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Homogenization in Perforated domains

Homogenizace na oblastech s dírami

zkrácená verze PhD Thesis

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1. Introduction

Theory of homogenization was developed for modeling media with a fine periodical structure. In a physical setting, homogenization means replacing a heterogeneous material by an equivalent homogeneous one, in mathematical setting it means approximating equations with highly oscillating coefficients by equations with constant ones.

The mathematical approach consists of considering a sequence of problems with a material with a more and more refined structure. Hence, we get a sequence of solutions. The principal question is: How does the sequence behave? Does the limit, the so called homogenized solution, exists? If so, how can it be characterized? This approach was first introduced by J.B. Keller (1973) and developed by I. Babuška (1975). More about the homogenization can be found in the monograph [BLP78] or in the textbook [CD99].

Other problems for which a similar approach can be used are problems defined on periodically perforated domains. Let Ω be a domain in $\mathbb{R}^N$ and let it be periodically perforated by holes. We shall construct a sequence of domains with an increasing number of holes and decreasing their volume. Again, we are interested in a behavior of the limit solution.

When we try to find the homogenized solution several difficulties occur. Some of them are common for both the case with and without holes. The following problem can illustrate the typical situation in the setting with no holes.

For $\varepsilon = 1, {}^1/_2, {}^1/_3, \dots$, let us assume a sequence of solutions $\{u_\varepsilon\}$ to a problem

$$\begin{cases} -\nabla \cdot (A_\varepsilon \nabla u_\varepsilon) &= f \quad \text{in} \quad \Omega, \\ u_\varepsilon &= 0 \quad \text{on} \quad \partial\Omega, \end{cases} \tag{1}$$

where $A_\varepsilon(x) = A\left(\frac{x}{\varepsilon}\right)$ and $A(y)$ is a Y-periodic function satisfying $0 < \alpha \leq$ $\leq A(y) \leq \beta$.

Weak formulation of this problem is:

$$\begin{cases} \text{Find } u_\varepsilon \in H_0^1(\Omega) \text{ such that} \\ \displaystyle\int_\Omega A_\varepsilon(x)\, \nabla u_\varepsilon(x) \cdot \nabla v(x)\, \mathrm{d}x = \int_\Omega f(x)\, v(x)\, \mathrm{d}x, \qquad \forall v \in H_0^1(\Omega). \end{cases} \tag{2}$$

For $A_\varepsilon \in L^\infty(\Omega)$, the domain Ω with a "good" boundary and $f \in L^2(\Omega)$, the unique weak solution u_ε exists and satisfies $\|u_\varepsilon\|_{H_0^1(\Omega)} \leq C$. Since the sequence $\{u_\varepsilon\}$ is bounded in $H_0^1(\Omega)$, it contains a weakly converging subsequence of gradients $\{\nabla u_\varepsilon\}$.

When we are tending to the limit, it turns out that the left-hand side of (2) contains a product of two weakly converging sequences, $\{A_\varepsilon\}$ and $\{\nabla u_\varepsilon\}$. In this case it is not possible to reach to the limit directly, since a limit of product need not to be a product of limits two weakly converging sequences.

In the past, several approaches to overcome this problem were developed.

- *Multiple-scale method* is summarized in monograph by A. Bensoussan, J. L. Lions and G. Papanicolaou [BLP78]. The method uses the asymptotic expansion of the solution u_ε to find the homogenized one.
- *Local energy method* (called also the oscillating test function method) was introduced by L. Tartar [Tar97] in the years 1977 and 1978. The method is based on a special choice of oscillating test functions in the weak formulation of the problem.
- *Two-scale convergence method* introduced by G. Nguetseng [Ngu89] in 1989 and developed by G. Allaire [All92] in 1992. In this method a new type of convergence is defined. The limit of two-scale convergent sequence has two variables, the second one describes local behavior. This method requires introducing a special space for test functions.
- *Periodic unfolding method* is an alternative approach to the two-scale convergence. It was introduced by J. Casado-Díaz [CD00] in 2000 and D. Ciorănescu, A. Damlamian and G. Griso [CDG02], L. Nechvátal [Nec04] and J. Franců [Fra10]. It removes problems with the choice of space for test functions, therefore it is more natural. A comprehensive survey of the application of this method to the problems in domains with holes is described by Ciorănescu, Damlamian, Donato, Griso and Zaki [Cio+12].

Let us turn our attention back to the problems defined on the domain with holes. In this case, one more problem arises. Let Ω_ε^* denotes a periodically perforated domain with period εY. For $\varepsilon \searrow 0$ the period is smaller and smaller and the domain is perforated by more and finer holes.

A model situation looks as follows: For $\varepsilon = 1, {}^1\!/_2, {}^1\!/_3, \ldots,$ let us assume a sequence $\{u_\varepsilon\}$, where u_ε is a solution of the problem

$$\begin{cases} -\Delta u_\varepsilon = f & \text{in} \quad \Omega_\varepsilon^*, \\ \quad u_\varepsilon = 0 & \text{on} \quad \partial\Omega_\varepsilon^*. \end{cases} \tag{3}$$

A weak formulation of the problem (3) is:

$$\begin{cases} \text{Find } u_\varepsilon \in H_0^1(\Omega_\varepsilon^*) \text{ such that} \\ \displaystyle\int_{\Omega_\varepsilon^*} \nabla u_\varepsilon(x) \cdot \nabla v_\varepsilon(x)\,\mathrm{d}x = \int_{\Omega_\varepsilon^*} f(x)\, v_\varepsilon(x)\,\mathrm{d}x, \qquad \forall v_\varepsilon \in H_0^1(\Omega_\varepsilon^*). \end{cases} \tag{4}$$

The problem is that each solution u_ε of problem (4) is defined on a different domain Ω_ε^*. Hence, it is not clear in which sense the convergence of the sequence $\{u_\varepsilon\}$ can be understood. Even if there existed some u_0 for which $\|u_\varepsilon - u_0\|_{H_0^1(\Omega_\varepsilon^*)} \to 0$, as $\varepsilon \searrow 0$, one could not speak about "convergence" (in a strong or weak sense) of the sequence $\{u_\varepsilon\}$.

Several methods to avoid this issue have been developed over time:

- Quite an intuitive approach is a construction of an *uniformly bounded extension operator* P_ε from $H_0^1(\Omega_\varepsilon^*)$ to $H_0^1(\Omega)$. Then, we can transform our problem of finding a "limit" of $\{u_\varepsilon\}$ by another one: Find a limit of the sequence $\{P_\varepsilon(u_\varepsilon)\}$ in the fixed space $H_0^1(\Omega)$.

  This approach has a limitation. The existence of operator P_ε depends on the boundary conditions of the problem (in the case that they are more complicated than in our model example) and also on the shape of the holes (for example they should have a sufficiently smooth boundary and should not intersect the boundary of Ω).

- Another approach is to use an *unfolding operator* to transform functions u_ε, resp. ∇u_ε defined on Ω_ε^* to the fixed domain $\Omega \times Y$.

As we shall see the *periodic unfolding method* is the technique which solves both problems mentioned above. This is the reason why the method is so suitable for problems defined on perforated domains.

Goal and contribution of the thesis

Let Ω be a bounded set, and Y a reference cell in $\mathbb{R}^N$. The unfolding operator $\mathcal{T}_\varepsilon$ maps a function in $L^p(\Omega)$ to a function in $L^p(\Omega \times Y)$.

The main disadvantage of an unfolding operator introduced in [CD00] and [CDG02] is that it does not conserve integrals. It means that in general for $u \in L^\infty(\Omega)$

$$\int\limits_{\Omega} u(x)\,\mathrm{d}x \neq \frac{1}{|Y|} \iint\limits_{\Omega\times Y} \mathcal{T}_\varepsilon(u)(x,y)\,\mathrm{d}x\,\mathrm{d}y. \tag{5}$$

It can be shown that the left-hand side of (5), for $u \geq 0$, is always grater or equal than its right-hand side. The equality holds only in limit, i.e. for $\varepsilon \to 0$.

This issue was removed by redefining this operator. The operator was improved by J. Franců and N.Svanstedt in [FS12]. This change simplifies the proofs and removes several difficulties and necessity of introducing "unfolding criterion for integrals" (see e.g. [CDG08]).

This thesis aims to prove properties of this improved unfolding operator, mainly the convergence for the sequence of gradients and applying an analogical approach to perforated domains. Finally, we apply this new operator to find a homogenized solution of the special family of the problems with an integral boundary condition and we present some numerical results.

The thesis intents to be self-contained work suitable as the first reading for engineers and applied mathematicians.

Related works

Homogenization on a periodically perforated domain for miscellaneous boundary value problems was treated by numerous authors. Let us mention some milestones in this area.

The Laplace equation with a homogeneous Dirichlet condition in the domain where the holes are regularly distributed and the size of the holes decreases when the number of the holes increases was studied by Murat and Ciorănescu [MC97]. They showed that even in this problem an interesting behavior of the limit solution occurs.

In this problem three different situations were identified. *The first situation* is when the size of holes decreases too quickly - quicker than the size of the cell period. Then u^ε converges to the solution of the Dirichlet problem in Ω. *The second situation* is when the size of holes decreases too slowly. Then u^ε converges to the zero function. *Between these two cases* there is one when the size of holes is critical, in that case an additional zero order term appears in the right-hand side of the limit equation.

In [MC97] there are quite strict assumptions on the distribution and shape of the holes. This limitation has been removed by Dal Maso and Garroni [MG94]. This break through made possible the solving the general case of homogeneous Dirichlet problems without any geometrical assumptions.

A problem with homogeneous Neumann boundary condition with some geometrical assumptions on holes was studied by Hruslov [Hru79].

Some assumptions on the size and shape of holes which are admissible for a periodic homogenization with Neumann boundary condition are given by Damlamian and Donato [DD02].

Classical situation is when the holes are distributed periodically and the ratio of material volume to the period volume is constant. This situation with a different type of boundary conditions has been described in numerous papers. Laplace equation with homogeneous mixed (Dirichlet and Neumann) boundary conditions was studied by Cardone, D'Apice and Maio [CDM02], elliptic equations with linear Robin resp. with non-linear conditions were studied by Ciorănescu, Donato and Zaki in [CDZ06] resp. in [CDZ07], elliptic equations with non-homogeneous mixed boundary conditions were studied by Esposito, D'Apice and Gaudiello [EDG02].

A problem on domains with holes which are distributed periodically and their size is diminishing with respect to the period (the so called *small holes*) was studied by Murat and Ciorănescu in [MC97] (homogeneous Dirichlet boundary conditions), and also by Conca and Donato in [CD88] (non-homogeneous Neumann boundary condition), by Ciorănescu and Ould Hammouda in [COH08] (elliptic equations with a non-homogeneous mixed boundary conditions), by Ould Hammouda in [OH11] (elliptic equations with non-homogeneous Neumann boundary).

A non-periodical behavior of the holes has been studied by Nguetseng in [Ngu04].

2. Periodic unfolding on perforated domains

2.1. Domain with holes

Let Ω be a bounded domain in $\mathbb{R}^N$ with Lipschitz boundary $\partial\Omega$. Let the *reference cell* Y in $\mathbb{R}^N$ be N-dimensional interval defined by

$$Y = \langle 0, l_1) \times \langle 0, l_2) \times \cdots \times \langle 0, l_N), \tag{6}$$

where $l_1, \ldots, l_N$ are fixed positive numbers.

Space $\mathbb{R}^N$ can be written as a union of the disjoint cells $Y_k = Y + k$, which are the cell Y shifted by vectors k, i.e.

$$\mathbb{R}^N = \bigcup_{k\in\mathcal{K}} (Y + k), \quad \mathcal{K} = \left\{ k \in \mathbb{R}^N \mid k = (\xi_1\, l_1, \xi_2\, l_2, \ldots, \xi_N\, l_N), \xi \in \mathbb{Z}^N \right\}$$

Let $T \subset Y$ be an open bounded set with a smooth boundary. This set represents reference holes in Y. The part of the reference cell Y occupied by a material is denoted by Y^*, i.e. $Y^* = Y \setminus \overline{T}$.

Furthermore, we consider so called scales $E = \{\varepsilon_k\}$, defined as followes:

Definition 2.1 (Scale)**.** A descending sequence $E = \{\varepsilon_k\}_{k=0}^{\infty}$ of positive numbers, such that $\varepsilon_k \searrow 0$ as $k \to \infty$, is called the *scale*.

In the following, as it is usual in the homogenization, all sequences will be denoted by the subscript ε_k, for example $\{a_{\varepsilon_k}\}$, or very often even only by the subscript ε, for example $\{a_\varepsilon\}$.

Now, we define ε-scaled system of the cells

$$Y^{*k}_{\varepsilon} = \varepsilon(Y^* + k), \quad k \in \mathcal{K}.$$

Similarly we define scaled system of the holes. Let us introduce function r, which determines how fast the shrinking of holes is. Let r be a positive increasing function, such that $\lim_{\varepsilon\to 0} r(\varepsilon) = 0$.

Then the set T_ε^k is defined as a translates and scaled image of T, so

$$T_\varepsilon^k = r(\varepsilon)(T+k), \quad k \in \mathcal{K}.$$

It is necessary to choose the function r in such a way that ensures that the holes are always inside the cells, i.e. $r(\varepsilon)\,T \subset \varepsilon\,Y \quad \forall\varepsilon$.

Furthermore, we suppose that, if the set T consists of more connected disjoint sets then these sets remain disjoint for all ε.

We can distinguish three typical kinds of behaviors of the holes. For that reason let us denote by θ_ε the ratio of the volume of material in cell and the volume of cell, i.e.

$$\theta_\varepsilon = \frac{|\varepsilon Y - r(\varepsilon)T|}{|\varepsilon Y|}.$$

The case when $r(\varepsilon) = \varepsilon$ is very classical, the ratio θ_ε is constant for all ε . The case when $\frac{r(\varepsilon)}{\varepsilon} \to 0$ as $\varepsilon \to 0$ is called *small holes*. In such case the volume of holes goes to zero quicker than the volume of material in the cell, i.e. $\theta_\varepsilon \to 1$ as $\varepsilon \to 0$. In the last case $\theta_\varepsilon \to 0$, which means that the shrinking of the holes is slower than the shrinking of the cells. An example of these three cases is on the Figure 1.

Let Ω_ε^* denote the part of Ω occupied by material. It is defined as Ω without holes T_ε^k, i.e.

$$\Omega_\varepsilon^* = \Omega \setminus T_\varepsilon, \quad \text{where} \quad T_\varepsilon = \bigcup_{k\in\mathcal{K}} \overline{T_\varepsilon^k}. \tag{7}$$

Furthermore we denote by $T_{\mathrm{int},\,\varepsilon}^i$, $i = 1,\ldots,m(\varepsilon)$, the "interior holes", they are such sets T_ε^k which are completely inside Ω and do not intersect the boundary $\partial\Omega$, i.e. $\overline{T_\varepsilon^k} \subset \Omega$. Their union is denoted by $T_{\mathrm{int},\,\varepsilon}$,

$$T_{\mathrm{int},\,\varepsilon} = \bigcup_{i=1}^{m(\varepsilon)} T_{\mathrm{int},\,\varepsilon}^i.$$

Let the sets T_ε^k which intersect the boundary be denoted by $T_{\mathrm{ext},\,\varepsilon}$, i.e. $T_{\mathrm{ext},\,\varepsilon} = \big(T_\varepsilon \setminus T_{\mathrm{int},\,\varepsilon}\big) \cap \Omega$, and $\partial_{\mathrm{ext}}\Omega_\varepsilon^*$ denote the exterior boundary of Ω_ε^*, i.e. $\partial_{\mathrm{ext}}\Omega_\varepsilon^* = \partial\Omega_\varepsilon^* \setminus \partial T_{\mathrm{int},\,\varepsilon}$.

2.2. Unfolding operator $\mathcal{T}_\varepsilon^*$ in perforated domains

First of all we define splitting of each point in $\mathbb{R}^N$ in two parts. The idea is analogical to the following one: each real number x can be uniquely split to the integer part $[x]$ and the fractional part $\{x\} \in (0,1)$. Since the disjoint cells Y_k cover whole $\mathbb{R}^N$, for each point $x \in \mathbb{R}^N$ it holds $x = [x]_Y + \{x\}_Y$, where $[x]_Y$

denotes the shift of the cell Y_k containing x, and $\{x\}_Y$ stands for the relative position of x with respect to the cell Y_k, i.e. $[x]_Y \in \mathcal{K}$ and $x - [x]_Y$ belongs to Y. Set $\{x\}_Y = x - [x]_Y$.

Using ε-scaled system of the cells Y_ε^k, the domain Ω can be split into two parts: $\widehat{\Omega}_\varepsilon$ and Λ_ε, and the domain Ω_ε^* into $\widehat{\Omega}_\varepsilon^*$ and Λ_ε^*, see Figure 3. The set $\widehat{\Omega}_\varepsilon$ contains cells Y_ε^k lying inside Ω, while the set Λ_ε is a strip on the boundary composed of cells Y_ε^k intersecting the boundary $\partial\Omega$. More precisely:

$$\Xi^\varepsilon = \left\{k \in \mathbb{R}^N \text{ s.t. } Y_\varepsilon^k \subset \overline{\Omega}\right\}, \quad \widehat{\Omega}_\varepsilon = \left(\bigcup_{k\in\Xi^\varepsilon} Y_\varepsilon^k\right) \cap \Omega, \quad \Lambda_\varepsilon = \Omega \setminus \widehat{\Omega}_\varepsilon,$$
$$\widehat{\Omega}_\varepsilon^* = \widehat{\Omega}_\varepsilon \setminus T_{\text{int},\varepsilon} \quad \text{and} \quad \Lambda_\varepsilon^* = \Omega_\varepsilon^* \setminus \widehat{\Omega}_\varepsilon^*, \tag{8}$$

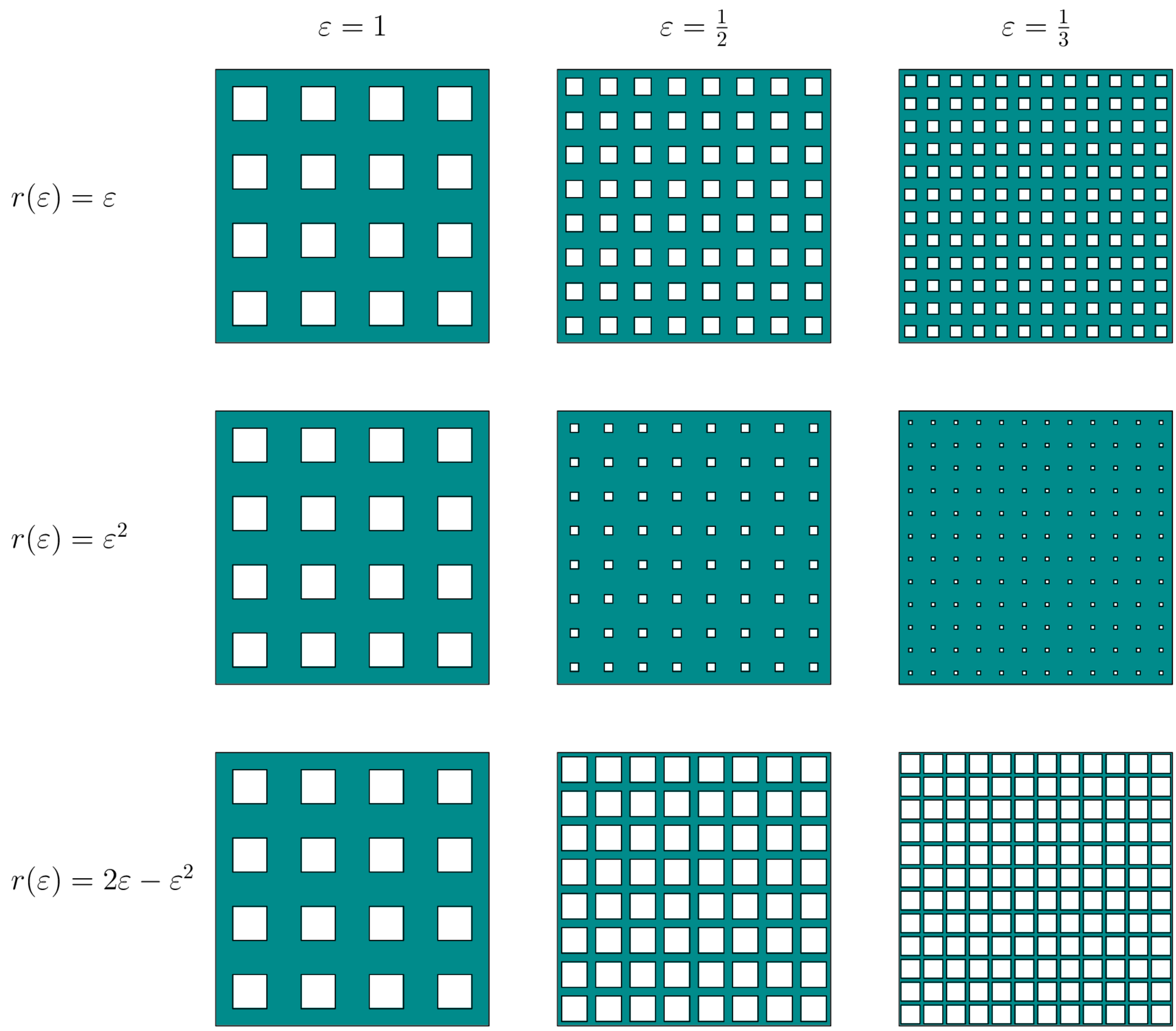

Figure 1: Example of three different behaviors of the holes depending on the choice of function r. A case on the middle line belongs to the cases called small holes.

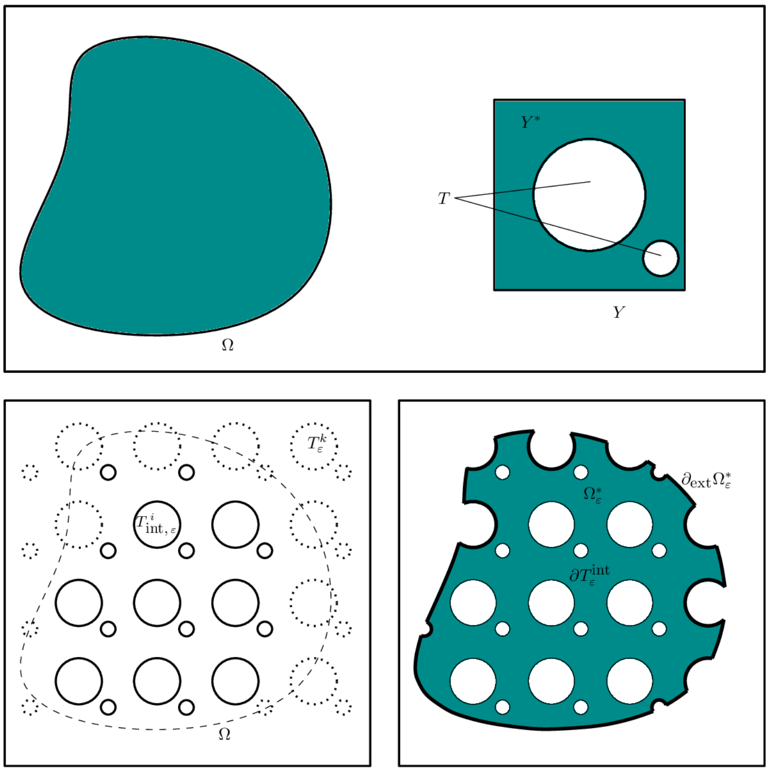

Figure 2: Periodically perforated domain. Upper: domain Ω and the reference cell; lower left: inner holes $T^i_{\text{int},\varepsilon}$; lower right: part of Ω occupied by material Ω_ε^* (marked by cyan), with its exterior boundary $\partial_{\text{ext}}\Omega_\varepsilon^*$ and interior boundary $\partial T_{\text{int},\varepsilon}$.

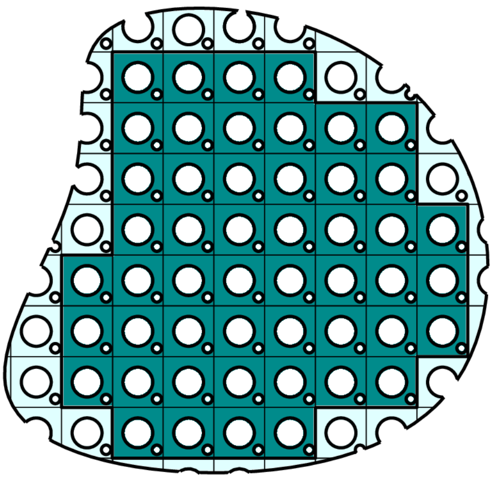

Figure 3: Domains Λ_ε^* (light) and $\widehat{\Omega}_\varepsilon^*$ (dark).

Now, we define the unfolding operator for perforated domains. In sequel, we cover the case where the ratio of the volume of material to the volume of cell is constant for all ε, i.e. the function $r(\varepsilon) = \varepsilon$.

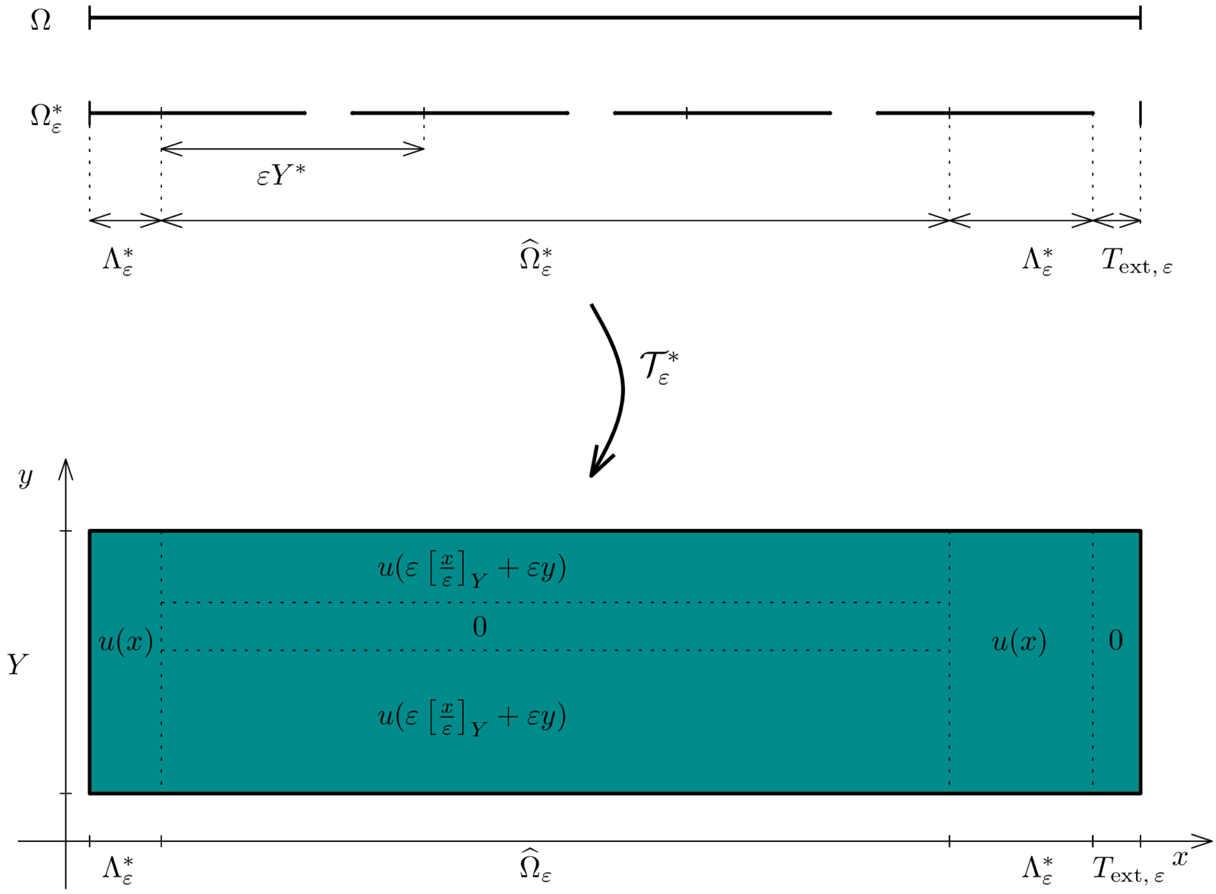

Figure 4: Example of the unfolding of a function $u(x)$ defined on a periodically perforated domain Ω^*_ε.

Definition 2.2 (Unfolding operator for perforated domains)**.** An operator $\mathcal{T}^*_\varepsilon$ maps a function $u : \Omega^*_\varepsilon \to \mathbb{R}$ to $\mathcal{T}^*_\varepsilon(u) : \Omega \times Y \to \mathbb{R}$, and is defined as follows:

$$\mathcal{T}^*_\varepsilon(u)(x,y) = \begin{cases} u\big(\varepsilon\big[\frac{x}{\varepsilon}\big]_Y + \varepsilon y\big) & \text{for } (x,y) \in \widehat{\Omega}_\varepsilon \times Y^*, \\ u(x) & \text{for } (x,y) \in \Lambda^*_\varepsilon \times Y, \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

For u defined on Ω^*_ε we denote its extension by zero into Ω by $\tilde{u}$. The same notation will be used for functions defined on $\Omega \times Y^*$ extended by zero into $\Omega \times Y$.

Theorem 2.3 (Properties of the unfolding operator for perforated domain)**.** *Let $\mathcal{T}^*_\varepsilon$ be the unfolding operator for perforated domains defined by (9). Then for all $\varepsilon \in E$ we have:*

(i) The operator $\mathcal{T}^*_\varepsilon$ is multiplicative, i.e. for all $u, v : \Omega^*_\varepsilon \to \mathbb{R}$ we have

$$\mathcal{T}^*_\varepsilon(u\,v) = \mathcal{T}^*_\varepsilon(u)\,\mathcal{T}^*_\varepsilon(v).$$

(ii) The unfolding operator $\mathcal{T}^*_\varepsilon$ is linear.

(iii) The unfolding operator $\mathcal{T}_\varepsilon^*$ *conserves the integral, i.e. for all* $u \in L^1(\Omega_\varepsilon^*)$ *one has*

$$\iint\limits_{\Omega\times Y} \mathcal{T}_\varepsilon^*(u)(x,y)\,\mathrm{d}x\,\mathrm{d}y = |Y| \int\limits_{\Omega_\varepsilon^*} u(x)\,\mathrm{d}x.$$

(iv) The unfolding operator $\mathcal{T}_\varepsilon^*$ *conserves the norm in the sense that for every* $u \in L^p(\Omega_\varepsilon^*)$, $p \in \langle 1, \infty)$, *it holds* $\|\mathcal{T}_\varepsilon^*(u)\|_{L^p(\Omega\times Y)} = |Y|^{\frac{1}{p}} \|u\|_{L^p(\Omega_\varepsilon^*)}$.

Thus $\mathcal{T}_\varepsilon^*$ *is bounded and its norm satisfies* $\|\mathcal{T}_\varepsilon^*\|_{\mathcal{L}(L^p(\Omega_\varepsilon^*),L^p(\Omega\times Y))} = |Y|^{\frac{1}{p}}$.

(v) $\mathcal{T}_\varepsilon^*$ *is continuous operator for* $L^p(\Omega_\varepsilon^*)$ *to* $L^p(\Omega \times Y)$, *where* $p \in \langle 1, \infty)$.

Using the unfolding operator, we define two-scale convergence.

Definition 2.4 (Two-scale convergence for perforated domains)**.** Let $\{u_\varepsilon\}$ be a sequence in $L^p(\Omega_\varepsilon^*)$ and $u_0 \in L^p(\Omega \times Y)$, $p \in \langle 1, \infty)$.

A sequence $\{u_\varepsilon\}$ *two-scale strongly (resp. weakly) converges* to u_0 in $L^p(\Omega)$ with respect to the scale E if the sequence $\{\mathcal{T}_\varepsilon^*(u_\varepsilon)\}$ converges to u_0 strongly (resp. weakly) in $L^p(\Omega \times Y)$.

The following theorem describes relations among convergences.

Theorem 2.5. *Let* $\{u_\varepsilon\}$ *be a sequence in* $L^p(\Omega_\varepsilon^*)$ *and* $u_0 \in L^p(\Omega \times Y)$, $p \in \langle 1, \infty)$. *Then*

(i) Any constant sequence $\{u\} \in L^p(\Omega)$ *strongly two-scale converges to*

$$u_0(x,y) = \begin{cases} u(x) & [x,y] \in \Omega \times Y^*, \\ 0 & \textit{otherwise.} \end{cases}$$

(ii) Any sequence $\{u_\varepsilon\} \in L^p(\Omega_\varepsilon^*)$ *two-scale converging (strongly or weakly) in* $L^p(\Omega)$ *is bounded in* $L^p(\Omega_\varepsilon^*)$.

(iii) If $\{u_\varepsilon\}$ *strongly two-scale converges to* u_0 *in* $L^p(\Omega)$, *then it weakly two-scale converges to the same limit.*

(iv) For $p \in (1, \infty)$, *if* $\{u_\varepsilon\}$ *weakly two-scale converges to* u_0 *in* $L^p(\Omega)$, *then its extension by zero converges weakly to* $u^*(x) = \frac{1}{|Y|}\int_{Y^*} u_0(x,y)\,\mathrm{d}y = \frac{|Y^*|}{|Y|}\mathcal{M}_{Y^*}(u_0)(x)$ *in* $L^p(\Omega)$.

2.3. Unfolding operator $\mathcal{T}_\varepsilon^*$ and gradients

Consider a function $u \in W^{1,p}(\Omega_\varepsilon^*)$. It is straightforward that

$$\mathcal{T}_\varepsilon^*(\nabla u) = \begin{cases} \frac{1}{\varepsilon}\nabla_y \mathcal{T}_\varepsilon^*(u) & \text{on } \widehat{\Omega}_\varepsilon \times Y^*, \\ \nabla u = \nabla \mathcal{T}_\varepsilon^*(u) & \text{on } \Lambda_\varepsilon^* \times Y, \\ 0 & \text{otherwise.} \end{cases} \tag{10}$$

Now we will state the main result about the convergence of an unfolded sequence of gradients $\{\mathcal{T}_\varepsilon^*(\nabla u_\varepsilon)\}$.

Theorem 2.6. *Let a sequence $\{u_\varepsilon\}$ be bounded in $W^{1,p}(\Omega_\varepsilon^*)$, for $p \in (1,\infty)$. Then, there exists a subsequence (still denoted $\{u_\varepsilon\}$) and functions $u_0 \in W^{1,p}(\Omega)$ and $u_0^* \in L^p(\Omega; W^{1,p}_{per}(Y))$ such that*

(i) $\mathcal{T}_\varepsilon^*(u_\varepsilon) \rightharpoonup u$ weakly in $L^p\big(\Omega; W^{1,p}(Y)\big)$, where

$$u(x,y) = \begin{cases} u_0(x) & [x,y] \in \Omega \times Y^*, \\ 0 & \textit{otherwise.} \end{cases}$$

(ii) $\mathcal{T}_\varepsilon^*(\nabla u_\varepsilon) \rightharpoonup \nabla u_0 + \nabla_y u_0^*$ weakly in $[L^p(\Omega \times Y)]^N$.

Moreover, $\mathcal{M}_Y(u_0^*) = 0$ and $u_0^* = -y^c \cdot \nabla u_0 \quad on \quad \Omega \times T$.

3. Application - Torsion problem

Study of elastic torsion of a bar leads to a problem described in [FNJ12; FR15]. Here, a more general problem is studied and the case of elastic torsion is obtained as an application.

Let us start with a definition:

Definition 3.1. Let $\alpha, \beta \in \mathbb{R}$, such that $0 < \alpha < \beta$. We say that a matrix function $A(x) = \big(a_{ij}^\varepsilon(x)\big) \in [L^\infty(\Omega)]^{N\times N}$ belongs to a set $M(\alpha, \beta, \Omega)$ if and only if

$$\begin{array}{lrcll} (i) & (A(x)\lambda, \lambda) & \geq & \alpha|\lambda|^2, & \text{(ellipticity)}, \\ (ii) & |A(x)\lambda| & \leq & \beta|\lambda|, & \text{(boundedness)}. \end{array} \tag{11}$$

$\forall \lambda \in \mathbb{R}^N$, a.e. in Ω.

Now we can state a boundary problem:

$$\left.\begin{array}{rcll} -\nabla \cdot (A^\varepsilon \nabla u_\varepsilon) & = & f & \text{in } \ \Omega_\varepsilon^*, \\ u_\varepsilon & = & 0 & \text{on } \ \partial_{\text{ext}}\Omega_\varepsilon^*, \\ u_\varepsilon & = & \text{const.} & \text{on } \ \partial T^i_{\text{int},\,\varepsilon}; \quad i = 1, \ldots, m(\varepsilon), \\ \displaystyle\int\limits_{\partial T^i_{\text{int},\,\varepsilon}} A^\varepsilon(x)\, \frac{\partial u_\varepsilon}{\partial n}(x)\,\mathrm{d}x & = & \displaystyle\int\limits_{T^i_{\text{int},\,\varepsilon}} f(x)\,\mathrm{d}x & \end{array}\right\} \tag{12}$$

where Ω^*_ε, $\partial_{\text{ext}}\Omega^*_\varepsilon$, $T^i_{\text{int},\,\varepsilon}$, etc. are defined in the beginning of the Section 2.1, $f \in L^2(\Omega)$, $A^\varepsilon(x) = \big(a^\varepsilon_{ij}(x)\big)_{i,j=1\dots N}$ is a matrix function from the set $M(\alpha,\beta,\Omega^*_\varepsilon)$, n is the outward-pointing unite normal (i.e. on the inner boundary, n is directed inward the holes), $m(\varepsilon)$ denotes number of interior holes.

For $f(x) = -2$ and $N = 2$ we get a torsion problem derived in [FR15].

Let us introduce the linear space

$$\mathcal{S}_\varepsilon(\Omega) = \Big\{ v \in H^1_0(\Omega);\ v = 0 \text{ in } \overline{T_{\text{ext},\,\varepsilon}},\ v = \text{const. in } \overline{T^i_{\text{int},\,\varepsilon}},\ i = 1,\dots,m(\varepsilon) \Big\}. \tag{13}$$

with the norm $\|v\|_{\mathcal{S}_\varepsilon(\Omega)} = \|\nabla v\|_{[L^2(\Omega^*_\varepsilon)]^N}$.

Weak formulation of the problem (12):

$$\begin{cases} \text{Find } u_\varepsilon \in \mathcal{S}_\varepsilon(\Omega) \text{ such that} \\ \displaystyle\int_{\Omega^*_\varepsilon} A^\varepsilon(x)\,\nabla u_\varepsilon(x)\cdot\nabla v(x)\,\mathrm{d}x = \int_\Omega f(x)\,v(x)\,\mathrm{d}x, \qquad \forall v \in \mathcal{S}_\varepsilon(\Omega). \end{cases} \tag{14}$$

Homogenized solution of the problem above is described by to following theorem:

Theorem 3.2. *Let u_ε be the solution of the problem (14). Assume that*

$$\mathcal{T}^*_\varepsilon(A^\varepsilon) \ \to\ A \quad \textit{a.e. in } \ \Omega \times Y \tag{15}$$

for a matrix $A = A(x,y)$ such that $A = (a_{ij})_{i,j=1\dots N} \in M(\alpha,\beta,\Omega\times Y)$.

Then, there exists $u_0 \in H^1_0(\Omega)$ and $u^*_0 \in L^2(\Omega, H^1_{\text{per}}(Y))$ such that

$$\|u_\varepsilon - u_0\|_{L^2(\Omega^*_\varepsilon)} \ \to\ 0,$$

$$\mathcal{T}^*_\varepsilon(u_\varepsilon) \ \rightharpoonup\ u \quad \textit{weakly in } L^2(\Omega, H^1(Y)), \ \textit{where}$$

$$u(x,y) = \begin{cases} u_0(x) & [x,y] \in \Omega \times Y^*, \\ 0 & \textit{otherwise.} \end{cases} \tag{16}$$

$$\mathcal{T}^*_\varepsilon(\nabla u_\varepsilon) \ \rightharpoonup\ \nabla u_0 + \nabla_y u^*_0 \quad \textit{weakly in } \big[L^2(\Omega\times Y)\big]^N, \ \textit{where}$$

$$\mathcal{M}_Y(u^*_0) = 0 \quad \textit{and} \quad u^*_0 = -y^c\cdot\nabla u_0 \quad \textit{on } \ \Omega \times T.$$

The pair (u_0, u_0^*) *is the unique solution of the problem:*

$$\begin{cases} \textit{Find } u_0 \in H_0^1(\Omega) \textit{ and } u_0^* \in L^2(\Omega, H^1_{\text{per}}(Y)) \textit{ such that} \\ \dfrac{1}{|Y|} \iint\limits_{\Omega\times Y^*} A(x,y)\,[\nabla u_0(x) + \nabla_y u_0^*(x,y)]\cdot[\nabla\Psi(x) + \nabla_y\Phi(x,y)]\,\mathrm{d}x\,\mathrm{d}y = \\ \qquad\qquad\qquad\qquad \int\limits_\Omega f(x)\,\Psi(x)\,\mathrm{d}x, \\ \forall\Psi \in H_0^1(\Omega), \\ \forall\Phi \in L^2(\Omega, H^1_{\text{per}}(Y)),\ \textit{such that } \Phi + y_c\cdot\nabla\Psi \textit{ is constant in } y \textit{ on } \Omega\times T. \end{cases} \tag{17}$$

4. Numerical examples

We present numerical example for dimension $N = 2$.

Let $x = (x_1, x_2) \in \Omega$ and $y = (y_1, y_2) \in Y$, where Ω is a simple domain in $\mathbb{R}^2$ and $Y = \langle 0, l_1) \times \langle 0, l_2)$, l_1, l_2 are real positive numbers. Vector function y^c has the form $y^c = (\,y_1^c,\, y_2^c\,)$. Furthermore, let us suppose that A is a function only in variable y, i.e. $A(x,y) = A(y)$.

We would like to solve the problem, derived in the Theorem 3.2:

$$\begin{cases} \text{Find } u_0 \in H_0^1(\Omega) \text{ and } u_0^* \in L^2(\Omega, H^1_{\text{per}}(Y)) \text{ such that} \\ \dfrac{1}{|Y|} \iint\limits_{\Omega\times Y^*} A(y)\,[\nabla u_0(x) + \nabla_y u_0^*(x,y)]\cdot[\nabla\Psi(x) + \nabla_y\Phi(x,y)]\,\mathrm{d}x\,\mathrm{d}y = \\ \qquad\qquad\qquad\qquad = \int\limits_\Omega f(x)\,\Psi(x)\,\mathrm{d}x, \\ \forall\Phi \in L^2(\Omega, H^1_{\text{per}}(Y)), \\ \forall\Psi \in H_0^1(\Omega),\ \text{s. t. } \Phi + y^c\cdot\nabla\Psi \text{ is const. in } y \text{ on } \Omega\times T, \\ \mathcal{M}_Y(u_0^*) = 0, \\ u_0^* = -y^c\cdot\nabla u_0 \ \text{ on } \ \Omega\times T. \end{cases} \tag{18}$$

We shall look for u_0, u_0^* in two steps. At first, we will compute auxiliary functions denoted $\hat{\chi}_1, \hat{\chi}_2$ and subsequently, using them, we will find homogenized solutions u_0, u_0^*.

Let us choose $\Psi(x) \equiv 0$ as a test function in (18). We suggest function u_0^* in the form

$$u_0^*(x,y) = -\hat{\chi}_1(y)\,\frac{\partial u_0}{\partial x_1}(x) - \hat{\chi}_2(y)\,\frac{\partial u_0}{\partial x_2}(x).$$

Then, (18) takes the form

$$\iint\limits_{\Omega\times Y^*} A\left[\frac{\partial u_0}{\partial x_1}\,\frac{\partial \Phi}{\partial y_1} + \frac{\partial u_0}{\partial x_2}\,\frac{\partial \Phi}{\partial y_2}\right] \mathrm{d}x\,\mathrm{d}y =$$

$$= \iint\limits_{\Omega\times Y^*} A\left[\frac{\partial \hat{\chi}_1}{\partial y_1}\,\frac{\partial u_0}{\partial x_1}\,\frac{\partial \Phi}{\partial y_1} + \frac{\partial \hat{\chi}_2}{\partial y_1}\,\frac{\partial u_0}{\partial x_2}\,\frac{\partial \Phi}{\partial y_1} + \frac{\partial \hat{\chi}_1}{\partial y_2}\,\frac{\partial u_0}{\partial x_1}\,\frac{\partial \Phi}{\partial y_2} + \frac{\partial \hat{\chi}_2}{\partial y_2}\,\frac{\partial u_0}{\partial x_2}\,\frac{\partial \Phi}{\partial y_2}\right] \mathrm{d}x\,\mathrm{d}y.$$

From this we see that the problem (18) is fulfilled when the auxiliary function $\hat{\chi}_i$, $i = 1,2$, satisfies

$$\int\limits_{Y^*} A\,\frac{\partial \Phi}{\partial y_i}\,\mathrm{d}y = \int\limits_{Y^*} A\,\nabla\hat{\chi}_i\cdot\nabla\Phi\,\mathrm{d}y,\ \forall\,\Phi \in L^2(\Omega, H^1_{\mathrm{per}}(Y)),\ \text{ s. t. } \Phi \text{ is const. in } y \text{ on } T.$$

Rewriting this, we derive the following problem

$$\begin{cases} \text{Find } \hat{\chi}_i \in H^1_{\mathrm{per}}(Y) \text{ such that} \\ \displaystyle\int\limits_{Y^*} A\,\nabla(\hat{\chi}_i - y_i)\cdot\nabla\Phi\,\mathrm{d}y = 0, \\ \forall\,\Phi \in L^2(\Omega, H^1_{\mathrm{per}}(Y)),\ \text{ s. t. } \Phi \text{ is constant in } y \text{ on } T, \\ \mathcal{M}_Y(\hat{\chi}_i) = 0,\ \hat{\chi}_i = -y_i^c \text{ on } T. \end{cases} \tag{19}$$

Now, let us choose as a test function in (18) a function $\Phi(x,y) = -\varphi_1(y)\,y^c\cdot\Psi(x)$, where $\Psi \in \mathcal{D}(\Omega)$, φ is Y-periodic function and $\varphi_1|_Y \in \mathcal{D}(Y)$, $\varphi_1 \equiv 1$ on T.

Then, (18) takes the form

$$\frac{1}{|Y|}\iint\limits_{\Omega\times Y^*} A\left[\nabla u_0 + \nabla_y\left(-\hat{\chi}_1\,\frac{\partial u_0}{\partial x_1} - \hat{\chi}_2\,\frac{\partial u_0}{\partial x_2}\right)\right]\cdot\left[\nabla\Psi - \nabla_y(\varphi_1\,y^c\cdot\Psi)\right]\mathrm{d}x\,\mathrm{d}y =$$

$$= \int\limits_{\Omega} f\,\Psi\,\mathrm{d}x. \tag{20}$$

Simple computations yield the problem

$$\begin{cases} \text{Find } u_0 \in H^1_0(\Omega) \text{ such that} \\ \dfrac{|Y^*|}{|Y|}\displaystyle\int\limits_{\Omega} \mathcal{A}\,\nabla u_0\cdot\nabla\Psi\,\mathrm{d}x\,\mathrm{d}y = \int\limits_{\Omega} f\,\Psi\,\mathrm{d}x, \quad \forall\Psi \in H^1_0(\Omega). \end{cases} \tag{21}$$

Where matrix $\mathcal{A}$ is given by $\mathcal{A} = (a_{ij})_{i,j=1,2}$

$$a_{11} = \int\limits_{Y^*} A\left[\left(1-\frac{\partial\hat{\chi}_1}{\partial y_1}\right)\left(1-\frac{\partial(y_1^c\varphi_1)}{\partial y_1}\right)+\frac{\partial\hat{\chi}_1}{\partial y_2}\frac{\partial(y_1^c\varphi_1)}{\partial y_2}\right]\mathrm{d}y, \qquad (22)$$

$$a_{12} = -\int\limits_{Y^*} A\left[\left(1-\frac{\partial\hat{\chi}_1}{\partial y_1}\right)\frac{\partial(y_2^c\varphi_1)}{\partial y_1}+\frac{\partial\hat{\chi}_1}{\partial y_2}\left(1-\frac{\partial(y_2^c\varphi_1)}{\partial y_2}\right)\right]\mathrm{d}y, \qquad (23)$$

$$a_{21} = -\int\limits_{Y^*} A\left[\left(1-\frac{\partial\hat{\chi}_2}{\partial y_2}\right)\frac{\partial(y_1^c\varphi_1)}{\partial y_2}+\frac{\partial\hat{\chi}_2}{\partial y_1}\left(1-\frac{\partial(y_1^c\varphi_1)}{\partial y_1}\right)\right]\mathrm{d}y, \qquad (24)$$

$$a_{22} = \int\limits_{Y^*} A\left[\left(1-\frac{\partial\hat{\chi}_2}{\partial y_2}\right)\left(1-\frac{\partial(y_2^c\varphi_1)}{\partial y_2}\right)+\frac{\partial\hat{\chi}_2}{\partial y_1}\frac{\partial(y_2^c\varphi_1)}{\partial y_1}\right]\mathrm{d}y. \qquad (25)$$

In the sequel, we present results of homogenization of the torsion problem derived in [FR15]. We assume $\Omega = (0,1)\times(0,1)$, reference cell $Y = \langle 0,1)\times\langle 0,1)$, reference hole $T = \left(\frac{1}{4},\frac{3}{4}\right)\times\left(\frac{1}{4},\frac{3}{4}\right)$. Torsion problem is obtained for $A(y) = 1$, $f(x) = -2$.

According to the behavior of the holes, we distinguish three cases. They were described in Section 2.1.

- First, let us present results for $r(\varepsilon) = \varepsilon$, as for this case the Theorem 3.2 and all results in this chapter were derived. The sequence of domains is shown on the upper line on Figure 1. In the first step, by solving problem (19) we get two auxiliary functions $\hat{\chi}_1$ (on Figure 5) and $\hat{\chi}_2$.

  In the second step the problem (21) is solved to obtain the homogenized solution. A comparison of functions u_ε and homogenized solution u_0 is on Figure 7. Graph of function $u_{1/4}$ is on Figure 6.

In the following two cases we only present numerical results without any theoretical result.

- For $r(\varepsilon) = \varepsilon^2$ (so called small holes), the results are on Figure 8. The sequence of domains is on the middle line on Figure 1.
- For $r(\varepsilon) = \varepsilon(2-\varepsilon)$, the results are on Figure 9. The sequence of domains is on the lower line on Figure 1.

The numerical results are obtained by finite element method implemented in MATLAB.

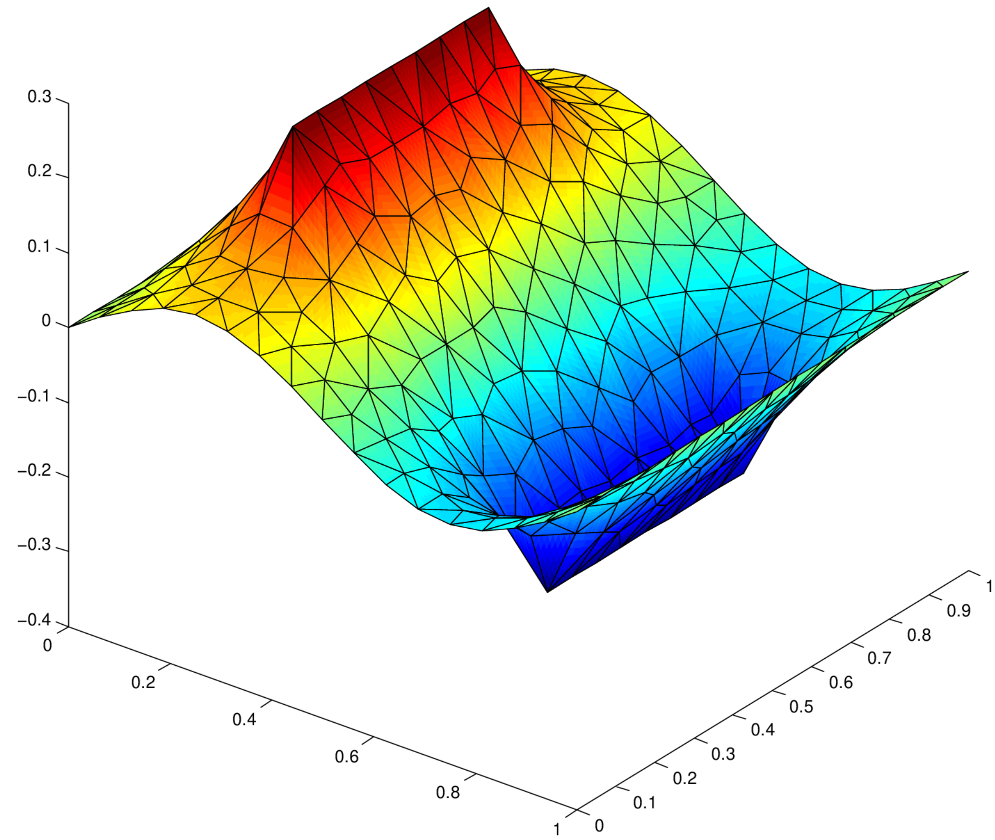

Figure 5: Auxiliary function $\hat{\chi}_1$.

5. Conclusion

In problems which are set on perforated domains Ω_ε^*, where the shape and distribution of holes depends on the parameter ε, it may be difficult to define convergence for the sequence of solutions $\{u_\varepsilon\}$. There exist some approaches to solve this difficulty but their usage is usually limited. Limiting factors are usually the shape of the perforations or boundary conditions on inner boundaries.

The two-scale convergence, the approach presented in this thesis, is based on periodic unfolding operator for perforated domains $\mathcal{T}_\varepsilon^*$. This method is suitable for periodically distributed holes. The unfolded sequence $\{\mathcal{T}_\varepsilon^*(u_\varepsilon)\}$ is defined on fixed domains which removes difficulties with the convergence.

This technique was applied to the problem describing torsion of the bar (and its more general version). We derived a homogenized equation defined on a simply connected domain (without holes). We also presented numerical aspect

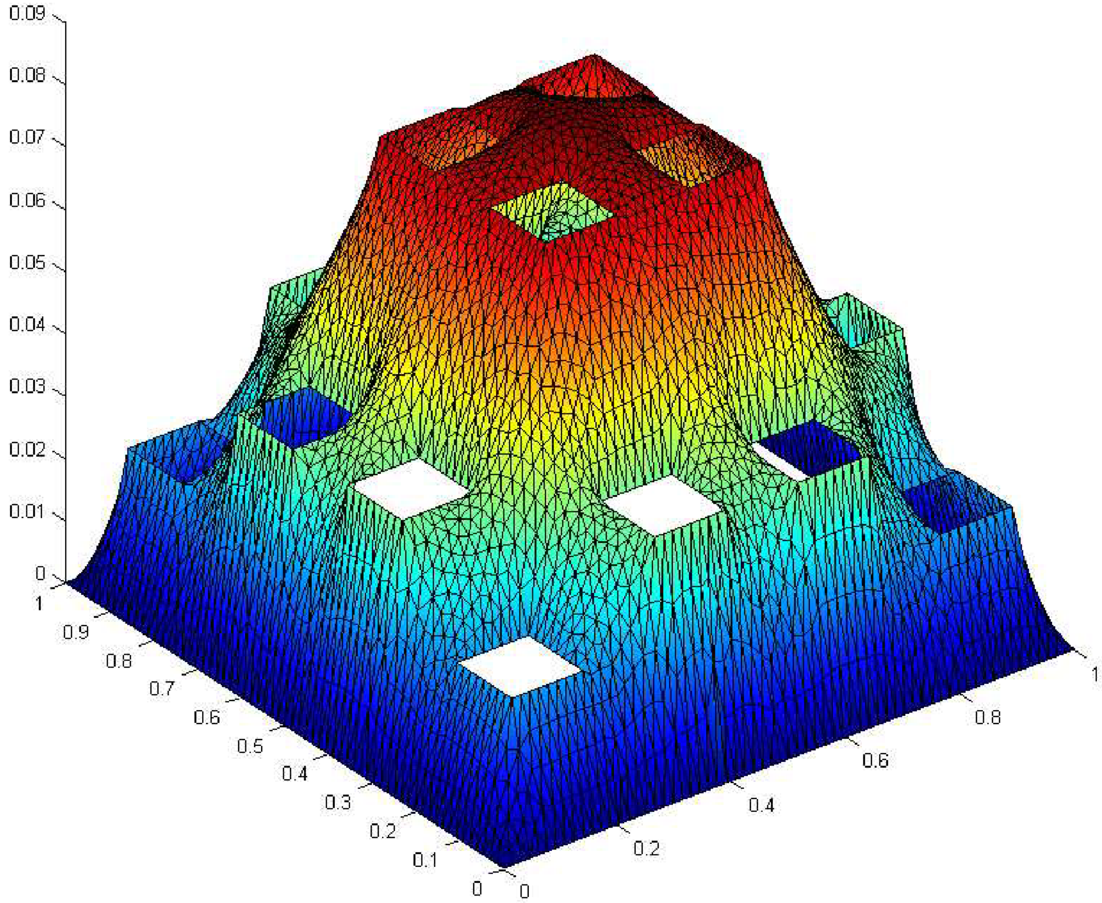

Figure 6: Graph of function $u_{1/4}$.

of solving such a homogenized problem and in the last section there are some numerical examples.

Moreover, we proved some interesting properties which make it suitable for more general situations than that presented here. Unfolding operator $\mathcal{T}_\varepsilon^*$, used in this thesis, is slightly different than the one used in e.g. [CD00], [CDG02]. This change in definition allowed us to prove some properties in a more elegant way.

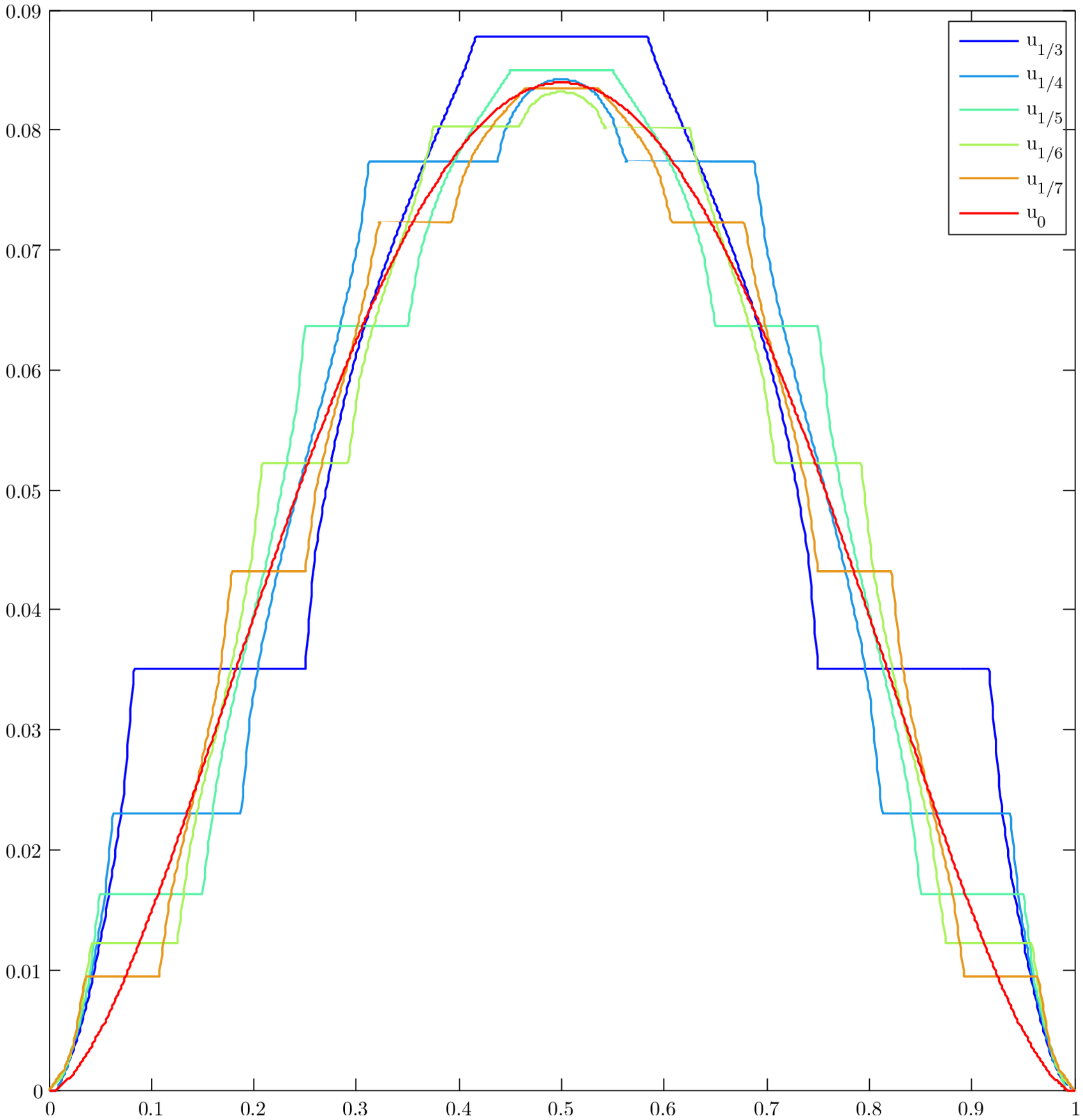

Figure 7: Diagonal cuts of functions u_ε, for $\varepsilon = {}^1/_3, {}^1/_4, {}^1/_5, {}^1/_6, {}^1/_7$, and of homogenized solution u_0, the behavior of holes is described by $r(\varepsilon) = \varepsilon$.

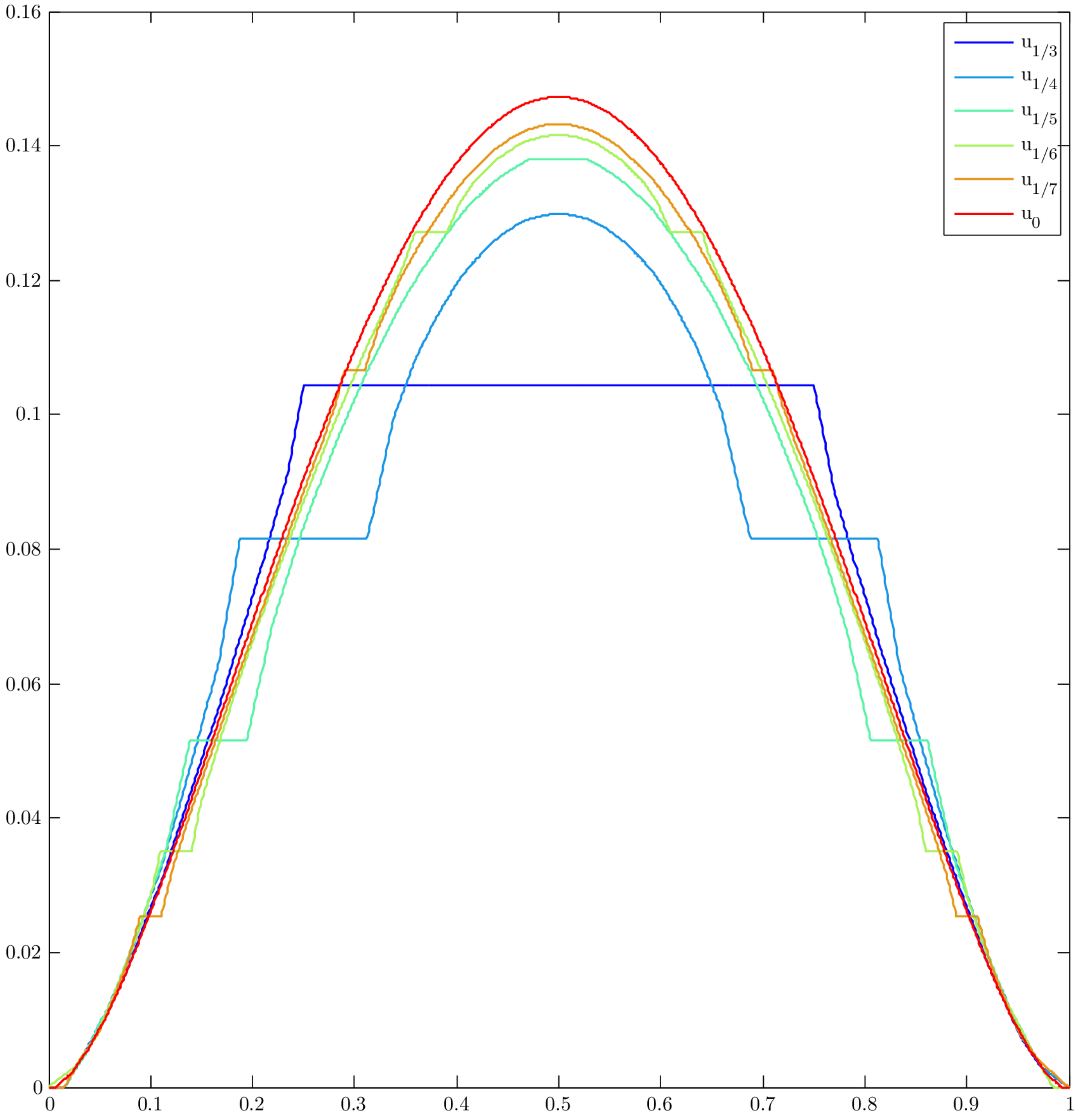

Figure 8: Diagonal cuts of functions u_ε, for $\varepsilon = {}^1/_3, {}^1/_4, {}^1/_5, {}^1/_6, {}^1/_7$, and solution u_0 of torsion problem on domain without holes (simply connected domain), the behavior of holes is described by $r(\varepsilon) = \varepsilon^2$.

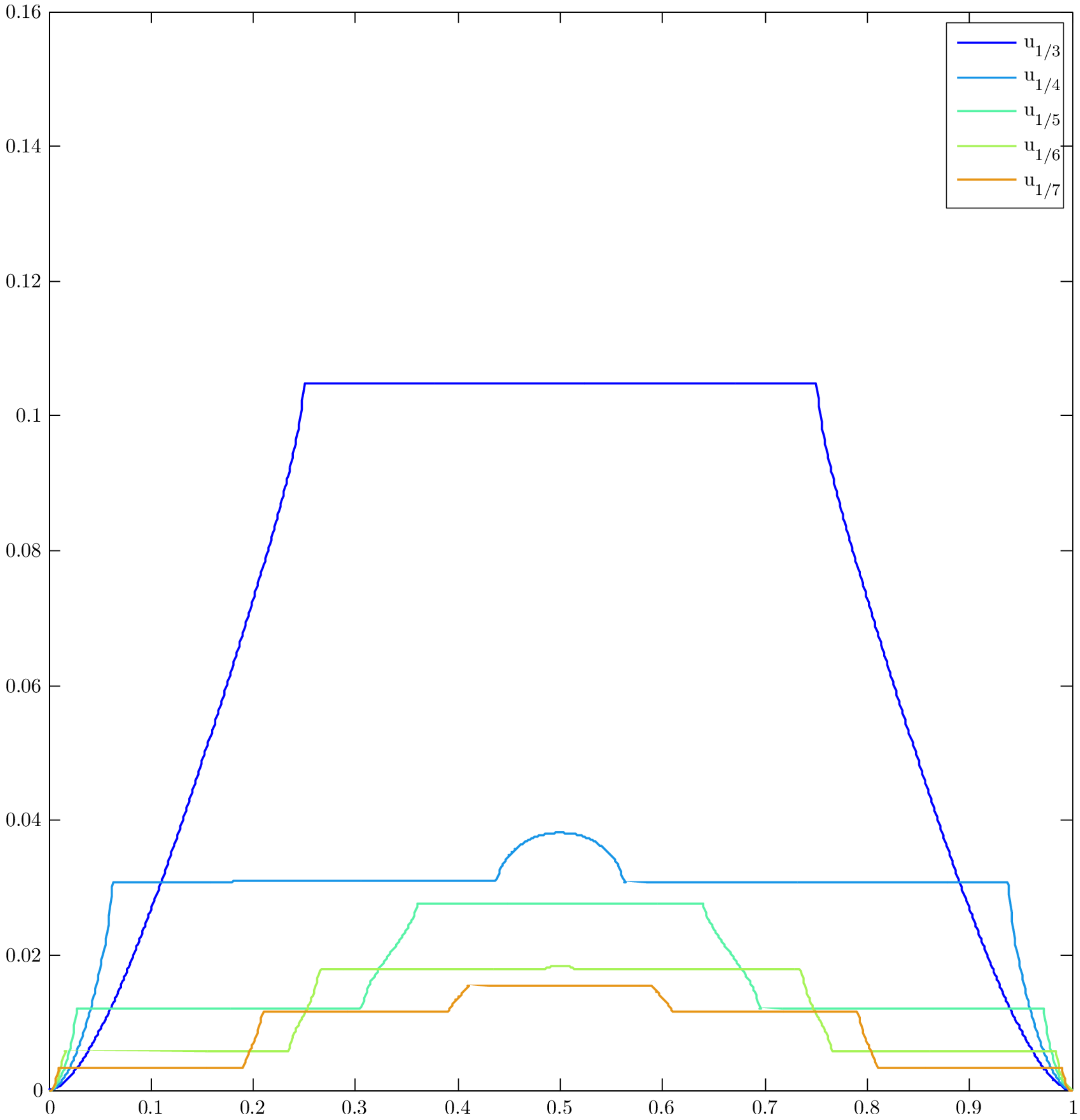

Figure 9: Diagonal cuts of functions u_ε, for $\varepsilon = {}^1/_3, {}^1/_4, {}^1/_5, {}^1/_6, {}^1/_7$, the behavior of holes is described by $r(\varepsilon) = \varepsilon(2 - \varepsilon)$.

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Education

PhD. studies 2009 - 2016.	**Brno University of Technology,** **Faculty of Mechanical Engineering,** Applied mathematics, Dissertation: Homogenization in perforated domains. Tutor: Prof. Jan Franců.
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Work experience

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2010, 2012	**University of Defence,** Part-time job: Designing a motion of a movement of a robot, modeling of the optimal strategy, modeling of a organization structure.

2009 - 2011	**INEX-SDA Brno**, project Cipísek, Coordinator of a voluntary project. Project helps socially handicapped children with integration to the society. In the project was approx. 30 children and 12 volunteers.
2008	**Honeywell** - Innovators Scholarship program, FEM analysis of a turbo-charger, optimization of a turbo-charger design, improving of a mesh algorithm.

University activities

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Abstract

The numerical solving of mathematical models describing the mechanical behavior of materials with a fine structure (composite materials, finely perforated materials etc.) usually requires huge computational performance. Hence in numerical modeling the original material is replaced by an equivalent homogeneous one.
In this work a two-scale convergence based on a periodical unfolding operator is used to find the homogenized material. The operator was for the first time used by J. Casado-Díaz. In this Ph.D. thesis, the operator is defined in a slightly different way which allows us to prove some of its new properties. The unfolding operator for functions defined on a perforated domain is defined analogically and its properties are proved. Finally, this operator is used to find the homogenized solution of a special family of problems with an integral boundary condition; some numerical results are presented.