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GAME THEORY IN WASTE MANAGEMENT

TEORIE HER V ODPADOVÉM HOSPODÁŘSTVÍ

SHORT VERSION OF DOCTORAL THESIS

TEZE DIZERTAČNÍ PRÁCE

AUTHOR — **Ing. IVAN ERYGANOV**

AUTOR PRÁCE

SUPERVISOR — **doc. Mgr. JAROSLAV HRDINA, Ph.D.**

ŠKOLITEL

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Abstract

Game theory handles tasks such as cooperation, competition, and self-regulation in the environment, where numerous agents with conflicting goals are involved. These conflicts of interest are extremely common, when dealing with environmental sustainability and circular economy. The Ph.D. thesis is devoted to applications of game theory in waste management, with an emphasis on Waste-to-Energy treatment of non-recyclable waste. After an introduction, the author's own contribution in the application of non-cooperative and cooperative games to problems of waste management is presented. In particular, this Ph.D. thesis is focused on the Waste-to-Energy plants' price-setting game and the waste producers' cost minimization game. Theoretical properties of these games are studied in detail. The original algorithms for bilevel optimization problems and dynamic coalition formation are proposed to solve the considered games. The case studies' results demonstrate rational outcomes of the conflicts and prove that the proposed approaches to the considered waste management problems are reasonable.

Abstrakt

Teorie her se zabývá tématy, jako je spolupráce, konkurence a seberegulace v prostředí, kde je zapojeno mnoho entit s protichůdnými cíly. Rozdílné zájmy jsou běžné při řešení environmentální udržitelnosti a oběhového hospodářství. Disertační práce je věnována aplikacím teorie her v odpadovém hospodářství s důrazem na energetické zpracování nerecyklovatelného odpadu. Po úvodu je uveden vlastní přínos autora v aplikaci nekooperativních a kooperativních her v oblasti odpadového hospodářství. Konkretně je Ph.D. práce zaměřena na hru o stanovení cen zařízeními pro energetické využití odpadů a hru o minimalizaci nákladů producentů odpadů. Jsou podrobně studovány teoretické vlastnosti těchto her. Pro řešení uvažovaných her jsou navrženy originální algoritmy pro problémy dvouúrovňové optimalizace a vytváření dynamických koalic. Výsledky případových studií ukazují racionální vyústění konfliktů a dokazují, že navrhované přístupy k uvažovaným problémům odpadového hospodářství jsou rozumné.

Keywords

game theory, optimization, waste management, decision-making, sustainability

Klíčová slova

teorie her, optimalizace, odpadové hospodářství, rozhodování, udržitelnost

Reference

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Game Theory in Waste Management

Declaration

I hereby declare that the Ph.D. thesis was prepared as an original work by the author under the supervision of doc. Mgr. Jaroslav Hrdina, Ph.D. Furthermore, I declare, that I cited all literature, publications, and other sources used for this document.

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Ivan Eryganov
September 27, 2023

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Contents

Chapter 1

Introduction

According to [19], game theory (GT) focuses on mathematical models of complex interactions among rational participants of the formalized conflict. GT enables the description of the natural and logical development of such conflicts. It anticipates possible outcomes of situations in which decision-makers with different goals are involved and can affect each other [17]. Among other applications, it can imitate rationality and optimize arbitrary complex engineering systems, where different system parts are considered to be players performing various, often conflicting, tasks. GT has become an essential framework in the past years, since the number of applications involves multiple users, where disagreements between them are incredibly likely or even unavoidable [15]. These disagreements are common to a wide range of disciplines such as economics, computer science, social sciences, or engineering. Among all these disciplines, the doctoral thesis is focused on sustainability research, circular economy (CE), and efficient green waste management (WM).

Well-planned WM is an essential part of CE, and behavioral modeling, describing the ever-changing decisions of the involved agents, is its key aspect [1]. The doctoral thesis is devoted to applications of cooperative GT CGT and non-cooperative GT NGT to WM problems, which are of critical importance for the modern society. The considered problems are the non-cooperative gate fee setting game between waste treatment facilities and the municipalities' cooperative waste treatment cost game.

The main goals of the doctoral thesis are:

- to present an overview of the GT theoretical concepts, with an emphasis on branches, solutions, and specific game types, which will be used later in the application section;
- to review recent applications of GT in environmental sustainability research within different fields in order to identify currently existing research gaps;
- to formulate and analyze WM problems using CGT and NGT;
- to design algorithms for solving these problems;
- to implement bilevel programming techniques into the price-setting problem.

According to the performed review, currently employed models in the considered area lack more sophisticated approaches, real data-based case studies, and are often limited to comparison of fully cooperative and non-cooperative cases, or to solution of simple matrix-form games. Thanks to cooperation on research projects with the Institute of Process Engineering, Faculty of Mechanical Engineering, Brno University of Technology, real data and operation conditions of WM networks, in the form of waste production, price levels, capacities and infrastructure, are available to experiment with designed approaches under conditions, that are maximally close to real ones. The proposed game theoretic approaches to the considered problems will be presented in the following chapter. Proofs of the theorems presented in the next chapter can be found in the full version of the thesis.

Chapter 2

Games in waste management

The main issues of WM are monitoring and regulation of the collection, transportation, treatment, and disposal of waste [1]. Whereas the recyclable waste fits perfectly into the design of CE closed production cycles, the non-recyclable fraction of mixed solid waste (MSW) cannot be utilized in the same way. However, the energy potential of non-recyclable waste can be restored through Waste-to-Energy (WtE) technology [11]. It is expected that WtE plants will play an important role in waste treatment under CE package CEP legislative changes [14]. Whereas in the past, incineration of MSW has been a source of substantial pollution, nowadays, due to the continuous development of WtE technology, WtE plants can serve as an environmentally friendly source of energy [21]. In [18], the WtE environmental impact has been thoroughly studied. The research concluded that WtE, as a combination of WM practice and electricity sources, can provide climate change benefits. However, if it is considered a renewable energy source solely, it cannot compete with other sources regarding greenhouse gas emissions. On the other side, it is more stable than wind power or solar energy [22]. Thus, the embedment of the WtE plants into cities' smart-energy grids might help to increase the sustainable production of energy and solve the problem of overwhelming energy demand expected in the near future [20].

2.1 Waste-to-energy plants price-setting

Expectedly, the actual capacities of already existing waste treatment facilities can be insufficient for efficient waste energy recovery in the future. Therefore, new waste treatment facilities will be needed [9]. The placement of a new WtE facility is strongly impacted by the existing infrastructure of the considered region and therefore does not suggest vast space for possible decisions. On the other side, the optimal capacity design brings numerous variants that should be assessed correctly. Such strategical decisions should be made with the help of suitable decision-making (DM) methods. Moreover, it should be supported by a reliable analysis of the current WM situation, since the accurate estimate of potential occupancy of capacity, and a realistic gate fee will enable to correctly anticipate return on investment and the financial feasibility of the whole project. However, in most operational research models employed in WM [2], gate fees are assumed to be external fixed parameters that have been set or optimized centrally. Such assumption neglects individual behaviors of WtE plants management and cannot describe a real conflict of interests in a waste treatment market. Therefore, there is an open problem of how to efficiently anticipate the gate fees, which will realistically reflect the WM network setting.

2.1.1 Problem statement

The detailed formulation of the considered problem can be described as follows. Consider the already built WM network. WtE plants with different capacities and waste producers (mainly cities or agglomerations) with different waste productions are presented in an area. Each WtE plant is interested in maximizing its income by setting the optimal gate fee, which will be sufficiently high or/and will attract waste producers. WtE plant income is presented as a product of its gate fee and the total amount of waste sent to this WtE plant by waste producers. The main assumption is that landfilling of utilizable waste is substantially limited, according to [3]. This fact forces waste producers to treat all produced non-recyclable waste using the services of WtE plants. Each waste producer's main interest is to reduce costs for waste treatment. These costs are represented as a product of the amount of waste sent to a particular WtE plant and the sum of gate fee and transportation costs. Another important assumption is that, whereas WtE plants located in an area are individually maximizing their income, waste producers are cooperatively minimizing their total waste treatment costs. The cooperating waste producers reflect the current trend when municipalities tend to create unions to lower their waste treatment costs [6].

Thus, the established task comprehends two distinct challenging steps:

- a solution of the price-setting bilevel programming problem with one WtE plant, maximizing its revenue on the upper level and cooperating waste producers, minimizing their total costs on the lower level;
- a determination of the Nash equilibrium (NE) of the price-setting normal form game between WtE plants.

Now, the mathematical formalization of the considered problem will be given.

2.1.2 Model and game

Let $N = \{1, \ldots, n\}$ be a set of WtE plants; $w_1^c, \ldots, w_n^c$ denotes their capacities and $C_1^g, \ldots, C_n^g$ denotes their strategy sets (sets of possible gate fees) with an element $c_j^g \in C_j^g, j \in N$. The set of producers is $M = \{1, \ldots, m\}$. Their waste productions are $w_1^p, \ldots, w_m^p$. Transportation costs are given by the matrix $\left[c_{i,j}^t\right]$, where $c_{i,j}^t$ represents the cost of waste transportation from the producer $i \in M$ to the plant $j \in N$. In the following expressions, $x_{i,j}$ denotes the amount of waste sent by the producer $i \in M$ to the WtE plant $j \in N$ in tonnes. For each WtE plant $j \in N$, the payoff function π_j is defined as

$$\pi_j\left(c_1^g, \ldots, c_n^g\right) = \sum_{i \in M} c_j^g x_{i,j}^*, \tag{2.1}$$

where $(x^*_{i,j})_{i\in M, j\in N} \in \{(x^*_{i,j})_{i\in M, j\in N}\}$, such that

$$\{(x^*_{i,j})_{i\in M, j\in N}\} = \arg \min_{x_{i,j}: i\in M, j\in N} \sum_{j\in N} \sum_{i\in M} \left(c^t_{i,j} + c^g_j\right) x_{i,j}, \tag{2.2}$$

$$s.t. \sum_{i\in M} x_{i,j} \le w^c_j, \ \forall j \in N, \tag{2.3}$$

$$\sum_{j\in N} x_{i,j} = w^p_i, \ \forall i \in M, \tag{2.4}$$

$$x_{i,j} \ge 0, \quad \forall i \ \in M, \quad \forall j \in N. \tag{2.5}$$

The $(x^*_{i,j})_{i\in M, j\in N}$ describe resulting non-negative (2.5) waste flows after cooperative minimization of total costs by cities (2.2) and the fact, that they have to dispose of all waste they produce (2.4) and cannot exceed capacities of WtE plants (2.3). The set $\{(x^*_{i,j})_{i\in M, j\in N}\}$ is not necessarily a singleton. To prevent ambiguity, in this work, a risk-averse leader, who wants to create a financial cushion, is considered. Thus, the worst possible waste distribution scenario $(x^*_{i,j})_{i\in M, j\in N}$ for the WtE plant will be taken among all possible arguments of optima of the above-presented mathematical programming problem. To make the problem of waste producers feasible, it is necessary to assume $\sum_{i\in N} w^c_i \ge \sum_{j\in M} w^p_j$. By now, two of three necessary elements of the normal form game of WtE plants have been established: the set of players $N = \{1, \ldots, n\}$ and their payoff functions $\pi_j(c^g_1, \ldots, c^g_n), \ j \in N$, have been defined. To thoroughly study the properties of the problem, the whole set of positive reals will be considered as a strategy space of possible gate fees. Thus, the considered game can be represented as a triple $G = (N, (\pi_j, C^g_j)_{j\in N})$, where $C^g_j = (0, \infty), \forall j \in N$.

The above-defined payoff functions are not differentiable or continuous. As a result, their derivatives cannot be described in order to analytically find the NE. Author's first paper on this topic [5] has considered applying best-response dynamics (BRD) to discrete sets of possible gate fees. Compared to the original work on this topic [16], the cardinality of the sets of possible gate fees for which equilibrium can be found was substantially enlarged. In [16], the NP-hard problem $MR_{j'}$ of setting the optimal price between one WtE plant and all waste producers has been solved by a simple combinatorial approach through simple iteration over all possible strategies. However, such an approach does not reflect reality, where WtE plants can choose from the continuous sets of gate fees. Then, an achieved equilibrium might seem artificial because players were not allowed to play optimal strategy and arbitrarily change it. This is the reason why we apply bilevel programming methods in the next section: it will enable us to consider continuous strategy spaces, find optima faster and better reflect reality.

The solution idea. The combination of the mixed-integer programming (MIP) reformulation proposed by Heilporn et al. [7] and of the idea analogical to [10] has inspired the development of a new heuristic approach providing the near-optimal solution for $MR_{j'}$. Whereas, in the latter work, the follower's behavior has been anticipated via small perturbations in flows, in the Ph.D. thesis, a completely new iterative solution approach is presented. It is suggested to neglect the idea of approximation of objective function derivatives. The proposed approach captures the followers' behavior via iterative update of their optimal flows after the solution of the risk-averse revenue maximization problem of the leader: the iterative adjustment of the lower level solution enables to estimate the optimal price of the upper level. The whole leader problem is formulated based on MIP reformulation proposed by [7] with novel additions, enabling the embedding of leader capacities constraints and new inequalities reflecting his ability to raise gate fees by neglecting some of the flows.

2.1.3 Finding the optimal gate fee

In this section, the previously introduced idea of finding the solution will be further formalized. In particular, the precise description of the proposed algorithm and commentary on it will be introduced.

Suggested approach

In this subsection, a heuristic algorithm for solving the original problem $MR_{j'}$, which is based on the approach presented in the previous subsection, is proposed. This suggested algorithm embeds the capacities of other WtE plants into a DM process and can be described as follows.

First step. Solve the problem $LP_{j'}WITHOUT$ and obtain information about the current state of the network without WtE plant j'.

$$\min_{x_{i,j}, i\in M, j\in N\setminus j'} \sum_{j\in N\setminus j'} \sum_{i\in M} \left(c^t_{i,j} + c^g_j\right) x_{i,j}, \tag{2.6}$$

$$\text{s.t.} \sum_{i\in M} x_{i,j} \le w^c_j, \forall j \in N \setminus j', \tag{2.7}$$

$$\sum_{j\in N\setminus j'} x_{i,j} = w^p_i, \ \forall i \in M, \tag{2.8}$$

$$x_{i,j} \ge 0, \ \ \forall i \ \in M, \ \ \forall j \in N \setminus j'. \tag{2.9}$$

Second step. Set $(x^{*,j'}_{i,j})_{i\in M, j\in N\setminus j'} \in \arg LP_{j'}WITHOUT$. Solve the problem $HNP_{j'}RA$ and consequently $HNP_{j'}RA\ FULL$ (formulations can be found in the full version of the work). The first two steps provide the main body of the algorithm with the relevant estimate of the network starting state and the gate fee $c^{start,g}_{j'} \in \arg HNP_{j'}RA\ FULL$ is the starting price in the iterative solution process. Currently, the capacity constraints hold for every WtE plant in the network.

Third step. Solve the $LP_{j'}$, corresponding to the lower-level problem in the original bilevel formulation $MR_{j'}$ with $c^g_{j'} = c^{start,g}_{j'}$, to obtain the current state of the network:

$$\min_{x_{i,j}, i \in M, j \in N} \sum_{j \in N} \sum_{i \in M} \left(c^t_{i,j} + c^g_j\right) x_{i,j}, \tag{2.10}$$

$$s.t. \sum_{i \in M} x_{i,j} \ \le w^c_j, \forall j \in N, \tag{2.11}$$

$$\sum_{j \in N} x_{i,j} \ = w^p_i, \ \forall i \in M, \tag{2.12}$$

$$x_{i,j} \ge 0, \ \ \forall i \ \in M, \ \ \forall j \in N. \tag{2.13}$$

In each iteration, this step corrects the reactions of the follower to the newly chosen $c^{start,g}_{j'}$, so that leader has actual information about current flows for the given gate fee.

Fourth step. Set $(x^{*,j'}_{i,j})_{i \in M, j \in N \setminus j'} \in \arg LP_{j'}$. Solve the problem $HNP_{j'}CAP$

$$\max_{c^g_{j'}, p^{i,j}, q^{i,j}, i \in M, j \in N} \sum_{i \in M} \sum_{j \in N} x^{*,j'}_{i,j} p^{i,j}, \tag{2.14}$$

$$s.t. \left(c^t_{i,j'} q^{i,j} + p^{i,j}\right) + \left(c^t_{i,j} + c^g_j\right)\left(1 - q^{i,j}\right) \le c^t_{i,j'} + c^g_{j'}, \ \forall i \in M, \forall j \in N \setminus j', \tag{2.15}$$

$$c^t_{i,j'} q^{i,j'} + p^{i,j'} \ + L^i(1 - q^{i,j'}) \le c^t_{i,j'} + c^g_{j'}, \forall i \in M, \tag{2.16}$$

$$c^g_{j'} - p^{i,j} \le N\left(1 - q^{i,j}\right), \ \forall i \in M, \forall j \in N, \tag{2.17}$$

$$p^{i,j} \le M^{i,j} q^{i,j}, \ \forall i \in M, \forall j \in N, \tag{2.18}$$

$$p^{i,j} \le c^g_{j'}, \ \forall i \in M, \forall j \in N, \tag{2.19}$$

$$p^{i,j} \ge 0, \ \forall i \in M, \forall j \in N, \tag{2.20}$$

$$q^{i,j} \in \{0, 1\}, \ \forall i \in M, \forall j \in N, \tag{2.21}$$

$$\sum_{i \in M} \sum_{j \in N} q^{i,j} x^{*,j'}_{i,j} \le w^c_{j'}, \tag{2.22}$$

where

$$M^{i,j} = \max\left\{0, c^t_{i,j} + c^g_j - c^t_{i,j'}\right\}, \ \forall i \in M, \forall j \in N \setminus j',$$

$$M^{i,j'} = \max\{0, L^i - c^t_{i,j'}\},$$

$$L^i = \min_{j \in N \setminus j'} \{c^t_{i,j} + c^g_j \mid c^t_{i,j} + c^g_j > c^t_{i,j'} + c^{start,g}_{j'}\},$$

and $N = \max M^{i,j}$. In case L^i is not defined due to emptiness of the underlying set, L^i can be set as sufficiently large number. Consequently, solve modification $HNP_{j'}CAP\ FULL$: modify flows and constraints analogous to $HNP_{j'}RA\ FULL$. These two problems describe the adaptation of the leader to the current flows that have been changed in the previous step. Novel, newly introduced constraint (2.16) reflects the possible choice of abandoning some of the current non-zero waste flows to j' in order to increase the price and potentially obtain higher revenue. Set $c^{opt,g}_{j'} \in$ arg $HNP_{j'}CAP\ FULL$.

Fifth step. Raise $c_{j'}^{opt,g}$ and solve $LP_{j'}$ with $c_{j'}^{g} = c_{j'}^{opt,g}$, until the first decrease in $\sum_{i \in M} x_{i,j'}^{*,j'}$, where $(x_{i,j}^{*,j'})_{i \in M, j \in N \setminus j'} \in \arg\ LP_{j'}$. This is a simple computational check in case the WtE plant j' might still be the best waste treatment option due to the filled capacities of the other plants.

Sixth step. If $c_{j'}^{opt,g}$ from the previous step guarantees greater revenue than $c_{j'}^{start,g}$, then set $c_{j'}^{start,g} = c_{j'}^{opt,g}$ and go back to the third step. Otherwise, the solution $c_{j'}^{start,g}$ is found, END. This is a classical search stop condition, where the main body of a cycle runs as long as it can find a better solution.

Commentary. The algorithm is meant to produce the optimal or near-optimal solution. To create an artificial upper bound for gate fees and to ensure the requirement that for every commodity exists the toll-free path from its origin to its destination, a WtE plant with a capacity that can meet waste production of the whole region has to be considered.

2.1.4 Exemplary case study

In this section, the Czech Republic exemplary case study will demonstrate how the proposed approach could be applied to design the optimal capacity of the future WtE plant. Moreover, the numerical results of the proposed bilevel programming heuristics algorithm will be discussed. It is assumed that in the Czech Republic, there are 16 WtE plants (the founding of 12 of them is currently planned). However, some waste producers from the Czech Republic might use the services of facilities in the nearby countries (Germany and Austria). To create an upper boundary on the possible gate fee and ensure the existence of the «toll-free» path, these facilities are represented as three WtE plants with a fixed gate fee of 100 €/t and the capacity corresponding to the total waste production of the whole Czech Republic.

To compete with these foreign facilities, it is planned to build one more WtE plant in the Czech Republic (WtE plant «Otrokovice»), and the question of optimal capacity design arises. To optimally estimate the capacity, it is advised to «place» this facility in the currently existing network and find the NE of the considered WtE plants price-setting game using the suggested approach: BRD based on the proposed bilevel programming heuristics. The resulting price state will enable the establishment of the waste flows and revenues of all WtE plants in the network. This process, iteratively repeated for each capacity design, will provide an image of the expected revenue of the planned facility, which can be compared to required investments. The starting point of the whole process for each WtE plant (except the foreign plants) is assumed to be the gate fee of 50 €/t, and the first capacity design is 25 kt/y. To computationally simplify the algorithm, the transportation costs are assumed to be integers. Productions, as well as capacities, are assumed to be annual.

Unfortunately, the BRD failed to find an NE during the first attempt. When the σ, defining stopping condition of the algorithm, is considered to be too small, the algorithm gets stuck in the cycle. This fact can be explained, by the hypothesis, that when continuous strategy sets are assumed, the change of the gate fee is expected to be always profitable. This would lead to non-existence of the fixed-point in best-response correspondences, and, as a result, the NE would cease to exist in a general game. This possible explanation will be studied in detail at the end of the section devoted to WtE price-setting. To overcome this complication, it is assumed that, when the norm of the difference vector is less than 1, no substantial change in the gate fees vector has occurred, and the algorithm will be stopped. This assumption will enable to prevent the cyclic nature of the price-setting game, when players successively lower their prices to obtain greater demand. Under the new precision assumption, the gate fee stable outcomes were computed for the suggested capacities from 25 kt to 350 kt with the step of 25 kt. The capacity usage and the estimated revenue of the planned WtE plant «Otrokovice» are presented in Table 2.1. The table confirms that

Table 2.1: Results for «Otrokovice»

Capacity [kt]	**Gate fee [€/t]**	**Obtained amount of waste [kt]**	**Employed capacity**	**Revenue [T€]**
25	68.8	6.54	26.17%	450.21
50	55.9	36.93	73.85%	2,064.21
75	54.6	67.47	89.97%	3,684.07
100	53.2	84.60	84.60%	4,500.81
125	52.9	103.14	82.51%	5,456.18
150	50.8	146.09	97.40%	7,421.55
175	50.5	152.88	87.36%	7,720.50
200	51.5	163.94	81.97%	8,442.81
225	49.3	163.94	72.86%	8,082.15
250	48.9	239.66	95.87%	11,719.57
275	47.6	265.91	96.69%	12,657.26
300	46.8	252.75	84.25%	11,828.56
325	48	265.91	81.82%	12,763.62
350	48.6	260.06	74.30%	12,638.91

the proposed model is reasonable: capacity increase causes a gradual decrease in gate fees for all of the considered WtE plants. Thus, in accordance with basic economy rules, the greater «supply» (capacity) leads to a lower price (gate fee). Clearly, to improve the reliability of the found solutions, the impact of the input parameters and initial point choice on the algorithm precision and speed of convergence should be studied in the future.

To choose an appropriate capacity design for a particular WtE project, the revenues from waste treatment have to be compared with the initial investments. For the sake of simplicity, the solved task does not consider operational costs and revenues related to heat and electricity selling. In the case of investment costs, it is important to reflect decreasing unit costs when increasing capacity. The costs for particular capacity variants are estimated by adopting the following formula from [4]:

$$I = I_R \frac{C}{C_R}^{0.75},$$

where I represents investments and C represents the capacity of the facility. Subscript R denotes the reference number. For the case presented herein, the reference numbers were set to $I_R = 4$ M€/y and $C_R = 100$ kt/y. Figure 2.1 illustrates the results for the considered capacity variants. The profitability of investment can be easily compared via ratios illustrated by a line. Figure 2.1 demonstrates that the greater capacity does not always guarantee a better ratio between revenue and investments. Thus, the market power induced by a greater capacity does not automatically ensure a greater return on investment but has phase-shifting properties. For example, only after trespassing the capacity of 225 kt/y the WtE plant again obtains an advantageous position on the WM market and can pursue a greater return on investment. The decision about the optimal capacity directly depends on the available capital for the investment. For example, if the maximal possible investment is around 7 M€/y, it is reasonable to invest less and build a WtE plant with a capacity of 150 kt/y. Now, suppose the management of the WtE plant can ensure greater resources for the investment. Then, it is more profitable to invest approximately 8 M€/y and build a facility with a capacity of $250 - 275$ kt/y (higher precision can be achieved by choice of the smaller step).

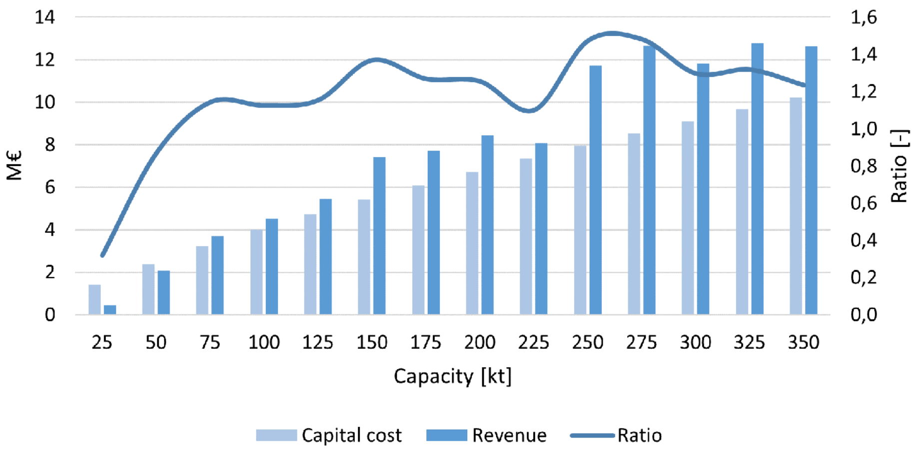

Figure 2.1: Ratio of revenue vs. capital cost

Numerical results of the heuristic algorithm

To verify that the algorithm is able to provide the optimal or near-optimal solution in realistic scenario, its performance has been compared to the classical enumeration. In particular, gate fee vectors from the last iteration of BRD have been used as an input describing fixed gate fees of competitors. Thus, 17 different cases (each for one of 17 competing WtE plants) have been calculated for 14 capacity designs. The heuristic failed to find an optimum solution only in 44 cases out of the considered 238, only 10 of which have led to a loss greater than 1%. Moreover, the largest difference between found optimum and the optimum established by the algorithm is 1.1.

2.1.5 Price-setting game and its properties

Our empirical results have pointed out possible non-existence of NE in problems of price-setting. This is why this section is devoted to analysis of the newly introduced class of price-setting games and to research on existence of NE in games of this type. In particular, we are interested in proving the fact that, under some real-world constraints and limitations, there might be no stable price state for sufficiently small artificial parameter ϵ from previous section. Before we define a price-setting game, a concept of market situation should be discussed.

Definition (Market situation)**.** The market situation

$$MS = (M, N, R, (t_{i,j})_{i \in N \cup R, j \in M}, (c_i)_{i \in N \cup R}, (d_j)_{j \in M}, x_{ref})$$

is defined by the set of customers $M = \{1, ..., m\}$, $|M| \geq 1$, the set of domestic producers $N = \{1, ...n\}$, $|N| \geq 2$, the set of foreign producers $R = \{n+1, ..., r\}$, $|R| \geq 1$, transportation costs per unit of goods $t_{i,j} \geq 0$, $\forall j \in M$, $\forall i \in N \cup R$, needed to transport unit of product from producer $i \in N \cup R$ to consumer $j \in M$, production capacities $c_i > 0$, $\forall i \in N \cup R$, of producers, and demands $d_j > 0$, $\forall j \in M$, of consumers. Foreign producers are participants of the market creating the reference price $x_{ref} > 0$.

Further, to simplify some expressions, we will use notation $\tilde{N} = N \cup R$. We also would like to describe role of x_{ref} in more details. In our study, the reference price x_{ref} is a price of a product on a foreign market, so, when the price on the domestic market exceeds the reference price (and potential transportation costs), it is more economic to import the product. Thus, it indeed creates «reference» for domestic producers and establishes price ceiling after trespassing which, domestic market begin to lose customers. Now, we can proceed to the definition of the price-setting game associated with a market situation.

Definition (Price-setting game)**.** Let us assume the market situation MS. Then, we define the price-setting game $G = (N, (X_i, \pi_i)_{i \in N})$ associated with MS as a game between players from a set N, where strategy of each player is represented as a price $x_i \in X_i = (0, \infty), \forall i \in N$. Elements of R are not part of the game itself, and they prices are fixed as $x_i = x_{ref}, \forall i \in R$. Then, each player's payoff function $\pi_i(x), i \in N$, is defined as

$$\pi_i(x) = \sum_{j \in M} x_i q^*_{i,j}, \text{ where } (q^*_{l,j})_{l \in \tilde{N}, j \in M,} \in Q,$$

$$s.t. \sum_{j \in M} x_i q^*_{i,j} \leq \sum_{j \in M} x_i q_{i,j}, \ \forall (q_{l,j})_{l \in \tilde{N}, j \in M} \in Q,$$

where set Q is defined as

$$Q = \arg \min_{q_{l,j}, l\in\tilde{N}, j\in M} \sum_{j\in M} \sum_{l\in\tilde{N}} (x_l + t_{l,j}) q_{l,j},$$

$$s.t. \sum_{j\in M} q_{l,j} \leq c_l, \ \forall l \in \tilde{N},$$

$$\sum_{l\in\tilde{N}} q_{l,j} = d_j, \ \forall j \in M,$$

$$q_{l,j} \geq 0, \ \forall j \in M, \ \forall l \in \tilde{N}.$$

Thus, domestic producers are independently maximizing their profits, whereas customers are minimizing their total costs, while aiming at completely satisfying their demands without capacity overruns. The above-defined game is designed to model markets with a high level of government interference, where costs, that occur during operation, are negligible compared to initial capital investments: this is why the payoff function does not involve fixed or variable costs. In order to ensure the correct definition of the payoff function, we have employed the already introduced pessimistic approach, i.e., that in the case of the existence of multiple solutions to the lower level customers' cost minimization problem, the solution, which is the most unfavorable to the producer i is chosen. The following assumption should be imposed on game in order to make its study reasonable:

$$c_i \geq \sum_{j\in M} d_j, \forall i \in R, \ (\textbf{boundness}).$$

Properties of the payoff function and the lower-level optimal solution. Assume some $i \in N$, fixed strategy profile (x_{-i}), and given x_{ref}. Then, let us describe $\sum_{j\in M} q^*_{i,j}$ as a function of x_i. Due to the nature of linear programming problems, their solutions are convex combinations of extreme points or directly extreme points (in case problems are bounded). This implies that $\sum_{j\in M} q^*_{i,j}$ as a function of x_i is non-increasing piece-wise constant and right continuous [13]. This properties should hold, since otherwise it will be a contradiction with optimality of $(q^*_{l,j})_{l\in\tilde{N}, j\in M}$ and its pessimistic property with respect to i. If this function will be multiplied by a variable $x_i > 0$, we will obtain a piece-wise linear (where each segment is increasing) and a right continuous payoff function $\pi_i(x_i)$ [13]. Now, the concept of NE in the considered class of games can be discussed.

Concept of δ-equilibrium

Unfortunately, the definition of the problem violates the existence of NE. For the above-defined payoff function, a more profitable strategy can always be found: it is sufficient to choose the price, which will shift the payoff closer to the peak of the «optimal» linear segment. The peak istelf is «absent»: in pessimistic approach it is only a limit of the payoff function from the left, which corresponds to an optimistic approach optimal solution (which does not have to be unique). Thus, player is always able to choose some sufficiently small $\delta > 0$, such that, for a fixed (x_{-i}), given x_{ref}, and arbitraty x_i

$$\pi_i(x_i^{opt} - \delta) \geq \pi_i(x_i)$$

where x_i^{opt} denotes the optimistic approach optimal price. However, if we assume, that players can be satisfied with the «nearly» optimal solution, then it is possible to define the following alternative to the pure NE concept.

Definition (δ-NE)**.** Let us assume a normal form game $G = (N, (X_i, \pi_i)_{i \in N})$ with $X_i = (0, \infty)$, $\forall i \in N$. Then, we define δ-NE, $\delta > 0$, as a strategy profile $\tilde{x} \in X_N$, such that $\tilde{x}_i = x^{\delta}_{\lim,i} - \delta$, where $x^{\delta}_{\lim,i}$ fulfills

$$\lim_{x_i \to x^{\delta -}_{\lim,i}} \pi_i(x_i, \tilde{x}_{-i}) \geq \pi_i(x_i, \tilde{x}_{-i}), \forall x_i \in (\delta, \infty).$$

This way we avoid the concept of the classical NE, replacing it with the strategy profile that might be arbitrarily close to a profile that is NE in a sense of limit.

Zero transportation costs

In this part, we consider only price-setting games G associated with MS, where $t_{i,j} = 0, \forall i \in \tilde{N}, j \in M$. Further, we will use notation $x_{\lim,i}(x_{-i})$ describing all $x_{\lim,i}$ such that

$$\lim_{x_i \to x^{-}_{\lim,i}} \pi_i(x_i, x_{-i}) \geq \pi_i(x_i, x_{-i}), \forall x_i \in X_i.$$

Notation $x^{\delta}_{\lim,i}(x_{-i})$ will be used analogically. Then, we can proceed to the following theorem on δ-NE existence for price-setting games associated with a particular group of MS with zero transportation costs.

Theorem 2.1.1. *For any zero transportation costs price-setting game G fulfilling boundness and*

$$\sum_{l \in N \setminus \{i\}} c_l > \sum_{j \in M} d_j, \forall i \in N, \ (\textbf{\textit{absence of dictator}}),$$

δ-NE exists for every δ, $\delta > 0$.

Absence of dictator ensure, that there is some amount of demand over which players might possibly compete. However, the theorem points out an interesting drawback of δ-NE for MS with this property: some strategy profiles are δ-NE only due to the fact, that players cannot play their optimal prices with respect to given price state. This problem does not occur when capacity dictator exists, as we will demonstrate in the following theorem.

Theorem 2.1.2. *Assume zero transportation costs MS fulfilling boundness and that $\exists i^* \in N$ such that*

$$\sum_{j \in M} d_j > c_{i^*}, \ \sum_{j \in M} d_j > \sum_{k \in N \setminus i^*} c_k \text{ and } \sum_{j \in M} d_j < \sum_{k \in N} c_k. \ (\textbf{\textit{existence of dictator}})$$

Then, for the associated price-setting game G, there $\exists\delta$ such that δ-NE does not exist.

The previous theorem has led us to the following corollary.

Corollary 2.1.3. Assume market situation MS fulfilling boundness and existence of dictator. Then, for the associated price-setting game G, there $\exists\gamma$, s.t. for all $\delta \in (0, \gamma)$, δ-NE ceases to exist.

Non-zero transportation costs complicate study of δ-NE existence representing important competitive advantage for some of the players. Thus, transportation costs brings asymmetry into the game and it is not possible to generalize the considerations established in Theorem 2.1.2 to prove problem with optimality of playing x_{ref}.

2.2 Waste producers' costs minimization

The upcoming CEP legal changes will substantially affect municipalities due to more complex and expensive waste treatment in the future. Thus, it is also essential to model and study the implementation of WtE technology from the municipalities point of view, considering their objectives of WM cost minimization. The way how municipalities financially handle new legal requirements will substantially impact sustainability of WtE plants and, as a result, of the energy produced there. To react to the up-coming legal changes, it is beneficial to create municipal unions, focused on the cooperation in WM. Such municipal unions help to lower waste treatment costs and to optimize waste collection. Whereas full cooperation axiomatically assumed in [8] can be considered as the most desirable outcome, it may not correspond to the realistic one due to circumstances/settings. In fact, such a centralized approach cannot properly model individual incentives of municipalities and interactions between them. This behavioral aspect becomes crucial during planning of municipal budgets and negotiations about the legal form of municipal units' cooperation. Therefore, it is necessary to study formation of municipal unions in a dynamic manner. Moreover, the distribution of resulting costs across municipal units should be assessed with respect to their locations and waste productions. Such cost analysis will enable to estimate future realistic WM tariffs, providing important information for municipal councils.

2.2.1 Problem definition

The general case of the problem considers a nonspecific area in which WtE plants with different capacities are situated. Waste producers (municipalities) with different locations and waste productions treat their waste using services of the available WtE plants. The model works with the already existing WM network. Assuming limited or banned landfilling, waste producers are forced to treat produced waste using services of WtE plants. Gate fees of WtE plants are assumed to be external fixed parameters (which can be obtained using approach from the previous section). Waste producers minimize their total waste treatment costs, consisting of transportation and waste processing costs. Cooperation occurs when instead of competing over the free capacities, some producers create union and reserve capacities of nearby WtE plants to waste producers with unfavorable locations. This enables them to reduce their waste treatment costs in exchange for the financial compensation, from which some of the cooperating waste producers, that have renounced these capacities, might substantially benefit. Now, the deduction of the appropriate value function v will be discussed in detail.

Deduction of the value function

The main idea was to propose the value function, which will reflect a realistic worst-case scenario of the WtE treatment costs minimization by an arbitrary municipal union. In the following mathematical programming problem, notation is given as follows: M is set of WtE plants, N is set of waste producers, S is coalition of municipalities (subset of N), $v(S)$ is value function of S (total annual waste treatment costs of S), remaining notation coincides with the model from the previous section. Then, waste producers' cost reduction game can be defined as a pair (N, v), where N is a set of waste producers and v is the value function defined as

$$v(S) = \min_{x_{i,j}, i \in S, j \in M} \sum_{j \in M} \sum_{i \in S} (c^t_{i,j} + c^g_j) x_{i,j}, \tag{2.23}$$

$$\text{s.t.} \sum_{i \in S} x_{i,j} \leq w^c_j - \sum_{i \in N \setminus S} x'^{,N \setminus S}_{i,j}, \ \forall j \in M, \tag{2.24}$$

$$\sum_{j \in M} x_{i,j} = w^p_i, \ \forall i \in S, \tag{2.25}$$

$$x_{i,j} \geq 0, \ \forall i \ \in S, \ \forall j \in M, \tag{2.26}$$

$$(x'^{,N \setminus S}_{i,j})_{i \in N \setminus S, \ j \in M} \in \arg costs_{N \setminus S} \tag{2.27}$$

$$costs_{N \setminus S} = \min_{x_{i,j}, i \in N \setminus S, \ j \in M} \sum_{j \in M} \sum_{i \in N \setminus S} \left(c^t_{i,j} + c^g_j\right) x_{i,j}, \tag{2.28}$$

$$s.t. \sum_{i \in N \setminus S} x_{i,j} \leq w^c_j, \ \forall j \in M, \tag{2.29}$$

$$\sum_{j \in M} x_{i,j} = w^p_i, \ \forall i \in N \setminus S, \tag{2.30}$$

$$x_{i,j} \geq 0, \ \forall i \ \in N \setminus S, \ \forall j \in M. \tag{2.31}$$

Each waste treatment costs component is represented as linear variable costs, where the amount of waste is multiplied by transportation cost and gate fee per ton of waste. Most of the constraints are the same as in lower-level problem of WtE plants' price-setting. For the sake of clarity, we describe their role once more. time Expression (2.23) represents the minimal amount of total costs, that can be achieved by coalition S. Constraints (2.25), (2.26), and (2.30), (2.31), ensure that all waste is treated, and forbid negative waste flows. Constraint (2.29) ensures, that the capacity of WtE plants cannot be exceeded, when computing optimal waste flows of coalition $N \setminus S$ in expression (2.28). Constraint (2.24) guarantees, that coalition S optimizes its waste flows on the capacities remaining after $N \setminus S$. This value function describes the pessimistic setting, in which the coalition S makes decision after the coalition $N \setminus S$, and is assumed to describe upper bound of WM costs of coalition S. The considered v has been originally presented in [16]. It is crucial to assume, that the total capacities of regional WtE plants should be greater than (or equal to) total waste production in a region. Thus, once more, the main assumption of the whole model is

$$\sum_{i \in N} w_i^p \leq \sum_{j \in M} w_j^c.$$

Cooperation enables municipality, which does not have WtE infrastructure and is distant from other WtE plants, to lower its waste treatment costs through negotiation with the closest municipality, that is situated near some WtE plant. The latter municipality can choose to treat its waste at another WtE facility to let the former municipality minimize its transportation costs (in real life, it is enough to subsidize transportation of former municipality). The part of occurred financial surplus, i.e., difference between the potential non-cooperative scenario costs and the real costs achieved through cooperation, can be then transferred to the latter municipality as a compensation. Now, we will study the theoretical properties of the considered game.

2.2.2 Properties of the game

Throughout the whole section, we make the following assumption:

- Each considered waste producers' cost reduction game (N, v) has unique solutions $(x_{i,j}^{\prime,S})_{i,\in S, j \in M} = \arg costs_S$, $\forall S \subseteq N$.

Though, this assumption might seem quite strong, it is necessary in order to be able to study properties of the considered game and compare the underlying linear programming problems. When solving practical problems, addition of sufficiently small random $\epsilon \in \mathbb{R}$ (positive as well as negative) to each considered transportation cost might help to create unique decrease directions to meet this assumption. We begin with the properties, that might have practical consequences with respect to costs distribution and coalition formation process during our case study.

Cohesivity and balancedness

When studying a cohesive game using merge and split rules in terms of utilitarian order and $\mathcal{D}_{hp}$ or $\mathcal{D}_p$ stability this property implies that if a merge and split process will start from N, then it will never split. Now, we can proceed to the main theorem on cohesivity of waste producers' cost reduction games.

Theorem 2.2.1. *General waste producers' cost reduction game (N, v) is cohesive.*

Thus, when playing as one large entity, total costs of the waste treatment in a region are as minimal as possible. However, Shapley value, that has been chosen as suitable distribution of waste treatment costs, does not necessarily belong to the core of the non-convex game. Therefore, it might be beneficial to consider the core distribution to compare this stable solution to the Shapley value. We have focused ourselves on finding point $(x_i)_{i \in N}$ of the core for every cost minimization game (N, v). By finding a such distribution, the balancedness of the general waste producers costs minimization game will be automatically proven. The cohesivity of the general game (N, v) has motivated us to study costs of each $i \in N$, when $v(N)$ is calculated, since it is the optimal partition with respect to social welfare. Then, the main theorem on core of the general waste producers cost reduction game can be established.

Theorem 2.2.2. *Let us assume waste producers cost reduction game (N, v). Further assume the costs distribution $(\hat{x}_i)_{i \in N}$ such that*

$$\hat{x}_i = \sum_{j \in M} (c^t_{i,j} + c^g_j) x'^{,N}_{i,j}.$$

Then, $(\hat{x}_i)_{i \in N} \in C(N, v)$.

The result of the previous theorem and equivalence between balancedness and core non-emptiness imply the following corollary.

Corollary 2.2.3. Every waste producers' cost reduction game (N, v) is balanced.

It is also important to study another important property, that might substantially impact the distributed dynamic coalition formation process.

Subadditivity

Unfortunately, subadditivity is not satisfied for all games of the considered type.

Lemma 2.2.4. Waste producers cost reduction games are not subadditive in general.

Unfortunately, it is rather challenging to establish some easily verifiable condition for subadditivity or convexity, since the relationship between $\sum_{i \in N \setminus (S \cup T)} x'^{,N \setminus S \cup T}_{i,j}$ and $\sum_{i \in N \setminus S} x'^{,N \setminus S}_{i,j} + \sum_{i \in N \setminus T} x'^{,N \setminus T}_{i,j}$ for some $j \in M$ can be hardly predicted.

Additivity

Since some games are not subadditive, it was decided to focus on studying a condition (put on input parameters of the game) that makes cooperation during the game non-trivial for at least one coalition. Thus, our aim is to establish easily verifiable condition, that will demonstrate if the game is or is not additive. At first, let us focus on the relationship between $\sum_{T \in \mathcal{P}} costs_T$ and $costs_S$ for arbitrary partition $\mathcal{P} \in \mathcal{P}_S$ of $S \subseteq N$.

Corollary 2.2.5. Assume a waste producers' cost reduction game (N, v). Then,

$$\sum_{i \in N} x'^{,i}_{i,j} \leq w^c_j, \forall j \in M \Leftrightarrow (N, v) \text{ is additive.}$$

Therfore, in waste producers' costs minimization game, cooperation might bring benefits, when for at least two waste producers the most economical optimistic option of the individual waste treatment becomes infeasible due to limited capacities.

Distributed dynamic coalition formation

Whereas the theoretical concepts provide necessary elements to formalize dynamic coalition formation, they do not explain, how outcome of such process should be computed. Moreover, a particular implementation of the merge and split process might directly affect a found stable outcome. In this work, the following implementation is suggested (the implementation has been programmed in MATLAB).

The initial coalition structure is assumed to correspond to the state with no cooperation among players. The merge rule is always applied as first and operates exclusively on pairs of coalitions. Coalitions to be merged are subsequently taken from a set of all available pairs of coalitions in coalition structure. If the merge operation is performed, coalition structure is updated, and merge rule application starts again. When no merge operation can be performed, the algorithm proceeds to the application of a split rule. It iterates over all coalitions in the coalitional structure and checks the split operation assumption for every partition of the currently processed coalition. Partitions are taken from a set of all possible partitions. If the split operation is performed, the coalition structure is updated, and the split rule continues to run. When no split operation can be performed, the process proceeds to merge rule application. If in one full cycle (one application of merge rule and one application of split rule) no merge or split have been performed, the merge and split algorithm ends. The ordering in combinatorial sets (set of all pairs of coalitions and set of all partitions) is obtained via the MATLAB function „nchoosek". The assumptions about a starting coalition structure and the application of merge rule on pairs of coalitions are aimed at sustaining computational complexity on the desired level. Since every game has been proven to be cohesive, if the merge and split process with respect to utilitarian order will start from N, it will not be splitted any more. In the case of strict cohesivity, any starting profile will lead us to N, if we consider not only pairs but all possible merges. Still, when no additional costs are assumed, the first merge operation might result into complete cooperation and the formation of the grand coalition. In such case, the large set of players will potentially lead to the combinatorial explosion during the split operation, since all possible partitions must be checked. To overcome this potential problem, it has been decided to embed additional cooperation costs into the considered approach. Such penalization might reflect increasing financial costs for retaining efficient communication between coalition participants and coordination of mutual actions.

Additional costs algorithm. In order to capture the impact of additional cooperation costs, the definition of value function has been modified to:

$$v^*(S) = v(S) + \sum_{i \in S} \sqrt{|S| - 1}\frac{p}{100}v(\{i\}).$$

The value function now represents the sum of the original value function and additional cooperation costs, which are represented as a sum of value function values corresponding to the individual micro-regions contained in $S \subseteq N$. The latter term is multiplied by a square root of coalition S size minus one to embed nonlinear penalization of greater coalitions $(v^*(\{i\}) = v(\{i\}))$. To obtain uniform coalition, the latter term is also multiplied by a penalization term $p \in [0, 100]$, which will be further used as an instrument to manipulate the coalition formation process. In practice, it is almost impossible to find a general cost function describing the costs of cooperation. It is intuitively clear, that it will have positive correlation with the cardinality of the coalition, therefore the proposed function is in line with the basic premise. The exact idea of the manipulation with penalization dwells in an algorithm, which is aimed at obtaining the collation structure with the maximal average coalition size, through iterative alternation of penalization decreases and increases. The design of the proposed algorithm is sketched in Figure 2.2. In Figure 2.2, p with the lower subscript represents particular value of penalization, k is step with which penalization changes in each iteration, $\mathcal{C}_j = \{S_1, \ldots, S_m\}$ is a particular coalition structure, and $a_{\mathcal{C}_j} = \frac{|N|}{m}$ is an average coalition size under structure $\mathcal{C}_j$. The structure $\mathcal{C}_{start}$ represents starting coalition structure for application of merge and split algorithm (it corresponds to fully non-cooperative case only during the first penalization decrease).

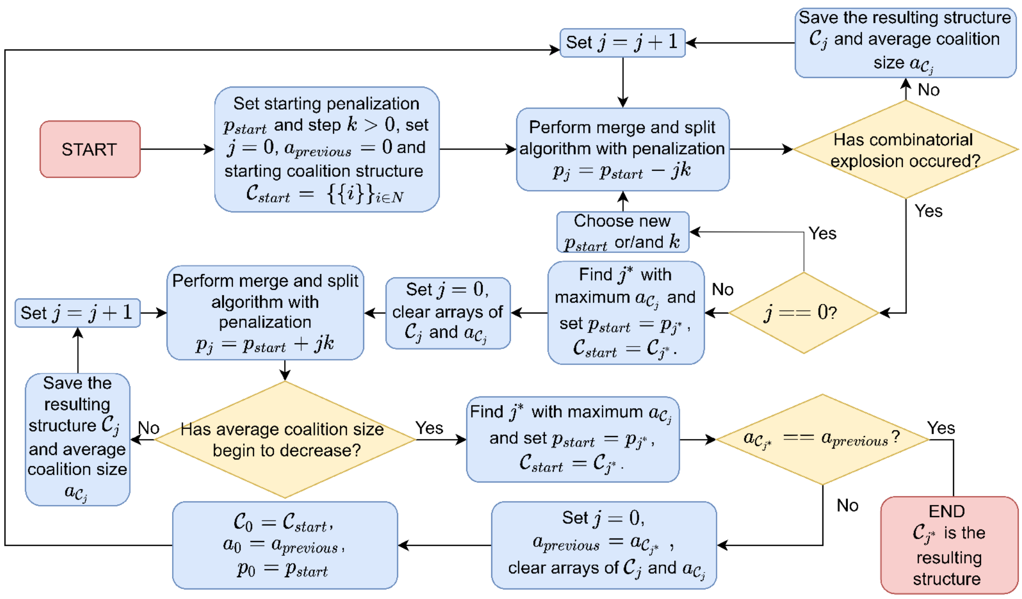

Figure 2.2: Penalization-based coalition formation algorithm

2.2.3 Case Study

The case study dwells in the application of the modified merge and split algorithm to the waste producers' cost game, where the set of players consists of 47 micro-regions (municipalities with extended authority), which are presented within three regions of the Czech Republic: the Zlín Region, the Olomouc Region, and the South Moravian

Region. In order to meet the requirement, that all waste meant for energy recovery can be handled by the Czech Republic's WM network, WtE plants, that do not exist, but are currently being planned, have also been assumed. This makes a total of seven WtE plants. The data on waste generation of the micro-regions has been provided by the Ministry of the Environment; financially sustainable gate fees, capacities, and transportation costs have been obtained from [12]. The additivity condition has been checked and the game in the considered setting is not inessential. As it was already mentioned, the initial coalition structure corresponds to the state with no cooperation among the micro-regions, i.e. the process starts with 47 disjoint coalitions, each represented by only one municipality. For the case study, starting penalization value has been set to 2 and the step has been set to 0.1. This relatively low penalization might be explained by a pessimistic setting of the problem, where only large coalitions might substantially reduce their total costs through cooperation. A schematic merge and split process for the penalization of 1.2 during first penalization decrease is depicted in Figure 2.3. The algorithm run information is presented in Table 2.2.

Table 2.2: Average coalition size changes

Penalization	1st decrease	1st increase	2nd decrease	2nd increase	3rd decrease
2	1.044				
1.9	1.119				
1.8	1.119				
1.7	1.119				
1.6	1.093				
1.5	1.119	1.306			
1.4	1.237	1.343		1.382	
1.3	1.424	1.382		1.382	
1.2	**1.469**	**1.469**	1.469	1.469	
1.1	Err		1.469	1.469	
1			**1.567**	**1.567**	**1.567**
0.9			Err		Err

The 3rd increase column has been omitted, since it fully copies the 2nd increase column. In each penalization increase step, few more iterations have been computed to ensure, that average coalition size is consistently decreasing. All resulting coalitions with the cardinality greater than one, can be considered as a steady and stable outcome. The map of the resulting structure is depicted in Figure 2.4.

Discussion

In this case study, the algorithm has enabled to create three "clusters", which attracted a certain number of micro-regions, due to substantial total costs decrease regardless of the applied penalization. These coalitions can be referred to as the most profitable, while other micro-regions are not interested in cooperation under

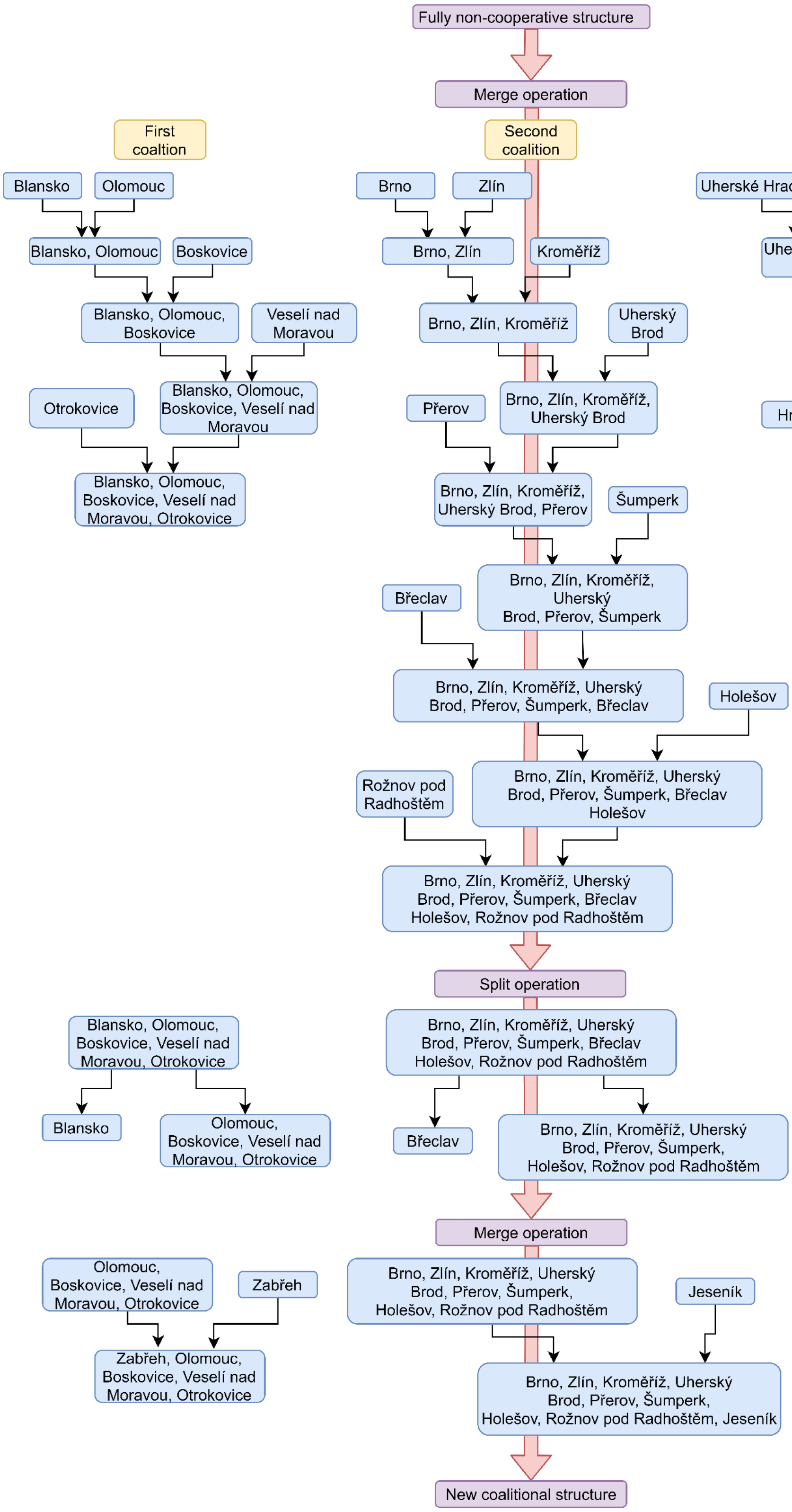

Figure 2.3: Merge and split full run for the penalization of 1.2

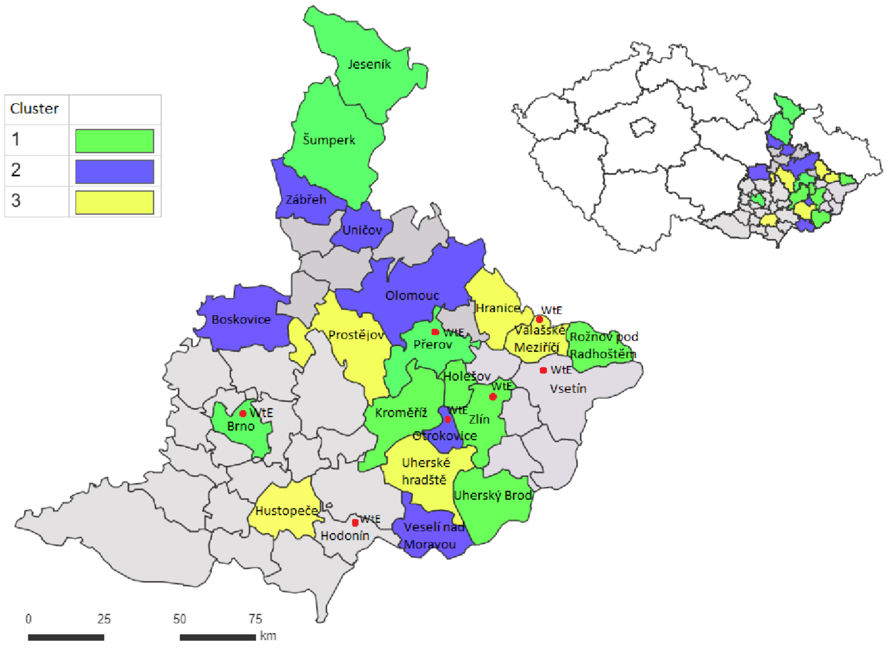

Figure 2.4: Map of the resulting municipal unions

additional cooperation costs. This implies, that cooperation cannot enable them sufficient compensation due to their waste productions and locations with respect to the WtE plants, which are indicated by red dots in Figure 2.4. These dots do not correspond to the real or planned location of WtE and only indicate their existence in a micro-region. Evident geographical inconsistencies in coalitions can be explained by the fact, that the considered micro-regions already represent aggregated smaller cities. Moreover, the planning of the waste collection is not taken into account in the model, which might promote cooperation between distant micro-regions.

Clearly, the proposed algorithm must be further improved to provide precise instructions in case of possible irregularities. A more comprehensive study of the development of average coalition size depending on penalization is also needed. Moreover, other “uniformity” metrics such as geometric or harmonic mean might be worth considering. The merging of pairs of coalitions remains the main disadvantage of the current implementation, but it is necessary to mitigate the risk of combinatorial explosion. When working with smaller player sets, merging three or more coalitions into one could also be considered.

The stable outcome. From Figure 2.4, it can be seen, that the resulting coalitions are not spatially consistent: cooperation of distant micro-regions can be profitable. Thus, formation of municipal unions cannot be solved solely intuitively based on geographical vicinity between subjects, as it is usually done in practice. The resulting coalitions have showed that micro-regions, where WtE facilities are situated, tend to be major players of their coalitions, around which other players are gathering. Due to assumed zero transportation costs, these “centers” tend to reserve capacities of their WtE plants to other participants of corresponding coalitions. While the greatest coalition consists of three such “centers” (“Brno”, “Přerov”, and “Zlín”), which explains its greater size, another coalition has occurred around “Otrokovice” with a WtE plant of large capacity and competitive gate fee for its region. The last coalition has been created around “Valašské Meziříčí” with a WtE plant, which, though of a relatively small capacity and higher gate fee, still provides possibility to achieve smaller waste treatment costs for local micro-regions. However, the existence of a WtE plant within a micro-region does not always guarantee that such micro-regions will attract others. For example, “Hodonín” micro-region, which has its own WtE facility, does not serve as a gathering “center” for any coalition. This fact can be explained by the fact that “Hodonín” is situated close to “Brno”, but its WtE plant is uncompetitive compared to “Brno” WtE plant. It can be concluded that obtained results lack irrationalities and the presented approach has potential in research on this topic. The case study results validate the proposed method and indicate, that the developed approach can be applied to locations with analogous demographical conditions.

The proposed distributions of waste treatment costs. The Shapley value has been chosen as a fair method of a total waste treatment cost distribution between micro-regions. Three possible scenarios have been considered to provide a better image about the role of cooperation in the presented problem. These scenarios are the following: I. fully non-cooperative case, II. fully cooperative case, III. stable outcome with non-cooperating outsiders (three proposed coalitions are considered and remaining micro-regions do not cooperate). For the sake of better comparison, all scenarios have been computed using the original function v. The suggested point of the core $C(N, v)$ has been calculated only for the fully cooperative case. The proposed costs distributions are presented in «Shapley values.docx» of Appendix. The results for the I. scenario are represented by total waste treatment costs per ton of waste. The results of scenarios II. and III. and the proposed core distribution are represented by percentual savings compared to the I. scenario. The sampling method has been employed to estimate the Shapley value of coalitions with cardinality greater than 7, where the sample size has been set to 10,000.

At first, it is necessary to emphasize that estimates of Shapley value in II. and III. scenarios are smaller than $v(\{i\})$ values of I. scenario. Thus, under both scenarios players were able to prosper from cooperation. Expectedly, micro-regions in which WtE plants are situated play a major role in their coalitions. This fact has also manifested itself through the suggested costs distributions. Mainly, micro-regions with production, which is smaller than capacity of their local WtE plant, can achieve substantial savings through cooperation. Other micro-regions in these coalitions, can also save considerable amount of money, especially if their waste production is high with respect to their geographical area. It can be concluded that micro-regions in which waste treatment facilities are situated and micro-regions with locally above-average waste generation should be maximally interested in cooperation and initiate the creation of municipal unions in order to substantially lower their waste treatment costs. While pursuing their own wealth, they can also reduce the financial impact of legal changes on the other micro-regions. As expected, global cooperation, corresponding to the II. scenario, is the most profitable outcome for everyone. According to the performed estimate, in case of global cooperation, all players can lower their total waste treatment costs. While the III. scenario represents an opportunity to lower waste treatment costs for members of the previously described coalitions, it should be noted, that it cannot offer such substantial savings that can be achieved through the II. scenario. It should be concluded that decision of the micro-regions to cooperate is based on all considered factors. Waste productions and locations play an equally eminent role in the process of coalition formation. The «attractiveness» of a micro-region in coalition formation is not guaranteed exclusively by existence of a nearby WtE or large waste production, rather it is a combination of both factors. There is obviously no intention to cooperate with micro-regions with small waste productions, since they cannot offer any benefits to their partners. Then after passing a certain threshold, where waste production becomes sufficient with respect to location of a microregion, attractiveness of the micro-region begins to grow. Due to the clear implication, that some micro-regions might play fundamental role in their coalitions, currently widely applied policy of equal waste treatment tariffs in municipal unions should be revised.

The proposed core distribution demonstrates that, in case of full cooperation, some waste producers are able to achieve enormous savings. They can save twice more than under the distribution proposed by Shapley value for fully cooperative scenario II. The large differences between the Shapley value and core point indicate that Shapley might not be stable distribution. However, it distributes costs in a more fair, uniform way. Indeed, in some cases stable core distribution provide savings comparable to III. scenario or does not provide any savings at all.

Chapter 3

Conclusion

In the Ph.D. thesis, application of GT approaches to problems of WtE treament of non-recyclable waste in WM networks has been demonstrated. The work has provided theoretical insight into domain of NGT and CGT. The latter branch has been discussed with respect to class of canonical coalitional games and coalition formation games. The performed review has enabled us to establish existing research gaps. These gaps have highlighted the contribution of this thesis. In particular, the autor's original research has been aimed at two types of games.

The WtE plants' price-setting problem has been thoroughly studied from two perspectives: setting the optimal prices for one WtE plant and the search for NE between WtE plants. The problem has been defined as a normal-form game of WtE plants, with gate fee as their strategies. Such a game has peculiar properties, wherein maximizing a player's payoff leads to a bilevel programming problem between one WtE plant and waste producers. However, these instances of bilevel optimization cannot be solved in polynomial time. After the extensive investigation of the bilevel optimization methods, the novel heuristic approach to solve the considered bilevel problem has been proposed. The approach considers that a simple iterative update of the lower-level linear problem solution provides sufficiently reliable estimates of waste flows, concerning which the optimization on the upper level is performed. Algorithm performance has been validated via testing and exemplary case study: it has been shown that it provides fast solutions to the considered problem and produces optimal solutions in approximately 60% of artificial scenarios and in nearly 85% of realistic cases. The research has also filled the gap in the current game-theoretic literature since the solution of the NP-hard optimization problem is only an instrument to find the NE in the WtE plants' network. Combined with the BRD algorithm, the heuristic enabled the search for NE under the assumption of continuous strategy sets. This approach should provide more realistic insight into the reaction of other WtE plants to changes in gate fees. Thus, the estimate of optimal waste flows and gate fees in the WM network provides more reliable input to decision-makers. The proposed method can be potentially applied to assess the feasibility of the investments in new WtE plants. In particular, the exemplary problem motivated by the Czech Republic data demonstrated how the approach could be applied in practice to design the capacity of the WtE plant. The optimal capacity of the facility, which is being planned in one of the regions, was proposed with respect to the analogous projects and actual waste production in the Czech Republic. The found stable gate fee outcomes exhibit economically reasonable behavior of waste treatment market participants, verifying that the developed tool can be used to simulate the market environment for the WtE facility. While solving the exemplary problem, the hypothesis about the non-existence of the NE in the considered game has been proposed. The existence of the NE has been studied for the whole class of the originally introduced price-setting games. Since the classical NE concept does not exist for the pessimistic setting, the author has proposed the modified concept of δ-NE. Existence of the δ-NE under different assumptions put on capacities and transportation costs has been studied.

The waste producer's cost reduction game has been defined to suggested the most suitable municipal unions for adaptation to new waste treatment legislative. The strong connection between the studied theoretical concepts and the real-world waste treatment problem has been showed. The cohesivity and balancedness of the studied class of games has been proven. Moreover, the easily verifiable necessary and sufficient condition of additivity has been established. The practical implications of the game properties has been discussed. The related research has provided concepts and instruments to study the formation of coalitions and distribution of costs for general TU-game with numerous players. The proposed method handles distributed coalition formation via merge and split rules under utilitarian order relation. In order to reasonably implement merge and split rules into the considered game, a cooperation costs model has been introduced. It has helped to achieve a more realistic outcome, which considers the possible suboptimality of the grand coalition and nonlinearly growing costs for creating a sustainable coalition of large number of players. The penalization percent has been used as the main instrument through which uniform coalition structure can be obtained and computational complexity can be retained at the desired level. The distribution of costs for the resulting coalition structure has been suggested on the basis of sampling Shapley value and the point of the core. Real WM data for the Czech Republic and distributed coalition formation between 47 micro-regions have been analyzed. After the application of the presented method, slightly less than half of micro-regions were engaged into some coalition under resulting coalition structure and their saves were varying from around 2% up to 8% compared to non-cooperative case. The estimated costs have provided an insight into how cooperation might affect the municipal budgets under transition from landfilling to WtE technology. The resulting coalitions can be viewed as a potential suggestion of which municipal unions should be formed. The case study data revealed that micro-regions possessing their own WtE infrastructure can substantially lower their total waste treatment costs via renouncing the capacities to other participants of the coalition. Brief sensitivity analysis has been performed, to assess impact of changes in waste production of the micro-regions (being the main source of the model variability) on the resulting costs of municipalities. The results demonstrated, that, when it is profitable for a municipality to cooperate, it tends to do so in majority of scenarios. Regarding the future research, we establish four possible directions:

- there is an opportunity to embed reconsideration of the waste flows with respect to capacities constraint into the heuristics from section 2.1 to improve the performance of the method;
- the detailed study of the possible generalization of Theorem 2.1.2 for arbitrary price-setting game;
- the estimation of the nucleolus for the waste producer's cost reduction game
- an embedment of waste collection within the established municipal unions into the waste producer's cost reduction game.

To summarize the whole work:

- the new price-setting approach, combining bilevel optimization techniques and GT, should help to ensure efficient and financially sustainable waste energy recovery;
- the presented coalition formation approach has a potential to serve as a basis for design of tariffs for different public services or for design of unions in arbitrary cost minimization problem, where cooperation between subjects is possible.

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