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Topic: Optimization of Transportation Routes between a Chosen Company and Its Clients

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DECLARATION OF ORIGINALITY

I declare that I compiled this diploma thesis on the topic "Optimization of Transportation Routes between a Chosen Company and Its Clients" on my own and that all the results presented come solely from my original work and expert consultations with RNDr. Petr Kučera. All literature and other sources I used to compile the Thesis are listed in the Bibliography and properly cited in the text.

In Prague on 14th April 2009

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(Author's signature)

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SUMMARY

This diploma's thesis on the topic "Optimization of Transportation Routes between a Chosen Company and Its Clients" is mainly based on the creation of new transportation routes for the distribution of ice-cream products. The conclusion identifies optimal paths for products delivery and at the same time attempts to accord with the requests of the company, suppliers and final consumers. The required result is reached by applying different methods of solving the "traveling salesman problem" in combination with logical categorization of information in a priority sequence.

Key terms: optimization of the transportation route, one-level transportation task, two-level transportation task, cyclic transportation task (traveling salesman problem), supplier, customer, initial vertex, edge.

Diplomová práce na téma optimalizace dopravních tras mezi firmou a jejími dodavateli a zákazníky je zaměřená především na vytvoření nových dodavatelských tras pro rozvoz zmrzlinových výrobků. Výsledkem je nalezení optimální trasy pro rozvoz zboží a zároveň přizpůsobení trasy všem požadavkům ze strany firmy, dodavatele i koncového zákazníka. Požadovaného výsledku je dosaženo prostřednictvím různých metod řešení problému obchodního cestujícího ve spojení s logickým tříděním získaných informací sestavených podle priorit. [12]

Klíčová slova: optimalizace dopravní sítě, jednostupňová dopravní úloha, dvoustupňová dopravní úloha, cyklická dopravní úloha (problém obchodního cestujícího), dodavatel, odběratel, výchozí uzel, hrana.

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1 Introduction

The topic for discussion is the optimization of transportation routes between a chosen company and its clients (suppliers and customers). The concept of optimization means searching for a preferable solution or searching for a new optimal solution which fits the given conditions. A transportation route is a path (in this case a cyclic path) connecting two or more places on which the vehicles move and perform the distribution of goods, the haulage of goods or the distribution and haulage of goods at the same time.

The optimization of transportation routes between companies and their suppliers and customers is the subject of much present discussion. The majority of big companies have their own department of logistics which deal with the problem of saving costs in the transportation and transferal of goods. The main objective of this department is not only to coordinate the distribution of goods but also to communicate with other departments (for example the customer service department) and deal with other problems connected with the transportation of goods (for example problems with distributors). Some smaller companies do not find it cost-efficient to have a department of logistics. These companies have to hire external logistic specialists or they will be unable to transport their goods optimally. In the worst cases these companies have neither logistics nor an optimal transportation system and their overall profits are reduced by the high costs of goods delivery. The choice of this topic is connected with the urgent need of a distributor of ice-cream to have new optimal transportation routes and this thesis endeavors to suggest a different system of ice-cream distribution which might be useful for this company in saving their costs and time and in improving the customer (supplier) service.

2 Objectives of Thesis and Methodology

The main objectives of this diploma thesis are to find a new optimal solution in transportation of goods, to follow and analyze the given parameters (requirements of distributors, company, customers and suppliers), to find a compromise solution fulfilling the priority requirements, and to find a solution which reduces costs and may prevent unanticipated problems which might occur on the route.

The hypothesis of the thesis is as follows:

There exists an optimal solution (set of cyclic transportation routes) for the distribution of ice-cream products in the given region which fits the given conditions, requirements of the company, of the distributor and of the customers.

The first step is to gather the basic information needed for the calculations connected to the selected topic. PK Frost, one of the distributors of Algida goods, was interested in the optimization of transportation routes. The intracompany file, used as a main source of information in the calculation, includes a description of the current system, a specific number of customers and their addresses, and the frequency of distribution (part of this Microsoft Office Excel document is shown as Table 1 in the supplements). It is also important to explain the conclusions which have been drawn from the documents and are important for the correct calculation. The other main sources of information are excerpts from interviews with ing. Luboš Krahulík of PK Frost and the current trade manager of Algida, ing. Jitka Šichová.

The second step is to gather information for the theoretical part of this thesis. The literature research is taken from seven expert publications, which deal with one-level transportation tasks, two-level transportation tasks and cyclic transportation tasks (the “traveling salesman problem”). These publications are helpful for the description of transportation tasks, types of methods and different approaches to the results. Other sources of information which describe new approaches to modifications of described methods will also be used and cited.

Official web pages are added as an alternative source of information about Algida and PK Frost. The gathering of information for this diploma thesis was carried out over a period of eight months.

The practical part of the thesis is the result of twelve methodical procedures. Many methods of calculation exist but the optimal one cannot be predicted. The description of the calculation consists of these methods:

- The Nearest Neighbour Method
- The Savings Method
- The Insertion Method
- The Convex Hull Method
- The Minimum Spanning Tree Method
- The Christofid Method
- The Nearest Merger Method
- The Exchange Method
- The Modified Savings Method
- The Habr Frequencies Approach
- The Mayer Method
- The FVL Method (Fernandez de la Vega – Luecker Method)

All methods presuppose knowledge of the distances between the cities which might be included in the route and the capacities of vehicles. At the request of the distribution company the calculation will be made without the second assumption concerning the capacities of vehicles because the capacity of vehicles owned by PK Frost is considerably higher than the amount of goods, which could be delivered in one day. In the calculation some of the methods are chosen to indicate new transportation routes for the delivery of Algida goods.

In the practical part there appears also a calculation based on special software which was created by ing. Šrámek for the informative and educational purposes. TSPKOSA software is used for the calculation by applying the Vogel method and the nearest neighbour method.

3 Literature Research

3.1 Distribution Tasks

Distribution tasks are ranked among the set of linear programming tasks. This set includes:

- One-level transportation tasks
- Two-level transportation tasks
- Assignment problems
- Generalized transportation tasks

[4]

The thesis consist of mathematical classification of distribution tasks, calculation by the application of different methods, classification of individual methods, explanation of possibilities how to solve a transportation task and finally a calculation of the practical element by the choice of an adequate method.

3.2 Mathematical Classification of Transportation Tasks

3.2.1 One-level Transportation Task

One-level transportation task deals with the creation of an optimal route for a transportation of the homogenous material from suppliers to customers allowing for the requirements of customers and suppliers with regard to the minimalization of transportation costs. [1]

In one-level transportation task the existence of a number (m) of suppliers signified for example from D_1 to D_m and each of them having a specific capacity of a vehicle from d_1 to d_m, is taken into consideration. There also exists an amount (n) of customers signified as S_1 to S_n who have requirements (specific amounts) with regard to the delivery of goods from s_1 to s_n. The costs on one unit of transported goods are marked as a transfer rate c_{ij} and an amount of a transferable commodity is marked with the symbol x_{ij}. [2] All data (amounts of goods) are marked on a table in which most of the calculations are made.

This is a general construction of a distribution (transportation) table:

		Customer					
		S_1	S_2	S_3	…	S_n	D_i
Supplier	D_1	c_{11} x_{11}	c_{12} x_{12}	c_{13} x_{13}	…	c_{1n} x_{1n}	D_1
	D_2	c_{21} x_{21}	c_{22} x_{22}	c_{23} x_{23}	…	c_{2n} x_{2n}	D_2
	D_3	c_{31} x_{31}	c_{32} x_{32}	c_{33} x_{33}	…	c_{3n} x_{3n}	D_3
	…	…	…	…	…	…	…
	D_m	c_{m1} x_{m1}	c_{m2} x_{m2}	c_{m3} x_{m3}	…	c_{mn} x_{mn}	D_m
s_j		s_1	s_2	s_3	…	s_n	

Table 1 Transportation table for the calculation of a one-level transportation task [2]

The presumption of the transportation task is its balance. The transportation task is balanced when the volumes of the capacities of the suppliers and the required volumes of the customers are equal. It is a condition for transportation task being solved. If a transportation task does not fit this condition at the beginning of the calculation it will have to be corrected by positioning "a slack supplier" or a "slack customer" to which zero rates are always assigned. [2], [1]

There are three steps in the solution of a transportation task. The first step is to find out a starting basic feasible solution by occupation of the exact $m + n - 1$ number of fields (the number of fields is identical to the number of basic variables) while keeping in mind a rule that other elements x_{ij} are equal to zero. Calculation of one-level transportation task can be made through one of the following most commonly used methods:

- The North-west Corner Method
- The Index Method
- The Vogel Approximation Method
- The Habr Frequencies Approach

The second step is aimed at checking the result of calculation with an optimum test by using a modified distribution method (MODI). This method demonstrates that the result of calculation is the optimal one. If an optimum test shows that the given task can have a new, better or other optimal solution then calculation continues with the third step. This step is a graphical demonstration called "Dantzig closed circuits". These circuits change the original solution and a better basic solution arises in a transportation table which has to be tested again by the test of optimum used in the second step. [2]

3.2.2 Assignment Problem

One of the distribution tasks is the assignment problem which involves the assignment of a certain group of elements to another group of elements while keeping in mind that this assignment should minimize total costs, total length of distribution route and generally reach an optimal solution. One of the characteristics of the assignment problem is the rule of the square matrix which states that the number of suppliers should equal the number of consumers and the capacities of suppliers and the requirements of customers should be equal to one. The symbology is the same as it was in the one-level transportation task. [1]

The assignment problem can be solved in six steps by the Hungarian method. The Hungarian method is an optimization algorithm which was first published in the year 1955. The procedure of the calculation is shown step by step below:

I. The first step consists of the creation of the reduced matrix of rates. This matrix contains zero in each row and column. This can be done by row or column reduction where in each row or column the lowest rate is subtracted from all values in the row (column).

II. In the second step the maximal count of the independent zeros has to be chosen. The independent zeros are those zeros which are alone in a row and column. These zeros have to be encircled in the matrix. Then the procedure continues by choosing other independent zeros which has the lowest number of zeros in a row (column). Every chosen zero has to be encircled and other zeros in the row (column) with the encircled zero have to be deleted. This process continues until the possibility of choosing the independent zeros exists.

III. When the number of independent zeros equals the number of rows (columns) then the solution is optimal and the result is in the ancient matrix on the places of the independent zeros (from the final matrix). The rates from the ancient matrix on the places of the independent zeros have to be added up and this sum will be the value of the target function.

IV. The verification of the amount of the independent zeros has to be made in the fourth step. If there are fewer zeros than ***n*** the verification is made by using the covering lines. The justice of the independent zeros´ choice is declared by König´s theorem, which states that the amount of the covering lines which can cover all zeros in the matrix equals the maximum amount of the independent zeros.

V. The secondary reduction of the matrix is made after the verification of the correctness. Firstly the element with the minimal value has to be chosen from the uncrossed elements; the elements which are crossed just once will remain unchanged and the elements which are crossed twice will be increased precisely by the value of the element with the minimal value. All other elements which are uncrossed will be decreased precisely by the value of the element with the minimal value.

VI. The last step is to repeat the same procedure as in steps II., III. and IV. until the optimal solution is reached. [5]

3.2.3 Two-level Transportation Task

There are two possible ways of solving a two-level transportation task. A balanced task can be solved by resolution into two one-level transportation tasks. [2]

In other case the implementation of a specific procedure which is based on similar principal as a one-level transportation task has to be made. This task requires the two-dimensional table for the calculation (illustrated below as a Table 2).

Task solving starts with the construction of the transportation table and continues with the choice of an appropriate approximation method. The calculation begins in the lower part of the table and moves upwards in accordance with the rules of one-level transportation task.

The second step requires an optimum test as usual. The criteria of optimality are gained from „duality principles". [3] If the test proves that the solution is not optimal then in the next step a new solution will have to be found. In this case it is possible to use "Dantzig closed circuits"again to make changes in the table, in a similar way, as in a one-level transportation task. [2]

The following table is the two-dimensional table table for the calculation of two-level transportation task:

	v_1	v_2	$\dots v_j \dots$	v_n			
	M_1	M_2	…….	M_n	F_i	a_i	
D_1	c_{11}	c_{12}		c_{1a}	0 y_1	a_1	u_1
D_2	c_{21}	c_{22}		c_{2a}	0 y_2	a_2	u_2
.			$\dots c_{ij} \dots$		. y_i	. .	. .
D_m	c_{m1}	c_{m2}		c_{mn}	y_m	a_m	u_m
F_j	q_1^0	q_2^0	$\dots q_j \dots$	q_n^0			
S_1	c_{11}	c_{12}		c_{1a}		b_1	w_1
S_2	c_{21}	c_{22}		c_{2a}		b_2	w_2
			$\dots c_{jk} \dots$			…	…
S_p	c_{p1}	c_{p2}		c_{pn}		b_p	w_p
d_j	d_1	d_2		d_n			
	v_1'	v_2'	…….	v_n'			

Table 2 Transportation table for the calculation of a two-level transportation task [2]

3.3 Optimization of Cyclic Routes

3.3.1 Cyclic Transportation Task (Traveling Salesman Problem)

Cyclic transportation tasks concur on allocation tasks in which the stocks are placed in the calculation and on regionalization tasks which optimize a creation of supply area assigned to a specific center or store. The allocation tasks, regionalization tasks and cyclic tasks are closely connected but cannot be solved in a complex way but only separately.

Traveling salesman problem is a task solved by finding an optimal route for vehicles which realize a cyclic transportation route where distribution network vertices are operated in some manner during the trip which in most cases starts and finishes in the starting vertex of the network. The result of optimization is a final minimal length of route with regard to the requirements of customers and the technological limitations of vehicles used for the transport.

Suppliers can use the cyclic routes for different kinds of activities: distribution of goods, haulage of goods or a combination of these two activities at the same time. The distribution of goods is important for supplying stores or restaurants and haulage is used, for example, for scrap collecting. The search for an optimal cyclic route arises from a given communication network such as, for example, roads with specified distances in meters or kilometers and from optimization of costs: petrol consumption, fee for rent of the car, vehicles reparations and the costs of the vehicle standing time. Time for distribution is also in some cases a limiting factor. When speaking, for example, about the distribution of leaflets with a special offer of goods valid on specific days, those leaflets have to be delivered before the first day when the goods can be bought. The distribution would be senseless if the delivery of the leaflets was delayed due to the insufficient working hours of the driver, or some technical fault, after the special offer was over.

There are many factors which can prevent goods being delivered on time:

- The amount of transported material
- The capacity of a vehicle
- The weight and volume of goods
- Time
- Other problems (technical problems – reparation of a vehicle, sudden illness or injury of a driver, traffic jam etc.)

The most important limiting factor for solving a "traveling salesman problem" is the amount of transported material. The requirements of customers and the capacity of the vehicles on a specific route have to be taken into account. The amount of goods can be expressed in weight or in volume units. Units of volume are used when weight is not an important factor in a fully loaded vehicle's haulage capacity.

The capacity of a vehicle has to be counted in the same units as the capacity of material or goods ordered by customers. There exist special cases where weight and volume of material is not a limiting factor (for example the distribution of postcards) or cases where the size of load has to be precisely determined (for example the transportation of the furniture). If more than one type of vehicle is available then a list of all different types of vehicles, their capacities, consumption and any other requirements important for their service have to be made.

Time is another important limiting factor in the construction of routes. If any kind of time limitation exists then the time aspect of specific routes has to be monitored very carefully. It is important to know how long the loading takes, the time needed for unloading and loading of goods in the customers' stores and the time needed for other service breaks (for example, the driver's lunch break). The time limitation can be expressed as time spent on the route (the driver's working hours), as time intervals at customers' sites (the opening hours of markets, restaurants or stores) and the opening hours of suppliers' stores. These intervals are generally known as "windows". It can happen that all vertices have the same time interval (for example all of the stores are open from 8 a.m. to 4 p.m.) or there can be also more time intervals (for example one store is open from 8 a.m. to 11 a.m. and another store is open from 11 a.m. to 8 p.m.). The existence of windows produce a situation in which some vertices have to be excluded from the route because the driver could not get there during the opening hours of this store or it can result in that the vehicle having to wait till the store is open. These windows have another big disadvantage, which is that time intervals can be unstable in connection to the different days in a week, season etc.. The construction of a route has to be flexible enough to be able to adapt to the instability of time intervals. There is also the possibility of constructing two or more different routes for two or more different seasons, different days in a week or for the different requirements of customers.

A general disadvantage in the construction of weekly transportation routes for distribution is that there always exists a possibility that goods will not be delivered for some reason or that the distribution will be severely delayed. This problem has to be solved very quickly otherwise the company which sells these products can lose many customers because of that (this is especially true of ice-cream products or products which are sold at a special price for only a limited period). The distribution network should be flexible enough to be able to react on unforeseeable problems which might arise on the route by adding some of the unattended vertices to the route for the next day.

3.3.2 Mathematical Classification of Optimization Tasks

Optimization tasks are classified within the following parameters:

- The number of transportation vehicles and types of vehicles
- The placement of vehicles (whether they leave each time from the same position - starting point - or leave each time from a different position)
- The capacity of vehicles (if it is limited or not and how much, the precise amount in weight or volume units)
- The nature of the demand for goods (determined or stochastic)
- The placement of the demand (on vertices, on edges or a combination of those two possibilities)

- Type of a network (directed, undirected, mixed)
- Type of service (the distribution of goods, haulage of goods or distribution and haulage of goods at the same time)
- Time windows (existence of time limitation in vertices – is it caused by one interval or by two or more intervals?) [5]

<u>Optimization tasks can be divided by the type of cycle, which operates within different parameters:</u>

A. **According to the position** - a cycle always starts in the same given vertex
   - a cycle has to go through a given vertex
   - a cycle goes through or starts each time in a different vertex
B. **According to the number of times** a cycle has to go through each edge, each vertex
C. **According to the type of parameters** which are known before the construction of a cycle (for example limited capacity of vehicles or time predisposition) [5]

Types of cycles are used for a mathematical classification of individual sorts of optimization tasks. Each type of task can be computed by using different types of methods but which of these methods is optimal cannot be uniquely determined because each method has advantages and also disadvantages. Types of optimization problems are differentiated as follows:

- Traveling salesman problem
- Multiple Traveling Salesman Problem
- Vehicle Routing Problem with One Standing Place
- Vehicle Routing Problem with Several Standing Places
- Postman Problem
- Capacity Postman Problem

[5]

3.3.3 Traveling Salesman Problem

Traveling salesman problem result from a graph $G = \{V, E, C\}$ where V stands for a set of vertices in this graph, E is a set of edges and a matrix of estimation is $C = \{c_{ij}\}$ where i, j belongs to set E. A further consideration is that vertex 1 represents a location of vehicles (a deposit), vertices in a set V form a location of commercial premises of customers (consumers) and edges in a set E represent communications evaluated by the distance of its incident vertices. The number of vertices is symbolized by n. The specification of bivalent variables of a model $x_{ij} = 1$ means that vertex i and vertex j lay consequently on a route. If x_{ij} equals zero then there will not be a connection between vertex i and vertex j, vertex j will not succeed to vertex i. The total sum of the length of edges between vertices indicates a value of an objective function.

The traveling salesman problem can be solved either by sequence or by parallel procedure. The sequence procedure generates a route by searching a vertex j which succeeds to vertex i. The parallel process connects different vertices into joins which finally together create a cyclic route. The sequence procedure guarantees very fast and simple calculation but the solution is not usually precise. The parallel process is computationally more demanding but the result is usually exact. [5] The possible methods of solving the "traveling salesman problem" are described in the following chapters.

3.3.4 Methods of Solving the Traveling Salesman Problem

3.3.4.1 The Nearest Neighbour Method

In this method the first step consists of a choice of one vertex as the starting point of a route. Another vertex which is the nearest one is chosen by using a matrix of distances and then joined to the first vertex. This procedure is repeated until all vertices appear once on a route. The complexity of the calculation is n^2 operations which have to be made where n stands for a number of nodes. If the initial vertex (for example a store belonging to the company) is not given the calculation can be repeated with a different starting point (different initial vertex). [5]

3.3.4.2 The Savings Method

This method requires to start with a calculation of a matrix of saving numbers $s_{ij} = d_{i1} + d_{1j} - d_{ij}$,

where $i, j = 2, 3, \ldots n$ of vertices, and $D = \{d_{ij}\}$ is a matrix of the shortest distances between vertices. Values s_{ij} gained from the matrix have to be put in a descending order and on the basis of this order joined into the final cyclic route beginning with the highest value. This method requires a number of calculations in the rank $n^2 log(n)$. [5]

3.3.4.3 The Insertion Method

The insertion method has several options for the calculation depending on the requirements and preferences of the creation of the transportation network. The different types of calculation are:

- The nearest insertion method
- The cheapest insertion method
- The random insertion method
- The fast insertion method

It is again not possible to predetermine which of these methods of the calculation will lead to the optimal solution. The approaches to the calculation are different in the way the vertices of the route are organized. Two methods are presented here for the purposes of comparison:

- <u>The nearest insertion method</u> requires the assignation of the initial vertex which is inscribed with *1*. Then vertex s which is the shortest distance from the initial vertex has to be joined into the cycle C which will now have thiis appearance $C \rightarrow 1 - s - 1$. Another vertex k for joining the route will be the vertex which is the shortest distance from the vertices which are already a part of the cyclic route and also the edge h which lies on C and has a minimal value of $c_{ik} + c_{kj} - c_{ij}$ has to be found. Then the vertex k can be added between the vertices i and j in the cycle C. The process of joining the vertices is repeated till all the vertices join the route and create the resultant graph

- The cheapest insertion method differs from the nearest insertion method by the fact that the vertex k is not a vertex which is the shortest distance from the edge h and the cycle C but it is the vertex which least prolongs the route. The amount of mathematical operations is in the rank $n^2 \log (n)$ which is the same rank as in the savings method. [5], [6]

3.3.4.4 The Convex Hull Method

The title of this method means that in the first step it is crucial to create the convex hull of the vertices in the graph which is the imaginary outline formed by vertices in the cycle *C,* in which the vertices are ordered in the same sequence as they appear in the convex hull. The calculation continues with a search for an edge h for which the result of the equation $c_{ik} + c_{kj} - c_{ij}$ has the minimal value which means that the length of the route from the cycle C to the vertex k is the lowest one. Then the vertex k is inserted between the vertices i and j in the cycle C. The procedure continues until all vertices are joined in the graph of the cycle C. [5]

This method cannot be used to solve the traveling salesman problem in all cases. It is recommended that this method be used for the calculation when the c_{ij} rates correspond with the euclidean distances of the vertices i and j. [6]

3.3.4.5 The Minimum Spanning Tree Method

The first step of this method is to search for a minimum spanning tree of a graph which has a symbology of the letter *T*. The Eulerian cycle originates from the doubling of the spanning tree edges and has to be transformed into the Hamiltonian cycle. Firstly the consequence of the vertices which lay on the Eulerian cycle has to be created and then the part of the consequence which begins and ends in the same vertex will be excluded. Other parts of the consequence form the solution. The calculation using this method has a complexity of n^2 mathematical operations. [5]

3.3.4.6 The Christofides Method

The Christofides method partly arises from the minimum spanning tree method because it also uses the spanning tree of the graph. The first step is to search for a minimum spanning tree of a graph *G* which is symbolized by the letter *T*. Then the vertices of the odd degree have to be marked and connected with the edges by using the perfect matching method with the minimal costs. The vertices will form the Eulerian cycle which will be again transformed into the Hamiltonian cycle preserving only the first occurences of the vertices. The Christofides method is very difficult for the calculation; it requires n^3 of the mathematical operations. The complexity has a positive impact on the estimation of the solution´s correctness which is lower than, or equal to 1,5. [5]

3.3.4.7 The Nearest Merger Method

The nearest merger method is based on connecting the individual cycles. Firstly, the elementary cycle is created from all the vertices. Then there an edge *(a, b)* has to be found, where $c_{a,b} = min\{c_{x,y}\}$; x and y stands for the vertices which are part of the different cycles. These two cycles are joined together and the procedure is repeated until all vertices form the Hamiltonian cycle. The complexity of the nearest merger method is in rank n^2 mathematical operations and the estimation of the accuracy of the result is lower than, or equal to two. [5]

3.3.4.8 The Exchange Method

The exchange method is the heuristic method based on reinforcement of the result by exchanging the edges. The new solution is reached by exchanging two edges from the cycle for two other edges (exchange 2-2). The exchange 2-2 starts with the choice of two edges from the Hamiltonian cycle which have the same orientation *(u, v)* and *(x, y)*. These edges have to be changed and the new solution will be *(u, y)* and *(v, x)*. [6] This process can also be applied to three or more edges.

3.3.4.9 The Modified Savings Method

The modified savings method is based on the savings method. The aim of this method is to create cycles with regard to the capacities of individual vertices (the amount of the transported goods) in one step. The problem appears when the amount of the transported goods exceeds the capacity of the vehicle. This method requires a calculation based on this equation:

$$s_{ij} = c_{i0} + c_{0j} - c_{ij} ,$$

where s_{ij} values have to be finally put in descending order. The connection between the nodes i and j is made with respect to the fact that the capacity of all cities which lie on the edge should not exceed the capacity of the vehicle. Each edge which fits this condition can be joined to the route. This procedure continues until all vertices form the final route or until the capacity of the vehicle would be exceeded by random connection of two vertices. Finally the initial vertex will be joined to the route and form the final cyclic route. [8]

3.3.4.10 The Habr Frequencies Approach

One of the biggest disadvantages of the savings method is that it compares a given edge with only one route where only one city is chosen as the substantial vertex for the further calculation. Habr created so-called frequencies which compare a given edge with all other edges, even the most distant ones. He applied them in approximation methods for different transportation problems and designed several methods of using frequencies for the traveling salesman problem.

The Habr frequency for the given edge is equal to the value of the equation:

$$F_{ij} = \sum\sum (c_{ij} + c_{kl} - c_{il} - c_{kj})$$

There also exists a modified frequency which makes the calculation simpler and is expressed by this equation:

$$F'_{ij} = c_{ij} - r_i - s_j,$$

where r_i a s_j stand for the arithmetic average of the i-th row costs and j-th column costs of the cycle C. F'_{ij} can be derived from F_{ij} by the linear transformation. The Habr frequencies attribute the same importance to all edges but in the case of the multiple traveling salesman problem the edges from and to the initial vertex are the most important because they are the most frequented. This is the reason why it is important to calculate with the probability of choosing a different edge to the edge which goes from or to the initial vertex. [8]

The new access to solving the traveling salesman problem can be reached by adopting the Habr frequencies method. The calculation starts again with the marking of the initial vertex with zero. In the second step the frequencies are calculated through the modified procedure which counts with the probability. The resultant values of the frequencies should be put in ascending order and with regard to this order the routes are created in the same way as in the savings method. The calculation continues until all the vertices form the final route or until the capacity of the vehicle would be exceeded by random connection of two vertices. Finally the initial vertex is joined to the route and form the final cyclic route. [8]

3.3.4.11 The Mayer Method

The Mayer method is also known as the tree approach but the procedure is different to the procedure of the utilization of trees for solving the traveling salesman problem. This method starts with the vertex which is the most distant from the central point (the initial vertex where stock is placed). Other vertices are added to the route with respect to the rule that the nearest vertex has to be joined first and this process is repeated until all vertices join the route or until the capacity of the vehicle is exceeded. This procedure is similar to the algorithm of the minimum spanning tree method. If the calculation cannot be finished because of the problem with the vehicle´s capacity then the procedure will be repeated from the first step where the most distant city will be chosen from the group of vertices which were not added into the route in the precedent calculation. The calculation continues until all vertices form the distribution network.

In cases where the calculation has to be interrupted as a result of the limitation of capacity the possibility of different procedure exists. Instead of searching for the most distant vertex from the group of vertices which are not already joined into the route it is possible to search for the vertex which has the lowest requirements for the capacity and can be joined to the route without exceeding the capacity. [3], [8]

3.3.4.12 The Fernandez de la Vega – Luecker Method

This method was originally created to solve the bin packing problem. In this problem there is a given amount of basic elements which should be placed in the minimal number of bins with respect to their capacities. The conversion of this problem to the multiple tours traveling salesman problem is as follows:

The basic elements represent the cities with their capacities and the bins represent a group of the individual routes. The basic elements should be divided into two categories according to their size. The bigger ones have to be placed into the bins in the optimal way, the smaller ones can be placed in two different ways. The first technique is based on the distance between cities (the most distant cities from the central point will be joined first to the route), the second technique is based on the capacities (the vertex which has the biggest requirements on the capacities will be joined first to the route).

[8]

4 Information about the Company and Description of the Problem

4.1 Basic Information about the Company

Unilever is one of the biggest producers of consumer goods (producing mainly food and cosmetics) in the world with an annual turnover higher than 41 billion Euros. This company has branches in 100 countries of the world in which more than 206 000 of people are employed. About 150 million consumers each day decide to buy a Unilever product.

Unilever is a major producer and supplier of consumer goods, not only in the world but also in the Czech Republic. Near Nelahozeves in Bohemia there is a production factory where all fats and margarines are produced. The popular brands Rama, Flora, Perla, Hera, Rama Créme Bonjour and Hellmann's are represented by this company. The Czech market is supplied by a part of an ice-cream portfolio from Algida in cooperation with local producers. Unilever also imports some products such as Lipton's tea and toiletries, for example Dove, Rexona, Axe, Sunsilk, Signal and also the cleaning materials Domestos and Cif. This company has already invested 6 billions of Czech crowns since they have entered the Czech market in the year 1991. [9] The department of ice-cream production, Algida, is the most important for the analysis of transportation routes in this thesis.

4.2 Information about the Distributor of Algida Goods

The distributor of Algida goods in northern Bohemia is the company PK Frost, Ltd. This company has a place of business in Dobrovíz, northwest of Prague, where the ice-cream is also stored. The distributor has its own vehicles which have a capacity of 800 – 1,000 liters and is specialized on region of distribution with more than 4,000 customers in northern Bohemia, northwest Bohemia, northeast Bohemia and Prague.

[11]

4.3 Requirements of the Company

Companies usually search for a distributor which can offer them the best conditions for transportation of their goods. General requirements consist of:

- Low transportation costs (the lowest rate per kilometer)
- Excellent services
- Sufficient number of vehicles
- Corresponding capacity of vehicles
- Extensive region of distribution

A chosen distribution company has to fit other specific conditions in connection with achieving a higher profit on the distribution of goods. Specific requirements for distributors of ice-cream products can be divided into 4 general areas:

1. **Marketing skills** – building of a good image of Algida company through customer relations management, meeting the customer's needs, propagation of the brand, being polite to customers etc.
2. **Sales techniques** – not only a person on the department of orders receiving but also a driver who distributes ice-cream products and has a personal contact with the customers should have sales techniques. Clients usually order the same types of products, changing only the amount of requested goods in relation to sales, weather and other factors. A driver's excellent sales techniques can also increase Algida's profit.

[10]

3. **Informative skills** – informative skills, together with sales and marketing skills, are very important because the distributor should be also able to inform a customer about new products and answer a customer's specific questions concerning description of the product and its qualities.
4. **Equipment of vehicles** - distribution of ice-cream products requires vehicles to have special freezing equipment, which is indispensable.

4.4 Requirements of the Distributor

The aim of a distributor is to be able to offer the best conditions to the company and became its sole contractor for a specific region. This aim can be reached by fulfilling different goals:

- To be able to offer a competitive price for a distribution which is dependent on price of petrol/oil/cars, costs connected with running a business, other indicators
- To be able to offer the best service which includes a sufficient number of vehicles (with a sufficient capacity) in a good actual running condition, flexibility of the distribution (in case of unforeseeable changes, for example in customers 'orders or quantity of customers), skilled drivers with sales techniques etc.

The main problem for PK Frost is the search for skilled drivers with sales techniques. It is difficult to find skilled drivers and there is not enough money to pay suitable salaries to drivers who also have sales techniques and can work as commercial agents as well as drivers. [12]

PK Frost proposed to hire some people in the position of sales agent. These would have the same route as drivers but would always go out one day before. They would pick the orders and the next day drivers would be able serve those customers who have made their orders one day before. This solution sounds perfect from the customer care point of view but on the other hand it appears to be too cost demanding. This solution would definitely increase profit but the question is, at what price?

4.5 Description of the Problem

While searching for a solution to this specific transportation task other exceptional parameters have to be defined. Those parameters consist of different conditions which are given and it is not possible to change them or unforeseen problems may arise:

- **Problem with ordering** – ice-cream is an impulse purchase consumer product. Demand for this good is volatile and is dependent on different factors (for example weather, season, market development, price – national, world, local, price set by other ice-cream producers etc.). Customers order different amounts of ice-cream each time in connection with these factors.
- **Capacity problem** – capacity problems are related to the problem of ordering. When speaking about this specific task to be solved the distributor has higher capacity of vehicles than amount of goods that is possible to deliver in one day (more than 40 customers per one day). The capacity of vehicles is so large because of the expectation of higher demand during the summer season.
- **Technical problems** – it is possible that some technical problems may arise on the route. These problems can have a different character:
  - Traffic jam
  - Car accident
  - Technical problems with vehicles
  - Sudden health problem of a driver who has to have longer breaks or may even be unable to continue with distribution
  - Other problems – a driver does not come to work because of disease, injury, or other reasons

In case that one of these problems appear it is important to deliver ice-cream to customers as soon as possible; that means that transportation routes should be interconnected. Goods can be delivered next day by another driver who transports on a route close to this one.

5 Optimization of Transportation Routes by Applying Different Methods

5.1 Supposed Appearance of Routes According to Requirements

On the basis of the requirements described in the preceding chapter the expected appearance of routes should correspond with a flabelliform scheme of routes (as can be seen below in Figure 1). Another presumption is that there are many fans with different lengths and capacities which overlap each other. Shorter routes can be used for many reasons:

- If driver finishes earlier then he will have a possibility to continue, using a shorter route
- If any technical problems occur on the route then the next day goods will be delivered to more costumers (on longer and on shorter route)
- Shorter routes are close to Prague where amounts of goods ordered is usually higher and there is also a higher chance of getting stuck in a traffic jam. Some drivers will travel only on those routes on which Prague is the biggest city

The expected appearance of overlapping fans can be seen on Figure 2.

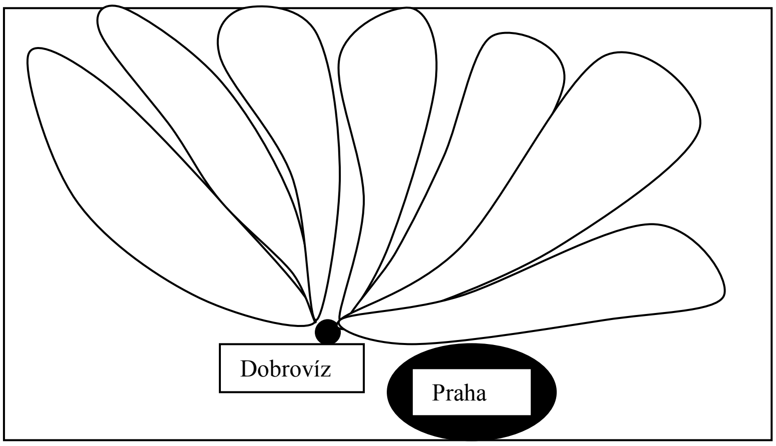

Figure 1: The supposed appearance of the transportation routes, Source: own sketch

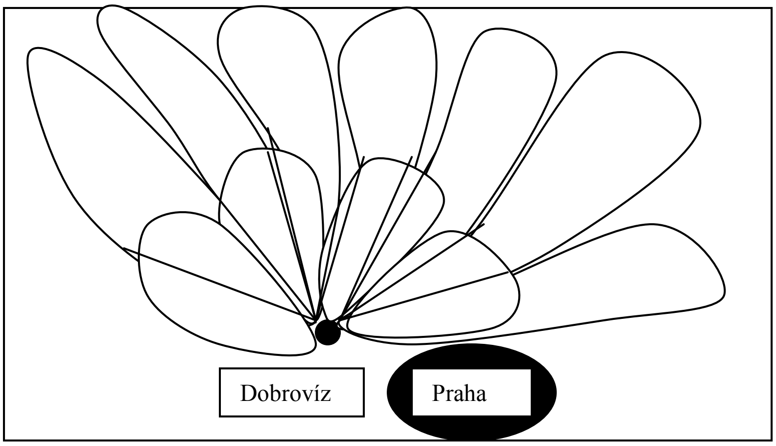

Figure 2: The modified appearance of the transportation routes, Source: own sketch

5.2 Division of Cities into Prepared Regions

PK Frost distributes ice-cream to more than three thousands customers. This number of cities could not be taken as a starting value for the determination of distances. For assorting this amount of cities and distances between them an individual department of logistics would be needed. This department would work on it for at least 6 months but there is a possible method of solving this problem. Cities can be divided into regions from a geographical point of view which would already respect the expected appearance of fans. These regions can not be described properly but it is possible to identify them as the north-western region, northern region, north-eastern region and south-eastern region in connection to their position on map with regard to the starting point, which is Dobrovíz.

5.3 Calculation by Applying Different Methods

5.3.1 Selected Region and Parameters of This Task

In connection to the previously mentioned technical and other problems, the precise parameters of this practical task have to be defined in this chapter.

Calculation by the application of different methods results from information about the distribution:

- The capacity of vehicles is dependent on the type of vehicle and varies from 800 to 1000 liters. The capacity is not a limiting factor because the maximal numbers of customers and their orders are much lower than the number of customers who can possibly be served in one day
- The outer dimensions of one pack of ice-cream are 30 centimeters in length, 10 centimeters in height and 20 centimeters in width. The dimensions of one pack have to be converted into liters because the capacity of vehicles is expressed in liters; so the volume of one pack is 6 liters
- The smallest vehicle can be loaded with approximately 133 packs and the largest vehicle with 166 packs of ice-cream
- The average amount of goods ordered by one customer is between two to three packs. An average order will be taken as the amount used for calculation because any real order will not be representative. An average order does not take into account fluctuation of temperature and seasonal changes which lead to different amounts of goods being ordered
- Higher costs for distribution caused by technical problems are precluded by specific appearance of routes which is the most important requirement for the calculation of this task

5.3.2 The Insertion Method

The first method of calculation is the insertion method. This method was chosen because it is an appropriate method when many cities and many customers are involved (as can be seen in Supplement 1). The insertion method makes it possible to find the city (customer) most distant from the initial store and then to place other cities (customers) in between those two cities (the initial vertex and the most distant vertex). This method makes it possible to determine the number of cities which could be added to the route with respect to the capacity of a vehicle, the maximal length of a route and the number of customers which could be possibly served in the course of one day.

The initial vertex is identical with the placement of the store and the place from which all drivers start their routes. The starting point is Dobrovíz which is marked as vertex 1. The most distant city on the first route is Chomutov which is marked as vertex n. This general equation is used when searching of cities (vertices j) which lie on the edge and the distances between them and two other cities (Dobrovíz and Chomutov):

$$d_{1j} + d_{jn} - d_{1n} \qquad [5]$$

The calculation of distances between cities placed on edge (1,n) is shown in the table bellow:

City	**Calculation**	**Number of customers**	**Frequency of distribution**
Peruc	38 + 48 - 80 = 6	4	1(7), 1(14), 2(28)
Most	73 + 25 - 80 = 18	102	20(7), 19(14), 63(28)
Černčice	46 + 41 - 80 = 7	3	2(7), 1(28)
Louny	46 + 35 - 80 = 1	22	10(7), 6(14), 6(28)
Lenešice	52 + 31 - 80 = 3	2	2 (28)
Jirkov	82 + 8 - 80 = 10	23	5(7), 5(14), 13(28)
Všehrdy u Chomutova	73 + 7 - 80 = 0	1	1(7)
Údlice	76 + 8 - 80 = 4	1	1(7)
Bečov	64 + 34 - 80 = 18	1	1(28)
Libčeves	62 + 40 - 80 = 22	4	1(7), 1(14), 2(28)
Havraň	72 + 18 - 80 = 20	3	3(28)

Braňany	79 + 35 - 80 = 34	1	1(14)
Nezabylice	71 + 8 - 80 = -1	1	1(28)
Blažim	62 + 23 - 80 = 5	1	1(28)
Spořice	80 + 4 - 80 = 4	1	1(14)
Panenský Týnec	35 + 45 - 80 = 0	2	2(7)
Přečáply	72 + 8 - 80 = 0	1	1(7)
Pátek u Loun	46 + 35 - 80 = 1	1	1(28)
Dobroměřice	51 + 34 - 80 = 5	2	1(7), 1(28)
Chlumčany	42 + 37 - 80 = -1	2	1(7), 1(28)
Černý Vůl	11 + 84 - 80 = 15	2	2(7)
Koštice nad Ohří	48 + 49 - 80 = 17	2	1(14), 1(28)
Libochovice	48 + 57 - 80 = 25	6	3(7), 3(14)
Výškov	57 + 27 - 80 = 4	1	1(28)
Budyně nad Ohří	42 + 64 - 80 = 26	3	1(7), 2(14)
Mšené Lázně	39 + 58 - 80 = 17	4	3(7), 1(14)
Obrnice	71 + 27 - 80 = 18	1	1(14)
Ředhošť	38 + 57 - 80 = 15	1	1(28)
Kozinec	16 + 82 - 80 = 18	2	1(14), 1(28)
Podsedice	71 + 48 - 80 = 39	1	1(28)
Želkovice	66 + 43 - 80 = 29	1	1(28)
Toužetín	38 + 41 - 80 = -1	1	1(28)
Pátek nad Ohří	43 + 48 - 80 = 11	1	1(28)
Vršovice	48 + 42 - 80 = 10	1	1(28)
Veltěže	47 + 41 - 80 = 8	1	1(28)
Třebívlice	67 + 46 - 80 = 33	1	1(28)
Černovice	82 + 5 - 80 = 7	1	1(28)
Zákolany	14 + 77 - 80 = 11	1	1(14)
Zvoleněves	21 + 67 - 80 = 8	1	1(7)
Nelahozeves	26 + 79 - 80 = 25	3	2(14), 1(28)
Středokluky	4 + 77 - 80 = 1	5	4(7), 1(14)
Kralupy nad Vltavou	23 + 79 - 80 = 22	24	11(7), 8(14), 5(28)
Libčice nad Vltavou	18 + 86 - 80 = 24	2	2(28)
Roztoky u Prahy	15 + 89 - 80 = 24	11	5(7), 3(14), 3(28)
Otvovice	17 + 80 - 80 = 17	2	2(7)
Veltrusy	27 + 81 - 80 = 28	6	2(7), 2(14), 2(28)
Tuchoměřice	6 + 79 - 80 = 5	3	2(7), 1(14)
Noutonice	10 + 78 - 80 = 8	1	1(14)

Holubice	17 + 81 - 80 = 18	1	1(7)
Dolany	21 + 83 - 80 = 24	1	1(14)
Kobylníky	31 + 52 - 80 = 3	1	1(14)
Okoř	9 + 77 - 80 = 6	2	1(14), 1(28)
Kněževes	4 + 79 - 80 = 3	2	2(28)
Hobšovice	28 + 68 - 80 = 16	1	1(28)
Statenice	9 + 83 - 80 = 12	1	1(14)
Blevice	18 + 74 - 80 = 12	1	1(14)
Hospozín	32 + 68 - 80 = 20	1	1(14)
Černuc	36 + 76 - 80 = 32	1	1(14)
Velvary	33 + 72 - 80 = 25	4	2(7), 2(14)
Koleč	16 + 72 - 80 = 8	2	1(14), 1(28)
Slatina	18 + 71 - 80 = 9	1	1(28)
Kamenný Most	24 + 70 - 80 = 14	1	1(28)
Knovíz	17 + 64 - 80 = 1	1	1(28)
Únětice	13 + 87 - 80 = 20	1	1(7)
Olovnice	21 + 76 - 80 = 17	1	1(14)
Podhořany	29 + 77 - 80 = 26	1	1(7)
Tursko	16 + 83 - 80 = 19	1	1(14)
Neuměřice	25 + 72 - 80 = 17	1	1(28)
Třebenice	67 + 51 - 80 = 38	1	1(14)
Sulec	38 + 41 - 80 = -1	1	1(7)
Stradonice	42 + 50 - 80 = 12	1	1(14)
Vtelno	70 + 28 - 80 = 18	1	1(28)
Velké Přílepy	11 + 83 - 80 = 14	1	1(14)
Svrkyně	12 + 80 - 80 = 12	1	1(28)
Dobrovíz (vertexl 1)		1	1(7)
Chomutov (vertex n)		79	14(7), 21(14), 44(28)
Total sum		**379**	**103(7), 99(14), 177(28)**

Table 3: The calculation by applying the Insertion method, Source: own calculation

The total sum of customers in this region is 379. There are 103 customers with a one week frequency of distribution who have their place of business in 30 cities. Frequencies of distribution are noted in the last column and are divided into three options:

- One week frequency 1(7) where the first number means the number of customers in a given city and the second number in brackets means 7 days
- Two weeks frequency 1(14)
- Four weeks frequency 1(28)

Only customers with one week frequency will be important for the optimization of routes in this task in the first north-western region.

The first vertex to join to the route is the vertex which has the lowest result of the equations in the second column of the table. Four nodes with the result of -1 appear in the table. These nodes represent Nezabylice, Chlumčany, Toužetín and Sulec town. Clumčany and Sulec should be added first on the edge because there is a one week frequency of distribution and both cities lie on the edge. There are 28 other cities, with 103 customers, which have to be added on the edge.

After joining vertices *j* to the route the calculation would continue in the same way by finding the distances of the other cities on four edges *$(1, j_1)$, (j_1, j_2), (j_2, n) a $(n, 1)$*.

It can be seen from the table of distances between cities that there are still too many cities in the region of calculation. An average driver can serve only 35 - 40 customers per day. There are two possible methods of adapting the calculation to this fact:

- There could be three or more optimal routes (instead of one) calculated by the same method from the table
- The hypothetical region could be narrowed down so that there will be fewer customers involved in the calculation; or the region could be divided into two overlapping regions as in Figure 2 above.

5.3.3 The Savings Method

This method covers the same region and the same cities which are dealt with in Table 3. This region is divided into four smaller regions because the number of cities in the ancient region exceeds the number of cities and customers which could be served by one driver in one day. In the first sub region the matrix of savings is derived from the general equation:

$$s_{ij} = d_{i1} + d_{1j} - d_{ij},$$

where i and j are vertices which should be joined. Figure 3 illustrates the division of the region into four sub regions:

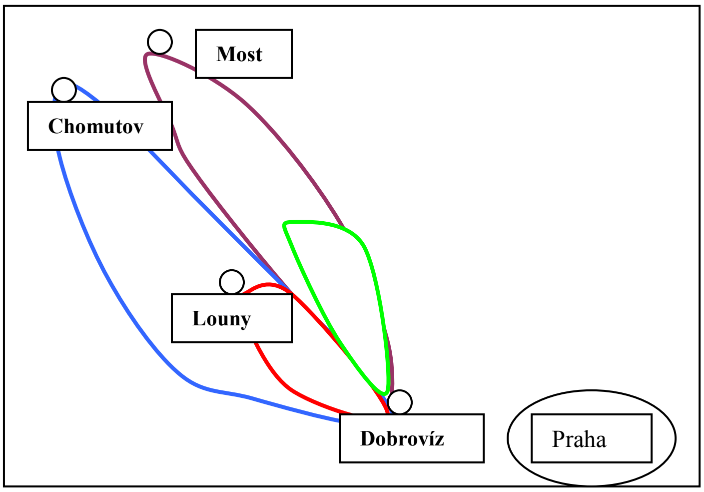

Figure 3: The supposed appearance of the transportation routes in a specific region of the distribution, Source: own sketch

The calculation starts with the route shown in red in Figure 3. The following cities appear on the route:

Peruc (2)	**Černý Vůl (7)**	Zákolany	Kobylníky	Koleč
Černčice (3)	Kozinec	**Zvoleněves (8)**	Okoř	Slatina
Louny (4)	Toužetín	**Středokluky (9)**	Kněževes	Kamenný Most
Panenský Týnec (5)	Vršovice	**Tuchoměřice (10)**	Statenice	Knovíz
Chlumčany (6)	Veltěže	Noutonice	Blevice	**Sulec (11)**
Velké Přílepy	Svrkyně			

Table 4: The list of vertices, Source: PK Frost, s.r.o. intracompany file

Red fields represent cities which can be included in the route because they fall into the one week distribution category. Other fields contain cities where the distribution is made once every 14 or 28 days. The numbers in brackets stand for significant vertex *i* or *j*.

The table below illustrates the savings matrix for the basic route:

0	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
0	**0**	74	71	64	68	5	32	6	5	68
0	74	**0**	88	70	83	5	32	6	5	75
0	71	88	**0**	70	83	5	32	6	5	75
0	64	70	70	**0**	69	5	32	6	5	70
0	68	83	83	69	**0**	5	32	6	5	76
0	5	5	5	5	5	**0**	10	5	12	6
0	32	32	32	32	32	10	**0**	6	5	33
0	6	6	6	6	6	5	6	**0**	5	6
0	5	5	5	5	5	12	5	5	**0**	6
0	68	75	75	70	76	6	33	6	6	**0**

Table 5a: The matrix of savings, Source: own calculation

The following step after the construction of the savings matrix consists of searching for the vertex which is nearest to the initial vertex. The first join is between Dobrovíz and Středokluky. Středokluky is listed in Table 4 as a vertex number 9 and this city is in the savings matrix on the 9th position in a row and in a column. The way to find another vertex to join the route is by marking the highest number in the 9th row and the 9th column (the 9th row and the 9th column are marked with a blue line and the highest number is highlighted in blue in Table 6). Due to the rounding of numbers when determining distances between cities there appear to be many vertices with the same value (the highest value of the 9th row and the 9th column is 6 which appears in seven vertices) in the savings matrix. This means that there are probably other alternative solutions which would be quite similar to the one which is calculated below. To find an optimal solution the values of distances between cities which are not rounded should be taken into account. The next vertex to join the route is vertex number 11 (Sulec).

Now the appearance of a route is:

1 – 9 – 11 – 1, which stands for cities:

Dobrovíz – Středokluky – Sulec – Dobrovíz

0	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
0	**0**	74	71	64	68	5	32	6	5	68
0	74	**0**	88	70	83	5	32	6	5	75
0	71	88	**0**	70	83	5	32	6	5	75
0	64	70	70	**0**	69	5	32	6	5	70
0	68	83	83	69	**0**	5	32	6	5	76
0	5	5	5	5	5	**0**	10	5	12	6
0	32	32	32	32	32	10	**0**	6	5	33
0	6	6	6	6	6	5	6	**0**	5	**6**
0	5	5	5	5	5	12	5	5	**0**	6
0	68	75	75	70	76	6	33	**6**	6	**0**

Table 5b: The matrix of savings, Source: own calculation

The procedure continues by searching for the highest number in the 11th row and the 11th column which is marked in Table 7 in green:

0	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
0	**0**	74	71	64	68	5	32	6	5	68
0	74	**0**	88	70	83	5	32	6	5	75
0	71	88	**0**	70	83	5	32	6	5	75
0	64	70	70	**0**	69	5	32	6	5	70
0	68	83	83	69	**0**	5	32	6	5	**76**
0	5	5	5	5	5	**0**	10	5	12	6
0	32	32	32	32	32	10	**0**	6	5	33
0	6	6	6	6	6	5	6	**0**	5	**6**
0	5	5	5	5	5	12	5	5	**0**	6
0	68	75	75	70	**76**	6	33	**6**	6	**0**

Table 5c: The matrix of savings, Source: own calculation

The process continues in the same way until all the vertices are joined in the cyclic transportation route. The final matrix of savings and the optimal route are illustrated bellow:

0	0	0	0	0	0	0	0	0	0	0
0	0	74	71	64	68	5	32	6	5	68
0	74	0	88	70	83	5	32	6	5	75
0	71	88	0	70	83	5	32	6	5	75
0	64	70	70	0	69	5	32	6	5	70
0	68	83	83	69	0	5	32	6	5	76
0	5	5	5	5	5	0	10	5	12	6
0	32	32	32	32	32	10	0	6	5	33
0	6	6	6	6	6	5	6	0	5	6
0	5	5	5	5	5	12	5	5	0	6
0	68	75	75	70	76	6	33	6	6	0

Table 5d: The matrix of savings, Source: own calculation

Dobrovíz – Středokluky – Sulec – Chlumčany – Louny – Černčice – Peruc – Panenský Týnec – Zvoleněves – Černý Vůl – Tuchoměřice – Dobrovíz (1 – 9 – 11 – 6 – 4 – 3 – 2 – 5 – 8 – 7 – 10 – 1)

The total length of this route is 129 kilometers; there are 11 cities and 26 customers to be served. The route was created effectively because there is enough space (in time for distribution and length of a route) to join other customers who are served only once every 14 or 28 days.

5.4 Calculation by Using the Computer Software

5.4.1 Description of the Computer Software

The computer software used for the calculation is TSPKOSA which is conceived as a freeware for private and educational purposes. The user is limited only by the technical instruments on which this software runs and possible limitations in connection to different versions of MS Excel. The application employs three methods of solving the traveling salesman problem:

- The Nearest Neighbour Method
- Vogel Approximation Method
- Hungarian Method

The software was created in the environment of Microsoft Visual Basic 6.3 and tested in Microsoft Excel 2002. The functionality is certified in all older versions of Microsoft Excel. The application requires entering the input data in the strictly defined form of matrix of values placed on the first list designated "List1". Data has to be put in to the first sheet beginning with the first cell. The size of the matrix (= number of rows, which means the number of cities) has to be entered in the first cell "A1". The TSPKOSA does not include any mechanism of results controlling but it is not important in this case because one result is already known from the previous chapter and can be compared with the result of the software application on the basis of the total length of a cyclic transportation route.

5.4.2 Procedure of the Calculation

The first step of the calculation after the installation of a software system as a complement into a Microsoft Office Excel sheet is the placement of a matrix of distances between cities into the first list of this document beginning with the cell A1 where the number 11 appears as can be seen in Figure 4. This number represents the size of the matrix.

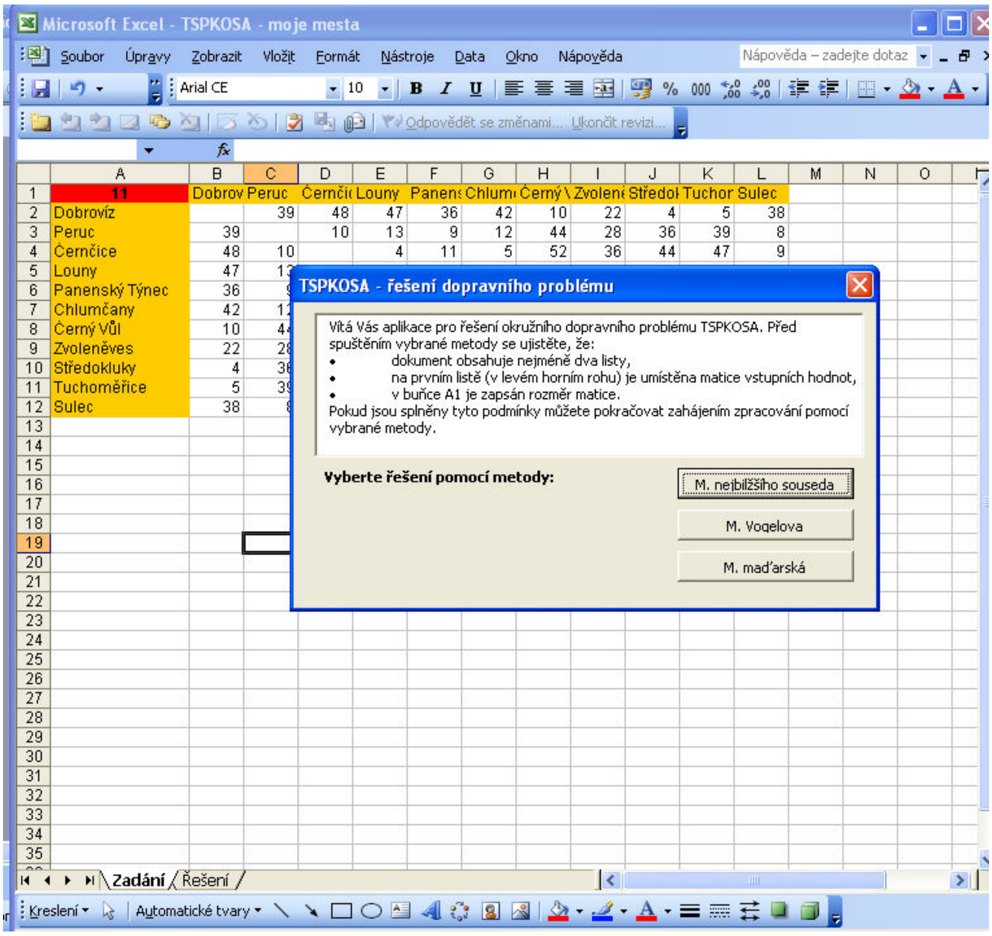

Figure 4: Input dialog, Source: TSPKOSA – input dialog

The second step is a choice of a TSPKOSA tool from a set of supplements which produces an input dialog which appears in Figure 4. The input dialog offers 3 methods of calculation and the result of the calculation can be found on the second list of the Microsoft Office Excel document.

- The result of the calculation by applying the nearest neighbour method

9	**0**	**5**	**Tuchoměřice**	**Dobrovíz**
0	**8**	**4**	**Dobrovíz**	**Středokluky**
8	**1**	**36**	**Středokluky**	**Peruc**
1	**10**	**8**	**Peruc**	**Sulec**
10	**4**	**3**	**Sulec**	**Panenský Týnec**
4	**2**	**11**	**Panenský Týnec**	**Černčice**
2	**3**	**4**	**Černčice**	**Louny**
3	**5**	**5**	**Louny**	**Chlumčany**
5	**7**	**31**	**Chlumčany**	**Zvoleněves**
7	**6**	**22**	**Zvoleněves**	**Černý Vůl**
6	**9**	**5**	**Černý Vůl**	**Tuchoměřice**
11		**134**		

Table 6: The calculation by applying the nearest neighbour method, Source: software calculation

The total length of this route is 134 kilometers; there are 11 cities and 26 customers to be served. The cyclic route is Dobrovíz – Středokluky – Peruc – Sulec – Panenský Týnec – Černčice – Louny – Chlumčany – Zvoleněves – Černý Vůl – Tuchoměřice – Dobrovíz.

- The result of the calculation by applying the Vogel approximation method

9	**0**	**5**	**Tuchoměřice**	**Dobrovíz**
0	**8**	**4**	**Dobrovíz**	**Středokluky**
8	**4**	**33**	**Středokluky**	**Panenský Týnec**
4	**10**	**3**	**Panenský Týnec**	**Sulec**
10	**5**	**4**	**Sulec**	**Chlumčany**
5	**3**	**5**	**Chlumčany**	**Louny**
3	**2**	**4**	**Louny**	**Černčice**
2	**1**	**10**	**Černčice**	**Peruc**
1	**7**	**28**	**Peruc**	**Zvoleněves**
7	**6**	**22**	**Zvoleněves**	**Černý Vůl**
6	**9**	**5**	**Černý Vůl**	**Tuchoměřice**
2		**123**		

Table 7: The calculation by applying the Vogel method, Source: software calculation

The total length of this route is 123 kilometers; there are 11 cities and 26 customers to be served. The cyclic route is Dobrovíz – Středokluky – Panenský Týnec – Sulec – Chlumčany – Louny – Černčice – Peruc – Zvoleněves – Černý Vůl – Tuchoměřice – Dobrovíz.

6 Analysis of Results

6.1 Results and Comparison of Different Methods of Calculation

All applied methods have advantages and disadvantages which are expressed below for each individual method:

The insertion method

+ is ideal for the determination of the region´s size which is adequate for the calculation

+ respects the requirements on the geographic appearance of the routes

- requires lengthy calculation

The savings method

+ the calculation in the savings matrix is lucid, mistakes can be avoided

+ the route is constructed from the initial vertex and it is easy to follow cities as vertices which are joined to the route and finally matched with the intitial vertex

- if there appear more cities on one path which have similar distances between each other and also to the initial vertex than more rates with the same value in the savings matrix may appear

- it does not respect any specific appearance of the routes

The nearest neighbour method

+ ensures that the vertices which are joined in the sequence have the shortest distance between each other

+ it can also be used when the specific geographic appearance of the route is required

- the final route may be longer than expected because it connects the nearest vertices but does not take account of their position

- the calculation and the appearance of the final route is not lucid

The Vogel method

+ usually produces an optimal or near-optimal solution

+ is the only method from those which were applied for the calculation which is parallel proceeding

- requires lengthy and difficult calculation

Method	The Savings Method	The Nearest Neighbour Method	The Vogel Method
Customers	26	26	26
Cities	11	11	11
Total distance	129	134	123

Table 8: The comparison of the results of the calculation, Source: own calculation

In cases where the substance of the optimality is the length of the route then the optimal method which should be applied is the Vogel method.

6.2 Comparison with an Old-fashioned System

The old-fashioned system of distribution contains the division of the customers among the drivers (each customer had the same driver for each distribution), frequency of distribution, and the day (in the week) when the goods should be delivered to the customer. The creation of a precise route decreased the costs of distribution, lowered the time cost, and created regulations for the distribution of Algida goods. The new system could facilitate the work of drivers, sales representatives and managers, who can follow the routes of the drivers and control their work and the functioning of the whole distribution network.

The old-fashioned system wasn't flexible enough to deal with unexpected problems which might appear during the distribution. The new system of distribution offers quick and low cost solutions to these problems. The problems are also prevented by the specific geographic appearance of the routes (the overlapping fans).

Another proposal for the improvement of the system of distribution is a change in the responsibility of the drivers. Each driver could have an individual region and could be responsible for the delivery of the ice-cream on time to all customers in this region. If there is any problem during the distribution, the driver can solve it by himself because he knows the customers personally, and his distribution area. It would be helpful if PK Frost invested money in a new employee who would be working in the store in Dobrovíz and would control the drivers and help them with their problems.

6.3 Recommendations

I would recommend two possible ways to develop the distribution network. Both strategies would have a positive impact on the quality of the distribution in the long run. Strategies are made with regard to the results of the optimization of the transportation routes and with regard to the amount of financial capital which is available for investment in the future development of the company.

1. STRATEGY A

This strategy presupposes investment in the new technologies to the tune of hundreds of thousands of Czech crowns. A substantial step would be the acquisition of the bespoken software application. On the basis of the results reached in the practical part of this thesis, this software should apply the Vogel approximation method for the optimization of transportation routes. The software does not require any experienced employees and is ready to use specifically for the distribution area which is supplied by PK Frost. Among the advantages of this strategy I would like to mention:

- Manageability of the software - does not require special skills of the employees
- Flexibility and adaptability of the software - in cases where the distribution company changes the contractors (or only adds or loses some customers), the software application will easily and quickly change the distribution routes on the basis of input data
- Independence – PK Frost would be an independent distributor and cooperate with any other company

One of the biggest disadvantages of this strategy is the price of the software application and other costs connected with the acquisition of the bespoken software.

2. STRATEGY B

This strategy presupposes investment the new technologies to the tune of tens of thousands of Czech crowns. It can provide a solution for at least the next five years. This strategy is not effective from the long term point of view because the problem of logistics is moving forward very fast and a decision about the software application which can optimize the transportation routes must be made without hesitation. The substance of this strategy is the acquisition of a very simple software application or simplification of the calculation by using the software which is already available (Microsoft Office tools for example). The calculation should be done by applying the

Insertion method for the determination of the region´s size and then by the Vogel approximation method. It has some advantages:

- The cheap investment - this type of software is not expensive
- The recovery of the investment – due to the low acquisition price, the investment is returnable in 2 years maximum
- The optimal method for the calculation is already known – it has been calculated and described in this diploma thesis

This strategy has substantial disadvantages:

- It is not effective to use this strategy over a long term period
- It requires experts or at least experienced employees who have experience with the calculation of the optimization of the transportation routes or understand the problem of logistics and are able to calculate the optimal solution by using only a simple software as the only utility which is available
- The software application is relatively cheap but other costs connected with the acquisition of the software and with the creation of the new transportation routes are very high
- It is a shortsighted solution which does not take the development of the company and rising number of contractors into account

It is not possible to explicitly recommend one of those strategies because an economic analysis of PK Frost is not available but I think that each company should try to apply strategy A because it can bring more benefits in the future.

7 Conclusions

This diploma thesis on the topic "Optimization of transportation routes between a chosen company and its clients" is composed of theoretical explanation of the problems of the transportation tasks and the practical example of solving the traveling salesman problem (the cyclic transportation task). The practical example deals with the distribution of Algida ice-cream by the distribution company PK Frost. The distributor does not have the optimal transportation system and the calculation showed few possibilities of optimizing the transportation network of PK Frost. These possibilities were formulated in strategies and the distribution company has an option to choose between those strategies.

The main objectives of this diploma thesis are to find a new optimal solution in transportation of goods, to follow and analyze the given parameters (requirements of distributors, company, customers and suppliers), to find a compromise solution fulfilling the priority requirements, and to find a solution which reduces costs and may prevent unanticipated problems which might occur on the route. This aim was fulfilled by finding a new access to the problem of the optimization of the transportation routes which divides the procedure into two steps. The first step applies the Insertion method in combination with the logical division of the cities according to the requirements of Algida (the ice-cream producer), PK Frost (the distributor of the ice-cream) and the customers. The new transportation routes have an appearance of overlapping fans. The second step of the calculation applies the Vogel approximation method which was found to be the least cost-demanding and usually produces an optimal or near-optimal solution.

The hypothesis of the diploma thesis was as follows:

There exists an optimal solution (set of cyclic transportation routes) for the distribution of ice-cream products in the given region which fits the given conditions, requirements of the company, of the distributor and of the customers.

The hypothesis is confirmed because the calculation, by applying different methods and comparison of these methods, resulted in the finding of the optimal solution. The calculation which reaches the optimal solution uses the Insertion method at the beginning (the determination of the region's size) and continues with the Vogel approximation method which generates the optimal solution.

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9 Supplements

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