Czech University of Life Sciences Prague

Faculty of Economics and Management

Department of Systems Engineering

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Bachelor Thesis

Multi-criteria decision making at an enterprise

Mariya Vassilyeva

CZECH UNIVERSITY OF LIFE SCIENCES PRAGUE

Faculty of Economics and Management

BACHELOR THESIS ASSIGNMENT

Mariya Vassilyeva

Economics and Management

Thesis title

Multi-criteria decision making at an enterprise

Objectives of thesis

The main goal of the thesis is to determine the service provider of mining and transport equipment for work at the coalmine using the methods of multi-criteria analysis on the example of a specific enterprise of the coal industry.

Methodology

The work consists of two parts: theoretical and practical. The theoretical part is devoted to multi-criteria analysis, methods of choosing a service provider, classification and solution of decision problems. The practical part describes the situation of decision-making and the choice of the best service provider including the decision criteria formulation and description, choice of methods and description of the decision process, calculations and recommendation of the chosen variant.

The proposed extent of the thesis

30-40 pages

Keywords

Multi-criteria analysis; decision-making; criteria; service provider.

Recommended information sources

ŠUBRT, T. *Ekonomicko-matematické metody*. Plzeň: Vydavatelství a nakladatelství Aleš Čeněk, 2011. ISBN 978-80-7380-345-2.

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Емельянов С. В., Ларичев О. И. Многокритериальные методы принятия решений. — М.: Знание, 1985. — 32 с.

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The Bachelor Thesis Supervisor

doc. Ing. Ludmila Dömeová, CSc.

Supervising department

Department of Systems Engineering

Electronic approval: 25. 1. 2020

doc. Ing. Tomáš Šubrt, Ph.D.

Head of department

Electronic approval: 13. 2. 2020

Ing. Martin Pelikán, Ph.D.

Dean

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Declaration

I declare that I have worked on my bachelor thesis titled "Multi-criteria decision making at an enterprise" by myself and I have used only the sources mentioned at the end of the thesis. As the author of the bachelor thesis, I declare that the thesis does not break copyrights of any their person.

In Prague on 18.03.2020 ______________________________

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Multi-criteria decision making at an enterprise

Abstract

The bachelor`s thesis is devoted to the application of various methods in multi- criteria analysis.

In the theoretical part, the concepts of multi-criteria analysis, methods for determining the weights of criteria and methods for determining alternatives are disclosed. Three methods (SAW, TOPSIS, and AHP) are described in more detail. In the practical part, the problem of a specific company is considered and ways to solve it are described, based on the methods that were previously presented in the theoretical part. Problem, choosing a service provider from a variety of offers. The result obtained, the best supplier of the company is determined.

Keywords: multi-criteria analysis, criteria, service provider.

Vícekritériální rozhodování v podniku

Abstrakt

Bakalářská práce je věnována použití různých metod v multi-kritéria analýzy.

V teoretické části jsou zveřejněny pojmy multikriteriální analýzy, metody pro stanovení hmotnosti kritérií a metody pro stanovení alternativ. Three methods (SAW, TOPSIS, and AHP) are described in more detail. V praktické části je zvažován problém konkrétní společnosti a způsoby jejího řešení jsou popsány na základě metod, které byly dříve prezentovány v teoretické části. Problém, výběr poskytovatele služeb z různých nabídek. Získaný výsledek, nejlepší dodavatel společnosti je určen.

Klíčová slova: multikriteriální analýza, kritéria, poskytovatele služeb

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1. Introduction

A person in his life is constantly faced with the problem of choice, with the need to make decisions in this choice.

In the everyday life of every person, in the activities of organizations or society as a whole, decision-making is an essential component that determines the future. A special process of human activity aimed at choosing the best option from possible actions is understood by decision-making [1].

The choice of a decision is determined by the results of the consequences analysis, but, unfortunately, it is very difficult to accurately calculate and evaluate the consequences for the vast majority of decisions made by a person. Within human capabilities, only an assumption about the result of a particular option plays a role. Due to the fact that it is not always possible to take into account all the factors affecting the result of the decision made, such an assumption may turn out to be incorrect.

At the present stage of the development of computer technology, computers are many times superior to humans in speed and accuracy of calculations, but on the other hand, people have the unique ability to quickly assess the situation, single out the main thing and discard the secondary one, measure conflicting estimates, and fill in the uncertainty with their own guesses.

But, despite this, the number of erroneous decisions is large, and together with the development of human society and globalization trends, the strength of their negative influence is growing. Therefore, once there has been a need to create tools that will help a person in making decisions, namely, decision-making support methods under several criteria.

All these methods are combined in the decision-making theory, which is the theoretical basis for this area. It was within its framework that methods that helped solve important life problems and underlay support and decision-making systems began to develop.

Decision-making support systems can be used for different problems and at different decision-making levels. So, for example, they can be useful for business in analyzing and forecasting the dynamics of the market environment, in developing the organization’s development strategy, in assessing the potential of an enterprise and choosing a service provider, etc.

2. Objectives and Methodology

2.1 Objectives

The main objective of the bachelor’s diploma paper is to select the necessary methods and ways for multi-criteria analysis and use them to support adopting the best solution to the real problem.

Partial objectives:

- Description of the situation, collection of real data, determination of the problem and the order of its solution using the selected methods, making the best decision;

- Theoretical research will describe the selected methods and serve as the basis for constructing a mathematical model.

2.2 Methodology

The bachelor's thesis is devoted to multi-criteria decision-making problems.

The paper consists of two main parts.

The first part is based on the study and analysis of the literature.

This part describes the concepts of the multi- criteria analysis, the principle of the sequential reduction of uncertainties, and more fully explains the methods used in MCDA:

- Criteria weights determination methods:

• Sequence method;

• Scoring method;

• Saaty method.

- Multi-criteria selection methods:

• Methods based on quantitative measurements: SAW-method, TOPSIS-method;

• Methods based on qualitative measurements, the results of which are transferred into a quantitative form: Analytical Hierarchy Process (AHP).

The second part of the paper is the selection of methods for multi-criteria analysis and the application of the selected methods in the practical solution of the problem set.

All calculations are based on data taken from the real coal industry enterprise.

3. Literature Review

3.1 Multiple Criteria Decision Making (MCDM)

Almost any kind of human activity is associated with situations where there are several opportunities and a person is free from these opportunities to choose any that is most suitable for him. The best choice problems are studied by the decision-making theory. With its help, one can learn to make choices more reasonably, effectively using available information about preferences. This theory helps avoid making obviously bad decisions and take into account possible negative consequences of an ill-conceived choice. Extremely wide and important from a practical point of view, the class of selection problems consists of multi-criteria problems in which the quality of a decision to be made is evaluated by several criteria at the same time. Multi-criteria decision-making (MCDM) is a discipline of operations research that explicitly evaluates multiple conflicting criteria in decision-making.

3.2 Multiple criteria decision analysis (MCDA)

Multi-criteria analysis of variants belongs to a set of techniques for MCDM.

MCDA is the general field of study which includes decision making in the presence of two or more conflicting objectives and/or decision analysis processes involving two or more attributes [2]. The general objective of MCDA is to assist a decision maker or a group of decision makers to choose the best alternative from a range of alternatives in an environment of conflicting and competing criteria. In recent years, several methods have been proposed to deal with MCDA problems.

MCDA methods differ, however, in the way the idea of multiple criteria is considered, the application and computation of weights, the mathematical algorithm utilised, the model to describe the system of preferences of the individual facing decision-making, the level of uncertainty embedded in the data set and the ability for stakeholders to participate in the process [3].

Currently, two types of multi-criteria decision analysis have been developed: qualitative and quantitative. A high-quality MCDA uses only a consultative process to make decisions based on many criteria. A qualitative approach may consider alternative decisions, however, the decision-making process itself is not formalized, and the influence of criteria on decision-making remains largely unobvious. A potential criticism of this approach is that the decision-making process is not transparent and is unlikely to be reproducible. Moreover, taking into account many heterogeneous factors is not subject to a predetermined algorithm, and the decision-making process is still subjective in nature. Quantitative MCDA allows developing a formalized decision-making support tool (i.e. sequence or categorizing alternatives) based on a preliminary assessment of the relative importance of different criteria (criteria weighting factors) and taking into account their values also expressed quantitatively. The development of a quantitative MCDA methodology to solve a specific problem includes the following components: 1) determining a set of criteria that are then combined into a single mathematical function, 2) weighting factors vector for the selected criteria, 3) scales for measuring the values of each of the criteria, 4) methods for classifying the assessment received for each decision as an appropriate priority level (if necessary). Quantitative MCDA methods differ in the ways of calculating the relative weights and the numerical values of the criteria, which are then used to make a decision.

3.2.1 Types of alternatives

Any selection problem begins with a review and description of the list of available alternatives or a set of solutions. One can say that it is their existence that creates the very need for decision-making.

Alternatives will also be called the set of permissible (possible) solutions of X. It is logical to assume that the minimum number of elements in this set is two, since this implies the existence of a choice. The upper limit cannot be clearly defined, in theory the number of suggested alternatives can be infinite, in practice, it is determined by reasonable sense and computing power. That is their nature.

There are several types of alternatives:

- Dominated alternative is an alternative that has at least one best criterion among other alternatives and at the same time, none of the criteria is the worse among other variants.

- Non-dominated alternative is an alternative that usually do not dominate each other. This alternative may be better in some criteria and in some be worst among all the alternatives.

- Ideal alternative is the best alternative among all the criteria. Usually there is no such ideal alternative, because it would automatically be the most effective alternative. Other alternatives will be dominated and the ideal alternative will be the optimal.

- Basal alternative is the opposite of the ideal alternative. Non-ideal alternative is the worst among all the criteria. As well as the ideal alternative usually does not exist.

- Optimal alternative exists in situation when only one solution is acceptable.
- Compromise alternative is non-dominated alternative which is recommended as solution of the decision making problem. The choice of the best compromise alternative depends on how it is determined, and its distance from the basal alternative. In situation where decision making process has more non-dominated alternatives, compromise alternative is the best solution.

3.2.2 The best solution.

The choice of the solution itself consists in highlighting among the X set the best (selected) option. It should be noted that the situation often arises when not one, but a whole set of solutions, which is a certain subset of the set of feasible X solutions, is selected.

The problems of multi-criteria selection have a certain complexity, which consists in the impossibility of a priori selection of the best option. The very concept of the “best” depends on the psychological perception of the situation by a person and on many factors that at the moment of the development of science and the mathematical apparatus cannot be taken into account in the model.

Let us denote the set of solutions to be selected as C (X). It is a solution to the choice problem and belongs to the set of feasible X solutions (is any of its subsets). Thus, to solve the choice problem means to find a subset of CX, $CX \subset X$. When the set of solutions to be selected does not contain a single element (i.e. it is empty), the actual selection does not occur, since no solution is selected. Such a situation is not of practical interest, since for the choice to take place the C (X) set should contain at least one element. With some problems, it can turn out to be infinite [4].

3.2.3 Decision maker.

The selection process is impossible without the presence of one who makes this choice, pursuing his own goals. A person (or an entire team subordinate to the achievement of the specific goal), who makes a choice and is fully responsible for its consequences, is called the decision maker (for short: DM). Moreover, in the framework of the problem, only those characteristics of the decision-maker that are involved in its solution, such as, for example, experience in this field and psychological features, are important.

3.2.4 Criteria

Criteria for assessing alternatives are indicators of their attractiveness (or unattractiveness) for participants in the selection process.

Criteria may be independent or dependent. Dependent criteria are those under which an alternative assessment by one of them determines (unambiguously or with a high degree of probability) an assessment by another criterion.

The complexity of decision-making problems is also affected by the number of criteria. With a small number of criteria (two or three), the problem of comparing the two alternatives is quite simple and transparent, the qualities of the criteria can be directly compared and a compromise can be worked out. With a large number of criteria, the problem becomes unobvious.

Depending on the quantifiable of the criterion, criteria divided into:

- Quantitative criteria are numerical values, which are objectively measurable.
- Qualitative criteria are values, which cannot be objectively measured. Usually values are estimated by decision makers and called subjective criteria. Criteria can be measured, if measured alternative is better, equal or worse than other alternatives.

Depending on the nature of the criterion, we distinguish:

- Maximization criteria – higher value is taken to be a better value. Decision maker prefers higher values to lower values.
- Minimization criteria – opposite situation, lower value is taken to be a better value. Decision maker prefers lower values to higher ones.

Requirements for the criteria can be formulated as follows:

Completeness - criteria should reflect all properties of the alternative that are essential and important for the decision maker.

Independence - criteria should not duplicate each other.

Universality - the criteria should be applicable to each alternative.

Quantity - the choice and quantity of criteria should not be very high, otherwise the model will be incomprehensible [5].

3.2.5 Multi-criteria problem.

The selection problem, which includes the set of feasible X solutions and the vector criterion f, is usually called the multi-criteria problem or the multi-criteria optimization problem.

Preference relationship. Let us consider two feasible solutions x' and x". We suppose that after presenting this pair of solutions to the decision maker, he chooses (prefers) the first of them. In this case they write x' $\succ_X$x".

The sign $\succ_X$ is used to denote the preferences of a given decision maker expressed by a preference relation.

Multi-criteria selection model.

The setting of any multi-criteria selection problem includes:

- the set of possible X solutions,
- the vector criterion f,
- the preference relation $\succ_X$.

The problem of the multi-criteria selection is to find the set of solutions to be selected CX, CX⊂X, taking into account its preference relation $\succ_X$ based on the given vector criterion f, which reflects the set of decision maker’s goals [4]

3.2.6 The principle of sequential reduction of uncertainty.

In the real problems of making managerial decisions to the selection stage, there is still a large uncertainty of information due to the presence of many situations and goals. Therefore, it is very difficult to immediately select the only solution from the set of formulated ones. In this regard, the principle of sequential reduction of uncertainty is used, which consists in the sequential narrowing of many solutions.

The narrowing of many alternatives occurs in several stages (Fig. 3.1).

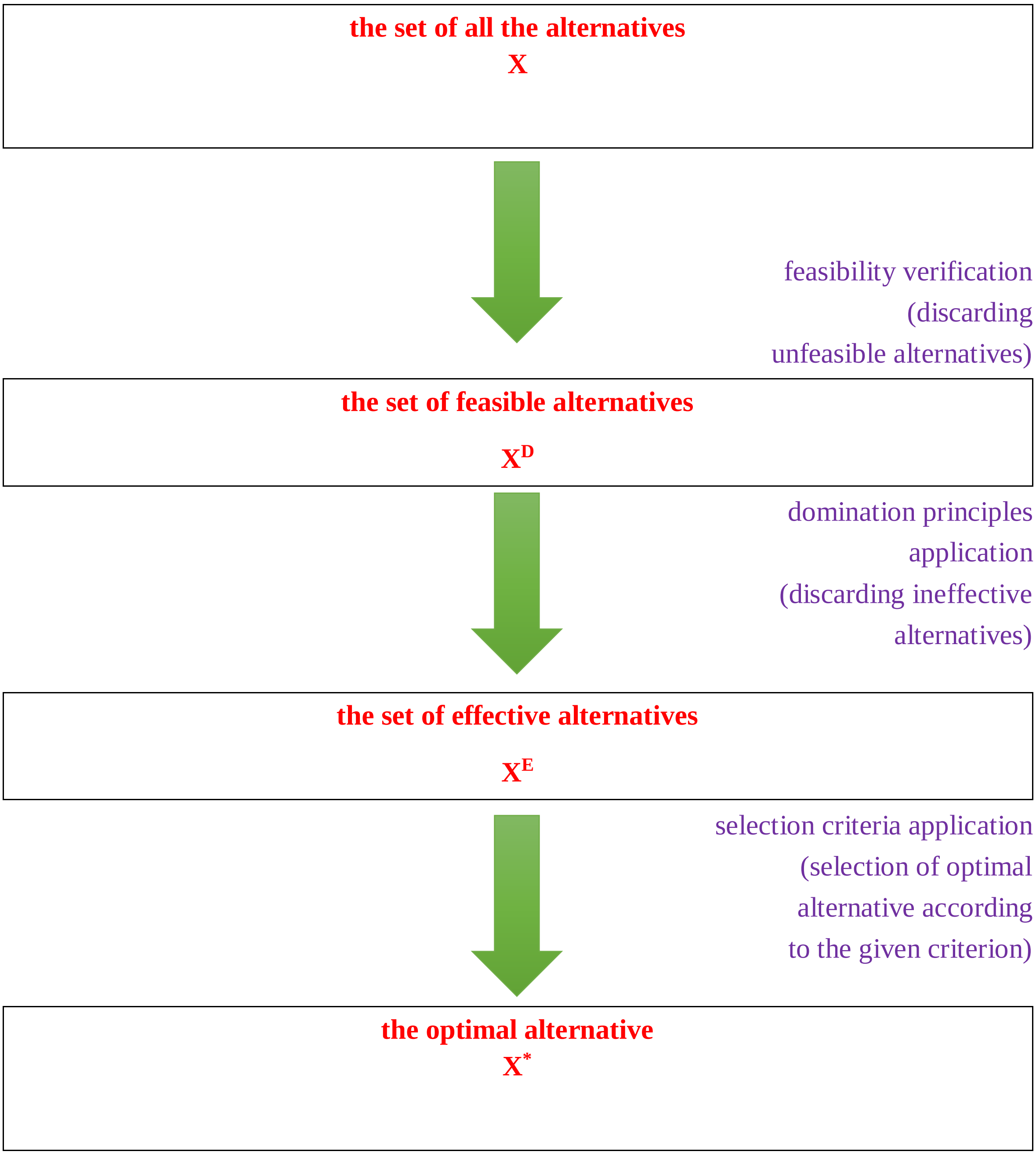


Fig. 3.1. The decision-making process by gradually narrowing down the set of alternatives.

On the first of all the original X alternatives, a set of feasible X^D is selected. Those that do not comply with the established restrictions are discarded. In the initial conditions of problems, as a rule, only feasible alternatives are already set, therefore, this stage is skipped. In practice, the decision maker has to independently verify the possibility of implementing one or another alternative and reject fundamentally impossible or unacceptable restrictions.

In the second stage, the so-called ineffective alternatives that are worse than all the others are discarded. Among them, there cannot be the best one. The remaining set are effective X^E alternatives, about which it is not yet possible to say that they are the best, but it is also impossible to find those that are clearly better than others are. To select such strategies, domination principles are applied.

The search for the optimal X* alternative is carried out at the third stage among the previously selected effective strategies. This is usually done using the so-called criteria.

As a result, it turns out that the optimal X* alternative refers to the set of effective X^E alternatives, which is a subset of the feasible X^D alternatives, which, in its turn, is a subset of all the possible X alternatives:

$$X^* \in X^E \subseteq X^D \subseteq X$$

Owing to the sequential verification of limitations, the application of the domination principles and criteria, it is possible to reduce the whole variety of options for action to one best strategy under the given conditions for the given decision maker [5].

3.3 Criteria weights determination methods

One can say that criteria weights are the weakest point in the criterion analysis problem. Weighting factors should qualitatively reflect the importance of relevant particular criteria. The li quantity determines the importance of $i-$ criterion of optimality and sets in quantitative measurement the preference of this criterion over other criteria of optimality. li weights factors should satisfy the condition of $\sum_{i=1}^{m} \lambda_i = 1$ i.e. the sum of all the criteria weights should be equal to 1.

Most often, weights are assigned based on an intuitive understanding of the relative importance of the criteria. However, studies show that a person (an expert) is not able to directly assign criteria to the correct numerical weights. Special procedures for obtaining weights are needed.

Let us consider some methods for calculating the weighting coefficients of the criteria.

3.3.1 Sequence Method

The sequence method is based on the fact that it is necessary to arrange particular criteria in order of their importance. Numeral 1 denotes the most important particular criterion; numeral 2 denotes the next most important particular criterion, etc. This sequence is transformed in such a way that rank 1 receives m (the number of particular criteria), rank 2 receives m-1, and so on till to rank m, which is assigned 1 [6].

Weighting factors are determined by the formula:

$$\lambda_i = \frac{r_i}{\sum_{i=1}^{m} r_i} \quad - (i=1,2,\ldots,m), \tag{3.1}$$

r_i is the value of the rank (or sum of ranks), i - is that of the criterion.

3.3.2 Scoring Method

This method is based on the fact that experts assess the importance of a particular criterion on a scale of [0-10]. It is allowed evaluating the importance by fractional values or attributing the same value from a selected scale by several criteria. The method uses a process similar to the sequence method to calculate criteria weights. [7]

Calculation Procedure:

1) building a matrix of estimates.
2) finding the sum of each row values.
3) calculating the weights for r_{ik}.
4) building the matrix of weights.
5) finding the sum of each ri column.
6) calculating the weighting coefficients according to the formula:

$$\lambda_i = \frac{r_i}{\sum_{i=1}^{m} r_i} \tag{3.2}$$

3.3.3 Saaty method

This method is applied when only one decision maker evaluates the problem. Saaty method is one of the most used methods of calculation of criterion weights. This method determines the inconsistency of the pairwise comparison matrix. Saaty method can be divided into two steps. The first step is determination of preferences between each pair of criteria and the second step is determination of criterion weights. The advantage of Saaty method is that decision maker can express their preferences verbally rather than numerically. [8] [9]

Usually for evaluation are used a nine-point scale scheme with values 1, 3, 5, 7, 9, but it is also possible to use intermediate values 2, 4, 6, 8. Even values are used to more accurately determine preferences.

1 – equal importance

3 – moderate importance

5 – strong importance

7 – very strong importance

9 – extreme importance

2, 4, 6, 8 – intermediate values

Decision maker compares all the pairs of criteria and writes preferences into the Saaty matrix C = (c_{ij}).

$$C = \begin{pmatrix} 1 & C_{12} & \cdots & C_{1n} \\ 1/C_{12} & 1 & \cdots & C_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ 1/C_{1n} & 1/C_{2n} & \cdots & 1 \end{pmatrix} \tag{3.3}$$

Elements of matrix c_{ij} are represented by preference value of i-th criterion against j-th criterion. The Saaty matrix is always a square matrix n x n. If the value of i-row and jcolumn is equal, then this preference is written as c_{ij} = 1. Otherwise, if j-th criterion is more preferable than i-th criterion, then the preference value is equal to the inverted value.

There are several methods of determination of the weights; most frequently used weighted geometric average of the Saaty matrix.

$$R_i = \sqrt[n]{\prod_{j=1}^{n} C_{ij}} \tag{3.4}$$

After normalizing averages, weights calculated by normalizing the R_i value.

$$w_i = \frac{R_i}{\sum_i^n R_i} \tag{3.5}$$

When filling out matrices of pairwise comparisons, a person can make mistakes. One of the possible mistakes is the violation of transitivity: from $c_{ij} > c_{jk}$, $c_{jk} > c_{is}$ may not follow $c_{ij} > c_{is}$. Secondly, there may be violations of the consistency of numerical judgments.

It is known that the consistency of a positive inverse symmetric matrix is equivalent to the requirement that its maximum eigenvalue λmax be equal to n. Note that the inequality λmax ≥ n is always true. The closer λmax is to n (the number of objects or types of actions in the matrix), the more consistent is the result. λmax is calculated by the formula:

$$\lambda_{max}= \sum_{i=1}^{n} \lambda_i, \quad \text{where} \qquad \lambda_i=\sum_{i=1}^{n} C_{ij} W_{ij} \tag{3.6}$$

One can also estimate the deviation from consistency by the difference λmax - n, divided by (n −1), this value will be called the consistency index (CI).

$$I_s = \frac{\lambda_{max}-n}{n-1} \tag{3.7}$$

The consistency index of the inversely symmetric matrix generated randomly on a scale from 1 to 9 with the corresponding inverse values of the elements is called the random index (RI).

Below are the matrix order (the first line) and the average RI (the second line):

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0,00	0,00	0,58	0,90	1,12	1,24	1,32	1,41	1,45	1,49	1,51	1,48	1,56	1,57	1,59

How bad the consistency for a particular problem is can be estimated by comparing the value of the quantity (CI) we obtained with its value from randomly selected judgments and the corresponding inverse values of a matrix of the same size (SI).

The ratio of the CI to the average SI for a matrix of the same order is called the consistency relation (CR). The CR value, which is less than or equal to 0.10 is considered acceptable. [9]

3.4 Methods for alternatives evaluation

The scientific discipline of decision-making under conditions of multi-criteria is relatively young: the first works in the framework of this discipline appeared in the 1970s, and references to the application of methods to solve practical problems - in the 1980s. [10] Despite this, more than seventy different methods have already been developed for solving multi-criteria problems. [11]

Here are the most popular ones:

• 1. Analytic Hierarchy Process (AHP)

• 2. Non-linear programming (NLP)

• 3. Compromise programming (CP)

• 4. Cooperative games theory (CGT)

• 5. Displaced ideal method (DISID)

• 6. ELECTRE method (ELEC)

• 7. Evaluation and sensitivity analysis program (ESAP)

• 8. Goal programming (CPU / GP)

• 9. Multi-attribute utility theory (MAUT)

• 10. Technique for Order Preference by Similarity to the Ideal Solution (TOPSIS)

• 11. Simple Additive Weighting (SAW)

• 10. Multi-criteria Q-Analysis (MCQA)

• 11. The probabilistic method of compromise development (PROTR)

• 12. The Zayonts-Wallenius method (Z-W)

• 13. STEM method

• 14. SWT method

• 15. PROMETHEE method (PRM)

A detailed review of all the existing methods does not seem necessary and possible in the framework of this paper, so we will consider in detail only three methods:

- Simple Additive Weighting (SAW);
- Technique for Order Preference by Similarity to the Ideal Solution (TOPSIS)
- Analytic Hierarchy Process (AHP).

Each of the methods reflects different approaches to solving the problem of selecting the best option from pre-selected alternatives. All three methods require the preliminary selection of a number of alternative options and the use of performance criteria that are different from each other and can be quantified. After various cross-comparisons of various options, taking into account this set of performance criteria, a balance of advantages and disadvantages, which is used to score alternatives, is derived. [12]

The SAW-method is considered the most simple and convenient. It is often used to obtain initial data for the comparative evaluation.

The TOPSIS-method offers a special, logical and structured approach to solving the problem. However, it is much more difficult to use.

The AHP method implies some freedom of choice when defining individual criteria in accordance with its own specific measurement scale.

3.4.1 SAW-method

The SAW-method is to quantitatively measure the significance of the criteria for each alternative, building on this basis a decision matrix A, from which a normalized decision matrix R is obtained, which determines the significance (weight) of each criterion, from which, in its turn, a generalized evaluation of each alternative is derived. Then, the option with the highest evaluation is selected to be the best.

The SAW-method calculation algorithm is as follows:

1. Let $C = \{c_1, c_2, ..., c_m\}$ be the set of criteria to be evaluated, $A = \{a_1, a_2, ..., a_m\}$ – the set of potential suppliers. The matrix X is built, where x_{ij} is the value of the c_i criterion for the a_j supplier:

$$X = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{pmatrix}$$

2. Searching for normalized values of the matrix of criteria estimates. To normalize the matrix of criteria estimates, we find the best x_{ij} values of the original matrix of $X = \{ x_{ij} \}$ criteria values, where the value of the c_i criterion of the $C = \{ c_i \}$ set for the a_j supplier of the $A = \{ a_j \}$ set is according to the following formulas:

$$p_{ij} = \frac{x_{ij} - x_j^{min}}{x_j^{max} - x_j^{min}}, \text{ if the criteria are maximized;} \tag{3.8}$$

$$p_{ij} = \frac{x_j^{max} - x_{ij}}{x_j^{max} - x_j^{min}}, \text{ if the criteria are minimized.} \tag{3.9}$$

The matrix of normalized criteria values takes the form of:

$$P = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ p_{m1} & p_{m2} & \cdots & p_{mn} \end{pmatrix}, \text{ i = 1,m ; j = 1,n} \tag{3.10}$$

3. For a more objective result, weight coefficients $w_i \in [0,1]$ are introduced for each criterion. These coefficients allow an assessment taking into account the priority and weight of the criteria. The sum of the coefficients of the specific gravity of all the criteria is 1:

$$\sum_{i=1}^{m} w_i = 1 \tag{3.11}$$

As a result, the matrix $\widetilde{P} = (\widetilde{P}_{ij})$ is obtained, the elements of which are found by the formula:

$$\widetilde{p} = w_i p_{ij} \tag{3.12}$$

Thus, the scoring calculation in general can be represented as follows:

$$r(a_j) = \sum_{i=1}^{m} \widetilde{p_{ij}}\ , i = \overline{1, m}\ , j = \overline{1, n} \qquad (3.13)$$

where x_{ij} is the normalized criterion evaluation;

w_i is the specific gravity of the criterion;

i is the number of the criterion,

j is the supplier's number.

The vector of scoring evaluations functions for a_j suppliers takes the form of:

$$R = \{r(a_1), r(a_2), \ldots, r(a_n)\} \qquad (3.14)$$

The optimal solution is determined by the highest value of the suppliers' scoring:

$$r^* = \max_{a_j \in A} r(a_j) \qquad (3.15)$$

3.4.2 TOPSIS-method

TOPSIS is a technology developed by Hwang and Yoon in 1981. This method is used to solve multi-criteria problems. The essence of the method is to search for alternatives whose evaluation values are the closest to the ideally positive solution and are the most distant from the ideally negative solution. An ideally positive solution is a vector of maximum values of the matrix of weighted alternatives estimates. An ideally negative solution, on the contrary, is a vector of minimum values. [13]

TOPSIS method algorithm:

1. Let $C=\{c_i\}$ be the set of criteria to be evaluated, $A=\{a_j\}$ - the set of potential suppliers on the basis of which the matrix of values of $X = (x_{ij})$ criteria is built.

To obtain a matrix of normalized values of $P = (p_{ij})$ criteria, the criteria are transferred into the dimensionless form by the formula:

$$p_{ij} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{n}(x_{ij}^2)}} \qquad (3.16)$$

2. Then, a matrix of weighted criteria values is constructed, where the weight coefficients are $w_i \in [0,1]$. The matrix of the normalized weighted values can be represented as:

$$\widetilde{P} = (w_i p_{ij}) = (\widetilde{p_{ij}}) \qquad (3.17)$$

3. The next stage is ideally positive and ideally negative solutions.

$$A^{+} = (max\ (\widetilde{p_{11}}), \ldots, max\ (\widetilde{p_{1n}})) = (\widetilde{p}_1^{+}, \ldots, \widetilde{p}_n^{+}) \quad (3.18)$$

$$A^{-} = (min\ (\widetilde{p_{11}}), \ldots, min\ (\widetilde{p_{1n}}) = (\widetilde{p}_1^{-}, \ldots, \widetilde{p}_n^{-}) \quad (3.19)$$

Then the distances from alternatives to ideally positive and ideally negative solutions are determined.

$$S_j^{+} = \sqrt{\sum_{j=1}^{n} (\widetilde{p}_1^{+} - \widetilde{p}_{ij})^2}\ , j = 1, \overline{\ }n \quad (3.20)$$

$$S_j^{-} = \sqrt{\sum_{j=1}^{n} (\widetilde{p}_1^{-} - \widetilde{p}_{ij})^2}\ , j = 1, \overline{\ }n \quad (3.21)$$

4. The final step will be to find a relative proximity to the ideally positive solution by the formula:

$$P_j^{+} = \frac{S_j^{-}}{S_j^{+} + S_j^{-}} \quad (3.22)$$

An alternative is chosen for which the value of relative proximity will be closer to 1.

3.4.3 AHP-method

AHP-method consists of three stages: decomposition of the problem into components; comparative assessment; synthesis of priority options [9; 14]. The decomposition of the problem is necessary to draw up a hierarchical scheme, in which the overall goal is correlated with alternative options for making decisions. A comparative assessment includes the construction of pairwise matrices and their comparison at two levels:

1) at a level where all the alternative options are compared in relation to each criterion;

2) at a level where the criteria relate to the final goal of the study.

At the level of criteria, a pairwise comparison of the criteria of $C=\{c_i\}$ set relative to each other is carried out. At the level of alternatives, the values of each individual c_i criterion are compared in pairs in relation to the $A=\{a_j\}$ providers. To establish the criteria values, the relative importance scale presented in Table 1 is used.

Table 1 - The scale of relative importance

Relative importance intensity	Definition
1	Equal importance
3	Moderate importance
5	Strong importance
7	Very strong importance
9	Extreme importance
2,4,6,8	Intermediate values

Analytic Hierarchy Process algorithm:

1. At the first stage, the set of pairwise compared elements is represented as a square matrix of n × n size:

$$U = \begin{pmatrix} u_{11} & u_{12} & \cdots & u_{1n} \\ u_{21} & u_{22} & \cdots & u_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ u_{n1} & u_{n2} & \cdots & u_{nn} \end{pmatrix} \qquad (3.23)$$

For any i and j, the relation, $u_{ij} = \frac{1}{u_{ij}}$, $u_{ij} \neq 0$ holds. The diagonal elements of the matrix are equal to 1. Thus, the matrix U can be reduced to

$$U = \begin{pmatrix} 1 & u_{12} & \cdots & u_{1n} \\ 1/u_{12} & 1 & \cdots & u_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ 1/u_{1n} & 1/u_{2n} & \cdots & 1 \end{pmatrix} \qquad (3.24)$$

2. The elements of the u_{ij} matrix are expressed as the ratios of the numerical weights w_1, w_2, ..., w_n in the following manner:

$u_{ij} = \frac{w_i}{w_j}$, $i,j = \overline{1,n}$

$$X = \begin{pmatrix} 1 & w_1/w_2 & \cdots & w_1/w_n \\ w_2/w_1 & 1 & \cdots & w_2/w_n \\ \cdots & \cdots & \cdots & \cdots \\ w_n/w_1 & w_n/w_2 & \cdots & 1 \end{pmatrix} \qquad (3.25)$$

If w_1, w_2, ..., w_n are not known in advance, then pairwise comparisons of elements are made using subjective judgments, numerically evaluated on a scale, and then the problem of finding the w component is solved.

3. To aggregate the opinions of experts, the geometric mean is calculated according to the following formula:

$$g_i = \sqrt[n]{\prod_{j=1}^{n} u_{ij}} \qquad (3.26)$$

where n is the number of elements considered. The components of the normalized priority vector are calculated by the formula:

$$y_i = \frac{g_i}{\sum_i^n g_i} \qquad (3.27)$$

4. Then, the generalized priorities for each supplier are found by the formula:

$$v(a_i) = \sum_{i=1}^{m} w_i^c \cdot \; y_{ij}^a \qquad (3.28)$$

where w_i^c are the components of the normalized priorities vector of the 1st level (criteria priorities values),

y_{ij}^a are the components of the normalized priorities vector of the 2nd level (the values of the supplier's priorities in relation to criteria).

5. Next, sequence and choosing the best solution (the highest numerical value of generalized priorities) is performed.

4. Practical Part

This part is devoted to the practical application of theory.

To write the paper, data were taken from the mining company of the city of Karaganda (Republic of Kazakhstan).

For calculations, the methods described in the theoretical part of the diploma thesis are used.

4.1 Company profile

The mining company “GDK PromTehnologiya” is engaged in the extraction of coal. For this, the development of opencast coal mines is carried out. In the paper, mining transport equipment (MTE) is used:

- excavators for overburden and mining operations;
- bulldozers for the formation of waste dumps and internal quarry roads;
- dump trucks for transportation of rock mass.

4.2 Company problem

In connection with the signing of a new contract for the supply of coal in the II quarter, the company needs to additionally produce 50,000 tons of coal.

The company’s goal and the expected result in this case is to obtain additional profit.

The problem is that the company does not have enough mining transport equipment to carry out these works on time (2 more excavators, 2 more bulldozers, 10 more heavy dump trucks are needed)

It is necessary to involve other organizations’ additional MTE in the work.

To solve this problem, the company announces a tender for outsourcing among enterprises that can perform these works.

The task of the company is to choose the best alternative, i.e. one that will allow getting the best result in achieving the goal set.

4.3 Action algorithm

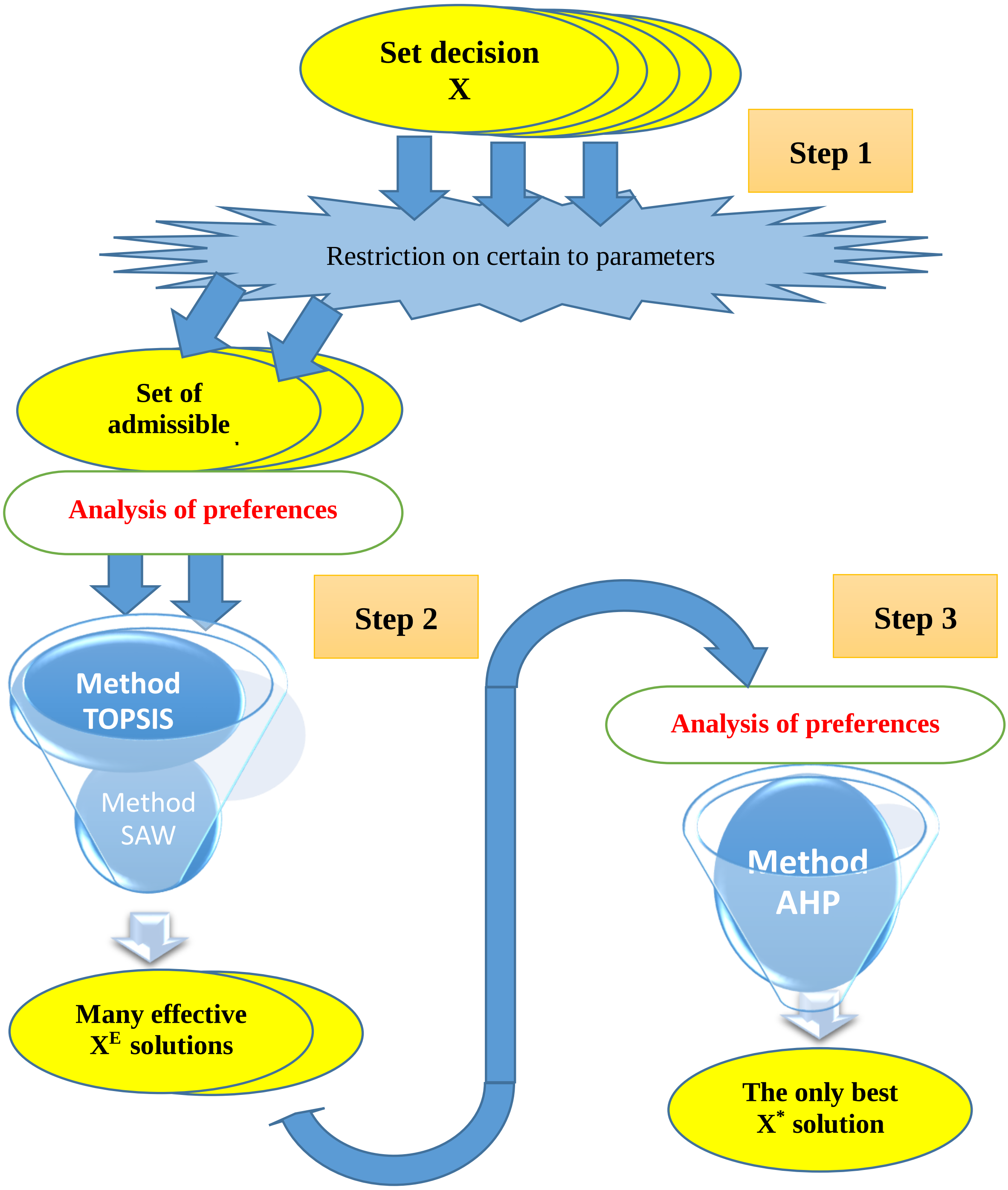
To select the most suitable service provider economically and technically, the company decided **to conduct a multi-criteria analysis** for further decision-making and conclusion of an appropriate contract.

To select the best alternative, an algorithm has been developed for conducting a multi-criteria analysis using the principle of sequential reduction of uncertainty, which consists in the sequential narrowing of many solutions.

A complete algorithm of actions using various methods in the analysis of preferences is presented in Picture 4.1

Picture 4.1

Algorithm
Multi-criteria analysis in service provider selection using the principle of consistent uncertainty reduction



4.3.1 Justification of the use of SAW, TOPSIS and AHP methods in multi-criteria analysis

In the modern methodology of mathematical modeling, to solve the problem in the analysis of preferences (step 2 and step 3), one can use three methods of multi-criteria decision making (MCDM): the Simple Additive Weighting method (SAW); the Technique for Order Preference by Similarity to the Ideal Solution (TOPSIS) and the Analytic Hierarchy Process (AHP) method. Each of the methods reflects different approaches to solving the problem of determining the service provider by choosing the best option from pre-selected alternatives. All three methods require the preliminary selection of a number of alternative options and the use of performance criteria that are different from each other and can be quantified. For example, the criteria for making a decision may be costs, reliability, staff qualifications. After various cross-comparisons of various options, taking into account this set of performance criteria, a balance of advantages and disadvantages, which is used to rank alternatives, is derived [12].

4.3.1.1 Methods SAW, TOPSIS

The use of two methods at once in the analysis at the second step of the algorithm is due to the desire to achieve the most accurate result in the selection of alternatives. The SAW method is considered the most simple and convenient. The TOPSIS method offers a special, logical and structured approach to solving the problem. It is much more difficult to use, but it makes it possible to comprehensively evaluate all alternative solutions to determine the level of readiness of the supplier by its degree of proximity to the ideal state.

However, both of these methods belong to a group of methods where there is a risk of a compensatory effect, i.e. low ratings of an object on one part of the criteria can be compensated by high ratings on the other part of the criteria.

Understanding this, we can conclude that as a result of applying these methods, an alternative can be identified that has very high, dominant ratings in one or two criteria, while the remaining indicators will be low. For example, a potential supplier may set a low price, which will give the alternative very high ratings in this part of the criteria, and the remaining indicators related to technical equipment and reliability of equipment will be weak. And if you choose this alternative, it will be very unprofitable for the company due to possible breakdowns, downtime and as a result of disruptions in the execution of work.

However, these two methods combined are still sufficient at this stage of selection to select several best alternatives from among the set of acceptable solutions, which will be considered at the next step.

4.3.1.2 Method AHP

At the last step of the algorithm, to select the best alternative from the remaining ones, we apply the AHP hierarchy analysis method.

Traditionally, the solution of the inhomogeneous production and economic problems we use analytic hierarchy process of Thomas Saaty. AHP allows you to take into account financial and non-financial quantitative and qualitative indicators, as well as to study the relationship between them. This method has also proved successful in comparing the performance of various economic entities using multi-factor performance criteria. The main principle laid down in the AHP is to compare a number of variables that are selected as determining factors for solving the task, with the significance of each variable. The undoubted advantage of AHP is that it can include more than one indicator and integrate all financial and non-financial indicators into one overall performance indicator.

In the process of applying the method, the consistency of the LPR's judgments is checked.

The advantage of the AHP method, which attracts the attention of many users, is the focus on comparing real alternatives.

As many note, the disadvantage of the method is the complexity of the procedure with a large number of alternatives considered. In the case of a small number of specified alternatives, it seems reasonable to direct the efforts of the LPR to compare these alternatives. This is the idea behind the AHP method.

So, the arguments for using the ANR method at this stage:

1) The method differs from the previous ones in the method of calculation and is not subject to the risk of a compensatory effect.

2) The method allows you to evaluate all multi- factor indicators and integrate them into a single overall indicator of the effectiveness of the alternative.

3) The small number of alternatives left for analysis at this stage.

4) The consistency of the LPR's judgments is checked.

Thus, the AHP method, given the overall goal of the task of determining the best service provider, with the specified multi-factor criteria, will allow you to most accurately choose the best alternative in terms of technical equipment, reliability and the offered price.

4.4 Step 1. Narrowing down the set of alternative solutions to the set of feasible solutions

4.4.1 Restrictions on certain parameters for service providers

To participate in the tender, the company identified the following restrictions on certain parameters:

- the availability of the necessary minimum equipment (2 excavators, 2 bulldozers, 10 heavy dump trucks);
- the price for the services rendered should not exceed 70 million tenge;
- the bucket capacity of each excavator should be at least 1.5 cubic meters.;
- mandatory availability of a repair base;
- presence of a qualified personnel;
- provision of all the reliable information at the request of the customer.

In accordance with these restrictions, 7 enterprises were selected from a variety of potential suppliers (Alternatives).

4.4.2 Alternatives (Options)

7 enterprises that have been selected by restrictions and will be Alternatives for making further decisions:

- Alternative (A1) – “Soyuz” LLP;
- Alternative (A2) – “Limma” LLP;
- Alternative (A3) – “Eksmash-20” LLP;
- Alternative (A4) – “Kazstal-ns” LLP;
- Alternative (A5) – “Sapsan group” LLP;
- Alternative (A6) – “Adal DorStroy” LLP;
- Alternative (A7) – “Astana Trans-2018” LLP

4.5 Step 2. Narrowing down the set of feasible solutions to the set of effective solutions

At this stage of multi-criteria analysis, we narrow down the set of feasible solutions to the set of effective solutions. For this purpose we use the TOPSIS-method and the SAW-method.

4.5.1 Criteria

To evaluate the selected alternatives, the company has defined such criteria as:

1) Price for services rendered (mln.tenge)

2) Depreciation of equipment (%)

3) The total volume of the bucket of two excavators (cubic meters)

4) Availability of additional replacement equipment in case of repair (%)

5) The remoteness of the repair base from the object of work (km)

6) Availability of the necessary stock of spare parts in own warehouse (%)

7) Staff qualification

8) The amount of equipment for working in two shifts (units)

9) The duration of the enterprise on the market for these services (months)

10) The reputation of the company in the industry

Based on the preferences of the company, it is clear that the bulk of the criteria (No. 2, 3, 4, 5, 6, 8) are focused on the technical readiness of the alternatives under consideration, so that the work would be carried out without downtime, in 2 shifts and be completed on time. In addition, the company wants to incur lower cash costs, so the criterion of price (No. 1) is minimized. Also, in order to reduce the risks of possible breakdowns, the wear of equipment should be minimal, and in case of failure of the equipment, the repair base should be as close as possible to the place of work, therefore criteria No. 2 and No. 5 are also minimized. According to other criteria, the company gives preference to the highest indicators, therefore they are maximized.

According to the terms of consideration of applications for the provision of services, all enterprises participating in the competition of suppliers provided the company with the necessary information and their price offers.

For the diploma project, the data on all the considered decision options upon written request were provided by the manager of the company “GDK PromTehnologiya” LLP and generated in Table 2.

Table 2 - List of suggested variants

	C_1 (mln.)	C_2 (%)	C_3 ($м^3$)	C_4 (%)	C_5 (км)	C_6 (%)	C_7	C_8 (units)	C_9 (months)	C_{10}
TOO «Soyuz»	66	7	4,06	75	20	100	Excellent	14	64	Excellent
TOO «Limma»	68	12	4,4	100	33	100	Very good	14	10	Very good
TOO «Eksmash-20»	65	10	3,8	100	15	80	Good	13	54	Good
TOO «Kazstal-ns»	62	15	3,36	50	8	95	Very good	12	46	Very good
TOO «Sapsan group»	68	5	3,72	75	40	75	Good	13	15	Good
TOO «Adal DorStroy»	65	12	3,36	75	26	90	Good	12	21	Very good
TOO «Astana Trans-2018»	70	10	4,26	100	31	100	Very good	14	13	Excellent

(Source: own processing)

4.5.2 Determination of the criteria weights (Saaty method)

To determine the weights of the criteria, we use the Saaty`s method, which was described in the theoretical part (Chapter 3.3.3).

This part of the decision-making process is the most important because the process is based on a multi-criteria assessment.

This method has been chosen because it is simple and allows studying a larger number of objects (compared, for example, with the scoring method) and with greater accuracy.

The main advantages of the method are as follows:

- it is allowed measuring the unevenly changing importance of indicators, which is so necessary for solving the majority of practical economic problems;

- an expert in the analysis process focuses not on all indicators at once, but only on two that are compared at any given moment, which facilitates the work and, therefore, helps to improve its quality;

- it is possible to get a large number of comparisons of each indicator with others, which increases the accuracy of the assessment and opens the possibility to study the quality of a larger number of sides of the object of study than using other methods.

The disadvantage of this method is the increase in the complexity of the procedure with an increase in the number of objects, the need to perform a huge number of pairwise comparisons, if one has to evaluate large groups, already at 12-15 objects the procedure becomes time-consuming.

After a pairwise comparison of the significance of the criteria, a matrix was compiled and weights were calculated. (Table 3)

Table 3 - Saaty matrix

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	R_i	W_i
C_1	1	1/2	2	3	7	5	5	1/3	8	8	2,5403	0,1643
C_2	2	1	3	3	5	4	4	1/2	8	8	2,9265	0,1892
C_3	1/2	1/3	1	2	6	5	5	1/3	7	7	1,9549	0,1264
C_4	1/3	1/3	1/2	1	5	4	3	1/5	7	7	1,4168	0,0916
C_5	1/7	1/5	1/6	1/5	1	1	1/3	1/7	3	3	0,4583	0,0296
C_6	1/5	1/4	1/5	1/4	1	1	1/2	1/8	5	4	0,5617	0,0363
C_7	1/5	1/4	1/5	1/3	3	2	1	1/6	6	5	0,7944	0,0514
C_8	3	2	3	5	7	8	6	1	9	9	4,3542	0,2816
C_9	1/8	1/8	1/7	1/7	1/3	1/5	1/6	1/9	1	1	0,2227	0,0144
C_{10}	1/8	1/8	1/7	1/7	1/3	1/4	1/5	1/9	1	1	0,2344	0,0152
$\sum$	**7,626**	**5,116**	**10,353**	**15,069**	**35,666**	**30,45**	**25,2**	**3,023**	**55**	**53**	**15,4642**	**1,0**
λ_i	**1,253**	**0,968**	**1,309**	**1,38**	**1,056**	**1,105**	**1,295**	**0,851**	**0,792**	**0,806**		

(Source: own processing)

Now let's prove the consistency of the matrix.

We find λmax by the formula (3.6) λmax = 10.815, then we find the consistency index by the formula (3.7)

$$I_s = \frac{10,815-10}{10-1} = 0,091$$

The random index for the tenth order matrix is 1.49, we find the consistency relation:

OS = 0,091 / 1,49 = 0,061 ≤ 0,10, therefore, the matrix is consistent.

4.5.3 Conducting a preference analysis using the TOPSIS-method

Given the variety of criteria, in this case, the assessment of service providers is based on the TOPSIS (The Technique for Order of Preference by Similarity to Ideal Solution) method (the method of ordered preference through similarities with the ideal solution). This comprehensive assessment method, based on distance calculation, is widely used in world practice for decision-making. In economic studies, the method is used to solve the problems of assessing the competitive advantages of a business and personnel management. For further calculations, we use the criteria for weights obtained by the Saaty method (Table 3).

Also, translate qualitative estimates into quantitative ones using the following scale:

Qualitative assessment	**Values on the scale**
Excellent	9
Very good	7
Good	5
Normally	3
Bad	1

Table 4- Input table for the Saaty`s method

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
A_1	66	7	4,06	75	20	100	9	14	64	9
A_2	68	12	4,4	100	33	100	7	14	10	7
A_3	65	10	3,8	100	15	80	5	13	54	5
A_4	62	15	3,36	50	8	95	7	12	46	7
A_5	68	5	3,72	75	40	75	5	13	15	5
A_6	65	12	3,36	75	26	90	5	12	21	7
A_7	70	10	4,26	100	31	100	7	14	13	9
Criteria weights	min	min	max	max	min	max	max	max	max	max
Criteria character	0,1643	0,1892	0,1264	0,0916	0,0296	0,0363	0,0514	0,2816	0,0144	0,0152

(Source: own processing)

We proceed to the construction of a normalized matrix of solutions.

In this step, all characteristics are reduced to dimensionless values. In calculations, we use the formula (3.16).

Table 5 - Normalized weighted decision matrix R

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
A_1	0,3761	0,2495	0,4197	0,3586	0,2824	0,4112	0,5170	0,4018	0,6380	0,4750
A_2	0,3875	0,4277	0,4549	0,4781	0,4660	0,4112	0,4021	0,4018	0,0997	0,3694
A_3	0,3704	0,3564	0,3928	0,4781	0,2118	0,3289	0,2872	0,3731	0,5383	0,2639
A_4	0,3533	0,5347	0,3474	0,2391	0,1130	0,3906	0,4021	0,3444	0,4586	0,3694
A_5	0,3875	0,1782	0,3846	0,3586	0,5648	0,3084	0,2872	0,3731	0,1495	0,2639
A_6	0,3704	0,4277	0,3474	0,3586	0,3671	0,3701	0,2872	0,3444	0,2093	0,3694
A_7	0,3989	0,3564	0,4404	0,4781	0,4377	0,4112	0,4021	0,4018	0,1296	0,4750

(Source: own processing)

Proceed to build a weighted normalized solution matrix W.

Table 6 – Normalized weighted decision matrix W

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
A_1	0,0618	0,0472	0,0538	0,0328	0,0084	0,0149	0,0266	0,1132	0,0092	0,0072
A_2	0,0637	0,0809	0,0575	0,0438	0,0138	0,0149	0,0207	0,1132	0,0014	0,0056
A_3	0,0609	0,0674	0,0496	0,0438	0,0063	0,0119	0,0148	0,1051	0,0077	0,0040
A_4	0,0581	0,1012	0,0439	0,0219	0,0033	0,0142	0,0207	0,0970	0,0066	0,0056
A_5	0,0637	0,0337	0,0486	0,0328	0,0167	0,0112	0,0148	0,1051	0,0021	0,0040
A_6	0,0609	0,0809	0,0439	0,0328	0,0109	0,0134	0,0148	0,0970	0,0030	0,0056
A_7	0,0655	0,0674	0,0557	0,0438	0,0130	0,0149	0,0207	0,1132	0,0019	0,0072

(Source: own processing)

The next step will be to determine a positive and negative ideal solution.

Table 7 - Positive and negative ideal solution.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
D	0,0581	0,0337	0,0575	0,0438	0,0033	0,0149	0,0266	0,1132	0,0092	0,0072
H	0,0655	0,1012	0,0439	0,0219	0,0167	0,0112	0,0148	0,0970	0,0014	0,0040

(Source: own processing)

Next, we evaluate the distances to ideal alternatives (d^+, d^-) and calculate the relative proximity to ideal solution (c_i).

Perform all calculations using formulas (3.20, 3.21, 3.22)

Table 8 - Order of alternatives (TOPSIS)

	d^+	d^-	c_i	rank
A_1	0,018885	0,060843	0,7631	1
A_2	0,049675	0,037441	0,4298	5
A_3	0,037964	0,043470	0,5338	4
A_4	0,074349	0,017542	0,1909	7
A_5	0,026274	0,069040	0,7243	2
A_6	0,055175	0,024406	0,3067	6
A_7	0,037093	0,045787	0,5525	3

(Source: own processing)

4.5.4 Conducting a preference analysis using the SAW-method

SAW-method is the *Simple Additive Weighting Method* (SAW). Using this method, the decision maker (DM) can get a general score for each alternative by multiplying the value on the scoring scale for each criterion value by the weight assigned to the criterion, and then summing up these values for all criteria. Thus, the decision maker gets the alternative with the highest score (the highest average weight), which is the answer to the decision-making problem.

For further calculations use the scale criteria obtained by Saaty´s method (Table 3).

Use matrix (Table 4) to apply the weighted sum method.

Table 4 – Matrix for applying a method

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
A_1	66	7	4,06	75	20	100	9	14	64	9
A_2	68	12	4,4	100	33	100	7	14	10	7
A_3	65	10	3,8	100	15	80	5	13	54	5
A_4	62	15	3,36	50	8	95	7	12	46	7
A_5	68	5	3,72	75	40	75	5	13	15	5
A_6	65	12	3,36	75	26	90	5	12	21	7
A_7	70	10	4,26	100	31	100	7	14	13	9
Criteria weights	min	min	max	max	min	max	max	max	max	max
Criteria character	0,1643	0,1892	0,1264	0,0916	0,0296	0,0363	0,0514	0,2816	0,0144	0,0152

(Source: own processing)

The next step will be to find standardized assessments of the criteria. For this purpose, the maximum and minimum estimates of the criteria are determined.

Table 9- Ideal and negative ideal alternatives

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
max	70	15	4,4	100	40	100	9	14	64	9
min	62	5	3,36	50	8	75	5	12	10	5
$X_j^{max} - X_j^{min}$	8	10	1,04	50	32	25	4	2	54	4

(Source: own sources)

The criteria in our case are cost or benefit, so the formula (3.8, 3.9) is used to normalize the estimates.

Table 10 –Normalized matrix

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
A_1	0,5	0,8	0,673	0,5	0,625	1	1	1	1	1
A_2	0,25	0,3	1	1	0,219	1	0,5	1	0	0,5
A_3	0,625	0,5	0,423	1	0,781	0,2	0	0,5	0,815	0
A_4	1	0	0	0	1	0,8	0,5	0	0,666	0,5
A_5	0,25	1	0,346	0,5	0	0	0	0,5	0,092	0
A_6	0,625	0,3	0	0,5	0,438	0,6	0	0	0,204	0,5
A_7	0	0,5	0,865	1	0,281	1	0,5	1	0,055	1

(Source: own processing)

The normalized criteria values are then multiplied by the weight factors

Table 11- Normalized matrix with weight factors

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
A_1	0,0822	0,1514	0,0851	0,0458	0,0185	0,0363	0,0514	0,2816	0,0144	0,0152
A_2	0,0411	0,0568	0,1264	0,0916	0,0065	0,0363	0,0257	0,2816	0	0,0076
A_3	0,1027	0,0946	0,0535	0,0916	0,0231	0,0073	0	0,1408	0,0117	0
A_4	0,1643	0	0	0	0,0296	0,0290	0,0257	0	0,0096	0,0076
A_5	0,0411	0,1892	0,0437	0,0458	0	0	0	0,1408	0,0013	0
A_6	0,1027	0,0568	0	0,0458	0,0130	0,0218	0	0	0,0029	0,0076
A_7	0	0,0946	0,1093	0,0916	0,0083	0,0363	0,0257	0,2816	0,0008	0,0152

(Source: own processing)

and the value of the aggregated trade-off function for each of the variants is determined. For this purpose we use formula (3.13).

We define the rating of alternatives. (Table 12)

Table 12- Order of alternatives (SAW)

	Tradeoff	rank
A_1	0,7819	1
A_2	0,6736	2
A_3	0,5253	4
A_4	0,2658	6
A_5	0,4619	5
A_6	0,2506	7
A_7	0,6634	3

(Source: own processing)

4.5.5 Determination of a set of effective solutions

The data of the alternatives scores obtained using 2 methods are summarized in a table.

Table 13- Summary table of the ranks of alternatives

	TOPSIS		SAW	
	c_i	rank	Tradeoff	rank
A_1	0,7631	1	0,7819	1
A_2	0,4298	5	0,6736	2
A_3	0,5338	4	0,5253	4
A_4	0,1909	7	0,2658	6
A_5	0,7243	2	0,4619	5
A_6	0,3067	6	0,2506	7
A_7	0,5525	3	0,6634	3

(Source: own processing)

The table shows the preference for Alternative A1, as well as A2 weakest Alternatives A4 and A6. At this stage, one could finally choose the best Alternative, which is Alternative A1, but it is also important for the company to know the second most preferred alternative, which will be the backup. Therefore, a decision is made to conduct a further analysis of preferences. To do this, we narrow the circle of Alternatives to 5, removing the weak Alternatives A4 and A6 from the list. And also reduce the number of criteria to 5, removing the least significant C5, C6, C7, C9, C10.

4.6 Step 3. The choice of a single solution from a set of effective solutions

At this stage, we select the best Alternative from the remaining set of effective solutions and determine the second most significant Alternative. To do this, we use the Analytic Hierarchy Process (AHP).

4.6.1 Criteria

The company narrowed the circle of criteria to evaluate the selected alternatives:

1) Price for services rendered (mln. tenge)

2) Depreciation of equipment (%)

3) The total volume of the bucket of two excavators (cubic meters)

4) Availability of additional replacement equipment in case of repair (%)

5) The amount of equipment for working in two shifts (units)

Criteria 1, 2 and 5 are minimized, the rest are maximized.

4.6.2 Determination of the criteria weights

To determine the weights of the criteria, we use the Saaty method, which was described in the theoretical part (chapter 3.3.3).

From the criteria left, the company manager determined the preferences according to the criteria. Based on these data, a matrix was compiled and weights were calculated. (Table 14)

Table 14- Saaty matrix 2

	C_1	C_2	C_3	C_4	C_5	$\prod_{j=1}^{n} C_{ij}$	R_i	W_i
C_1	1	0,5	2	3	0,333	0,999	0,9998	0,1649
C_2	2	1	3	3	0,5	9	1,5518	0,2560
C_3	0,5	0,333	1	2	0,333	0,111	0,6443	0,1063
C_4	0,333	0,333	0,5	1	0,2	0,0111	0,4065	0,0671
C_5	3	2	3	5	1	90	2,4595	0,4057
$\sum$	6,533	4,166	9,5	14,0	2,366		6,0619	1
λ_i	1,077	1,066	1,010	0,939	0,960			

(Source: own processing)

Now let's prove the consistency of the matrix.

We find λmax by the formula (3.6) λmax = 5,052 then we find the consistency index by the formula (3.7)

$I_s = \frac{5{,}052-5}{5-1} = 0{,}013$

The random index for the tenth order matrix is 1,12, we find the consistency relation: OS = 0,013 / 1,12 = 0,012 ≤ 0,10, therefore, the matrix is consistent.

4.6.3 Conducting a preference analysis using the Analytic Hierarchy Process (AHP)

At the level of alternatives, we will compare the values of each individual CI criterion relative to the suppliers of the set $A=\{a_j\}$.

We calculate the components of the normalized priority vector of the 2nd level (the values of suppliers ' priorities relative to the criteria) using the formulas (3.26), (3.27).

Table 15- Matrix of pair comparisons of suppliers according to the criterion “Price for services rendered”

	A_1	A_2	A_3	A_5	A_7	$\prod_{j=1}^{n} u_{ij}$	g_i	y_i
A_1	1	3	0,333	3	5	15	1,719	0,246
A_2	0,333	1	0,2	1	3	0,2	0,725	0,104
A_3	3	5	1	5	7	525	3,500	0,501
A_5	0,333	1	0,2	1	3	0,2	0,725	0,104
A_7	0,2	0,333	0,143	0,333	1	0,0032	0,317	0,045
$\sum$	4,866	10,333	1,876	10,333	19,0		6,986	1
λ_i	1,197	1,075	0,939	1,075	0,855			

(Source: own processing)

Now let’s prove the consistency of the matrix.

We find λmax by the formula (3.6) λmax = 5,141, then we find the consistency index by the formula (3.7)

$I_s = \frac{5{,}141-5}{5-1} = 0{,}013$

The random index for the tenth order matrix is 1.49, we find the consistency relation: OS = 0,013/1,12 = 0,0311 ≤ 0,10, therefore, the matrix is consistent.

Table 16 – Matrix of pair comparisons of suppliers according to the criterion "Depreciation of equipment"

	A_1	A_2	A_3	A_5	A_7	$\prod_{j=1}^{n} u_{ij}$	g_i	y_i
A_1	1	5	3	0,5	3	22,5	1,864	0,298
A_2	0,2	1	0,5	0,143	0,5	0,0072	0,372	0,059
A_3	0,333	2	1	0,2	1	0,133	0,668	0,107
A_5	2	7	5	1	5	140	2,687	0,429
A_7	0,333	2	1	0,2	1	0,133	0,668	0,107
$\sum$	3,866	17,0	10,5	2,043	10,5		6,259	1
λ_i	1,152	1,003	1,124	0,876	1,124			

(Source: own processing)

Now let's prove the consistency of the matrix.

We find λmax by the formula (3.6) λmax = 5,279, then we find the consistency index by the formula (3.7)

$$I_s = \frac{5,279-5}{5-1} = 0,069$$

The random index for the tenth order matrix is 1.49, we find the consistency relation: OS = 0,069/1,12 = 0,061≤ 0,10, therefore, the matrix is consistent.

Table 17 – Matrix of pair comparisons of suppliers according to the criterion "The total volume of the bucket of two excavators"

	A_1	A_2	A_3	A_5	A_7	$\prod_{j=1}^{n} u_{ij}$	g_i	y_i
A_1	1	0,25	2	3	0,5	0,75	0,944	0,136
A_2	4	1	6	7	2	336	3,2	0,460
A_3	0,5	0,167	1	2	0,2	0,0334	0,507	0,073
A_5	0,333	0,143	0,5	1	0,167	0,0040	0,331	0,047
A_7	2	0,5	5	6	1	30	1,974	0,284
$\sum$	7,833	2,06	14,5	19	3,867		6,956	1
λ_i	1,065	0,948	1,059	0,893	1,098			

(Source: own processing)

Now let's prove the consistency of the matrix.

We find λmax by the formula (3.6) λmax = 5,063, then we find the consistency index by the formula (3.7)

$$I_s = \frac{5,063-5}{5-1} = 0,016$$

The random index for the tenth order matrix is 1.49, we find the consistency relation:

OS = 0,016/1,12 = 0,014 ≤ 0,10, therefore, the matrix is consistent.

Table 18 – Matrix of pair comparisons of suppliers according to the criterion “Availability of additional replacement equipment in case of repair”

	A_1	A_2	A_3	A_5	A_7	$\prod_{j=1}^{n} u_{ij}$	g_i	y_i
A_1	1	0,333	0,333	1	0,333	0,037	0,517	0,0905
A_2	3	1	1	3	1	9	1,552	0,273
A_3	3	1	1	3	1	9	1,552	0,273
A_5	1	0,333	0,333	1	0,333	0,037	0,517	0,0905
A_7	3	1	1	3	1	9	1,552	0,273
$\sum$	11,0	3,666	3,666	11	3,666		5,69	1
λ_i	1,012	0,997	0,997	1,012	0,997			

(Source: own processing)

Now let’s prove the consistency of the matrix.

We find λmax by the formula (3.6) λmax = 5,015, then we find the consistency index by the formula (3.7)

$$I_s = \frac{5{,}015-5}{5-1} = 0{,}004$$

The random index for the tenth order matrix is 1.49, we find the consistency relation: OS = 0,004/1,12 = 0,0036 ≤ 0,10, therefore, the matrix is consistent.

Table 19 – Matrix of pair comparisons of suppliers according to the criterion “The amount of equipment for working in two shifts”

	A_1	A_2	A_3	A_5	A_7	$\prod_{j=1}^{n} u_{ij}$	g_i	y_i
A_1	1	1	2	2	1	4	1,319	0,25
A_2	1	1	2	2	1	4	1,319	0,25
A_3	0,5	0,5	1	1	0,5	0,125	0,66	0,125
A_5	0,5	0,5	1	1	0,5	0,125	0,66	0,125
A_7	1	1	2	2	1	4	1,319	0,25
$\sum$	4,0	4,0	8,0	8,0	4,0		5,277	1
λ_i	1	1	1	1	1			

(Source: own processing)

Now let’s prove the consistency of the matrix.

We find λmax by the formula (3.6) λmax = 5,0, then we find the consistency index by the formula (3.7)

$$I_s = \frac{5,0-5}{5-1} = 0$$

The random index for the tenth order matrix is 1.49, we find the consistency relation: OS = 0/1,12 = 0 ≤ 0,10, therefore, the matrix is consistent.

Table 20- Calculate generic supplier priorities

Alternatives	Criteria					The generalized priorities
	C_1	C_2	C_3	C_4	C_5	
A_1	0,0406	0,0763	0,0145	0,0061	0,1014	0,2389
A_2	0,0171	0,0151	0,0489	0,0183	0,1014	0,2008
A_3	0,0826	0,0274	0,0078	0,0183	0,0507	0,1868
A_5	0,0171	0,1098	0,0050	0,0061	0,0507	0,1887
A_7	0,0074	0,0274	0,0302	0,0183	0,1014	0,1847
Criteria character	0,1649	0,2560	0,1063	0,0671	0,4057	

(Source: own processing)

4.6.4 Determine the best solution

Table 21- Order of alternatives

	Tradeoff	rank
A_1	0,2389	1
A_2	0,2008	2
A_3	0,1868	4
A_5	0,1887	3
A_7	0,1847	5

(Source: own processing)

According to the analysis, the most preferred Alternative is Alternative A1, the second most important Alternative is Alternative A2.

5. Conclusion

A multi-criteria analysis of the choice of service providers was carried out using the principle of sequential reduction of uncertainty with 1 method for determining criteria weights (Saaty method) and 3 methods for evaluating alternatives (TOPSIS-method, SAW-method, AHP-method).

Based on the results of the analysis, the most optimal service provider for the company was determined – “Soyuz” LLP, the second (backup) by preference was “Limma” LLP.

Before starting the analysis, it was clear that Alternative No. 1 and Alternative No. 2 were not dominant in almost all the given criteria. But it was an integrated approach, using three different assessment methods that made it possible to identify the most technically prepared suppliers that meet the requirements of the company both in reliability and price. Using only one method in the analysis could give a result that did not meet all the stated requirements.

For example, during the analysis by the TOPSIS method, the Alternative No. 5 with low scores on most criteria was at the second place in the ranking.

However, this was not enough in the complex, as further calculations showed.

Conclusion:

The collected materials and the calculations made in this diploma paper showed that multicriteria decision making analysis using the principle of successive reduction of uncertainty, including three methods for evaluating alternatives (TOPSIS Method, SAW Method, AHP Method) can be an effective tool in supporting decision-making by coal enterprises industry when choosing service providers.

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