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MASTER THESIS

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The problem-solving ability of students in" British international school of Olomouc", based on the Polya method

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Supervisor: Mgr. Peng Danping, Ph.D.

Declaration

The thesis entitled '*The problem-solving ability of students in" British international school of Olomouc," based on the Polya method*' has been undertaken by me at the Faculty of Education, Palacky University Olomouc, under the supervision of Mgr. Peng Danping, Ph.D.

I declare that the information in this thesis is the result of my own research, and it has not previously been submitted to any other institution. Any content derived from the work of others is properly referenced and cited.

Maryam Pourpakdelfekr

Olomouc, 20 April 2023 Signature: …………..

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Abstract

This study was carried out by a teacher-researcher who conducted observations and interviews with students attending the British international school in Olomouc over one month. The primary objective of the research was to investigate the students' problem-solving skills with respect to the Polya model. To accomplish this, the students were tasked with solving ten mathematics word problems that reflected their mathematical knowledge and grade level. Following the completion of the tasks, the data collected was analyzed using the Polya problem-solving model to determine the frequency with which the students used problem-solving indicators. Results indicated that the students experienced difficulties using all the steps of the Polya model, particularly in the first step of understanding the problem. However, the students correctly performed the processes of devising a plan, executing the problem-solving procedure, and performing the calculations; many failed to write the final conclusions and engage in rechecking accurately. This study highlights a potential gap in the problem-solving skills of a significant proportion of the students, as they failed to meet three out of the six indicators defined in this research.

Keywords: Mathematics word problems, Polya model, Problem-solving skills, Interviews, International school

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Chapter 1: Introduction

1.1. Rationale of the Study

Children and adults both have ways of dealing with interpersonal problems, some productive and others ineffective, some incorporating a concern for the interests and rights of the other person involved, and some concentrating only on self-interests. Learning how to deal with problems successfully, productively, and constructively is one of the most crucial tasks of human development, yet many children do not learn it well, and others even reach adulthood with severe shortcomings in this domain (Battistich et al., 1989). Breakthroughs in the modern era's science and technology influence society's life, structure, education, and all other aspects of human development. In this regard, it is vital to train students who can adapt to change, think critically, be innovative, develop practical solutions to problems, and contribute to the society in which they live. It develops pupils' character and behavior for a successful future. This will be feasible by teaching children problem-solving abilities at an early age (Behlol et al., 2018; Sungur & Bal, 2016).

The problem-solving process requires cognitive, affective, and behavioral qualities. These abilities involve numerous operations like social and academic adjustment of the individual, self-confidence, decision-making styles, practical communication skills, trial and error, analysis, synthesis, discovering the cause and effect relationships, and learning ideas and principles. Individuals employ old information and learn new knowledge while conducting these activities (Sungur & Bal, 2016). Problem-solving can be classified into two main categories; creative and critical thinking. Critical thinking is concerned with analyzing ideas that might be used to solve a problem, whereas creative thinking is focused on producing ideas that could be used to solve a problem (R. Mayer & Wittrock, 2006).

George Polya is the founder of the modern problem-solving emphasis in mathematics education. He was a well-known mathematician who wrote several books on the topic, the most notable of which was "How to Solve." Polya proposed the four-step problem-solving process in this book. According to him, problem-solving is a strategy to increase the skill of mathematical problem-solving. It encourages students to become independent explorers and use this strategy to tackle daily mathematical challenges (Polya, 1973; Yuan, 2013).

1.2. Statement of the problem

The problem is that problem-solving is a crucial aspect of teaching mathematics, yet it is also a major challenge for students. Research has shown that students experience anxiety and stress when attempting to solve mathematical problems, which can negatively impact their perception of the subject. In this thesis, the four-step problem-solving approach proposed by Polya is employed to address this issue. The aim is to enable students to develop their critical and creative thinking skills, as well as their ability to analyze, synthesize, assess, and create. Problem-solving is crucial in mathematics education as it enables students to fulfill the functional, logical, and aesthetic purposes of learning mathematics.

1.3. Objectives of the study

The primary purpose of this study is to assess the abilities of students to solve mathematical questions based on the four Polya steps. It is believed that the outcomes of this study will be used as assessment material for future learning, particularly problem-solving subjects.

1.4. Research questions

To perform the research, the following questions were posed:

1. To what extent do the British International School of Olomouc students apply the four steps of the Polya method when solving math problems?
2. How does the frequency of use of the Polya method vary across different grade levels of students at the British International School of Olomouc?
3. What specific steps of the Polya method do students at the British International School of Olomouc struggle with and/or use the most when solving math problems?

1.5. Significant of the study

Comprehension of the problem-solving process can aid students in processing information and constructing knowledge regarding the physical and social worlds that surround them. Once students acquire an understanding of the problem-solving process, they can devise discoveries, think creatively, resolve practical problems, assess and interpret observations, enhance their thinking process, and resolve problems appropriately. Additionally, comprehending the problem-solving process makes school education more relevant to life. As a result, the comprehension of the problem-solving process bears a beneficial impact not only on mathematics learning results but on education holistically (Sinaga et al., 2023).

The study's findings can provide valuable insights for educators and educational activists to improve the quality of problem-solving skills based on Polya's steps and word problem types.

Chapter 2: Literature Review

2.1. Understanding a problem/problem-solving

A problem, synonymous with trouble, is defined as an issue that must be considered, explored, understood, and achieved to a conclusion (Sungur & Bal, 2016). Duncker (1945, p.1) described a problem as follows (Duncker, 1945):

"A problem arises when a living creature has a goal but does not know how this goal is to be reached. Whenever one cannot go from a given situation to the desired situation simply by action, then there has to be recourse to thinking. Such thinking has the task of devising some action, which may mediate between the existing and desired situations".

A polish scientist, W. Oko, describes a didactic problem as a practical or theoretical obstacle that students must answer individually via their own active investigation. Generally, the cornerstone of this problem is a systematic and purposefully arranged environment in which the child aims to deal with the challenges in line with the specific demands, which is gaining new knowledge and experiences. Willingness to deal with the problem is a state in which an individual approaches the assessment of the problem's circumstances and nature and places a high value on them. This is critical in the educational field since the problems assigned to students should be ones that they embrace willingly, and if not, the students should be motivated. There are only two ways to be motivated to overcome a challenge in the educational sphere. The first scenario is to create a setting that inspires and energizes the student and generates a condition in which the student feels compelled to learn more about the problem and its solution. The interest in problem-solving must be aroused in order to meet the demand caused by unfamiliarity, which is referred to as "cognitive need." The need is met by resolving the problem and acquiring the necessary information. The second way is to employ exterior stimuli to cause interior motives and resources to meet the individual's inner needs. Instead of wanting to solve the problem and satisfying one's needs for its solutions, the individual focuses on the problem's effective solution. Since the problem merely plays a vicarious role in this scenario, the purpose remains external to the problem. The student solves the problem to reach the goal, and the problem-solution becomes a resource. This is not as advantageous in terms of educational outcomes as when passion for the topic itself arises. It is common for educational settings to use non-natural stimuli, students do not experience grades in their normal lives (Dostál, 2015).

Finding an appropriate approach to describe an issue is an important part of problem-solving. Problem-solving research has a long and rich history. Many early psychologists studied the mental processes that occur during critical thinking and problem-solving. While they achieved some important discoveries on the nature of thinking and problem-solving, modern scholars have mostly ignored their work. Problem-solving research did not take its present form until the 1960s when Herbert Simon and his colleagues began examining human beings solving complex problems. By using both protocols and computational modeling, Newell and Simon (1972) proposed a comprehensive theory that is still at the core of modern problem-solving theory (Dunbar, 1998). According to Newell & Simon (1972), problem-solving consists of a search through a problem space. A problem space consists of an initial state, a goal state, and a collection of operators that may be used to move the solver from one state to the next. When a solver encounters a problem, he will highlight specific aspects of the problem and utilize these aspects to decide what to do when exploring a problem space (Newell & Simon, 1972).

To put it another way, problem-solving is cognitive processing aimed at achieving a goal when no clear solution approach exists (R. E. Mayer, 1991). Consistent with this definition presented by Mayer (1990), there are four fundamental aspects of problem-solving. Initially, problem-solving is cognitive, which means it happens internally in the problem solver's cognitive system and can only be deduced indirectly from the problem solver's actions. Secondly, problem-solving is a process that includes the problem solver's cognitive system representing and manipulating knowledge.

Furthermore, problem-solving is directed, which means that the problem-solver's objectives influence the problem-solver's cognitive processes. Lastly, problem-solving is personal, in that the problem solver's particular knowledge and skills influence the complexity or simplicity with which constraints to solutions can be overcome. To summarize, problem-solving is cognitive processing aimed at changing a given circumstance into the desired state when no evident solution exists (R. E. Mayer, 1990).

Problem-solving can be accomplished by using the senses and mind. However, several aspects influence a problem's resolution. Individual intellect, ideas, feelings, beliefs, personality traits, traditions and norms, self-confidence, lifetime experiences, and family influence are examples of these. The most significant part of education is that students may enhance their problem-solving approach (way of thinking). Many skills are required in the problem-solving process, including planning, analyzing information, applying methods, checking outcomes, attempting different strategies, and being inclined to accept help from others and colleagues (Franestian et al., 2020). In other words, problem-solving skill is an intellectual activity in analyzing, translating, logic, forecasting, assessing, and reflecting in order to develop solutions from students' own knowledge and experience (Riska Dwi Anggraini et al., 2022). The instructor should investigate pupils' problem-solving abilities. This implies that before developing problem-solving abilities, the instructor must first evaluate how far pupils' problem-solving capacity has progressed through a study of student challenges in solving issues (Nurkaeti, 2018a). Correspondingly, the elementary school years are critical for the development of this skill. To strengthen the child's problem-solving capabilities, he must be confronted with a variety of situations and encouraged to concentrate. It is also essential to motivate him to avoid false solutions of alternative solutions by restricting the problem's boundaries and to provide him with opportunities to describe the alternatives he developed to address the problem without judging him. Hence, students' problem-solving skills should be improved to be creative, self-confident, and critical thinkers and to become individuals who discover realistic answers to challenges they meet. This is extremely important for their psychological and social growth (Sungur & Bal, 2016).

2.2. Type of problems

Researchers have distinguished between two sorts of problems; well-structured (well-defined) and ill-structured (ill-defined) problems. The majority of problems faced in formal education are well-structured. Well-structured issues generally provide all aspects of the problem, employ a limited number of rules and principles arranged in a predictive and prescriptive order, offer accurate, convergent responses, and have a desired, prescribed solution procedure. On the other hand, ill-structured problems are the kind of challenges faced in daily practice, and the solver is not aware of the operators, the aim, or the current state. These problems are more difficult to solve since they include numerous possible solutions, vaguely defined or ambiguous goals and limitations, various solution routes, and multiple criteria for assessing solutions.

The complexity of problems also varies and is determined by the breadth of knowledge necessary to solve the problem, the amount of prior knowledge, the complication of the problem-solution techniques, and the problem's relational complexity[1]. Ill-structured problems are often more complex, yet, there are also several well-structured problems, such as chess. Dynamicity may be considered as another dimension of problem complexity. The relations between variables or factors alter over time in dynamic problems. Changes in one aspect might

[1] Is defined as the number of relations that required to be processed in parallel during a problem solving process (Jonassen, 2010).

generate variable changes in other factors, which can significantly alter the nature of the problem. The more complicated these relations are, the more difficult any solution will be. Ill-structured problems are more dynamic compared to well-structured challenges that are more static (Jonassen, 2010).

Routine and non-routine problems are other classifications of problems. A routine problem is one for which the solver already has a solution technique prepared. For instance, if a pupil was taught the process for the long division of whole numbers, a new long division problem resembles a routine problem (R. Mayer & Wittrock, 2006). Any effort to answer routine problems is meant to improve essential skills, particularly arithmetic ability using four basic operations in mathematics: addition, subtraction, multiplication, and division (In'am, 2014). A non-routine problem, on the other hand, is one for which the problem solver lacks a previously prepared solution technique and develops a novel approach to problem-solving. As a result, the definition of a routine or non-routine problem depends on the learner's knowledge, while the description of a well-structured or ill-structured problem is not. Although common issues are at the heart of many educational lessons, significant real-world challenges are generally non-routine (R. Mayer & Wittrock, 2006).

2.3. Impact of attitudes on learning

Attitude refers to a person's learned tendency to respond positively or negatively to an issue, circumstance, theory, or another individual (Sarmah & Puri, 2014). Attitudes develop in pupils as a function of their learning environment and appear as a form of cognition, affect, and action. Therefore, while designing materials, students' attitude toward a learning environment is a significant aspect that impacts the efficacy of instruction and learning outcomes. Attitudes may alter and evolve over time, and a good attitude can boost pupils' learning once created. A negative attitude, on the other hand, prevents successful learning and, as a result, decreases the learning outcome and, subsequently, performance. Students' attitude toward the class and factors that are associated with the class is one of the barriers to learning mathematics. Depending on the particular student, the influence of attitude on mathematics achievement might be beneficial or destructive. Course content, lecturer, students' preparation, presentation style, learning, and teaching atmosphere are all factors to consider. Continuity and change of attitude are intimately tied to the state of the variables in the context of teaching/learning (Çelik, 2018; Mazana et al., 2019).

According to (Syyeda, 2016), the term 'attitude' has been defined in various ways, including one-dimensional, bi-dimensional, and multi-dimensional. In light of various definitions, she designed a multi-dimensional framework that considers affect, cognition, and behavioral components for this study. Affect is expected to include the subject's emotions, beliefs, and vision. Emotions are feelings of happiness or pleasure in studying a subject, as opposed to finding it uninteresting. Beliefs are students' self-concepts as mathematics learners and their confidence in their skills, while vision is their view of the matter. The cognitive component comprises students' perceptions of the subject's current usefulness in daily life as well as its relevancy for future careers and life. Behavioral components can refer to students' behavior, commitment, and performance in class. Attitude components are not mutually exclusive; they intersect, overlap, and form clusters. They share similar attributes and impact one another (Syyeda, 2016). By measuring the following aspects, one can understand the attitude of pupils toward mathematics:

1. Self-confidence, anxiety, enjoyment (affect)
2. Internal motivation (behavior)
3. Perceived usefulness (cognition)

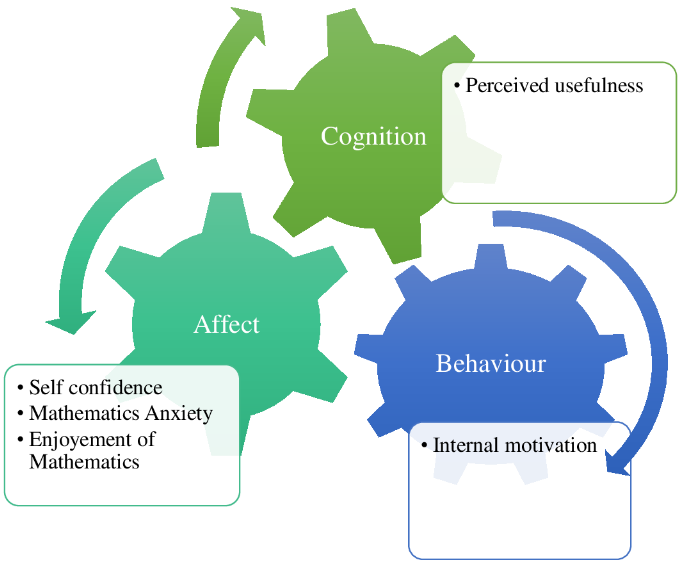


Figure 1. Components of attitudes (Syyeda, 2016)

2.3.1. Affect

Self-confidence

Self-confidence in mathematics relates to students' impressions of themselves as math learners, which include beliefs about one's capacity to learn and function well in mathematics. Students' self-confidence varies from one period of their lives to the next. This suggests that self-confidence is not a solid entity that may be favorably impacted and motivated. Promoting a student's feeling of capability, therefore encouraging the learner, is a simple strategy for educators to enhance the learner's self-confidence. Furthermore, students who are determined to succeed and have a sense of potential and self-confidence are more likely to take learning seriously. As a result, they will engage in more challenging activities to expand their knowledge and talents in order to foster successful learning.

Consequently, if a student lacks self-confidence, instructors, parents, and peers should be able to help boost the student's confidence (Van der Bergh, 2013). Kaskens et al. (2020) noted that improving young children's self-confidence is critical for their mathematical development. They discovered a significant correlation between children's math self-concept and mathematical proficiency, demonstrating the importance of providing the optimal opportunity for children to learn math early and feel confident concerning their math learning (Kaskens et al., 2020). This study aligns with Bernard & Senjayawati (2019), which stated an association between pupils' self-confidence and their mathematics comprehension abilities (Bernard & Senjayawati, 2019).

Mathematics anxiety

Mathematics anxiety has been defined as "a feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of mathematical problems…" (Richardson & Suinn, 1972). It refers to unfavorable attitudes that happen when certain pupils encounter mathematical issues and expresses as losing their mind and being frightened, nervous, afraid, and powerless (Keziah et al., 2021). Students with higher math anxiety tend to avoid math-related courses and careers and do not perform well in math. As a result, math anxiety is frequently detrimental to pupils' math learning and fulfillment of professional goals in scientific areas (Quintero et al., 2022). Keziah et al. (2021) believe that math anxiety levels do not substantially impact children's mathematical performance (Keziah et al., 2021). On the other hand, different studies find that mathematics anxiety is a psychological component affecting students' success and general practices (Paul & Ngirande, 2014; Quintero et al., 2022). So lecturers should endeavor to comprehend mathematics anxiety and adapt teaching and learning tactics to assist students in overcoming fear (Paul & Ngirande, 2014).

Enjoyment of Mathematics

The affective components of interest are referred to as emotions. Positive emotions happen when an individual interacts with a topic of interest, while negative emissions do not. One of the most often expressed pleasant feelings in the classroom is enjoyment. Students' engagement and re-engagement with pleasant content can be influenced by enjoyment as a positive activating emotion. In this approach, enjoyment may not only accompany but also positively affect interest growth. The findings of a study revealed that enjoyment during a teaching unit depended on students' initial interest in math, and students who enjoyed their classes were less bored than others (Schukajlow, 2015). The more pupils like mathematics, they tend to participate in problem-solving, thereby improving their learning and performance. Due to the relation between enjoyment, students' learning, and performance, it is worthwhile to assess students' levels of mathematics enjoyment in order to follow students' learning and performance (Mazana et al., 2019).

2.3.2. Behavior

Internal motivation

There are two types of motivation in general, external and internal motivation. External influences such as family, teachers, and the environment provide extrinsic motivation, which might take the shape of compliments, presents, outstanding grades, incentives, or any other external reasons. On the other hand, intrinsic motivation encourages a person to do something for their self-interest and pleasure (Sengodan & Iksan, 2012). Internal motivation is the inherent power within human nature that encourages people to seek out and tackle new challenges; they are keen to learn even when no external rewards are offered (Adamma et al., 2018). According to self-determination theory (Deci & Ryan, 2013), students' intrinsic motivation is dependent on the accomplishment of their needs for competence, autonomy, and relatedness. Students who are intrinsically motivated by a mathematical task because it is enjoyable; prefer and appreciate the activity for reasons that are inherent in the activity. Pupils' internal motivation is generally strong at the start of elementary school but decreases throughout the first several years of education (Heyder et al., 2020).

2.3.3. Cognition

Perceived usefulness

The perception of pupils about the necessity of a topic in their daily lives and future careers is defined as perceived usefulness, which positively correlates to their attitude towards the subject. Students who understand math's vital role in their lives are more willing to make full use of it, thus benefiting their performance (Mazana et al., 2019).

2.4. Activity-based learning

Students' attitudes toward activity-based learning are essential in the educational process since if a student is not positively oriented and liable to activity-based learning, they will not attain the purpose and objectives of the school lesson (Çelik, 2018). The interaction between lecturers and students is critical to student motivation. Teaching via interaction can widen students' thinking horizons. Activity-based learning is a learning process in which students are continually engaged rather than sitting as passive recipients. Active-based learning differs from chalk-and-talk teaching methods in two ways:

- The active participation and involvement of students in classrooms
- Cooperation among students in a learning environment

The primary focus of activity-based learning is that learning should be centered on undertaking hands-on, mind-on, or heart-on experiments and activities, improving creative and critical thinking abilities. However, it will only work when pupils are sufficiently motivated to reach their full potential. The most beneficial and successful way of teaching, particularly for complicated topics, is to engage students in interactive activities. Such classes have a beneficial impact on student performance and motivation and assist lecturers in controlling chaotic behavior more smartly. Students' critical thinking and creative skills are developed by implementing various exercises in the classroom (Anwer, 2019). There are several sorts of activities, including research-based, practical investigations, problem-solving, project work, and suchlike, that help in the teaching and learning process. These activities assist pupils in developing their understanding of mathematical concepts and procedures, allow them to discover themselves, encourage cause-and-effect thinking, and reduce reliance on authority. This approach places students at the center, provides more profound learning opportunities, causes students to use and enjoy mathematics by taking notes and discussing, and generally increases their encouragement (Çelik, 2018). Among the activity-based learning approaches, our focus in this thesis is on the problem- solving skills.

2.5. The Czech Curriculum in Mathematics

The Czech education system places significant importance on mathematics education in schools, with a well-structured and comprehensive curriculum designed to promote a thorough understanding of mathematical concepts and practical skills application. The curriculum is divided into six key areas, namely algebra, geometry, functions, statistics, probability, and calculus with the first level covering basic concepts and skills, and the higher levels building on this foundation (*Ministry of Education, Youth and Sports of the Czech Republic*, 2023).

In primary schools, students are introduced to mathematical concepts through practical tasks and games, while in secondary schools, the curriculum becomes more theoretical, focusing on the abstract concepts and proofs. The goal is to provide students with a solid foundation in mathematics and develop their critical thinking skills (Škultéty, 2019b).

The Czech Republic has consistently performed well in international mathematics assessments such as the Programme for International Student Assessment (PISA), with students ranking above the OECD average. This reflects the country's strong emphasis on mathematics education in schools (*European Commission Education and Training*, 2023).

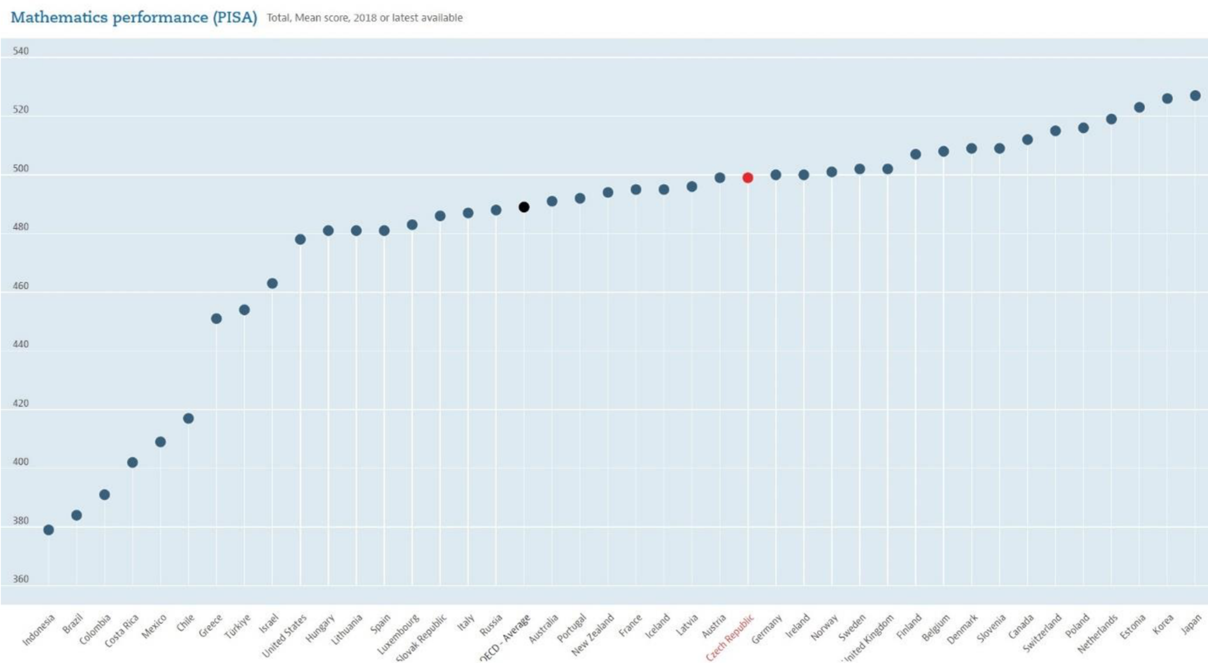


Figure 2. Mathematics Performance (PISA) (PISA: Programme for International Student Assessment, 2023)

2.6. Mathematical Problem-solving skills

Mathematics as a discipline encourages students to logically thinking and provides tools to express abstract thoughts in quantitative and sensible ways. It aids people in accomplishing their daily tasks and serves as a foundation for the evolution of several natural and social sciences disciplines. It promotes precision, focus, reasoning, analytical thinking, creative thinking, and intellectual independence (Behlol et al., 2018). A physicist, Feynan, believes that nature may be expressed by using mathematical symbols that help to comprehend and describe objects in the universe. If a country wishes to educate individuals capable of creating knowledge for development and improvement, it must ensure that the proper foundation is supplied in elementary and secondary schools through mathematics studies (Grolier Educational (Firm), 2004).

There are many distinct sorts of students in a mathematics class. Those individuals who enjoy working with mathematics and hence have an inner motivation for the subject. Students who are not interested in mathematics but who thrive at it. Finally, there are students who struggle with the topic, some of whom may be motivated to study more, but the majority of them believe that mathematics is a complex and boring subject; hence they are less engaged in studying, leading to poor mathematical problem-solving qualities (Thente, 2019). Students often face difficulty throughout the problem-solving process; specifically, mathematical word problems make pupils believe they are too abstract and ambiguous. Despite the increased emphasis on problem-solving skills, teachers usually struggle to educate students on how to approach problems and apply appropriate mathematical tools (Harskamp & Suhre, 2006). The difficulties derive partly from inefficient and limited teaching techniques, which are exacerbated for primary school instructors who are not subject matter (mathematics) teachers. As a result, many students struggle with arithmetic problem-solving, which has a detrimental impact on their willingness to learn this subject (Chadli et al., 2019).

Lester (1994) argues that good problem solvers differ from bad problem solvers in at least five significant ways (Lester, 1994):

1. Good problem solvers know more, and their knowledge is well-linked and rich in schemata;
2. They focus on structural characteristics of issues, while bad problem solvers focus on surface features;
3. Good problem solvers are more aware of their skills and flaws;
4. They are better at monitoring and managing their problem-solving efforts;

5. They are more interested in developing intelligent solutions to problems than poor problem solvers.

The ability to communicate mathematical ideas effectively is a crucial skill for students to develop in order to excel in mathematics. This encompasses the ability to use both language and mathematical representations to express and interpret mathematical concepts, either orally or in written form. However, it is important to note that different students may have varying proficiency levels in mathematical communication, particularly when it comes to problem-solving. The findings of the study by Puspa et al. (2019) indicate that there are notable differences in the mathematical communication abilities of students when it comes to problem-solving. Some students are more detail-oriented in their approach, with a solid ability to remember and apply concepts that have been taught.

On the other hand, others tend to take a more concise and precise approach to problem-solving. In other words, each student brings their own unique strengths and weaknesses to the table when it comes to mathematical communication skills. Overall, the findings of this study highlight the importance of developing strong mathematical communication skills in students in order to support their success in mathematics and beyond (Puspa et al., 2019).

One of the goals of learning mathematics in schools is to solve issues, which includes the ability to comprehend, explore, and collect problems, construct mathematical models, solve models, and interpret results. Before learning abstract mathematical concepts, pupils must be directly involved in actual experiences, then asked to focus on identifying mathematical concepts and relating them to current knowledge so that they feel participated in the discovery of mathematical concepts (Lestari et al., 2018). Accordingly, students must grasp and comprehend the cognitive process or procedures to achieve the results and get used to solving more complex problems while solving mathematical problems (Daulay & Ruhaimah, 2019). Alternatively stated, Mathematical Problem Solving Ability (MPSA) is the ability to develop, suggest and investigate problems, to gather, organize, and evaluate issues from a mathematical perspective, identify appropriate techniques, use the knowledge and skills acquired, and represent and monitor the mathematical thinking process (Lestari et al., 2018). In mathematics, problem-solving involves all activities related to problems that use mathematical language, problem-solving methods, and the application of mathematical ability to solve problems. Each stage in problem-solving has unique qualities that vary from one situation to the next and should be understood prior to tackling a problem. Understanding a problem's features may help discover an acceptable and intended solution. Some features of mathematical problem-solving include (In'am, 2014):

1. Appropriate techniques are required;
2. Possessed knowledge is vital in producing incorrect solutions;
3. Problem-solving skill levels have a significant impact on the accuracy and applicability of the findings;
4. Problem-solving is not dependent on memory;
5. Each problem requires a distinct strategy;
6. Different ways should be learned and understood in order to achieve suitable ad desired solutions;
7. Knowledge and abilities in applying learned mathematical concepts and principles are instrumental in problem-solving.

Dynamic visualizations are considered solid learning tools in many scientific fields and have been proven beneficial for improving academic competency and increasing student engagement, such as developing scientific thinking, spatial abilities, etc. (Wagner & Schnotz, 2017). Kohen et al. (2019) investigate middle school students' self-efficacy and mathematical problem-solving skills based on dynamic or static visualization. The findings demonstrated the beneficial impact of dynamic visualization training. Students' mathematical problem-solving skills improved both immediately following the intervention and a few months later, reflecting more robust conceptual and procedural knowledge (Kohen et al., 2019).

George Polya, based on his experiences as a mathematician and mathematics teacher, proposed Polya's theory. He believed that problem-solving has two objectives. The first is to resolve a specific mathematical issue. However, the second goal is to improve students' thinking and skills so they can deal with challenges they may experience in the future on their own outside the classroom (Hensberry & Jacobbe, 2012). According to Polya, the mathematical problem-solving strategy has four steps, namely (Daulay & Ruhaimah, 2019; Dostál, 2015; Polya & Conway, 1973; Winarso et al., 2022):

1. Comprehending and describing the problem. At this stage, students are asked to understand what is known and asked in the problem and write it on the response sheet.
2. Planning and identifying solutions to the problem. Students convert strategies or examples from the problem story into mathematical forms, like creating variables and formulas.

3. Implementing a problem plan. Pupils showed their abilities to count, use mathematics and employ suitable problem-solving procedures.
4. Assessing and rechecking the solution. Students review the answers to solved problems to make conclusions.

Problem-solving skills form a crucial basis for advancements in science and technology, and therefore, several studies have been conducted to investigate and enhance them. By observing test results, several authors figured out that employing the Polya learning theory helped increase students' mathematical problem-solving ability compared to students whose learning does not employ the Polya learning theory (Daulay & Ruhaimah, 2019; Rosiyanti et al., 2021). Based on the same theory, (Maulyda et al., 2019a) found that teachers performed properly in gathering information on the problem, carrying out the approach as planned, and performing calculations (Maulyda et al., 2019a). Okafor (2019) explored that Polya's problem-solving approach improves student performance in physics problems more than traditional problem-solving techniques (Okafor, 2019). Nurkaeti (2018), in her study, revealed that students face several challenges when solving mathematical problems using Polya's strategy. She emphasized that teachers need to provide students with a range of problem-solving experiences to help them develop their ability to apply mathematical concepts in diverse situations. This will not only enhance their mathematical skills but also support the development of their higher-order thinking skills, which will benefit them in various aspects of their lives (Nurkaeti, 2018b). In a similar study by Karlina (2022), the mathematical problem-solving ability of two low-achieving fifth-semester students at STKIP Paris Barantai Kotabaru in solving story problems was analyzed. The results revealed that the research subjects faced several difficulties in each of the four stages of problem-solving (Karlina, 2022). Based on the results of the study by Siregar et al. (2018), there are four main types of student errors in solving pedagogic problems using the Polya procedure (Siregar et al., 2018):

1. Making mistakes in answering the questions due to misunderstanding the problem
2. Making mistakes in planning the learning process based on constructivism due to a lack of understanding of how to design the learning
3. Choosing an improper learning tool due to a lack of recognition of what kind of learning tool is relevant to use
4. Making an error in sorting and arranging the learning process due to a lack of understanding of how to apply the learning model effectively.

In agreement with the previous research, Winarso et al. (2022) stated that they obtained an error in grasping the problem by 31%, making a strategy by 11%, carrying out the method by 9%, and reviewing by 33%. So mistakes arise when students do not perform precise calculations, do not learn the formula, and cannot distinguish between sequence and series (Winarso et al., 2022).

Maulyda et al. (2019) proved that teachers can also face challenges solving mathematical problems using the Polya method. This study uses Polya's method to explore the problem-solving abilities of primary school teachers in Mataram City, Indonesia. The results of this study highlight the need for professional development for primary school mathematics teachers to improve their problem-solving abilities and enhance their understanding and application of Polya's method. By doing so, teachers will be better equipped to support their students in developing strong problem-solving skills and achieving success in mathematics (Maulyda et al., 2019).

Nurhayanti et al. (2020), in an interesting study, tried to describe the mathematical problem-solving abilities of fifth-grade students based on gender. The findings reveal differences in problem-solving between girls and boys, although not significantly. Both genders with low initial mathematical competence could not use Polya steps to solve mathematical problems. As a result, additional learning strategies and habituation are required for them to fully address the challenge (Nurhayanti et al., 2020).

Chadli et al. (2019) addressed the question of whether pupils benefited from game-based learning techniques based on Polya's problem-solving strategy by offering support at each level to assist average and low-achieving second-grade elementary students in improving their abilities in solving basic word-based addition and subtraction questions and increasing their courage to pursue. This paper emphasized breaking down the problem-solving process into steps so that students could better comprehend the semantics and context of mathematical word problems. Their findings suggested that playing games could effectively enhance the development of mathematical skills, especially in word problem-solving. However, this implies the need for adequate training and the instructor's presence in the classroom to guide, monitor, and evaluate the student's learning (Chadli et al., 2019).

2.7. Problem-solving skills in the Czech Republic

Czech pupils' performance in mathematics was statistically similar to the average of the OECD countries. However, a long-term comparison revealed a statistically significant decline in the country's mathematics performance. In contrast, other European countries with similar economies and historical-cultural backgrounds, such as Estonia, Poland, and Slovenia, scored higher or significantly higher in mathematics. The differences in performance between the Czech Republic and other European countries in mathematics may be attributed to various factors, including the fact that the Czech Republic has the lowest number of mathematics lessons and lacks general mathematical goals and competencies. Additionally, there is a lack of emphasis on the home preparation of pupils, the absence of unified testing, and insufficiently detailed learning content and outcomes (Moravcová et al., 2019).

Škultéty & Hodaňová (2020) conducted a case study with the aim of identifying the level of readiness and success of mathematics teaching students in the Czech Republic in solving non-standard tasks. The results indicate that while students generally understand the concept of non-standard tasks and work with them occasionally, their ability to solve such tasks is inadequate. Many students face difficulties approaching and solving these tasks, suggesting that they struggle to incorporate non-standard tasks effectively in their future mathematics teaching practices due to a lack of understanding. This lack of comprehension may also contribute to the relatively lower popularity of mathematics among students. Alternatively, some may attempt to incorporate non-standard tasks in their teaching but may face difficulties explaining the solutions. It is crucial to note that if students of mathematics teaching do not comprehend the tasks adequately, it will be challenging for their pupils to understand them. This, in turn, may negatively impact the relationship of the students with mathematics (Škultéty & Hodaňová, 2020). The negative attitude towards mathematics among Czech pupils may also contribute to their poor performance on non-standard types of mathematics tasks. One potential cause of the low success rate of pupils on these tasks is the prevalent approach to mathematical education, which emphasizes the formation and consolidation of algorithmic practices based on standard tasks. This approach leads pupils to apply learned methods without thoughtful consideration, as teachers typically derive the algorithm from a model example, and pupils acquire solutions to similar tasks. To facilitate the quicker acquisition, teachers purposefully select type-like tasks to reinforce the given algorithm (Škultéty, 2019a).

Chapter 3: Methodology

3.1. Polya problem-solving model

The problem-solving process is an essential component of every mathematical lecture. Each mathematical acquisition is related to the acquisition of the problem-solving process, so this skill is attempted to be presented. However, problem-solving is the driving force behind students' challenges in mathematics. It has been proved that students’ anxiety and stress have a detrimental impact on their perception of this subject (Üredi & Kösece, 2016). As mentioned in the previous chapter, this thesis has been designed in accordance with Polya’s four-step problem-solving approach that works systematically to solve a mathematical problem.

By problem-solving, students can comprehensively understand theorizing, discovering, testing, analyzing, synthesizing, assessing, and creating. Many studies have demonstrated the benefits of problem-solving methods on various faculties of mind, such as critical and creative thinking and the ability to observe, correlate and address difficulties in daily life. The problem-solving approach is an essential educational tool in mathematics since it is the means by which the functional, logical, and aesthetic purpose of teaching mathematics can be fulfilled (Polya, 1973).

3.1.1. Step 1- Understanding the problem

In the first stage of this approach, the given problem is understood in the context of the provided data. Based on the nature of the problem, several questions, figures, and diagrams may be asked and illustrated to aid comprehension.

To understand the first step, Polya gave an example of finding the diagonal of a rectangular parallelepiped, which length, width, and height are unknown. Before beginning the first stage, students should be familiar with Pythagoras’ theorem and its application in plan Geometry. For teachers who wish to employ this strategy before addressing a problem, students must have a prior understanding of the problem's variables, which may not be entirely fresh to them. To provide a clear perspective of the topic, the instructor may use specific examples so that students may understand the actual problem and what is actually presented. Using the example of a classroom in the shape of a rectangle parallelepiped, of which the measurements can be determined, can make the problem more interesting and real. In brief, three types of basic information are necessary to address the problem at this stage (Polya, 1973):

1. What is the provided data?
2. What is asked to be found?
3. What are the conditions?

3.1.2. Step 2- Devising a plan

In the second phase, students are driven to discover connections between the facts provided and the unknown. This level gives a more in-depth knowledge of the situation, so pupils are asked about a similar problem. If they become aware of a similar problem, they are asked to recollect and solve it. Understanding the problem and developing a plan may be a long and challenging process. A 'bright idea' resulting from spontaneous inspiration or the outcome of a series of experiments can be used to construct a problem plan. In an unbiased manner, bright ideas can be created with pupils' and their tutor's discussion. In order to assist pupils in developing bright ideas, the teacher must consider themselves as students and begin the conversation by asking the students, "Do you have an example relevant to this problem?" choosing the appropriate option among the various relevant problems is the challenge of this stage. Sometimes the idea must be justified according to the problem. By asking the following questions, the teacher may help students achieve the right vision (Polya, 1973):

1. Do you know or have an example of understanding the unknown?
2. Are you capable of resolving such a problem?
3. Take your time before recalling a similar problem

A bright idea may be achieved if critical thinking is always used in observing every problem, whereas creative thinking is developed via thinking (In'am, 2014). Besides the bright idea, plans can be developed through an 'auxiliary problem.' An auxiliary problem is a problem that we explore not for its own sake but in the expectation that it may help us address our original issue. Reaching the answer to the actual problem is the goal, and the auxiliary problem is the means by which we hope to get there. One of the benefits of the auxiliary problem is that it is fresh and provides undiscovered possibilities; we chose it because we are tired of the main problem, all methods that appear to have been exhausted (Polya, 1945).

Polya (1973) provides an example in which an inset tries to enter a room through a closed window, and the insect tries time after time but does not attempt the next open window through which it enters the room. Conversely, humans are more intelligent, and by using their brains, they address obstacles directly and indirectly. Therefore, in order to deal with a problem directly, we occasionally require an auxiliary example (Polya, 1973). Table 1 summarizes the benefits and risks of using auxiliary examples to solve the original problem.

Benefit	Risk
Helping in resolving the original problem	Wasting time and energy
Saving time	Decisions are made based on judgments
Giving insight into the original problem	There are no deciding factors for the auxiliary problem
Easy to follow the procedures	The use of equivalent problems has a risk
Easy and basic calculations	Difficulty in distinguishing between problems

Table 1. Benefits and risk of auxiliary example (Polya, 1973)

As a result, the most critical phase in problem-solving is devising a plan. This phase goes from the original problem to the auxiliary example and is known as convertible reduction, bilateral reduction, or equivalent reduction. Heuristics is another way to achieve a bright idea or an auxiliary problem. Heuristics are general guidelines for solving complex problems. The central aspect of problem-solving is solving the current issue in mathematics using a method that has been applied elsewhere (Polya, 1973).

The Heuristics are designed to assist students in thinking outside of the box while tackling an issue. Using heuristics enables students to explain their thought processes, evaluate them, and ultimately gain flexible thinking and problem-solving abilities that they may use in the future. Students should employ Polya's heuristics throughout the problem-solving process to assist them in building their critical thinking and reflective skills and support their interpretation of situations (Hensberry & Jacobbe, 2012). Heuristics play an essential role in developing a plan for solving mathematical problems; it aids in discovering a novel solution to the original challenge. Not all heuristics can be used to address a problem; in fact, we sometimes apply heuristics based on our own experiences or by observing others (Polya, 1973).

3.1.3. Step 3- Carrying out the plan

Following careful considerations in the previous step, what was chosen is now applied to obtain a solution. Creating a plan and conceiving an idea for a solution is a time-consuming process. However, carrying out the plan is more effortless. The plan specifies the general framework for implementation, and we must collect data to determine whether or not it is functioning. It is beneficial to the lecturer if the students construct a plan to address the problem since much of the work has been accomplished. The main risk at this point is that students may forget the plan, although this happens when they borrow it; otherwise, they will be satisfied with their plan. Students may get at the proper plan intuitively or formally, but every phase of the plan must be clear for implementation. While pupils carry out the procedure, the teacher must act as a facilitator (Polya, 1973).

3.1.4. Step 4- Looking Back

At this stage, students must confirm their answer by implementing it in a new situation, pursue new arguments and attempt to reconfirm their findings by substituting the known with the unknown. In order to strengthen their knowledge and improve the skills to solve such examples independently in their everyday living, students should re-evaluate and reassess the findings and process of solution. The teacher must clarify that this solution to the problem is not the final destination but a breakthrough that must be reached. To improve further, the teacher should ask the participants the following questions (Polya, 1973):

- Did you recheck the findings step by step?
- Did you understand all of the variables in terms of known and unknown?
- Can you arrive at a different conclusion?
- Are you able to apply this technique to other examples?
- Can you change the condition among the variables?
- Can you provide an example from your daily life?

3.2. Case of study

The research was conducted at the International School of Olomouc (iSchool). iSchool was established in 2003 as a new branch of the International School of Prague in Olomouc for the children of international managers working at the Philips Hranice factory. The school has nearly 90 students and offers international educational programs at three levels of preschool (2-5 years), primary (5-11 years), and lower secondary (11-15 years) (*International School Olomouc*, 2022).

Figure 3. International School of Olomouc (*International School Olomouc*, 2022)

3.3. Design of the study

This study used a qualitative descriptive approach based on observation, comprehension of meaning, and assessment. Qualitative methods are research processes that generate qualitative data, words, or notes of individuals or their observed behavior (Creswell & Guetterman, 2018). The following research tools were used in this study:

1. Test to assess students' mathematical problem-solving abilities
2. Interviews to assess the validity of students' written responses to the test
3. Documentation as evidence that the research was conducted

Interviews help the researcher to acquire a description of mathematical problem-solving abilities based on the student's cognitive style. The steps of solving mathematical questions are not always observable from the answer sheet, and not everything in the student's thoughts is stated on the exam paper, including the four steps of the Polya method. Interviews were conducted using an audio recorder. Pictures, videos, notes, and transcripts are different ways of documentation (Karlina, 2022).

3.4. Study group

The study population included students at the international school of Olomouc in the 2022-2023 academic year, with 60 participants in 8 different grades. Although all students were included in the selection process, the researcher had specific criteria in mind to ensure the study's effectiveness. These criteria included selecting students who have a relatively good understanding of the math lesson and possess effective communication skills when explaining their problem-solving approach.

3.5. Selection of content for the study and data collection

Word problem questions suitable for the research were chosen from different sources, including *Education.com*, 2023; *K12 Math Worksheets* 2023; *Math Only Math*, 2023; *Math-Drills*, 2023, which was in line with the Cambridge Assessment International Education. A sample of each grade questions are provided in Appendix A.

After collecting data from tests and interviews, the triangulation method was conducted. According to Natow (2020), triangulation can be used when the researcher employs more than one type of qualitative data collection approach, such as interviews, observation, and documents. Triangulation has been used for various purposes, including improving the validity of a study (reflecting a postpositivist viewpoint) or illuminating different perspectives and power imbalances (evidencing a more constructivist or critical framework).

3.6. Data analysis

The data analysis technique was used to analyze the outcomes of the students' responses according to the four-step Polya model. The four steps are as follows (Karlina, 2022):

1. Understanding the problem
   The student must recognize what is known and requested. Essential notes from diagrams, tables, graphs, or pictures can be provided to assist them in understanding the problem and getting a general perspective of its solution.
2. Devising a plan
   Students must be able to identify the relationship between data in one question and theorems or concepts taught and combine them to solve the problem. If they are stuck, they must be assisted in viewing the situation from an alternative perspective.
3. Carrying out the plan
   Each step in addressing the problem is checked to determine whether it is accurate. The findings must also be checked to ensure they are the desired outcomes.

4. Looking back

   The last step can be considered the most important part of problem-solving. After obtaining the completion results, they must be evaluated and double-checked to verify that no alternatives are ignored.

Based on the above steps, the researcher defined six indicators to assess the problem-solving abilities of participants.

Problem-solving indicators based on the Polya model	**Indicators code**
1. Understanding the problem	Y1: Writing the information provided in the problem
	Y2: Writing the problem statement based on the given information
2. Devising a plan	Y3: Developing the mathematical models or equations required to solve the problem.
3. Carry out the plan	Y4: Implementing the problem-solving strategy
	Y5: Performing accurate calculations
4. Looking back	Y6: Presenting a clear and accurate conclusion

Table 2. Problem-solving indicators

Chapter 4: Results

4.1. Overview of study

The study was carried out at the British school in Olomouc, where the total number of participants amounted to 60 individuals. The age range of the participants spanned from 3rd to 9th grade, with the highest number of students in the 6th year and the lowest in the 9th year. The pie chart below represents the percentage distribution of students across each grade.

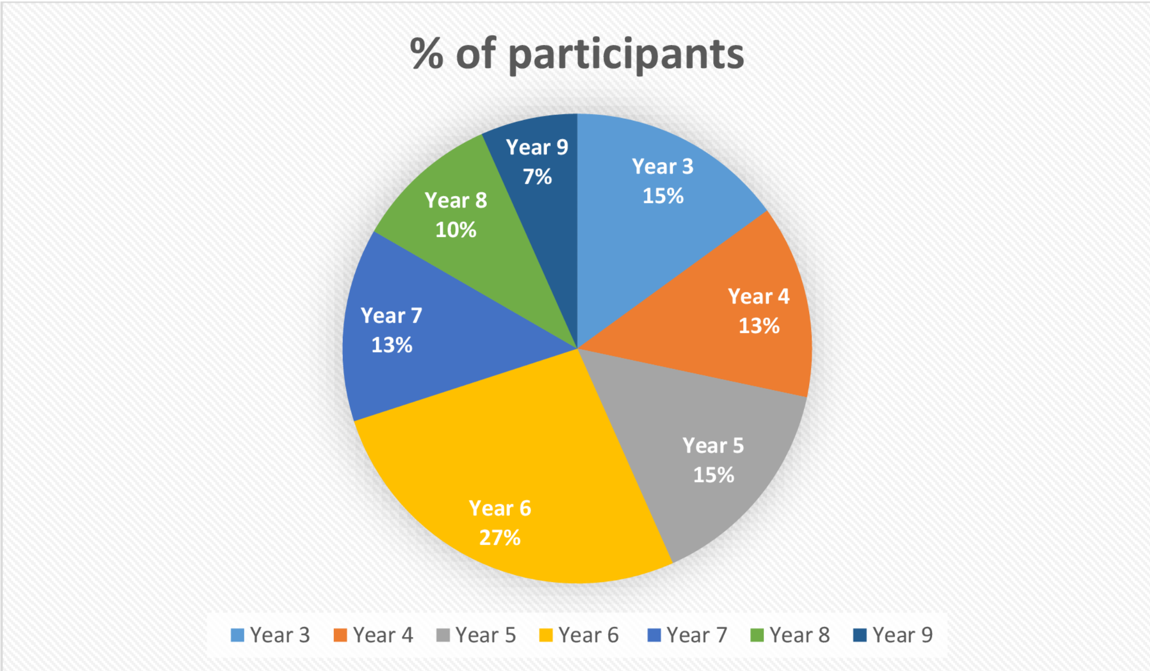


Figure 4. Percentage of students participating in the study

During a two-week period, the teacher-researcher prepared word problem questions tailored to each grade's cognitive level and mathematical knowledge. Subsequently, a period of one month was dedicated to the collection of data from all 60 participants in the study and followed by interviews with students.

Figure 5. Recorded documents during the study procedure (Captured by the author)

4.2. Description of the Results

Data collected from the questions were analyzed for the number of correct answers, and the score was obtained using the problem-solving indicators. In light of the equitability of each indicator (Nardo et al., 2005), uniform weighting was applied to all indicators, whereby a student who employs all problem-solving indicators for a given question will attain a score of 6. Using an instrument scale criterion of 0-1, as presented in the table below:

Score	Criteria
1	Correct use of an indicator in a correct manner: either in written form or oral communication
0	No use of an indicator

Table 3. Scoring criteria

The interviews were conducted randomly during the exam to determine the intentions and problem-solving strategies of the students. The teacher-researcher recorded their observations in a journal which were based on both verbal and non-verbal communications. The total score obtained from the assessment was converted into a percentage to enable comparison of the students' performance in meeting Polya's problem-solving steps. The outcomes will be presented for each grade level individually, followed by a comprehensive result later.

4.2.1. Results of Year 3

Based on the results of 10 questions provided to 9 students in year 3, it can be concluded that the students have a good understanding of mathematics, with 81 correct answers out of 100 responses. The bar chart below compares the number of correct and incorrect answers for each student.

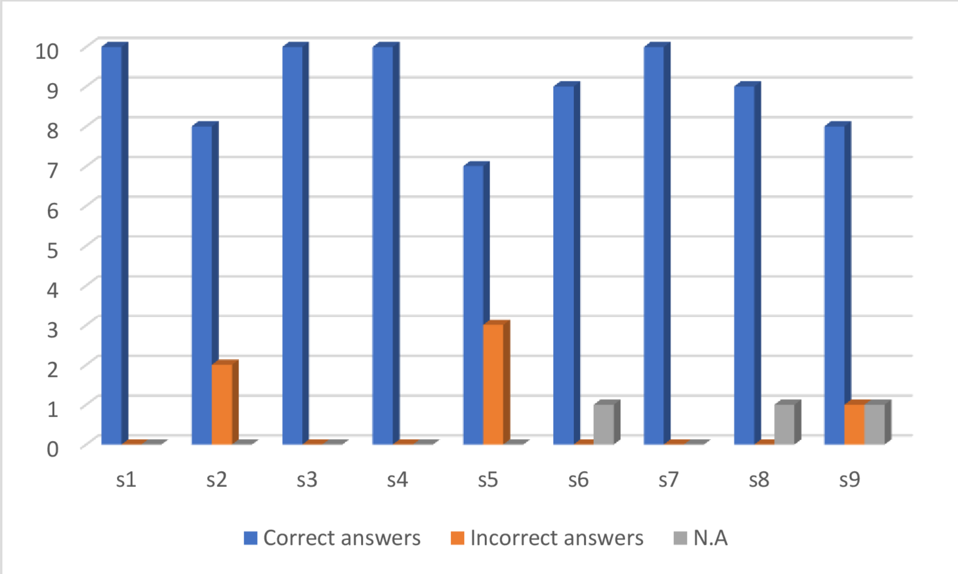


Figure 6. Number of correct, incorrect, and no answer of 9 students in the third year

After analyzing the problem-solving indicators of the Polya method, it was found that the two highest scores were 26, while the lowest score was 7. The research results indicate that the subjects could not achieve even 50% of the problem-solving goals based on the Polya method (as determined by the indicators presented in Table 3). After conducting a detailed investigation of each question, it was discovered that the students performed relatively well in devising a plan (Y3) and carrying out the plan (Y4 and Y5). The table below illustrates the strengths and weaknesses of students in year three in solving mathematical problems using the Polya method.

	Y1	Y2	Y3	Y4	Y5	Y6
Question 1	0	0	4	4	9	0
Question 2	0	0	4	4	5	0
Question 3	0	0	3	3	7	0
Question 4	0	0	4	4	8	0
Question 5	0	0	4	4	8	0
Question 6	0	0	4	4	9	0
Question 7	0	0	1	1	9	0
Question 8	0	0	0	0	7	0
Question 9	0	0	4	3	6	0
Question 10	0	0	3	3	9	0
Overall	0	0	31	30	77	0

Table 4. Frequency of each indicator for each question used by students of year three

After analyzing the strengths and weaknesses of students in each question, the researcher conducted a further comprehensive analysis of the subjects' answers to gain a deeper understanding of the underlying causes of the mistakes made across various questions.

Sample 1: The student's problem-solving approach (Y3) was deemed inappropriate as she subtracted the number of pizzas from the number of slices instead of multiplying the number of slices per pizza by the number of pizzas. This led to an incorrect solution.

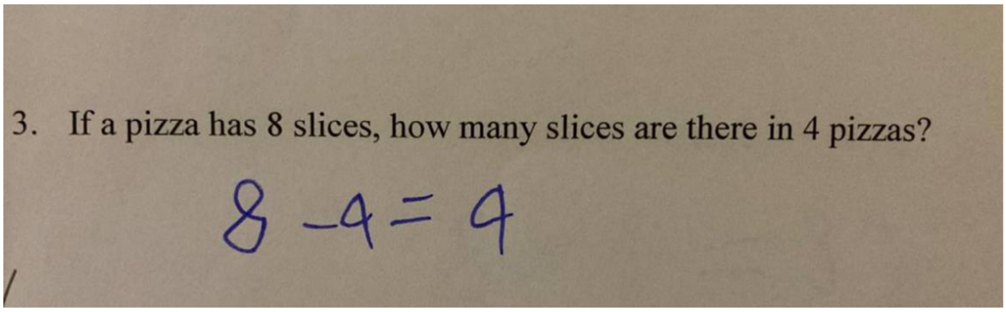


Figure 7. Sample of an incorrect solution in the third year, 1 (Captured by the author)

Sample 2: The student made an error in comprehending the question (Y2), as the question asked about the total number of sold cupcakes, but the student mistakenly thought it was asking about the number of cupcakes left.

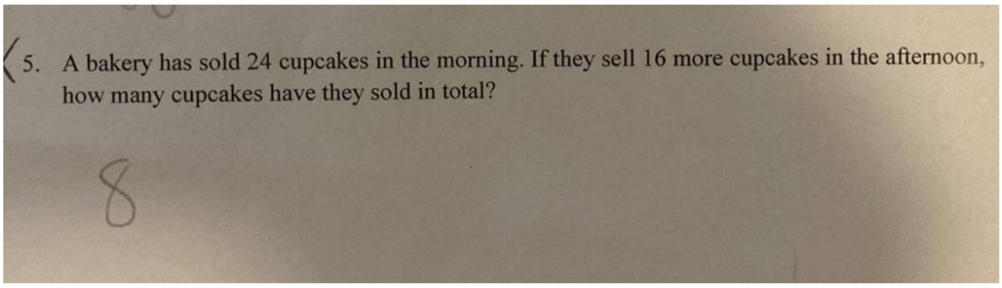


Figure 8. Sample of an incorrect solution in the third year, 2 (Captured by the author)

Sample 3: The student encountered difficulty with the first step of recognizing the pattern (Y1), which subsequently hindered her progress in solving the problem.

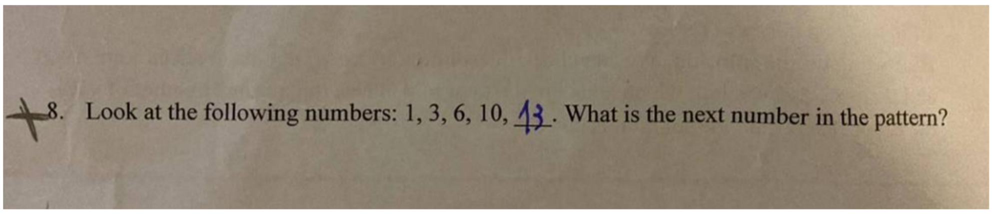


Figure 9. Sample of an incorrect solution in the third year, 3 (Captured by the author)

Sample 4: The student successfully devised a plan to solve the question correctly but was unable to execute the procedure, resulting in an inability to arrive at the final answer.

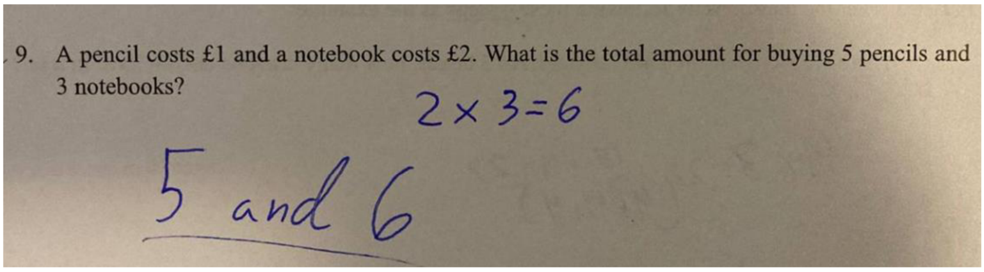


Figure 10. Sample of an incorrect solution in the third year, 4 (Captured by the author)

4.2.2. Results of Year 4

A cohort of eight students in year four attempted to answer ten questions; however, none of them managed to answer all of them correctly. A visual representation of the number of accurate and inaccurate responses is provided in the figure below to offer an overview of their performance.

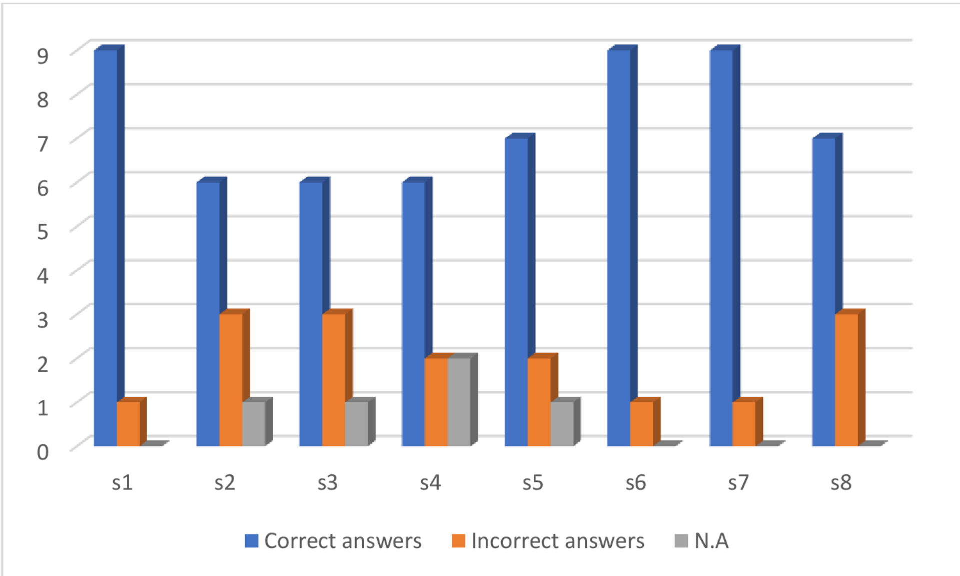
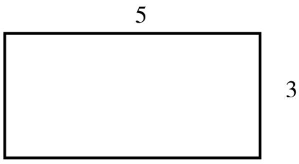


Figure 11. Number of correct, incorrect, and no answer of 8 students in the fourth year

Based on the problem-solving indicators, the students achieved a maximum and minimum percentage of 23.33 and 10, respectively. Following the preliminary analysis, the researcher investigated the answers in more detail and discovered that the question below posed the most significant challenge for the students.

Question 10. Here is a rectangle:



A larger rectangle is made using three of the same rectangles. Calculate the perimeter of the larger rectangle.

The majority of students failed to utilize any of the steps outlined in Polya's problem-solving model, resulting in only one out of eight students accurately solving the problem. Upon analyzing written responses and conducting random interviews, the researcher determined that the lack of comprehending the problem and devising a plan were the primary cause of the mistake.

In *Sample 1*, for instance, the student accurately comprehended the problem statement but faced difficulties in the "Devising a Plan" phase of Polya's problem-solving model (Y3). The student was unable to create a precise mathematical model (the perimeter of a rectangle), which consequently led to incorrect calculations, procedures, and answers.

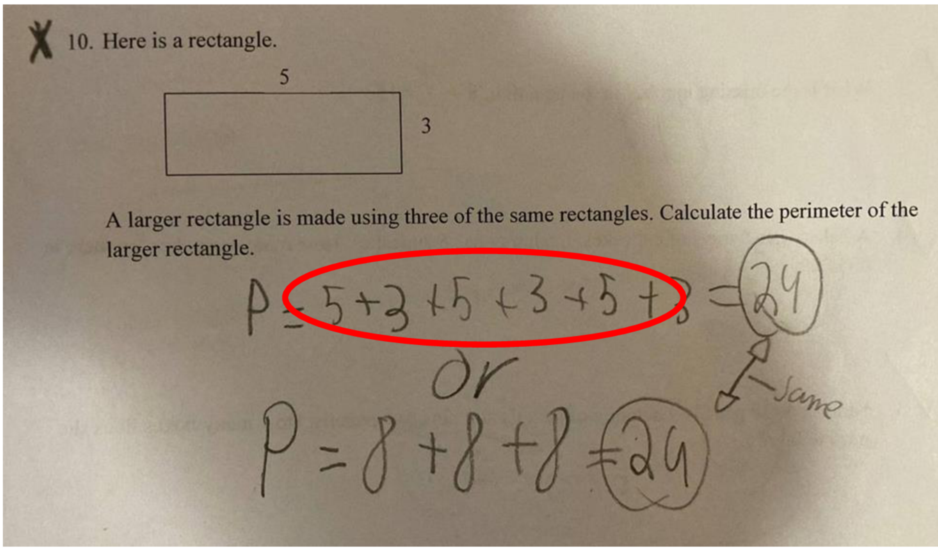


Figure 12. Sample of an incorrect solution in the fourth year, 1 (Captured by the author)

The sample mentioned above underwent an interview as follows:

> M: David, can you explain this problem to me, please? What is the problem about, and what is asking for?
>
> D: In this problem, we have a rectangle. Its length is 5, and its width is 3.
>
> M: Great, now can you tell me what we are supposed to calculate?

D: (Thinking for about 30-40 seconds and trying to find words to explain for me)… we should calculate the perimeter of…of…

M: of this rectangle? (Pointing to the image)

D: No, larger than this.

M: and do we know how much bigger?

D: Yesss…(checking one more time by moving the pencil under the lines)… 3 times bigger (while pointing out the correct part)

M: Well done, David, then?

D: The perimeter of the smaller rectangle is 5 plus 3 equals 8. I should add it 3 times, so 8 plus 8 plus 8 equals 24.

M: Mmm, David, what is the perimeter?

D: Perimeter is the total distance around a shape.

M: Very good, and how can you calculate the perimeter of a shape?

D: We have to add the length of all the sides.

M: So, can you check your calculations once again?

D: (Looking at the rectangle carefully), oooooohhhh, the perimeter is 5 plus 5 plus 3 plus 3.

M: Which equals?

D: … 16

M: Ok, and the perimeter of the larger one?

D: 16 plus 16 plus 16…(writing down the numbers, using place value to add them)… 48.

M: That's it.

Sample 2. In the same question, a different student utilized only the visual information provided and disregarded the accompanying textual information. This would be considered as understanding the problem (Y1) as the student could identify and use the relevant information to attempt a solution.

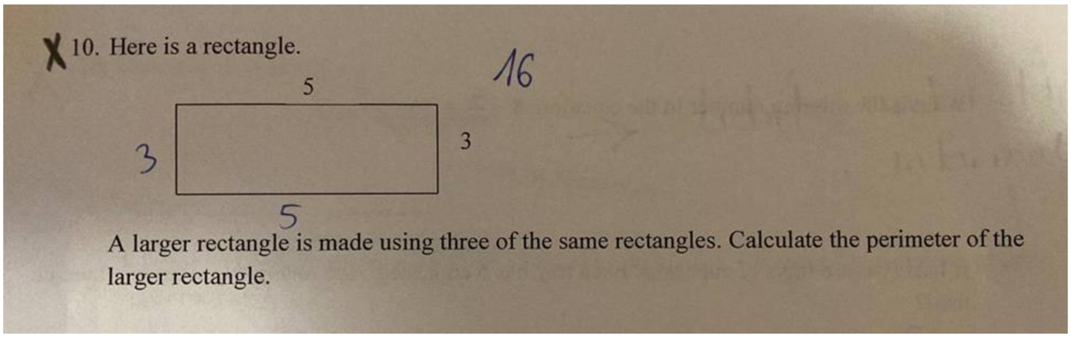


Figure 13. Sample of an incorrect solution in the fourth year, 2 (Captured by the author)

According to the table, it can be observed that fourth-grade students demonstrated limited use of Polya's problem-solving model indicators, particularly the first and second indicators. However, the data reveals that they were more skilled in the fifth indicator, which suggests their primary focus was on accurately performing mathematical computations.

	Y1	Y2	Y3	Y4	Y5	Y6
Question 1	0	0	1	1	7	3
Question 2	0	0	0	0	8	0
Question 3	0	0	0	0	8	0
Question 4	0	0	1	1	8	2
Question 5	0	0	2	2	7	1
Question 6	0	0	2	2	4	0
Question 7	1	1	1	1	7	3
Question 8	0	0	0	0	6	0
Question 9	0	0	0	0	3	0
Question 10	0	0	0	0	1	0
Overall	1	1	7	7	59	9

Table 5. Frequency of each indicator for each question used by students of year four

Sample 3. Both Y1 and Y2 indicators are zero since there is no mention of the problem's information and statement. The diagrams presented merely illustrate the student's process of devising and executing a solution for the problem.

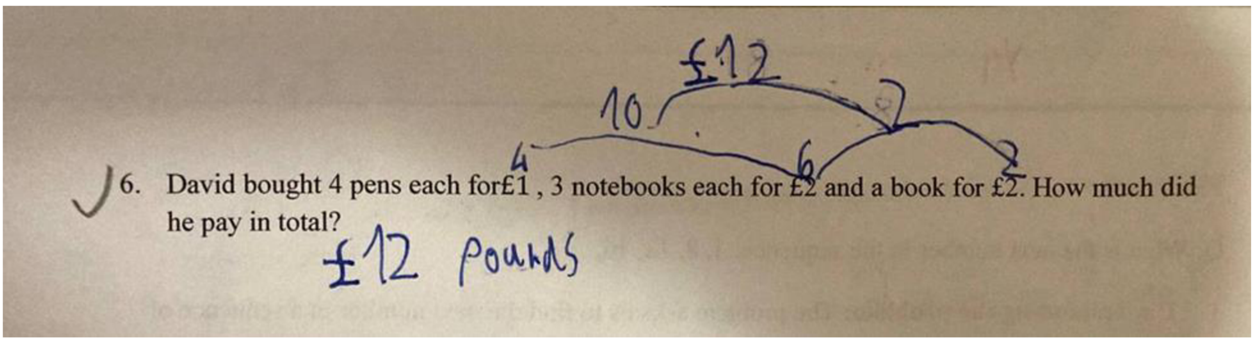


Figure 14. Sample of a correct solution in the fourth year, 3 (Captured by the author)

Sample 4. The answer shows the mistake in the first indicator, Y1, as the student failed to comprehend the information presented in the problem statement regarding "writing different numbers." Following a further inquiry from the researcher and a request to review the question and answer, the student realized that she had missed crucial information while reading the problem.

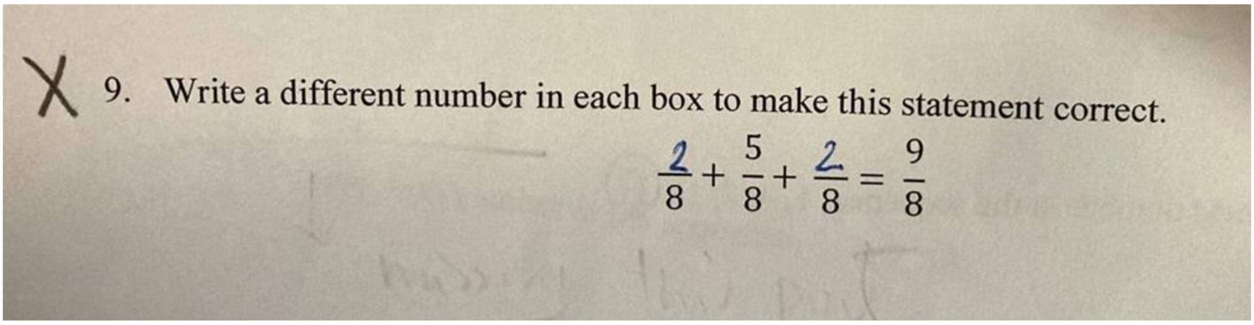


Figure 15. Sample of an incorrect solution in the fourth year, 4 (Captured by the author)

4.2.3. Results of Year 5

Out of the nine participants in the fifth grade, five pupils were able to provide accurate responses to all ten questions.

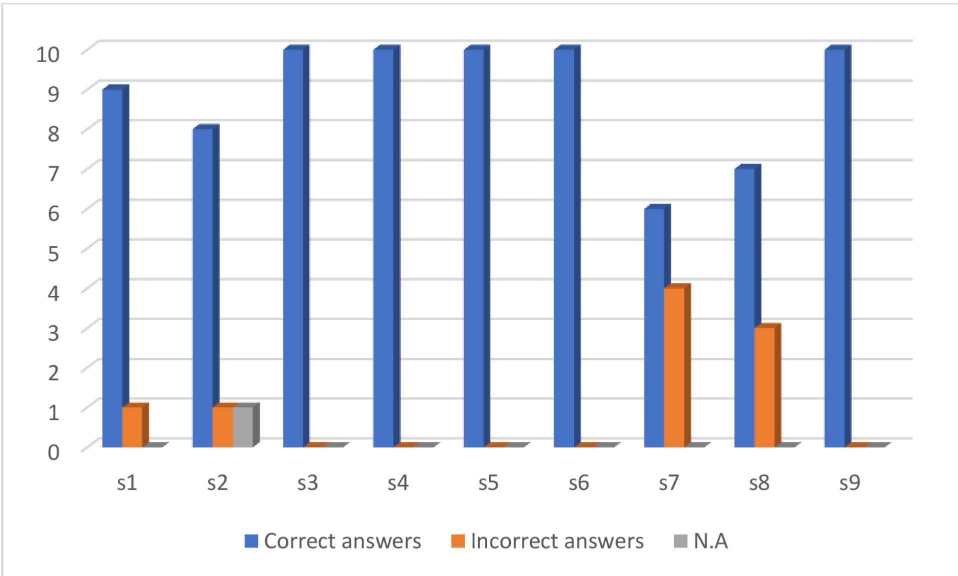


Figure 16. Number of correct, incorrect, and no answer of 9 students in the fifth year

Following the promising initial outcomes, the researcher was keen to investigate whether the pupils in this cohort employed more problem-solving indicators based on the Polya model compared to previous grades. To the researcher's surprise, two individuals achieved a score of

91.6%, while the lowest score was 16.6%. It was observed that the majority of students in this grade followed more steps outlined in the Polya model, indicating a higher level of proficiency in problem-solving skills.

	Y1	Y2	Y3	Y4	Y5	Y6
Question 1	2	2	2	4	7	2
Question 2	2	2	6	6	9	5
Question 3	2	2	6	5	8	3
Question 4	2	2	4	4	7	3
Question 5	2	2	6	6	9	5
Question 6	0	0	7	7	7	7
Question 7	0	0	2	3	7	0
Question 8	2	2	4	4	9	3
Question 9	2	2	5	5	6	3
Question 10	4	3	4	5	8	3
Overall	18	17	46	49	77	34

Table 6. Frequency of each indicator for each question used by students of year five

According to the final results, the analysis reveals that a significant number of students have demonstrated a strong understanding of mathematical problem-solving. Specifically, they have demonstrated the ability to formulate an appropriate plan of action, implement it effectively, and competently perform relevant mathematical operations accurately, thereby highlighting their proficiency in this area.

Sample 1: Student number one had only one mistake, prompting the researcher to request an explanation of her response. Upon examination, it became apparent that she had misinterpreted the question and believed the question was asking about the fraction of total balls over blue balls. This misunderstanding, referred to as Y2, led to the mistake mentioned above.

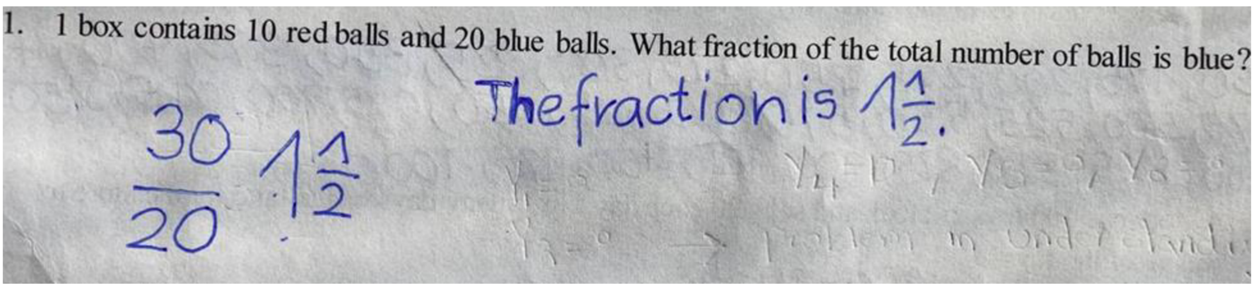


Figure 17. Sample of an incorrect solution in the fifth year, 1 (Captured by the author)

Sample 2: The student in question has successfully developed a mathematical model and devised a plan, despite omitting the information outlined in the question (Y1) and the question statement (Y2). The student carried out their plan accurately and arrived at the correct final answer. However, he has not mentioned his conclusion clearly.

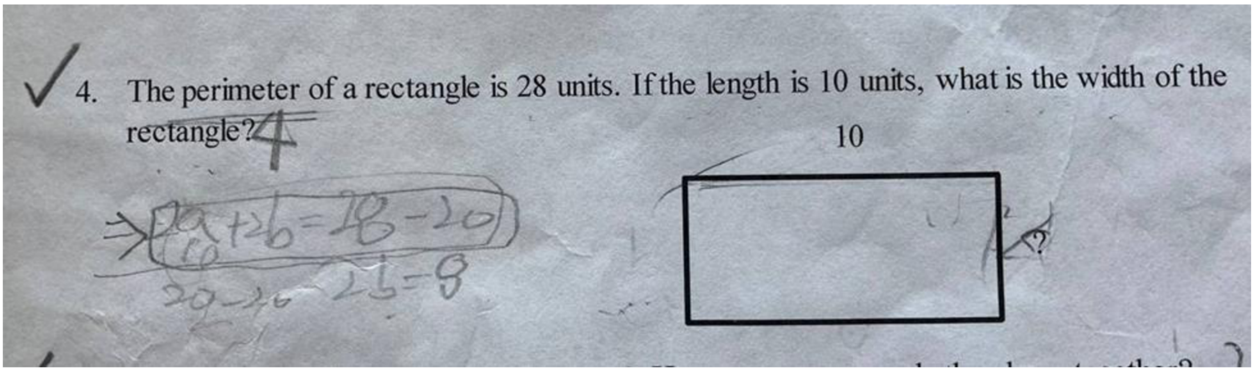


Figure 18. Sample of a correct solution in the fifth year, 2 (Captured by the author)

Sample 3: Student number four successfully developed and implemented a plan (Y3, Y4, and Y5) to address the problem; however, the student neglected to incorporate indicators related to comprehension of the problem and reviewing their work (Y1, Y2, and Y6). Consequently, this resulted in a loss of points for these indicators in the present question.

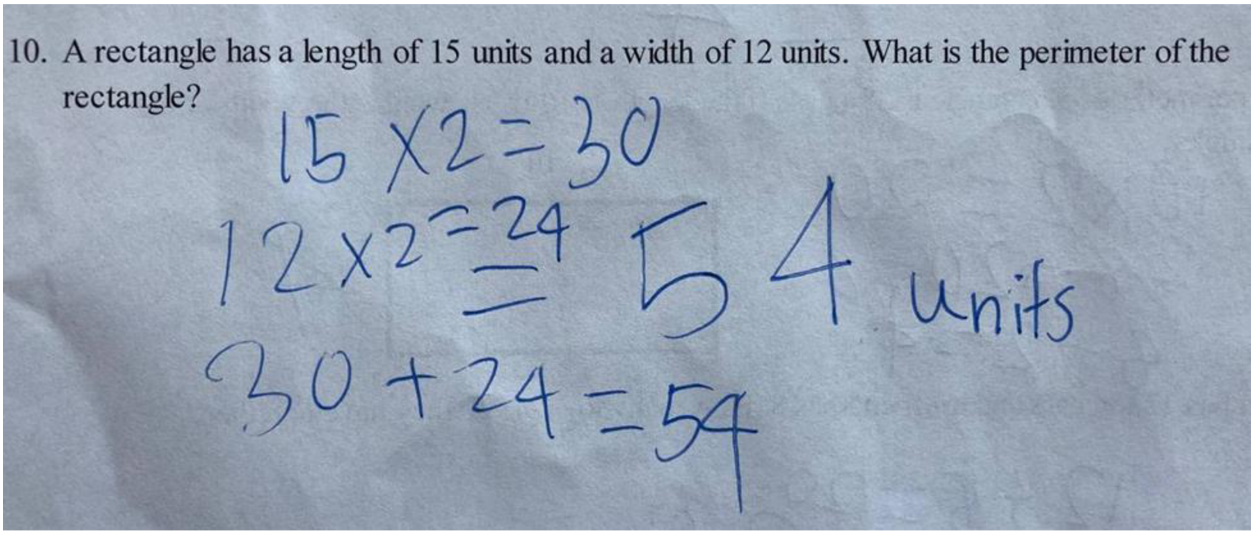


Figure 19. Sample of a correct solution in the fifth year, 3 (Captured by the author)

Sample 4: This sample is a remarkable example of the successful implementation of the Polya problem-solving model. By expertly following each step of the model, the student achieved a perfect score of 6 out of 6 for the given question, highlighting their exceptional competency in utilizing this methodology.

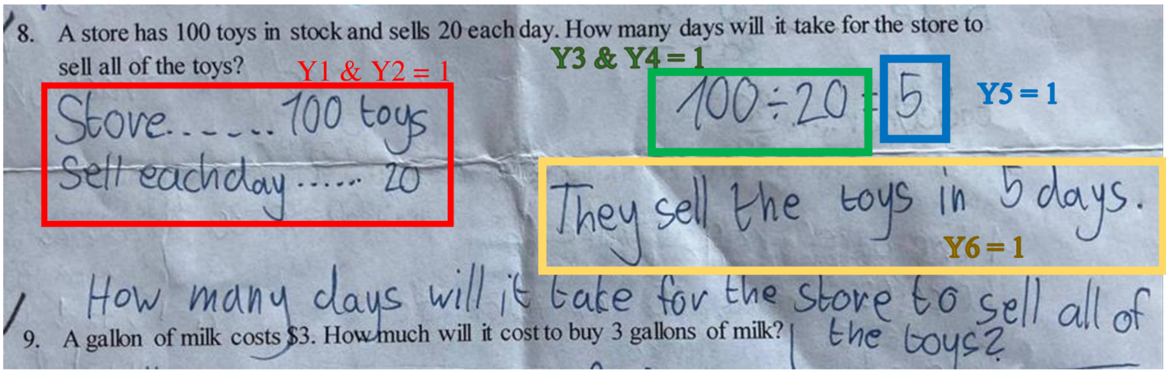


Figure 20. Sample of a correct solution in the fifth year, 4 (Captured by the author)

4.2.4. Results of Year 6

This particular class constituted 27% of the total participants and, with 16 students, it holds the title of the largest class in the school. It is noteworthy that only one student was able to successfully solve all of the questions provided, and interestingly, two students answered more questions incorrectly than correctly.

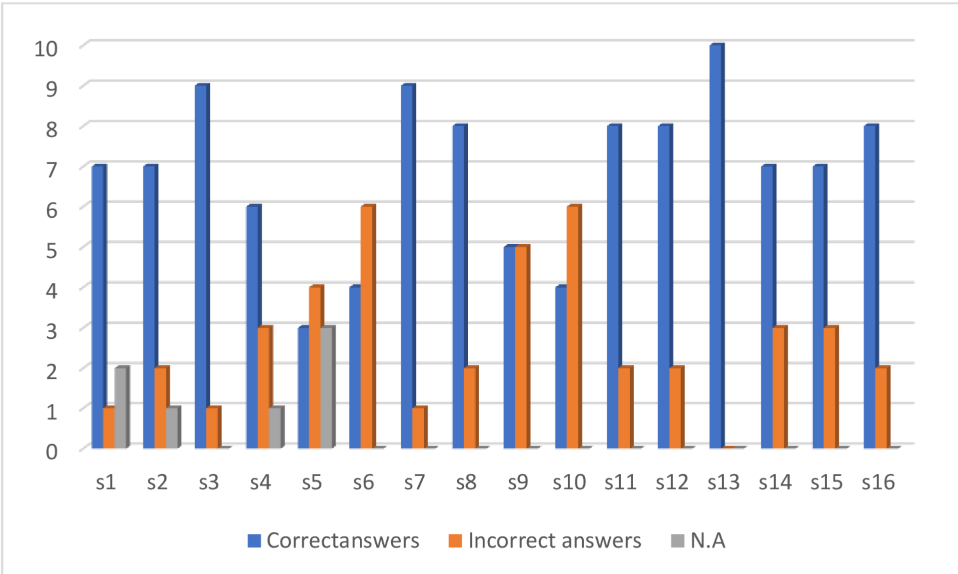


Figure 21. Number of correct, incorrect, and no answer of 16 students in the sixth year

Within this grade, an average of 44.5% of six problem-solving indicators were utilized, with 11 students achieving a score above this average. The highest score attained was 60, while the lowest score recorded was 10. The researcher conducted further examination of the results, and the findings are presented in the following table.

	Y1	Y2	Y3	Y4	Y5	Y6
Question 1	0	0	15	13	13	9
Question 2	0	0	15	15	16	9
Question 3	0	0	15	15	16	12
Question 4	5	0	9	9	9	5
Question 5	0	0	11	1	2	0
Question 6	0	0	10	8	8	2
Question 7	1	1	16	16	16	14
Question 8	2	3	12	12	13	8
Question 9	0	0	10	10	11	9
Question 10	0	0	10	7	9	5
Overall	8	4	123	106	113	73

Table 7. Frequency of each indicator for each question used by students of year six

Sample 1. The following sample proved to be challenging, with 14 individuals answering it incorrectly and only two able to provide a partial answer. The difficulty in answering the question did not stem from its complexity but rather from misreading the problem, leading to a fundamental misunderstanding of the task at hand. Precisely, students encountered difficulty with the first step of Polya's problem-solving method, indicating a lack of full comprehension regarding the nature of the question.

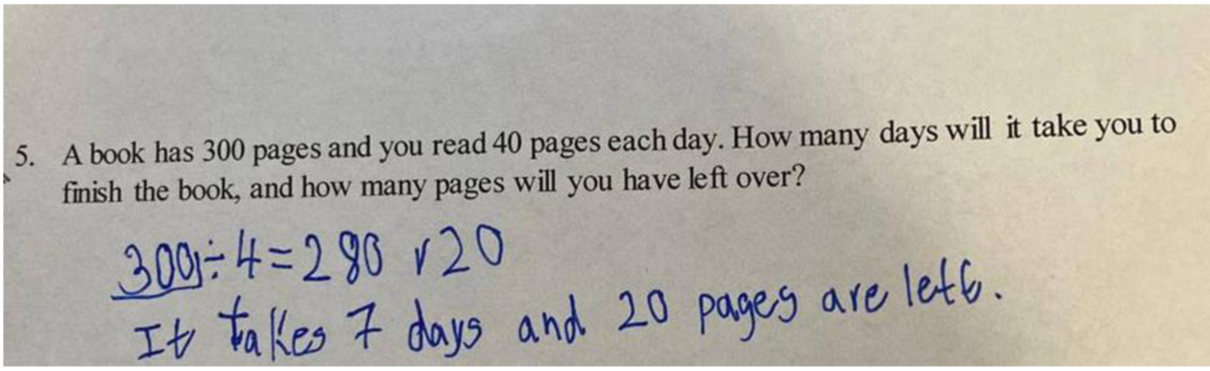
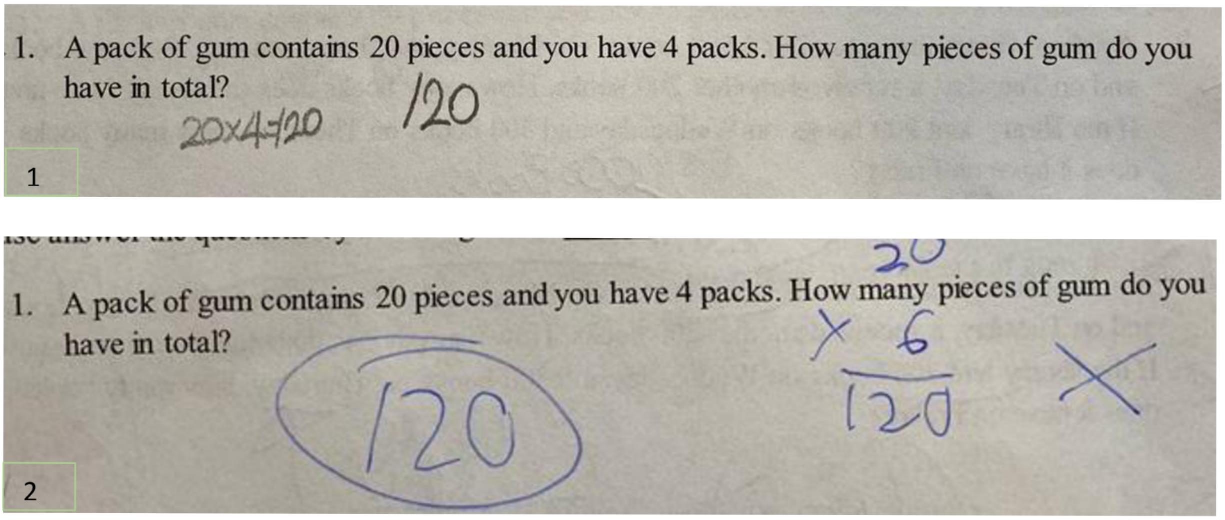


Figure 22. Sample of an incorrect solution in the sixth year, 1 (Captured by the author)

Sample 2. Two students answered the first question incorrectly, prompting the researcher to compare their responses. Upon review, it was discovered that both students failed to write the problem's information and the question's statement in their answers, resulting in an incorrect response from the second student. Although both students had devised a plan to correctly multiply, the first student executed the calculations incorrectly. If the students had taken the time to review their responses, they might have recognized their mistakes.


1. A pack of gum contains 20 pieces and you have 4 packs. How many pieces of gum do you have in total?

20x4=120 120

1

1. A pack of gum contains 20 pieces and you have 4 packs. How many pieces of gum do you have in total?

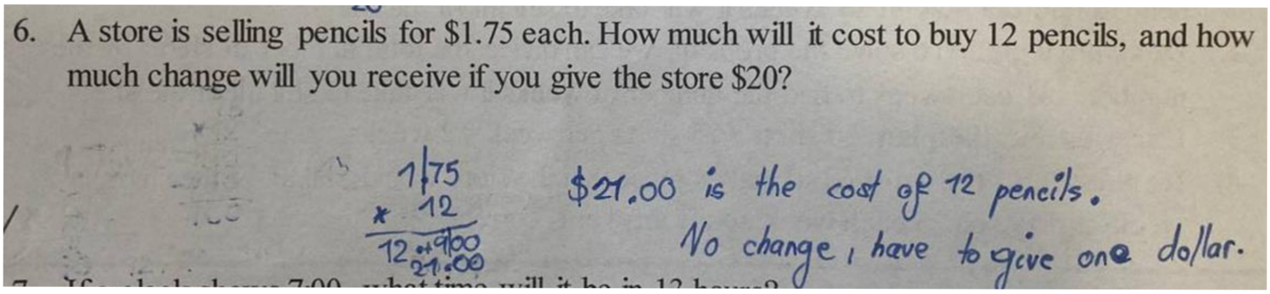
120

20 × 6 = 120

2

Figure 23. Samples of an incorrect solution in the sixth year, 2 (Captured by the author)

Sample 3. Despite six students correctly identifying the first part of the problem, they left the second part blank due to a lack of experience in evaluating their answers (Y6). In contrast, one student performed the procedure accurately, checked their answer for accuracy, and successfully answered the second part by providing a correct conclusion.


6. A store is selling pencils for $1.75 each. How much will it cost to buy 12 pencils, and how much change will you receive if you give the store $20?

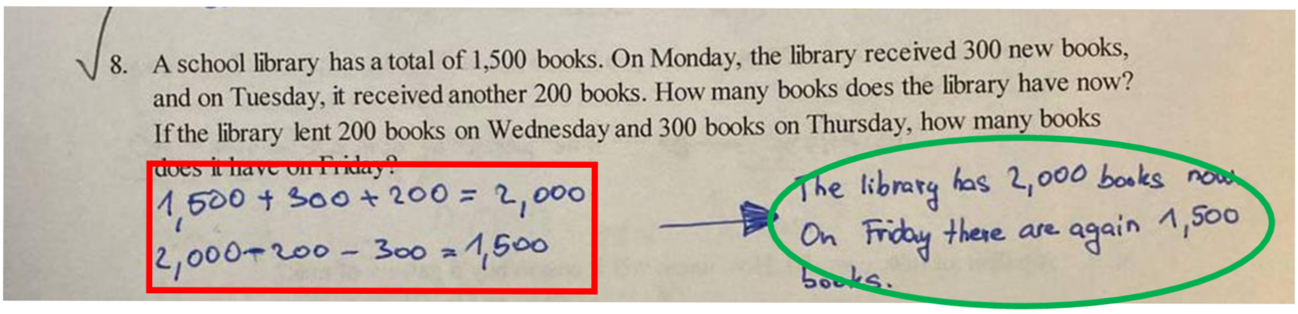
1.75 × 12 = 21.00

$21.00 is the cost of 12 pencils.

No change, have to give one dollar.

Figure 24. Sample of a correct solution in the sixth year, 3 (Captured by the author)

Sample 4. The student's response indicates that they thoroughly followed all four steps of the Polya method, starting with a clear understanding of the problem, devising a plan, executing the procedure accurately, and finally reflecting on the solution by checking their answer.


8. A school library has a total of 1,500 books. On Monday, the library received 300 new books, and on Tuesday, it received another 200 books. How many books does the library have now? If the library lent 200 books on Wednesday and 300 books on Thursday, how many books does it have on Friday?

1,500 + 300 + 200 = 2,000

2,000 − 200 − 300 = 1,500

The library has 2,000 books now. On Friday there are again 1,500 books.

Figure 25. Sample of a correct solution in the sixth year, 4 (Captured by the author)

Sample 5. The respondent appeared to be confused about the given information and the inquiry (Y1 & Y2). Upon further inquiry, it was revealed that the respondent had difficulty interpreting the phrase *'a number is increased by'* due to a lack of understanding of the term 'increase' (Y1). Additionally, the respondent's notes indicated a lack of comprehension regarding what was being asked and what was expected of them, leading to an inability to use a suitable approach to solve the problem.

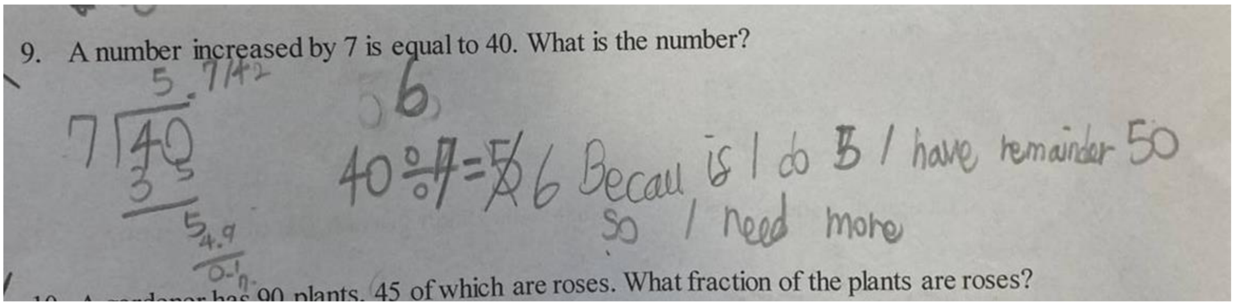


Figure 26. Sample of an incorrect solution in the sixth year, 5 (Captured by the author)

Sample 6. The student demonstrated proficiency in the first step of the Polya method by comprehending and paraphrasing the given information in their response. However, such cases may arise due to either a misunderstanding of the problem (Y2), resulting in an incorrect approach to the question, or an inability to identify a suitable strategy or technique and a lack of necessary knowledge (Y3). Upon inquiry, it was revealed that the student lacked understanding of fractions despite being taught previously, which impeded their progress in solving the problem.

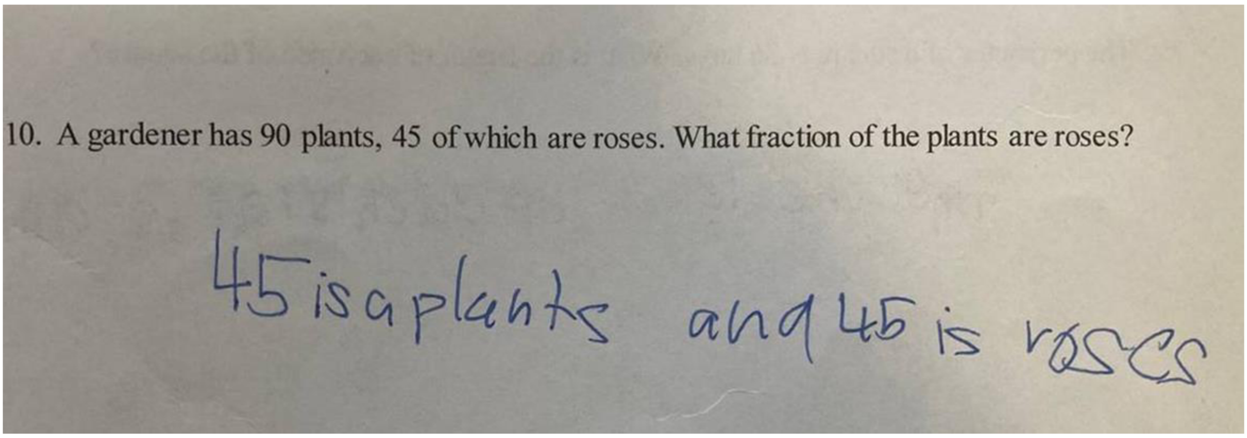


Figure 27. Sample of an incorrect solution in the sixth year, 6 (Captured by the author)

4.2.5. Results of Year 7

The 7th grade has a total of 8 individuals, out of which seven students have provided correct responses for at least 50% of the questions.

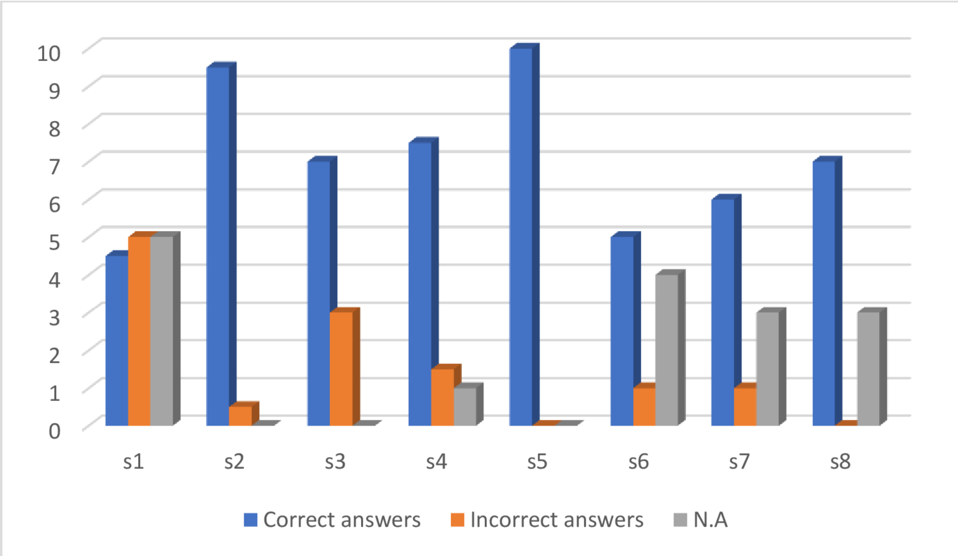


Figure 28. Number of correct, incorrect, and no answer of 8 students in the seventh year

The class had an average utilization rate of 26.6% for the problem-solving indicators. Student number five, who answered all questions correctly, obtained the highest score of 33. Conversely, student number one used only 10% of the indicators. Upon analyzing each question in depth, it was revealed that Y3, Y4, and Y5 indicators were used more often, while other indicators were utilized less frequently.

	Y1	Y2	Y3	Y4	Y5	Y6
Question 1	1	1	4	4	6	1
Question 2	1	1	5	5	8	2
Question 3	2	1	4	3	4	2
Question 4	0	0	3	3	8	0
Question 5	0	0	1	1	8	0
Question 6	0	0	2	2	4	0
Question 7	0	0	2	5	8	0
Question 8	0	0	2	2	4	0
Question 9	1	1	1	1	4	0
Question 10	0	0	2	3	3	2
Overall	5	4	26	29	57	7

Table 8. Frequency of each indicator for each question used by students of year seven

Sample 1. The individual recorded the cuboid's length, width, and height as required information (Y1) and indicated that they were asked to compute the cuboid's volume (Y2).

However, they were unable to proceed with finding the answer due to forgetting the formula for calculating the volume.

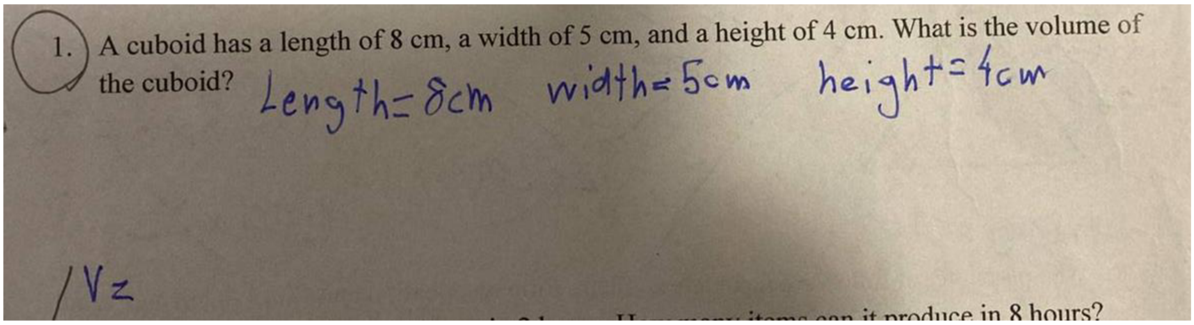


Figure 29. Sample of a student unable to devise a plan in the seventh year, 1

Sample 2. At first, it seemed like the student faced difficulties understanding the problem (Y2) and finding a strategy to solve it (Y3). However, during the interview, it was discovered that she had overlooked a crucial aspect of the problem (Y1) because of inadequate attention to the instructions.

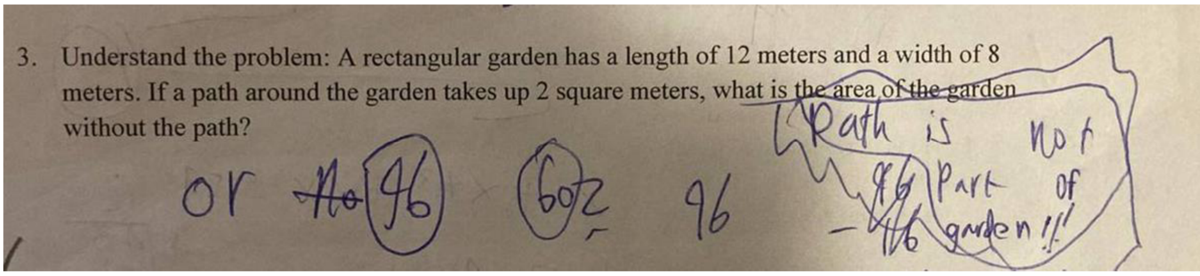


Figure 30. Sample of an incorrect solution in the seventh year, 2 (Captured by the author)

An interview was conducted regarding the sample mentioned above:

M: Anni, your answer is 96. Can you explain to me how did you get that, please?

A: 12 times 8 equals 96.

M: and why did you decide to multiply these two numbers?

A: Because it is a rectangle miss Maryam, and the area of the rectangle is length times width.

M: Good, our garden is a rectangle, and we can find its area, as you mentioned. But what does the question ask us? The area of the garden?

A: Yes

M: Can you read this part for me? (Pointing to the last part of the question)

A: What is the area of the garden without the path?

M: So ‘the area of the garden’ and ‘the area of the garden without a path’ are the same?

A: Mmmmmm, I don’t know. It’s confusing. It might be a tricky point because the path is not part of the garden.

M: Well, Anni, don’t make it too complicated. Here, in this question, we assume it’s part of the garden. With this information, tell me, what should you do to find the area of the garden without the path?

A: I think I should subtract the path area from the total area. But we don’t have any information about the path’s sides to find its area.

M: Are you sure? Let’s read the question again, loudly.

A: A rectangular garden has a length of 12 meters and a width of 8 meters. If a path around the garden takes up 2 square meters….. AAAAAA, I got it miss Maryam. The area of the path is 2. I’m so stupid. I didn’t read this part before.

M: Are you sure 2 is the area of the path? For example, it’s not a side of that?

A: Yes, miss Maryam, because it says 2 square meters. Square meters is the unit of the area.

M: Well done. Now tell me what is the final answer?

A: The total area which is, let me find it, Aha 96 square meters minus 2 square meters, 94. The answer is 94 miss Maryam.

M: Thank you

Sample 3. The student's assumption that part (b) of the problem was similar to part (a) without thoroughly reading and comprehending part (b) led to oversight of the relevant information and question statement. Consequently, an incorrect plan leads to the wrong problem–solving.

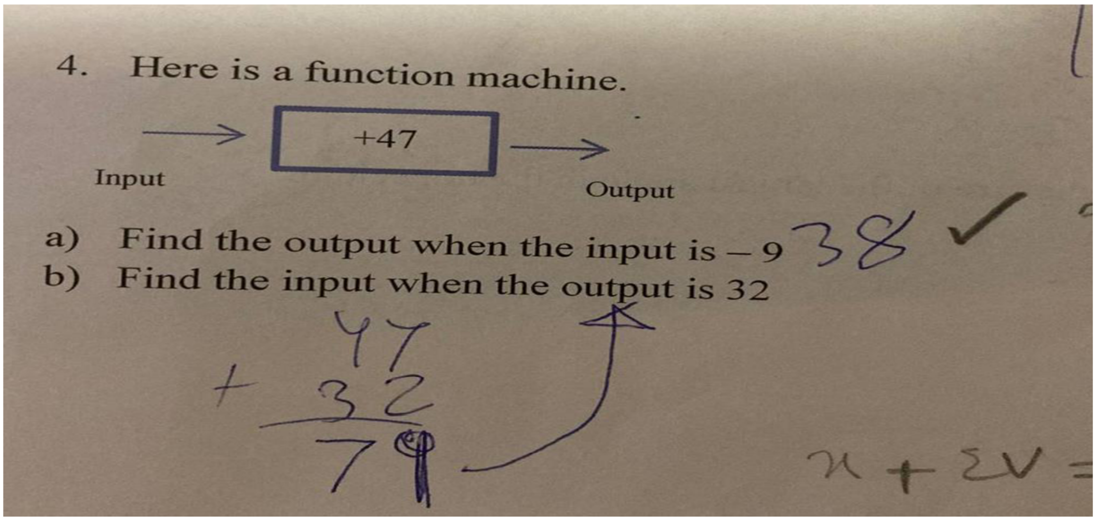


Figure 31. Sample of an incorrect solution in the seventh year, 3 (Captured by the author)

Sample 4. The student's method of arriving at the final solution was deemed incorrect since they did not pay attention to the correct order of operations (Y4) and struggled to perform accurate calculations (Y5).

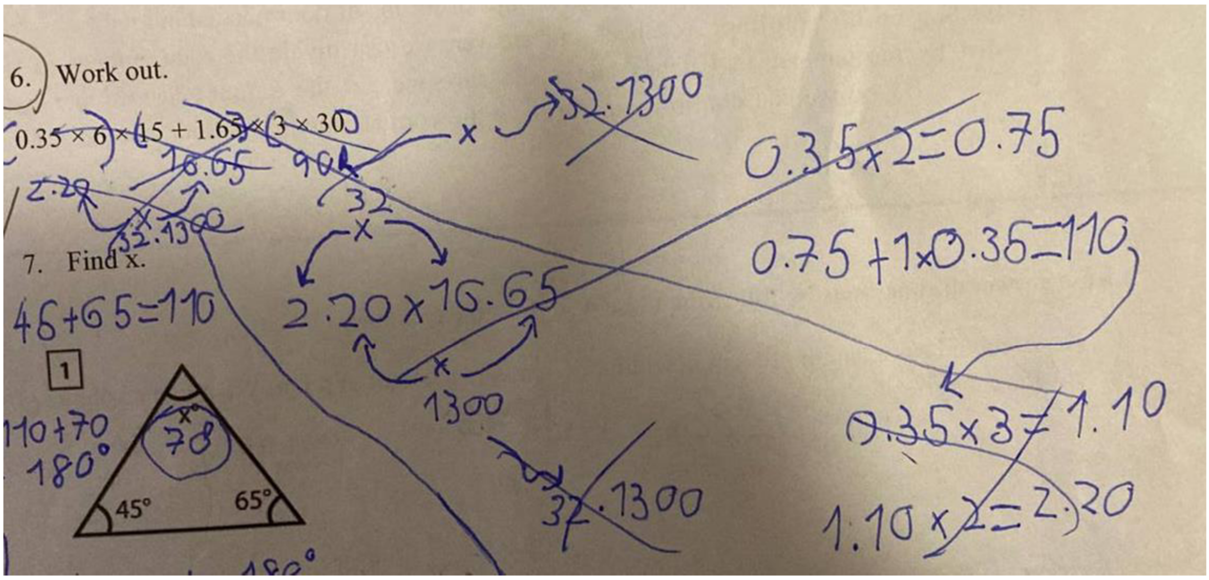


Figure 32. Sample of an incorrect solution in the seventh year, 4 (Captured by the author)

Sample 5. It is evident that the student encountered challenges in comprehending the information related to the rectangle's perimeter (Y1) and realizing that they were asked to determine the length and width of the rectangle (Y2).

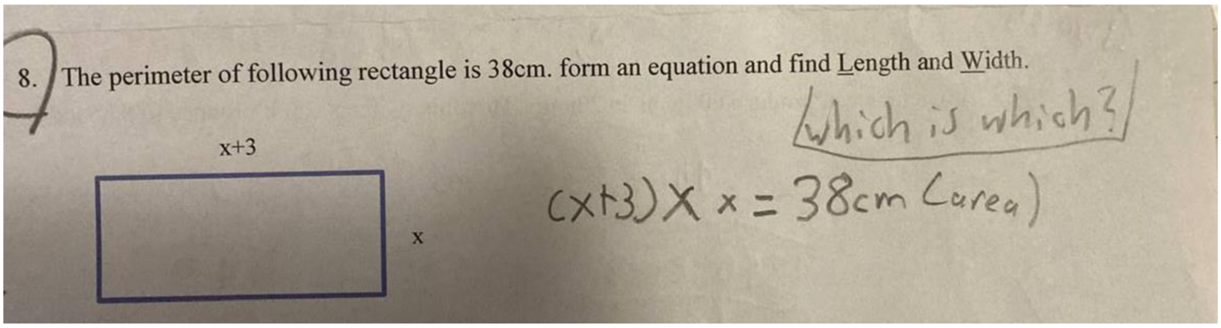


Figure 33. Sample of an incorrect solution in the seventh year, 5 (Captured by the author)

4.2.6. Results of Year 8

This class with six students enrolled in the 8th grade, is the second smallest among all the classes in the school. On average, the academic performance of this class is good, except two students answered only 20% of the questions correctly.

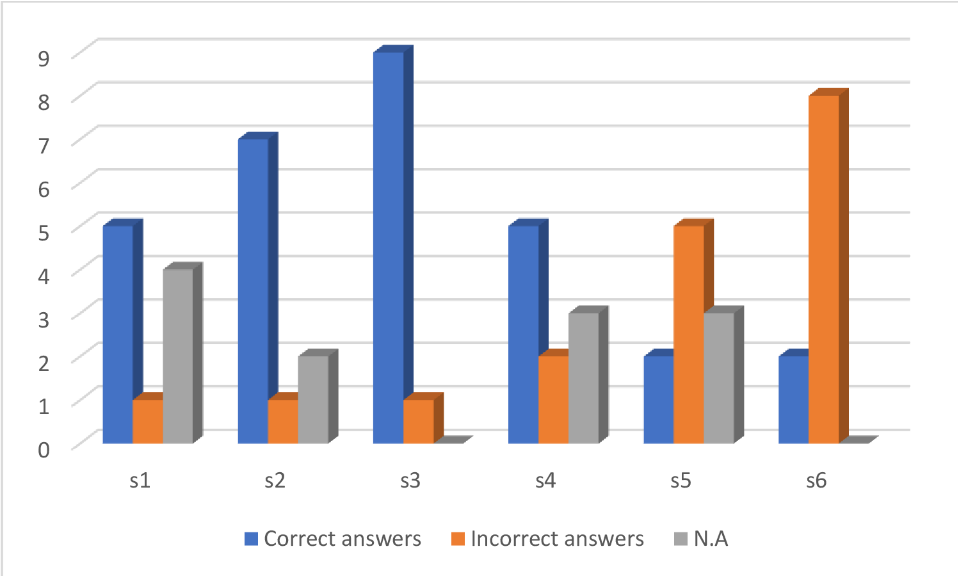


Figure 34. Number of correct, incorrect, and no answer of 6 students in the eighth year

The problem-solving indicators used by the students in question range from 11 to 26 out of 60, with the highest scores being 26 and 23. Indicators Y2, Y1, and Y6 were the least utilized compared to the other indicators.

	Y1	Y2	Y3	Y4	Y5	Y6
Question 1	1	0	4	4	4	0
Question 2	1	0	0	0	2	2
Question 3	1	1	4	3	4	1
Question 4	1	0	1	0	3	0
Question 5	0	0	6	6	5	1
Question 6	0	0	5	4	3	1
Question 7	0	0	3	2	2	1
Question 8	0	0	1	1	2	1
Question 9	0	0	2	2	3	0
Question 10	2	1	2	2	2	1
Overall	6	2	28	24	30	8

Table 9. Frequency of each indicator for each question used by students of year eight

Sample 1: Despite the student's successful adoption of a problem-solving approach, there appears to be a challenge in accurately calculating and performing the necessary computations, as demonstrated by the example (1500÷75=20) (Y5).

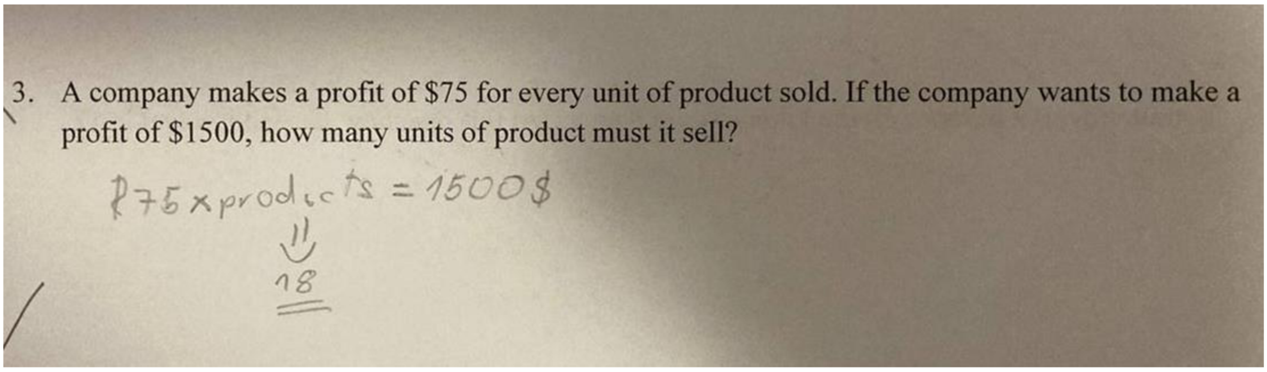


Figure 35. Sample of a correct solution/ an incorrect answer in the eighth year, 1 (Captured by the author)

Sample 2: The student attempted to initiate the problem-solving process by documenting the given information. However, it is apparent that they encountered difficulties in accurately analyzing the data provided (Y1).

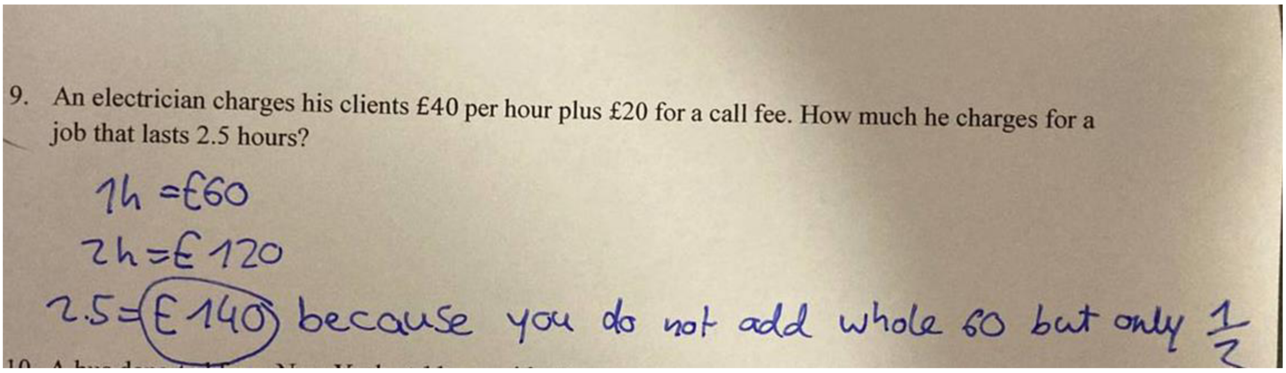


Figure 36. Sample of an incorrect solution in the eighth year, 2 (Captured by the author)

Sample 3: The student devised a correct plan (the total angle sum of the quadrilateral is 360°); however, they were unable to carry out the procedure due to forgetting two other angles.

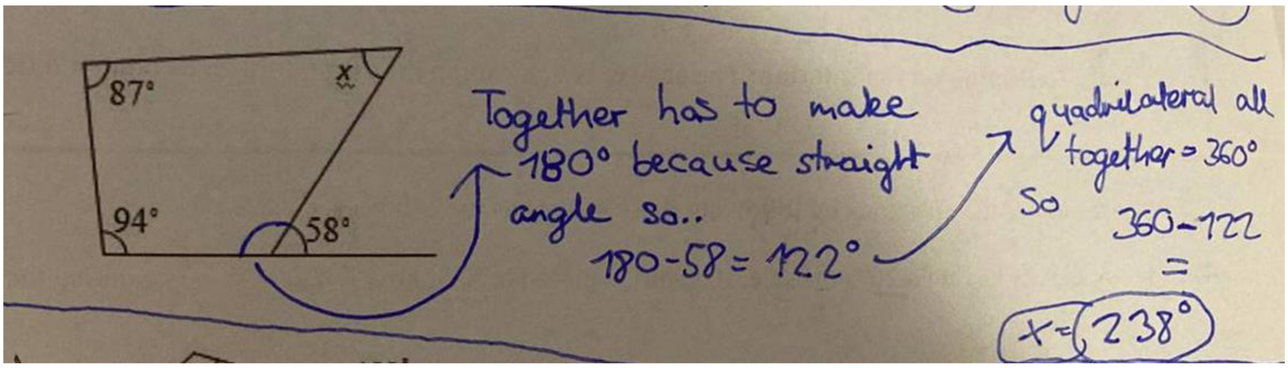


Figure 37. Sample of an incorrect solution in the eighth year, 3 (Captured by the author)

4.2.7. Results of Year 9

The last group of students represents the smallest class, comprising only 7% of the total participants in the school. Among the respondents, 50 percent achieved a perfect score of 10 out of 10 on the test, while the remaining 50 percent scored at least 7 out of 10 questions correctly.

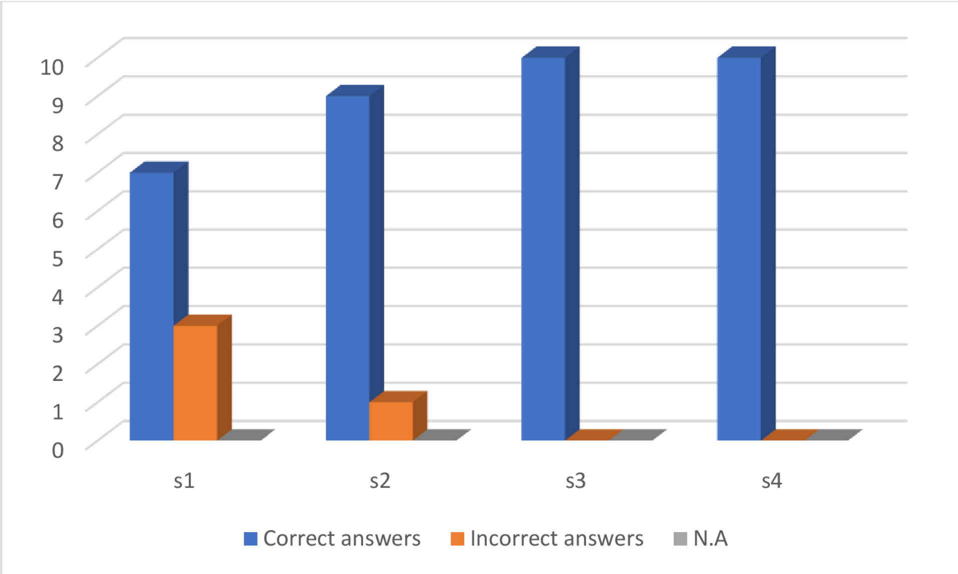


Figure 38. Number of correct, incorrect, and no answer of 4 students in the ninth year

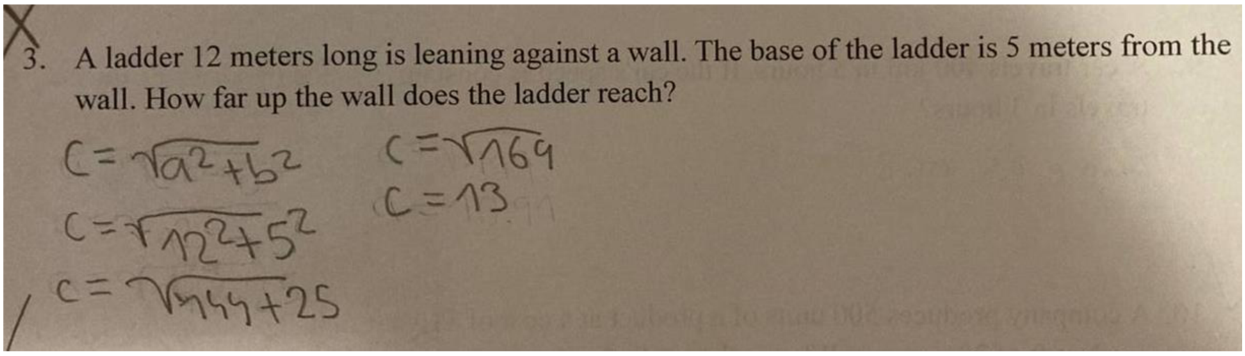
The last class demonstrated remarkable performance in utilizing the Polya problem-solving indicators. The percentage of indicators used by the oldest group of students varied from 40 to 61.6, and the average of 48 percent.

The Y3, Y4, and Y5 indicators were employed by all or at least two students in every question, while surprisingly, none of the students used Y2 in any of the questions. Furthermore, only three students used Y6 in three different questions.

	Y1	Y2	Y3	Y4	Y5	Y6
Question 1	0	0	4	4	4	0
Question 2	0	0	4	4	4	0
Question 3	0	0	4	3	3	0
Question 4	0	0	4	4	4	0
Question 5	0	0	3	3	4	0
Question 6	0	0	2	3	3	1
Question 7	2	0	4	3	3	0
Question 8	2	0	3	3	4	1
Question 9	1	0	4	4	4	0
Question 10	2	0	3	3	3	1
Overall	7	0	35	34	36	3

Table 10. Frequency of each indicator for each question used by students of year nine

Sample 1: The student was able to recognize that the ladder placed against the wall formed a right triangle and correctly utilized the Pythagorean theorem (Y3) to solve the problem. Nonetheless, she couldn't proceed with the problem-solving because of difficulty in distinguishing the data and what the problem was asking for, which is a consequence of misinterpreting the problem's information.


3. A ladder 12 meters long is leaning against a wall. The base of the ladder is 5 meters from the wall. How far up the wall does the ladder reach?

$C = \sqrt{a^2+b^2}$

$C = \sqrt{12^2+5^2}$

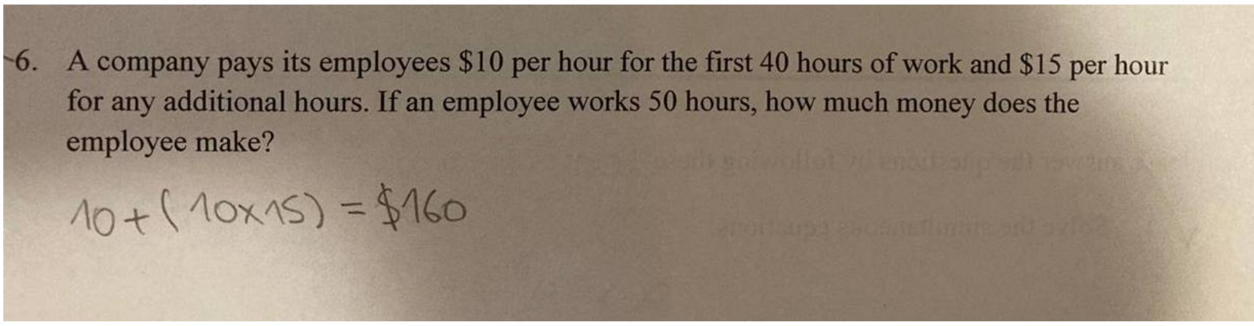
$C = \sqrt{144+25}$

$C = \sqrt{169}$

$C = 13$

Figure 39. Sample of an incorrect solution in the ninth year, 1 (Captured by the author)

Sample 2: The individual in question missed the word 'per hour' while reading and assumed that the payment for the first four hours was $10 when in fact, it was $10 per hour. Despite this error, she was able to comprehend the second part of the problem and employed an appropriate plan to solve it. However, the individual arrived at an incorrect solution due to an error (Y1).


6. A company pays its employees $10 per hour for the first 40 hours of work and $15 per hour for any additional hours. If an employee works 50 hours, how much money does the employee make?

10 + (10×15) = $160

Figure 40. Sample of an incorrect solution in the ninth year, 2 (Captured by the author)

4.3. Comprehensive results

Analysis of using problem-solving indicators was conducted among the students at the British School of Olomouc. Based on the number of indicators used to solve mathematical problems, students attain a numerical grade, as evidenced in table 11, presented below. This visual representation depicts the aggregated results of all students, ranging from the minimal score of six to the maximal score of 55.

	Grade 3 (n=9)	Grade 4 (n=8)	Grade 5 (n=9)	Grade 6 (n=16)	Grade 7 (n=8)	Grade 8 (n=6)	Grade 9 (n=4)
Aggregated results of each student	11	11	31	24	6	15	24
	23	6	14	29	10	26	26
	29	14	22	34	11	23	37
	14	9	29	27	23	11	28
	12	7	55	6	33	12	
	15	10	55	15	17	11	
	33	16	16	35	17		
	33	11	10	36	11		
	17		13	27			
				17			
				31			
				20			
				36			
				34			
				28			
				28			

Table 11. Aggregated score achieved by students of different grades

The aspect of understanding a problem can be measured by two indicators: 1) the ability to write the information provided in the problem, and 2) the ability to write the problem statement based on the given information. Based on our analysis, we identified three levels of understanding among students:

1. Students comprehend the problem and can accurately identify what is provided and what is asked.
2. Students do not fully understand the problem, they can still identify what is provided but cannot write the problem statement.
3. Students lack an understanding of the problem and are unable to recognize what information is provided and what is the problem statement.

Three indicators can measure aspects of devising and carrying out a plan: 1) developing the mathematical models or equations required to solve the problem. 2) implementing the problem-solving strategy, and 3) performing accurate calculations. According to the analysis, the following observations were made:

1. Students have the ability to devise and carry out problem-solving strategies.

2. Students can create a problem-solving plan but may face challenges in executing it effectively to solve the problem.
3. Students are unable to formulate a plan for problem-solving and lack the ability to implement such a plan.

An aspect of looking back can be measured by one indicator, 1) presenting a clear and accurate conclusion.

After completing the assessment, the total score was tabulated and then transformed into a percentage value. This was done to facilitate a more effective comparison of the student's individual performance in fulfilling each of the Polya problem-solving steps. In addition to enabling comparison of individual student performance, percentage scores can also be used to compare the overall performance of different groups or classes. This helped the researcher to identify areas where additional support or instruction may be necessary and to develop more effective teaching strategies.

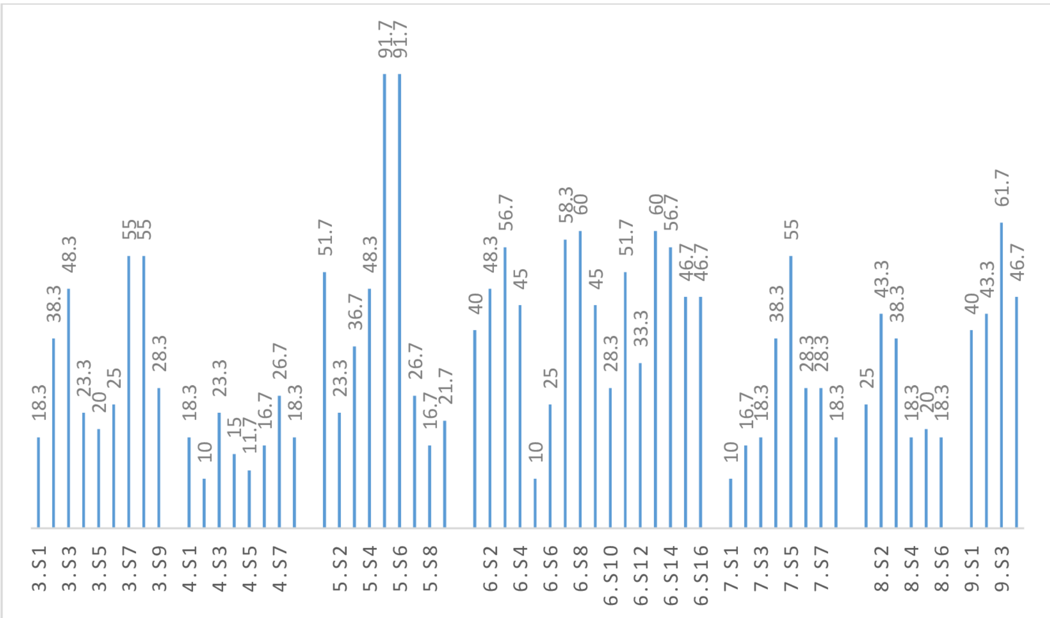


Figure 41. The percentage of using problem-solving indicators (Y: %, X: student of each grade)

Based on the results of the written tests and interviews, some weaknesses were found in common based on the students' ability to solve the problem in different grades.

Firstly, there are some errors in determining what is known during the stage of understanding the problem. It was observed that some students are not precise in interpreting or comprehending the sentence in question, leading to mistakes. In other words, some participants lack a clear understanding of the questions and are not used to mentioning what is known and asked in the answer sheet.

Second, the students confront difficulties in devising an effective problem-solving approach due to their tendency to rely on conventional procedures. Consequently, students may become confused when asked to apply the appropriate concepts and mathematical formulas to solve a problem.

Finally, the students encounter challenges in looking back on the accuracy of their problem-solving solutions. Such difficulties in assessing the correctness of their answers arise due to wrong comprehension, formulation of the problem-solving plan, and implementation of the problem-solving process. Hence, students will not encounter issues in evaluating the accuracy of their answers if they accurately comprehend the problem, formulate an effective problem-solving plan, and implement the problem-solving process correctly.

Chapter 5: Discussion

5.1. Overview of study

Over the course of one month, a research study was conducted by a researcher involving a total of 60 students from the British School of Olomouc who were in grades three to nine. The study involved administering ten word problem questions to the students, which they answered by demonstrating their proficiency in applying the Polya problem-solving model. Additionally, the teacher conducted random interviews with the respondents to gain further insights into the students' problem-solving approaches and recorded her observations in a notebook. The researcher assessed each question and counted the number of problem-solving indicators utilized. Furthermore, the researcher recorded the number of problem-solving indicators employed by each student to gain a comprehensive understanding of their individual problem-solving abilities.

It should be noted that due to varying levels of mathematical knowledge among students of different grades, making direct comparisons may not be appropriate. Nevertheless, it can be observed that the subjects did not perform optimally in utilizing all the steps of the Polya model for problem-solving, and some even failed to solve the problems correctly. Notably, a significant number of students faced challenges in the first step of understanding the problem, which was measured using two indicators of writing the information given in the question and writing the problem statement. However, the processes of devising a plan, executing the problem-solving procedure, and performing the calculations were carried out correctly. Nonetheless, many students failed to accurately write the final conclusions and engage in rechecking. Consequently, based on the data collected and analyzed, it can be concluded that a significant proportion of the students were unable to meet three out of the six indicators that were defined in this research, indicating a potential gap in their problem-solving skills.

The number of students	60
Number of questions	10
Lowest score achieved	6 out of 60
Highest score achieved	55 out of 60

Table 12. Overview of the study

5.2. Research questions

The results obtained from the study's sixty participants may potentially provide answers to the research inquiries posited in the first chapter:

1- To what extent do the British International School of Olomouc students apply the four steps of the Polya method when solving math problems?

It is worth noting that students possess varying levels of mathematical problem-solving abilities. Within the cohort of students involved in the study, it was observed that three individuals achieved the lowest score of 10%, while two individuals obtained the highest score of 91.6%. Furthermore, a group of seven students from four different grade levels achieved a score of 18.33%, which represented the mode of the study.

2- How does the frequency of use of the Polya method vary across different grade levels of students at the British International School of Olomouc?

As can be deduced from the data presented in Figure 41, the performance of students at the British School of Olomouc is generally similar, with higher grade levels not necessarily translating into higher scores in problem-solving.

Notably, two students in grade five achieved the highest score of 55 out of 60, while one student in grade nine achieved a score of 37. Intriguingly, upon comparing the youngest participants in grade three with the oldest respondents in grade nine, it was observed that three students in year three outperformed their counterparts in year nine. These results suggest that the frequency of utilizing the Polya problem-solving method does not appear to be positively associated with the grade level of students.

The present study's outcomes align with those of the investigation conducted by (Puspa et al., 2019), in which it was observed that the problem-solving skills of junior high school students display substantial variation. Some students tend to adopt a comprehensive and thorough approach, demonstrating a keen attention to detail and an ability to retain information effectively, while others display a more brief and straightforward style. Additionally, the results reveal that there are individual differences in communication and problem-solving skills among pupils, indicating that each student possesses a unique set of strengths and weaknesses (Puspa et al., 2019). (Sinaga et al., 2023) also found that the academic achievement of students in mathematics learning is not significantly affected by their grade level, while it is observed that there is a notable impact of grade level on their problem-solving comprehension (Sinaga et al., 2023).

However, it should be noted that the findings presented by Riyadi et al. (2021) contradicted the outcomes mentioned earlier. They found that the fifth-grade pupils exhibited significantly better performance compared to the third and fourth-grade students, highlighting their remarkable proficiency in logical, abstract, and proportional thinking. Furthermore, they found that the students could efficiently solve word problems with minimal difficulty (Riyadi et al., 2021).

3- What specific steps of the Polya method do students at the British International School of Olomouc struggle with and/or use the most when solving math problems?

Polya's problem-solving approach outlines a series of steps to help students tackle mathematical problems effectively. However, despite this framework, many students faced obstacles when attempting to solve problems. One of the most significant challenges encountered by students is difficulty in understanding the problem statement itself. This can arise from a variety of factors, such as a lack of familiarity with the type of problem being presented or insufficient exposure to problem-solving techniques.

Another hurdle that students often face is in developing an effective problem-solving plan. They may struggle to identify the relevant mathematical concepts to apply or have difficulty breaking down the problem into manageable steps. Even once a plan has been developed and the calculation process begins, students can still encounter issues. They may find it challenging to track their progress accurately or to identify any mistakes in their calculations.

As a result, students may neglect certain problem-solving indicators, such as 1, 2, and 6. These indicators are writing problem information, writing the problem statement, and summarizing their findings. These three indicators are often skipped or not included in problem-solving even though they can help students to keep track of their progress and ensure that they are on the right track throughout the problem-solving process.

The current study's findings echo those of Nurkaeti's (2018) research, which also examined the challenges that elementary school students encounter when solving mathematical problems, particularly in the form of word problems. Nurkaeti's investigation identified several obstacles that students face, including comprehending the problem statement, identifying relevant mathematical concepts or formulas, making connections between different mathematical ideas, and verifying the accuracy of their answers through questioning. These challenges are further compounded by the fact that many students do not encounter word problems as frequently in their routine practice as they do with straightforward algorithmic calculations. As a result, students may not have the requisite skills to solve problems effectively when presented with such questions (Nurkaeti, 2018).

Moreover, it is noteworthy that even with a small sample size, Karlina's (2022) research highlights the importance of individual differences in mathematical problem-solving abilities among students. The study's findings indicate that the mathematical problem-solving abilities of the two research subjects were insufficient, as evidenced by the analysis of their answers and interview responses. Specifically, during the understanding problem stage, both subjects struggled with comprehending the problem statement and interpreting it accurately. Furthermore, during the planning stage, the two subjects employed inappropriate strategies or made errors in their planning approach. Additionally, in the carrying out the plan stage, they made errors in the calculation process, resulting in incorrect answers. To improve students' ability to solve mathematical problems, it is vital to expose them to a broad range of questions and provide extensive opportunities to practice. Emphasis should also be placed on reinforcing basic mathematical concepts to ensure that students have a solid foundation on which to build their problem-solving skills (Karlina, 2022).

5.3. Recommendations

The mathematical problem-solving abilities of students at International British school of Olomouc require attention as per the findings. The development of problem-solving skills is a crucial aspect of mathematics learning, particularly for preparing the 21st-century generation. Therefore, it is imperative to focus on nurturing and enhancing students' problem-solving abilities. Polya's problem-solving steps have been proposed, but students still encounter difficulties such as comprehending the problem, writing the problem statement, and rechecking the conclusion. These challenges can be attributed to factors such as the lack of familiarity with word problems and insufficient problem-solving development in the learning process.

Instructors who wish to implement a specific problem-solving approach in their classes should exercise patience during the initial stages. It is common for students to lack a structured approach to answering questions, even when using well-known problem-solving methods like Polya's. Some students may initially resist the first step of this method, which involves rewriting all the information provided in the question under the "understanding the problem" stage. This resistance often manifests as questioning the necessity of repeating the given information, as they fail to recognize how it can aid them in finding the solution. It takes several exercises for students to appreciate the value of this method. Nevertheless, students' appreciation of this method typically requires extensive practice until it becomes a habit (Wickramasinghe & Valles Jr, 2015).

Based on the study's findings, it is recommended that educators pay close attention to students' abilities when planning learning methods, models, and strategies. This can help devise approaches that stimulate and motivate students to solve problems effectively (Taneo & Kusumah, 2021).

Students must be exposed to various problem types, remarkably open-ended and non-routine ones that allow exploration. Alternatively, teaching approaches such as problem-based, open-ended, mathematical realistic, and others should be implemented to offer students more opportunities to practice mathematical problem-solving (Nurkaeti, 2018).

According to the research results, it is advisable for mathematics instructors to prioritize assisting students in comprehending the methodologies and techniques involved in problem-solving instead of focusing solely on arriving at the correct answer to the given problem (Lupahla, 2014). Mathematics teachers should integrate the problem-solving approach into all mathematics learning materials and ensure students understand the process before assigning them mathematics tasks to improve their mathematics learning results. Teachers should carefully select appropriate learning tools and media to support this approach in mathematics learning. Students who struggle with mathematics problem-solving and have lower learning results should work collaboratively with those who have higher results to improve their mathematics knowledge and cognitive abilities(Sinaga et al., 2023).

This study's outcomes can serve as a reflective tool for teachers at school to improve their problem-solving skills and apply the formulated rules to teach students to solve mathematical problems effectively (Maulyda et al., 2019). Furthermore, through the education office agency, the local government should develop and implement a primary school mathematics education curriculum based on a problem-solving learning model. This would enhance students' problem-solving abilities and help them to apply mathematical concepts to real-world situations, better preparing them for future endeavors (Sinaga et al., 2023).

5.4. Limitations

The present study has some limitations, including the fact that it was conducted only in the International School of Olomouc and the sample size was too small. Therefore, the research findings cannot provide a comprehensive profile of primary and secondary students in Olomouc. Future researchers should broaden the scope of the study to cover a more extensive area, such as the provincial or even national level, to improve the generalizability of the research results.

It should be noted that a significant proportion of the participants in the study did not have English as their first language. This fact may have contributed to the inability of many students to complete the initial stage of problem-solving successfully. However, it is essential to acknowledge that, according to Pillos et al. (2020) findings, proficiency in both general and mathematical language is essential for students to comprehend and solve mathematical word problems effectively. In cases where students lack adequate mathematical proficiency, regardless of the language of instruction, whether it is their native language or English, the absence of a foundation in relational understanding, or a combination of poor English proficiency and underdeveloped mathematical discourse in their native language may exist. They believe that the primary obstacle lies with the student's mathematical skills rather than the language of instruction. It is suggested that future researchers replicate this study in both languages to investigate the impact of the mother tongue on problem-solving abilities.

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Appendix A

Following is the mathematical questions given to the students of each grade.

Year 3

(1) There are 9 crayons in a box. If you have 3 boxes, how many crayons do you have in total?

(2) A bag of candy has 50 pieces. If 17 pieces are taken out of the bag, how many pieces are left in the bag?

(3) If a pizza has 8 slices, how many slices are there in 4 pizzas?

(4) If a toy store has 20 toy cars and each toy car costs $3, how much money will you need to buy all the toy cars?

(5) A bakery has sold 24 cupcakes in the morning. If they sell 16 more cupcakes in the afternoon, how many cupcakes have they sold in total?

(6) A family has 4 people. If they have 3 guests over for dinner, how many people are there in total?

(7) Look at the following numbers: 2, 4, 6, 8, ___. What is the next number in the pattern?

(8) Look at the following numbers: 1, 3, 6, 10, ___. What is the next number in the pattern?

(9) A pencil costs £1 and a notebook costs £2. What is the total amount for buying 5 pencils and 3 notebooks?

(10) Tiffany had to complete 2 pages of homework. Each page had 9 problems on it. How many problems did she have to complete total?

Year 4

(1) If it takes 3 apples to make a pie and you have 15 apples, how many pies can you make?

(2) What is the next number in the sequence: 1, 4, 7, 10,...

(3) What is the missing number in the equation: 8 + __ = 15

(4) A bakery has 6 trays of cupcakes. Each tray has 6 cupcakes. How many cupcakes are there in total?

(5) Sara needs 30 pencils. Each box of pencils contains five pencils. How many boxes does she need to buy?

(6) David bought 4 pens each for£1 , 3 notebooks each for £2 and a book for £2. How much did he pay in total?

(7) If a pizza has 8 slices, and 4 people will share it, how many slices will each person get?

(8) My starting number is 5

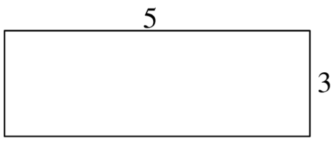
My term to term rule is +4

Write the first 5 numbers in the sequence.

(9) Write a different number in each box to make this statement correct.

$$\frac{\ }{8} + \frac{5}{8} + \frac{\ }{8} = \frac{9}{8}$$

(10) Here is a rectangle.



A larger rectangle is made using three of the same rectangles. Calculate the perimeter of the larger rectangle.

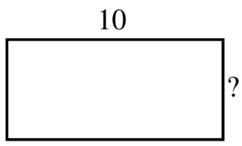
Year 5

(1) 1 box contains 10 red balls and 20 blue balls. What fraction of the total number of balls is blue?

(2) A baker needs to make 60 cookies. He has already made 30. How many more cookies does he need to make?

(3) If a pizza costs $12 and you have $48, how many pizzas can you buy?

(4) The perimeter of a rectangle is 28 units. If the length is 10 units, what is the width of the rectangle?



(5) John has 12 toy cars and his friend has 8. How many toy cars do they have together?

(6) Sofia has a counter. It is black on one side and white on the other. She tosses her counter. The counter lands with the white side up. She says: If I toss the counter again, it will land black side up because last time it landed white side up.

Tick (□) to show if Sofia is correct. Yes No, Explain how you know.

(7) Here is part of a sequence. 0 , 25 , 50, … , 100 , …

The sequence continues in the same way. Write the missing numbers.

(8) A store has 100 toys in stock and sells 20 each day. How many days will it take for the store to sell all of the toys?

(9) A gallon of milk costs $3. How much will it cost to buy 3 gallons of milk?

(10) A rectangle has a length of 15 units and a width of 12 units. What is the perimeter of the rectangle?

Year 6

(1) A pack of gum contains 20 pieces, and you have 4 packs. How many pieces of gum do you have in total?

(2) A gallon of milk costs $4. How much will it cost to buy 5 gallons of milk?

(3) The perimeter of a square is 36 units. What is the length of each side of the square?

(4) A store has 150 toys in stock and sells 25 each day. On the first day, the store also received a shipment of 50 toys. How many toys will the store have in stock after 3 days?

(5) A book has 300 pages and you read 40 pages each day. How many days will it take you to finish the book, and how many pages will you have left over?

(6) A store is selling pencils for $1.75 each. How much will it cost to buy 12 pencils, and how much change will you receive if you give the store $20?

(7) If a clock shows 7:00, what time will it be in 13 hours?

(8) A school library has a total of 1,500 books. On Monday, the library received 300 new books, and on Tuesday, it received another 200 books. How many books does the library have now? If the library lent 200 books on Wednesday and 300 books on Thursday, how many books does it have on Friday?

(9) A number increased by 7 is equal to 40. What is the number?

(10) A gardener has 90 plants, 45 of which are roses. What fraction of the plants are roses?

Year 7

(1) A cuboid has a length of 8 cm, a width of 5 cm, and a height of 4 cm. What is the volume of the cuboid?

(2) A machine can produce 60 items in 2 hours. How many items can it produce in 8 hours?

(3) Understand the problem: A rectangular garden has a length of 12 meters and a width of 8 meters. If a path around the garden takes up 2 square meters, what is the area of the garden without the path?

(4) Here is a function machine.

Input $\rightarrow$ $\boxed{+47}$ $\rightarrow$ Output

a) Find the output when the input is – 9

b) Find the input when the output is 32

(5) Here are the first four terms in a sequence.

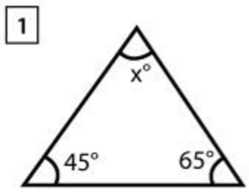
6, 1, -4, -9…

a) Write down the next two terms in the sequence.

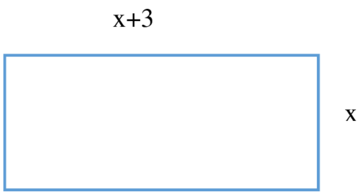
b) Describe the term-to-term rule for the sequence.

(6) Work out. $0.35 \times 6 \times 15 + 1.65 \times 3 \times 30$

(7) Find x.

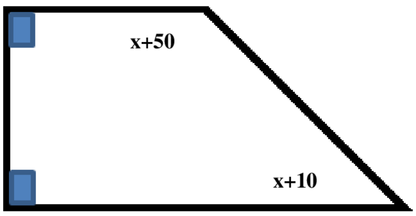


(8) The perimeter of following rectangle is 38cm. form an equation and find Length and Width.



(9) Sarah is x years old. Thomas is 3 years older than Sarah. David is twice as old as Sarah. The total of their ages is 51. Find all their ages.

(10) Shown is a trapezium. Calculate the size of the largest angle in the trapezium.



Year 8

(1) A store is having a 20% off sale. If a shirt is priced at $25, how much will it cost during the sale?

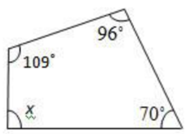
(2) A gardener has 20 rose bushes. If he wants to plant them in rows with an equal number of bushes in each row, what is the maximum number of rows he can create?

(3) A company makes a profit of $75 for every unit of product sold. If the company wants to make a profit of $1500, how many units of product must it sell?

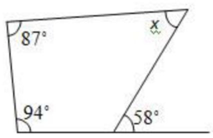
(4) The sum of two numbers is 28. If one number is 4 more than the other, what are the two numbers?

(5) Calculate the value of x for the following figures:

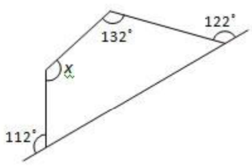
a)



b)



c)



(6) The length of a rectangular field is three times its width. The perimeter of the field is 840 square meters. What is the area of the field?

(7) A train travels a distance of 480 kilometres in 8 hours. At this rate, how far will it travel in 12 hours?

(8) Two numbers have a product of 56 and a sum of 18. What are the numbers?

(9) An electrician charges his clients £40 per hour plus £20 for a call fee. How much he charges for a job that lasts 2.5 hours?

(10) A bus departed from New York at 11 pm with 44 passengers. An hour later, a few passengers got off at New Jersey. The number of passengers who boarded the bus at New Jersey were twice the number of passengers who got off the bus. If the bus has 50 passengers now, how many passengers got off at New Jersey?

Year 9

(1) Solve the simultaneous equations. Show your work.

$7x + y = 50$

$4x + y = 23$

(2) A microwave oven normally costs $160. In a sale, there is a discount of 15%. Work out the sale price of the microwave oven.

(3) A ladder 12 meters long is leaning against a wall. The base of the ladder is 5 meters from the wall. How far up the wall does the ladder reach?

(4) A rectangular box has a volume of 96 cubic centimeters. If the length of the box is 6 cm and the height is 4 cm, what is the width of the box?

(5) A cylindrical tank has a radius of 6 meters and a height of 10 meters. If the tank is filled with water to a depth of 8 meters, what is the volume of water in the tank?

(6) A company pays its employees $10 per hour for the first 40 hours of work and $15 per hour for any additional hours. If an employee works 50 hours, how much money does the employee make?

(7) A car travels 60 km/h for the first half of a trip and 80 km/h for the second half of the trip. If the trip is 600 km in total, how long does it take the car to complete the trip?

(8) A car travels 400 km in 5 hours. If the car's speed is constant, what is the distance the car travels in 3 hours?

(9) The area of a rectangle is 119 〖cm〗^2. The length of the shorter side of the rectangle is (5x + 2) cm. The longer side of the rectangle is 7 cm. Find the value of x.

(10) A company produces 500 units of a product at the cost of $10 per unit. The company sells the product for $20 per unit. What is the profit for the company?