BRNO UNIVERSITY OF TECHNOLOGY

VYSOKÉ UČENÍ TECHNICKÉ V BRNĚ

FACULTY OF MECHANICAL ENGINEERING

FAKULTA STROJNÍHO INŽENÝRSTVÍ

INSTITUTE OF SOLID MECHANICS, MECHATRONICS AND BIOMECHANICS

ÚSTAV MECHANIKY TĚLES, MECHATRONIKY A BIOMECHANIKY

STRESS-STRAIN ANALYSIS OF THE RAILWAY BRIDGE

NAPJATOSTNĚ DEFORMAČNÍ ANALÝZA ŽELEZNIČNÍHO MOSTU

BACHELOR'S THESIS

BAKALÁŘSKÁ PRÁCE

AUTHOR | Ali Mohammed Ali Hasan Al-Qubati

AUTOR PRÁCE

SUPERVISOR | doc. Ing. Vladimír Fuis, Ph.D.

VEDOUCÍ PRÁCE

BRNO 2019

VYSOKÉ UČENÍ TECHNICKÉ V BRNĚ
FAKULTA STROJNÍHO INŽENÝRSTVÍ

Zadání bakalářské práce

Ústav:	Ústav mechaniky těles, mechatroniky a biomechaniky
Student:	**Ali Mohammed Ali Hasan Al-Qubati**
Studijní program:	Aplikované vědy v inženýrství
Studijní obor:	Mechatronika
Vedoucí práce:	**doc. Ing. Vladimír Fuis, Ph.D.**
Akademický rok:	2018/19

Ředitel ústavu Vám v souladu se zákonem č.111/1998 o vysokých školách a se Studijním a zkušebním řádem VUT v Brně určuje následující téma bakalářské práce:

Napjatostně deformační analýza železničního mostu

Stručná charakteristika problematiky úkolu:

Prutové soustavy se běžně používají k modelování chování mostů, jeřábů, stožárů a ostatních technických objektů, které jsou vyrobeny z prutových těles a splňují předpoklady kladené na prutové soustavy.

Cíle bakalářské práce:

1. Literární rešerše.
2. Vytvoření výpočtového modelu prutové soustavy mostu a provedení napjatostně deformační analýzy při statické zatěžování. Optimalizace příčných průřezů.
3. Verifikace vybraného analytického výpočtu numerickým řešením.

Seznam doporučené literatury:

JANÍČEK, P., ONDRÁČEK, E., VRBKA, J. a BURŠA, J. Body Mechanics: Flexibility and Strength I, Academic Publishing House CERM, s.r.o., Brno, 2004, ISBN 80-214-2592-x, (in Czech).

FLORIAN, Z., PŘIKRYL, K., ONDRÁČEK, E. Body Mechanics: Statics, Brno: PC-DIR, 1995, ISBN 80-214-0694-1, (in Czech).

Termín odevzdání bakalářské práce je stanoven časovým plánem akademického roku 2018/19

V Brně, dne

L. S.

prof. Ing. Jindřich Petruška, CSc. ředitel ústavu	doc. Ing. Jaroslav Katolický, Ph.D. děkan fakulty

Abstrakt:

Tato práce se zabývá především napjatostně deformační analýzou železničního mostu, který je příkladem prutové soustavy.

Práce byla rozdělena do třech hlavních částí. V první části je představena teorie a vztahy, které jsou důležité pro celou práci. Druhou částí je analýza mostu a výpočet bezpečnosti s vlakem i bez něj. Poslední část se zabývá porovnáním výsledků z analytického řešení s výsledky z numerického řešení z programu ANSYS.

Absstract :

This thesis was mainly about Analyzing the stresses and strain of a railway bridge which is an example of a truss structure.

The thesis was divided into three main part. The first part is introducing the theory and the important relations that will help us through the whole thesis. The second part is analyzing the bridge and calculating the safety factors with and without the train. Last part is about comparing the results from the analytical way with the results from the numerically using ANSYS.

Kličová slova:

Prut, prutová soustava , železniční most, napětostně analyza, deformační analyza.

Key words:

Bar, system of bars, truss system, stress analysis, strain analysis, railway bridge.

Bibliografická citace:

AL-QUBATI, Ali Mohammed Ali Hasan. *Napjatostně deformační analýza železničního mostu.* Brno, 2019. Bakalářská práce. Vysoké učení technické v Brně, Fakulta strojního inženýrství, Ústav mechaniky těles, mechatroniky a biomechaniky. Vedoucí práce Vladimír Fuis.

BIBLIOGRAPHIC CITATION:

AL-QUBATI, Ali Mohammed Ali Hasan. *Stress-strain analysis of the railway bridge.* Brno, 2019. Bachelor's Thesis. Vysoké učení technické v Brně, Fakulta strojního inženýrství, Institute of Solid Mechanics, Mechatronics and Biomechanics. Supervisor Vladimír Fuis.

Declaration:

I declare that have written my bachelor thesis with the subject of (Stress-strain analysis of the railway bridge) individually with the instructions of the supervisor. I declare that I have not violated any copy rights and all the literatures used are detailed on the source section.

Brno: …………………….   ………………………………………

Ali Al-Qubati

Acknowledgement:

Firstly, I would like to express my appreciations to doc. Ing. Vladimír Fuis, Ph.D. for his advices and instructions that helped me accomplish this work.

I would like to send my heartfelt thanks to my parents and family members who have been my main support throughout my studies.

Last but not least, I would like to thank my friends who helped me accomplish this thesis, some of which are Zaid Al-Dailami and Tomáš Adamic.

Content:

1.Introduction... 7

2.Objectives 8

3. theorical part 10

3.2. Bars..10

3.2.1. Definition...10

3.2.2 Bar assumptions...10

3.3 Geometrical characteristics of the cross-section...12

3.3.1 Cross-section area..12

3.3.2 Static (linear) moment...12

3.3.3 Moment of inertia...12

3.3.4 Properties of moment of inertia...13

3.3.5 Saint Venant's principle...13

3.3.6 Uses of Saint Venant's principle 14

3.4. System of bars...14

3.4.1. System of bar assumptions...14

3.4.2. Types of bar system...14

3.5. Methods for solving the system of bar..15

3.5.1. Method of joints...15

3.5.2. Method of sections..15

3.6. Simple tension and compression...15

3.6.1. Geometrical relations...15

3.7. deformation characteristic of the centerline 17

3.8. Castigliano's theorem 17

3.9. limit state..18

3.9.1. Factors influencing the limit states 18

3.9.2. Types of limit states..18

3.9.3. Limit state of deformation……………………………………………………………………18
3.9.4. Limit state of elasticity…………………………………………………………………………19
3.9.5. Limit state of buckling…………………………………………………………………………19
4. Analytical calculations………………………………………………………………………………23
4.1. Stress and strain of the truss construction…………………………………………………23
4.1.1. Rods numbering and nods naming……………………………………………………23
4.2. Cross-sections…………………………………………………………………………………………24
4.3. The deck of the bridge………………………………………………………………………………25
4.4. Material character of the bridge…………………………………………………………………26
4.5. Static analysis…………………………………………………………………………………………26
4.5.1. External static analysis…………………………………………………………………………26
4.5.2. External static analysis…………………………………………………………………………27
4.6. load on the bridge from its own weight…………………………………………………………27
4.7. Normal forces and normal stresses………………………………………………………………28
4.8. Deflection of joints……………………………………………………………………………………36
4.9. Load from train…………………………………………………………………………………………38
4.9.1. Case 1…………………………………………………………………………………………………39
4.9.2. Case 2…………………………………………………………………………………………………42
4.9.3. Case 3…………………………………………………………………………………………………45
4.9.4. Case 4…………………………………………………………………………………………………48
4.10. Checking the buckling of the bridge……………………………………………………………51
5. Finite element method for solving…………………………………………………………………55
5.1. The original structure…………………………………………………………………………………55
5.2. Case 1……………………………………………………………………………………………………57
5.3. Case 2……………………………………………………………………………………………………59
5.4. Case 3……………………………………………………………………………………………………61
5.5. Case 4……………………………………………………………………………………………………63

6. Conclusion ……64
Reference………66
List of attachments……..67

1. Introduction

Truss structure is a structure made of bars, and they are useful for the fact that they are strong durable and cheap to build. Some examples for a truss structure are cranes, watchtowers, masts and railway bridges which we are going to study in this thesis. The bridge presented in this thesis is considered to be a virtual bridge inspired from the bridge that connects both sides of the Vlára river and is located on the borders between Slovakia and Czech Republic, in a city called Horné srnie. The data we have taken from the bridge are the outer dimentions and the shapes of the ross-sections. In this thesis, the stress and strain analyses of the mentioned bridge with the knowledge of statics, atrength of materails and finite elemnt method. Moreover, we have used Matlab for math calculations and equations solving, Inventor for the bridge, joints and cross-sections drawing, and ANSYS for analyzing the bridge and comparing the result.

Pic.1 ridge across Vlára river

2. Objectives

The first aim of this thesis is the literature search .The second aim of this thesis is to do a stress and strain analysis of the bride mentioned, compute the normal forces and deflection of the bridge analytically under static load- wither load is from the own weight of the bridge or form a train- and to optimal the cross-section areas of the bridge bars. The third aim of this thesis is to analysze the stress and strain of the bridge using the program of Finite Element Method – like ANSYS which is used in this thesis- and compare the results from both methods, the analetical and neumerical way.

3.Theorical part

The theorical part is mainly summarized from source [1]

3.1. Basic characteristics of mechanics of materials

Mechanics of material is a science that uses the knowledge of statics, mathematics, physics, material science material design in order to help the designer chose the appropriate shape and dimensions of machines and structures with respect to safety, economy and lifetime.

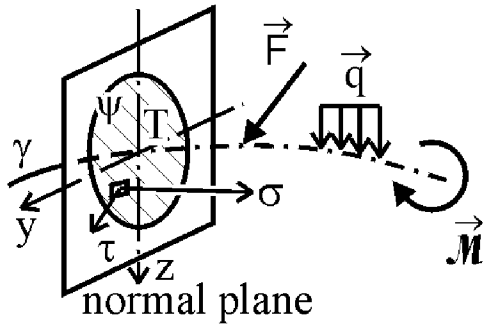
3.2. Bars

3.2.1 Definition: it is the simplest part of a real body that needs to meet certain assumptions with geometry, deformations, loads, supports and stress states to be applicable**.**

3.2.2 Bar assumptions

Assumptions concerning geometry:

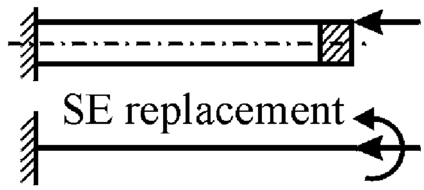
- The bar is defined by its centerline γ and the cross-section ψ of that bar.
- The bar line should be continuous with a finite length.
- The cross-section should be continues.



Pic.2 Cross section and centerline of a bar

Assumptions concerning supports and loads:

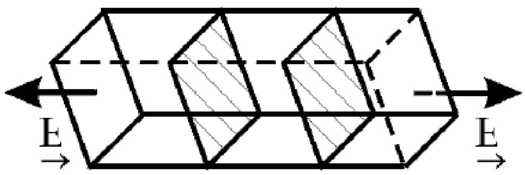
- The supports restrict only the points within the center line.
- The loads are concentrated on the centerline.



Pic.3 Replace the bar by the centerline

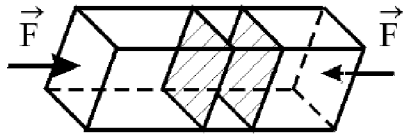
Assumptions concerning deformation:

- The centerline stays smooth and continuous during the deformation
- The cross-sections remain planer and perpendicular to the centerline no matter what the type of the deformation is.
  a) Tension (drawing the two ends far from each other)



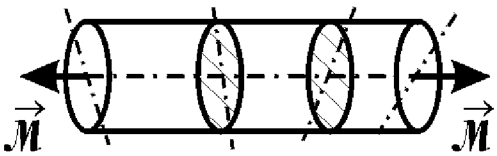
Pic.4 Tension

b) Compression (drawing the two ends far from each other)



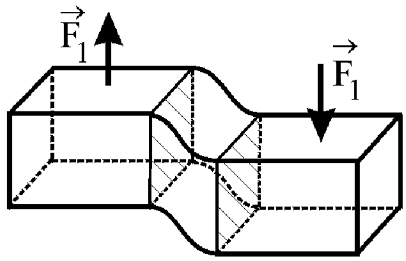
Pic.5 Compression

c) Torsion (rotating the ends of the body around an axis perpendicular to the cross-section and haing the centerline undeformed)



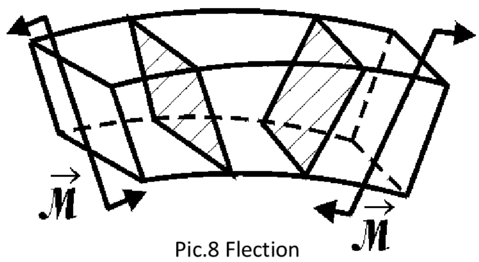
Pic.6 Torsion

d) Shear (shift the cross-sections perpendicular to the centerline)



Pic.7 Shear

e) Flection (rotating the ends of the body around an axis lying on the cross-section and haing the centerline deformed)

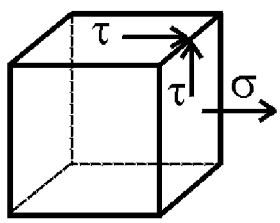


Pic.8 Flection

Assumptions concerning stress states

- The stres state in any point in the bar is defined by a the normal and shear stresses and all other components equal zero because of semetry ,

$$T\sigma = \begin{pmatrix} \sigma & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ or } \begin{pmatrix} \sigma & 0 & \tau \\ 0 & 0 & 0 \\ \tau & 0 & 0 \end{pmatrix}$$



Pic.9 Segment with stresses

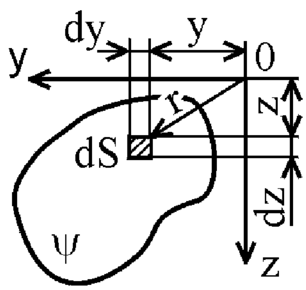
3.3 Geometrical characteristics of the cross-section

The geometrical characteristics characterizes the cross-section that is used for calculating the stress and deformation of the body. The characteristics of the cross-section is divided into two main parts:

Dependent: it depends on the coordinate system like the linear moment, moment of inertia and the center of gravity.

Independent: it does not depend on the coordinate system like the area, circumference, and the center of mass.

3.3.1 Cross-section area: the cross-section area does not depend on the coordinate system, and is calculated by this equation:



$$S = \int_{\psi} dS = \iint_{\psi} dydz \qquad [\text{m}^2]$$

Pic.10 Cross-section with distances from the axis

3.3.2 Static (linear) moment: the static moment is dependent on the coordinate system**,** and is calculated by these equations:

$$U_y = \int_{\psi} zdS \qquad , U_z = \int_{\psi} ydS \qquad [\text{m}^3]$$

y is the distance of the infinitesimal cross-section area dS from the Z axis.

Z is the distance of the infinitesimal cross-section area dS from the Y axis .

3.3.3 Moment of inertia: it is also called the quadratic moment and is dependent on the coordinate system**.**

- Axial moment of inertia

$$J_y = \int_{\psi} z^2 dS \qquad [\text{m}^4]$$

$$J_z = \int_{\psi} y^2 dS \qquad [\text{m}^4]$$

The axial moment of inertia is used in the calculations of the stresses and deformations in bending.

y is the distance of the infinitesimal cross-section area dS from the Z axis.

Z is the distance of the infinitesimal cross-section area dS from the Y axis.

- Derivation

$$J_{yz} = \int_{\psi} yz\, dS \qquad [\mathrm{m}^4]$$

This type of moment of inertia is used to the direction of principal axes.

- Polar

$$J_p = \int_{\psi} r^2 dS \qquad [\mathrm{m}^4]$$

The polar moment of inertia is are used in the calculations of the stresses and deformations in torsion.

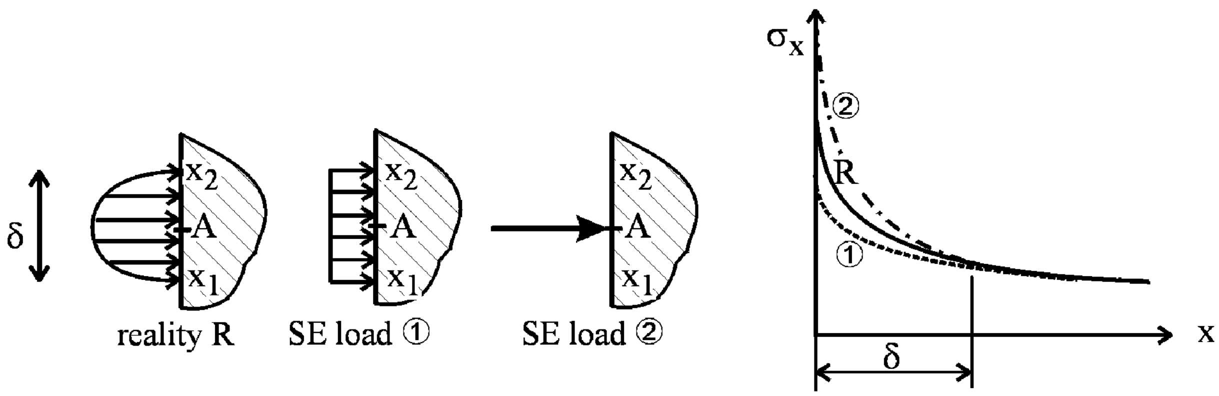
r is the distance of the infinitesimal cross-section area dS from the origin of the coordinate system and is equal to $\sqrt{y^2 + z^2}$

3.3.4 Properties of moment of inertia:

- **Additive**: If the cross-section is divided to several parts, the moment of inertia is then equals to the sum of the moments of inertia of each part of the cross-section.
- **Signs:** the axial and polar moment of inertia are always positive numbers, but the derivation moment can be any real number.
- **Symmetric:** if the cross-section has two symmetrical section, then the moment of entia of both sections to the axis of symmetry are equal. The derivation moments are also equal in magnitude with different signs.

3.3.5 Saint Venant's principle:

The real system of load acting on a body is substituted by an equivalent force, couple or moment that acts on the same region is statically equivalent to the original to the original one. This substitution causes the same stresses on the body as the original one.



Pic.11 Load distribution in Venants princip

3.3.6 Uses of Saint Venant's principle:

- Use computational model of loads correctly.
- Use computational model of contacts between bodies correctly.
- Proves incorrectness of some substitutes.

3.4. System of bars

3.4.1. System of bar assumptions:

- The joints between the bars are rotary joints
- The system consists or bars only that are connected with joints (nodes).
- External load operates only on the joints
- Every bar is connected with at least two other bars from each end so that it doesn't move
- The bar system is connected with the external body with a rotary joint.

3.4.2. Types of bar system:

- Statically determinant externally:
  When we consider the whole system as one body and the number of unknown parameters is the same as the number of equations.

$$\nu = \mu_{ex}$$

Where ν is the number of equations and μ_{ex} is the external unknowns.

- Statically determinant internally:
  For the system to be statically determinant internally, the number of internal unknown parameters is equal to the number of equations for solving them.
  To check whether the system is statically determinant internally we use these relations:

$$2k - 3 = p\ldots\ldots 2D$$

$$3k - 6 = p\ldots\ldots 3D$$

Where K is the number of joint (nodes) and K is the number of the bars.

- Statically indeterminant bar system:

When the number of unknown parameters is more than the number of equations, we say that the system is statically indeterminant and we need some boundary conditions to solve.

And the number of boundary conditions can be calculated by:

$$sex = \mu ex - \nu$$

3.5. Methods for solving the system of bar

There are two main ways to solve a system of bars:

3.5.1. Method of joints: in this method we draw the free body diagram of every joint in the bar system and for each joint we apply the equations of equilibrium

$$\sum F_X = 0$$
$$\sum y = 0$$
$$\sum M_0 = 0$$

and then we solve all the equations mathematically.

3.5.2. Method of sections: in this method we cut the system from a specific place and for the internal forces we apply the same equations of equilibrium we used before.

3.6. Simple tension and compression

Tension/compression is a type of load on a straight prismatic bar, if:

- Bar assumptions are applied.
- Cross-section may change their magnitude but not the shape.
- Normal force is the only non zero component of the inner forces.
- Deformations are not substantial in the viewpoint of the static equilibrium equations.

3.6.1. Geometrical relations:

a) The length and the angular displacement are calculated by these relations

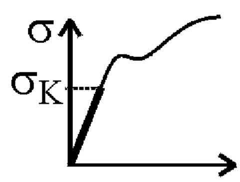
$$\varepsilon_x = \frac{du}{dx}$$

Where *du* is the displacement and ε_x is the strain in the x direction, and because the displacement is the same for all points of the cross-section, the strain ε_x is also the same though the cross-section.
The angular strains are all zero because the cross-section remains perpendicular to the centerline.

$$\gamma_{xy}=0 \text{ , } \gamma_{xz} = 0$$

b) For a homogeneous linear body hooks low is applied, which states that the dependency between the stress σ and the strain ε is approximately constant in the whole range of the elastic deformation.

$$\sigma = E\varepsilon$$



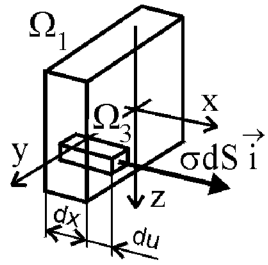
Pic.12 Stress strain graph

Where E is the elastic modulus, and σ is the normal stress which also calculated by this relation:

$$\sigma = \frac{N}{S}$$

where N is the normal force and S is the cross-section area of the bar.

c) Strain energy: for a beam made of elastic material, the deformation A done during the loading is equal to the revisable strain energy W



Pic.13 Segment volume illustrating energy

$$A = \Delta W$$

$$dW = \frac{1}{2}\frac{\sigma^2}{E} dSdx$$

We substitute σ with $N/_S$ and we get:

$$dW = \frac{1}{2}\frac{N^2}{ES^2} dSdx$$

Then we integrate it for the whole cross-section area ψ ,

$$dW = \int_{\psi} \frac{1}{2}\frac{N^2}{ES^2} dSdx$$

We then get

$$dW = \frac{N^2}{2ES} dx$$

We then integrate it for the length, and we get:

$$W_l = \int_0^l \frac{N^2}{2ES} dx = \frac{N^2 l}{2ES}$$

This is the final formula of the energy strain applied for the whole length of the bar.

3.7. Deformation characteristic of the centerline

the most common deformation of a centerline of a body that is loaded in tension or compression is to be in the direction parallel to the centerline.

We have stated previously that the strain is calculated by $\mathrm{du}/_{dx}$, and for this formula we can say that

$$du= \varepsilon\, dx$$

we substitute the strain ε with $\sigma/_{E}$ we get that

$$du=\frac{\sigma}{E}\, dx$$

we then substitute σ with $N/_{S}$ and integrate du to get:

$$U=\int_0^x \frac{\mathrm{N}}{E\,S}\,dx=\frac{\mathrm{N\,x}}{E\,S}$$

where ES is called the stiffness of the cross-section in tension, and x is the length from the bar of our calculations (usually the whole length).

3.8. Castigliano's theorem

Castigliano's theorem states that for linear structures, the partial derivation of the total strain energy with respect to any force F is equal to the displacement of the point that the force is acting on, in the direction of that force. Castigliano's theorem is very important and very practical because it enables us to calculate the deformations of any linear elastic body.

$$U=\frac{\partial W}{\partial F}$$

We have stated before that W is equal to $\mathrm{N}^2 l/_{2ES}$, therefore,

$$U=\frac{Nl}{ES}\frac{\partial N}{\partial F}$$

For multiple bars:

$$U=\sum_{i=1}^{n}\frac{N_i l_i}{E_i S_i}\frac{\partial N_i}{\partial F_j}$$

Castigliano's theorem also states that for linear structures, the partial derivation of the total strain energy with respect to any Moment M is equal to the turning angle of the point that the moment is acting on, in the direction of that moment.

$$\alpha = \frac{\partial W}{\partial M}$$

3.9. limit state

''Limit state is such one of the possible operational states of the body (system) which brings either a qualitative change in the ability of the body (system, structure) to perform some of its intended functions or an absolute loss of its functionality.'' [1]

From the previous sentence we can say that the limit state is a value that describes the safety of a machine or a system, and by which we can decide if the machine or system is well functioning or it is out of operation.

3.9.1. Factors influencing the limit states:

- Some of the external factors:
  a) Mechanical load (static, dynamic).
  b) Temperature load.
  c) Energetic field (electrical, magnetic field)
  d) Wrong manipulation.

- Some of the internal factors:
  a) Inefficient choice of the material.
  b) Welding operations.
  c) Inefficient design.
  d) Inefficient production.

3.9.2. Types of limit states:

There are a lot of the limit state types some of which are:

- Limit states of deformation.
- Limit state of elasticity.
- Limit states of buckling.

3.9.3. Limit state of deformation:

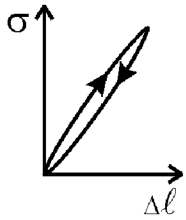
It is one of the operational states when the body exceeds the allowable deformation value, where the allowable deformation value is the value after which the machine stops functioning properly. $k_D = \frac{U_{limited}}{U_{max}}$

The unallowable value of deformation can be elastic as well as permanent deformation.

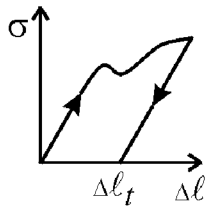
3.9.4. Limit state of elasticity:

It is one of the operational states where the first measurable plastic deformation occurs.

If the body is being loaded from the nonzero state, and then unloaded back to zero state, the deformation might be either very small that it can't be measured, or big enough that the deformation can be measured and plastic deformation occurs.



Pic.14 Unmeasurable deformation



Pic.15 Measurable deformation

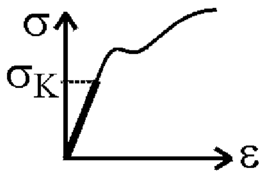
The factor of safety according to the limit states of elasticity is calculated by this formula:

$$k_K = \frac{\sigma_K}{\sigma_{max}}$$

Where K_K is the factor of safety,

σ_K is the yield stress

σ_{max} is the maximum stress of the body



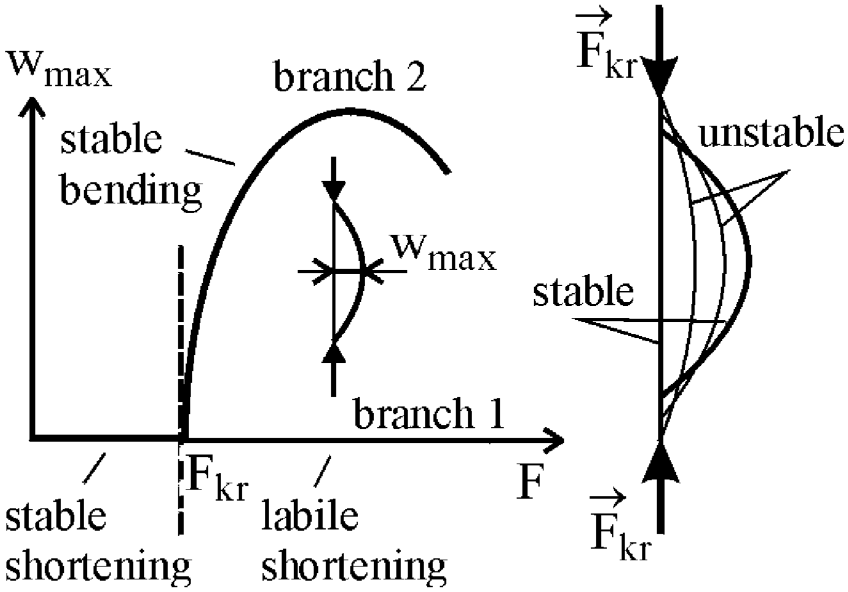
If $k_K > 1$ ……………the body didn't exceed the limit state.

If $k_K < 1$ ……………the body exceeded the limit state, and plastic deformation will occur.

3.9.5. Limit state of buckling:

It is one of the operational states where the body loses its equilibrium shape and becomes unstable, and the shape changes to another stabled geometry.

Buckling can occur only on compressed bodies and checking the buckling state is quite difficult where the shape, the length, the cross-section area, the type of supports and the critical load, where the critical load is used to determine if the body will be stable or not.

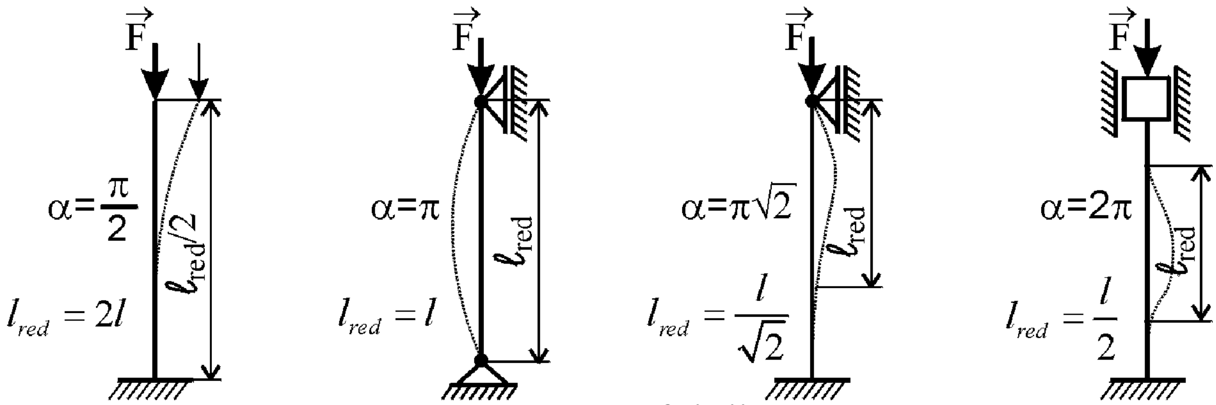


Pic.16 Buckling curve

From the previous picture we can tell that F_{kr} is very essential for determining the stability of the object and it is calculated by this formula:

$$F_{kr} = \alpha^2 \frac{EJ}{l^2}$$

Where α is taken from the supports of the object, and l is the length of the bar



Pic.17 Supports constant for buckling

If F is the force that the object is compressed by and F_{kr} is the critical force of that object then ,

a) $F < F_{kr}$ .... the bar is in stable shortening and there are no deflections.
b) $F > F_{kr}$ .... the bar is either in labile shortening, or it is bended and then became in stable equilibrium.
c) $F = F_{kr}$ .... it is the point when the bar changes from stable to unstable state , and that point is then called the equilibrium bifurcation point.

The factor of safety according to the limit states of buckling, by which we can also determine the stability of the bar: $k_V = \frac{F_{kr}}{F}$

In the previous sections we were dealing with ideal material with no plastic deformation but, in real material, the material might have the buckling before that plastic deformation, therefore we should check what happens earlier, and if the limit state of buckling is before limit states of elasticity.

For the checking we interduce the slenderness ratio which is calculated by this equation:

$$\lambda = \frac{l}{i}$$

Where i is called radius of gyration, and is calculated by this equation:

$$i = \sqrt{\frac{J}{S}}$$

For the critical slenderness ratio, we use this equation:

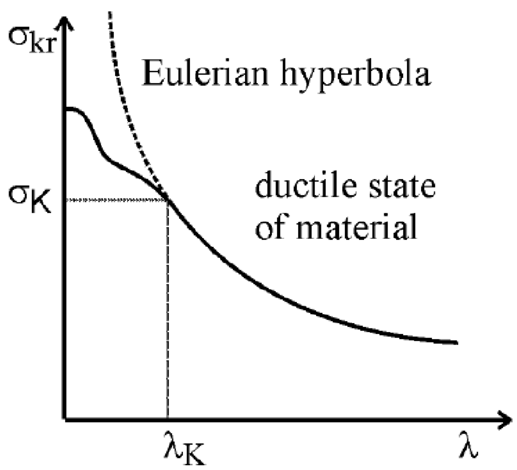
$$\lambda_{kr} = \alpha \sqrt{\frac{E}{\sigma_k}}$$

For calculating the normal stress in the equilibrium bifurcation point:

$$\sigma_{kr} = \frac{F_{kr}}{S} = \alpha^2 \frac{E\,J}{l^2 S} = \alpha^2 \frac{E\,J}{l^2} = \alpha^2 \frac{E}{\lambda^2}$$

Therefore, the critical slenderness ratio, we use this equation:

$$\lambda_{kr} = \alpha \sqrt{\frac{E}{\sigma_k}}$$



Pic.18 Eulerian hyperbola

When comparing the λ with the λ_{kr} we get one of these cases:

a) $\lambda > \lambda_{kr}$ the limit sate of backing deceives.

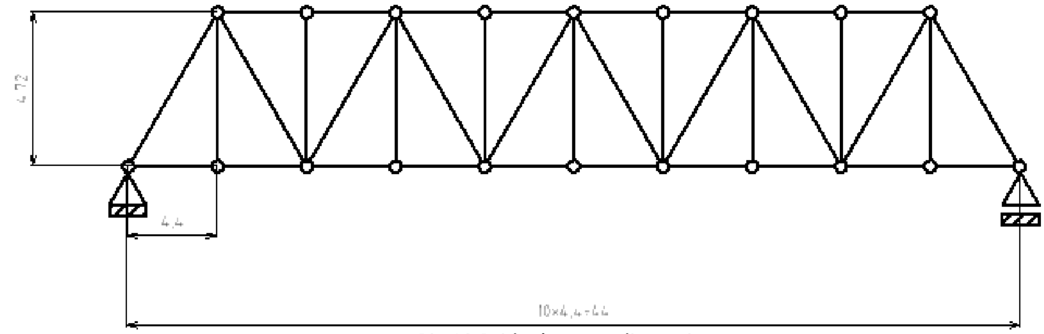
$F_{kr} = \alpha^2 \frac{E\,J}{l^2}$ and the factor of safety according to buckling is $k_V = \frac{F_{kr}}{F}$

b) $\lambda < \lambda_{kr}$ the limit sate of elasticity deceives,
and the factor of safety according to the limit state of elasticity will be $k_K = \frac{\sigma_K}{\sigma_{max}}$

4. Analytical calculations

4.1. Stress and strain of the truss construction

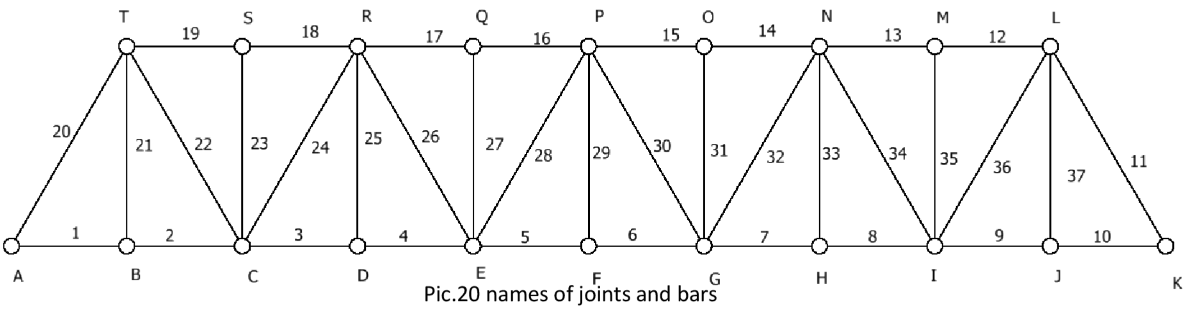
The bridge we are studying contains of 37 rods that we have simplified to links so that each bar with only be either in tension or in compression, and because the bridge is symmetrical, we all only study one side of it, which is also going to be applied for the other side.



Pic.19 Bridge’s outer dimensions

the total length of the bridge is 44 meters, the hight is 4.72 meters and the width is 5 meters. The bottom part is devided to 10 parts and each part is 4.4 miters long, the vertical rod is 4.72 meters long, the tilted rod is 6,452 meters and the angle between the rods is 47 degrees. The bridge is supported from the left by a pin support which means that the bridge does not move horizontally or vertically but it can rotate around perpendicular axis. From the right, it is supported by a roller support which means that it moves horizontally and rotates around its perpendicular axis.

4.1.1. Rods numbering and nods naming:



Pic.20 names of joints and bars

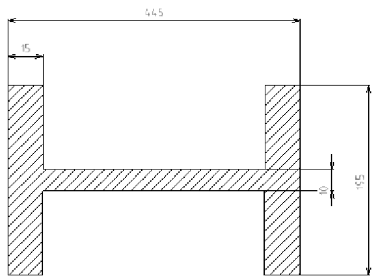
We have named the nodes by the alphabets starting from left to right and we numbered the bars from 1 to 37. The table below shows each rod and its length in meters.

L1	4.4	L6	4.4	L11	6.4	L16	4.4	L21	4.7
L2	4.4	L7	4.4	L12	4.4	L17	4.4	L22	6.4
L3	4.4	L8	4.4	L13	4.4	L18	4.4	L23	4.7
L4	4.4	L9	4.4	L14	4.4	L19	4.4	L24	6.4
L5	4.4	L10	4.4	L15	4.4	L20	6.4	L25	4.7

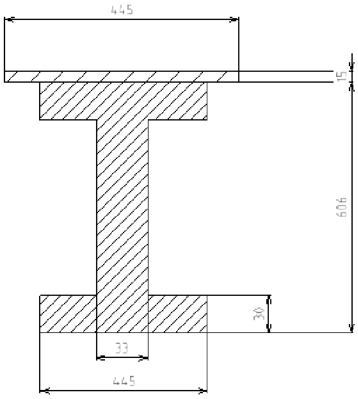
L26	6.4	L31	4.7	L36	6.4
L27	4.7	L32	6.4	L37	4.7
L28	6.4	L33	4.7		
L29	4.7	L34	6.4		
L30	6.4	L35	4.7		

4.2. Cross-sections

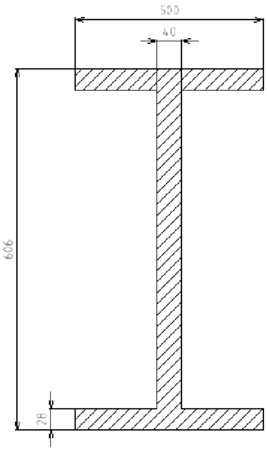
The outer shapes of the cross-sections were drawn according the bridge. The dimensions and the areas of these cross sections we gained by comparing the cross sections of the bachelor thesis of the past years that similar to this one and chose the ones that are mostly close to the cross sections of the bridge we have. In this bridge there are four types of cross-section. The horizontal cross-section on the bottom part of the bridge has an area of $S_{vd} = 50000\ mm^2$, the horizontal cross-section on the upper part of the bridge has an area of $S_{vn} = 49593\ mm^2$, the vertical bars have cross-sections section with an area of $S_s = 10000\ mm^2$ for each, and the tilted bars have cross-sections with an area of $S_c = 20550\ mm^2$. The shapes and dimensions of the cross-section of these cross-sections are shown on the pictures below with the dimensions written in millimeters. [2],[3],[4],[5],[6]



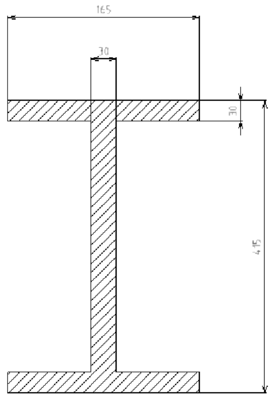
Pic.21 Cross-section of tilted rods



Pic22. Cross-section of upper horizontal rods



Pic.23. Cross-section of bottom horizontal rods



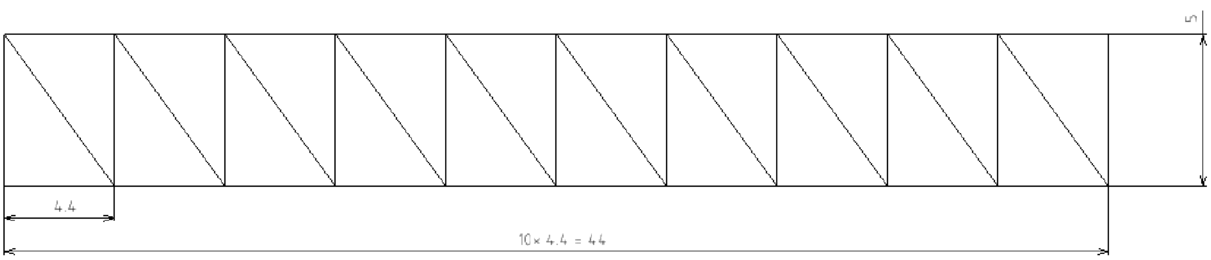
Pic.24 Cross-section of vertical rods

The table below shows each bar with its cross-section area in m^2.

S1	0.0500	S6	0.0500	S11	0.0206	S16	0.0496	S21	0.0100
S2	0.0500	S7	0.0500	S12	0.0496	S17	0.0496	S22	0.0206
S3	0.0500	S8	0.0500	S13	0.0496	S18	0.0496	S23	0.0100
S4	0.0500	S9	0.0500	S14	0.0496	S19	0.0496	S24	0.0206
S5	0.0500	S10	0.0500	S15	0.0496	S20	0.0206	S25	0.0100

S26	0.0206	S31	0.0100	S36	0.0206
S27	0.0100	S32	0.0206	S37	0.0100
S28	0.0206	S33	0.0100		
S29	0.0100	S34	0.0206		
S30	0.0206	S35	0.0100		

4.3. The deck of the bridge



Pic.25 Deck of the bridge

The deck of the bridge connects the two sides of the bridge, and the picture above shows the shape and the dimensions of the deck where the dimensions are written in meters. The cross-section area of the tilted bars of the deck is S_{mc}= 50000 mm^2 with a length of 6.66 m, and the vertical bars of the deck have also a cross-section area of S_{ms} = 50000 mm^2, and length of 5 meters.

4.4. Material character of the bridge

The bridge is made of steel with the following parameters:

R_e = 210 [MPa]………. yield strength

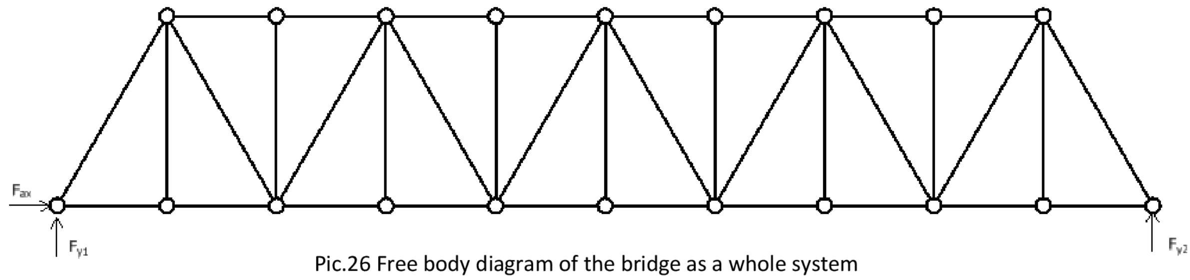
R_m = 370 [MPa]……ultimate strength

E = 210 [GPa]……. young's modulus

ρ = 7850 [$^{Kg}/_{m^3}$]………density

4.5. Static analysis

4.5.1. External static analysis: we consider the whole system as one body and draw the free body diagram with the reaction forces from the supports.



Pic.26 Free body diagram of the bridge as a whole system

The unknown parameters are NP = {F_{y1}, F_{y2}, F_{ax}} and all three static equilibrium equation for 2D bodies are applicable.

$$\mu_{ex} = 3,$$

$$\nu = 3$$

$$\nu = \mu_{ex}$$

The system is statically determinant externally.

4.5.2. External static analysis: the system has P=37 bar, and K=20 node.

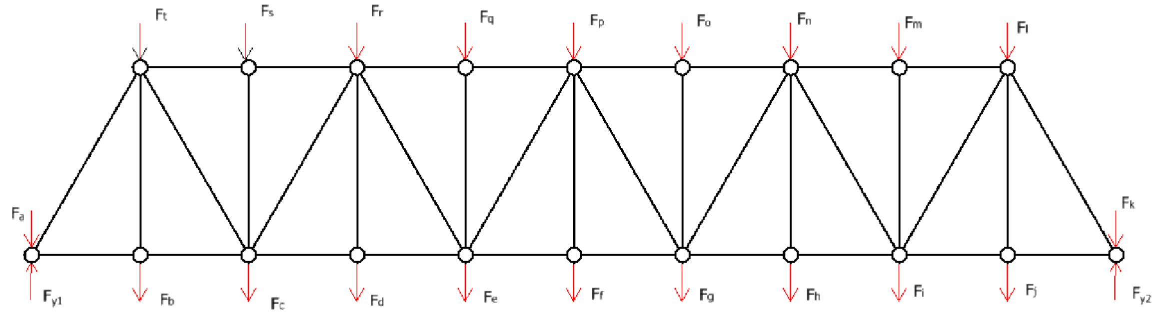
2k – 3 = p

2*20 - 3 = 37 =P

The system is statically determinant internally.

4.6. load on the bridge from its own weight

The weight of the bridge is a very big factor to make in consideration when calculating the normal forces and deformation of the bridge, therefore we have calculated the gravitational force on each joint of the bridge. For calculating the mas of the bar and thus the gravitational force we have used the density and volume of the bar.



Pic.27 Gravitational forces from bridge's weight

Every bar on the bridge has two joints connecting them to the other bars, as well as the deck which also have bars with joints on each end, therefore the force from the bar mass will be distributed equally on both joints, the same way shown in the example bellow for force F_f .

$$F_g = \frac{1}{2}\, g\, \mathcal{f}\, (S_5 * L_5 + S_6 * L_6 + S_{29} * L_{29} + S_{mc} * L_{ms} + S_{mv} * L_{mv})$$

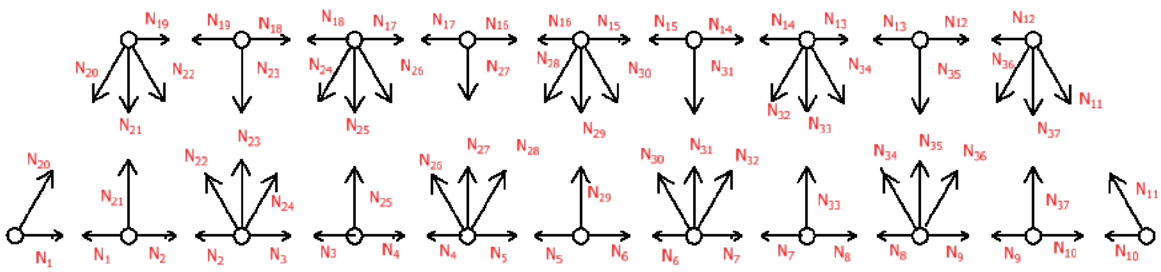
Where g is the acceleration due to gravity and is 9.912 $m/_{s^2}$,L_{mv} , L_{mc} are the length of horizontal and tilted bars of the dech respectivly.

With the same way we have calculated for the rest of the joints as shown in the table:

Gravitational forces [N]	
F_a	21832
F_b	41207
F_c	48677
F_d	41207
F_e	48677
F_f	41207
F_g	48677
F_h	41207
F_i	48677
F_j	41207
F_k	34654
F_l	17689
F_m	18621
F_n	26091
F_o	18621
F_p	26091
F_q	18621
F_r	26091
F_s	18621
F_t	17689

4.7. Normal forces and normal stresses

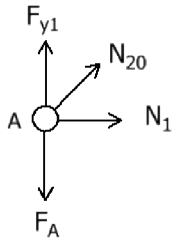
We have drawn the free body diagram of every joint (node), and all the normal forces that can be cased by the loads.



Pic.28 Free body diagram of the joints

To solve this truss system, we will use the method of joints, where we write all applicable static equations of equilibrium for each node. The applicable static equations of equilibrium for each node are two, and we have 20 nodes, which means that we will have 40 equations of equilibrium.

Node A:

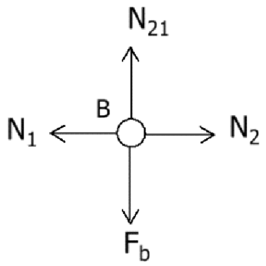


Eqn1 …. $N_1 + N_{20}\cos(\alpha) = 0$

Eqn2…. $F_{y1} + N_{20}\sin(\alpha) – F_a = 0$

Pic.29 Free body diagram of the joint A

Node B:

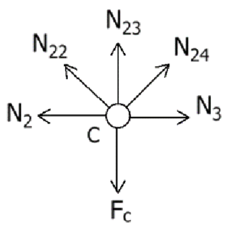


Eqn3 …. $N_2 – N_1 = 0$

Eqn4…. $N_{21} – F_b = 0$

Pic.30 Free body diagram of the joint B

Node C:

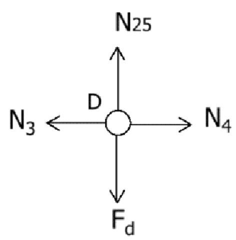


Eqn5 …. $N_3 – N_2 + N_{24}\cos(\alpha) - N_{22}\cos(\alpha) = 0$

Eqn6…. $N_{23} - F_c + N_{24}\sin(\alpha) + N_{22}\sin(\alpha) = 0$

Pic.31 Free body diagram of the joint C

Node D:

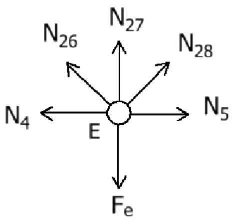


Eqn7 …. $N_4 – N_3 = 0$

Eqn8…. $N_{25} – F_d = 0$

Pic.32 Free body diagram of the joint D

Node E:

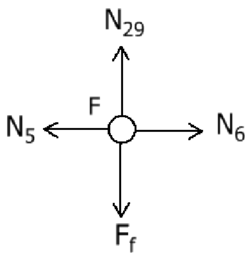


Eqn9 .... $N_5 - N_4 + N_{28}\cos(\alpha) - N_{26}\cos(\alpha) = 0$

Eqn10.... $N_{27} - F_e + N_{28}\sin(\alpha) + N_{26}\sin(\alpha) = 0$

Pic.33 Free body diagram of the joint E

Node F:

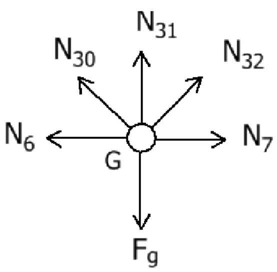


Eqn11 .... $N_6 - N_5 = 0$

Eqn12.... $N_{29} - F_f = 0$

Pic.34 Free body diagram of the joint F

Node G:

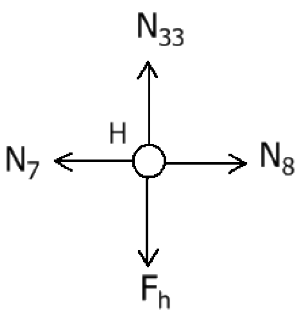


Eqn13 .... $N_7 - N_6 + N_{32}\cos(\alpha) - N_{30}\cos(\alpha) = 0$

Eqn14.... $N_{31} - F_g + N_{32}\sin(\alpha) + N_{30}\sin(\alpha) = 0$

Pic.35 Free body diagram of the joint G

Node H

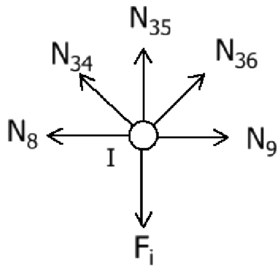


Eqn15 .... $N_8 - N_7 = 0$

Eqn16.... $N_{33} - F_h = 0$

Pic.36 Free body diagram of the joint H

Node I:

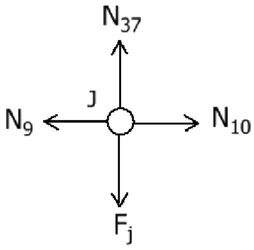


Eqn17 …. $N_9 - N_8 + N_{36}\cos(\alpha) - N_{34}\cos(\alpha) = 0$

Eqn18…. $N_{35} - F_i + N_{36}\sin(\alpha) + N_{34}\sin(\alpha) = 0$

Pic.37 Free body diagram of the joint I

Node J:

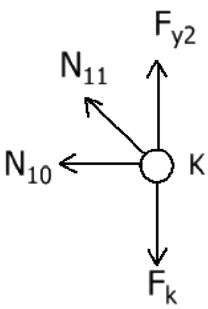


Eqn19 …. $N_{10} - N_9 = 0$

Eqn20…. $N_{37} - F_j = 0$

Pic.38 Free body diagram of the joint J

Node K:

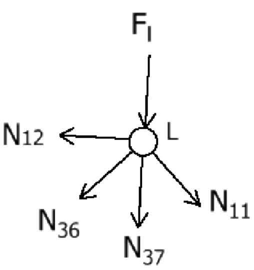


Eqn21 …. $-N_{10} - N_{11}\cos(\alpha) = 0$

Eqn22 …. $F_{y2} + N_{11}\sin(\alpha) - F_k = 0$

Pic.39 Free body diagram of the joint K

Node L:

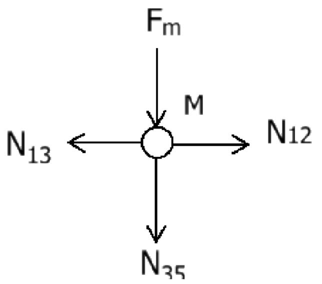


Eqn23 …. $N_{11}\cos(\alpha) - N_{36}\cos(\alpha) - N_{12} = 0$

Eqn24…. $-N_{37} - N_{11}\sin(\alpha) - N_{36}\sin(\alpha) - F_l = 0$

Pic.40 Free body diagram of the joint L

Node M:

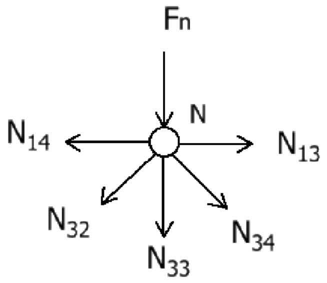


Eqn25 .... $N_{12} - N_{13} = 0$

Eqn26.... $-N_{35} - F_m = 0$

Pic.41 Free body diagram of the joint M

Node N:

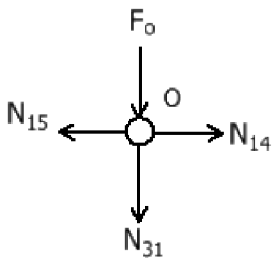


Eqn27 .... $N_{34}\cos(\alpha) - N_{32}\cos(\alpha) - N_{14} + N_{13} = 0$

Eqn28.... $- N_{34}\sin(\alpha) - N_{32}\sin(\alpha) - N_{35} - F_n = 0$

Pic.42 Free body diagram of the joint N

Node O:

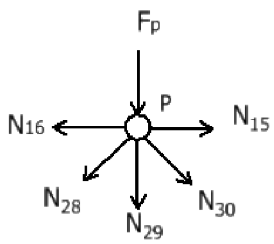


Eqn29 .... $N_{14} - N_{15} = 0$

Eqn30.... $-N_{31} - F_o = 0$

Pic.43 Free body diagram of the joint O

Node P:

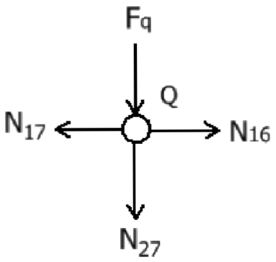


Eqn31 .... $N_{30}\cos(\alpha) - N_{28}\cos(\alpha) - N_{16} + N_{15} = 0$

Eqn32.... $- N_{30}\sin(\alpha) - N_{28}\sin(\alpha) - N_{29} - F_p = 0$

Pic.44 Free body diagram of the joint P

Node Q:

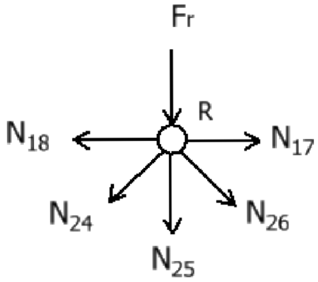


Eqn33 …. $N_{16} - N_{17} = 0$

Eqn34…. $-N_{27} - F_q = 0$

Pic.45 Free body diagram of the joint Q

Node R:

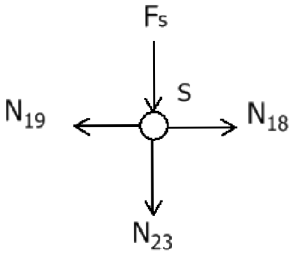


Eqn35 …. $N_{26}\cos(\alpha) - N_{24}\cos(\alpha) - N_{18} + N_{17} = 0$

Eqn36…. $- N_{36}\sin(\alpha) - N_{24}\sin(\alpha) - N_{25} - F_r = 0$

Pic.46 Free body diagram of the joint R

Node S:

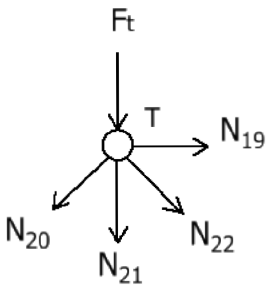


Eqn37 …. $N_{18} - N_{19} = 0$

Eqn38…. $-N_{23} - F_s = 0$

Pic.47 Free body diagram of the joint S

Node T:



Eqn39 …. $N_{19} + N_{22}\cos(\alpha) - N_{20}\cos(\alpha) = 0$

Eqn40…. $- N_{22}\sin(\alpha) - N_{20}\sin(\alpha) - N_{21} - F_t = 0$

Pic.48 Free body diagram of the joint T

To solve these 40 equations with the 40 unknown parameters (37 normal forces, and 3 reaction forces from the supports) we have used Matlab.

```
%% solution
solu=solve(eqn2,eqn1,eqn3,eqn4,eqn8,eqn12,eqn16,eqn20,eqn22,eqn21,eqn26,...
    eqn30,eqn34,eqn38,eqn40,eqn39,eqn37,eqn6,eqn23,eqn25,eqn18,eqn28,...
    eqn14,eqn32,eqn10,eqn36,eqn5,eqn7,eqn9,eqn11,eqn13,eqn15,eqn17,eqn27,...
    eqn29,eqn31,eqn33,eqn24,eqn35,eqn19);

%% normal forces
N20=vpa(solu.N20);N1=vpa(solu.N1);N2=vpa(solu.N2);N21=vpa(solu.N21);N25=vpa(solu.N25);
N29=vpa(solu.N29);N33=vpa(solu.N33);N37=vpa(solu.N37);N11=vpa(solu.N11); N10=vpa(solu.N10);
N35=vpa(solu.N35);N31=vpa(solu.N31);N27=vpa(solu.N27);N28=vpa(solu.N28);N22=vpa(solu.N22);
N19=vpa(solu.N19);N18=vpa(solu.N18);N36=vpa(solu.N36);N12=vpa(solu.N12);N13=vpa(solu.N13);
N34=vpa(solu.N34);N32=vpa(solu.N32);N30=vpa(solu.N30);N26=vpa(solu.N26);N24=vpa(solu.N24);
N23=vpa(solu.N23);N3=vpa(solu.N3);N4=vpa(solu.N4);N5=vpa(solu.N5);N6=vpa(solu.N6);
N7=vpa(solu.N7);N8=vpa(solu.N8);N9=vpa(solu.N9);N14=vpa(solu.N14);N15=vpa(solu.N15);
N16=vpa(solu.N16);N17=vpa(solu.N17);Fy1=vpa(solu.Fy1);Fy2=vpa(solu.Fy2);
```

Pic.49 Part of MATLAB code calculating normal forces

and the results we got for normal forces and thus stress are listed in this table:

Normal Forces [N]					
N1	274473	N18	-494043	N35	-18621
N2	274473	N19	-494043	N36	322012.01
N3	650879	N20	-402529	N37	41207
N4	650879	N21	41207		
N5	776348	N22	322012		
N6	776348	N23	-18621		
N7	650879	N24	-230009		
N8	650879	N25	41207		
N9	274473	N26	138005		
N10	274473	N27	-18621		
N11	-402529	N28	-46002		
N12	-494043	N29	41207		
N13	-494043	N30	-46002		
N14	-744981	N31	-18621		
N15	-744981	N32	138005		
N16	-744981	N33	41207		
N17	-744981	N34	-230009		

The results of the reaction forces calculated in Newtons are:

Fy1	316271
Fy2	329093

Then we have calculated the normal stress of each bar by the equation:

$$\sigma = \frac{N}{S}$$

and the results we got are listed in the table below:

Stress [MPa]							
σ1	5.53	σ11	-9.88	σ21	4.12	σ31	6.72
σ2	5.53	σ12	-9.88	σ22	-1.86	σ32	-2.24
σ3	13.12	σ13	-14.90	σ23	4.12	σ33	-2.24
σ4	13.12	σ14	-14.90	σ24	-1.86	σ34	6.72
σ5	15.65	σ15	-14.90	σ25	4.12	σ35	-11.19
σ6	15.65	σ16	-14.90	σ26	-1.86	σ36	15.67
σ7	13.12	σ17	-9.88	σ27	4.12	σ37	-19.59
σ8	13.12	σ18	-9.88	σ28	-19.59		
σ9	5.53	σ19	4.12	σ29	15.67		
σ10	5.53	σ20	-1.86	σ30	-11.19		

From the tables we can see that the maximum stress is -19.59 [MPa] on bars number 28. and 37, and the absolute value is $\sigma_{max} = 19.59\ [Mpa]$.

Therefore, for calculating the factor of safety according to the limit state of elasticity we will use this equation:

$$k_K = \frac{\sigma_K}{\sigma_{max}} = \frac{210}{19.59} = 10.72$$

4.8. Deflection of joints

For calculating the deflection, we will use this equation

$$U = \sum_{i=1}^{n} \frac{N_i l_i}{E_i S_i} \frac{\partial N_i}{\partial F_j}$$

We have used Matlab to calculate the deflection of the joints and that was by expressing all normal force with the gravitational forces acting on the bottom part of the bridge, the we calculated the partial derivation of these normal forces according to the forces we have expressed the normal forces by.

```
for i=1:size(N)
    Wa=Wa+(((N(i)*Length_matrix(i))/(E*CrossSection_matrix(i)))*Diff1(i));
    Wb=Wb+(((N(i)*Length_matrix(i))/(E*CrossSection_matrix(i)))*Diff2(i));
    Wc=Wc+(((N(i)*Length_matrix(i))/(E*CrossSection_matrix(i)))*Diff3(i));
    Wd=Wd+(((N(i)*Length_matrix(i))/(E*CrossSection_matrix(i)))*Diff4(i));
    We=We+(((N(i)*Length_matrix(i))/(E*CrossSection_matrix(i)))*Diff5(i));
    Wf=Wf+(((N(i)*Length_matrix(i))/(E*CrossSection_matrix(i)))*Diff6(i));
    Wg=Wg+(((N(i)*Length_matrix(i))/(E*CrossSection_matrix(i)))*Diff7(i));
    Wh=Wh+(((N(i)*Length_matrix(i))/(E*CrossSection_matrix(i)))*Diff8(i));
    Wi=Wi+(((N(i)*Length_matrix(i))/(E*CrossSection_matrix(i)))*Diff9(i));
    Wj=Wj+(((N(i)*Length_matrix(i))/(E*CrossSection_matrix(i)))*Diff10(i));
    Wk=Wk+(((N(i)*Length_matrix(i))/(E*CrossSection_matrix(i)))*Diff11(i));
end
```

Pic.50 Part of MATLAB code for deflection calculation

The values we got from Matlab are:

Deflection [mm]	
Wa	0.00
Wb	2.92
Wc	5.27
Wd	7.23
We	8.31
Wf	8.80
Wj	8.31
Wh	7.23
Wi	5.27
Wj	2.92
Wk	0.00

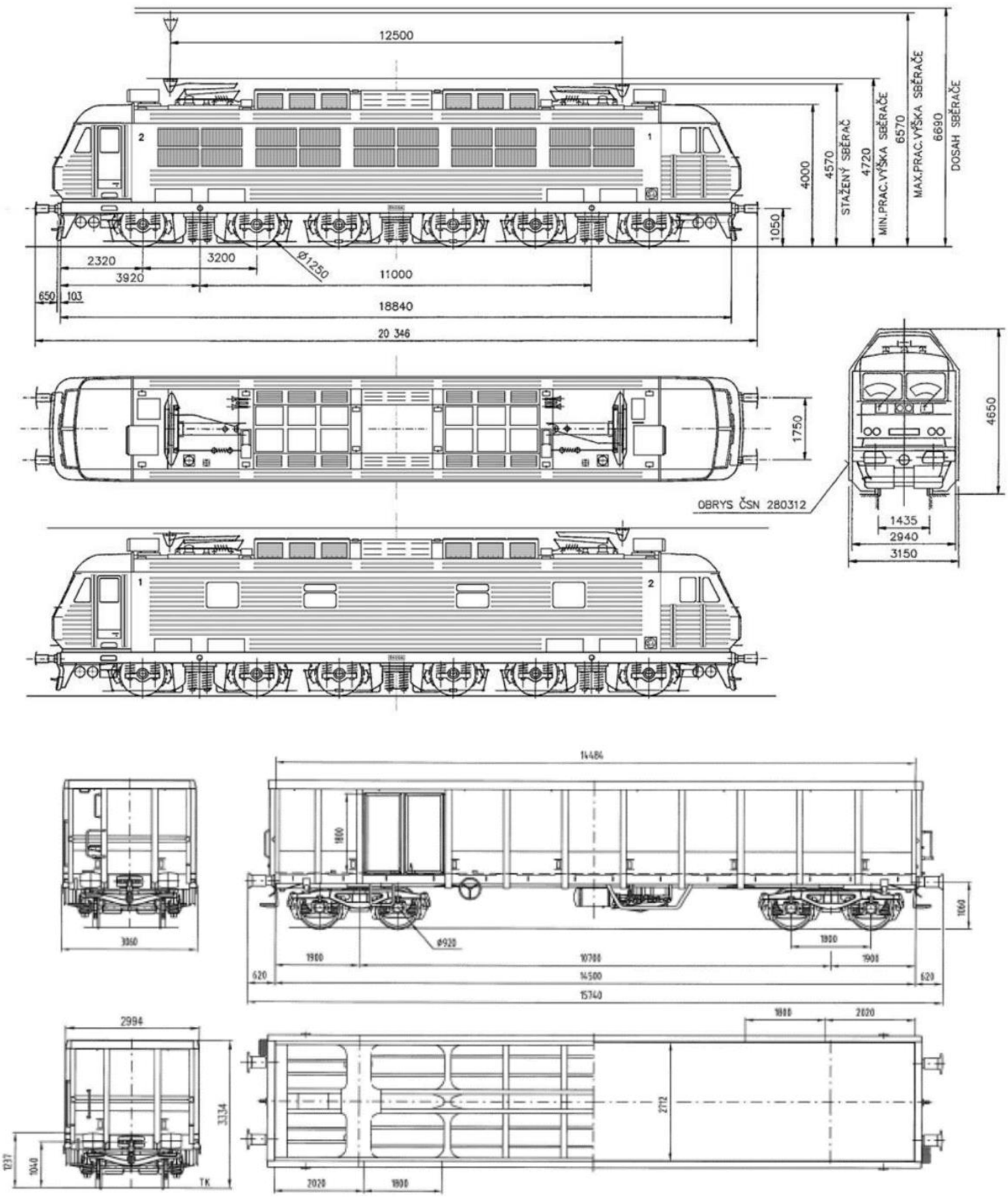
From the table we can see that the deflection values are symmetrical, and the maximum deflection value is 8.8 mm on joint F which is the on the middle.

4.9. Load from train

The train we are considering in this thesis has the following data [2]:

Locomotive: the locomotive has a weight of m_{lok} = 123 tons.

Wagon: Empty wagon has a weight of 27 tons and the maximum weight it is m_{vag} = 63 tons.



Pic.51 Train dimensions

For calculating the weight of the locomotive and wagon we should consider the symmetry and the number of the wheels.

We will only consider half of the locomotive because we are studying only one side of the bridge, and we know that the locomotive has 6 wheels, therefore the gravitational force that one wheel of the locomotive can apply on the bridge is:

$$F_{lok} = \frac{m_{lok} \cdot g}{2 \cdot 6} = \frac{123 \cdot 10^3 \cdot 9.812}{2 \cdot 6} = 100553\ N$$

We will also consider half of the locomotive, and we know that the wagon has 4 wheels, therefore the gravitational force that one wheel of the wagon can apply on the bridge is:

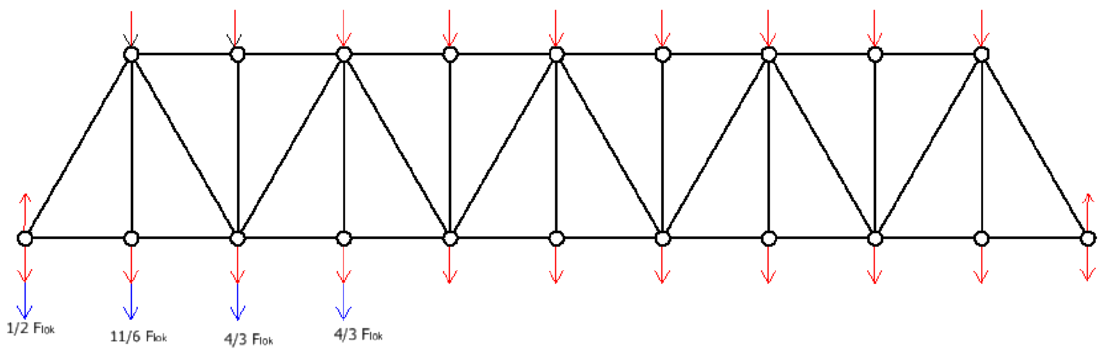
$$F_{lok} = \frac{m_{vag} \cdot g}{2 \cdot 4} = \frac{63 \cdot 10^3 \cdot 9.812}{2 \cdot 4} = 77254\ N$$

We will study the impact of the train on the train in four position of the train, when the train have passed a quarter from the bridge length , when the train is on the middle of the bridge, when the train has passed three quarters of the train length and lastly when the train is in the end of the bridge.

The gravitational force of the train will be only applied on the joints of the bridge. If the wheel is directly located on the joint, the gravitational force of the train on that wheel will be added on that joint. If the wheel is located on the bar, the force will be distributed on the two ends according to the distance from that end, for example if the wheel is located 1/3 of the bar length far from one end a that bar , then the force on that end will be 2/3 more than the force on the second end.

4.9.1. Case 1

When the train has passed quarter of the bridge length, forces from the locomotive only will be applied on the bridge, and these forces will act on the joints A, B, C, D as shown on the picture below:



Pic.52 Bridge with train in position 1

The gravitational force on each joint of the bridge in case 1:

Gravitational forces [N]			
Fa	72108	Fk	34654
Fb	225553	Fl	17689
Fc	182747	Fm	18621
Fd	175277	Fn	26091
Fe	48677	Fo	18621
Ff	41207	Fp	26091
Fg	48677	Fq	18621
Fh	41207	Fr	26091
Fi	48677	Fs	18621
fj	41207	Ft	17689

By the same way we have done before we have calculated the normal force and the normal stress of each bar of the bridge in case 1:

Normal Forces [N]			
N1	616601	N21	225553
N2	616601	N22	571742
N3	1208596	N23	-18621
N4	1208596	N24	-296450
N5	1174717	N25	175277
N6	1174717	N26	21159
N7	889901	N27	-18621
N8	889901	N28	70844
N9	354147	N29	41207
N10	354147	N30	-162848
N11	-519375	N31	-18621
N12	-653391	N32	254851
N13	-653391	N33	41207
N14	-1063676	N34	-346854
N15	-1063676	N35	-18621
N16	-1223024	N36	438858
N17	-1223024	N37	41207
N18	-1006455		
N19	-1006455		
N20	-904279		

The maximum tension force is 1208596.06 N and the maximum compression force is

- 1223024 N and the maximum absolute force is **1223024 N**

The reaction forces on the bridge from the supports calculated in Newtons:

Fy1	733563
Fy2	414563

The normal stresses of each bar of the bridge in case 1 are listed on the table below:

Stress [MPa]			
σ1	12.43	σ21	17.53
σ2	12.43	σ22	-1.86
σ3	24.37	σ23	4.12
σ4	24.37	σ24	-1.86
σ5	23.69	σ25	4.12
σ6	23.69	σ26	-1.86
σ7	17.94	σ27	4.12
σ8	17.94	σ28	-44.00
σ9	7.14	σ29	27.82
σ10	7.14	σ30	-14.43
σ11	-13.07	σ31	1.03
σ12	-13.07	σ32	3.45
σ13	-21.27	σ33	-7.92
σ14	-21.27	σ34	12.40
σ15	-24.46	σ35	-16.88
σ16	-24.46	σ36	21.36
σ17	-20.13	σ37	-25.27
σ18	-20.13		
σ19	22.56		
σ20	-1.86		

The maximum stress in tension is 27.82 [MPa] and the maximum stress in compression is

- 44 [MPa] and the maximum absolute force is **44 [MPa].**

The factor of safety according to the limit state of elasticity:

$$k_K = \frac{\sigma_K}{\sigma_{max}} = \frac{210}{44} = 4.8$$

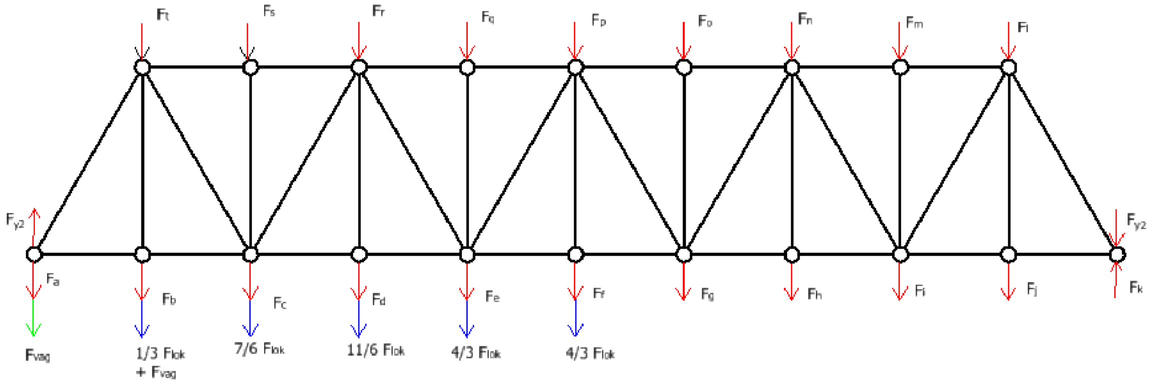
The deflection of joints of the bridge in case 1:

Deflection [mm]	
Wa	0.00
Wb	5.80
Wc	9.42
Wd	12.58
We	13.46
Wf	13.66
Wj	12.58
Wh	10.65
Wi	7.65
Wj	4.14
Wk	0.00

The maximum deflection is **13.66 mm**, and as we can see that the deflection is not symmetrical anymore and that is because the train load is not apllied on the bridge symmetrically.

4.9.2. Case 2

When the train is at the middle of the bridge length, the load of the locomotive and part of the first wagon will be applied on the bridge, and these forces will act on the joints A, B, C, D, E, F as shown on the picture below:



Pic.53 Bridge with train in position 2

The gravitational force on each joint of the bridge in case 2:

Gravitational forces [N]			
Fa	99086	Fk	34654
Fb	151978	Fl	17689
Fc	165988	Fm	18621
Fd	225553	Fn	26091
Fe	182747	Fo	18621
Ff	175277	Fp	26091
Fg	48677	Fq	18621
Fh	41207	Fr	26091
Fi	48677	Fs	18621
fj	41207	Ft	17689

By the same way we have done before, we have calculated the normal force and the normal stress of each bar of the bridge in case 2:

Normal Forces [N]			
N1	712659	N21	151978
N2	712659	N22	813200
N3	1649564	N23	-18621
N4	1649564	N24	-560821
N5	1757505	N25	225553
N6	1757505	N26	216797
N7	1239574	N27	-18621
N8	1239574	N28	58494
N9	470704	N29	175277
N10	470704	N30	-333786
N11	-690313	N31	-18621
N12	-886506	N32	425789
N13	-886506	N33	41207
N14	-1529907	N34	-517793
N15	-1529907	N35	-18621
N16	-1797391	N36	609796
N17	-1797391	N37	41207
N18	-1267157		
N19	-1267157		
N20	-1045153		

The maximum tension force is 1757505 N and the maximum compression force is

-1797391 N and the maximum absolute force is **1797391 N**

The reaction forces on the bridge from the support calculated in Newtons:

Fy1	863587
Fy2	539599

The normal stresses of each bar of the bridge in case 2 are listed on the table below:

Stress [MPa]			
σ1	14.37	σ21	22.56
σ2	14.37	σ22	-1.86
σ3	33.26	σ23	17.53
σ4	33.26	σ24	-1.86
σ5	35.44	σ25	4.12
σ6	35.44	σ26	-1.86
σ7	24.99	σ27	4.12
σ8	24.99	σ28	-50.86
σ9	9.49	σ29	39.57
σ10	9.49	σ30	-27.29
σ11	-17.73	σ31	10.55
σ12	-17.73	σ32	2.85
σ13	-30.60	σ33	-16.24
σ14	-30.60	σ34	20.72
σ15	-35.95	σ35	-25.20
σ16	-35.95	σ36	29.67
σ17	-25.34	σ37	-33.59
σ18	-25.34		
σ19	15.20		
σ20	-1.86		

The maximum stress in tension is 39.57 [MPa] and the maximum stress in compression is - 50.86 [MPa] and the maximum absolute force is **50.86 [MPa].**

The factor of safety according to the limit state of elasticity:

$$k_K = \frac{\sigma_K}{\sigma_{max}} = \frac{210}{50.86} = 4.13$$

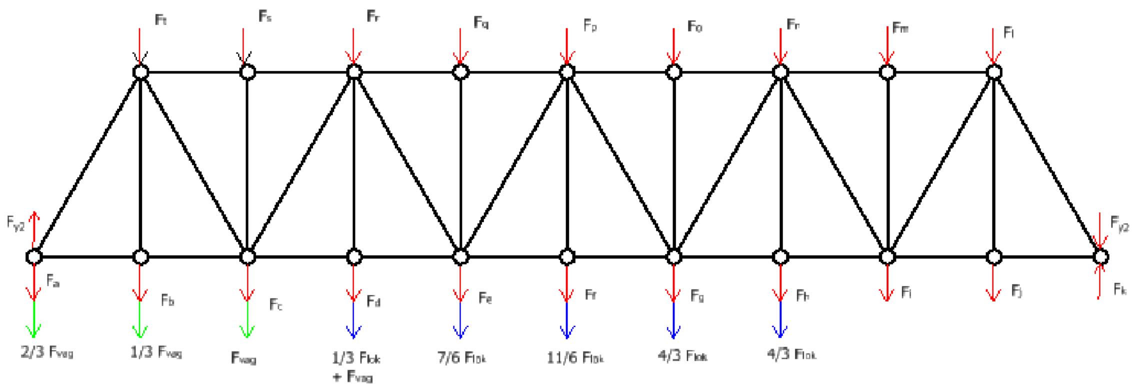
The deflection of joints of the bridge in case 2:

Deflection [mm]	
Wa	0.000
Wb	7.167
Wc	12.619
Wd	17.406
We	19.186
Wf	19.89
Wj	17.865
Wh	14.935
Wi	10.664
Wj	5.691
Wk	0.000

The maximum deflection is **19.89 mm**.

4.9.3. Case 3

When the train has passes three quarters of the bridge length, the load of the locomotive and part of the first wagon will be applied on the bridge, and these forces will act on the joints A, B, C, D, E, F, G, H as shown on the picture below:



Pic.54 Bridge with train in position 3

The gravitational force on each joint of the bridge in case 3:

Gravitational forces [N]			
Fa	73335	Fk	34654
Fb	66958	Fl	17689
Fc	125931	Fm	18621
Fd	151978	Fn	26091
Fe	189451	Fo	18621
Ff	225553	Fp	26091
Fg	182747	Fq	18621
Fh	175277	Fr	26091
Fi	48677	Fs	18621
fj	41207	Ft	17689

By the same way we have done before, we have calculated the normal force and the normal stress of each bar of the bridge in case 3:

Normal Forces [N]			
N1	678115	N21	66958
N2	678115	N22	878771
N3	1741782	N23	-18621
N4	1741782	N24	-681153
N5	2144749	N25	151978
N6	2144749	N26	437714
N7	1696881	N27	-18621
N8	1696881	N28	-153258
N9	623140	N29	225553
N10	623140	N30	-190766
N11	-913868	N31	-18621
N12	-1191378	N32	466057
N13	-1191378	N33	175277
N14	-2014672	N34	-741348
N15	-2014672	N35	-18621
N16	-2040247	N36	833351
N17	-2040247	N37	41207
N18	-1277324		
N19	-1277324		
N20	-994493		

The maximum tension force is 2144749 N and the maximum compression force is

-2040247 N and the maximum absolute force is **2144749 N**

The reaction forces on the bridge from the supports calculated in Newtons:

Fy1	800779
Fy2	703124

The normal stresses of each bar of the bridge in case 3 are listed on the table below:

Stress [MPa]			
σ1	13.67	σ21	15.20
σ2	13.67	σ22	-1.86
σ3	35.12	σ23	22.56
σ4	35.12	σ24	-1.86
σ5	43.25	σ25	17.53
σ6	43.25	σ26	-1.86
σ7	34.22	σ27	4.12
σ8	34.22	σ28	-48.39
σ9	12.57	σ29	42.76
σ10	12.57	σ30	-33.15
σ11	-23.83	σ31	21.30
σ12	-23.83	σ32	-7.46
σ13	-40.29	σ33	-9.28
σ14	-40.29	σ34	22.68
σ15	-40.80	σ35	-36.08
σ16	-40.80	σ36	40.55
σ17	-25.55	σ37	-44.47
σ18	-25.55		
σ19	6.70		
σ20	-1.86		

The maximum stress in tension is 43.25 [MPa] and the maximum stress in compression is - 48.39 [MPa] and the maximum absolute force is **48.39 [MPa].**

The factor of safety according to the limit state of elasticity:

$$k_K = \frac{\sigma_K}{\sigma_{max}} = \frac{210}{48.39} = 4.34$$

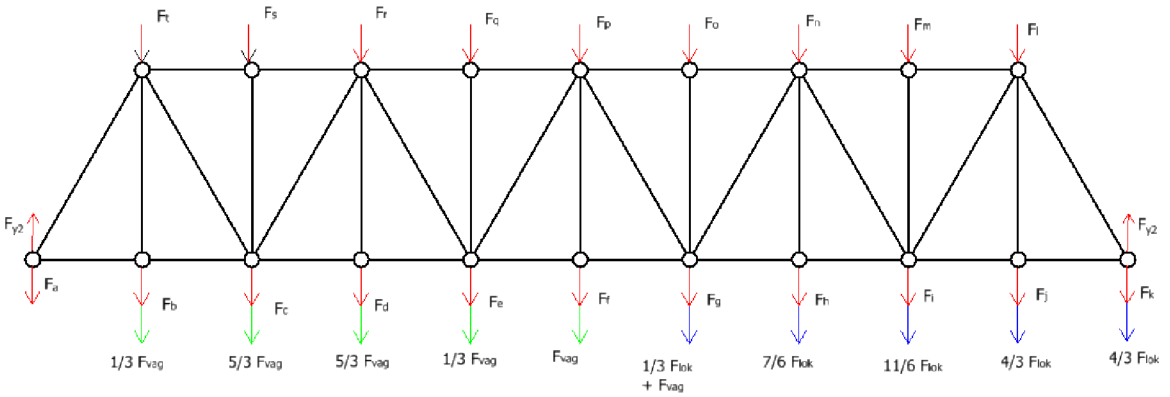
The deflection of joints of the bridge in case 3:

Deflection [mm]	
Wa	0.000
Wb	7.491
Wc	13.914
Wd	19.419
We	22.381
Wf	24.003
Wj	22.232
Wh	19.211
Wi	13.513
Wj	7.175
Wk	0.000

The maximum deflection is **24 mm**.

4.9.4. Case 4

When the train is at the end of the bridge length, the load of the locomotive, the first wagon and part of the second will be applied on the bridge, and these forces will act on the joints A, B, C, D, E, F, G, H, I, J, K as shown on the picture below:



Pic.55 Bridge with train in position 4

The gravitational force on each joint of the bridge in case 4:

Gravitational forces [N]			
Fa	21832	Fk	168724
Fb	66958	Fl	17689
Fc	177433	Fm	18621
Fd	169963	Fn	26091
Fe	74428	Fo	18621
Ff	118461	Fp	26091
Fg	159448	Fq	18621
Fh	158518	Fr	26091
Fi	233023	Fs	18621
fj	175277	Ft	17689

By the same way we have done before, we have calculated the normal force and the normal stress of each bar of the bridge in case 4:

Normal Forces [N]			
N1	647502	N21	66958
N2	647502	N22	833876
N3	1601934	N23	-18621
N4	1601934	N24	-565849
N5	1921348	N25	169963
N6	1921348	N26	297823
N7	1718530	N27	-18621
N8	1718530	N28	-170615
N9	770957	N29	118461
N10	770957	N30	-27003
N11	-1130650	N31	-18621
N12	-1362033	N32	270442
N13	-1362033	N33	158518
N14	-1902936	N34	-522821
N15	-1902936	N35	-18621
N16	-1805011	N36	866845
N17	-1805011	N37	175277
N18	-1216098		
N19	-1216098		
N20	-949597		

The maximum tension force is 1921348 N and the maximum compression force is -1902936 N and the maximum absolute force is **1921348 N**

The reaction forces on the bridge from the supports calculated in Newtons:

Fy1	716436
Fy2	995764

The normal stresses of each bar of the bridge in case 4 are listed on the table below:

Stresss [MPa]			
σ1	13.06	σ21	17.00
σ2	13.06	σ22	-1.86
σ3	32.30	σ23	11.85
σ4	32.30	σ24	-1.86
σ5	38.74	σ25	15.85
σ6	38.74	σ26	-1.86
σ7	34.65	σ27	17.53
σ8	34.65	σ28	-46.21
σ9	15.55	σ29	40.58
σ10	15.55	σ30	-27.54
σ11	-27.24	σ31	14.49
σ12	-27.24	σ32	-8.30
σ13	-38.06	σ33	-1.31
σ14	-38.06	σ34	13.16
σ15	-36.10	σ35	-25.44
σ16	-36.10	σ36	42.18
σ17	-24.32	σ37	-55.02
σ18	-24.32		
σ19	6.70		
σ20	-1.86		

The maximum stress in tension is 42.18 [MPa] and the maximum stress in compression is - 55.02 [MPa] and the maximum absolute force is **55.02 [MPa].**

The factor of safety according to the limit state of elasticity:

$$k_K = \frac{\sigma_K}{\sigma_{max}} = \frac{210}{55.02} = 3.82$$

The deflection of joints of the bridge in case 4:

Deflection [mm]	
Wa	0.000
Wb	7.072
Wc	13.101
Wd	18.155
We	20.646
Wf	22.103
Wj	21.121
Wh	18.765
Wi	13.838
Wj	7.884
Wk	0.000

The maximum deflection is **22.1 mm**.

4.10. Checking the buckling of the bridge

For calculating the factor of safety, we will use this equation:

$$k_V = \frac{F_{kr}}{F}$$

Where F_{kr} is the critical force and is calculated by this equation:

$$F_{kr} = \alpha^2 \frac{E\,J}{l^2}$$

Where l is the length is the length of the bar, α depends on the supports of the bar which is equal to π in our bridge, and J is the quadratic moment. The table below shows the quadratic moment of each type of bar calculated in mm^4:

	Jy [mm^4]	Jz [mm^4]
Horizontal up	4.69E+08	3.07E+09
Horizontal Bottom	5.86E+08	2.90E+09
vertical	3.30E+08	1.86E+07
tilted	4.79E+08	2.33E+07

For the calculation we will only use the minimum quadratic moment to get the minimum critical force.

The table below shows all the bars that have been in compression by either the bridges weight or by the load of the train. The maximum normal force (minimum with the minus) of the compressed bar is chosen. In the last column of the table we have changed the values of the maximum forces to absolute value because the factor of safety should always be positive value.

Bar number	Bar length [m]	N [N]	N-case 1 [N]	N-case 1 [N]	N-case 1 [N]	N-case 1 [N]	Min-N [N]	Absolut-N [N]
11	6.4	-402529	-519375	-690313	-913868	-1130650	-1130650	1130650
12	4.4	-494043	-653391	-886506	-1191378	-1362033	-1362033	1362033
13	4.4	-494043	-653391	-886506	-1191378	-1362033	-1362033	1362033
14	4.4	-744981	-1063676	-1529907	-2014672	-1902936	-2014672	2014672
15	4.4	-744981	-1063676	-1529907	-2014672	-1902936	-2014672	2014672
16	4.4	-744981	-1223024	-1797391	-2040247	-1805011	-2040247	2040247
17	4.4	-744981	-1223024	-1797391	-2040247	-1805011	-2040247	2040247
18	4.4	-494043	-1006455	-1267157	-1277324	-1216098	-1277324	1277324
19	4.4	-494043	-1006455	-1267157	-1277324	-1216098	-1277324	1277324
20	6.4	-402529	-904279	-1045153	-994493	-949597	-1045153	1045153
23	4.7	-18621	-18621	-18621	-18621	-18621	-18621	18621
24	6.4	-230009	-296450	-560821	-681153	-565849	-681153	681153
27	4.7	-18621	-18621	-18621	-18621	-18621	-18621	18621
28	6.4	-46002	70844	58494	-153258	-170615	-170615	170615
30	6.4	-46002	-162848	-333786	-190766	-27003	-333786	333786
31	4.7	-18621	-18621	-18621	-18621	-18621	-18621	18621
34	6.4	-230009	-346854	-517793	-741348	-522821	-741348	741348
35	4.7	-18621	-18621	-18621	-18621	-18621	-18621	18621

On this table we have listed the forces we got from the previous table, the minimum quadratic moment of each bar, the critical force of each bar and lastly the safety factor.

Bar number	Absulut-N [N]	J_{min} [mm^4]	F_{kr} [N]	k_v
11	1130650	2.33E+07	1179003	1.04
12	1362033	4.69E+08	50209573	36.86
13	1362033	4.69E+08	50209573	36.86
14	2014672	4.69E+08	50209573	24.92
15	2014672	4.69E+08	50209573	24.92
16	2040247	4.69E+08	50209573	24.61
17	2040247	4.69E+08	50209573	24.61
18	1277324	4.69E+08	50209573	39.31
19	1277324	4.69E+08	50209573	39.31
20	1045153	2.33E+07	1179003	1.13
23	18621	1.86E+07	1745164	93.72
24	681153	2.33E+07	1179003	1.73
27	18621	1.86E+07	1745164	93.72
28	170615	2.33E+07	1179003	6.91
30	333786	2.33E+07	1179003	3.53
31	18621	1.86E+07	1745164	93.72
34	741348	2.33E+07	1179003	1.59
35	18621	1.86E+07	1745164	93.72

We can clearly see that the minimum safety factor according to buckling is 1.04 in bar 11 which is a tilted bar, and the rest of the safety factors of the other tilted bars vary form 6.91 – 1.04 which is quite low. The reason behind this is that our bridge is a virtual bridge the cross-sections of which are made by us.

For the factor of safety to be 2 we will use this equation to get the critical force:

$K_V = \frac{F_{kr}}{F} \Longrightarrow F_{kr} = 2 \cdot F \Longrightarrow F_{kr} = 2 \cdot 1130650 =$

$$F_{kr} = 2261300\ N$$

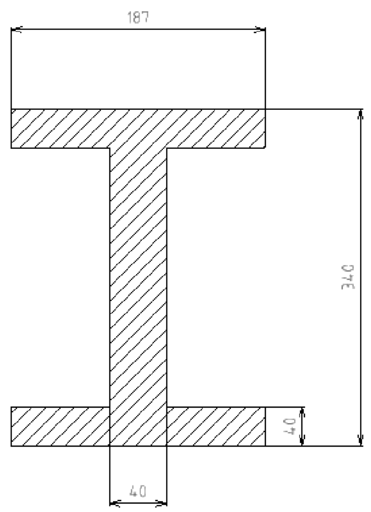
To get the quadratic moment, we will use this equatuon

$$F_{kr} = \alpha^2 \frac{E\,J}{l^2} \Longrightarrow J = \frac{F_{kr} \cdot l^2}{\alpha^2 \cdot E} \Longrightarrow J = \frac{2261300 \cdot 6.4^2}{(\pi^2) \cdot 210 \cdot 10^9}$$

$\boldsymbol{J = 4.47 \cdot 10^{-5}}$ **[m⁴]**

$\boldsymbol{J_{op} = 4.47 \cdot 10^{7}}$ **[mm⁴]**

We have designed the cross section of the tilted bars again so that the quadratic moment changes to be close to the optimal value with keeping the area of the cross section.



Pic.56 Optimal cross-section

The new cross-section has the folowing quadratic moments:

Jy [mm^4]	Jz [mm^4]
4 E+08	4.5E+07

The area of the new cross-section is 0.0254 m^2 which is not very different that the original value 0.0205 m^2, therefore the difference of normal forces, stresses and deflections is not going to be very large.

The minimum factor of safety:

$$F_{kr} = \alpha^2 \frac{E\,J}{l^2} = \alpha^2 \frac{E\,J}{l^2} = (\pi^2)\frac{210 \cdot 10^9 \cdot 4.5 \cdot 10^7 \cdot 10^{-12}}{6.454^2} = 2251744 \text{ N}$$

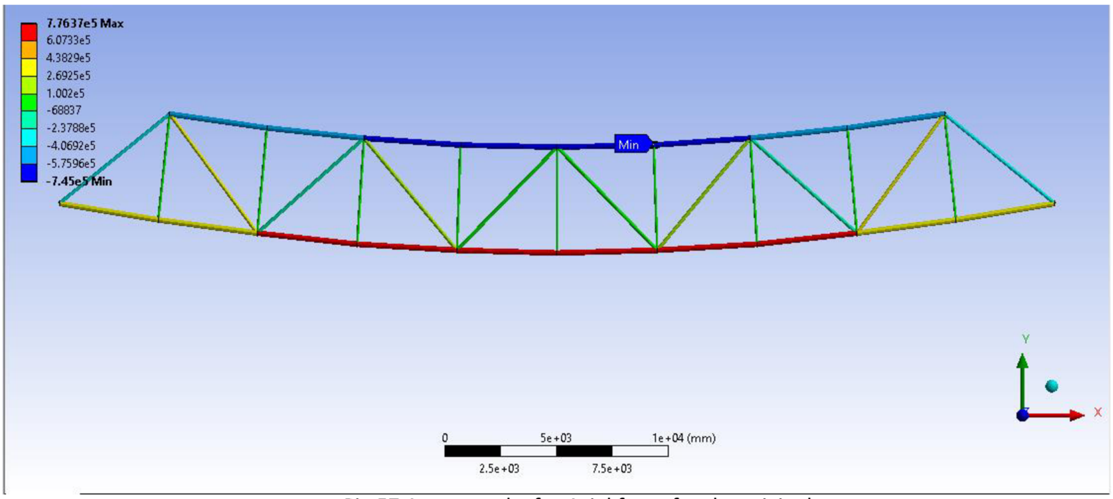
$$k_V = \frac{F_{kr}}{F} = \frac{2251744}{1130650} = 2$$

The factor of safety according to buckling is equal to F_{v4} = **2 .**

5. Finite element method for solving

The program used for this solution is ANSYS Workbench, we have drawn the geometry and set the bars to links. This solution was done for the four case of the train and the original structure.

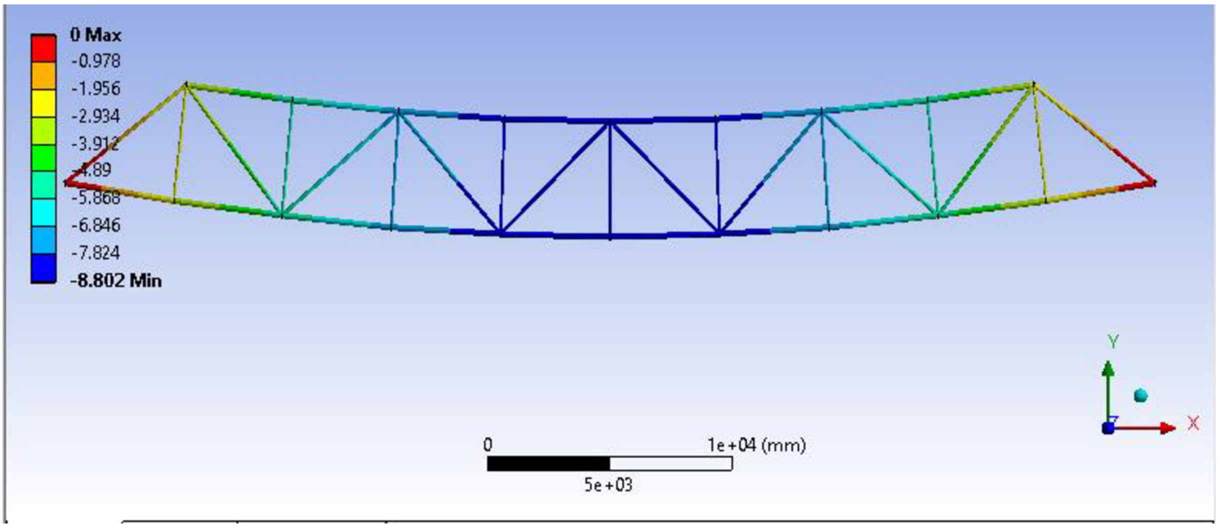
5.1. The original structure



Pic.57 Ansys results for Axial force for the original stuructue

Compare the normal forces of the original structure that we have calculated analytically with the forces from the ANSYS:

Normal Forces [N]		FEM	Normal Forces [N]		FEM
N1	274473	2.75E+05	N20	-402529	-4.03E+05
N2	274473	2.75E+05	N21	41207	4.12E+04
N3	650879	6.51E+05	N22	322012	3.22E+05
N4	650879	6.51E+05	N23	-18621	-1.86E+04
N5	776348	7.76E+05	N24	-230009	-2.30E+05
N6	776348	7.76E+05	N25	41207	4.12E+04
N7	650879	6.51E+05	N26	138005	1.38E+05
N8	650879	5.51E+05	N27	-18621	-1.86E+05
N9	274473	2.75E+05	N28	-46002	-4.60E+05
N10	274473	2.75E+05	N29	41207	-4.12E+04
N11	-402529	-4.03E+05	N30	-46002	-4.60E+04
N12	-494043	-4.94E+05	N31	-18621	-1.86E+04
N13	-494043	-4.94E+05	N32	138005	1.38E+05
N14	-744981	-7.45E+05	N33	41207	4.12E+04
N15	-744981	-7.45E+05	N34	-230009	-2.30E+05
N16	-744981	-7.45E+05	N35	-18621	-1.86E+04
N17	-744981	-7.45E+05	N36	322012	3.22E+05
N18	-494043	-4.49E+05	N37	41207	4.12E+04
N19	-494043	-4.49E+05			



Pic.58 Ansys results for deformation for the original structure

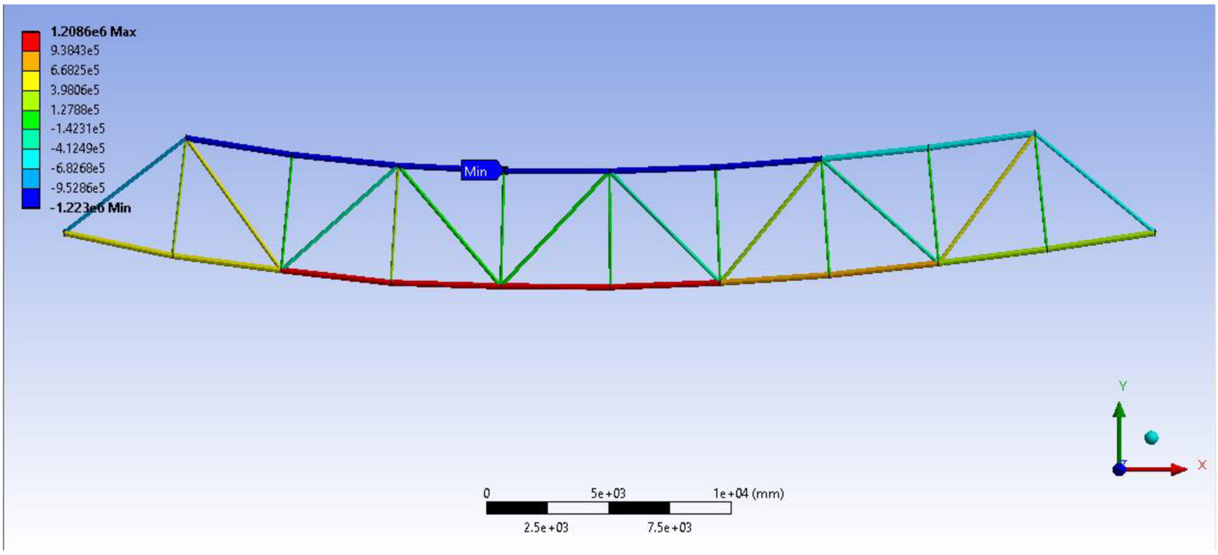
Comparison of the deflections of the original structure that we have calculated analytically, and the deflection from the ANSYS:

Deflection [mm]		FEM
Wa	0.000	0.000
Wb	2.918	-2.918
Wc	5.273	-5.271
Wd	7.234	-7.233
We	8.314	-8.312
Wf	8.804	-8.802
Wj	8.314	-8.312
Wh	7.234	-7.233
Wi	5.273	-5.271
Wj	2.918	-2.918
Wk	0.000	0.000

The maximum difference between the analytical and the numerical values is 0.002 mm.

5.2. Case 1

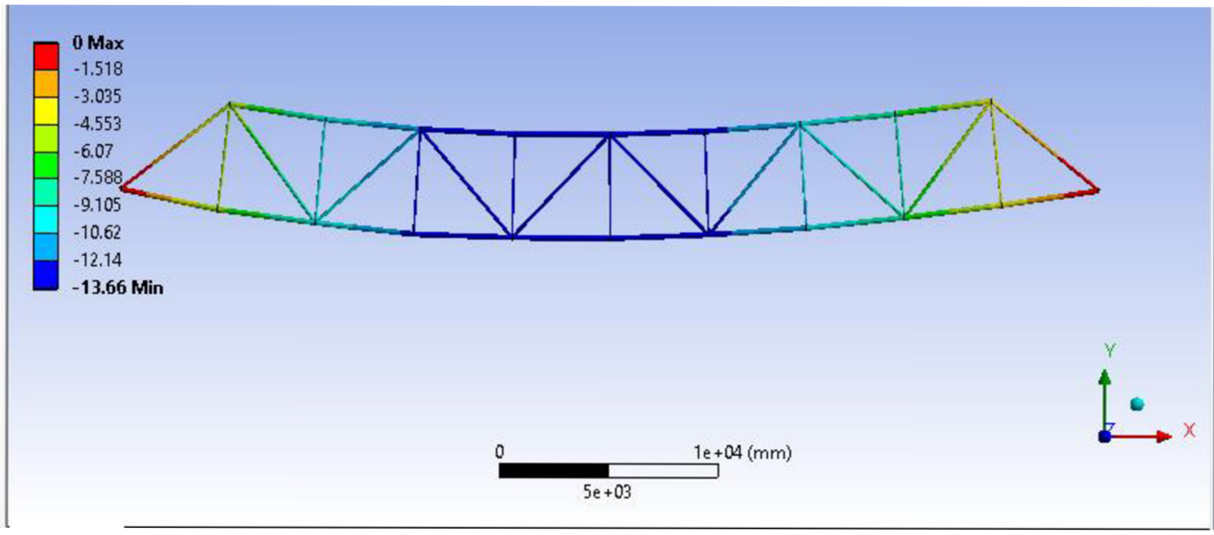
where the train has passed quarter of the bridge length



Pic.59 Ansys results for Axial force for

Compare the normal forces of the bridge in case 1that we have calculated analytically with the forces from the ANSYS:

Normal Forces [N]		FEM	Normal Forces [N]		FEM
N1	616601	6.17E+05	N20	-904279	-9.04E+05
N2	616601	6.17E+05	N21	225553	2.26E+05
N3	1208596	1.21E+06	N22	571742	5.72E+05
N4	1208596	1.21E+06	N23	-18621	-1.86E+04
N5	1174717	1.18E+06	N24	-296450	-2.97E+05
N6	1174717	1.18E+06	N25	175277	1.75E+05
N7	889901	8.89E+05	N26	21159	2.12E+04
N8	889901	8.89E+05	N27	-18621	-1.86E+04
N9	354147	3.54E+05	N28	70844	7.08E+04
N10	354147	3.54E+05	N29	41207	4.12E+04
N11	-519375	-5.19E+05	N30	-162848	-1.63E+05
N12	-653391	-6.53E+05	N31	-18621	-1.86E+04
N13	-653391	-6.53E+05	N32	254851	2.55E+05
N14	-1063676	-1.06E+06	N33	41207	4.12E+04
N15	-1063676	-1.06E+06	N34	-346854	-3.47E+05
N16	-1223024	-1.22E+06	N35	-18621	-1.86E+04
N17	-1223024	-1.22E+06	N36	438858	4.39E+05
N18	-1006455	-1.01E+06	N37	41207	4.12E+04
N19	-1006455	-1.01E+06			



Pic.59 Ansys results for deformation for case1

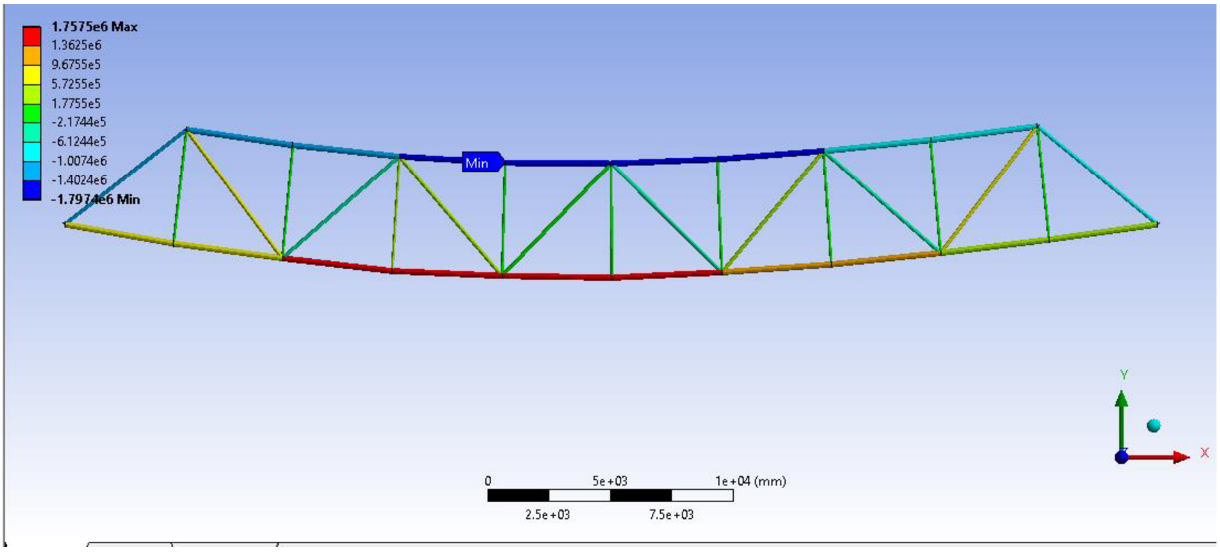
Comparison of the deflections of the bridge in case 1 that we have calculated analytically, and the deflection from the ANSYS:

Deflection [mm]		FEM
Wa	0.000	0.000
Wb	5.797	-5.796
Wc	9.418	-9.416
Wd	12.584	-12.581
We	13.455	-13.452
Wf	13.661	-13.658
Wg	12.575	-12.572
Wh	10.649	-10.646
Wi	7.653	-7.651
Wj	4.140	-4.139
Wk	0.000	0.000

The maximum difference between the analytical and the numerical values is 0.003 mm.

5.3. Case 2

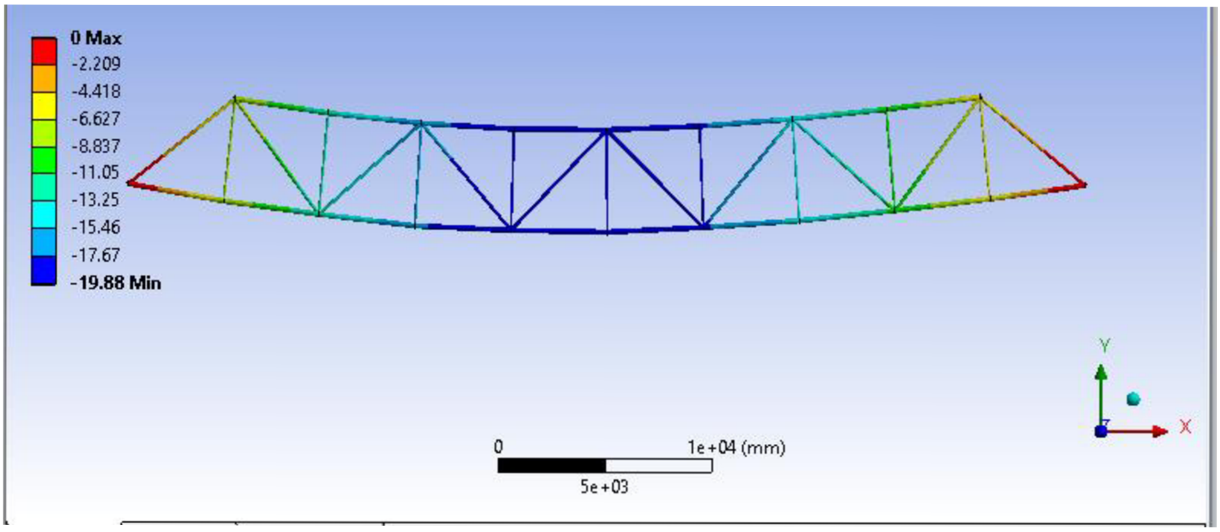
where the train is at the middle of the bridge



Pic.60 Ansys results for axial force for case 2

Compare the normal forces of the bridge in case 2 that we have calculated analytically with the forces from the ANSYS:

Normal Forces [N]		FEM	Normal Forces [N]		FEM
N1	712659	7.13E+05	N20	-1045153	-1.05E+06
N2	712659	7.13E+05	N21	151978	1.52E+05
N3	1649564	1.65E+06	N22	813200	8.13E+05
N4	1649564	1.65E+06	N23	-18621	-1.86E+04
N5	1757505	1.76E+06	N24	-560821	-5.61E+05
N6	1757505	1.76E+06	N25	225553	2.26E+05
N7	1239574	1.24E+06	N26	216797	2.17E+05
N8	1239574	1.24E+06	N27	-18621	-1.86E+04
N9	470704	4.71E+05	N28	58494	5.85E+04
N10	470704	4.71E+05	N29	175277	1.75E+05
N11	-690313	6.90E+05	N30	-333786	-3.34E+05
N12	-886506	-8.87E+05	N31	-18621	1.86E+04
N13	-886506	-8.87E+05	N32	425789	4.26E+05
N14	-1529907	-1.53E+06	N33	41207	4.12E+04
N15	-1529907	-1.53E+06	N34	-517793	-5.18E+05
N16	-1797391	-1.80E+06	N35	-18621	-1.86E+04
N17	-1797391	-1.80E+06	N36	609796	6.10E+05
N18	-1267157	-1.27E+00	N37	41207	4.12E+04
N19	-1267157	-1.27E+06			



Pic.61 Ansys results for deformation for case 2

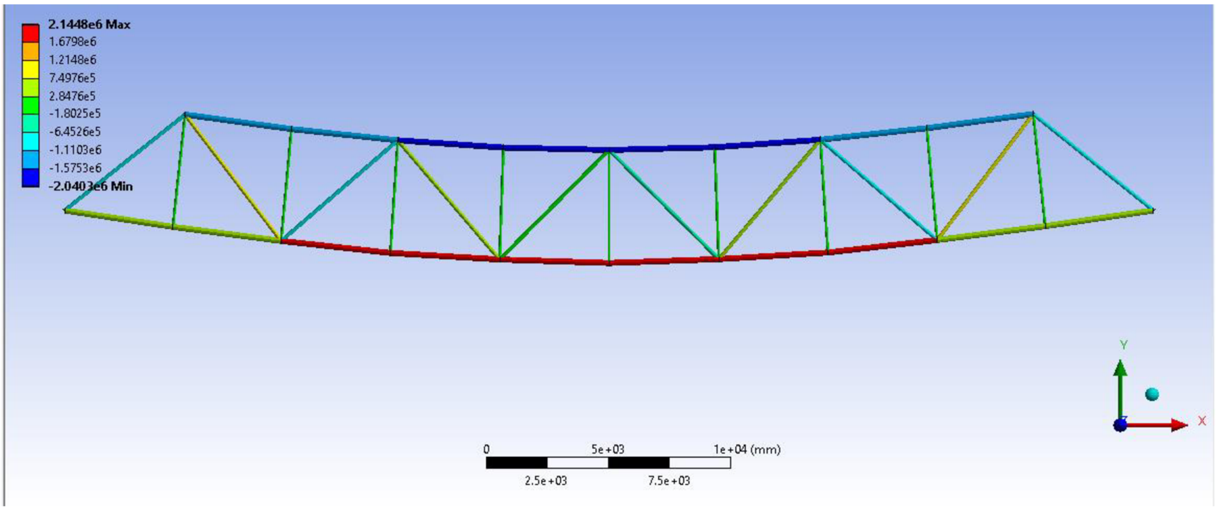
Comparison of the deflections of the bridge in case 2 that we have calculated analytically, and the deflection from the ANSYS:

Deflection [mm]		FEM
Wa	0.000	0.000
Wb	7.167	-7.165
Wc	12.619	-12.616
Wd	17.406	-17.401
We	19.186	-19.181
Wf	19.887	-19.882
Wj	17.865	-17.860
Wh	14.935	-14.932
Wi	10.664	-10.661
Wj	5.691	-5.689
Wk	0.000	0.000

The maximum difference between the analytical and the numerical values is 0.005 mm.

5.4. Case 3

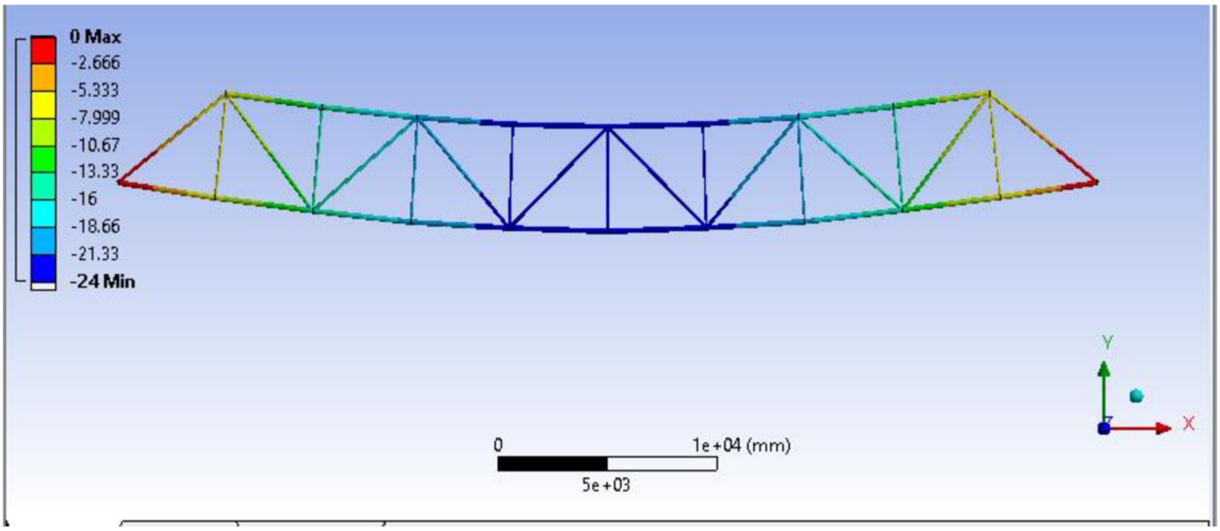
where the train has passed three quarters of the bridge length



Pic.62 Ansys results for Axial force for case 3

Compare the normal forces of the bridge in case 3 that we have calculated analytically with the forces from the ANSYS:

Normal Forces [N]		FEM	Normal Forces [N]		FEM
N1	678115	6.78E+05	N20	-994493	-9.95E+05
N2	678115	6.78E+05	N21	66958	6.70E+04
N3	1741782	1.74E+06	N22	878771	8.79E+05
N4	1741782	1.74E+06	N23	-18621	-1.86E+04
N5	2144749	2.15E+06	N24	-681153	-6.81E+05
N6	2144749	2.15E+06	N25	151978	1.52E+05
N7	1696881	1.70E+06	N26	437714	4.38E+05
N8	1696881	1.70E+06	N27	-18621	-1.86E+04
N9	623140	6.23E+05	N28	-153258	-1.53E+05
N10	623140	6.23E+05	N29	225553	2.26E+05
N11	-913868	-9.14E+05	N30	-190766	-1.91E+05
N12	-1191378	1.19E+06	N31	-18621	-1.86E+04
N13	-1191378	-1.19E+06	N32	466057	4.66E+05
N14	-2014672	-2.02E+06	N33	175277	1.75E+05
N15	-2014672	-2.02E+06	N34	-741348	-7.41E+05
N16	-2040247	-2.04E+06	N35	-18621	-1.86E+04
N17	-2040247	-2.04E+06	N36	833351	8.33E+05
N18	-1277324	-1.28E+06	N37	41207	4.12E+04
N19	-1277324	-1.28E+06			



Pic.63 Ansys results for deformation for case 3

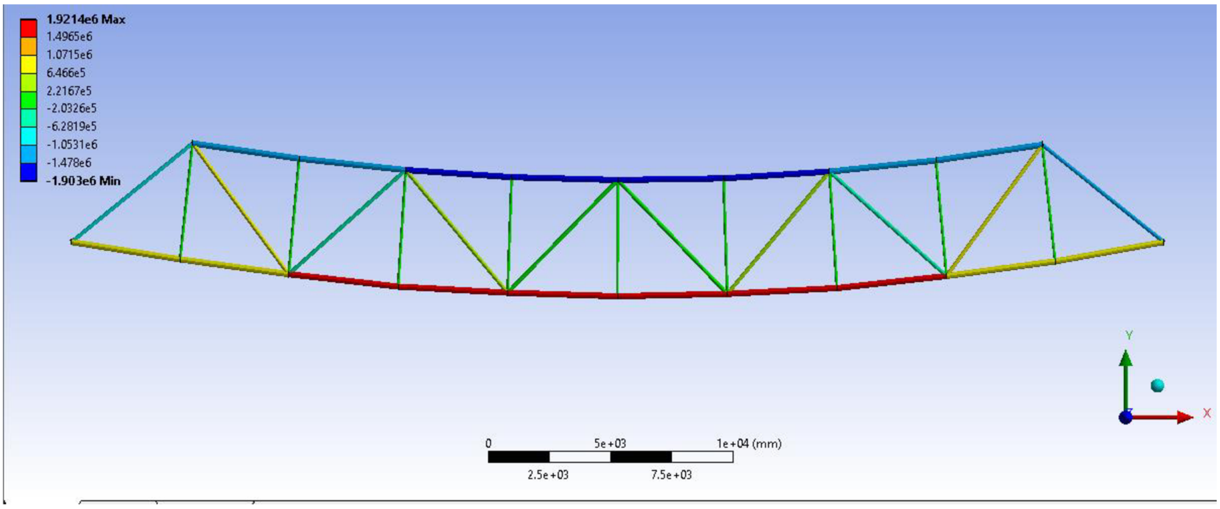
Comparison of the deflections of the bridge in case 3 that we have calculated analytically, and the deflection from the ANSYS:

Deflection [mm]		FEM
Wa	0.00	0.00
Wb	7.49	-7.49
Wc	13.91	-13.91
Wd	19.42	-19.41
We	22.38	-22.38
Wf	24.00	-24.00
Wj	22.23	-22.23
Wh	19.21	-19.21
Wi	13.51	-13.51
Wj	7.17	-7.17
Wk	0.00	0.00

The maximum difference between the analytical and the numerical values is 0.006 mm.

5.5. Case 4

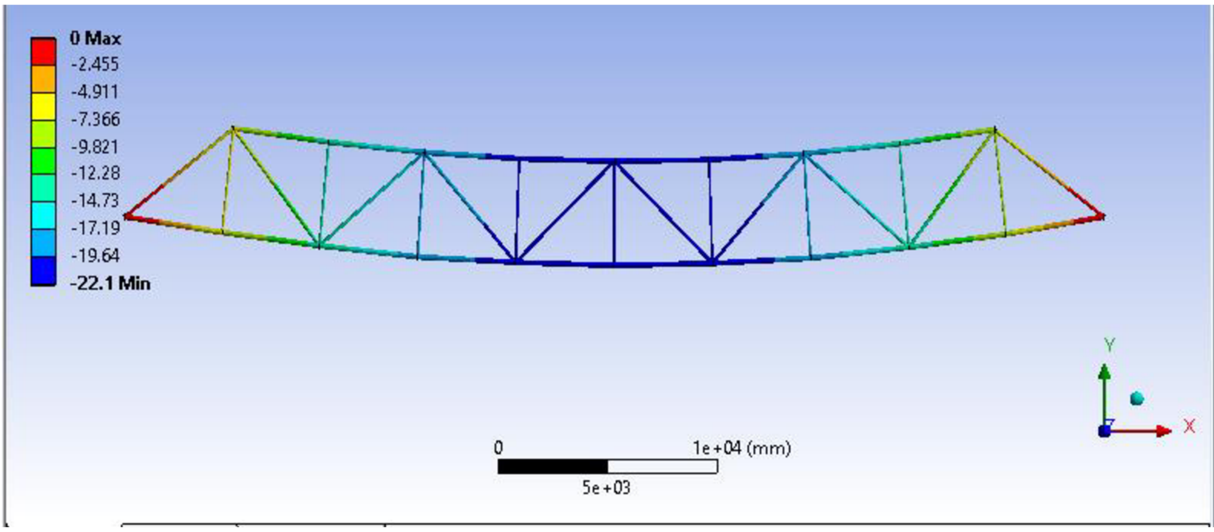
Where the train is at the end of the bridge



Pic.64 Ansys results for axial force for case 4

Compare the normal forces of the bridge in case 4 that we have calculated analytically with the forces from the ANSYS:

Normal Forces [N]		FEM	Normal Forces [N]		FEM
N1	647502	6.48E+05	N20	-949597	-9.50E+05
N2	647502	6.48E+05	N21	66958	6.70E+04
N3	1601934	1.60E+06	N22	833876	8.34E+05
N4	1601934	1.60E+06	N23	-18621	-1.86E+07
N5	1921348	1.92E+06	N24	-565849	-5.66E+08
N6	1921348	1.92E+06	N25	169963	1.70E+05
N7	1718530	1.72E+06	N26	297823	2.98E+05
N8	1718530	1.17E+06	N27	-18621	-1.86E+04
N9	770957	7.71E+05	N28	-170615	-1.71E+05
N10	770957	7.71E+05	N29	118461	1.19E+05
N11	-1130650	-1.13E+06	N30	-27003	-2.70E+04
N12	-1362033	-1.36E+06	N31	-18621	-1.86E+04
N13	-1362033	-1.36E+06	N32	270442	2.70E+05
N14	-1902936	-1.90E+06	N33	158518	1.59E+05
N15	-1902936	-1.90E+06	N34	-522821	-5.23E+05
N16	-1805011	-1.81E+06	N35	-18621	-1.86E+04
N17	-1805011	-1.81E+06	N36	866845	8.67E+05
N18	-1216098	-1.22E+06	N37	175277	1.75E+05
N19	-1216098	-1.22E+06			



Pic.64 Ansys results for deformation for case 4

Comparison of the deflections of the bridge in case 4 that we have calculated analytically, and the deflection from the ANSYS:

Deflection [mm]		FEM
Wa	0.000	0.000
Wb	7.072	-7.070
Wc	13.101	-13.097
Wd	18.155	-18.151
We	20.646	-20.641
Wf	22.103	-22.097
Wj	21.121	-21.116
Wh	18.765	-18.761
Wi	13.838	-13.834
Wj	7.884	-7.882
Wk	0.000	0.000

The maximum difference between the analytical and the numerical values is 0.006 mm.

6. Conclusion

The main goal from this thesis was to do a stress and strain analysis of a truss system. The type of truss system we have used in this thesis was a virtual bridge whose basic dimensions (like, length, Height, width and the shape of cross-sections) were mostly taken from a bridge that is located on the borders between Czech Republic and Slovakia, in a city called Horné srnie. This bridge connects both sides of Vlára river and is used for trains to pass throw it to the other side of the river.

The cross-sections of the virtual bridge are then drawn to match the cross-sections of the mentions bridge as much as possible. To do so, we have checked the bachelor thesis of the past years that are similar to ours and compared the input data of the bridges there with the data we have from the mentioned bridge. Then we have chosen the most similar bridge to our virtual to take the areas of the cross sections from. For the analysis we have firstly written the most important theory and all the equations and the relations that have been used throw out the thesis.

For the stress and strain analysis, we have firstly considered the load of the bars without any outer effects, like ice or wind load. We then used the method of joints where we have drawn the free body diagram of every joint of the bridge. Then we have used Matlab for solving these equations to get the normal forces ,and from the normal forces we have calculated the stress of each bar of the bridge, where the maximum stress was -19.59 MPa in bar number 28 and 37. the factor of safety according to the limit states of elasticity was 10.72 and the maximum deflection was 8.8 mm in joint F. The factor of safety is quite height because the load from the train is not put on the bridge yet and this will put the bridge in a much higher stress.

The we have introduced the stress and strain analysis of the bridge when a train passes throw it. We did not consider the force to be dynamic force, we considered it as a static force in four different cases according to the position of the train. First case is when the train has past quarter of the length of the bridge. Second case is when the train has past to the middle of the bridge. Third case is when the train has passed three quarters of the bridge. Last case is when the train is at the end of the bridge.

From the analysis of the first case we got that the maximum stress in compression was bar number 28 where its stress was -44 MPa, the maximum stress in tension was bar number 29 where the maximum stress was 27.82 MPa. The safety factor according to the limit states of elasticity was 4.8 and the maximum deflection was 13.66 mm in joint F. In this case the stress is mostly concentrated on the center of the bridge where 28 is a tilted bar and 29 is a vertical bar.

In the second case, the maximum stress in compression was -50,86 MPa in bar number 28 and the maximum stress in tension was 39.57 MPa in bar number 29. the factor of safety according to the limit state of elasticity was 4.13 and the maximum deflection was 19.89 mm in joint F. In this case, the most stressed place is at the center of the bridge.

The analysis of the third case shows that the maximum stress in compression was -48,39 MPa in bar number 28 and the maximum stress in tension was 43.25 MPa in bar number 5 and 6. the factor of safety according to the limit state of elasticity was 4.34 and the maximum deflection was 24 mm in joint F. In this case, the most stressed place is at the center of the bridge but the bars that are mostly stressed are the two center horizontal down ones and the vertical bar between them.

In the last case, the maximum stress in compression was -55 MPa in bar number 36 and the maximum stress in tension was 42.18 MPa in bar number 37. the factor of safety according to the limit state of elasticity was 3.82 and the maximum deflection was 22.1 mm in joint F. This case is the dangerous case where the factor of safety is the lowest among all and the most stressed place is at the end of the bridge at bar 36 and 37.

Then we have calculated the buckling of the bridge. We have checked all compressed bars form the own weight and from the load of the train in all different cases. Then we have calculated the factor of safety according to buckling of all compressed bars and we got that the minimum factor of safety was 1 in a tilted bar number 11 which is considered to be very low. Then we have changed the cross section of the tilted bars in a way that they the quadratic moment will change to increase the factor of safety to be 2 with keeping the cross-section area as close to possible to the original one so that the stresses and the normal forces will not be dramatically changed.

As the last part, we have done the analysis of all the cases before for the bridge with the train load in all different position and the analysis of the own wight using ANSYS, and compared the results we got from the analytical way and the resulted from the numerical way(ANSYS). The results were quite similar and there was no big difference between both results where the maximum difference in deflection we got from both ways was 0.006 mm.

Reference

[1] BURŠA, J., JANÍČEK, P., HORNÍKOVÁ, J., ŠANDERA, P.: Pružnost a pevnost

[on-line]. Dostupné z <http://beta.fme.vutbr.cz/cpp/> ISBN 80-7204-268-8.

[2] SVOBODA, J. Deformačně napěťová analýza železničního mostu přes řeku Odru. Brno: Vysoké učení technické v Brně, Fakulta strojního inženýrství, 2015. 65 s. Vedoucí bakalářské práce doc. Ing. Vladimír Fuis, Ph.D

[3] GAGO, V. Napjatostně deformační analýza železničního mostu v Brně Černovicích. Brno: Vysoké učení technické v Brně, Fakulta strojního inženýrství, 2018. 68 s. Vedoucí bakalářské práce doc. Ing. Vladimír Fuis, Ph.D

[4] KOVÁŘ, J. Deformačně napěťová analýza železničního mostu přes řeku Svratku. Brno: Vysoké učení technické v Brně, Fakulta strojního inženýrství, 2016. 71 s. Vedoucí bakalářské práce doc. Ing. Vladimír Fuis, Ph.D

[5] ŠČERBA, B. Napjatostně deformační analýza mostu v Třineckých železárnách. Brno: Vysoké učení technické v Brně, Fakulta strojního inženýrství, 2018. 92 s. Vedoucí bakalářské práce doc. Ing. Vladimír Fuis, Ph.D.

[6] TRÁVNÍČEK, L. Deformačně napěťová analýza železničního mostu přes řeku Moravu. Brno: Vysoké učení technické v Brně, Fakulta strojního inženýrství, 2015. 68 s. Vedoucí bakalářské práce doc. Ing. Vladimír Fuis, Ph.D

List of attachment

Exel_File

Sheet1 …………………Data for the main structure of the bridge.

Variant A………………Data for case 1.

Variant B………………Data for case 2.

Variant C………………Data for case 3.

Variant D………………Data for case 4.

Matlab _file

First code……………… Code for the main structure of the bridge.

Variant A……………… Code for case 1.

Variant B……………… Code for case 2.

Variant C……………… Code for case 3.

Variant D……………… Code for case 4.

Deformation………… Function for partial derivation