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Bachelor Thesis

Selection of a rock polishing machine using MCDM

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Faculty of Economics and Management

BACHELOR THESIS ASSIGNMENT

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Thesis title

Selection of a rock polishing machine using MCDM

Objectives of thesis

The main objective of the thesis is to determine best rock polishing machine based on criteria provided by Yakutian Art School using established multi-criteria analysis methods.

Methodology

There will be two parts introduced first by the theoretical and then followed by the practical section. The theoretical part is devoted to multi-criteria analysis, its history and uses, and methods of choice and selection. The practical part describes the implementation of MCDM and selection process of the best rock polishing machine, and the selection of criteria, details of the machine, methodology of measurements, details of the process, and final selection will all be found in this dedicated part.

The proposed extent of the thesis

30-40

Keywords

Multi-criteria analysis; decision-making; weighted sum approach;

Recommended information sources

Belton, V. and Stewart, T., 2002. Multiple criteria decision analysis : an integrated approach. Boston: Kluwer.

Dodgson, J.S.; Spackman, M.; Pearman, A.; Phillips, L.D.Multi-Criteria Analysis: A Manual; Department for Communities and Local Government: London, UK, 2009; Volume 11

Köksalan, M., Wallenius, J. and Zionts, S., 2011. Multiple criteria decision making: from early history to the 21st century. Hackensack, N.J.: World scientific.

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Declaration

I declare that I have worked on my bachelor thesis titled " Selection of a rock polishing machine using MCDM" by myself and I have used only the sources mentioned at the end of the thesis. As the author of the bachelor thesis, I declare that the thesis does not break copyrights of any their person.

In Prague on 14.03.2021 ____________________________

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I would like to thank Ing. Robert Hlavatý, Ph.D. from the bottom of my heart for his valuable advice and support during my work on this thesis.

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Selection of a rock polishing machine using MCDM

Abstract

This bachelor thesis deals with the selection of a rock polishing machine for the Yakutian Art School. The school currently has rock polishing machines that do not fit the current requirements because they are dated and old.

In the beginning of the theoretical part of this thesis, the issue of the decision-making process will be outlined. The model of multiple criteria decision analysis (MCDA) will be described and its elements will be specified. This section explains the method of determining the weights of the criteria and the method for selecting a suitable, efficient alternative using multiple criteria decision-making (MCDM).

The practical part of this thesis presents the decision-making problem the Yakutian Art School faces and its specifics. All following elements of this model are specified and described.

The pairwise comparison method of Saaty is used to determine criteria weight through the preferences of the decision maker of this process. After the modelling, the concept of dominance is used to reduce the set of alternatives. The calculation leads to the selection of an efficient alternative which is done through a weighted sum approach. Based on all calculations, the option with the greatest utility for the Art School is selected and recommended to the Yakutian Art School. Finally, a sensitivity analysis is performed to test the ordering of alternatives based on the change of the price criterion. The price of one of the alternatives changes when given a discount from the seller and it is considered prior to the final solution of this thesis.

Keywords: weighted sum approach, sensitivity analysis, consistency, decision-making process, pairwise comparison method, MCDM, rock polishing machine, criteria weight, efficient alternative

Vícekriteriální rozhodování k výběru stroje na leštění hornin

Souhrn

Tato bakalářská práce se zabývá výběrem stroje na leštění hornin pro Jakutskou uměleckou školu. Škola má v současné době lešticí stroje, které neodpovídají současným požadavkům, protože jsou zastaralé.

Na začátku teoretické části této práce bude nastíněna problematika rozhodovacího procesu. Bude popsán model vícekriteriální analýzy variant (MCDA) a budou specifikovány jeho prvky. Tato část popisuje metodu určování vah kritérií a metodu pro výběr vhodné a eficientní alternativy pomocí rozhodování na základě více kritérií (MCDM).

Praktická část této práce představuje problém rozhodování, který řeší jakutská umělecká škola, a jeho specifika.

K určení vah kritérií prostřednictvím preferencí rozhodovatele je použita Saatyho metoda. Po určení vah kritérií následuje redukce množiny alternativ na základě konceptu dominance alternativ. Následný výpočet vede k výběru kompromisní alternativy, který je proveden pomocí metody váženého součtu. Na základě výpočtu je vybrána možnost s největším užitkem pro uměleckou školu a doporučena jakutské umělecké škole. Nakonec je provedena analýza citlivosti, která testuje, jakým způsobem váha kritéria cena ovlivňuje výsledné pořadí alternativ. Cena jedné z alternativ se mění, když je poskytnuta sleva od prodejce a tento scénář je v praktické části také popsán.

Klíčová slova: metoda váženého součtu, analýza citlivosti, konzistence, rozhodovací proces, metoda párového porovnání, MCDM, stroj na leštění hornin, váha kritérií, kompromisní varianta

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1. Introduction

When a person studies, they make the decision to do this. If they make coffee, it was their choice. The clothes they wear, once more options of how to act present themselves. There are multiple levels of criteria that a person can go through in order to arrive at their final selection. Color, size, time, day or night, warm or cold, opinion and bias, the opinion of others, the image of the final state within the imagination, the effort needed, the consequences, the necessity or lack of, the payoff in the end, and past experience: these are things that influence a number of choices. In business this is the same. Well, perhaps the factors of a choice are different, but a decision-making process is present nonetheless. “Decisions are often, as in our data, the focal point of organizational meetings. If decisions are defined as outcomes, however, they may not actually be produced in the meetings at all and, even if they are, objectively identifying them may be difficult” (Bolander and Sandberg, 2013). The things that must be decided prior to opening a business, during operation hours, outside of operation hours, long term, short term, suddenly, and when an obstacle is met are some of the most important moments in a business. These choices effect the productivity of workers and can either harm or help efficiency in a workspace. Managerial Decision-Making(MDM) involves operating a team of people in order to find the most fluid and constructive methods of satisfying a business’s needs. MDM “comprises nothing more than calculating the output of… normative models” (Rode, 1997). Employee decision making “is considered key for organizations,” and it can range from understanding the hierarchy in a business to understanding all parts of a required job and where that job might end and begin (Bolander and Sandberg, 2013). Corporate decision making involves understanding the interconnected mechanisms that are a corporate chain and how pulling or pushing on each part might affect another. “For decision-making purposes, the corporation is said to consist of a board of directors, which manages-the corporation's business; officers, who, as agents of the board, execute its bidding; and shareholders, who elect the board and determine "major corporate actions," or "fundamental" or "extraordinary" or "unusual" changes” (Eisenberg, 1969). Consumers weigh as to whether or not a purchase is amenable, viable, useful, or if it satisfies demand.

To make the right choice and ensure maximum satisfaction from available products in the market, consumers, as thinking actors, guide themselves via a decision-making process. This process consists of many steps and includes internal and external factors that influence the final decision. The basic steps that take place in the consumer decision-making process are need recognition, pre-purchase search, evaluation of alternatives, purchase decision, and post-purchase behavior (Kumra, 2007).

The criteria for making a decision is vast but ultimately it is something that can be understood by looking from the point of view of a person who is making a decision.

2. Objectives and Methodology

2.1 Objectives

The main objective of the thesis is to determine the best rock polishing machine for the Yakutian Art School using established multi-criteria analysis methods.

This thesis is analyzing different types of rock polishing machines compared to established criteria made by a decision maker from the Yakutian Art School.

2.2 Methodology

This thesis has two parts introduced first by the theoretical and then followed by the practical section. The theoretical part is based on scientific literature. This part is devoted to multi-criteria analysis, its history and uses, description on its elements, and methods of choice and selection that will be used in the practical part.

The practical part starts with the description of the profile and the problem the Yakutian Art School faces. It continues with the establishment of the criteria and sets of alternatives of this process. When the main variables (criteria, alternatives) of the process are determined, it proceeds with the calculation of the criteria weights by using the pairwise comparison method. This method compares the criteria to create a ratio matrix of the criteria preferences which shows the weight of each criterion, the sum of which is 1.

After the criteria weights determination, this paper moves to the consistency test to make sure that the matrix doesn't have contradictions in the criteria preferences. Beyond this, the test of dominance is used to reduce alternatives which are inferior to dominated alternatives.

The Weighted Sum Approach is used to determine the general utility of alternatives, the ordering of which from most efficient to the least preferred being based on the approach.

In the end, the change of the ordering of alternatives will be tested through the change of the value (weight) of the price criterion using sensitivity analysis. This analysis will show how the outcome of the weighted sum approach will change if the School has a big budget (low weight on criterion price) and when the School's budget is low (bigger weight on criterion price).

In addition, the new price of one of the alternatives changes with a generous discount offer from the supplier, which will come into consideration before the announcement of the final recommendation for the purchase that should made by the Yakutian Art School.

3. Literature review

This thesis will discuss the difficulties and problems of the decision-making processes. First, decision-making processes will be characterized in general and then they will be defined various methods solving decision - making processes.

3.1 Decision-making process

According to Eisenfuhr (2011) decision-making is the process of making a choice from a number of alternatives to achieve a desired outcome.

“Decision making is the study of identifying and choosing alternatives based on the values and preferences of the decision maker. Making a decision implies that there are alternative choices to be considered, and in such a case we want not only to identify as many of these alternatives as possible but to choose the one that best fits with our goals, objectives, desires, values, and so on.” (Harris, 2012)

3.1.1 Structure of decision-making

The decision-making process was broken by into six parts by (Schoenfeld, 2011).

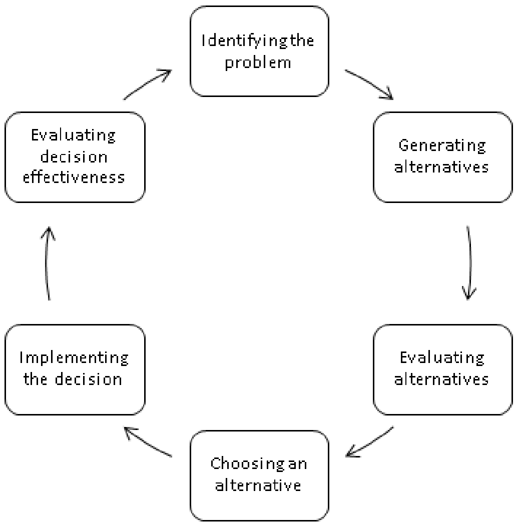


Figure 1 The decision-making process. (Schoenfeld, 2011)

They are:

1. The first step is to identify the problem that will be addressed and the decision that needs to be made. Organizing these based on criteria is fundamental to the future steps taken.

Understanding both the objective the problem addresses and the elements around which the operation takes place are vital.

2. The next step is to generate alternatives. These come from the goals and ideals established by the associated organization. Each alternative should have a wide and detailed amount of data gathered. The more alternatives generated the better multi-criteria decision making will work.

3. At this stage should all the alternatives be evaluated. Are the selections reasonable, satisfactory to demand, and what impact will each selection have upon the operators: these are the questions that should be addressed.

4. At the fourth stage the decision maker or makers need to select an alternative that best fits the criteria and goals established. At this phase the people involved need to rely on experience and knowledge as well as insight into the problem to make their selection.

5. The next stage is to implement the decision which hinges on a lot of factors and could fail without proper action. Communication of the selection to subordinates and proper resources should be made available to enact this stage. Responsibilities of all acting parties in this stage should be made known.

6. Once implemented, the effectiveness of the decision should be evaluated. This important stage could lead to further decisions and help the decision maker in future decisions. Poorly evaluated data or bad information could lead to a new alternative being selected.

3.2 Multiple criteria decision-making

"Multiple Criteria Decision Making (MCDM) or Multiple Criteria Decision Analysis (MCDA) is a sub-discipline of operations research with foundations in economics, management, mathematics, and psychology, and it is concerned with complex decision-making situations involving multiple and conflicting criteria, objectives, or attributes that have to combined or aggregated in order to choose or rank between alternatives." (La Torre, n.d.)

When there are a range of options that are available to solve an issue, in selecting a product, or satisfy a demand then there is a range of comparisons that must be made. Price, quality, make, and model are just some of the words used in order to create a table that shows positives and negatives that each choice brings. Sometimes people go through these in their head, but this will inevitably cause them to overlook certain factors that would otherwise influence their decision and lead to a stronger conclusion.

Multicriteria Decision Making is a more complex way of putting available options together that is made for the purpose of arriving at and implementing decisions. It is an assistant tool that can be used to confront complex decision problems. It helps to eliminate false assumptions and provide a clearer reason for why a final decision is made. (Kahraman, Onar and Oztaysi, 2015)

This is an important process when the decision is being made for a group’s selection process who must weigh all parts of what they are evaluating. An individual too is aided by the creation of a MCDM chart by allowing them not only to take cost into account but also those options that might affect their own efficiency and longevity of what is purchased.

Regardless of who, the person deciding can build their analysis based on criteria that is more obvious to what is at hand or on things which are secondary results. These obvious criteria refers to price, make, and model but the less obvious refers to user friendly or experience based tasks. MCDM has been increasingly used to support healthcare workers as they tackle a range of issues and to help form a final decision that is unique to their field. Broadly speaking MCDA is concerned with those moments where a choice is presented with multiple effective options available. Each is openly categorized, measured, and weighed against each other in order to arrive and the strongest or most fitting option present. (Delvin, 2019)

MCDM is about giving structure to the categories and providing a chart that can be based on importance of data from a wide range so that each selection is weighed to arrive at suitable outcomes. Governments, doctors, engineers, and individuals often have an array of paths that lead them to an action, which in turn leads to an outcome, that involve several influencing factors. (Köksalan and Wallenius, 2014)

These factors might be things such as how an office is going to divide the labor of filing or drafting a business document, assessing which employees have the best skills needed to accomplish the task, and calculating the current work load of the individuals present.

“In the decision - making context, this would imply some sort of standard by which one particular choice or course of action could be judged to be more desirable than another. Consideration of different choices or courses of action becomes a multiple criteria decision making (MCDM) problem when there exist a number of such standards which conflict to a substantial extent." (Belton and Stewart, 2002)

The fact remains that people are given choices when they are presented with any task on how to approach or where to begin. The best choice isn't always obvious nor is it often the first one that is made. Applying theory helps one learn to make the proper choice and can even influence future decisions in the process of an endeavor. There comes a view of what is right and what is wrong with the different paths made available. If there is no right or wrong there can be an easier, a better, or a more reasonable option that stands out based on multiple directions of criteria analysis. Multi-criteria decision-making breaks down and simplifies the choices presented and directly assists in obtaining a proper decision. MCDM is a means, for those who cannot decide, to act. This is a theory of approaching behavior in order to facilitate and make the act of decision making narrower when a broad range is presented

3.2.1 History of MCDM

For as long as man has lived there have been choices made. The choices of the past can be seen in history books. Some of the choices that are made in history have involved people moving quickly into acts that led to disaster, which wouldn't have been had they weighed opportunities against each other. Multi-criteria analysis isn't named until the American statesman Benjamin Franklin used it in making some of his important decisions. Difficulties in the selection of criteria and evaluation of alternatives hindered the growth of MCDM; however, in the 1950's Abraham Charnes and William Cooper pushed the discipline further with the introduction of goal or target programming. In the period following this there was an explosion of material written about multicriteria decision-making. Vilfredo Pareto, an Italian economist, aggregated conflicting criteria into a single index, and he was the first to introduce efficiency into his tests.

In France in the 1960's, Bernard Roy designed the ELECTRE method. This method was used to select the best action of a set of actions as well as ranking and sorting problems (Köksalan, Wallenius, Zionts, 2011).

In the 1970's, R.W. Saaty developed the Analytic Hierarchy Process (AHP) which helped to organize criteria along complexity, measurement, and synthesis. The ease of use and broad application led to wider use of multi-criteria decision making. (Saaty, 1987; Forman, 2001)

In the 1980's, MCDM began to solidify as a study and works using it broadened. Application to geographical information systems (GIS) used MCDM to discuss problems of space and area (Chakhar and Mousseau, 2008).

Since the 1950's the study had begun to take off and by the 90's was generally accepted by a large following. Its application found uses in government programs, institutional research, military, and of course economics. Saaty is likely the one most responsible for this spread in the United States and abroad. His easy to use AHP method brought decision making analysis to a wider audience that had previously been available due to analysis often involving complex criteria and the necessity of expert opinions. Now, from its first mention almost two hundred years prior, the study spread like wildfire.

3.3 Multi-criteria decision analysis

According to (Tecle and Duckstein, 1994), MCDA is a study which includes decision making when there are two or more conflicting objectives or goals, or an analysis process involving more than one attribute that can be compared. The objective of this process is to eliminate false or unfit alternatives and lead to the best of the alternatives presented.

MCDA is an assistant decision maker or makers which are responsible for sifting through data to eliminate data that conflicts. But what are the criteria and how does one arrive at what criteria are important enough to be evaluated? There are multiple methods that help to deal with the wide and varied choices that face people. MCDA methods are different based on the data observed, the original evaluation of the individual making the decision, how weight is given to each criteria, computations, levels of uncertainty of the data, and whether or not there are multiple people who are involved in the decision making process (Pietersen, 2007)

"MCDA is both an approach and a set of techniques, with the goal of providing an overall ordering of options, from the most preferred to the least preferred option. The options may differ in the extent to which they achieve several objectives, and no one option will be obviously best in achieving all objectives. In addition, some conflict or trade-off is usually evident amongst the objectives; options that are more beneficial are also usually more costly, for example. Costs and benefits typically conflict, but so can short-term benefits compared to long-term ones, and risks may be greater for the otherwise more beneficial options." (Dodgson, Spackman, Pearman, Phillips, 2009)

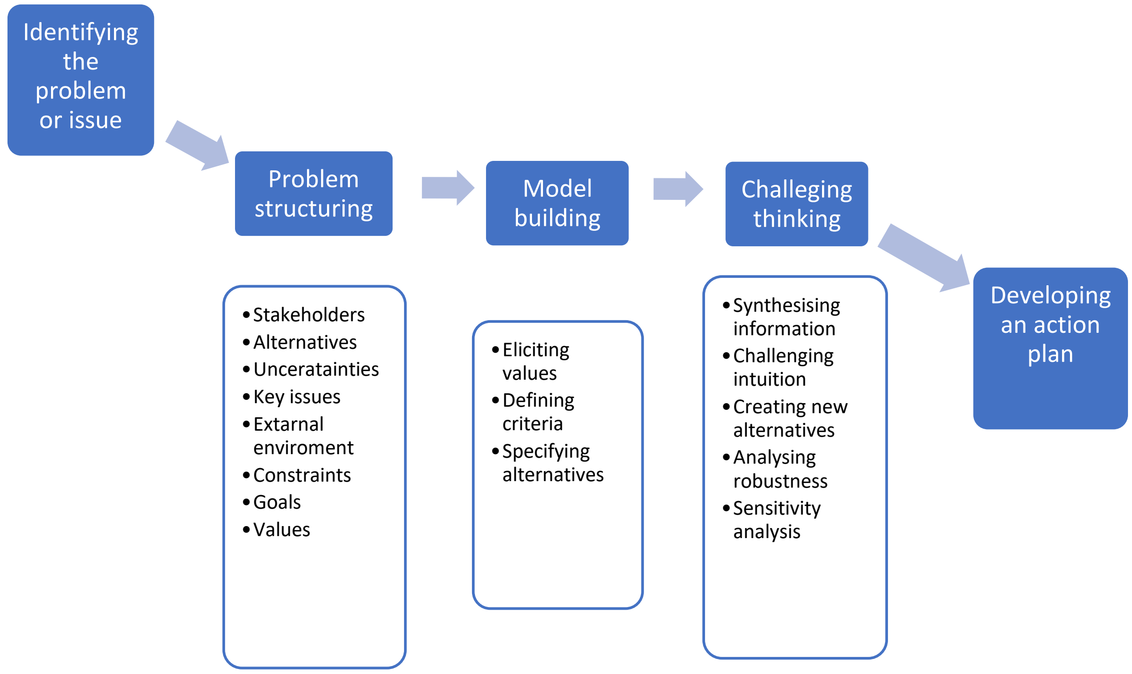


Figure 2 Overview of the MCDM process. (Belton and Stewart, 2002)

"The main steps of multiple criteria decision making are the following:

(a) establishing system evaluation criteria that relate system capabilities to goals;

(b) developing alternative systems for attaining the goals (generating alternatives);

(c) evaluating alternatives in terms of criteria (the values of the criterion functions);

(d) applying a normative multicriteria analysis method;

(e) accepting one alternative as ''optimal'' (preferred);

(f) if the final solution is not accepted, gather new information and go into the next iteration of multi-criteria optimization" (Jahanshahloo, Lotfi and Izadikhah, 2006)

3.4 Multiple Criteria Decision-Making Analysis Modeling

a. A value system must be established that is represented in number values. The scale established is then applied to each category established to produce totals and, based off totals, the strength of an option can be established.

b. A set of goals that are sought and the minimum levels to satisfy the demand based on the number values is created. Which number is highest isn't always the best and the option that best fits the goal is sought.
c. Systems of hierarchy are then built with options set side by side to provide alternatives based on strength. The total strength value is based both on the established goal and the number system chart. (Belton and Stewart, 2002)

3.4.1 Alternatives

Once the options are made known and the possible solutions are established, it is time to look at how each alternative can be broken down. Because there is competition in all fields and there is a variety of options in any choice made, possibly since forever, the necessity of multi-criteria decision-making is made clear. Choices or alternatives are called "permissible" or "possible" due to the nature of existing options and can be represented in the number "X". Due to there being a choice the minimum number available to pick from is 2. The maximum are those things that have been made known in the process of creating a decision. For statistics of large-scale numbers or running a series of possible alternatives based on scenario, the choices are limited to the power of the computational device used.

3.4.2 Dominance

When one option performs as well in other criteria yet performs better in one in comparison, it is said to have dominance over other choices. There are cases where one might be better than all but it is more usual to see options holding dominance to some and not others. When there is dominance, it is wise to see whether or not this satisfies the goal set or if there are hidden factors that might further influence choice. Dominance does not necessarily mean that the option will be selected but that other choices can be eliminated in the decision-making process. (Dodgson, Spackman, Pearman, Phillips, 2009)

Dominance is established through the number value system created and relating these numbers to other alternatives.

"In the absence of any preferential information, the only possible operation on the performance table is to eliminate the dominated alternatives. Let a and b be two alternatives from A and F a family of criteria. The alternative a dominates the alternative b in respect to F, noted $a\Delta b$, if and only if:

$$g_j(a) \geq g_j(b); \; j \in F \qquad (1)$$

with at least one strict inequality. Then, an alternative a from A is said to be efficient or admissible or Pareto optimal if and only if there is no other alternative b in A such that: $b\Delta a$." (Chakhar and Mousseau, 2008).

1. A dominant choice is established as an option that meets the ideal value of at least one of the number values while also not having any choices being worse in comparison to other alternatives
2. A non-dominant choice does not have any one criteria that meets the desired goal but my not be "worse" either. There are other options that will have a better value in one of the criteria

3.4.3 Ideal and negative-ideal alternatives

Ideal alternative

According to (Brožová, Houška and Šubrt, 2003) The ideal alternative is the best possible estimate according to all criteria. It is often a hypothetical as normally no true ideal exists within the criteria. It is good to have an ideal when comparing alternatives to get an idea of quality.

The ideal choice is the one which satisfies the goals established from the number values. It is not often that such exists. Life is usually not ideal and compromises need be made.

Negative-ideal alternative

The negative-ideal alternative is the opposite of the ideal alternative . The values of negative-ideal alternatives hold the worst values in the criteria.

As well as the ideal alternative, this usually does not exist and is a hypothetical used to eliminate choices that might conform the envisioned non-ideal.

3.4.4 Efficient alternative

A efficient alternative is an alternative that is selected from all non-dominated alternatives as a solution to the problem. There are several methods to obtaining the efficient alternative. A sum of values, differences in value scale, or comparing each individual criteria score. (Brožová, Houška and Šubrt, 2003)

“The compromise solution is a feasible solution, which is the closest to the ideal, and a compromise means an agreement established by mutual concessions.” (Opricovic and Tzeng, 2004)

Where there are a large number of non-dominated alternatives, the efficient alternative stands out along the lines established. Other means of choosing the compromise include closeness to the ideal and distance from the negative-ideal.

3.4.5 Criteria

“Every decision we ever take requires the balancing of multiple factors (I.e. “criteria” in the above sense) - sometimes explicitly, sometimes without conscious thought – so that in one sense everyone is well practiced in multicriteria decision making.” (Belton and Stewart, 2002)

“Thus, identifying criteria requires considering the underlying reasons for the organization’s existence, and the core values that the organization serves.” (Dodgson, Spackman, Pearman, Phillips, 2009)

The criteria are divided according to their nature into (Brožová, Houška and Šubrt, 2003):

- Maximizing - Higher value is taken to be a better value. The decision maker prefers higher values to lower values.

- Minimizing - The opposite of the maximizing criterion, and thus the lower the value of the criteria the less preferred.

For some criteria, it is not possible to decide whether they are maximizing or minimizing.

For example, criteria whose values are expressed verbally. According to quantifiability, criteria are divided into (Brožová, Houška and Šubrt, 2003):

- quantitative - The criteria are expressed in a measurable numerical value.

- qualitative – The criteria are not expressed in a measurable value. Using various methods such as the scoring method or the ranking method, these criteria need to be quantified.

Requirements for the criteria can be formulated as follows (Dodgson, Spackman, Pearman, Phillips, 2009):

Completeness - Criteria should reflect all properties of the alternative that are essential and important for the decision maker.

Redundancy - Unimportant or duplicate criteria should be filtered early on to not clutter the decision-making process. Removal of some criteria could be useful building an economy of time and analysis. Removal should be carefully weighed nonetheless.

Operationality – Each option should be able to be evaluated based on all criteria made available. This should be possible regardless of whether the assessments are objective (weight, distance, etc...) or judgmental (expert opinion).

Mutual independence of preferences – Each option must be preferentially independent of others. This means that the preference score on the criterion be able to be assessed without knowing the values of others. If they are not independent then criteria may be able to be combined into a single value or to see if the option meets acceptable performance levels and can be rejected.

Double counting - Most often associated with decisions made in the public sector, double counting comes when groups are evaluating populations in different areas and offering values that represent the same basic criteria. Judgement on whether or not double counting has occurred does not come out of objective observation but understanding what the values counted represent. It is necessary to not allow double counting because this can bias the decision in the end.

Size - Over filling a study with too many criteria slows the analysis process and makes communicating what impacts a decision difficult. Eliminations of criteria that do not have importance or are inconsistent need to be made.

Impacts occurring over time – This has to do with whether or not target completion of processes over time will be completed in the time of a target completion date. Completing on time leads to high scores, slightly behind schedule lower, and far behind the lowest possible score. Understanding the time in which an analysis is taking place is necessary to differentiate between temporary and long-term effects.

3.4.6 Decision maker

The selection process is impossible without the presence of one who makes this choice, pursuing his own goals. A person or an entire team, who make a choice and are fully responsible for its consequences, are called the decision maker (for short: DM). Moreover, in the framework of the problem, only those characteristics of the decision-maker that are involved in its solution, such as experience in this field and psychological features, are important.

"Decision makers usually handle the trade – offs between objectives by evaluating the alternatives under consideration based on the explicit weighting of criteria relevant to the overarching decision – in order to, depending on the application, rank (or prioritize) or choose between the alternatives" (Hansen and Delvin, 2019)

"Effective decision makers are keenly aware of the importance of properly identifying the problem and understanding the problem situation." (Lunenburg, 2010)

3.5 Criteria weights determination method

When deciding, the so-called preference of the criterion is important, which indicates how important this criterion is compared to others. The determination of preferences is very often influenced by a subjective opinion decision-maker, which can thus negatively affect the solution. However, this approach is considerable measures disadvantageous, wisely chosen preferences lead to a very good solution. According to the author it is possible preferences may be expressed in the various ways described below, or may not be known at all. However, some methods do not work with them.

When considering multi-criteria analysis, it is important to understand the set of criteria that represent the points of view which determine selection. Each type of point of view holds different value to the decision maker. Information gathered by the decision maker, how it is represented, and the usage and effects factor into the overall choice. (Solymosi and Dombi, 1986)

Weighing criteria is an important aspect during analysis of alternatives. Stakeholders in a business each have a different point of view in respect to the criteria which makes it very tedious and difficult to establish a shared value system of weighing criteria. Such disagreements might slow down the selection process if left unresolved. The criteria weighing comes in establishing a single system of weights over several selected criteria emerging from different stakeholders or decision makers in a group. "The criteria weighting problem faces four critical challenges:

— Evidence of validity for criteria weights.

— Transparency of DMs' participation in criteria weighting.

— Scalability of criteria weighting.

— Acceptable Cognitive load on DMs.

The criteria weighting technique should be able to address this hierarchy of criteria. The Fourth most important challenge is about the amount of cognitive load that a technique poses on the DM. If majority of DMs find it difficult to use the technique for weighting the criteria, then the technique has less chances to be used in the process. Whereas, if a technique which poses less cognitive load on the DMs, can easily win over others and find acceptability in the approach, even if it provides less accurate results." (Shukla and Auriol, 2013)

Table 1: Classification of weighting methods (Odu, 2019)

Weighting methods		
Subjective weighting methods	**Objective weighting methods**	**Integrated weighted methods**
Point allocation	Entropy method	Multiplication synthesis
Direct rating	Criteria Importance Through Inter-criteria Correlation (CRITIC)	Additive synthesis
Ranking method	Mean weight	Optimal weighting based on sum of squares
Pairwise comparison (AHP)	Standard deviation	Optimal weighting based on relational coefficient of graduation
Ratio method	Statistical variance procedure	
Swing method	Ideal point method	
Delphi method		
Nominal group technique		
Simple Multi-attribute Ranking Technique (SMART)		

Furthermore, only the method that was used in the practical part of this bachelor thesis is described - Pairwise comparison method.

3.5.1 Pairwise comparison method

"The pairwise comparison method was developed by Saaty (1980) in the context of the analytic hierarchy process (AHP). This method involves pairwise comparisons to create a ratio matrix. It takes as an input the pairwise comparisons and produces the relative weight as output. Specifically, the weights are determined by normalizing the eigenvector associated with the maximum eigenvalue of the (reciprocal) ratio matrix." (Malczewski, 1999)

This is used to analyze and compare populations in pairs to see if they are different from one another. It is also used to compare each criterion and determine the level of preferences to keep mutual independence. (Odu, 2019)

Usually for evaluation are used a nine-point scale scheme with values 1, 3, 5, 7, 9, but it is also possible to use intermediate values 2, 4, 6, 8. The individual degrees of preference are listed below and we can also find the rating scale directly in the work of Thomas L. Saaty from 1990.

Table 2: Scales for expressing preferences in the Saaty method

1	equal importance *i* and *j*
3	moderate importance *i* over *j*
5	strong importance *i* over *j*
7	very strong importance *i* over *j*
9	extreme importance *i* over *j*

The evaluation is usually performed by only one expert. Decision maker compares all the pairs of criteria and writes preferences into the Saaty matrix $C = (C_{ij})$.

$$\begin{pmatrix} 1 & C_{12} & \dots & C_{1n} \\ 1/C_{12} & 1 & \dots & C_{2n} \\ \dots & \dots & \dots & \dots \\ 1/C_{1n} & 1/C_{2n} & \dots & 1 \end{pmatrix} \quad (2)$$

Elements of matrix C_{ij} are represented by preference value of i-th criterion against j-th criterion. If the value of i-row and j-column is equal, then this preference is written as C_{ij}=

1. If i-th criterion is weakly preferred over j-th criterion, then C_{ij}= 3, at strong preference of i-th criterion over j-th criterion then C_{ij}= 5, and etc. by the table above. If j-th criterion is preferred over i-th criterion, the value of the C_{ij} will be the inverse value which is C_{ij}= $1/3$, C_{ij}= $1/5$, etc.

The Saaty matrix is always a square matrix $n \ x \ n$, diagonals of which are equal to value 1, because each criterion is equivalent to itself.

According to Šubrt (2011), there are several ways to determine the weights. The most frequently used method is to calculate using a standardized geometric mean of the Saaty matrix.

$$b_i = \sqrt[n]{\prod_{j=1}^{n} c_{ij}} \tag{3}$$

After normalizing averages, weights are calculated by normalizing the b_i value.

$$v_i = \frac{b_i}{\sum_{i=1}^{n} b_i} \tag{4}$$

In most cases, to make the elements of the Saaty matrix perfectly consistent, i.e. that the relationship is $C_{hj} = C_{hi}$ x C_{ij} does apply to all $h, i, j = 1, 2, \dots, n$. A degree of consistency can be measured with the consistency relation (CR). To determine it, it is necessary to calculate the consistency index first (CI), according to formula (5), where l_{max} is the largest eigenvalue of the Saaty's matrix and *n* number of criteria.

$$CI = \frac{l_{max} - n}{n - 1} \tag{5}$$

The acquired value CI, is needed to adjust in the next step. The ratio of the CI to the average RI for a matrix of the same order is called the consistency relation (CR). A CR value less than or equal to 0.1 is considered acceptable (Saaty, 1980). The formula is:

$$CR = \frac{CI}{RI} \tag{6}$$

The random index value (RI) is determined by Saaty (2008) and are contained in the following tables.

Table 3: The random index value RI (part 1)

n	1	2	3	4	5	6	7	8
RI	0	0	0,52	0,89	1,11	1,25	1,35	1,40

Table 4: The random index value RI (part 2)

n	9	10	11	12	13	14	15
RI	1,45	1,49	1,52	1,54	1,56	1,58	1,59

3.6 Method of alternative evaluation

3.6.1 Weighted Sum Approach

The Weighted Sum Approach WSA (also known as simple additive weighing (SAW) method).

The Weighted Sum Approach is the best known and most widely used method of dealing with MADM (Multiple Attribute Decision Making) situations. The basic concept of the WSM is to find the weighted sum of performance ratings on each alternative on all attributes. To apply weighted sum approach to MCDM problems, a fundamental requirement is to aggregate criteria such that an overall decision function is formed (represented as a scalar value). However, this process highlights the need to overcome incommensurability of criteria, due to which different criteria cannot be combined into a single decision function. Therefore, it is necessary to convert all criteria to a unit-less, uniform scale. This is done by normalizing each criterion such that the value of the criterion lie in a 0 – 1 range, where 0 – negative-ideal alternative, 1 – ideal alternative. (Rehman and A., 2017)

Steps of the WSA:

1. Determine ideal H = $(h_1, h_2, \dots h_n)$ and negative-ideal alternatives D = $(d_1, d_2, \dots d_n)$

2. Second step is to calculate the criteria matrix R:

$$r_{ij} = \frac{y_{ij} - d_j}{h_j - d_j} \quad (7)$$

Where

y_{ij} – the actual value of the original matrix

d_j – the negative-ideal alternative for j-th criteria

h_j – the ideal alternative for j-th criteria

3. Last step is to calculate the whole utility function:

$$\sum_{j=1}^{K} w_j r_{ij} \quad (8)$$

Where w_j – weight of the j-th criteria. The Weighted Sum Approach determines the overall benefit of each alternative. For this reason, you can find a compromise option or the alternatives with the highest utility, and organize a set of alternatives from the best to the worst. (Šubrt a kol., 2015)

4. Practical part

The following part of the bachelor thesis will deal with the solution itself by using multi-criteria decision analysis, the main goal of which is to select a suitable rock polishing machine for the Yakutian Art School. There are several choices that offer the service and each will be measured under criteria that is established through direct communication with members of the Yakutian Art School.

4.1 School profile

The Yakutian Art School was established in 1945. In 45', after the war, Stalin gave the order to create and implement this school. The school is free to enrolled students who met score requirements, but it does require payment from students who didn't accomplish the final state exam with enough points to get in.

At the moment there are 4 main faculties: design, art, decorative applied art and ethno cultural works, and the faculty of restoration. All faculties require special equipment for their work. The rock polishing machine is needed in the decorative applied art and ethno cultural work department.

4.2 School's problem

Currently the rock polishing machines do not fit the current requirements because they are dated and old. The school has decided to acquire new machines that will perform better and have more output. The amount needed is 2.

4.3 Requirements

As mentioned above, the Yakutian Art School has the decorative applied art faculty which is in need of new polishing machines. It has specified requirements for a new machine which will fulfil the requirements of the school. From the following requirements, the decision maker established criteria which will help in the following multiple criteria decision – making process. In the following chapters the criteria will be discussed and explained, the primary goal of which is to find the machine that will fulfil the school's requirements.

The main reason of the need of polishing machine is to keep up with the state and health standards of Russia. The new standard for machines is to have an exhaust fan which will protect the working area from the dust and small pieces of polishing materials.

New machines should fit the size of less than the width of 610 mm and the length of 1500 mm. These measures are required for machines to be placed in the selected classroom.

The power of the engine and the power of the exhaust fan are required to be large. The principle of the increase in power is a larger output.

To keep the classroom clean and fully fulfil the health standards, the machine should have the protection screen in front of the worker. Ideally, the protection screen should cover the full working area to prevent it from the leak of the dust to the classroom.

The two polishing motors are preferable over one polishing motor in the machine. Two spindles give two working places for two people; also, it can help one worker to work faster with 2 motors rather than with only 1 as he would need different sized nozzles at his work at the same time.

After making all criteria clear, the decision maker made a list of preferred machines:

- SO-TEC 3 (NEW)
- SO-TEC STANDART
- SO-TEC DELUXE
- VSH Single
- SO-TEC 4
- SO-TEC 2
- SO-TEC 4/2

4.4 Alternatives

Below will be described all alternatives that were chosen by the decision maker. All alternatives will be marked as (a_1 - a_7), the new labels will be in the following tables.

4.4.1 SO-TEC 3 (NEW) (a_1)

This rock polishing machine with two working places is used for the hand polishing of jewelry. The machine has a built-in dust collector with a power of 1,1 kw. The dust is going through the process of filtration and stays in filter bags. The power of the engine is

750 w. The rotation speed is 2800 turns per minute. The machine has a front protection to save the worker’s vision from dust and small pieces of rock.

Figure 3: SO-TEC 3 (NEW) (a_1); source: ruta.ru

4.4.2 SO-TEC STANDART (a_2)

This model is intendent for large and small jewelry production. It has two working places. The size of the machine is lowered but the function remains the same, these changes will make it fit in a small working areas and keep productivity high. The power of the engine is 1000 w. The power of the exhaust fan is 1100 w. with a rotation speed of 2800 turns per minute. The weight is 90 kilograms.

Figure 4: SO-TEC STANDART (a_2); source: sapphire.ru

4.4.3 VSC Single (a_3)

The polishing machine VSC has a built-in exhaust fan to collect dust. The filtered dust stays in separated filter bags for each working place. The desk is made from high-quality wood and covered with a highly protective plastic cover. The polishing motor has an electric speed controller. The power of the engine is 1,1 kw. The weight is 147 kilogram.

Figure 5: VSC Single (a_3); source: ruta.ru

4.4.4 SO-TEC DELUXE (a_4)

This model is intendent for large and small jewelry production. It has two working places for operative work. The power of the engine is 1000 w. The power of the exhaust fan is 1100 w. The dust is going through the process of filtration and stays in filter bags. The weight is 140 kilograms.

Figure 6: SO-TEC DELUXE (a_4); source: sapphire.ru

4.4.5 SO-TEC 4 (NEW) (a_5)

This polishing machine is equipped with two working stations. The machine is made for hand polishing of items from different materials: rock, metal, or wood. The polishing machine has a built-in exhaust fan with a power of 11000 w. The filtered dust is collected in two separated filter bags (one for each working place).

Figure 7: SO-TEC 4 (NEW) (a_5); source: ruta.ru

4.4.6 SO-TEC 2 (NEW) (a_6)

This polishing machine is equipped with two places of work too. Each has a filter bag for dust filtered through the exhaust fan. The weight is 90 kilograms. The power of the engine is 750 w. The power of the exhaust fan is 550 w.

Figure 8: SO-TEC 2 (NEW) (a_6); source: ruta.ru

4.4.7 SO-TEC 4/2 (NEW) (a_7)

This machine has two working places. The working place is isolated and fully covered to protect the worker's vision from any flying material and the area around from dust. The power of the engine is 745 w. The power of the exhaust fan is 1,1 Kw. It has a rotation speed of 2800 turns per minute. The weight is 140 kilograms.

Figure 9: SO-TEC 4/2 (NEW) (a_7); source: ruta.ru

4.4.8 CHS-62FL (a_8)

This polishing machine comes with an exhaust fan and filter bags to collect the dust. The polishing area is fully isolated. The power of the engine is 370 watt and the power of the exhaust fan is 550 watt. The machine has only one working place.

Figure 10: CHS-62FL (a_8); source: sapphire.ru

4.4.9 EC Single (a_9)

This polishing machine from EC series comes with one working space. The machine has an exhaust fan with a few separated filter bags. The desk is made of high-quality wood and covered with a highly protective transparent plastic cover on the worker's side.

Figure 11: EC Single (a_9); source: ruta.ru

4.5 Criteria

In the previous chapter we discussed the requirements from the Yakutian Art School. Based on that a decision maker made the criteria list which will be used in the decision-making process in choosing the rock polishing machine. All criteria will be marked as (C_1-C_7) and will wear the new mark in following tables.

4.5.1 Price (C_1)

The principle of this decision-making process is to minimize the cost. The price will be shown in Russian rubles (RUB).

4.5.2 Dust Collector (exhaust fan) (C_2)

A dust collector is needed to stop the dust from filling the room and creating an unhealthy environment in the classroom. This criterion was established by health inspectors. It will keep the air clean from dust created in the polishing process.

4.5.3 Two Work-Stations (C_3)

This will be useful for more output. One person can set multiple modes on the machines and polish under different specifications, or two students can work simultaneously which will save time.

4.5.4 Enclosed work area (full protection) (C_4)

In order to keep the rocks from damaging the user and to keep a safe work environment the use of an enclosed polisher is necessary.

4.5.5 Vacuum engine power (C_5)

The higher power output the better and faster the engine will work to clean the air and maintain a healthy work environment.

4.5.6 Polishing engine power (C_6)

This has to do with how many times the polishing machine tumbles the rocks and the speed at which the polishing occurs. The greater the power the better.

4.5.7 Compact size (C_7)

The size is important because the use of the machine will at times be done by one person or people who are not used to working with large equipment; also, it must fit in a space in the classroom without taking up workspace. Specifications of a width of 610 and a length of 1500 mm are given.

4.6 Decision maker

The person who is taking the role of the decision maker of this process is a technician, restorer, and a lecturer of the decorative applied art and ethno cultural works of the Yakutian Art School. The decision maker is a professional jewelry maker and the winner of multiple ice, wood, and snow carving competitions worldwide. As he firsthand knows about the field, he will help to find the most suitable rock polishing machine for the Art School.

4.7 Criteria matrix

In order to achieve the greatest possible clarity in the decision-making process, it is appropriate to organize the obtained data into a table. In the case of this decision-making processes, criteria (in the table named as C_1-C_7) are filled in the criteria matrix which contains a list of possible alternatives (in the table named as $a_1 – a_9$) of solutions. Criteria were chosen by the decision maker. A list of all possible alternatives and a list of all decision criteria have already been given in the previous chapters, namely Section 4.3 Requirements for the rock polishing machine and Section 4.5 Criteria.

Alternatives and their values are shown in the table below.

Table 5: Criteria matrix

	c_1	c_2	c_3	c_4	c_5	c_6	c_7
a_1	180 000	yes	yes	no	1100	750	yes
a_2	157 560	yes	yes	no	1100	1000	yes
a_3	720 000	yes	no	no	1100	1000	no
a_4	196 305	yes	yes	no	1100	1100	no
a_5	202 000	yes	yes	no	1100	1000	no
a_6	132 000	yes	yes	no	550	750	yes
a_7	274 850	yes	yes	yes	1100	745	no
a_8	78 540	yes	no	yes	370	550	yes
a_9	330 000	yes	no	no	370	370	yes

From the table above, the **c_2 – dust collector** is part of all polishing machines on this list. As was mentioned in section 4.5.2, the availability of the exhaust fan is necessary to fulfil the health state conditions. This criterion doesn't affect the decision-making process because of the constant value in all alternatives of the process. The **c_2 – dust collector** will be eliminated from the table because it doesn't change the value. The word-described criteria **c_2, c_3, c_4, c_7** will be transformed into binary form: yes – 1, no – 0.

4.8 Dominance

Before starting the main calculations, it is preferable to identify the dominant choice if it is clearly visible at the first look.

Table 6: Test of dominance

	c_1	c_3	c_4	c_5	c_6	c_7
a_1	180 000	1	0	1100	750	1
a_2	157 560	1	0	1100	1000	1
a_3	720 000	0	0	1100	1000	0
a_4	196 305	1	0	1100	1100	0
a_5	202 000	1	0	1100	1000	0
a_6	132 000	1	0	550	750	1
a_7	274 850	1	1	1100	745	0
a_8	78 540	0	1	370	550	1
a_9	330 000	0	0	370	370	1
FORMAT	MIN	MAX	MAX	MAX	MAX	MAX

The table 6 shows that machine $\mathbf{a_2}$ is dominating over polishing machine $\mathbf{a_5}$. The price and the size values are better in the alternative $\mathbf{a_2}$ than in alternative $\mathbf{a_5}$. Other criteria are equal. The alternative $\mathbf{a_2}$ is also dominating over $\mathbf{a_3}$ with a better value of criteria: price and two work-stations, and size. It is also dominating over $\mathbf{a_1}$ with greater values of criteria: price and engine power.

The alternative $\mathbf{a_8}$ is dominating over $\mathbf{a_9}$ with criteria: price, full protection, and the engine power.

The alternatives $\mathbf{a_1, a_3, a_5, a_9}$ will be excluded from the table and will not participate in following calculations.

Table 7: After the test of dominance

	c_1	c_3	c_4	c_5	c_6	c_7
a_2	157 560	1	0	1100	1000	1
a_4	196 305	1	0	1100	1100	0
a_6	132 000	1	0	550	750	1
a_7	274 850	1	1	1100	745	0
a_8	78 540	0	1	370	550	1
FORMAT	MIN	MAX	MAX	MAX	MAX	MAX

4.9 Pairwise comparison method

The decision maker made his preferences of criteria.

The following table will show his decision using table 2. from section 3.5.1 which will be put into the Saaty matrix below.

Table 8: Saaty matrix

	C_1	C_3	C_4	C_5	C_6	C_7	b_1	v_1
C_1	1	0,2	7	3	1	7	1,757	0,225
C_3	5	1	3	1	1	5	2,054	0,263
C_4	0,143	0,333	1	0,2	0,143	0,143	0,241	0,031
C_5	0,333	1	5	1	1	5	1,424	0,182
C_6	1	1	7	1	1	7	1,913	0,245
C_7	0,143	0,2	7	0,2	0,143	1	0,423	0,054
						$\sum$	7,812	1

After weight determination, it is important to prove the consistency of the matrix.
The consistency index should be less than 0,1. Only then will the matrix be acceptable.

The CR using formula (5) and (6) of section 3.5.1 is equal to 0,21 and unfortunately this value is not acceptable. The CR value 0,21 shows that there are contradictions between criteria evaluations.

I asked the decision maker to reconsider and adjust his criteria preferences in the Saaty matrix. The new matrix looks like this:

Table 9: The new Saaty matrix

	C_1	C_3	C_4	C_5	C_6	C_7	b_1	v_1
C_1	1	0,33	5	1	0,33	7	1,254	0,145
C_3	3	1	7	3	1	9	2,877	0,331
C_4	0,2	0,143	1	0,2	0,143	0,143	0,221	0,025
C_5	1	0,333	5	1	0,333	5	1,185	0,137
C_6	3	1	7	3	1	7	2,76	0,318
C_7	0,143	0,111	7	0,2	0,143	1	0,383	0,044
						∑	8,68	1

After the reconsideration of the matrix evaluation the new values are (formulas (5) and (6)):

$$l_{max} = 6{,}73$$

$$CI = \frac{6{,}73 - 6}{6 - 1} = 0{,}146$$

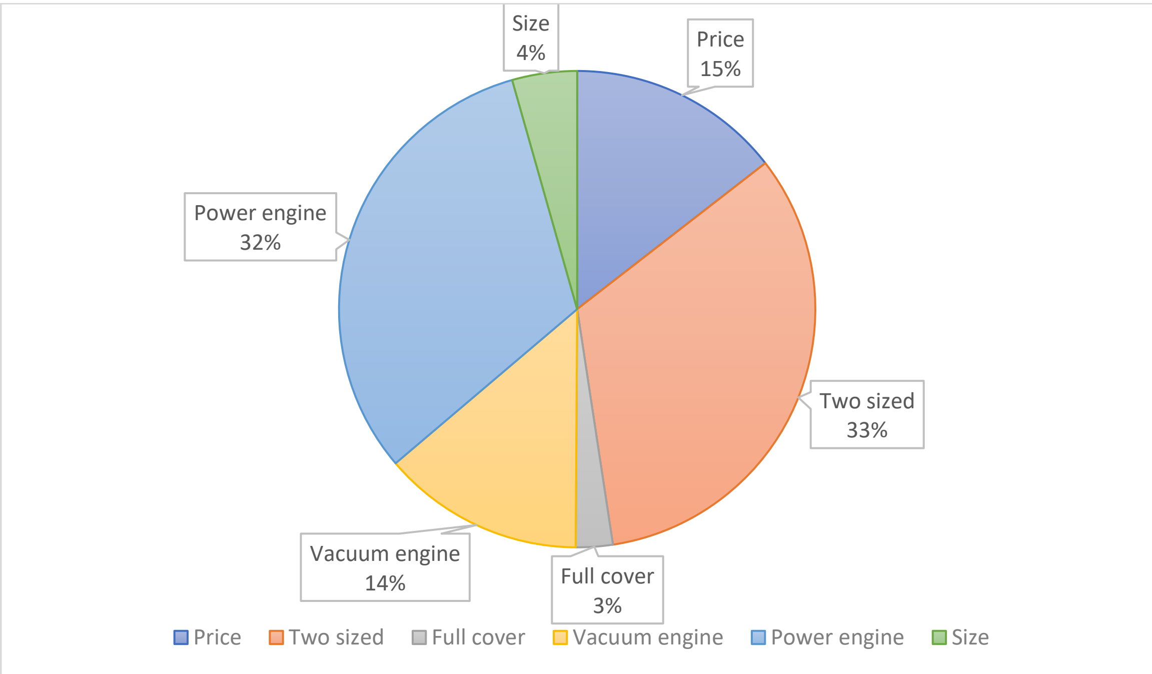
$$CR = \frac{0{,}146}{1{,}25} = 0{,}11$$

The new value has a consistency of 0,11 of the matrix which is not consistent according to the textbook value (<0,1) but it is an acceptable improvement compared to the previous version of table 8.

From table 9, the weight of each criteria is identified. The biggest value among all criteria is an availability of two work-stations on the polishing machine (33,1%). The next is polishing engine power (31,8%) and then price (14,5%).

The lowest criterion in importance is the enclosed working area (2,5%).

All weights are shown in the pie chart for a visual understanding.



Graph 1: Pie chart of criteria weights

4.10 Weighted Sum Approach

After the criteria weight determination and the elimination of alternatives using the dominance test, it is time to find the best alternative applying the Weighted Sum Approach. This method will find the efficient alternative which will give the biggest utility to the Art School.

The decision maker can get the general score of each alternative by multiplying each normalized value of the alternative by each weight of the criteria and summing up these values for all alternatives.

For the following calculations the scale criteria obtained by Pairwise comparison method will be used (Table 9).

Table 10: Matrix for applying a method

	c_1	c_3	c_4	c_5	c_6	c_7
a_2	157 560	1	0	1100	1000	1
a_4	196 305	1	0	1100	1100	0
a_6	132 000	1	0	550	750	1
a_7	274 850	1	1	1100	745	0
a_8	78 540	0	1	370	550	1
format	MIN	MAX	MAX	MAX	MAX	MAX
weights	0,145	0,331	0,025	0,137	0,318	0,044

The next step of the approach is to standardize table values. For this purpose, the ideal and negative-ideal values of criteria are determined.

Table 11: Ideal and Negative-ideal alternatives

	c_1	c_3	c_4	c_5	c_6	c_7
Ideal	78 540	1	1	1100	1100	1
Negative-ideal	274 850	0	0	370	550	0
$X_J^{Ideal} - X_J^{\text{Negative-ideal}}$	-196 310	1	1	730	550	1
format	MIN	MAX	MAX	MAX	MAX	MAX

After determining the differences of ideal and negative-ideal values, it is time to normalize all table values using formula (7).

Table 12: Normalized matrix

	c_1	c_3	c_4	c_5	c_6	c_7
a_2	0,597	1	0	1	0,818	1
a_4	0,400	1	0	1	1	0
a_6	0,728	1	0	0,247	0,364	1
a_7	0	1	1	1	0,355	0
a_8	1	0	1	0	0	1
format	MAX	MAX	MAX	MAX	MAX	MAX
weights	0,145	0,331	0,025	0,137	0,318	0,044

The normalized criteria values are then multiplied by the criteria weight.

Table 13: Normalized matrix multiplied by weights

	c_1	c_3	c_4	c_5	c_6	c_7
a_2	0,087	0,331	0	0,137	0,260	0,044
a_4	0,058	0,331	0	0,137	0,318	0
a_6	0,106	0,331	0	0,034	0,116	0,044
a_7	0	0,331	0,025	0,137	0,113	0
a_8	0,145	0	0,025	0	0	0,044
format	MAX	MAX	MAX	MAX	MAX	MAX
weights	0,145	0,331	0,025	0,137	0,318	0,044

The final step is calculating the utility of each alternative using formula (8).

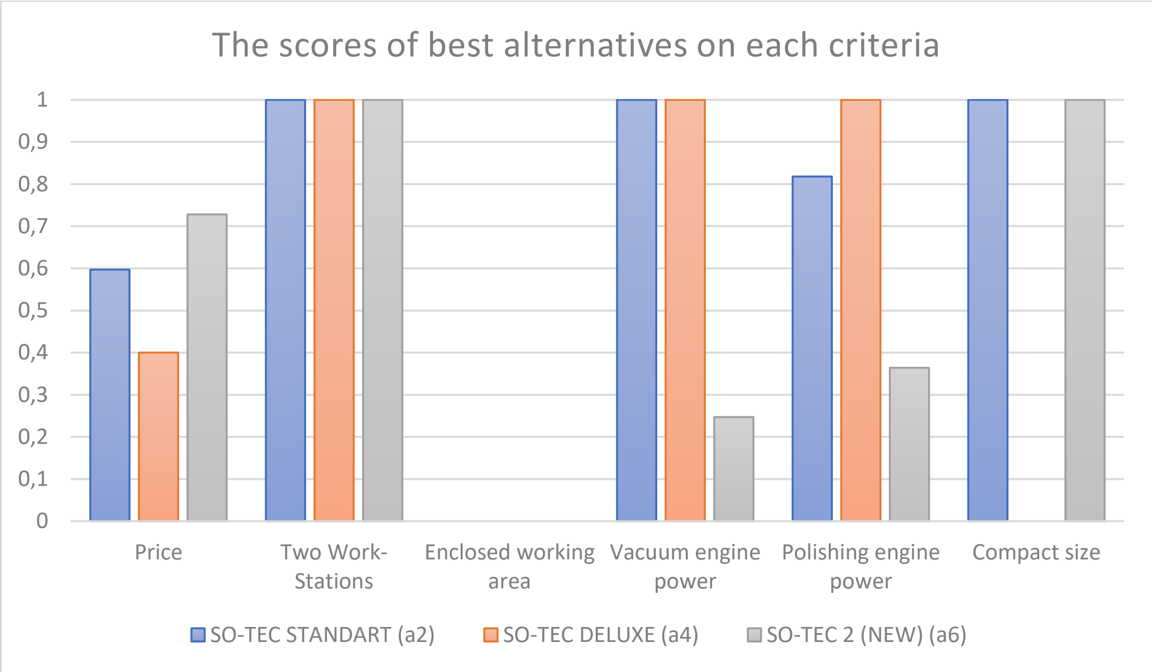
Table 14: Order of alternatives

	Utility	rank
a_2	0,859	1
a_4	0,844	2
a_6	0,630	3
a_7	0,606	4
a_8	0,214	5

The rank of each alternative is identified. The alternative with the biggest utility is chosen as the efficient alternative.

Table 14 shows the general score of each alternative. The **SO-TEC STANDART (a_2)** is an efficient alternative with the biggest utility value 0,859. This model is intendent for large and small jewelry productions. It has two working places for operative work. The size of the machine is lowered but the function stays the same. These changes will make it fit in the selected classroom and keep productivity up. The power of the engine is 1000 w. The power of the exhaust fan is 1100 w. with a rotation speed of 2800 turns per minute. The weight is 90 kilograms. The dust is going through the process of filtration and stays in filter bags. The price of the machine is 157 560 RUB.

The second place was taken by **SO-TEC DELUXE (a_4)** with a utility score of 0,844. The third best alternative is **SO-TEC 2 (NEW) (a_6)** with a general utility score of 0,63.



Graph 2: The score of best alternatives of each criterion

Graph 2 represents the utility of the three best alternatives in each criterion. SO-TEC 2 (NEW) (a_6) is better in the criterion of price. All of the alternatives have two work-stations, none of them have an enclosed working area, only a protective screen in front of the worker. Vacuum and power engines values are better in SO-TEC STANDART (a_2) and SO-TEC DELUXE (a_4). The size of SO-TEC STANDART (a_2) and SO-TEC 2 (NEW) (a_6) allows them to be placed in the original classroom.

After considering all calculations above, the SO-TEC STANDART (a_2) will be recommended to the Yakutian Art School.

4.11 Sensitivity analysis

What if something will change? For that reason, the sensitivity analysis was performed. The focus of the analysis will be on criteria “price” which can have a different value depending on the financial situation of the school. If the school’s budget is big then the weight of the price in this decision goes down, and if it is small this raises the weight.

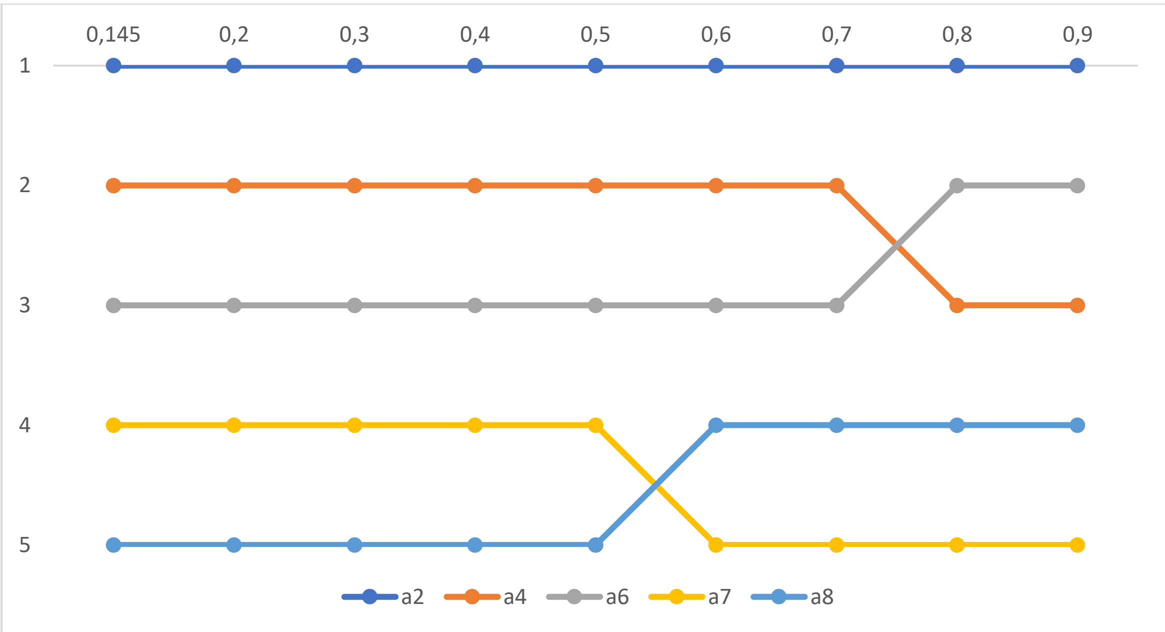
In the following calculations, the change of the efficient alternative by the change of the price weight will be shown. The price weight will be gradually increase by 0,1 until it reaches 0,9 (value 1 will mean that there’re no other criteria in the process). The starting and original value is 0,145.

The change of the weight of one criterion will change others.

The table below shows the change of the rank (found by the weighted sum approach) caused by change of the weight of the price - **c_1**. The value of this weight was gradually increased by 0,1 (except the first increase from the original and first value of the scale).

Table 15: The order of alternatives after the change of the weight

weight	**0,145**	**0,2**	**0,3**	**0,4**	**0,5**	**0,6**	**0,7**	**0,8**	**0,9**
				rank					
a_2	1	1	1	1	1	1	1	1	1
a_4	2	2	2	2	2	2	2	3	3
a_6	3	3	3	3	3	3	3	2	2
a_7	4	4	4	4	4	5	5	5	5
a_8	5	5	5	5	5	4	4	4	4



Graph 3: The change of the ranking of alternatives depending on the weight of the price

The graph and the table clearly show that the alternative **SO-TEC STANDART (a_2)** is an absolute compromise choice. If price's weight will increase or be the only criteria in this decision-making process, the efficient alternative will stay the same - **a_2.**

There are only slight changes in this analysis. The alternative **a_4** and **a_6** will change places from the price's weight at a value of 0,8. The alternative **a_7** will take the last place start from the price's weight at 0,6, and **a_8** will take his place from then.

4.12 Change in price

The seller of the polishing machine gave the Art school an offer. The offer states that they are willing to give a discount of 20% for buying 2 pieces but only for the polishing machine **SO-TEC 4/2 (NEW) (a_7)** which has an original price of 274 850 RUB. The new price of this machine will be 219 880 RUB. We should consider this generous offer in our process and calculate the new outcome.

The following calculation will test the change of the efficient alternative but the decrease of the price of the alternative **a_7**.

The original estimated weights of the criteria will stay the same.

Table 16: Change of the price of the alternative a_7

	c_1	**c_3**	**c_4**	**c_5**	**c_6**	**c_7**
a_2	157 560	1	0	1100	1000	1
a_4	196 305	1	0	1100	1100	0
a_6	132 000	1	0	550	750	1
a_7	219 880	1	1	1100	745	0
a_8	78 540	0	1	370	550	1
format	MIN	MAX	MAX	MAX	MAX	MAX
weights	0,145	0,331	0,025	0,137	0,318	0,044

As the biggest possible number of price changed, it is necessary to recalculate the $X_J^{Ideal} - X_J^{\text{Negative-ideal}}$ of criteria **c_1**.

Table 17: Ideal and Negative-ideal alternatives after the change of a_7

	c_1	c_3	c_4	c_5	c_6	c_7
Ideal	78 540	1	1	1100	1100	1
Negative-ideal	219 880	0	0	370	550	0
$X_J^{Ideal} - X_J^{Negative-ideal}$	-141 340	1	1	730	550	1
format	MIN	MAX	MAX	MAX	MAX	MAX

The new table of utility will look like this:

Table 18: The order of alternatives after the change of the price of a_7

	c_1	c_3	c_4	c_5	c_6	c_7	Utility	rank
a_2	0,087	0,331	0	0,137	0,260	0,044	0,836	1
a_4	0,058	0,331	0	0,137	0,318	0	0,810	2
a_6	0,106	0,331	0	0,034	0,116	0,044	0,615	3
a_7	0	0,331	0,025	0,137	0,113	0	0,606	4
a_8	0,145	0	0,025	0	0	0,044	0,214	5
format	MAX	MAX	MAX	MAX	MAX	MAX		
weights	0,145	0,331	0,025	0,137	0,318	0,044		

The decrease of the price of the **a_7** will not affect the rank of alternatives. The alternative **a_7** was the negative-ideal alternative of the criteria and it stayed the same even if the price decreased which means that alternatives will never be efficient unless his price will change significantly.

The polishing machine **SO-TEC STANDART (a_2)** will be recommended for a purchase to the Yakutian Art School.

5. Conclusion

This bachelor thesis deals with a real decision-making problem of the Yakutian Art School. This school was established in 1945, after World War II, when Stalin gave the order to create and implement the Art school in the region. Currently the rock polishing machines do not fit the current health state requirements. The new standard for machines is to have an exhaust fan which will protect the working area from the dust and small pieces of polishing materials. The school is in need of two machines and they are looking for the worthiest choice to fulfill their need.

The goal of this thesis was to find a suitable alternative or alternatives to close the need for two new machines for the decorative applied and ethno cultural works faculty. The criteria weights were determined by Saaty's pairwise comparison method. The criteria matrix went through consistency test twice to get close to the Saaty's required number of 0,1. The set of alternatives were checked by the test of dominance and a few of the alternatives were reduced. The final calculation of the determination of an efficient alternative was provided by the Weighted Sum Approach.

After the calculation, the alternative SO-TEC STANDART was chosen as the recommended alternative based on the general utility score performed by the Weighted Sum Approach. The school was in need of two machines which could possibly be fulfilled by two different kinds. For example, if there were two top machines of equal or close to equal utility, the value of each would be worth considering.

The sensitivity analysis was performed to create a graph of changes in ranking of alternatives by the change of the weight of the price criterion. The change of this particular criterion was chosen for the reason of possible financial changes the school might face. How the outcome will change depends on the financial ability of the Art School. The result showed that the SO-TEC STANDART is the most efficient alternative on every scale of the price weighing (0,145 – 1) where 0,145 is an original weight and 1 means that there is no other criteria. In addition, the machine SO-TEC 4/2 (NEW) has a change in price thanks to an offered discount. This offer was analyzed but it didn't change the efficient alternative.

In summary, the machine SO-TEC STANDART will be recommended to the Yakutian Art School to be purchased to fulfill two available places. This machine is a highly efficient alternative compared to the others and the change of the price weight and the offered discount for another alternative changes nothing in this decision.

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