Czech University of Life Sciences Prague

Faculty of Economics and Management

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Bachelor Thesis

Application of Advanced Mathematical Methods in Economics in Practice: Multivariate Linear Regression for Bolivia's Economic Growth

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Application of Advanced Mathematical Methods in Economics in Practice: Multivariate Linear Regression for Bolivia's Economic Growth

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Objectives of thesis

The main goal of the bachelor's thesis is to make a linear regression model for economic growth (real GDP growth) in the Plurinational State of Bolivia between 1991 and 2023. A partial goal concerns evaluating the period between 1991 and 2023 in terms of the country's economic development, predicting the future development of the variable, and identifying the proper variables for the model.

Methodology

The bachelor's thesis has two parts: the first is theoretical, and the second is practical.

In the first part that is a theoretical one, the thesis uses qualitative (document study and literature review). This part provides an overview of the implemented method (linear regression model) and the relevant background of economic growth in Bolivia.

In the practical part, the quantitative (time series, descriptive analysis and linear regression analysis) approaches are implemented. The application, analysis, prediction and description of the results are implemented in that part of the thesis. Secondary data from the IMF and the World Bank is used. Gretl is the key software of the investigation.

The proposed extent of the thesis

40-50 pages

Keywords

Bolivia, economic growth, economy, developing economy, South America.

Recommended information sources

ERIKSSON, Clas. Economic growth and the environment: an introduction to the theory. Oxford: Oxford University Press, 2013. ISBN 978-0-19-966389-7.

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Declaration

I declare that I have worked on my bachelor thesis titled " Application of Advanced Mathematical Methods in Economics in Practice: Multivariate Linear Regression for Bolivia’s Economic Growth" by myself, and I have used only the sources mentioned at the end of the thesis. As the author of the bachelor thesis, I declare that the thesis does not break any copyrights.

In Prague on 15.03.2025 _____

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Application of Advanced Mathematical Methods in Economics in Practice: Multivariate Linear Regression for Bolivia’s Economic Growth

Abstract

The thesis concludes that unemployment is the only significant among the four hypothesized determinants of economic growth in Bolivia—unemployment, inflation, presidential elections, and economic crises. Bolivia's central economic challenge over the past 33 years (1991-2023) was inflation, which was stabilized by pegging the exchange rate to the USD. This helped maintain economic growth, with annual increases typically above 4%, except during the 2020 pandemic recession. While the estimated model has strong statistical properties, it leaves a portion of variation unexplained, suggesting potential improvements by adding more predictors, extending the time series, and employing more complex estimation techniques to enhance the empirical analysis and provide better recommendations for Bolivia’s economic policy.

Keywords: Bolivia, economic growth, economy, developing economy, South America

Aplikace pokročilých matematických metod v ekonomii v praxi: vícerozměrná lineární regrese pro ekonomický růst Bolívie

Abstrakt

Tato práce dochází k závěru, že nezaměstnanost je jediným významným faktorem mezi čtyřmi předpokládanými determinanty ekonomického růstu v Bolívii—nezaměstnaností, inflací, prezidentskými volbami a ekonomickými krizemi. Hlavní ekonomickou výzvou Bolívie za posledních 33 let (1991–2023) byla inflace, která byla stabilizována navázáním směnného kurzu na americký dolar. To pomohlo udržet ekonomický růst, přičemž roční přírůstky obvykle přesahovaly 4 %, s výjimkou recese během pandemie v roce 2020. Ačkoli odhadovaný model vykazuje silné statistické vlastnosti, ponechává část variability nevysvětlenou, což naznačuje možnost zlepšení přidáním dalších prediktorů, prodloužením časové řady a použitím složitějších odhadovacích technik. Tyto kroky by mohly přispět k lepší empirické analýze a poskytnout vhodnější doporučení pro ekonomickou politiku Bolívie.

Klíčová slova: Bolívie, hospodářský růst, ekonomika, rozvojová ekonomika, Jižní Amerika

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1 Introduction

Bolivia is a land-locked mountainous country located in South America, and as of 2024, it is classified as a developing one with a total population of around 12 million. In recent years, Bolivia has struggled with attaining political stability and entering the path of sustained economic growth over time.

The phenomenon led to the incumbent government marking achieving economic growth and development as one of the most crucial points during their time in office. At the same time, achieving economic growth is not a straightforward objective where the economic theory clearly indicates that there are infinitely many mechanisms, both endogenous and exogenous, that influence economic growth in a given country.

This bachelor thesis investigates the causes and drivers of economic growth in Bolivia using a multivariate linear regression, which is a widespread tool used in econometrics for identifying the causal effect of a selected set of variables on the dependent one (Hamilton, 2020). The employed multivariate linear regression addresses the socio-economic development of Bolivia in the years between 1991 and 2023, which helps to build a representative sample.

The bachelor thesis has both theoretical and practical contributions, where the theoretical contributions are concentrated on expanding the state of knowledge about general drivers of economic growth. The practical contributions will lead to formulating recommendations for the government of Bolivia on how they can achieve better economic output in the near future (upcoming decades of the 21st century).

The bachelor thesis is split into seven essential chapters. Chapter 1 provided an introduction to the bachelor thesis and its main contents. Chapter 2 proceeds to the objectives and methodology, where Chapter 2.1 provides an overview of the main goals and aims, and Chapter 2.2 focuses on a more detailed description of the thesis' methodology. Chapter 3 forms the theoretical basis of the work, where Chapter 3.1 provides an overview of what economic growth is, Chapter 3.2 addresses the Bolivian economy, and Chapter 3.3 pays closer attention to theoretic catalyzers of economic growth. Chapter 4 is dedicated to the

empirical part of the work, where Chapter 4.1 provides an overview of the economic model, Chapter 4.2 describes the econometric one, Chapters 4.3 – 4.5 address technical details of the estimation and, more importantly, present the estimation results. Chapter 5 is dedicated to the results and discussion, and Chapter 6 concludes the investigation. Chapter 7 presents the list of sources used.

2 Objectives and Methodology

2.1 Objectives

The main goal of the bachelor's thesis is to make a linear regression model for economic growth (real GDP growth) in the Plurinational State of Bolivia between 1991 and 2023. A partial goal concerns evaluating the period between 1991 and 2023 in terms of the country's economic development, predicting the future development of the variable, and identifying the proper variables for the model.

2.2 Methodology

The methodology of this thesis integrates both theoretical and empirical approaches to analyze the determinants of economic growth in Bolivia between 1991 and 2023 using multivariate linear regression. The theoretical framework is built upon a literature review, which provides a comprehensive understanding of economic growth, its key drivers, Bolivia's economic structure, and the linear regression estimation. This foundation supports the selection of relevant variables and informs the construction of the econometric model. The empirical component relies on a quantitative approach, where real GDP growth serves as the dependent variable, and four independent variables - unemployment, inflation, presidential elections, and economic crises are analyzed for their influence on economic performance. Secondary data is sourced from the World Bank and the IMF, ensuring reliability and consistency. Gretl software is used for data analysis, estimation, and verification of statistical assumptions.

The modeling process begins with the formulation of an economic model based on theoretical expectations about how these variables impact GDP growth. The econometric model is then specified in a linear form, incorporating the selected independent variables. The Ordinary Least Squares (OLS) estimation technique is applied to obtain parameter estimates, where the computational process in matrix form is described in Equation 1.

$$\begin{matrix}\beta_1 \\ \beta_{\dots} \\ \beta_n\end{matrix} = (X^T X)^{-1} X^T Y \qquad (1)$$

The model's statistical validity is assessed through hypothesis testing, including t-tests for individual coefficients and an F-test for overall model significance. The coefficient of determination (R^2) and adjusted R^2 are used to measure the explanatory power of the model. Diagnostic tests for heteroscedasticity, autocorrelation, and normality of residuals ensure that the assumptions of OLS regression are met, where White's test is used for the first, Breusch-Godfrey test for the second, and Jarque-Bera test for the third. Additionally, the stationarity of the variables is evaluated using the Augmented Dickey-Fuller (ADF) test.

3 Literature Review

The literature review of the bachelor thesis is split into four sections. The first section addresses the concept of economic growth, its definition, and the very idea. The second section continues with an overview of Bolivia, its economy, and its specialization. The third section proceeds to the causes of economic growth according to relevant literature. The final section is dedicated to the linear regression estimation and fundamental properties of linear regression models based on time series data.

3.1 Economic Growth – Definition and Idea

Economic growth as a separate phenomenon has, for the first time, entered the economic theory largely thanks to the publication of "The Wealth of Nations" by Adam Smith in 1776, which became the cornerstone of modern economic studies. Adam Smith paid explicit attention to the output and believed that the main driver of any significant increase in the national production of a given state is investment (Lewis, 2013). Nevertheless, as the development of the classical theory of the economy continued, other authors equally emphasized the importance of economic growth but believed it to stem from entirely different sources, which was the case of the theory of comparative advantage developed by David Ricardo (Aghion & Howitt, 2008).

Nevertheless, the conventional economic theory did not pay much attention to economic growth as such before the start of The Great Depression in 1929, where the subsequent shift in the economic policy employed by states (the term that was known as "the end of the minimal state") led to an increased interest to the idea of economic growth (Lavoie, 2014). Before The Great Depression and the devastation of the global economic system (not taking into consideration states not primarily integrated into the world economy), it was believed that government intervention in the economy should be kept at a minimum since "the invisible hand" of the market will correct and account for all imperfections and market failures (Blinder, 2008). The inability of the American and world economies to come to grips with the recession of 1929 led to the adoption of a Keynesian-type of economic policy where the government was actively engaged in providing fiscal stimuli in order to increase consumption and pull the economy out of the crisis (Coddington, 2013).

Therefore, Keynesian economics is based mainly on the works of John Maynard Keynes, as well as The Great Depression and the New Deal policy of Franklin D. Roosevelt, which rekindled the interest in economic growth and equally led to the ascension of macroeconomics as a separate discipline (Snowdon & Vane, 2005). In the newly emerged discipline, economic growth and achieving macroeconomic stability became the leitmotivs. Nevertheless, it became pretty apparent to researchers and scientists dedicated to macroeconomics that achieving economic growth without actually addressing side issues is simply impossible due to the fact that in macroeconomics, everything is dependent on each other (De Vroey, 2016). The earlier statement is certainly in line with Walrasian economics and, notably, his most well-known theory about the general equilibrium (De Vroey, 2007). According to Walrasian economics, all economic processes are dependent on each other, and a slight change in one market inevitably triggers changes in others, practically implying that there has to be a general equilibrium achieved throughout all possible markets (Walker, 2006).

Effectively speaking, in contemporary economic science, the definition of economic growth is as follows – economic growth is an increase in the output of a given country over a given period of time (Barro, 1997; Hubbard et al., 2014). From the perspective of macroeconomics, the most common way of analyzing economic growth is by looking at quarterly data since they offer a higher frequency and a lower level of aggregation than annual observations (Gujarati, 2009).

Another essential aspect in relation to economic growth is the main indicators used for measuring the phenomenon. The most common way to measure economic growth is by calculating the chain index of the gross domestic product (GDP), which is the indicator encompassing the total output of a given country (Coyle, 2015; Hubbard et al., 2014). The indicator itself has two approaches for calculating, where the simplified approach contains just two aspects – price level and output (Masood, 2016), which makes the GDP calculation extremely easy with the formula shown in Equation 2:

$$GDP = P * Q \tag{2}$$

At the same time, this approach is rarely feasible, and it does not really allow governments to appropriately and, more importantly, precisely determine the level of output (Lepenies, 2016). For this purpose, the most common modern way of measuring the GDP is through the so-called expenditure approach (Mitchell et al., 2019), which involves the calculation based on the formula from equation 3:

$$GDP = C + I + G + (X - M) \tag{3}$$

In this formula, C stands for the final consumption expenditure of inhabitants, I represents the private consumption, G depicts the government investment, X stands for the total export, and M is the total import (Mitchell et al., 2019). The final part of the formula is also conventionally known as the net exports. In addition to the classical GDP used for describing and measuring economic growth in a given nation, there are also two alternatives – the gross national product (GNP) and gross national income (GNI). In essence, the two supplementary indicators depict more or less similar phenomena but have their minor exceptions (Goodwin et al., 2022). In GNP, the focus is put not on the territorial boundaries of a given state but on the origin of producers, focusing only on the country of a given producer's origin. GNI follows the same idea as the GDP but also takes into consideration outcoming and incoming transfers, which are also known as remittances (Goodwin et al., 2022).

3.2 Bolivia and the Economy

The Bolivian economy (Figure 1 presents the position of Bolivia on the world map) represents a very particular mixture of dependence on natural resources, state-driven initiatives, and a growing concern with diversifying its economic base. At the center of South America lies Bolivia, which has a rich natural resource base dominated by natural gas and minerals such as tin, silver, and lithium. These natural resources have historically formed the very backbone of economic activities in this country, where mining has always held the central place since colonial times (Morales, 2010). Bolivia has recently positioned itself as one of the key protagonists in the world market for lithium due to its immense reserves within the Salar de Uyuni-the, the world's largest salt flat. In effect, it places Bolivia as an

increasingly important player in any lithium-ion battery supply that spans from electric vehicle companies to renewable energy storage manufacturers (Kehoe et al., 2019).

Traditionally, Bolivia has employed an active governmental hand in the country's economy. Much of it came through former president Evo Morales, whose nearly two-decade presidential run extended from 2006 into 2019 (McNelly, 2020). Based on this approach, the elected administration, following influences from socialism, started putting an emphasis on governmental control over specific sectors such as hydrocarbon or mining-based economic sectors (Webber, 2018). It became poignantly illustrated, if not fully realized, in the nationalization of the gas and oil sectors in 2006, with heavy public investment into infrastructure, health, and education, state revenues radically improved. The policy was developed to create a period of maintained economic growth and reduction of poverty. Bolivia reported more than a 4% annual GDP growth rate on average during Morales's presidency (Brown, 2023).

Figure 1. Bolivia on the map.

Source: Britannica, 2024

Despite these feats, some structural challenges still beset the Bolivian economy. Being highly dependent on commodity exports, it is vulnerable to world price fluctuations. For example, the slump in commodity prices in the mid-2010s has contributed to slower growth and fiscal pressures. Besides, there is little industrialization and diversification in Bolivia's economy due to heavy dependence on extractive industries (Anaya, 2020). The poor infrastructure, technological lags, and small domestic market have restricted its efforts toward manufacturing and agricultural development (Morales, 2001).

Over the last couple of decades, Bolivia has been pursuing economic diversification and deeper regional integration as possible solutions to these challenges. Indeed, investment in renewable energy, especially solar and wind projects, reveals Bolivia's commitment to sustainable development (Klein, 2021). Other value-addition initiatives pursued are adding value to lithium resources at home instead of exporting raw minerals. This will develop more industries and job opportunities within the country (Ferguson, 2018).

The informal economy is another dominant characteristic in the Bolivian economy, which commands the lion's share in the sphere of employment and economic activities. While it is an employment source for many people, it raises some obstacles regarding taxation and social security. Informality may be overcome only with comprehensive reforms in areas like access to credit, education, and vocational training for small entrepreneurs (Benkraiem et al., 2019).

The prospects for Bolivia are inextricably linked with political stability and policy continuity. The political turbulence of recent years, marked by the contested 2019 election and a transitional government, has underlined the fragile balance between state control and market openness (Arauco & Pozo, 2023). Looking ahead, the ability of Bolivia to navigate these dynamics while addressing its structural vulnerabilities will be key to its pursuit of sustainable and inclusive growth (Oxford Analytica, 2024). By utilizing its rich resources, investing in human capital, and promoting diversification of the economy, Bolivia could, therefore, have every chance to realize a bright future.

3.3 Causes of Economic Growth

Growth is a multidimensional development factor, propelled by interlinked factors that can be broadly put as supply-side and demand-side determinants of growth. Understanding these is crucial for policymakers who are desirous of pursuing sustainable and inclusive development. Even as the theories of growth have evolved over time, the fundamental drivers of growth remain embedded in capital accumulation, labor force dynamics, technological innovation, institutional quality, and external factors (Iskandar et al., 2023).

Capital accumulation is the bedrock of economic growth. This involves investment in physical capital, such as infrastructure, machinery, and buildings, and human capital, which refers to the education, skills, and health of the labor force. Countries with more of these kinds of investments tend to have faster economic growth since capital is a factor of productivity and an enabler for economies to produce more goods and services. For example, investment in modern transport networks lowers transaction costs and facilitates access to markets, while education and training provide workers with the ability to innovate and adapt to technological change (Mabrouki, 2023).

Labor force participation and demographic trends are equally important. A larger and more active labor force is conducive to economic growth because it raises the productive capacity of an economy. However, the quality of the labor force is as important as its size. More literacy rates and specialized skill sets would put countries in a better competitive position within the global economy. The age distribution of a nation influences its growth. A young population fuels growth through a demographic dividend if there are enough employment and education opportunities for this workforce.

Technological development is one of the significant factors behind sustained economic growth. Innovations and technological progress raise productivity as a given input can now be used more efficiently (Acemoglu & Restepo, 2018). Wider dissemination of technology in the economy enhances competitiveness, cuts costs, and allows for higher levels of production and income. New technologies can originate from indigenous R&D processes but can also be borrowed from more advanced trading partners under a liberal trade policy (Acemoglu & Restepo, 2018).

The quality of the institution is, therefore, a preponderant determinant that underpins all the other determinants. Good institutions promote the rule of law, property rights, and the enforcement of contracts, an environment propitious for investment and innovation. Political stability and transparent governance reduce uncertainty, inspiring domestic and foreign investors to commit resources (Acemoglu, 2008). Weak institutions, corruption, and policy instability deter investment and stifle growth, even in resource-rich countries (Acemoglu, 2008).

Trade and foreign investment are two other important exogenous factors explaining economic growth. An open economy internationally integrated into world markets enjoys more significant opportunities for trade, access to technology, and efficiency (Forte & Moura, 2013). For example, export-oriented development strategies have proved very successful in achieving high growth rates in countries like South Korea and Singapore. FDI also means capital, technology, and management for the host country (Berthélemy & Demurger, 2000).

The definition of macroeconomic stability, which is another critical enabler of growth, is low inflation, stable exchange rates, and sustainable fiscal policies, or the general set of conditions favorable to investment and consumption (Vasylieva et al., 2018). High levels of debt crowd out private investment and strongly disturb economic stability (Vasylieva et al., 2018).

3.4 Linear Regression Estimation

Linear regression estimation is a fundamental tool used in statistics and econometrics, where the method is used to evaluate relationships between variables. According to the relevant literature, there are currently two methods of estimating linear regression models that yield identical structural parameters – the ordinary least squares method or OLS and the maximum likelihood method. At the same time, the majority of the contemporary publications and investigations employing linear regression estimation rely on OLS due to its lower computational burden (Gujarati, 2009).

Despite the lower degree of computational burden, linear regression estimation is still a demanding process as there is a series of assumptions that have to be verified and complied with (Gujarati, 2009). Those assumptions are: normality of residuals (due to t- and F-tests demanding normality), absence of heteroscedasticity of residuals or the presence of constant variance of residuals, absence of serial correlation or autocorrelation, no correlation between the error term and independent variables, zero mean for the error term, linear parameters (in case of non-linear models, the parameters are traditionally linearized using logarithms), no multicollinearity (no high correlation between independent predictors), and that there is random sampling of observations (Gujarati, 2009). At the same time, the list of initial assumptions can increase in the case of more complex estimation procedures involving panel (longitudinal) data (Gujarati, 2009).

On the other hand, there are equal ways of solving violations of the above-mentioned assumptions, where in the case of autocorrelation and heteroscedasticity, there is an option of substituting conventional standard errors with robust ones, such as HAC (Gujarati, 2009). When it comes to the violation of the normality assumption, it is recommended to increase the sample size as non-normality is a classic problem in finite samples, whereas it is less visible in samples involving a higher number of observations (Gujarati, 2009).

4 Practical Part

4.1 Economic Model

By now, it should be pretty evident that the thesis seeks to fulfill the primary goal of identifying determinants causing the economic growth of Bolivia to either go up or down. Based on the literature review and the overview of the Bolivian economy, it was possible to come up with a series of hypothetical determinants that can be viewed as significant factors influencing the economic growth of the country.

The variable that will represent economic growth in this thesis is the real GDP growth, which was calculated as the chain index of the underlying level variable obtained from the World Bank database. Focusing on the real GDP will help to isolate the effect of selected determinants, whereby the results will be less prone to bias occurring from the effect of unobserved factors on the variable in nominal terms. The original variable was expressed in constant USD using the price level from the year 2015. The resulting variable is in percentages, showing a percentual increase in the GDP from the previous year in the current year.

The first factor used in the analysis is unemployment. The most evident indicator that helps to address and measure unemployment is the unemployment rate, which is exactly the variable chosen and obtained from the World Bank. The methodology of the indicator takes into consideration the approach from the ILO, addressing thus people forming part of the labor force that is currently unemployed. Unemployment interacts with economic growth through the labor channel, where labor is one of the key factors of production. Where higher unemployment diminishes the GDP of a country, so the supposed link is negative.

The second factor that often goes hand in hand with unemployment is inflation. High inflation diminishes the purchasing power of consumers and decreases consumption, driving economic growth down. Due to the hypothetical link described above, it was decided to introduce the variable as it is expected to be an important predictor of the GDP with a negative effect on the dependent variable. The World Bank is the source of data.

Bolivia is a country in South America that is evidently quite dependent on the USA, but this dependency did not prompt Bolivia to let go of its monetary independence and replace the official currency with the USD, unlike Ecuador and a bunch of Central American states. Moreover, Bolivia is a country extremely dependent on exports, which by default increases the importance of the exchange rate for the country's well-being and smooth economic growth. Therefore, the exchange rate is selected as the third factor, and the link is expected to be positive, which is not a fully conventional approach. Usually, a depreciating exchange rate (the one that goes up) is expected to contribute to a conventional economy negatively – of course, it should drive exports up since they become cheaper, but this increment in the GDP is compensated by a negative force of inflation, where the depreciated currency results in a higher price paid for goods bought by the country. However, this situation is not something observed in an export-oriented economy, where an increment in exports from the depreciation of the currency is likely higher than the negative effect of inflation, resulting in a growth of the GDP stock. The data is once again obtained from the World Bank.

Moreover, Bolivia is a democratic country, and unfortunately, the country is subject to a quite turbulent political environment, which is a common situation in South America. This turbulence, which is especially visible every time before major elections, prompted the inclusion of a variable that will reflect this aspect. The most logical way to do that would be an introduction of a dummy variable showing presidential elections, which was exactly the step taken with variable taking 1 in case presidential elections take place in a given year and zero if no presidential elections are held this year. The effect should be a negative one as it increases uncertainty and drives the political environment to become even more unstable.

Subsequently, continuing the narrative about the export orientation of Bolivia, it is important to specify that gold contributes to approximately 20% of the country's annual exports, meaning that this commodity is central to Bolivian economic growth. However, it is a common situation that fluctuations in the international prices of commodities either drive up or diminish the GDP, making countries specializing in trade in natural resources quite vulnerable to this aspect. To address this, the international annual price of gold is selected as a predictor, and the source of data is FRED.

The final determinant is introduced to smooth the fitted curve and account for other shocks, where economies of all countries are extremely vulnerable to major economic crises. Those crises are often labeled as "black swan" events, which is explained by their unpredictable nature. Over 33 years between 1991 and 2023 that are analyzed, there were only two major crises that hit the economy of Bolivia – The Great Recession of 2008 and the COVID-19 pandemic of 2020. Therefore, there is also a dummy variable reflecting economic crises, taking the value of 1 whenever an above-mentioned crisis took place and 0 for years without a major international crisis.

Based on the described logic, the aim of the thesis, and the available data, it is possible to create a model found in Equation 4:

$$\boldsymbol{Y_t = f(X_{1t}, X_{2t}, X_{3t}, X_{4t}, X_{5t}, X_{6t})} \quad (4)$$

- $\boldsymbol{Y_t}$ is the real GDP growth in Bolivia in %.
- $\boldsymbol{X_{1t}}$ is the unemployment rate in Bolivia in %.
- $\boldsymbol{X_{2t}}$ is the inflation rate (CPI) in Bolivia in %.
- $\boldsymbol{X_{3t}}$ is the exchange rate in Bolivia in boliviano per USD.
- $\boldsymbol{X_{4t}}$ is the presidential elections dummy variable.
- $\boldsymbol{X_{5t}}$ is the international price of gold in USD per ounce.
- $\boldsymbol{X_{6t}}$ is the economic crisis dummy variable.

4.2 Econometric Model

The second step that follows is the creation of an econometric model. There are two fundamental differences between the economic and econometric models. The economic model is a prerequisite step for which it is needed to have economic relationships and effects

in mind, while the econometric model connects the hypothetical relations from the economic model to mathematics and statistics, introducing a structural form and also mentioning the stochastic component, that is, the error term.

A desired option for the dependent variable is, of course, an exponential form, which will also entail an exponential form for the predictors, but there are two limitations preventing this from happening – the implemented GDP variable is not a stock one but a percentual change, making it mathematically impossible to add a logarithm of the GDP then to linearize the model to implement the OLS, and the second limitation are two dummy variables, where it will equally be mathematically impossible to implement a logarithm transformation to them. Therefore, the selected form is linear, which is still likely an ideal compromise solution. The model takes the form specified in Equation 5:

$$\boldsymbol{Y}_t = \beta_1 + \beta_2 \boldsymbol{X}_{1t} + \beta_3 \boldsymbol{X}_{2t} + \beta_4 \boldsymbol{X}_{3t} + \beta_5 \boldsymbol{X}_{4t} + \beta_6 \boldsymbol{X}_{5t} + \beta_7 \boldsymbol{X}_{6t} + \boldsymbol{\eta_t} \qquad (5)$$

- $\boldsymbol{Y_t}$ is the real GDP growth in Bolivia in %.
- $\boldsymbol{X_{1t}}$ is the unemployment rate in Bolivia in %.
- $\boldsymbol{X_{2t}}$ is the inflation rate (CPI) in Bolivia in %.
- $\boldsymbol{X_{3t}}$ is the exchange rate in Bolivia in boliviano per USD.
- $\boldsymbol{X_{4t}}$ is the presidential elections dummy variable.
- $\boldsymbol{X_{5t}}$ is the international price of gold in USD per ounce.
- $\boldsymbol{X_{6t}}$ is the economic crisis dummy variable.
- β_1, β_2, β_3, β_4, β_5, β_6, and β_7 are parameters showing the effect of the selected factors on $\boldsymbol{Y}$.
- t is an indicator of an annual observation.

- η is the error term.

Betas are the key output of the thesis, where those coefficients will show the effect of the selected predictors with the exception of the first beta, which is an intercept term showing the average value of the dependent variable under zero values for other predictors. The evaluation of those parameters will happen during hypothesis testing (t-tests), and it will allow us to compare the significance of predictors between each other and draw key findings about the most important determinants of economic growth in Bolivia.

Nevertheless, the estimated coefficients and the tests performed might not be correct even despite the implementation of a complex mathematical procedure via Gretl that will ensure the absence of the human factor and computational issues that could arise under manual processing of data. The problem with the OLS is that it relies on various assumptions, some of which are easy to achieve, e.g., linear parameters and more observations than independent variables, while other assumptions are more complex. The estimation of linear regression is based on the integral BLUE assumption, implying that coefficients are best, linear, and unbiased, where the third component is achieved mainly through creating a nearly perfect model that will include all the required variables.

Additionally, there is a need to ensure that not just the parameters are unbiased but also the standard errors that are derived from the covariance matrix. This is harder to achieve because data is nearly never perfect, and so are the models that inevitably drop a factor or two due to either data unavailability or conceptual issues. One such assumption is the assumption about a constant variance that says that the variance of the error term should be constant at different values of independent variables. Otherwise, it distorts the standard errors and does not allow to properly interpret the results of the hypothesis testing since it is simply not correct. Another assumption is the assumption about autocorrelation, which is, in essence, a dependency of error term over different values of independent variables, and the effect is equally destructive for standard errors. Usually, if those assumptions are violated, modified or robust standard errors are then introduced, and it helps to estimate non-biased standard errors. The final assumption's violation (normality assumption) does not lead to biased standard errors, but it raises concerns about whether it is possible to apply statistical

tests at all because, in the same manner as the OLS method, those tests have assumptions, and normality is one of them.

4.3 Data Collection and Analysis

This chapter proceeds to the data collection and its analysis. Table 1 is a complete overview of the selected data.

Table 1. Collected data for Bolivia's economic growth.

Year	GDP	Unemployment	Inflation	Exchange Rate	Elections	Gold	Economic Crisis
	%	%	%	BOB/USD	dummy	USD/ounce	dummy
1991	0.05	2.93	21.45	3.58	0.00	362.34	0.00
1992	0.02	3.03	12.06	3.90	0.00	343.87	0.00
1993	0.04	3.05	8.53	4.27	1.00	360.05	0.00
1994	0.05	3.08	7.87	4.62	0.00	384.16	0.00
1995	0.05	2.67	10.19	4.80	0.00	384.07	0.00
1996	0.04	2.38	12.43	5.07	0.00	387.73	0.00
1997	0.05	2.08	4.71	5.25	1.00	331.00	0.00
1998	0.05	2.12	7.67	5.51	0.00	294.12	0.00
1999	0.00	2.30	2.16	5.81	0.00	278.86	0.00
2000	0.03	2.44	4.61	6.18	0.00	279.29	0.00
2001	0.02	2.63	1.59	6.61	0.00	271.19	0.00
2002	0.02	2.61	0.93	7.17	1.00	310.08	0.00
2003	0.03	2.66	3.34	7.66	0.00	363.83	0.00
2004	0.04	2.62	4.44	7.94	0.00	409.53	0.00
2005	0.04	2.60	5.39	8.07	1.00	444.99	0.00
2006	0.05	2.58	4.28	8.01	0.00	604.34	0.00
2007	0.05	2.62	8.71	7.85	0.00	696.43	0.00
2008	0.06	2.60	14.01	7.24	0.00	872.37	1.00
2009	0.03	2.97	3.35	7.02	1.00	973.66	0.00
2010	0.04	2.61	2.50	7.02	0.00	1226.66	0.00
2011	0.05	2.26	9.88	6.94	0.00	1573.16	0.00
2012	0.05	2.07	4.52	6.91	0.00	1668.86	0.00
2013	0.07	2.44	5.74	6.91	0.00	1409.51	0.00
2014	0.05	2.02	5.77	6.91	1.00	1226.06	0.00
2015	0.05	3.11	4.06	6.91	0.00	1158.86	0.00
2016	0.04	3.50	3.62	6.91	0.00	1251.92	0.00
2017	0.04	3.66	2.82	6.91	0.00	1260.39	0.00
2018	0.04	3.52	2.27	6.91	0.00	1268.93	0.00

2019	0.02	3.68	1.84	6.91	1.00	1393.34	0.00
2020	-0.09	7.90	0.94	6.91	1.00	1773.73	1.00
2021	0.06	5.09	0.74	6.91	0.00	1798.89	0.00
2022	0.04	3.55	1.75	6.91	0.00	1801.87	0.00
2023	0.02	3.08	2.58	6.91	0.00	1943.00	0.00

Source: The World Bank (2024) and FRED (2024).

In the next step, a more detailed overview of data is generated, which is all about descriptive statistics, where Gretl was used to calculate them. Figure 2 presents the key statistics for all variables.

Figure 2. Summary statistics.

	Mean	Median	Minimum	Maximum
GDP	0.036981	0.042693	−0.087379	0.067960
Unemployment	2.9824	2.6260	2.0210	7.9030
Inflation	5.6585	4.4374	0.73738	21.447
ExchangeRate	6.4677	6.9100	3.5806	8.0661
Elections	0.24242	0.0000	0.0000	1.0000
Gold	882.03	696.43	271.19	1943.0
EconomicCrisis	0.060606	0.0000	0.0000	1.0000

	Std. Dev.	C.V.	Skewness	Ex. kurtosis
GDP	0.026443	0.71504	−3.2600	13.002
Unemployment	1.0771	0.36115	3.1704	11.614
Inflation	4.5462	0.80343	1.5557	2.6311
ExchangeRate	1.1927	0.18440	−0.94896	−0.027766
Elections	0.43519	1.7952	1.2021	−0.55500
Gold	571.16	0.64755	0.41392	−1.3417
EconomicCrisis	0.24231	3.9980	3.6830	11.565

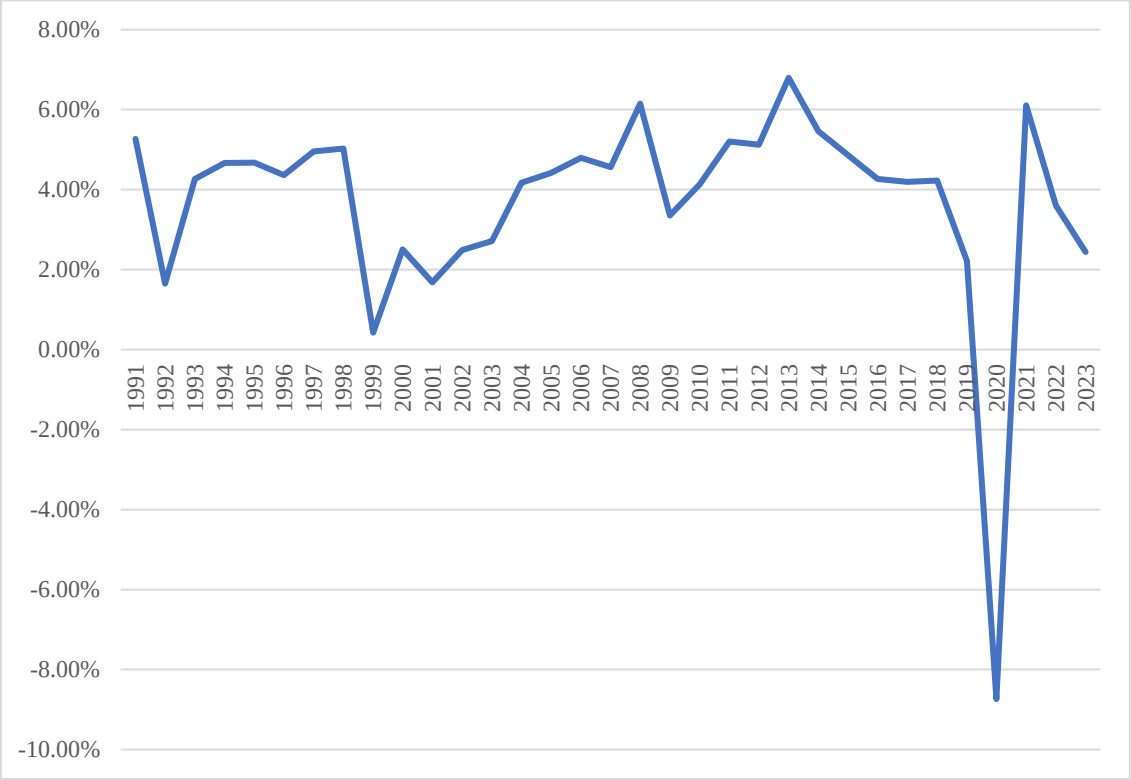
	5% perc.	95% perc.	IQ range	Missing obs.
GDP	−0.023225	0.063428	0.023822	0
Unemployment	2.0525	5.9332	0.65650	0
Inflation	0.87100	16.239	5.8133	0
ExchangeRate	3.8045	8.0280	1.3571	0
Elections	0.0000	1.0000	0.50000	0
Gold	276.56	1844.2	969.94	0
EconomicCrisis	0.0000	1.0000	0.0000	0

Source: author's own contribution in Gretl based on The World Bank (2024) and FRED (2024).

In the remaining part of the chapter, each selected variable's trajectory over time is assessed using Excel charts (apart from the dummy variables whose behavior is visible in

Table 1). Figure 3 presents the historical trajectory of Bolivian economic growth in percentages between 1991 and 2023.

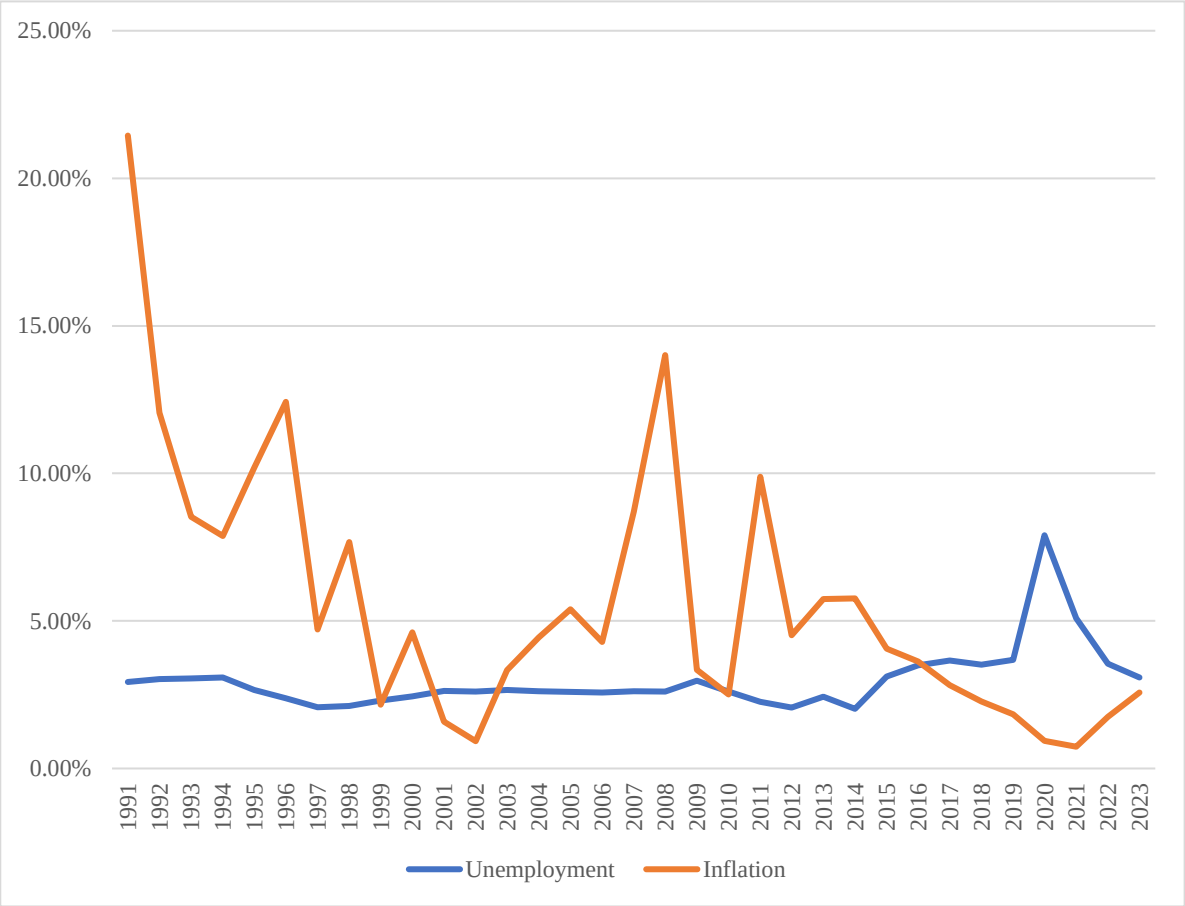
Figure 3. Bolivian economic growth (1991 - 2023).



Source: author's own contribution in Excel based on The World Bank (2024) and FRED (2024).

Bolivia experienced a very good period of development between 1991 and 2023, where there was no single year when the economy experienced a recession, with the exception of the first pandemic year in 2020. This is a sign that, over time, Bolivia will manage to industrialize and substantially increase its economic output in the world economy, also allowing the country to potentially join the group of the wealthiest countries in the Western hemisphere if the development does not change drastically. There are years when the increase is not that fast, e.g., in 1999 (caused by the overall economic turmoil of South America, notably in Brazil and Argentina, are one of the key trading partners of Bolivia) and 2023 (caused mainly by political instability and uncertainty about the future democratic trajectory). Figure 4 will introduce the unemployment rate and the inflation rate, projecting them on the same graph.

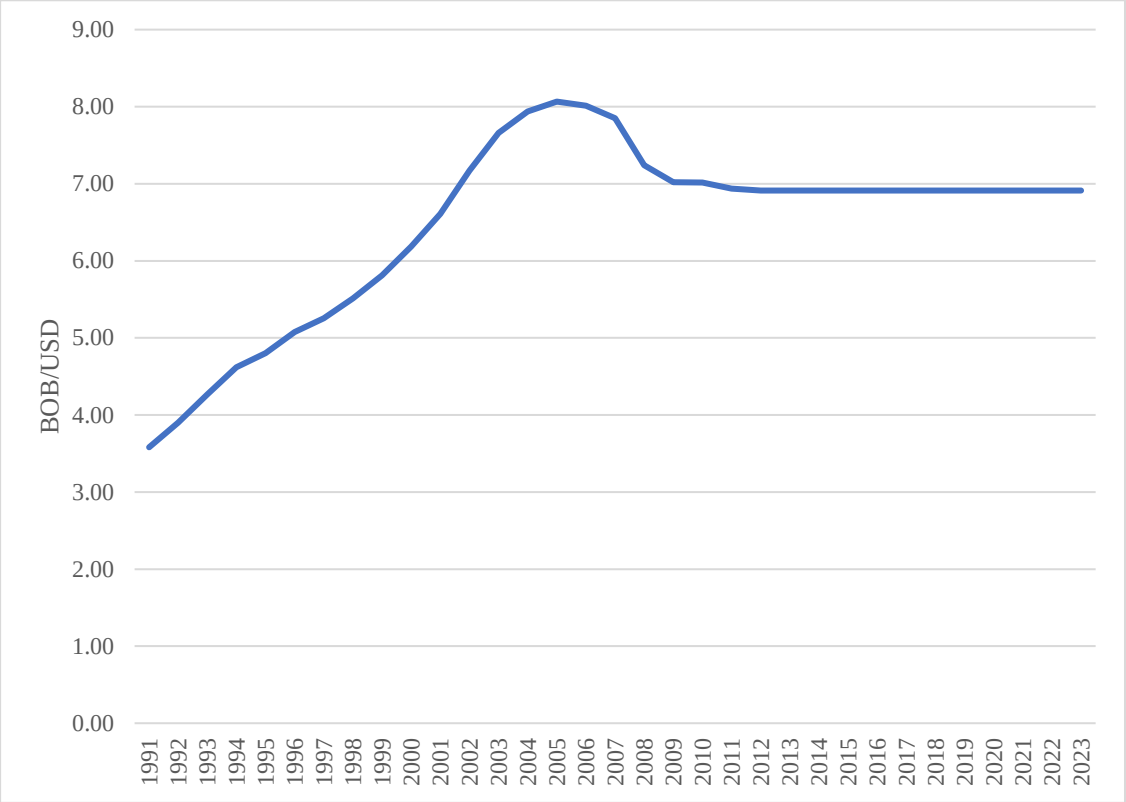
Figure 4. Bolivian inflation and unemployment (1991 – 2023).



Source: author's own contribution in Excel based on The World Bank (2024) and FRED (2024).

Unemployment was visibly a lesser problem than inflation. A very static and stable curve of unemployment is a sign of a firm public employment policy, which is unsurprising given the prevalence of left-wing politicians who are especially interested in public programs. On the other hand, a pervasive situation in such countries where public policy is a preeminent aspect of government operations is soaring inflation, which is often caused by budget deficits and excessive printing of money, which was one of the reasons for the growth disaster of Venezuela. Bolivia partially suffered from a similar issue, but most of the problems came from the Great Recession and the negative situation in financial markets that resulted in extreme price instability in Bolivia. Compared with the 90s, 00s, and 10s, the new decade is seemingly much calmer and more adequate in terms of price stability. Figure 5 presents Bolivia's exchange rate.

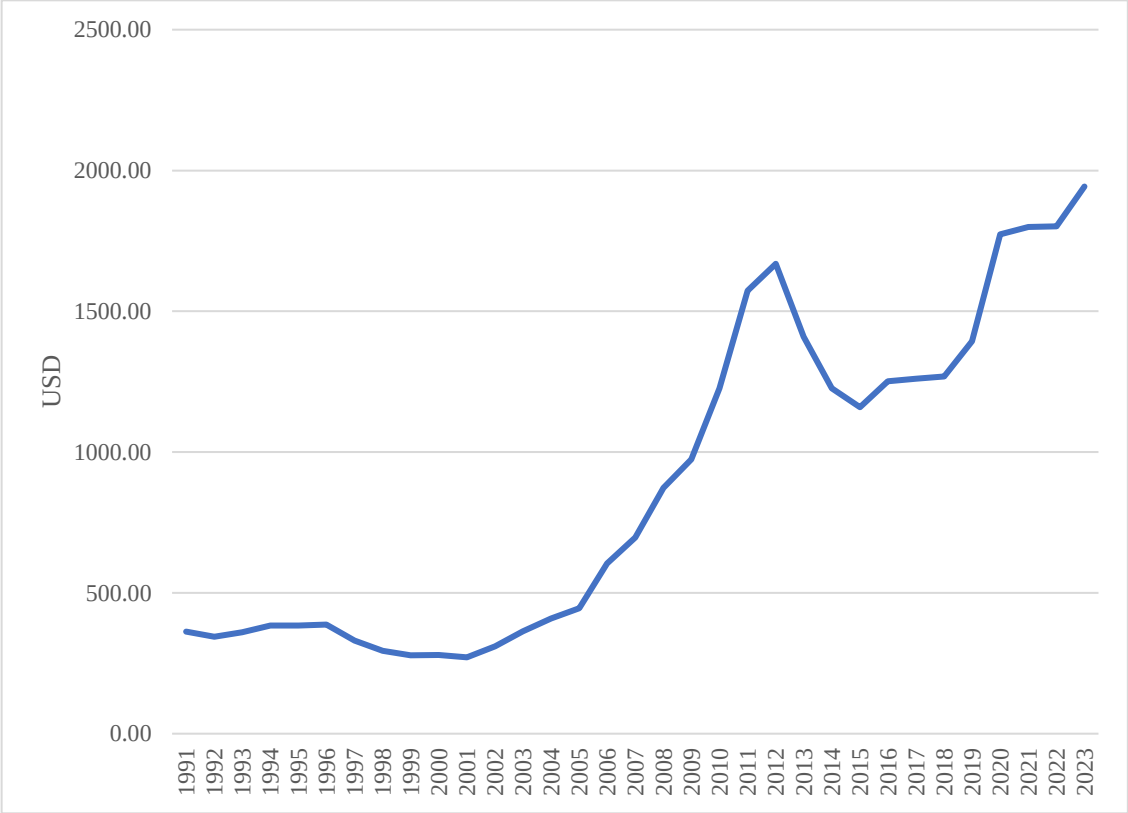
Figure 5. Bolivian exchange rate (1991 – 2023).



Source: author's own contribution in Excel based on The World Bank (2024) and FRED (2024).

The Bolivian exchange rate's trajectory is not thoroughly typical for a country with a floating exchange rate. Since 2009, the rate has largely not moved and remained static. This is a consequence of pegging a domestic currency to another more powerful one, where a role model for Bolivia became the USD. In other words, the country did not really want to give up its monetary independence, but it pegged the Boliviano to the USD and controls for balance of payments shocks using foreign reserves, not allowing thus the currency to fall or increase from the set benchmark of 6.91 Bolivianos per one USD. The situation can change if the reserves wear out, but this is not a realistic scenario for a country that exports one of the key commodities – petroleum gas and gold. This might explain why, in recent years, Bolivia's problems with inflation did not exist; it simply avoided the negative effect of an increase in the price of raw materials imported from neighboring countries and other countries. Figure 6 will present the trajectory of the price of gold per ounce.

Figure 6. International gold price (1991 - 2023).



Source: author's own contribution in Excel based on The World Bank (2024) and FRED (2024).

Gold was gaining in price, which is a consequence of its scarce nature and the fact that this investment option is viewed as a safe haven against inflation. Moreover, in some countries with tight currency holding restrictions, people outrightly save their money against inflation by buying gold, which is especially important in countries with depreciating currency. The mentioned situation fruitfully contributed to the international price of gold, benefitting gold-exporting countries all over the world.

The final component of this chapter is the projection of a correlation matrix that will not just support the description of data and the relationship between the variables but also contribute to verifying if there is multicollinearity or not. Table 2 is the table of the relationships between the independent variables, while the relationship between the independent variables and the dependent one is subject to the analysis starting in the next chapter. The table confirms that no multicollinearity is present in this study.

Table 2. Relationships between the independent variables.

	Unemployment	Inflation	Exchange Rate	Elections	Gold	Economic Crisis
Unemployment	1.00	-0.30	0.08	0.20	0.47	0.54
Inflation	-0.30	1.00	-0.59	-0.22	-0.38	0.10
Exchange Rate	0.08	-0.59	1.00	0.05	0.41	0.13
Elections	0.20	-0.22	0.05	1.00	-0.03	0.15
Gold	0.47	-0.38	0.41	-0.03	1.00	0.20
Economic Crisis	0.54	0.10	0.13	0.15	0.20	1.00

Source: author's own contribution in Excel based on The World Bank (2024) and FRED (2024).

On top of that, Table 3 presents the table of correlations between the independent variables and the dependent ones.

Table 3. Correlations between Y and Xs.

	Unemployment	Inflation	Exchange Rate	Elections	Gold	Economic Crisis
GDP	-0.69	0.34	-0.02	-0.30	-0.09	-0.49

Source: author's own contribution in Excel based on The World Bank (2024) and FRED (2024).

It is visible that the two variables that correlate the most with Y are unemployment and economic crisis. After performing the correlation analysis, the data analysis part finishes with the verification of the stationarity of each variable (excluding the dummy ones, i.e.,

elections and economic crisis). Table 4 presents the results of the stationarity test (the lag length is selected automatically based on Akaike criteria), including the null hypotheses, P value, and the result of the testing.

Table 4. Unit-root testing.

Ho: GDP is non-stationary (contains a unit root)	Ho: unemployment is non-stationary (contains a unit root)	Ho: inflation is non-stationary (contains a unit root)	Ho: The exchange rate is non-stationary (contains a unit root)	Ho: gold is non-stationary (contains a unit root)
Ha: GDP is stationary (does not contain a unit root)	Ha: unemployment is stationary (does not contain a unit root)	Ha: inflation is stationary (does not contain a unit root)	Ha: The exchange rate is stationary (does not contain a unit root)	Ha: gold is stationary (does not contain a unit root)
0.001 < 0.05, GDP is stationary	0.087 < 0.05, unemployment is non-stationary	0.001 < 0.05, inflation is stationary	0.75 > 0.05, the exchange rate is non-stationary	0.70 > 0.05, gold is non-stationary

Source: author's own contribution in Excel based on The World Bank (2024) and FRED (2024).

Additionally, evidence for the performed tests can be found in Figures 11, 12, 13, 14, and 15 in the list of appendices. There are currently three non-stationary variables – gold, exchange rate, and unemployment. For the first two, this does not present a significant problem as they are shortly omitted from the estimation (see section 4.4), while the non-stationarity of unemployment is a bigger problem, potentially leading to the issue of spurious regression (Gujarati, 2009). This is further discussed in the limitations of the investigation.

4.4 Modeling

During the preliminary testing, it was identified that the variables exchange rate and gold do not behave themselves in the assumed way, and there is also a violation in the sign of the estimated parameters vis-à-vis the correlation coefficients, which resulted in the dropping of those variables. The resulting econometric model can be found in Equation 6:

$$\boldsymbol{Y}_t = \beta_1 + \beta_2 \boldsymbol{X}_{1t} + \beta_3 \boldsymbol{X}_{2t} + \beta_5 \boldsymbol{X}_{4t} + \beta_7 \boldsymbol{X}_{6t} + \boldsymbol{\eta_t} \quad (6)$$

Modeling is performed in two steps – the first step introduces the estimated coefficients using the OLS approach, and the second step is the interpretation of the coefficients that were estimated, comparing their signs to the assumptions from Chapter 4.1. Figure 7 introduces the resulting model.

Figure 7. Bolivian economic growth OLS model.

```
Model 1: OLS, using observations 1991-2023 (T = 33)
Dependent variable: GDP

                  coefficient   std. error    t-ratio   p-value
  ------------------------------------------------------------
  const            0.0700134    0.0148196      4.724    5.89e-05 ***
  Unemployment    -0.0119979    0.00412494    -2.909    0.0070   ***
  Inflation        0.00108057   0.000839959    1.286    0.2088
  Elections       -0.00785347   0.00805544    -0.9749   0.3379
  EconomicCrisis  -0.0240847    0.0176612     -1.364    0.1835

Mean dependent var   0.036981   S.D. dependent var   0.026443
Sum squared resid    0.010122   S.E. of regression   0.019013
R-squared            0.547656   Adjusted R-squared   0.483036
F(4, 28)             8.474963   P-value(F)           0.000130
Log-likelihood       86.65343   Akaike criterion    -163.3069
Schwarz criterion   -155.8243   Hannan-Quinn        -160.7892
rho                  0.100844   Durbin-Watson        1.785180

Excluding the constant, p-value was highest for variable 5 (Elections)
```

Source: author's own contribution in Gretl based on The World Bank (2024) and FRED (2024).

The model describing the real GDP growth in Bolivia is in Equation 7:

$$\boldsymbol{Y}_t = 0.07 - 0.011\boldsymbol{X}_{1t} + 0.001\boldsymbol{X}_{2t} - 0.007\boldsymbol{X}_{4t} - 0.024\boldsymbol{X}_{6t} + \boldsymbol{\eta_t} \quad (7)$$

The interpretation:

- If the unemployment rate in Bolivia increases by one percentage point, the GDP growth will diminish by 0.011 percentage points, ceteris paribus.

- If the inflation rate in Bolivia increases by one percentage point, the GDP growth will increase by 0.001 percentage points, ceteris paribus.

- On average, the GDP growth in the year of presidential elections in Bolivia is 0.007 percentage points lower.

- On average, if there is a major international economic crisis, the real GDP of Bolivia will decrease by 0.024 percentage points.

At first, the results are highly satisfying as there is a high correspondence between the expected and the resulting outcomes. Nevertheless, one coefficient is different from the assumption – inflation, whose positive sign is explained by the early development of the Bolivian economy, when the country grew at a high pace even despite soaring inflation.

Despite this observation that should be viewed with a certain grain of salt, it is likely that inflation has a stimulating effect on the Bolivian economy up to a particular level, but once it becomes uncontrollable, the situation will likely change. Another reason why the sign is positive is the actual income, which might have been increasing, which could be explained by incomes rising higher than inflation even despite high inflation. The next chapter proceeds with the verification and the application of the model.

4.5 Verification and Application

The first step is the verification of the significance of the whole model, which is done following the procedure of applying the F-test in Table 5. The significance level is 5 percent, at which the model is significant as per the testing results.

Table 5. F-test.

Ho: the model is not significant.
Ha: the model is significant.
0.00013 < 0.05, the null hypothesis is rejected.

Source: author's own contribution based on The World Bank (2024) and FRED (2024).

The subsequent step is the test of statistical significance of individual parameters, which is done in Table 6, confirming that only 3 out of 7 parameters are statistically significant – unemployment, inflation, and the international price of gold.

Table 6. T-tests.

Ho: constant is not significant	Ho: unemployment is not significant	Ho: inflation is not significant	Ho: elections dummy is not significant	Ho: crisis dummy is not significant
Ha: constant is significant	Ha: unemployment is significant	Ha: inflation is significant	Ha: elections dummy is significant	Ha: crisis dummy is significant
0.001 < 0.05, significant	0.007 < 0.05, significant	0.20 > 0.05, insignificant	0.33 > 0.05, insignificant	0.18 > 0.05, insignificant

Source: author's own contribution based on The World Bank (2024) and FRED (2024).

The third step involves the verification of other statistical properties, such as the coefficient of determination and the adjusted coefficient of determination. The goal of the thesis is the evaluation of the causal effect, so not astonishingly high figures for the ordinary coefficient of determination and the adjusted one of 0.54 and 0.48, respectively, are not so critical, but they still indicate that the highest share of variation in the real GDP growth in Bolivia was explained by the model.

Verification taking place in Figure 8, which is the fourth step, is more important as it will explicitly say if modifications have to be made in the model to estimate standard errors.

Source: author's own contribution in Gretl based on The World Bank (2024) and

Figure 8. Heteroscedasticity, autocorrelation and normality.

```
White's test for heteroskedasticity -
  Null hypothesis: heteroskedasticity not present
  Test statistic: LM = 24.5045
  with p-value = P(Chi-square(10) > 24.5045) = 0.00636815

LM test for autocorrelation up to order 1 -
  Null hypothesis: no autocorrelation
  Test statistic: LMF = 0.2753
  with p-value = P(F(1, 27) > 0.2753) = 0.60408

Test for normality of residual -
  Null hypothesis: error is normally distributed
  Test statistic: Chi-square(2) = 7.21651
  with p-value = 0.0270991
```

FRED (2024).

Variance is not constant, but autocorrelation is not present, which is a piece of evidence that standard errors are biased. Residuals are not normally distributed, which raises doubts about the additional validity of the results of the statistical inference introduced earlier. The model is re-estimated using robust standard errors (HAC) in Figure 9 to display unbiased standard errors.

Figure 9. Re-estimated model.

```
Model 2: OLS, using observations 1991-2023 (T = 33)
Dependent variable: GDP
HAC standard errors, bandwidth 2 (Bartlett kernel)

                  coefficient   std. error    t-ratio   p-value
  ------------------------------------------------------------
  const            0.0700134    0.0188823     3.708    0.0009  ***
  Unemployment    -0.0119979    0.00535403   -2.241    0.0331  **
  Inflation        0.00108057   0.000791652   1.365    0.1831
  Elections       -0.00785347   0.00684648   -1.147    0.2611
  EconomicCrisis  -0.0240847    0.0307688    -0.7828   0.4403

Mean dependent var   0.036981   S.D. dependent var   0.026443
Sum squared resid    0.010122   S.E. of regression   0.019013
R-squared            0.547656   Adjusted R-squared   0.483036
F(4, 28)             8.632364   P-value(F)           0.000114
Log-likelihood       86.65343   Akaike criterion    -163.3069
Schwarz criterion   -155.8243   Hannan-Quinn        -160.7892
rho                  0.100844   Durbin-Watson        1.785180

Excluding the constant, p-value was highest for variable 7 (EconomicCrisis)
```

Source: author's own contribution in Gretl based on The World Bank (2024) and FRED (2024).

As a result, there are still two significant predictors – constant and unemployment. The remaining ones are insignificant. Figure 10 is the final piece of the puzzle, visualizing the two curves and also projecting the 95% confidence interval for the fitted curve.

Figure 10. 95% confidence interval for the fitted curve.

Source: author's own contribution in Gretl based on The World Bank (2024) and FRED (2024).

5 Results and Discussion

The practical part of the bachelor thesis clearly revealed what was originally under question in this investigation. Thanks to the implementation of the OLS as a part of a comprehensive econometric estimation searching for determinants of economic growth in Bolivia, it was identified that the most important predictors and the only ones significant in the model containing unemployment, inflation, elections dummy, and crises dummy is unemployment.

One significant predictor confirms that the economy of Bolivia is fundamentally dependent on the smooth employment of people, which is likely influenced by large public spending, which is especially typical for leftist governments, which was the case in Bolivia for many years. The second objective was the description of the economic development in Bolivia, which was also accomplished as a part of the time series analysis. Unemployment was developing in a highly favorable way with really high national employment. Inflation was more complex, especially in the early years and the years when major economic crises were happening. However, after pegging the currency to the USD, the situation stabilized, and by the beginning of the 20s, inflation did not pose any serious threats to the country. The GDP growth of Bolivia was almost flawless and largely above the threshold of 4 percent for every year with no recessions at all, with the exception of the COVID-19 pandemic's first year, 2020.

An interesting finding connects the political instability of Bolivia to economic growth, where the effect of presidential elections and the turmoil that they cause is a negative one but not significant. As for major crises, the effect is more extensive, but the variable is also not substantial. All in all, Bolivia is an example of an up-and-coming economy that is, however, dependent on exports of raw materials, which is a major risk factor. Nevertheless, if the country manages to reallocate funds obtained from exports to the development of the public sector, infrastructure, and incentives for people to develop businesses, Bolivia can emerge as an exemplary country that, despite its natural abundance, did not make itself highly vulnerable to commodity prices fluctuations.

A significant limitation is the sample size. In statistics, as the sample size increases, the estimated parameters converge to the valid population parameter, which is a desired situation. This estimation had only 33 observations, which is not an ideal figure, and a higher number of observations could be achieved if not for data unavailability and, more specifically, for the unemployment variable with no data available prior to 1991. Nevertheless, Bolivia emerged relatively recently as a significant, promising South American economy, and as the years pass, it will naturally increase the number of available years, so reattempting to create a model in the future can be a good idea. Additionally, new investigations into the same matter can consider more complex methods that will be more comprehensive and robust, such as accounting, for instance, for stationarity and non-stationarity of the considered variables, which was not done in this thesis.

Another limitation is related to the mixed results of the stationarity testing, where unemployment was non-stationary, whereas the remaining indicators were stationarity. This potentially leads to the problem of spurious regression, according to Gujarati (2009), and this is something that should be addressed in future research by employing alternative methods of estimation, such as distributed lag autoregressive models (ARDL). Finally, some of the predictors can simultaneously influence one another, which opens room for using a 2SLS estimation that will address the occurred endogeneity bias.

6 Conclusion

The conclusion to the thesis stresses that out of 4 hypothetically essential determinants of economic growth in Bolivia, including unemployment, inflation, occurrence of presidential elections, and occurrence of economic crises, the significant one is just unemployment, which is highly logical from the economic perspective. Unemployment and, more importantly, the opposite phenomenon of employment can be viewed as wheels of economic growth because higher employment and lower unemployment, as a consequence, lead to the generation of wealth and higher consumption.

As for the development of economic variables in the recent period of 33 years in 1991 – 2023, it is identified that the weakest point of Bolivia was inflation that was out of control in the 90s, 00s, and early 10s, but pegging the exchange rate to the USD at the rate of 6.91 BOB helped the country to stabilize the price of imports and made the country less vulnerable to exchange rate shocks typical for export-oriented countries. The country's rate of economic growth was nearly flawless, with an annual increase almost every year above 4 percent and no economic recession at all, with the exception of the year 2020, which was obscured by the coronavirus pandemic.

The estimated model has favorable statistical properties, but improvements can also be made. First, the model explains the largest portion of variation in the dependent variable, but approximately 30% to 40% was not explained, which suggests that more predictors can be added to the model. Second, the model has a relatively short time series length of only 33 years, which can be improved by returning back to the investigation once a sufficient number of years pass.

Third, the model can profit from employing a more complex technique of estimation that will account for different stationarity statuses of the considered variables (ARDL, for instance), where some variables are not stationary, and other variables are stationary. Additionally, employing the 2SLS method can account for the potential endogeneity of the predictors. If all the mentioned aspects are taken into consideration, it is expected that the quality of the thesis and of this empirical analysis will go up substantially, opening new

grounds for recommendations for the Plurinational State of Bolivia, as well as contributing to the relevant theory about the predictors of economic growth.

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8 Appendices

8.1 List of Pictures

8.2 List of Tables

8.3 List of Abbreviations

BOB	Bolivian Boliviano

USD	United States Dollar
GDP	Gross Domestic Product
CPI	Consumer Price Index
FRED	Federal Reserve Economic Database
OLS	Ordinary Least Squares
GNI	Gross National Income
GNP	Gross National Product
FDI	Foreign Direct Investment
R&D	Research and Development
BLUE	Best Linear Unbiased Estimate
OLS	Ordinary Least Squares
ADF	Augmented Dickey-Fuller Test
ARDL	Autoregressive Distributed Lag
2SLS	Two-Stage Least Squares

8.4 Additional Testing Results

Figure 11. ADF for GDP.

```
Dickey–Fuller test for GDP
sample size 32
unit–root null hypothesis: a = 1

  with constant and trend
  model: (1–L)y = b0 + b1*t + (a–1)*y(–1) + e
  estimated value of (a – 1): –0.954958
  test statistic: tau_ct(1) = –5.15511
  p–value 0.001123
  1st–order autocorrelation coeff. for e: 0.012
```

Source: author's own contribution in Gretl based on The World Bank (2024) and FRED (2024).

Figure 12. ADF for unemployment.

```
Dickey-Fuller test for Unemployment
sample size 32
unit-root null hypothesis: a = 1

  with constant and trend
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + e
  estimated value of (a - 1): -0.540321
  test statistic: tau_ct(1) = -3.28166
  p-value 0.08748
  1st-order autocorrelation coeff. for e: 0.031
```

Source: author's own contribution in Gretl based on The World Bank (2024) and FRED (2024).

Figure 13. ADF for inflation.

```
Dickey-Fuller test for Inflation
sample size 32
unit-root null hypothesis: a = 1

  with constant and trend
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + e
  estimated value of (a - 1): -0.705557
  test statistic: tau_ct(1) = -4.99565
  p-value 0.001682
  1st-order autocorrelation coeff. for e: -0.028
```

Source: author's own contribution in Gretl based on The World Bank (2024) and FRED (2024).

Figure 14. ADF for exchange rate.

```
Dickey-Fuller test for ExchangeRate
sample size 32
unit-root null hypothesis: a = 1

  with constant and trend
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + e
  estimated value of (a - 1): -0.0629244
  test statistic: tau_ct(1) = -1.64037
  p-value 0.7539
  1st-order autocorrelation coeff. for e: 0.716
```

Source: author's own contribution in Gretl based on The World Bank (2024) and FRED (2024).

Figure 15. ADF for gold.

```
Dickey-Fuller test for Gold
sample size 32
unit-root null hypothesis: a = 1

  with constant and trend
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + e
  estimated value of (a - 1): -0.16064
  test statistic: tau_ct(1) = -1.74575
  p-value 0.7069
  1st-order autocorrelation coeff. for e: 0.451
```

Source: author's own contribution in Gretl based on The World Bank (2024) and FRED (2024).