	139 980	-35 020	-210 020	-210 020	-210 020
Hedged bal.	-210 020	-210 020	-210 020	-210 020	-35 020	139 980

Table 4.3: Plain Vanilla Option Calculation

The cost of this solution is 210 020 CZK and company will break even at exchange rate of 25,55 CZK/EUR.

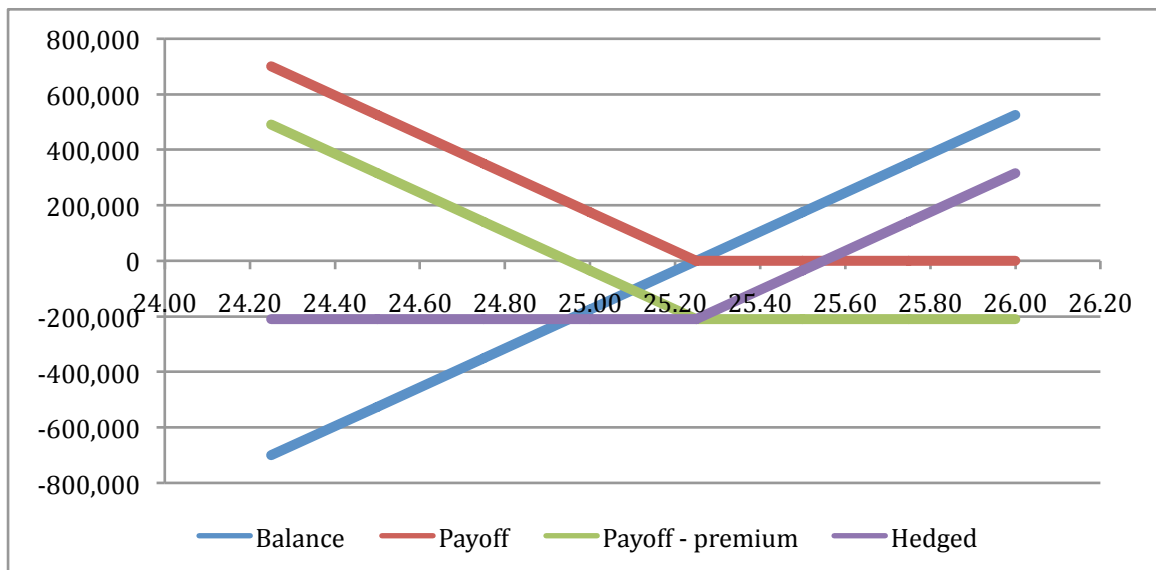


Chart 4.2: Plain Vanilla Option solution

Positives:

This solution moves all risk from the option holder to the option writer for the cost of the option premium.

Negatives:

The option premium is expensive.

The vanilla option cannot be additionally customized.

4.2 Proposal #2: Asian Average Rate Put Option

The company is going to buy only one option at this moment and secure all the cash flow at once. In this case the payoff and hedged solution is not dependent only on the strike price and the exchange rate at the moment of the expiration, but also on the averaged rate.

Black-Scholes formula for the Asian options is used to price the option. Details are stated in following table 4.4.

Asian Average Rate Put Option	
Underlying in €	700 000
Spot	25,25
Strike	25,25
Volatility	3,55%
Maturity	1
Nr of averaging	12
Premium CZK	0,2047
Premium tot CZK:	143 290

Table 4.4: Asian Average Rate Put Option

The overall option price is lower than Vanilla European Put option. On the other hand the payoff is calculated differently. It is not the difference between the strike and the spot at the expiration, but it is the difference between the strike and the averaged exchange rates in 12 months. The volatility of this averaged rate is lower, therefore the risk for the issuer is lower and the premium is lower too. Next three tables are expressing payoffs under the different averaged rates.

Exchange rate		24,25	24,50	25,00	25,50	26,00
Balance		-700 000	-525 000	-175 000	175 000	525 000
Payoff avg rate #1	24,50	525 000	525 000	525 000	525 000	525 000
Payoff avg #1 - premium		381 710	381 710	381 710	381 710	381 710
Hedged #1		-318 290	-143 290	206 710	556 710	906 710

Table 4.5: Asian Average Rate Put option #1

Exchange rate		24,25	24,50	25,00	25,50	26,00
Balance		-700 000	-525 000	-175 000	175 000	525 000
Payoff avg rate #2	24,85	280 000	280 000	280 000	280 000	280 000
Payoff avg #2 - premium		136 710	136 710	136 710	136 710	136 710
Hedged #2		-563 290	-388 290	-38 290	311 710	661 710

Table 4.6: Asian Average Rate Put option #2

Exchange rate		24,25	24,50	25,00	25,50	26,00
Balance		-700 000	-525 000	-175 000	175 000	525 000
Payoff avg rate #3	25,25	0,00	0,00	0,00	0,00	0,00
Payoff avg #3 - premium		-143 290	-143 290	-143 290	-143 290	-143 290
Hedged #3		-843 290	-668 290	-318 290	31 710	381 710

Table 4.7: Asian Average Rate Put option #3

The table is divided into the three parts. Each part has a different averaged rate. The payoff is constant in every part of the table but it differs within the parts. The payoff has been calculated for averaged rates of 25,50; 24,85 and 25,25 CZK/EUR. The table is showing that balance hasn't been really hedged. Although, the average price is dependent on the spot price, but it's lower volatility is bringing more risk for the holder of the option. Several situations can take place.

The best situation for the holder would be steady evaluation of the CZK followed by the fast depreciation in the last month. That would result in low average rate, thus high payoff and the holder would be able to profit from the weak CZK also (Table 4.5)

In contrast, the worst scenario for the holder would be exactly opposite. Steady depreciation of the CZK followed with the great appreciation in last month would generate great loss. The option would end up out-of-the-money with average rate above the strike and the spot price (Table 4.7).

Following three charts show payoff differences.

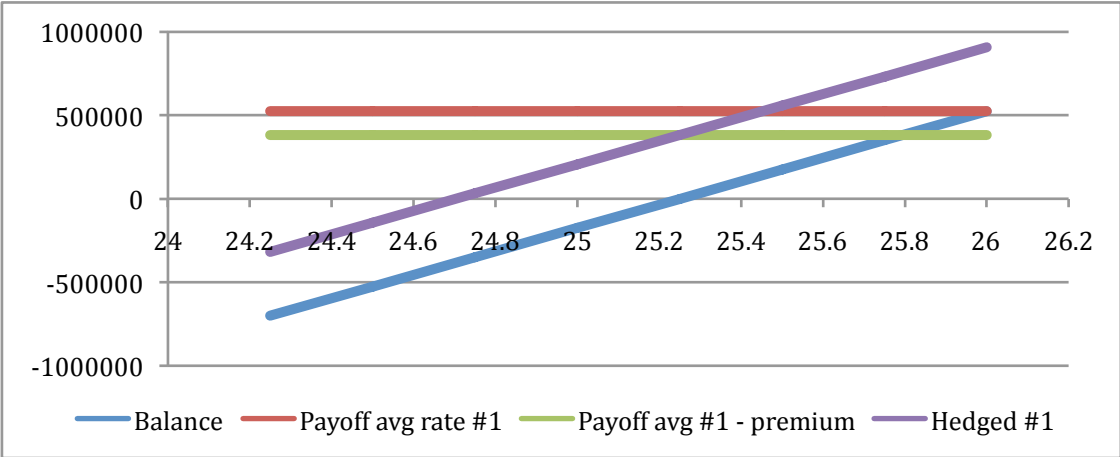


Chart 4.3: Asian Average Rate Put option #1

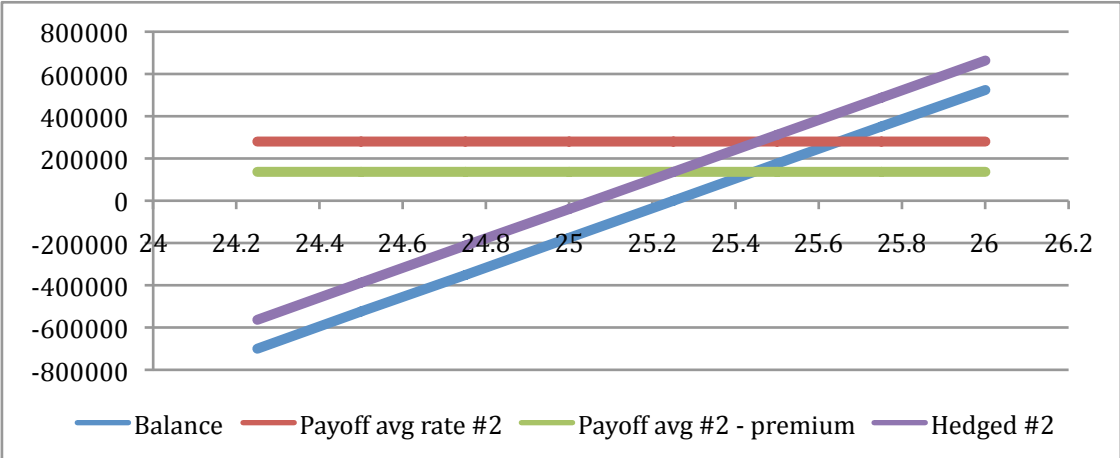


Chart 4.4: Asian Average Rate Put option #2

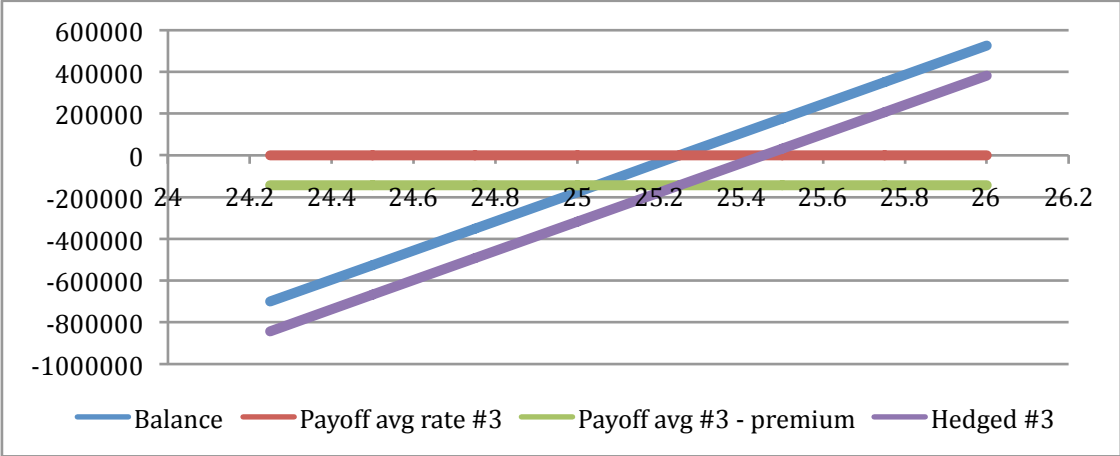


Chart 4.5: Asian Average Rate Put option #3

Positives:

Underlying of the option is less volatile; hence the premium is slightly lower.

Negatives:

This option is not really suitable to hedge this kind of risk.

Highly speculative when used to hedge this particular risk.

It could be therefore recognized, that Average Rate Asian option is not really suitable to hedge this kind of risk. The company wants to secure exchange rate at the certain time not average price for the certain period. A buyer of the option can assume that average rate will follow the development of the exchange rate, but fast movements of the rate right at the expiration can change whole situation. To be concluded, hedging with the Asian option would be highly speculative.

4.3 Proposal #3: Down-and-Out Barrier Put Option

The barrier options are very similar to the vanilla options in the way of payoff calculation. On one hand, this solution is bringing more risk for the holder of the option but on the other hand, the holder can customize this amount of the risk. First proposed solution is to hedge the risk down to the level of exchange rate of 24,25 CZK/EUR. If the holder decides so, the option will knock out worthless if the exchange rate drops below this barrier. Due to this high risk for the holder, the option premium is much lower than it's vanilla counterpart. The company is purchasing two Down-and-Out Barrier Put options. First expires in six months. Second one expires in one year.

Details of the option are stated in the following tables 4.8 and 4.9.

Black-Scholes formula for Barrier options is used to calculate option premium.

Down-and-Out Barrier Put #1	
Underlying in €	300 000
Spot	25,25
Strike	25,25
Volatility	3,55%
Maturity	0,5
Lower barrier	24,25
Premium CZK	0,1396
Premium tot CZK:	41 880

Table 4.8: Down-and-Out Barrier Put #1

Down-and-Out Barrier Put #2	
Underlying in €	400 000
Spot	25,25
Strike	25,25
Volatility	3,55%
Maturity	1
Lower barrier	24,25
Premium CZK	0,0893
Premium tot CZK:	35 720

Table 4.9: Down-and-Out Barrier Put #2

The total price for the option premiums is 77 600 CZK that is much lower than vanilla or Asian option. The option premium for the longer second option is lower as there is bigger probability of knocking out the option. That is opposite to the vanilla option, which is getting more expensive with the longer lifetime. In addition, if the barrier were set up lower, the premium would be higher for both options.

The following table 4.10 is showing the calculations.

Exchange rate	24,00	24,25	24,26	25,00	26,00
Balance	-875 000	-700 000	-693 000	175 000	525 000
Payoff	0	0	693 000	175 000	0
Payoff - premium	-77 600	-77 600	615 400	97 400	-77 600
Hedged	-952 600	-777 600	-77 600	-77 600	447 400

Table 4.10: Down-and-Out Put options calculation

The cost of this solution is 77 600 CZK that is 63% less than the vanilla option. The company will break even at 25,36 that is 0,19 less than in case of vanilla option. Following chart demonstrates payoffs.

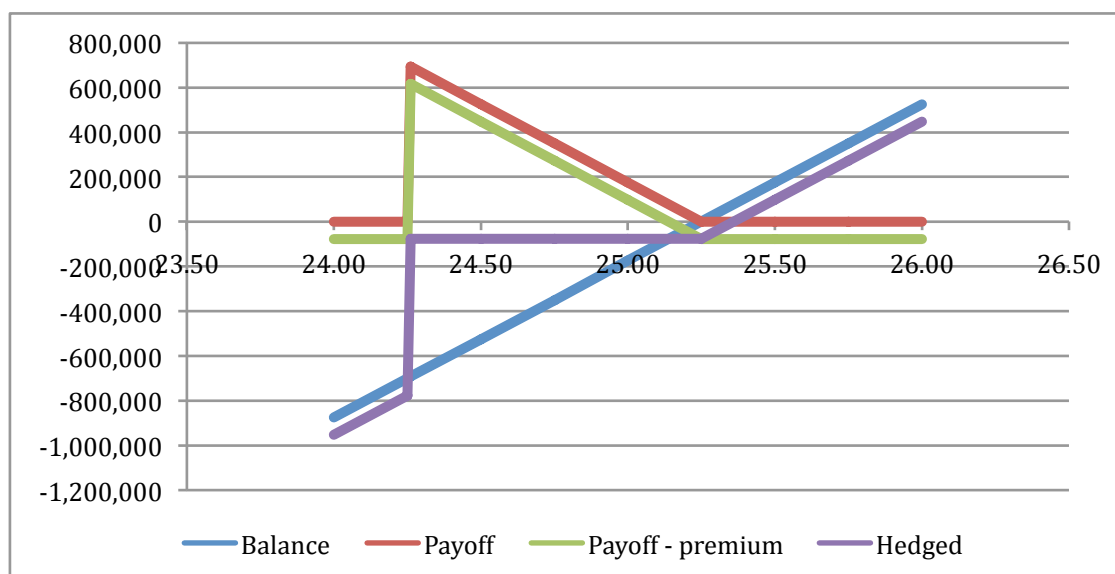


Chart 4.6: Down-and-Out Barrier Put options

This kind of barrier option will hedge the exchange rate only within small interval. Exchange rate doesn't need to be hedged over the 25,25 CZK/EUR, but it is not hedged for rates below 24,25 CZK/EUR. Therefore, it is much more risky solution for the holder of the option. On the other hand this solution is much less expensive than hedging with vanilla option.

Positives:

The option premium is much cheaper than vanilla option's as it's only 37% of the regular price.

The break-even point is lower by 0,19 down at 25,36 CZK/EUR.

This solution is highly customizable. The holder can set up the barrier at his will. The lower the barrier is, the higher is the premium.

Negatives:

There is a high risk that option will be knocked out worthless. Thus, the holder will lose both premium and hedging.

4.4 Proposal #4: Down-and-In Barrier Put Option

A different barrier option is proposed for this solution. Instead of risking knocking out the option, the holder is going to accept little loss and risk that option will be not activated until the exchange rate drops below certain level. The strike price stays at the 25,25 CZK/EUR level but the barrier has been set up to the 24,75 exchange rate. If the exchange rate reaches this level at any time of the lifetime, the option will be activated. Otherwise it never will.

Due to this additional risk for the holder, the premium is going to be slightly lower as the risk of not reaching the barrier is also low.

The company is going to acquire two options with different time of expiration. First one is set up to six months, as the second one is set up to 12 months expiration.

The following tables 4.11 and 4.12 contains details of both options.

Black-Scholes model used for further calculations.

Down-and-In Barrier Put #1	
Underlying in €	300 000
Spot	25,25
Strike	25,25
Volatility	3,55%
Maturity	0,5
Lower barrier	24,75
Premium CZK	0,2242
Premium tot CZK:	67 260

Table 4.11: Down-and-In Put #1

Down-and-In Barrier Put #2	
Underlying in €	400 000
Spot	25,25
Strike	25,25
Volatility	3,55%
Maturity	1
Lower barrier	24,75
Premium CZK	0,3308
Premium tot CZK:	132 320

Table 4.12: Down-and-In Put #2

The total price for the both options has been calculated to 199 580 CZK that is only 10 440 CZK less that equals to 5% discount. Therefore, the probability of this option not to be activated is only 5%. The premium of the longer option is also higher

as there is bigger probability that option will be activated and ends up in-the-money. The price of the Down-and-In #2 is 47,5% higher than Down-and-In #1.

The payoffs are shown in following table. Black-Scholes model has been used to calculate option's premium.

Exchange rate	24,50	24,75	24,76	25,50	25,75	26,00
Balance	-525 000	-350 000	-343 000	175 000	350 000	525 000
Payoff	525 000	350 000	0 (343 000)	0	0	0
Payoff - premium	325 420	150 420	-199 580 (143 420)	-199580	-199580	-199580
Hedged	199 580	199 580	-542 580 (-199 580)	-24 580	150 420	325 420

Table 4.13: Down-and-In Put option calculation

The company will break even at 25,535 that is only 0,015 less than in case of solution #1. In addition, the company will generate loss down to exchange rate 24,75 CZK/USD that will be approaching - 549 580 CZK from the right. If the option is activated, the payoff will be executed even within exchange rate of 24,75 CZK/EUR and 25,25 CZK/EUR (the amounts are in the brackets).

The following chart is expressing the payoff development.

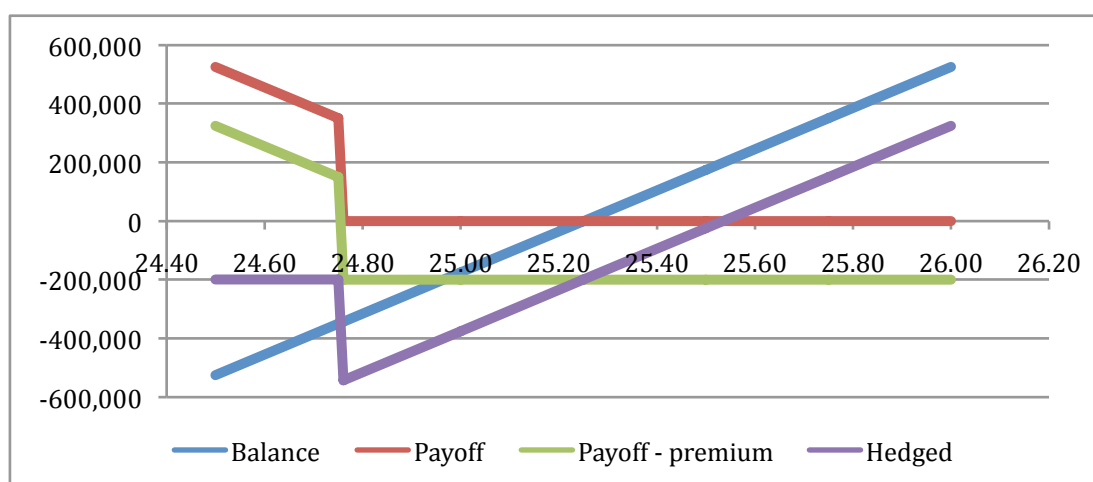


Chart 4.7: Down-and-In Put option

The Down-and-In Put option is hedging much more of the risk, therefore its much more expensive. It cost 121 980 CZK more than Down-and-Out option, so the option is 2,57 times more expensive.

Positives:

This solution is highly customizable. The holder can set up the barrier at his will. The lower the barrier will be the lower the premium will be also.

This option is removing a large amount of risk.

Negatives:

It is only 5% cheaper solution than vanilla option and it is 2,57 times more expensive than Down-and-Out proposed solution.

There is small probability that option will be never activated, thus holder will have to accept bigger loss.

4.5 Proposal #5: Cash-or-Nothing Binary Put Option

A Cash-or-Nothing Put option is the last of the proposed solution to hedge the exchange rate risk. This option is similar with the Down-and-In barrier option; only the payoff is not rising with the falling exchange rate. Instead of that the payoff is constant amount of cash. Strike price of the binary option is similar to the barrier of the barrier option, therefore it has been set up to the 24,75 so both solutions can be compared easily.

Due to the constant payoff, the premium of the option is going to be lower.

The company is going to purchase two Cash-or-Nothing binary options. First one has six months lifetime. Second binary option is going to expire in twelve months. Both option are written for same cash amount of 250 000 CZK to be easily compared.

Black-Scholes model has been used for valuation of the options.

The following tables 4.14 and 4.15 contain details about both binary options.

Cash-or-Nothing Put Option	
Underlying in €	300 000
Spot	25,25
Strike	24,75
Volatility	3,55%
Maturity	1
Cash amount CZK	250 000
Premium CZK	52 778
Premium tot CZK:	52 778

Table 4.14: Cash-or-Nothing Put #1

Cash-or-Nothing Put Option	
Underlying in €	400 000
Spot	25,25
Strike	24,75
Volatility	3,55%
Maturity	1
Cash amount CZK	250000
Premium CZK	69 595
Premium tot CZK:	69 595

Table 4.15 Cash-or-Nothing Put #2

The total price of the options is 122 373 CZK that is 38,7% less than Down-and-In barrier option and 41,7% less expensive than vanilla option counterpart. The premium of the longer option is higher as there is higher probability that exchange rate will reach 24,75 CZK/EUR.

The calculation of the payoff is stated in the following table 4.16.

Exchange r.	24,25	24,50	24,75	24,76	25,00	25,50
Balance	-700 000	-525 000	-350 000	-343 000	-175 000	175 000
Payoff	500 000	500 000	500 000	0	0	0
Payoff - premium	377 627	377 627	377 627	-122 373	-122 373	-122373
Hedged	-322 373	-147 373	27 627	-465 373	-297 373	52 627

Table 4.16: Cash-or-Nothing Put option

The company will break even at exchange rate of 25,425 CZK/EUR that is 0,125 less than vanilla option and 0,11 less than Down-and-In Put option.

The following chart is expressing payoff of the option.

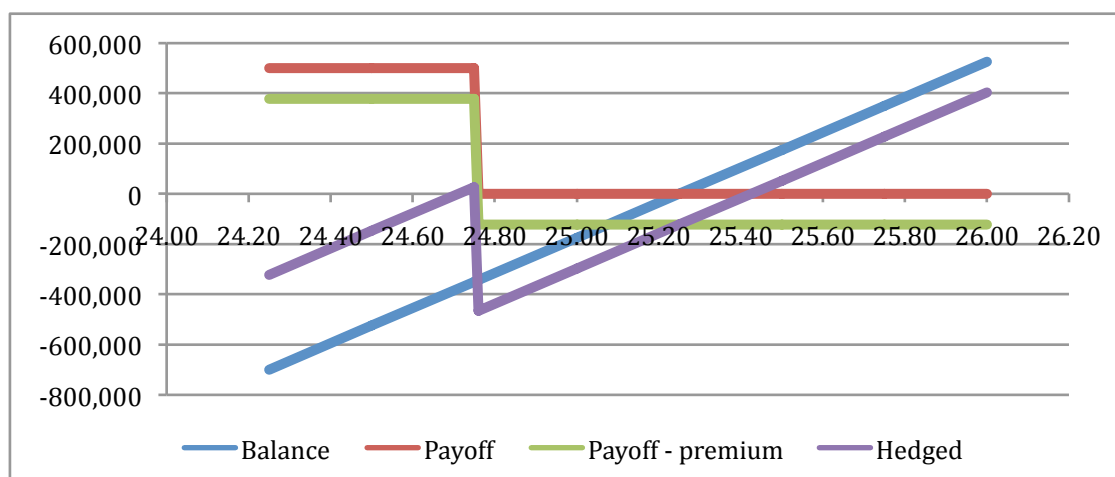


Table 4.8: Cash-or-Nothing Put option payoff

The Cash-or-Nothing put option doesn't fully hedge the risk. Instead of that the compensation is provided to the holder if the exchange rate drops below the level of strike price. It needs to be pointed out that this compensation doesn't grow as the exchange rates falls, therefore the additional loss is generated as the exchange rate falls.

Positives:

The price of the option premium is low in compare with other previously analyzed exotic options.

The binary option is highly customizable; strike price and cash amount can be set at holders will.

Negatives:

The risk cannot be properly hedged with binary options.

The binary option is highly speculative investment instrument.

4.6 Summary

All of the options are evaluated by several features such as option premium price, break-even exchange rate, level of hedging, ability of customization and overall feasibility to hedge the risk.

Following table gives executive summary to all proposals.

	#1	#2	#3	#4	#5
Premium price	210 020	143 290	77 600	199 580	122 373
Break-even exch. Rate	25,55	N/A	25,36	25,535	24,425
Level of hedging	Very high	<i>Poor</i>	Fair	High	Fair
Customization	<i>Poor</i>	Fair	Very high	Very high	High
Overall feasibility	High	<i>Poor</i>	High	High	Fair

Table 4.17: Summarization

(Scale: Very high, high, fair, poor)

The Asian option (#2) proved to be the less feasible for hedging this kind of risk. The option would be better used for needs of hedging average rates during certain periods of time. It's been found highly speculative to hedge two incomes. The price of the option is 32% lower than Vanilla option. The author doesn't recommend using Asian options to hedge this kind of risks.

The binary option (#5) proved to be highly speculative also. The investor is not really hedged because the hedging is limited by fixed payoff that doesn't evolve with the exchange rate. On the other hand, the price is the second lowest from proposed

solutions. The author doesn't recommend using Binary options to hedge this kind of risk.

The Plain Vanilla option (#1) is the most expensive but it's providing the best level of hedging. Nevertheless, it is not an exotic option, but it has been included for comparison purposes.

The Down-and-Out Barrier option (#3) is the second best from compared examples of path-dependent exotic options. This option has the lowest option premium, but on the other hand it's very risky. The level of the risk respective option premium is highly customizable by the barrier. Therefore, the holder can sacrifice some of the highly improbable scenarios in favor of option premium discount. To be concluded, the author would recommend using barrier options for hedging purposes of this kind.

The Up-and-In Barrier option (#4) is providing almost same level of the hedging as the Vanilla option. The premium calculated in the analysis is high as the barrier is set very safe. Very high level of customization, high level of hedging capability and high level of overall feasibility is making this option really suitable candidate for Vanilla option substitute. As well as with the previous type of the option, the author would recommend using the barrier option for hedging.

5 Conclusion

This diploma thesis analyses feasible usage of the chosen exotic options as an investment instrument. Asian options, barrier options and binary options were evaluated and compared with the plain vanilla option counterpart.

The exotic options proved to be more flexible and cheaper in all of the executed analyses and tests. Although they may look as more affordable alternative to the vanilla options, they carry certain limitations. This is compensated by the discount of the option premium.

In addition, the exotic options are taking supplemental features into the action. For instance, the path-dependent options analyzed in this thesis are dependent on the price development of the underlying asset during whole lifetime of the option. That means that path taken by the underlying has significant influence on the payoff at the maturity.

Therefore, the speculating investor with better knowledge of the further development of the asset could easily build his strategy by using proper instruments. Asian options are able to generate additional yield if the underlying changes the direction before the expiration. The Down-and-out Barrier option must not cross the barrier during its lifetime or it will turn worthless, whereas the American type of Binary option is very similar but the payoff is fixed amount of cash or a asset.

Incorporating the Black-Sholes option pricing model to the analyses proved that discount relates to the amount of risk, which is not transferred to the option issuer. For example, the price of the vanilla option is equal to the combination of one Barrier Knock-in and one Knock-out option.

The best result of all the proposals were carried out by Barrier options, therefore they proved to be the most suitable for hedging as a substitute for the vanilla option in case of this particular example. They can be customized better than Vanilla options. On the other hand, since the Barrier option doesn't transfer all the risk, the holder has to be aware and prepare his strategy wisely.

Asian options are not feasible to hedge this kind of risk. They are more suitable to hedge the situations where there is a need to secure average rate. The binary options are similar to barrier options but since they have fixed payoff, the ability of hedging is significantly compromised. Therefore they should be used only for speculations.

Exotic options proved to be more flexible and customizable. If any company chooses exotic option instead of the vanilla option for hedging their exchange rate risks, they will need to build the strategy more carefully. In addition, they will save resources on the price of the instrument. This will help to improve profitability, hence competitiveness.

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